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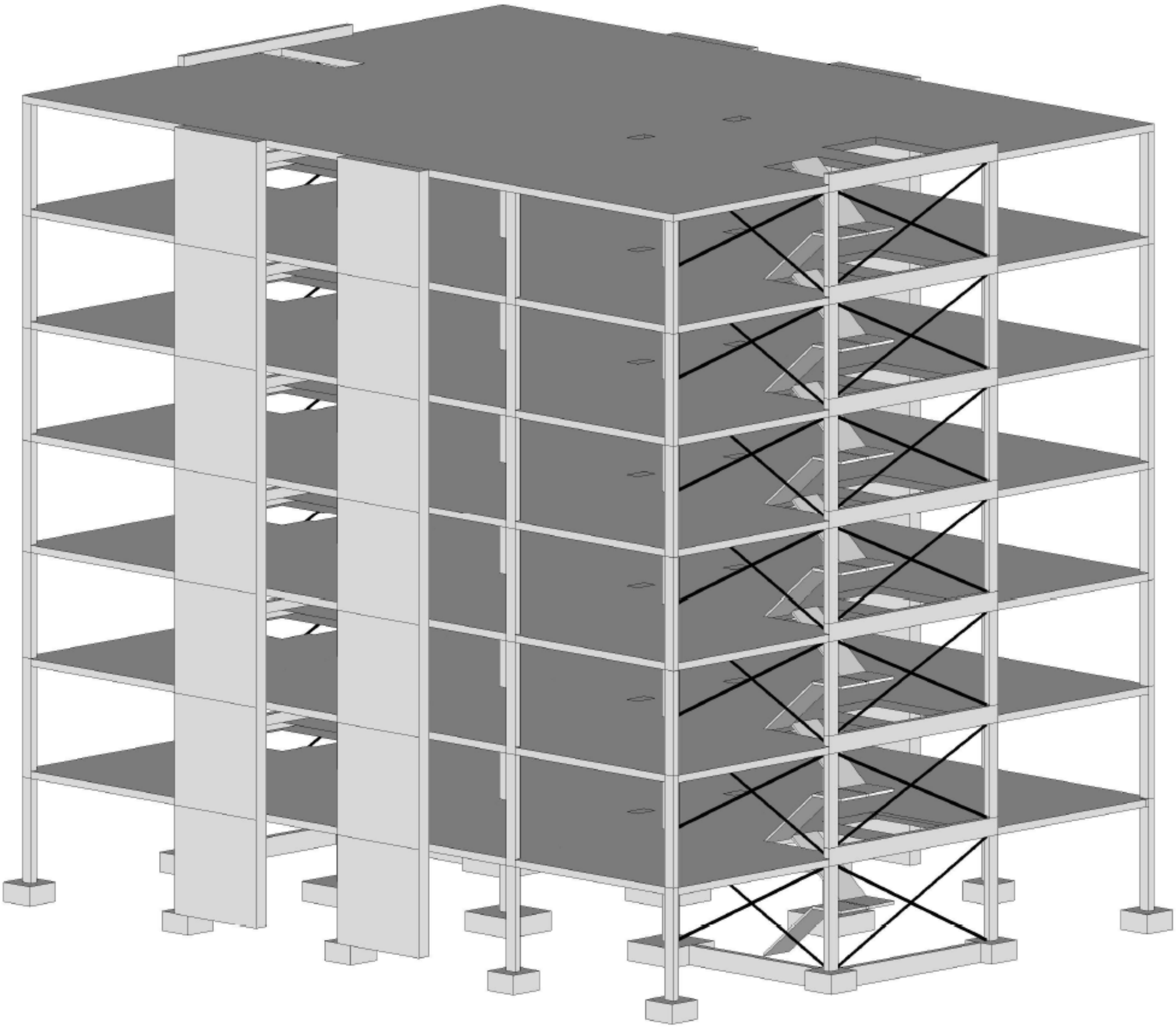
MASTER'S THESIS

Response of Primary and Secondary Systems under Dynamic Excitation



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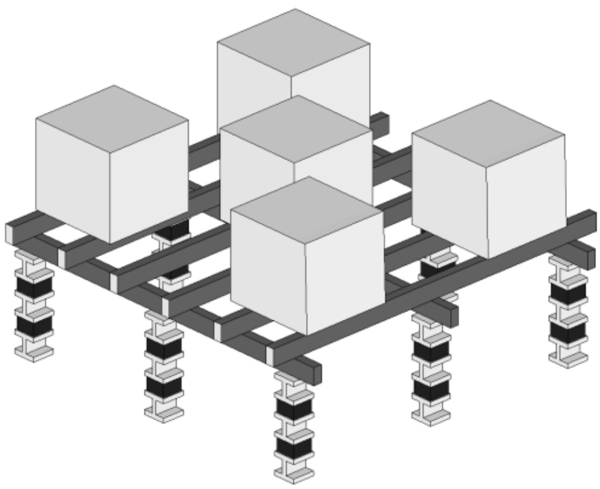

Writer:
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June 14, 2010

Preface

This thesis ends a two years long master's degree in Construction and Materials at the University of Stavanger. The thesis consists of dynamic analyses of the Cardington Building, which was built inside an old Zeppelin hangar near Bedford, UK. The building was built in 1998, and was a part of the European Concrete Building Project (ECBP). Its goal was to provide improved design codes, especially for the dynamic properties of the concrete structure.

In the thesis a finite element model of the Cardington building is made using the FE program Ruaumoko. The FE model is calibrated to experimental test results obtained by Jónas Thór Snæbjörnsson, Ódinn Thórarinsson and Símon Ólafsson, in 2000 [2].

The purpose of this thesis was to test and verify the application of a traditional floor response spectra methodology used to estimate seismic design actions for secondary structures and investigate the effect of structural parameters such as damping and natural frequency on the evaluated response.

The thesis consists of

- an introduction to the dynamics of structures
- a presentation of the Cardington building and the secondary systems
- an overview of the FE model for the primary system and the secondary systems
- analyses performed with harmonic and earthquake excitation
- a comparison of the obtained results to the Eurocode 8 standard

I want to take this opportunity to thank my faculty supervisor, Professor Jónas Thór Snæbjörnsson, for his valuable help and support. This thesis would not be the same without his comments and suggestions.

University of Stavanger, June 14, 2010

Terje Fritzman

Table of contents

Summary

Engineers have gained much knowledge on how a primary system behaves under harmonic excitation and earthquakes, but they do not know as much about how a secondary system affects the behavior. In seismic areas, buildings (primary system) are built to withstand the impact of the earthquake, but the effects on secondary systems are often not taken into consideration when the structure is designed.

In the past 40 years a lot of research has been done on developing methods to analyze the non-structural elements during seismic excitation. However, these methods were mainly focused on the safety of critical equipment in, for example, power plants. In the later years it has been shown that the non-structural elements in conventional buildings also should be taken into consideration in the earthquake design process.

A floor response spectrum from the primary structure is often used to estimate the dynamic response of the secondary system, but one problem by doing so is that it does not take into consideration the effect of the interaction between the two systems.

The so-called Cardington Building, the primary system used in this thesis, was a seven storey in-situ concrete building erected inside an old Zeppelin hanger which housed the Cardington Laboratory near Bedford, UK, owned and operated by the BRE (Building Research Establishment Ltd). The building was built in 1998, and was a part of the European Concrete Building Project (ECBP). It was constructed like an office block, and its goal was to provide improved design codes, especially for the dynamic properties of the concrete structure.

The analyses in this thesis are based on a numerical model of the Cardington Building made in a finite element program called Ruaumoko. The model is calibrated against experimental results from full scale tests performed by Jónas Thór Snæbjörnsson, Ódinn Thórarinsson and Símon Ólafsson in 2000 [2].

The FE model in this thesis is exposed to harmonic excitation in the first tests and excited by earthquakes in the second tests. Also, the seismic coefficients obtained from the earthquake excitations have been compared with the seismic coefficients calculated by equations from the Eurocode 8 standard.

From the harmonic excitation tests it is seen that the secondary systems with their natural frequencies inside the resonant frequency range of the primary system get a much higher displacement than the ones outside the resonant frequency range, but in exchange the displacement of the primary system is reduced. For secondary system 6 the displacement of the primary system drops 34 % for the fourth floor and 35 % for the top floor (Figure 5.10 and Figure 5.11). This is true for secondary systems with both 0.8 % damping and 5.0 % damping.

When the earthquake tests are performed it is seen that the floor response spectra method gives values that are generally higher than the values computed from the numerical model when the secondary system is attached to the primary system, see Table 5.10. It is seen that the natural frequency and the damping ratio of the secondary system affects the response generated by the two methods of measuring in different ways.

For secondary systems with natural frequencies far away from the resonant frequency range of the primary system tends to get a much higher response for the floor response spectra method (≈30 %). This pattern is seen for all damping ratios (0.08 %, 2.0 % and 5.0 %).

For secondary systems with natural frequencies just outside the resonant frequency range of the primary system the response for the floor response spectra method compares very well to the results from the case when the secondary system is attached to the primary system. This is true for all damping ratios tested.

The problems appear when the secondary systems with natural frequencies inside the resonant frequency range of the primary system are analyzed by the floor response spectra method. The response for a low damping ratio (0.08 %) makes the amplitudes much higher for the floor response spectra method (≈40 %), and the shape of the response is different.

If the secondary system has at least a medium damping ratio (2.0 %) the differences in the amplitudes are not big, and the response of the two methods compare well to each other.

The secondary systems with a high damping ratio (5.0 %) will make the amplitudes of the floor response lower than the ones measured from the analysis where the secondary system is mounted on the primary system. This means that the real response is higher than the one obtained from the floor response method. Standards, like Eurocode 8, normally use a damping ratio of 5.0 %, which makes this a notable effect.

In spite of the various shortcomings of the floor response approach discussed above it is found to give a reasonably good description of the overall response of the secondary system.

Eurocode 8 is the standard for design of structures for earthquake resistance, and includes an equation that calculates the seismic coefficients for secondary systems. This calculated seismic coefficient has been found to correspond well to the floor response method's seismic coefficients, but the actual seismic coefficients (generated by the analysis where the secondary system is attached to the primary system) did not correspond as well, especially for secondary system no. 6 and 7. These are the two secondary systems with natural frequencies inside the natural frequency range of the primary system. The actual seismic coefficient for secondary system 6 during earthquake 2000kav is twice as high as the seismic coefficient calculated by the Eurocode 8 standard (Figure 6.2).

The Eurocode 8 standard states that the equation should only to be used for secondary systems that are not very important and / or dangerous. This makes the error less severe, but the equation should be representative none the less. The seismic coefficient at the natural frequency of the primary system is too low.

1. Introduction

The object of this study is to investigate how a primary system (PS) and a secondary system (SS) work together under dynamic excitation. The primary system is normally what keeps the building standing, in other words, is the load-bearing structure. The secondary system can be both load-bearing (structural) and non load-bearing (non-structural). Typical non load-bearing secondary systems would be any type of equipment in the building, e.g., computer systems, heavy machinery and storage tanks. Typical load-bearing secondary systems are stairways, piping systems, ducts and other minor structures.

Engineers have gained much knowledge on how a primary system behaves under harmonic excitation and earthquakes, but they do not know as much about how a secondary system affects the behavior. In seismic areas, buildings (PS) are built to withstand the impact of the earthquake, but secondary systems are often not taken into consideration when the structure is designed. The main concern for the engineer is to prevent structural failure first and the secondary systems are often added after the building is erected. If a machine got broken due to an earthquake in a factory and the production had to be stopped, the company could lose millions. If a machine got broken in a power plant or a hospital, the outcome could be catastrophic.

To avoid this from happening, a floor response spectra method is often used to estimate the dynamic response of the secondary system. A floor response spectrum means that the primary structure is excited alone, and the response of the floor where the secondary system is going to be located, is recorded. This floor response is then applied as a ground motion on the secondary system. One problem by doing so is that it does not take into consideration the effect of the interaction between the two.

This study will investigate the suitability of this method. In order to do so, a numerical model of the primary and secondary system is made using a FEM (finite element method) program called Ruaumoko [1]. Then, an analysis of the behavior of the primary and secondary system will be performed both jointly and separately. The analysis will include both harmonic and earthquake excitation.

The thesis also includes a testing of a method used by the Eurocode 8 standard to estimate seismic design response spectra for secondary structures.

The building that this thesis will investigate is the Cardington building. It was built in a hangar, the BRE Cardington Laboratory, near Bedford, UK. Actual recordings of excitations on a secondary system have already been measured for several stiffness values and masses. The thesis will be based on full scale tests performed by Jónas Thór Snæbjörnsson, Ódinn Thórarinsson and Símon Ólafsson in 2000 [2].

2. Theoretical Background

The theoretical background is explained fully in Appendix A. The following chapters are a summary of the most important methods and equations.

2.1 Single Degree of Freedom

2.1.1 Equation of motion

Simple structures can be modeled as a single-degree-of-freedom (SDOF) system. A water tank with a massless tower is the most common example, along with the idealized one-story frame. The structure is defined by a system that consist of a mass, spring, damper and force (Figure 2.2).

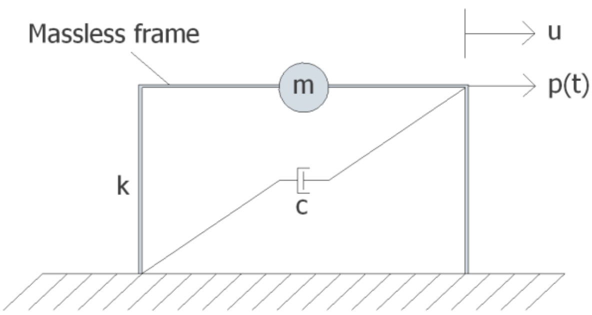

Figure 2.2. Idealized one-story frame.

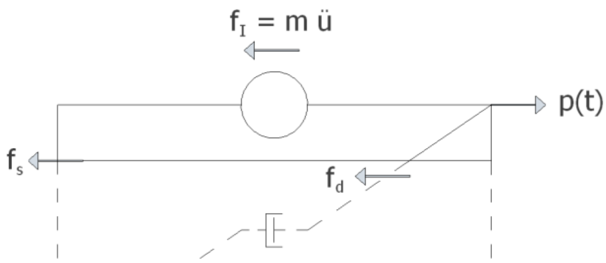

Figure 2.1. Free-body diagram of the one-story frame.

By applying Newton's Second Law on the system and drawing the free-body diagram, the equation of motion is defined (Appendix A: Single Degree of Freedom**Feil! Fant ikke referansekilden.**).

$$m\,\ddot{u} + c\,\dot{u} + k\,u = p(t) \qquad \textbf{(2.1)}$$

where m is the mass, k is the stiffness, c is the damping, $\ddot{u}$ is the acceleration, $\dot{u}$ is velocity and u is displacement.

2.1.2 Stiffness

The stiffness, k, can be determined, if the top beam is rigid, in the following way ([7],page 9):

$$k = \frac{12\,E\,I}{L^3} \qquad \textbf{(2.2)}$$

where E is the Young's modulus (modulus of elasticity), I is the moment of inertia (second moment of area) and L is the length of the element.

2.2 Multiple Degree of Freedom (2DOF)

2.2.1 Equation of motion

The SDOF system can be expanded into a two degree of freedom system which is usually expressed in a matrix form ([7], page 345-348):

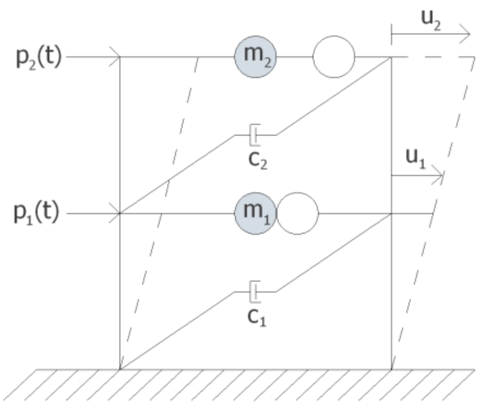

Figure 2.4. Two-story frame.

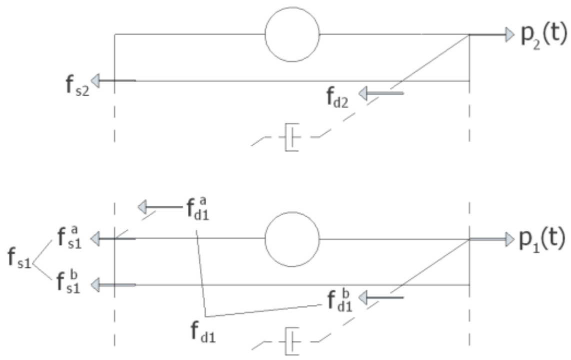

Figure 2.3. Free-body diagram of the two-story frame.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} f_{d1} \\ f_{d2} \end{bmatrix} + \begin{bmatrix} f_{s1} \\ f_{s2} \end{bmatrix} = \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} \text{ or, in matrix notation } \boldsymbol{m}\,\ddot{\boldsymbol{u}} + \boldsymbol{f_d} + \boldsymbol{f_s} = \boldsymbol{p}(t) \quad \textbf{(2.3) (2.4)}$$

where $\boldsymbol{m}$ is the lumped mass matrix. To find the damping matrix, $\boldsymbol{f_d}$, and the stiffness matrix, $\boldsymbol{f_s}$, the free body diagram is used. It is shown in “Appendix A: MDF” that the damping and stiffness matrix is as follows:

$$\boldsymbol{f_s} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{k}\,\boldsymbol{u} \qquad \boldsymbol{f_d} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \boldsymbol{c}\,\dot{\boldsymbol{u}} \qquad \textbf{(2.5) (2.6)}$$

Substituting equation (**2.5**) and (**2.6**) into (**2.3**):

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} \qquad \textbf{(2.7)}$$

or in matrix notation:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}} + \boldsymbol{c}\,\dot{\boldsymbol{u}} + \boldsymbol{k}\,\boldsymbol{u} = \boldsymbol{p}(t) \qquad \textbf{(2.8)}$$

The equation of motion looks the same as for the SDOF, but it is now in matrix form and the equation is *coupled* with regard to the damping and stiffness.

To include more DOF, the same methodology can be applied again.

2.3 Free vibration

To find the natural frequencies of the system, one must look at the structure when it is undergoing free vibration. Free vibration means that if the structure is pulled out of its equilibrium position it can vibrate without any external dynamic forcing ($\boldsymbol{p}(t) = 0$). A convenient start is to look at undamped vibrations, which means that **c** = **0** for the time being.

The free vibration of a linear undamped multiple degree of freedom system is then ([7], page 39):

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = \boldsymbol{0} \qquad \textbf{(2.9)}$$

in which $\boldsymbol{0}$ is a zero vector. A solution to the free vibration can be described as:

$$\boldsymbol{u}(t) = q_n(t)\,\phi_n \qquad (\,n \leq \text{Number of degrees of freedom} = \text{N}\,) \qquad \textbf{(2.10)}$$

where ϕ_n are the natural modes of the system, and $q_n(t)$ is a harmonic function.

Taking the second time derivative of $\boldsymbol{u}(t)$ and substituting the solution for both $\boldsymbol{u}(t)$ and $\ddot{\boldsymbol{u}}(t)$ into the free vibration equation (see Appendix A: Free vibration) gives:

$$[\,-{\omega_n}^2\,\boldsymbol{m}\;\phi_n + \boldsymbol{k}\,\phi_n\,]\,q_n(t) = \boldsymbol{0} \qquad \textbf{(2.11)}$$

This equation can be satisfied in two ways. One way is saying that $q_n(t) = 0$, but that implies $\boldsymbol{u}(t) = \boldsymbol{0}$, which means that there is no motion. This is called a trivial solution, and does not give a useful result. Instead, let the equation $-{\omega_n}^2\,\boldsymbol{m}\;\phi_n + \boldsymbol{k}\,\phi_n$ equal to 0. In other words, the following equation must be satisfied:

$$\boldsymbol{k}\,\phi_n = {\omega_n}^2\,\boldsymbol{m}\;\phi_n \qquad (\,\boldsymbol{k} = {\omega_n}^2\,\boldsymbol{m}\,) \qquad \textbf{(2.12)}$$

This is called the *matrix eigenvalue problem*.

The equation can be written as follows:

$$[\,\boldsymbol{k} - {\omega_n}^2\,\boldsymbol{m}\,]\,\phi_n = \boldsymbol{0} \qquad \textbf{(2.13)}$$

Setting $\phi_n = 0$ gives another trivial solution, but the equation has a nontrivial solution if:

$$det\,[\,\boldsymbol{k} - {\omega_n}^2\,\boldsymbol{m}\,] = \boldsymbol{0} \qquad \textbf{(2.14)}$$

From this equation, the natural frequencies are calculated, and to get the natural mode shapes, ϕ_n, the known natural frequencies are inserted into equation (**2.13**) and solved for ϕ_n. However, this does not give the absolute values for the vectors ϕ_n, but only the amplitude shape of the vector. This means that it can be scaled. A popular way of scaling is called normalization of modes, and is discussed in Appendix A: Normalization of modes.

2.4 Orthogonality of modes

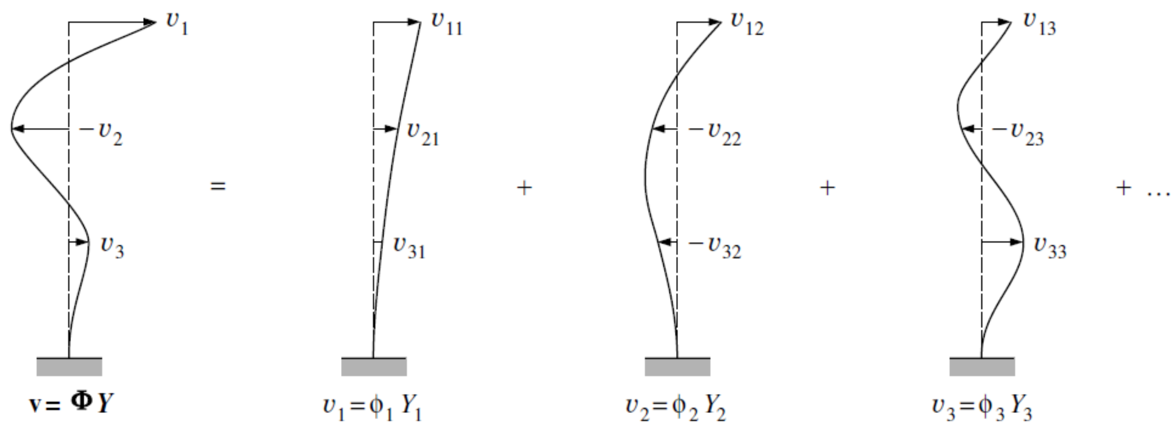

Figure 2.5. Representing deflections as sum of modal components ([3], page 220).

The displacement for any structure can be split up into modal contributions based on the normal mode shapes of the structure. For one of the modal components (u_n), the displacements are given by multiplying the mode shape vector (ϕ_n) with a time dependent modal amplitude (Y_n) ([3], page 220-223):

$$u_n = \phi_n \, \mathrm{Y}_n \tag{2.15}$$

The normal modes can then be superimposed to get the total displacement. The sum of all the displacements from the modal components gives the total displacement:

$$\boldsymbol{u} = \phi_1 \, \mathrm{Y}_1 + \phi_2 \, \mathrm{Y}_2 + \cdots + \phi_n \, \mathrm{Y}_n = \sum_{n=1}^{N} \phi_n \, \mathrm{Y}_n \tag{2.16}$$

or, in matrix notation:

$$\boldsymbol{u} = \boldsymbol{\Phi} \, \mathbf{Y} \qquad \text{where } \boldsymbol{\Phi} = \begin{bmatrix} \phi_{11} & \cdots & \phi_{1N} \\ \vdots & \ddots & \vdots \\ \phi_{N1} & \cdots & \phi_{NN} \end{bmatrix} = modal\ matrix \tag{2.17}$$

By applying this superimposing method on the MDOF equation of motion, the equations get *uncoupled*. The derivation is shown in Appendix A: Orthogonality of Modes. The *coupled* 2DOF equation from chapter 2.2 can be *uncoupled* and the result is the following equation:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{Y}_1(t) \\ \ddot{Y}_2(t) \end{bmatrix} + \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \begin{bmatrix} \dot{Y}_1(t) \\ \dot{Y}_2(t) \end{bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} Y_1(t) \\ Y_2(t) \end{bmatrix} = \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix} \quad \textbf{(2.18)}$$

or on a more compact form:

$$M_n\, \ddot{Y}_n(t) + C_n\, \dot{Y}_n(t) + K_n\, Y_n(t) = P_n(t) \quad \textbf{(2.19)}$$

where M_n, C_n, K_n are the diagonal elements:

$$M_n = \phi_n^T\, \boldsymbol{m}\, \phi_n \qquad C_n = \phi_n^T\, \boldsymbol{c}\, \phi_n \qquad K_n = \phi_n^T\, \boldsymbol{k}\, \phi_n \quad \textbf{(2.20) (2.21) (2.22)}$$

and the force is defined in the same way:

$$P_n(t) = \phi_n^T\, \boldsymbol{p}(t) \quad \textbf{(2.23)}$$

Normally, the *uncoupled* equation of motion is divided by the generalized mass since it is more convenient and physically reasonable to express the equation with the damping ratio, because it is a variable that can be measured experimentally or estimated with good precision.

$$\ddot{Y}_n(t) + 2\, \xi_n\, \omega_n\, \dot{Y}_n(t) + {\omega_n}^2\, Y_n(t) = \frac{P_n(t)}{M_n} \quad \textbf{(2.24)}$$

where ξ_n is the critical damping ratio, defined as:

$$\xi_n = \frac{C_n}{2\, M_n\, \omega_n} \quad \textbf{(2.25)}$$

If ξ_n is equal to 1, it is a state of critical damping, which explains the name.

The equation of motion is *uncoupled*, which means that it can be expressed as n SDOF systems. To obtain the total response of the MDOF system, the n SDOF equations are solved and their effects are superimposed.

2.5 Forced vibration – Harmonic excitation

Since the MDOF system has been uncoupled, the same equations that are valid for the SDOF system are also valid for the MDOF system. Therefore the derivation can be done for a SDOF system.

2.5.1 Undamped harmonic excitation

The system is subjected to a harmonically varying load $p(t)$. The load is of sine-wave form with an amplitude p_0 and circular frequency ω ([3], page 33-35). To simplify the solution process the undamped system is investigated first. The derivations for the *general solution* are shown in Appendix A: Harmonic excitation.

If the system starts at rest, the *general solution* response becomes:

$$u(t) = \frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right](\sin\omega t - \beta \sin\omega_n t) \quad \textbf{(2.26)}$$

where $p_0/k = u_{st}$ is the displacement the system would get if only affected by the static load p_0. $[1/1-\beta^2]$ is the magnification factor representing an amplification effect based on the applied excitation. $\frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\sin\omega t$ is called the steady-state response, and $\frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\beta\sin\omega_n t$ is called the transient response. The latter one will disappear when damping is included. This means that very often in practical cases with damping the dominating response in the first second(s), if it is big enough to matter, is based on the transient response, and later only on the steady-state response.

Both the transient and steady state response are graphed in the figure below. One thing that is worth mentioning is that the two responses in the case below are out of phase with each other. This gives the total response a beating effect.

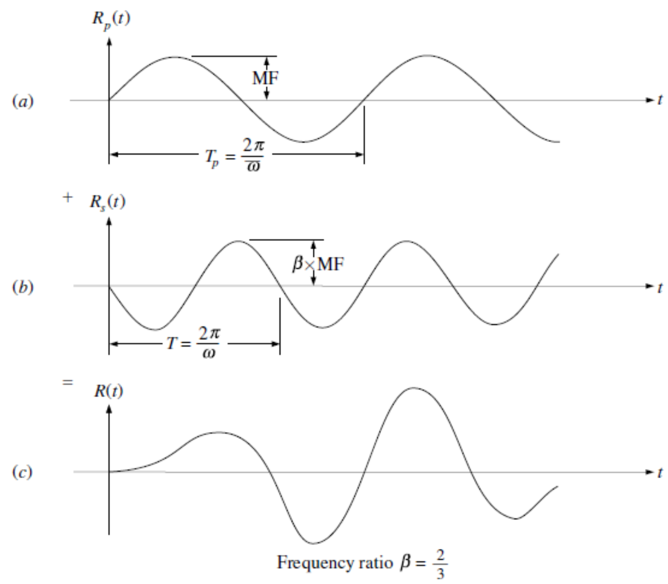

Figure 2.6. a) steady state response; b) transient response; c) total response ([3], page 35)

2.5.2 Harmonic excitation with viscous damping

It can be shown ([3], page 36-37) that the viscously damped total response is:

$$u(t) = [\,A\ \cos\omega_D\, t + \ B\ \sin\omega_D\, t\,]\, e^{-\xi\omega t} + \frac{p_0}{k}\left[\frac{1}{(1-\beta^2)^2+(2\,\xi\,\beta)^2}\right][\,(1-\beta^2)\sin\omega\, t - 2\,\xi\,\beta\cos\omega\, t\,] \qquad \textbf{(2.27)}$$

where $\omega_D = \omega\sqrt{1-\xi^2}$ and $e^{-\xi\omega t}$ governs the rate of decay in the transient response. If we graph the transient response (first term on the right hand side) and the steady state response (last term), we can see that the transient response disappears relatively rapidly within the first four periods.

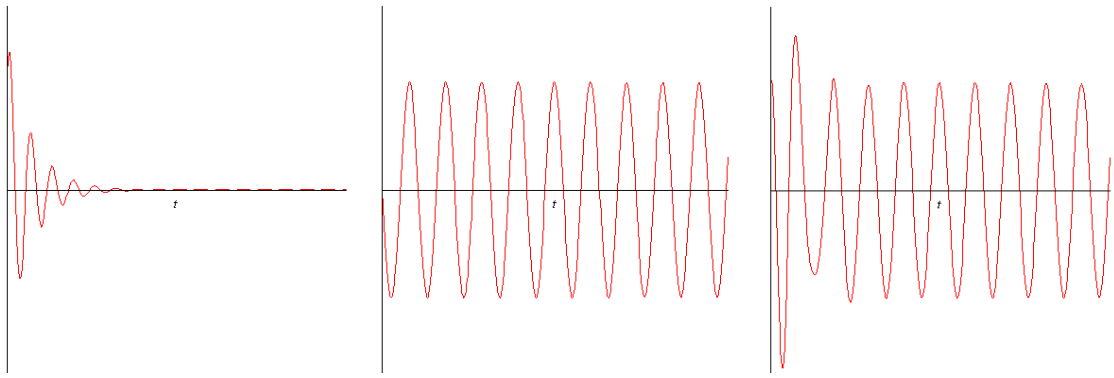

Figure 2.7. Left) Transient response; Middle) Steady State response; Right) Total response.

2.6 Damping

2.6.1 Evaluation of Viscous Damping

Buildings, as well as most structures are underdamped, i.e., $\xi \ll 1$. When a structure is underdamped the free vibration decays out with time as the structure slowly losses its energy until the oscillations stop.

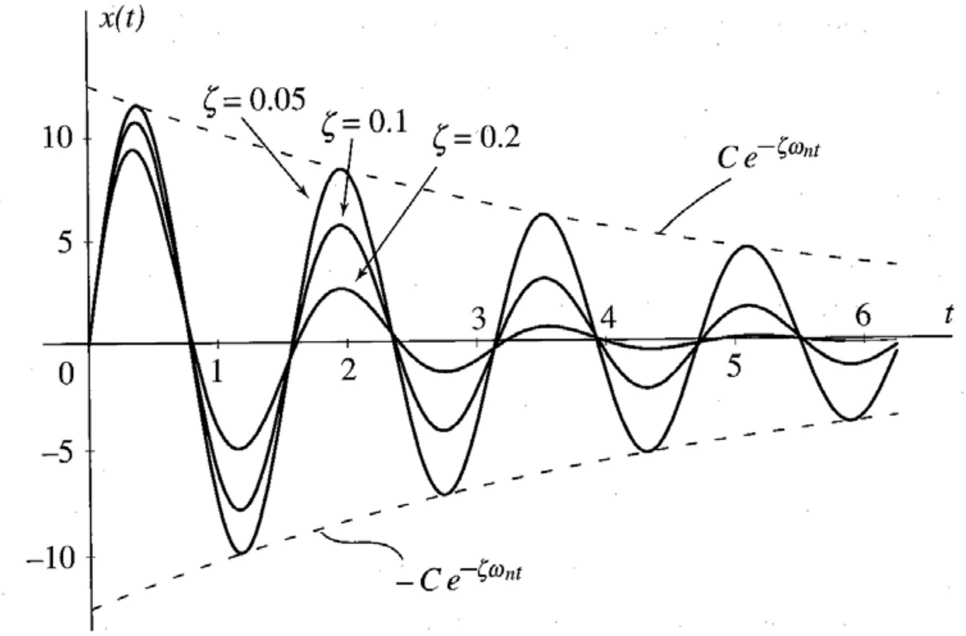

Figure 2.8. Response of an underdamped system ([4],page 91).

There are many ways of determining the damping ratio of a system. Two of the most common ones are the Half-power bandwidth method ([7], page 83) and the Free-Vibration Decay method ([5], page 35).

2.6.1.1 Half-power bandwidth method

The Half-power bandwidth method uses the resonant amplitude, or rather $1/\sqrt{2}$ times the resonant amplitude, to calculate the damping ratio.

The equation is written as follows: $\xi = \omega_b - \omega_a/2\omega_n$, where ω_a and ω_b are the forcing frequencies on either side of the resonant frequency, ω_n, at which the amplitude is $1/\sqrt{2}$ times the resonant amplitude. (Anil K. Chopra [7], *Dynamics of Structures*, 2007, page 83)

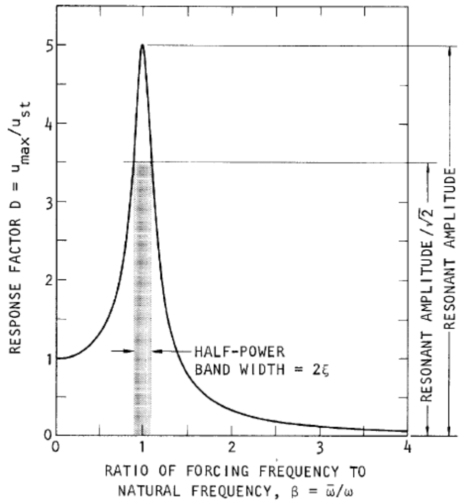

Figure 2.9. Definition of half-power bandwidth ([8], page 34).

2.6.1.2 The Free-Vibration Decay Method

The Free-Vibration Decay Method is the simplest and most used method for measuring damping. To use this method, one has to excite a system and then suddenly stop the excitation. The next step is to evaluate the free vibration response. The rate of decay of oscillation is the interesting factor. By using two peak amplitude values and the number of oscillations between them, one can determine the damping ratio of the system by using the equation below. For derivations see Appendix A: The Free-Vibration Decay Method ([5], page 35).

$$\xi = \frac{1}{2\,\pi} \ln\left(\frac{x_i}{x_j}\right) \frac{1}{N} \qquad \textbf{(2.28)}$$

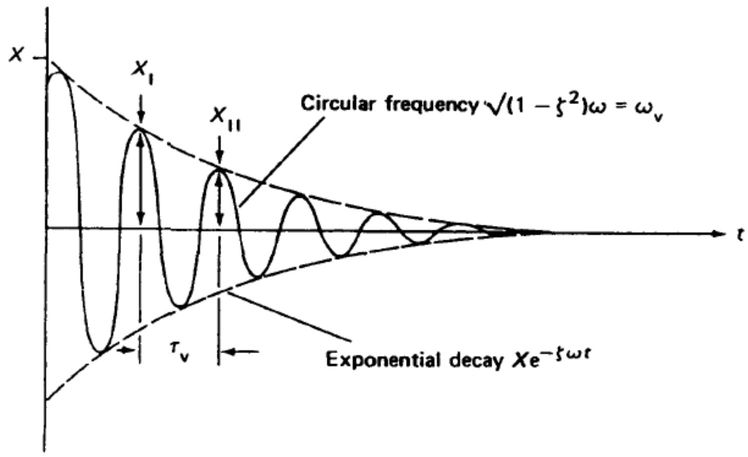

Figure 2.10. Vibration Decay ([5], page 35)

2.6.2 Rayleigh Damping

Rayleigh damping is the traditional damping model used in step-by-step or time-history programs. The reason why it is so popular is that the Rayleigh damping only uses two variables and all matrices are already computed from the analysis. The matrices used are the mass matrix and the stiffness matrix ([6], page 4).

$$C = \alpha M + \beta K \tag{2.29}$$

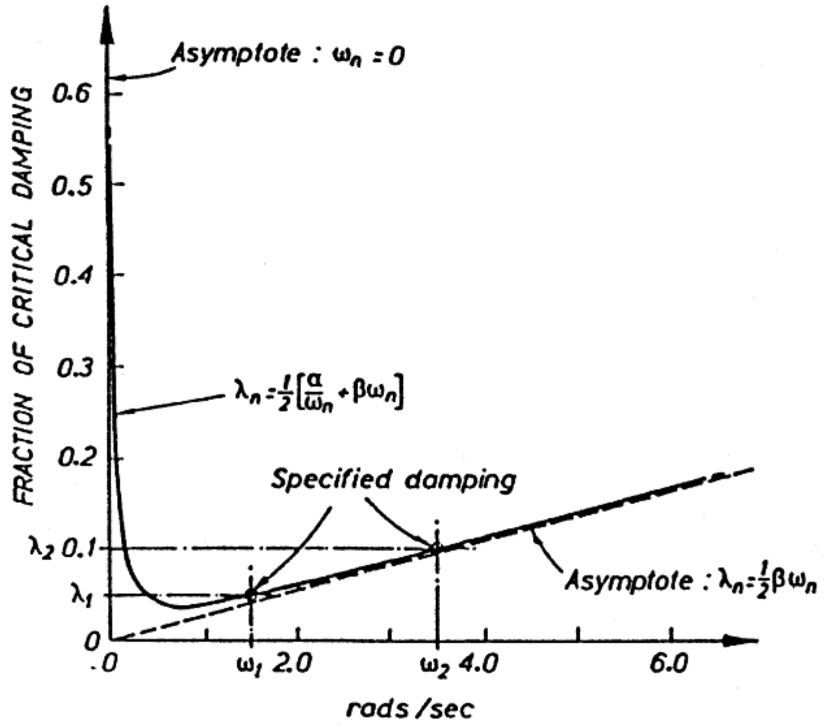

Figure 2.11. Rayleigh Damping Model. ([6], page 3)

2.6.3 Caughey Damping

The Caughey Damping Model is used when one has to have the same damping ratio on a great number of modes. The damping matrix can be expressed as follows ([6], page 6):

$$\boldsymbol{C} = \boldsymbol{M} \sum_{b=0}^{N-1+q} \boldsymbol{a_b} \ [\boldsymbol{M}^{-1}\boldsymbol{K}]^{\boldsymbol{b}} \tag{2.30}$$

where q is any integer and N is the number of nodes at which damping is specified.

The original equation of the Caughey Damping Model is awkward to use for any computation. Wilson and Penzien have developed a more simple way of obtaining the same damping model. It uses the properties of orthogonality of the mode shapes, which is done in chapter 2.4: Orthogonality of modes. It uses the generalized mass of the ith mode:

$$m_i = \phi_i^T \boldsymbol{M} \, \phi_i \tag{2.31}$$

The same way the generalized damping is computed:

$$c_i = 2 \, \lambda_i \, m_i \, \omega_i = \phi_i^T \, \boldsymbol{C} \, \phi_i \tag{2.32}$$

where λ is the *critical* damping ratio.

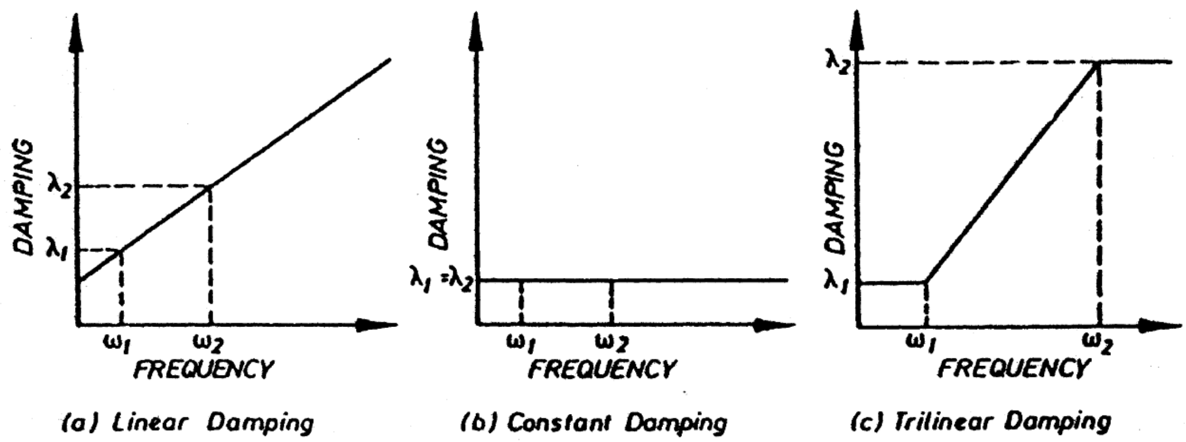

Figure 2.12. Caughey Damping Model: Linear or Tri-linear Damping Model. ([6], page 6)

2.6.4 Effect of a Secondary System: Dynamic Vibration Absorber

When a single mass, without damping, is excited by a harmonic force with a frequency equal to the natural frequency of the main mass, the response is infinite (resonance). Now a second mass is introduced to the system, an absorber mass, which is tuned to have the same natural frequency as the main mass. By doing so, the response of the main mass at its natural frequency becomes zero. This happens because the absorber mass changes the 1DOF into a 2DOF which has two resonance frequencies, neither of which equals the original resonance frequency of the main mass. The absorber mass *absorbs* the motion of the main mass [9].

The response at the two new natural frequencies is also infinite, which may not be a problem when a machine is running at working frequency, but it may cause problems during startup and shutdown. When damping is introduced, the response is no longer infinite at the natural frequencies, but the response of the main mass is no longer zero at the original resonance frequency [9].

This phenomenon occurs in the results of this report, and an example of the occurrence is shown in Figure 2.13. The green curve represents the response of the Primary System (Main mass) when it is excited with a frequency equal to its natural frequency. The blue curve represents the response of the Primary System with a Secondary System attached.

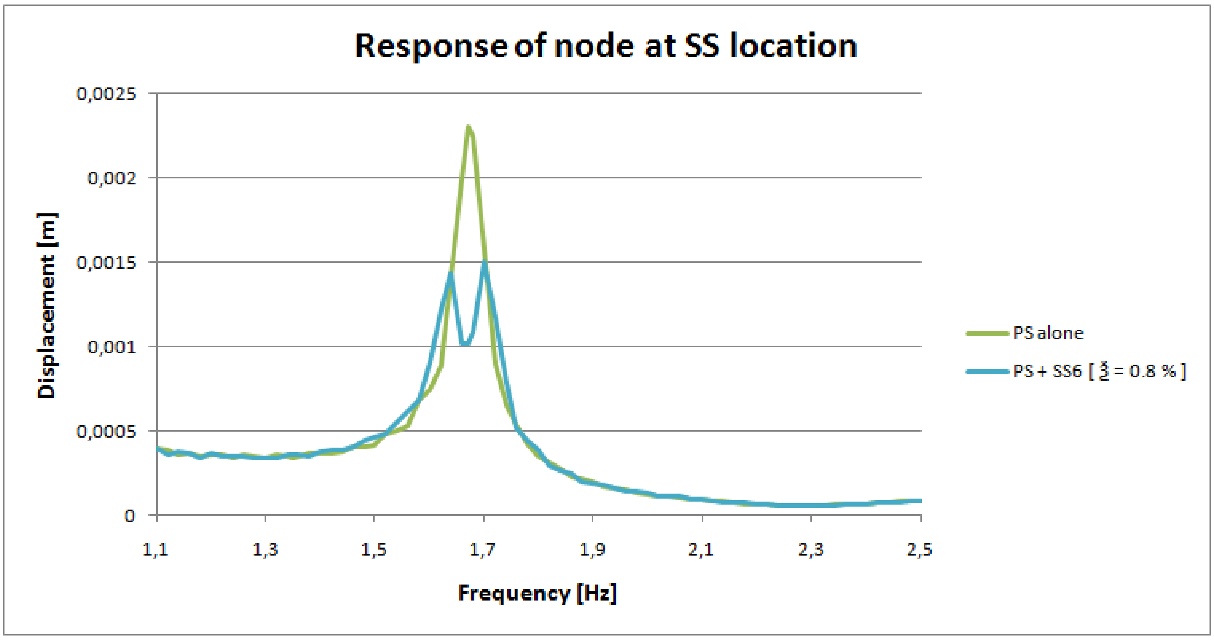

Figure 2.13. A response function showing the damping effect when the SS system is attached.

2.7 Solutions of the equation of motion

There are various ways of obtaining the solution to the equation of motion, e.g., Duhamel's Integral, Frequency-Response Method and Direct Integration of the equation of motion.

Duhamel's Integral is often preferred for arbitrary loading, see Appendix A: Duhamel's integral.

Direct integration of the equation of motion is the method used by the FEM program, Ruaumoko; hence this is the method that is explained in detail in this chapter.

2.7.1 Direct integration of the equation of motion

In 1959, N. M. Newmark developed several methods for solving differential equations numerically by integration. The two most well know methods are the average acceleration and the linear acceleration. Newmark's methods are time-stepping methods ([6], page 46-47).

The Newmark Constant Average Acceleration method is often used in computer programs. Ruaumoko, the FEM program used in this thesis, is no exception. The method is a step-by-step integration of the equation of motion. The acceleration is assumed to be constant during the time step, i.e., time t and time $t + \Delta t$. This gives the following relation between the accelerations:

$$\ddot{u} = \frac{\ddot{u}(t)+\ddot{u}(t+\Delta t)}{2} \tag{2.33}$$

Integrating with respect to time over the time-step Δt gives the velocity and displacement. By rearranging and letting the increment in the displacement be the variable, the increment in the acceleration is:

$$\Delta\ddot{u} = \ddot{u}(t + \Delta t) - \ddot{u}(t) = \frac{4\,\Delta u}{(\Delta t)^2} - \frac{4\,\dot{u}(t)}{\Delta t} - 2\,\ddot{u}(t) \tag{2.34}$$

and the increment in the velocity is:

$$\Delta\dot{u} = \dot{u}(t + \Delta t) - \dot{u}(t) = \frac{2\,\Delta u}{\Delta t} - 2\,\dot{u}(t) \tag{2.35}$$

Substituting the acceleration and velocity into the equation of motion at time $t + \Delta t$ gives:

$$m\,[\,\ddot{u}(t) + \Delta\ddot{u}\,] + \;c\,[\,\dot{u}(t) + \Delta\dot{u}\,] + \;k\,[\,u(t) + \Delta u\,] = p(t + \Delta t) \tag{2.36}$$

The stiffness term can be rewritten as:

$$k(t + \Delta t)[\,u(t) + \Delta u\,] = k(t)[\,u(t)\,] + k_t[\,\Delta u\,] = F_{Elastic}(t) + k_t[\,\Delta u\,] \tag{2.37}$$

where $k(t)$ is the secant stiffness matrix at time t, the elastic forces are the nodal equivalent of the member forces at time t and k_t is the current tangent stiffness matrix.

The damping term may also be rewritten similarly to the stiffness term:

$$c(t+\Delta t)[\,\dot{u}(t)+\Delta\dot{u}\,] = c(t)[\,\dot{u}(t)\,] + c_t[\,\Delta\dot{u}\,] = F_{Damping}(t) + c_t[\,\Delta\dot{u}\,] \qquad \textbf{(2.38)}$$

where the damping forces are at time t and the matrix c_t is the current tangent damping matrix.

Substituting the two above equations into the equation of motion gives:

$$m\,[\,\Delta\ddot{u}\,] + c_t[\,\Delta\dot{u}\,] + k_t[\,\Delta u\,] = p(t+\Delta t) - m\,[\,\ddot{u}(t)\,] - F_{Damping}(t) - F_{Elastic}(t) \qquad \textbf{(2.39)}$$

Inserting equation (**2.34**) and (**2.35**) into equation (**2.39**) gives:

$$\left[\frac{4}{(\Delta t)^2}\,m + \frac{2}{\Delta t}c_t + k_t\right][\,\Delta u\,] = p(t+\Delta t) + m\left[\ddot{u}(t) + \frac{4}{\Delta t}\,\dot{u}(t)\right] + 2\,c_t[\,\dot{u}(t)\,] - F_{Damping} - F_{Elastic} \qquad \textbf{(2.40)}$$

If the damping matrix is constant (does not change with time), the above equation can be simplified to the following:

$$\left[\frac{4}{(\Delta t)^2}\,m + \frac{2}{\Delta t}c_t + k_t\right][\,\Delta u\,] = p(t+\Delta t) + m\left[\ddot{u}(t) + \frac{4}{\Delta t}\,\dot{u}(t)\right] + F_{Damping} - F_{Elastic} \qquad \textbf{(2.41)}$$

From this equation the incremental displacement can be calculated, hence the displacement, velocity and acceleration vectors, along with the member forces, can be updated. When the damping matrix and the stiffness matrix are updated, the whole process can be repeated to find the next incremental displacement.

The Newmark Constant Average Acceleration method is unconditionally stable. However, the time step should be small (0.01-0.02 sec) to minimize numerical errors.

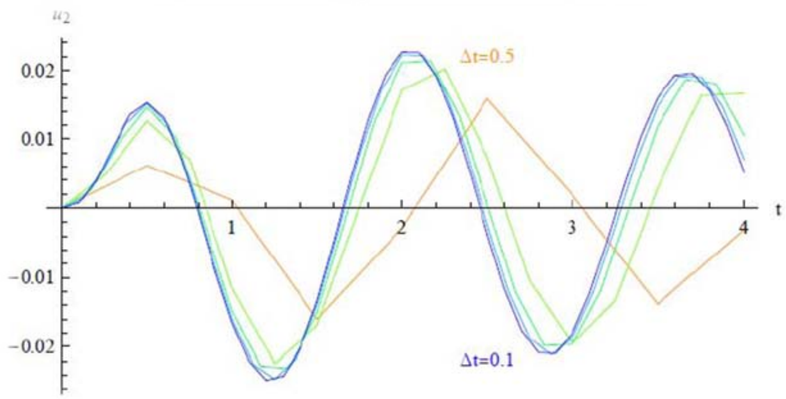

Figure 2.14. Newmark Constant Average Acceleration method - Time steps.

2.8 Ground excitation

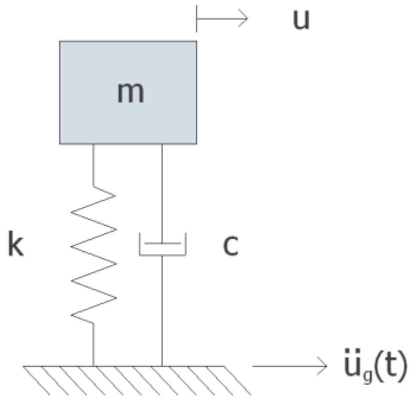

Figure 2.15. Ground excitation $\ddot{u}_g$.

The equation of motion for a structure subjected to an earthquake ground motion is ([7], page 203):

$$m\,\ddot{u} + c\,\dot{u} + k\,u = -\,m\,\ddot{u}_g(t) \qquad \textbf{(2.42)}$$

or divided by the mass:

$$\ddot{u} + 2\,\xi\,\omega\;\dot{u} + \omega^2\,u = -\,\ddot{u}_g(t) \qquad \textbf{(2.43)}$$

The solution to the equation of motion can be solved by using Duhamel's integral (Appendix A: Duhamel's integral):

$$u(t) = -\,\frac{1}{\omega_D}\int_0^t \ddot{u}_g(\tau)\;e^{-\xi\omega_n(t-\tau)}\;\sin[\omega_D\,(t-\tau)]\;d\tau \qquad \textbf{(2.44)}$$

2.9 Response spectrum

A response spectrum is basically a plot of the peak response or peak steady-state response of a structure with varying natural frequency. In a system where the excitation is harmonic, the steady-state response is usually the preferred response, i.e., the transient response is not of interest. If a system undergoes an earthquake load, the peak (transient) response is the one desired.

The response spectrum is a very well known concept in earthquake engineering. It was already initiated in 1932 by M. A. Biot [9]. It is especially useful when you are in a seismic area. Given the earthquake ground motion, the response spectrum is made by exposing systems with different natural frequencies to the same ground motion. The damping ratio is fixed, but by running multiple response spectra with different damping ratios you cover the relevant damping values.

The peak amplitudes in the response spectrum indicate that the excitation frequency hits the natural frequency of one of the modes.

The figure below is an example of a response spectrum. This specific spectrum shows the displacement plotted against frequency. The spectrum belongs to secondary system no. 3, which has been calculated in the analysis part of this thesis (Chapter 5).

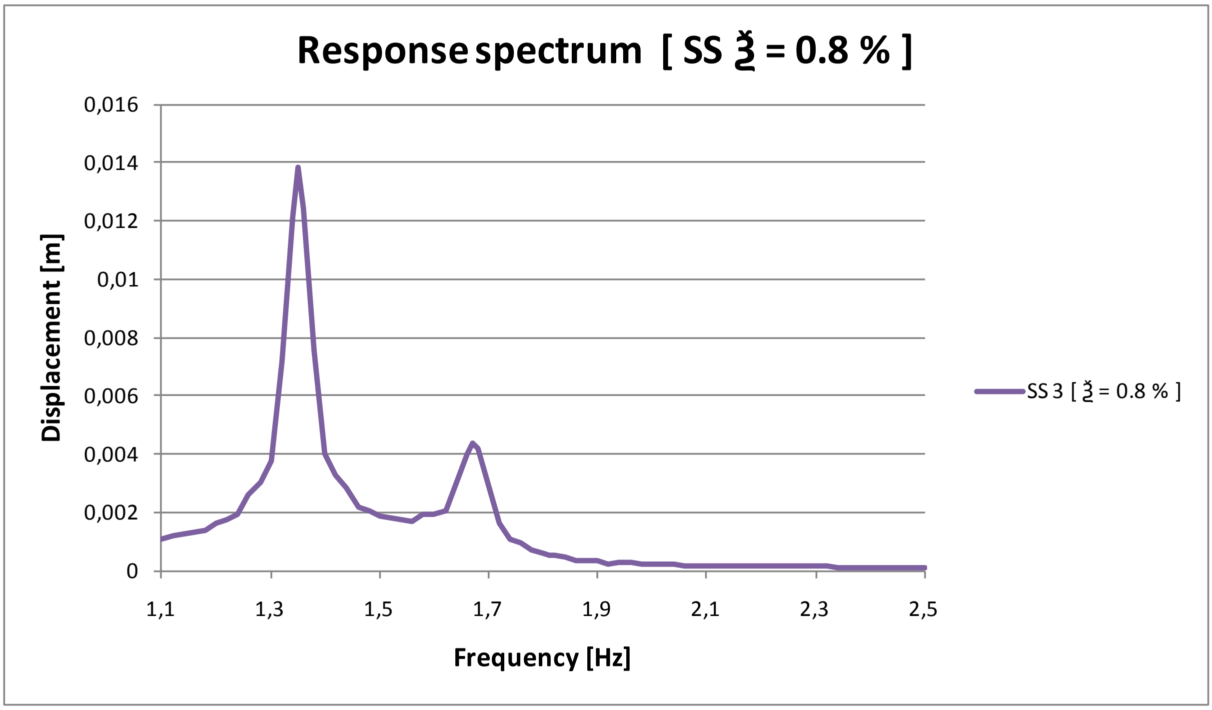

Figure 2.16. An example of a response spectrum.

3. The Cardington Building

3.1 Primary System

The so-called Cardington Building, our primary system, was a seven storey in-situ concrete building erected inside an old Zeppelin hanger which housed the Cardington Laboratory near Bedford, UK, owned and operated by the BRE (Building Research Establishment Ltd). The building was built in 1998, and was a part of the European Concrete Building Project (ECBP). It was constructed like an office block, and its goal was to provide improved design codes, especially for the dynamic properties of the concrete structure.

Figure 3.1. The old Zeppelin hangar near Bedford, UK.

Figure 3.2. The Cardington Building

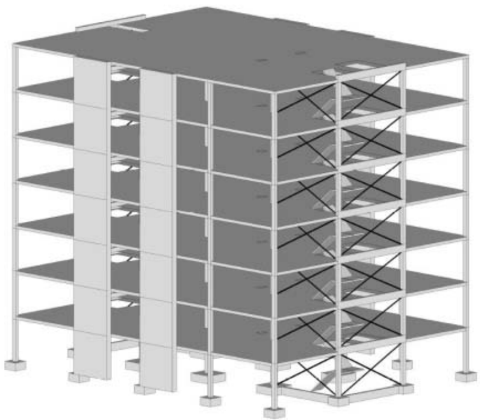

Figure 3.4. . A 3D model of the Cardington Building.

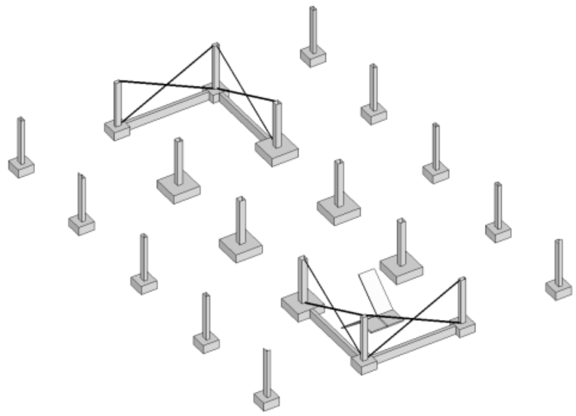

Figure 3.3. Ground level with foundations.

The building rests on foundation blocks under every column. The inner foundations dimensions are 2x2 m and the outer foundations dimension are 1.2x1.2 m. The inner columns are 0.4x0.4 m and the outer columns are 0.25x0.4 m. From the ground floor up to the third floor the columns are C70 concrete. From the third floor to the roof the columns are C30 concrete. The center-to-center distance for the columns is 7.5 m. There are five columns in the east-west direction (z direction) and four in the north-south direction (x direction). This gives a total of twenty columns and an area of 30x22.5 m.

The floors consist of 0.25 m thick concrete slabs which are all C30 concrete. The columns give a clear height of 3.5 m on each floor, which give a total height of 26.25 m.

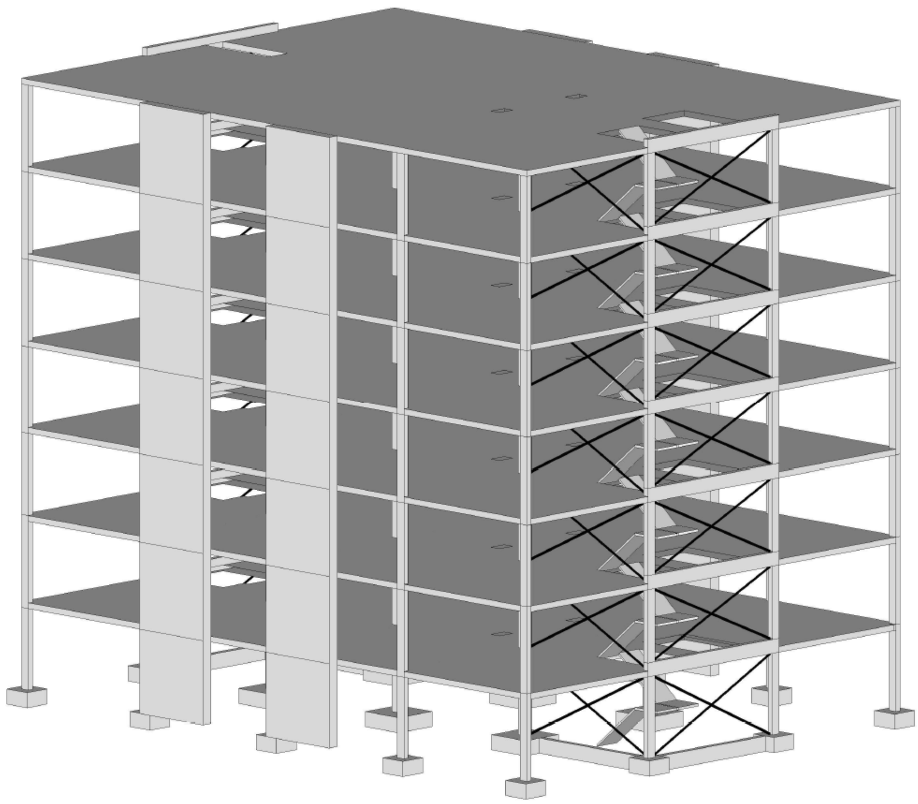

Figure 3.5. A 3D Model of the Cardington Building.

There is no external cladding, walls or other partitions in the building, but there are two large brick panels mounted on both sides in the north-south direction. To provide lateral stiffness against wind and accidental loading there are cross bracings mounted around the staircase and elevator shaft.

At the staircase and at the elevator shaft a beam of 0.65 m height is constructed. This has been done to give some extra stiffness to the building floors to counteract the weakening effects caused by the staircase and elevator shaft openings.

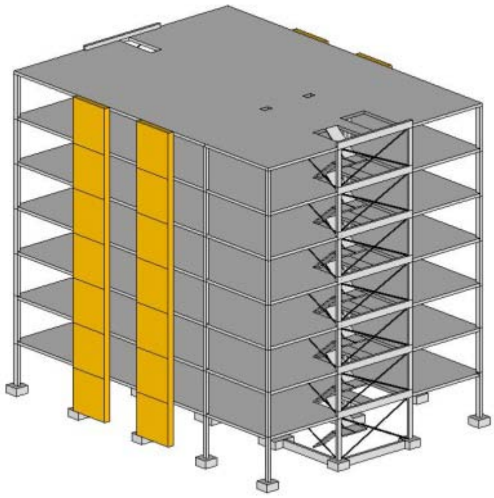

Figure 3.7. The brick panels are shown in yellow.

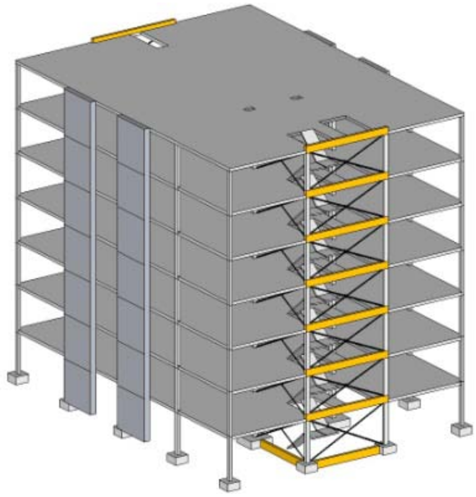

Figure 3.6. The beams are shown in yellow

To simulate service load, 96 sandbags are placed on each floor. Each bag weighs around 1100 kg. They are placed on the six middle bays, in a 4x4 pattern. The six middle bays consist of two bays in the longitudinal direction and three bays in the transverse direction.

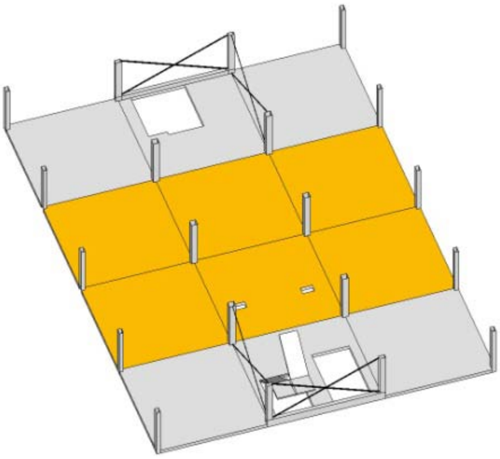

Figure 3.9. The location of the sandbags are shown in yellow.

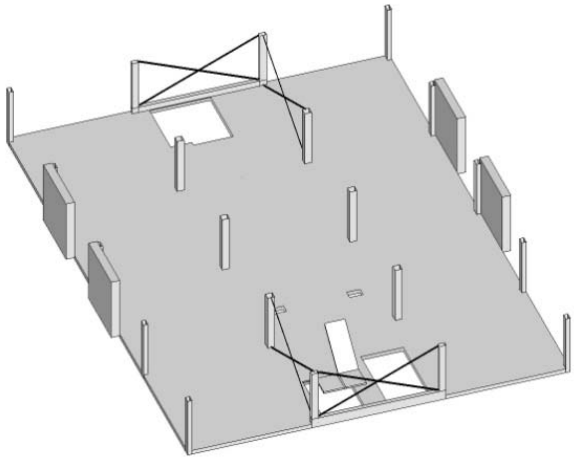

Figure 3.8. A typical floor

The building is relatively symmetric, but the staircase and the elevator shaft create some asymmetry since they are not exactly the same size. Also the brick partitions on the side of the building are located slightly off centre.

3.2 Secondary system

The secondary system is located at the 4th floor. It consists of sandbags, aluminum beams, rubber pads and UC-203 steel sections. The size of the table is 3x3 m, i.e., the length of the nine aluminum beams are 3 m. The rubber pads are 120 mm thick and 180 mm wide, bolted together with the steel sections, making the height of the columns equal to 0.85 m. The aluminum beams dimensions are 150x80 mm. The sandbags weigh approximately 1100 kg each, which is the same as the ones placed on the floors of the primary system.

In this thesis six different secondary systems will be tested, each with a unique natural frequency. The variables that change between the systems are the number of columns, the orientation of the columns and the number of sandbags. The orientation of the columns matters because the steel sections have different stiffness properties in each direction because they have the shape of an I-beam.

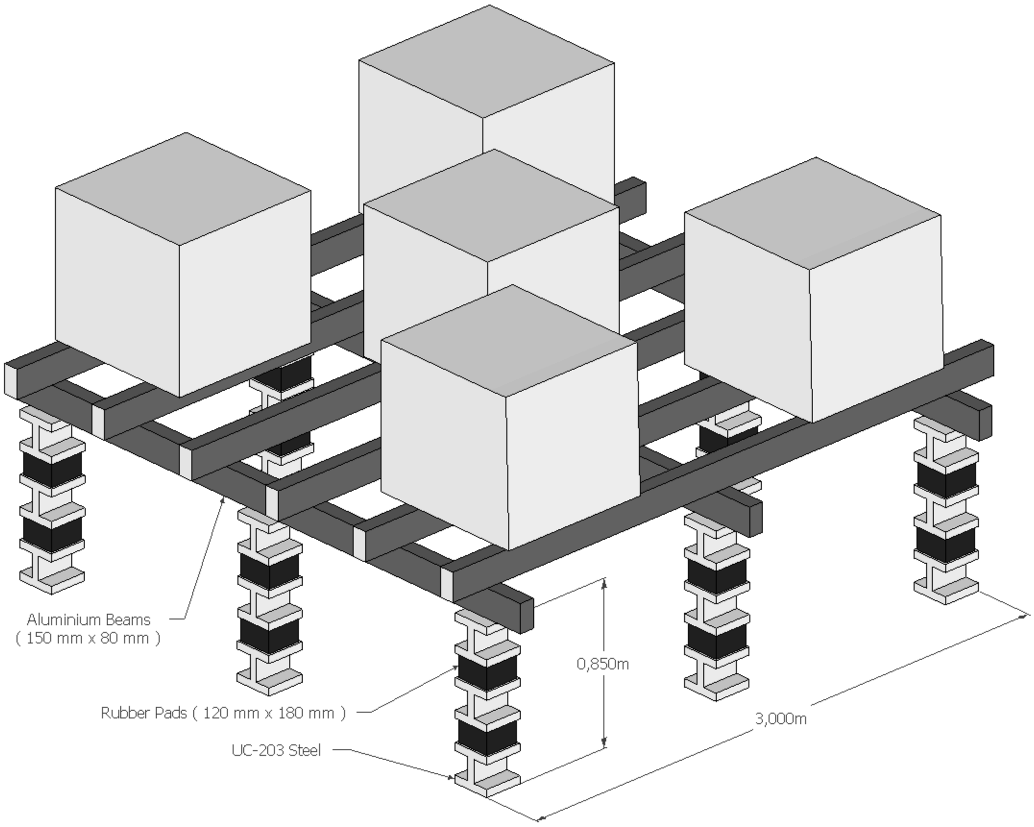

Figure 3.10. The secondary system with its dimensions.

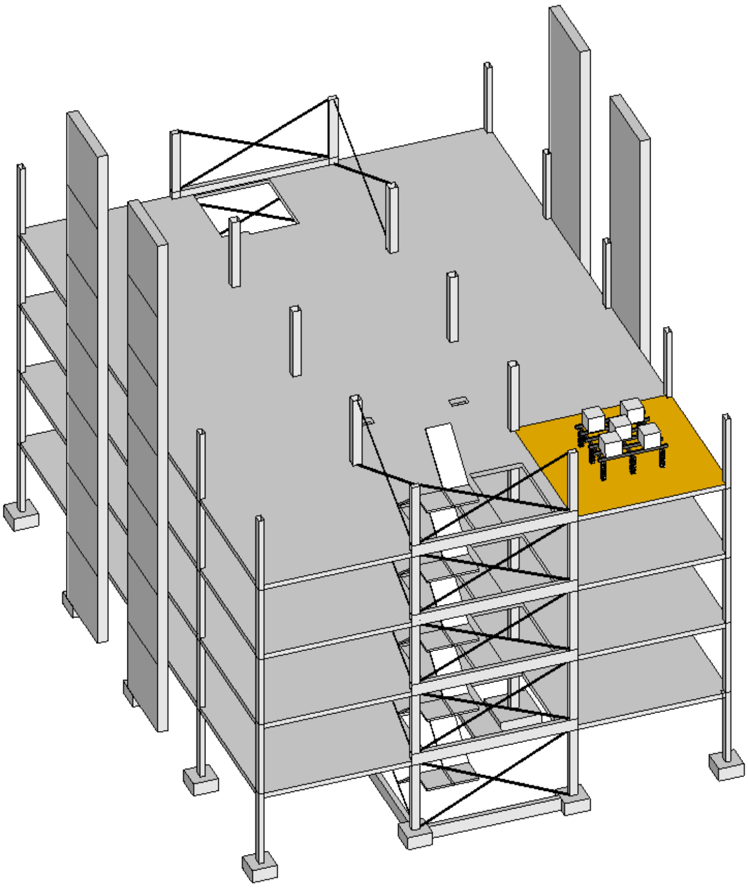

Figure 3.11. The location of the secondary system is shown in yellow.

Figure 3.12. The secondary system used in the experiments performed by J.T. Snæbjörnsson et al. in 2000 [2].

3.3 The vibration generator

The force action on the building is created by four large counter-rotating eccentric weight vibration generators. The generator has one mass that rotates clockwise and another mass that rotates counterclockwise. Because of this the generator can produce a unidirectional sinusoidal force. They are positioned at each corner of the roof (Figure 3.13). Their orientation and rotation frequency determines which mode shape of the building is excited. The system works within the frequency range from 0.3 to 20 Hz, with a resolution of 0.001 Hz.

The force applied varies with the rotational frequency of the eccentric weights. The force the two masses produce can be calculated with the following equation:

$$p(t) = (m_e\, e\, \omega^2) \sin \omega\, t$$

where m_e is the combined mass of the eccentric masses, e is the eccentricity, ω is the rotational excitation frequency and t is the time. The mass and the eccentricity do not change, i.e., the force varies with the excitation frequency. This means that for a response spectrum based on the force generated by the machine has to be normalized with respect to the force, because the force at a low excitation frequency is not the same at a high excitation frequency.

Figure 3.13. The vibration generator used by J.T. Snæbjörnsson et al. in 2000 [2].

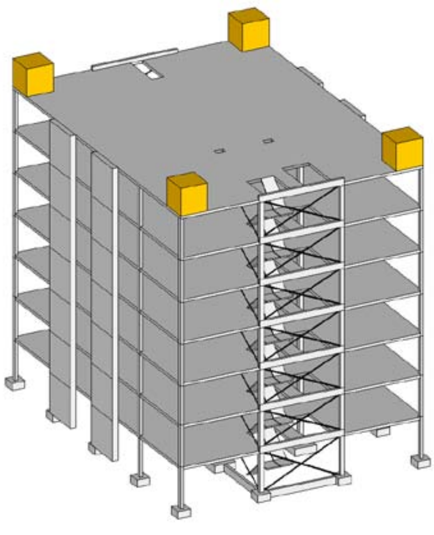

Figure 3.14. The location of the vibration generators are shown in yellow.

4. FEM Model

4.1 Ruaumoko

For the structural modeling a FE program called Ruaumoko [1] is used. The program has been written at the University of Canterbury under the supervision of Dr. Athol Carr. The program is specifically designed to analyze structures, both elastic and inelastic, that are subjected to earthquake and dynamic excitation.

Ruaumoko uses Newmark Constant Average Acceleration Method to calculate the results, which is explained in chapter 2.7.1. All analyses are elastic, performed with a lumped mass matrix, and uses “user specified modal damping”. This damping model gives the user the possibility to choose a different damping ratio for each mode shape.

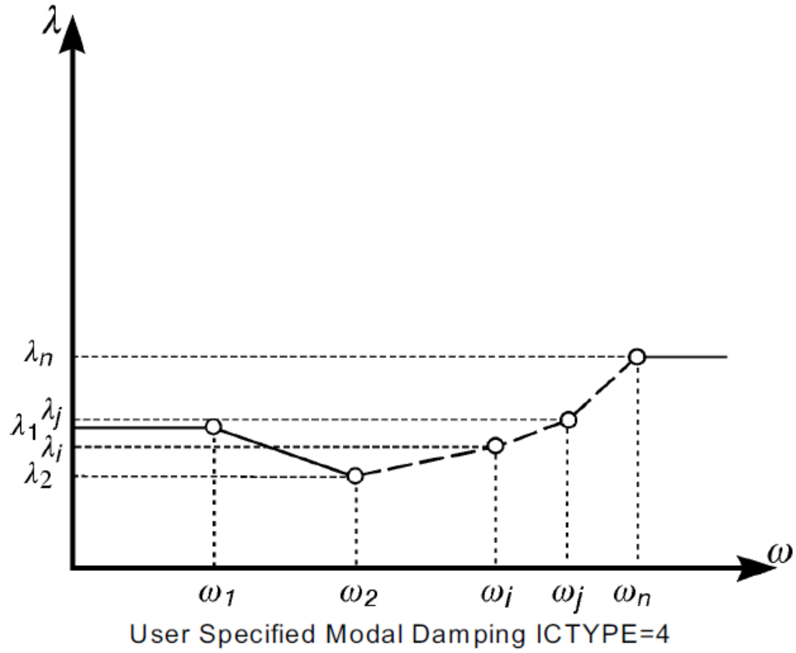

Figure 4.1. User Specified Modal Damping [11].

4.2 Primary system

4.2.1 Modeling of the primary system

The elements used in this analysis are the following:

- Columns: BEAM elements
- Floors: QUADRILATERAL finite elements
- Beams: BEAM elements
- Cross bracing: BEAM elements, with the ends pinned
- Brick panels: QUADRILATERAL finite elements

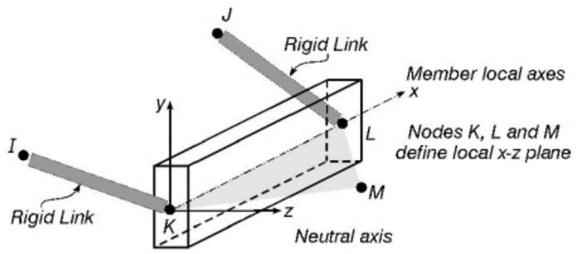

Figure 4.3. The BEAM element ([11], page 23).

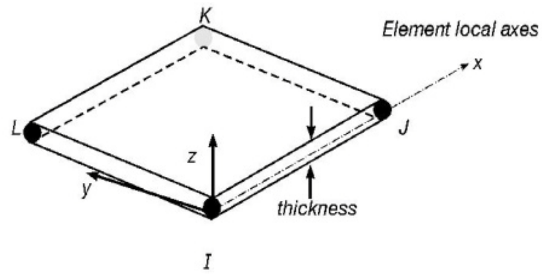

Figure 4.2. The QUADRILATERAL finite element ([11], page 83).

All beam elements are Giberson one-component elements, which are general quadratic beam elements. That means that every member can be expressed by a quadratic equation, e.g., $ax^2 + bx + c = 0$.

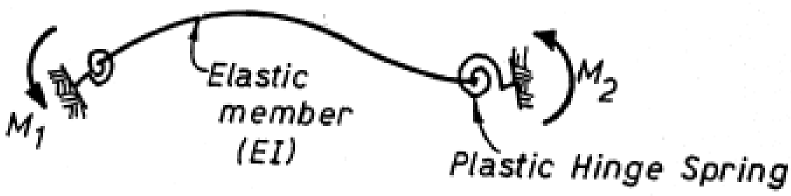

Figure 4.4. Giberson one-component elements ([11], page 29)

A total of 478 nodes and 577 elements were used. Base blocks were placed between the ground floor and the columns. These rotational and translational degrees of freedom at the bottom of the base blocks were then fixed.

A simplification has been made to make the modeling easier. The precast stairs have not been modeled with elements, but have instead been added as lumped mass in the respective nodes around the stair openings.

The service loads, i.e., the sandbags, were added as lumped masses at the neighboring nodes. In the 4x4 array, the mass of four bags was placed in the middle node, two bags at the edges and one bag in each corner.

The FE model of the primary system, with its simplifications, is shown in the figures below.

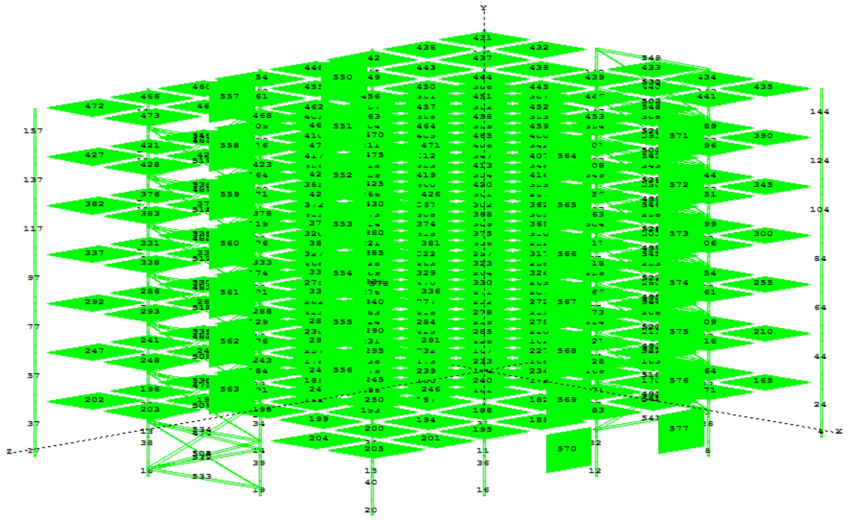

Figure 4.5. The FE model of the primary system.

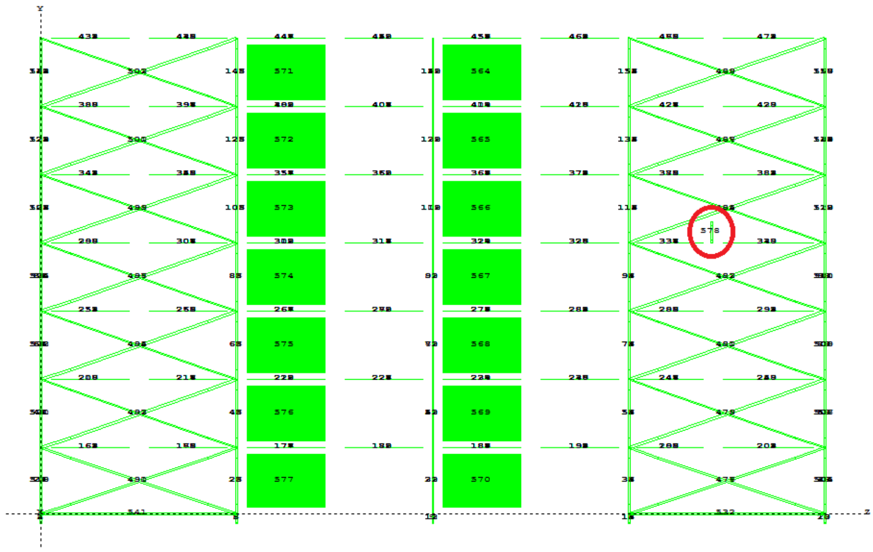

Figure 4.6. Side view of the FE model showing the location of the secondary system (east-west plane).

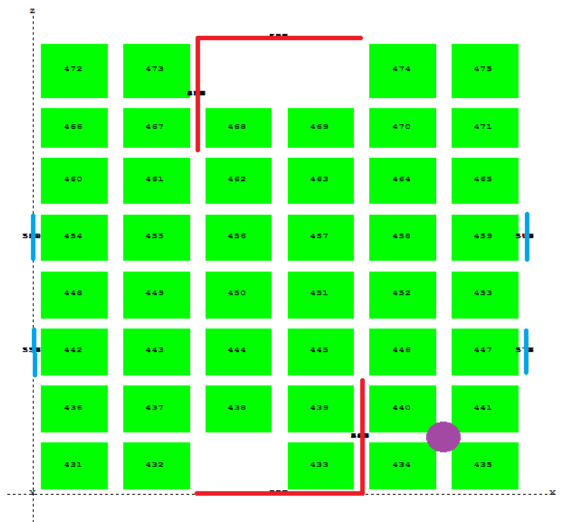

Figure 4.7. Plan view of the FE model showing the location of the floor elements (green), the brick panels (blue), the cross bracings (red) and the secondary system's location (purple dot).

4.1.1 Eigenfrequency analysis of the primary system

In order to get the eigenfrequency (natural frequency) of the primary system, an eigenfrequency analysis of the system is performed in Ruaumoko. This means no loading is applied, just the systems own weight. The eigenfrequency analysis gives the mode shapes and the natural frequencies. Each mode shape corresponds to a natural frequency. The 1st mode shape is in the x-direction, the 2nd in the z-direction and the 3rd is rotation in the x-z plane (around the y-axis). The 4th mode shape is again in the x-direction, the 5th in the z-direction and the 6th is rotation in the x-z plane. This pattern repeats itself throughout.

The normal mode this thesis is going to investigate is mode no. 5, which has a natural frequency of 1.67 Hz and a translational mode shape parallel to the short side of the building. The reason for this is that the secondary system was designed to have natural frequencies between 1 and 2 Hz and this frequency range was the focus of the full scale experiment from 2000 [2], from which the measured recordings are well documented.

The first test run by BRE in 1999 is shown in Table 4.1. This is an early test and some changes were made in the building after this testing. The main alteration is the addition of the brick panels. The adding of these bricks gives the structure some added mass without contributing too much to the stiffness. Because of this, the structure gets a longer period, which means a lower natural frequency ($\omega = \sqrt{k/m}$). This can be seen in the later experiment performed by J.T. Snæbjörnsson et al. in 2000 [2]. This experiment focused on the 4th and 5th mode shape and their measured natural frequencies can be found in Table 4.2. The natural frequency of mode 5 in this experiment was $1{,}6733\ Hz$. By comparing this frequency to the earlier experiment by BRE [12] where the natural frequency was $1{,}7760\ Hz$ for the same mode shape, it indicates that the brick panels, in addition to some minor adjustments made to the building, lowered the natural frequency by $0{,}1027\ Hz$.

Table 4.1. BRE test - Natural Frequencies and CDR of the primary system [12].

Mode	Direction of vibration	Natural Frequency (Hz)	Critical Damping ratio (%)
1	x - North-South	0.567	0.86
2	z - East-West	0.604	0.74
3	Rotation	0.752	0.86
4	x - North-South	1.652	1.50
5	z - East-West	1.776	1.40
6	Rotation	2.162	1.08

Table 4.2. Snæbjörnsson et al. - Natural Frequencies and CDR of the primary system ([2], page 10).

Date	System no.	North-South vibration		East-West vibration	
		NF (Hz)	CDR (%)	NF (Hz)	CDR (%)
15.09.2003	1			1.6675	0.95
17.09.2003	2			1.6613	0.10
18.09.2003	3			1.6733	0.65
19.09.2003	5	1.5232	1.24		
20.09.2003	6			1.6776	0.92
20.09.2003	7			1.6900	0.77
20.09.2003	8			1.6703	0.76
21.09.2003	9	1.5182	0.91	1.6773	0.73
	Median	1.5207	0.01075	1.6733	0.0076
	Stand. dev.			0.0091	0.0028

The natural frequencies generated by the FE model are shown in Table 4.3. The model has been calibrated so that the 4th and the 5th natural frequency correspond to the natural frequencies experimentally measured by J.T. Snæbjörnsson et al. [2] (see Table 4.2). The natural frequency values of other modes also correspond reasonable well to the values measured by the BRE (see Table 4.1), considering the physical changes of the building.

Table 4.3. Natural Frequencies for the primary system generated by the FE model.

Mode	Direction of vibration	Natural Frequency (Hz)	Period (sec)
1	x - North-South	0.5314	1.882
2	z - East-West	0.5734	1.744
3	Rotation	0.7105	1.407
4	x - North-South	1.5350	6.514
5	z - East-West	1.6750	5.969
6	Rotation	2.0460	4.887

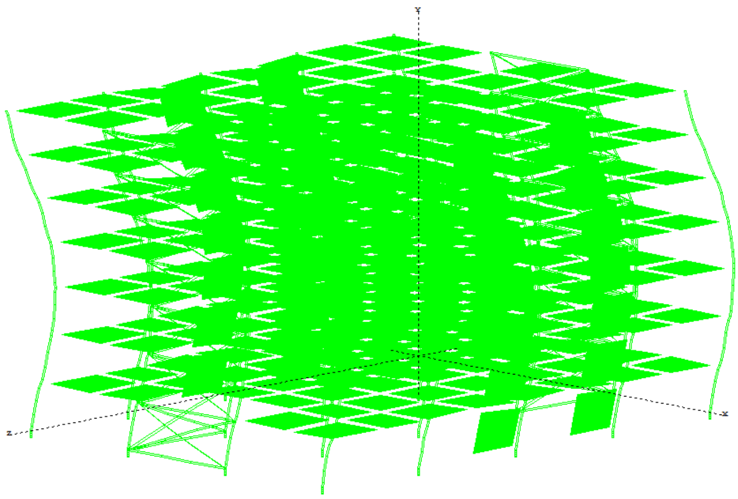

Figure 4.8. The fifth mode shape (east-west direction).

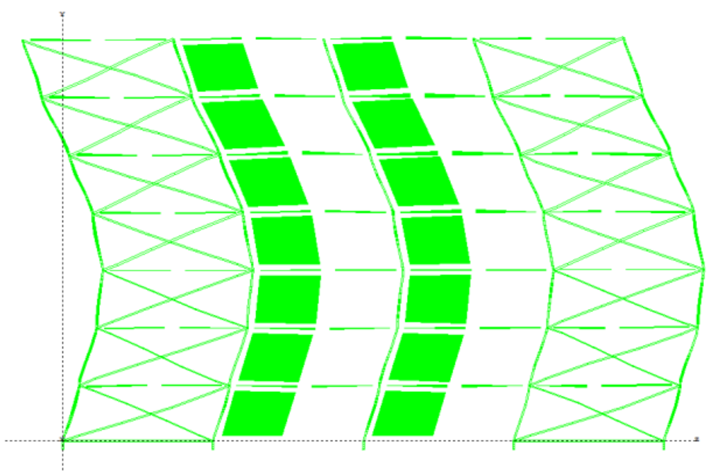

Figure 4.9. The fifth mode shape (east-west plane).

4.2 Secondary system

4.2.1 Modeling

The secondary system is modeled as a single DOF system. The system consists of a beam element with a lumped mass in the upper node. The beam element is modeled as massless (zero density); hence it does not contribute to the mass of the system.

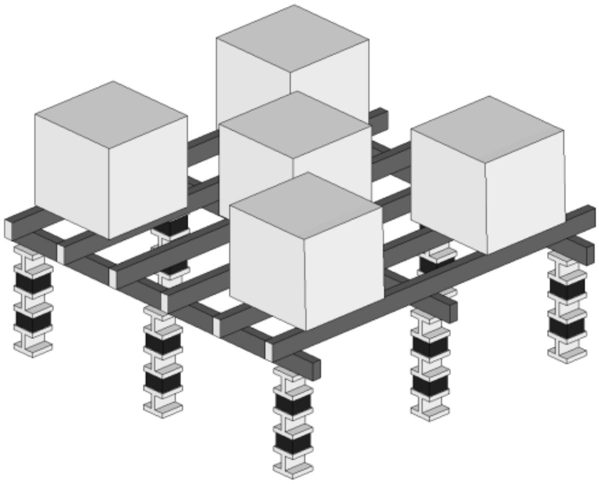

Figure 4.11. The secondary system.

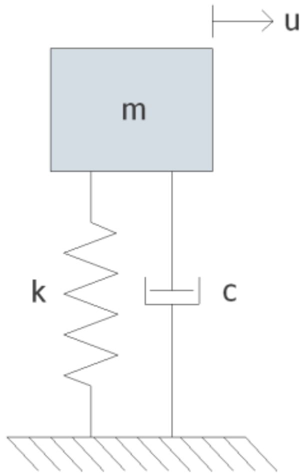

Figure 4.10. Simple model of the SS.

4.3.2 Eigenfrequency analysis

Six different secondary systems are considered (see Table 4.4). Each system has a different natural frequency. The systems of interest are the ones that have been excited by an east-west vibration, i.e., number 1, 2, 3, 6, 7 and 8/9 using the reference numbers from Table 4.4. The stiffness for each system varies with the orientation of the columns, thus the stiffness corresponding to each secondary system has been calculated by knowing that $k = \omega^2 m$ (**2.12**) and $k = \frac{12\,E\,I}{L^3}$ (**2.2**).

Table 4.4. Snæbjörnsson et al. [2] – The secondary systems natural frequencies and CDR.

Date	System no.	North-South vibration		East-West vibration		No of sandbags on table
		NF (Hz)	CDR (%)	NF (Hz)	CDR (%)	
14.09.2003	1	2.4536	1.64	1.2024	0.42	1
16.09.2003	2			1.8092	0.66	1
18.09.2003	3			1.3513	0.07	2
19.09.2003	5	1.4315	1.70			5
20.09.2003	6			1.6684	0.08	3
20.09.2003	7			1.6999	5.77	4
21.09.2003	8 / 9	1.5053	2.41	1.9577	2.28	5

4.3 Decay test to check the damping ratio

In order to check that the damping ratios inserted into the FEM model is correct, one has to perform a damping ratio test. There exists several different methods of identifying the damping ratio, but the most common ones are the Resonant Amplification method ([3], page 53) and the Free-Vibration Decay method ([5], page 35), which are explained in chapter 2.6.1. The easiest one to use when you have the availability of a FEM model is the Free-Vibration Decay method. A decay test means that the building is excited for a while, then the excitation stops and the free vibration response is analyzed.

To perform the decay test, the harmonic excitation has been stopped after 16 seconds and the free vibration response has been analyzed. The Cardington building has been excited by frequencies equal to the natural frequency of the primary system and the secondary systems. By doing so, one gets the damping ratio of the primary system and the secondary systems, because when a structure is excited with its natural frequency, it is mainly that specific structure that moves and everything else stays almost at rest.

The decay test is performed by following the method explained in chapter 2.6.1 (The Free-Vibration Decay Method). An example of the use of this method is shown in Figure 4.12. The Cardington building is excited, with SS1 in place, at the natural frequency of the secondary system (1.20 Hz), which gives (see figure below):

$x_i = 1.073 \times 10^{-2}$ (Peak amplitude at ≈ 20 sec)
$x_j = 0.801 \times 10^{-2}$ (Peak amplitude at ≈ 29 sec)
$N = 11$ (Number of peak amplitudes between the two amplitudes)

To calculate the damping ratio, equation **2.28** is applied:

$$\xi = \frac{1}{2\,\pi} \ln\left(\frac{x_i}{x_j}\right)\frac{1}{N} = \frac{1}{2\,\pi} \ln\left(\frac{1.073 \times 10^{-2}}{0.801 \times 10^{-2}}\right)\frac{1}{11} = 4.2299 \times 10^{-3}$$

The decay test gives a damping ratio of 0.423 % for secondary system 1, which is approximately the input damping ratio (0.42 %).

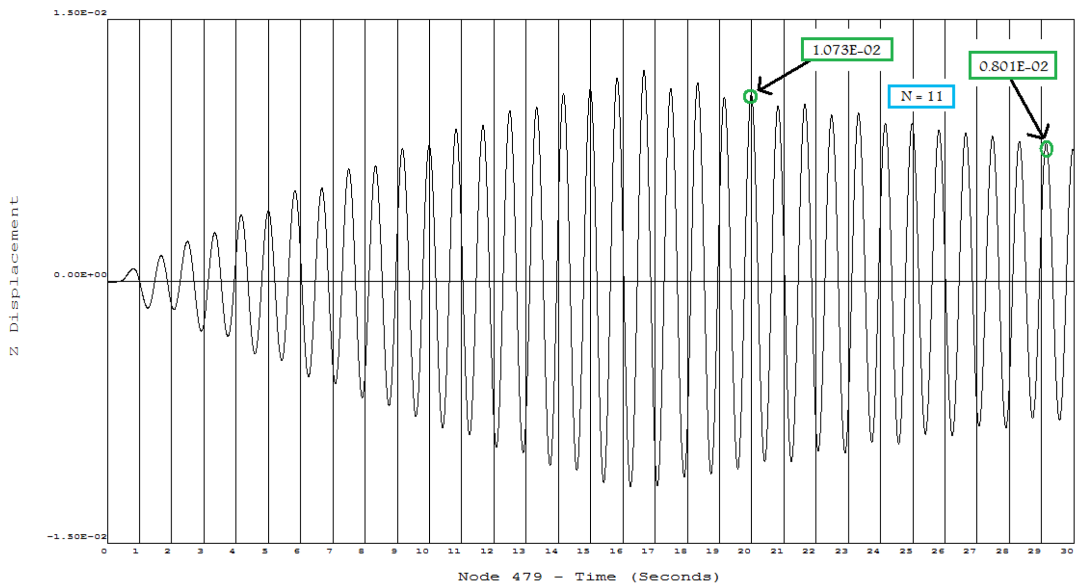

Figure 4.12. Free-vibration decay of SS1 excitated at 1.20 Hz.

4.4 Comparison of calculated results with previously measured results

In the dynamic full scale testing of the interaction between secondary and primary systems undertaken by J.T. Snæbjörnsson et al. [2] in 2000, nine different secondary systems were tested using forced vibration techniques where a vibration generator was used to excite the building. The harmonic forcing generated was dependent on the rotational frequency of the rotating masses, i.e. the excitation frequency. It is there for not possible to produce a harmonic excitation of constant forcing as has been used in the FE-model runs.

Figure 4.13 and 4.14 show the comparison between the full scale test results and the FE-model results for secondary system 3 and secondary system 2 respectively. The frequency response function for the floor, which the secondary system is standing on, is also shown. As can be noted the displacement frequency response is normalized with the forcing to make the test curves and the model curves comparable.

Although there are clear differences, the overall behaviour is similar. The calculated curves are not as spiky as the experimentally measured curves at the natural frequencies. This may be due to several reasons.

Firstly the damping ratios of the experimentally measured results may in reality be smaller than the ones used in the FE-modelling. One thing that may contribute to such discrepancy may for instance be the notable variations in the damping ratio of primary structure during the period of testing as can be seen in Table 4.2. These variations in both damping and natural frequency are likely caused by a change in the ambient temperature. Similar variations in the damping of the secondary structure are to be expected within the duration of the test and for instance slight variations in the characteristics of the rubber elements and their pre-stressing may have notable influence.

Secondly, it may influence this comparison, especially the peak comparison, that the resolution in frequency in the FE-model study is constant. That is, the frequency of the forcing is varied in steps of constant frequency difference. In the full-scale testing on the other hand the frequency resolution is increased close to resonance and much smaller frequency steps are used in that frequency range than for the excitation frequencies outside the resonance peak.

Thirdly, since there is some time since the full-scale test was undertaken the scaling of the forced vibration test curves may be lacking some details.

However, in spite of the differences noted, the calculated results are overall acceptably close to the measured results and it can be concluded that the FE-model is representative for the Cardington Building and the secondary systems tested.

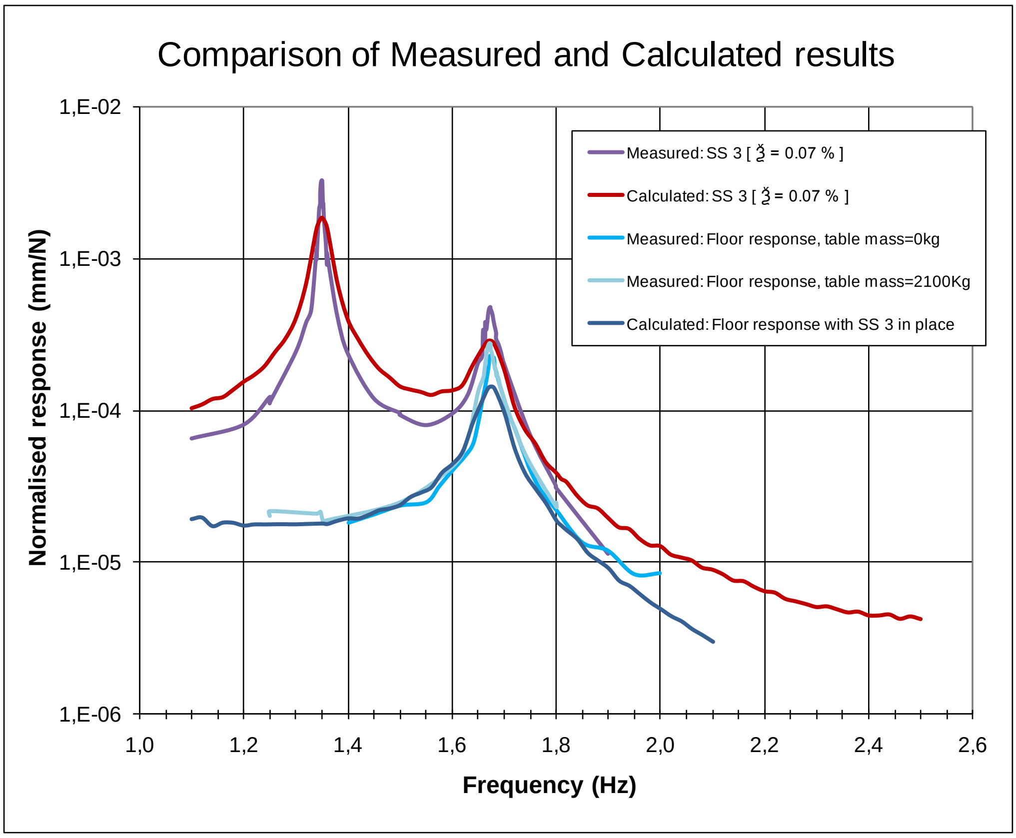

Figure 4.13. Comparison of experimentally measured results [2] and calculated results from Ruaumoko for secondary system 3. The x-axis is excitation frequency and the y-axis is the displacement divided by the excitation force.

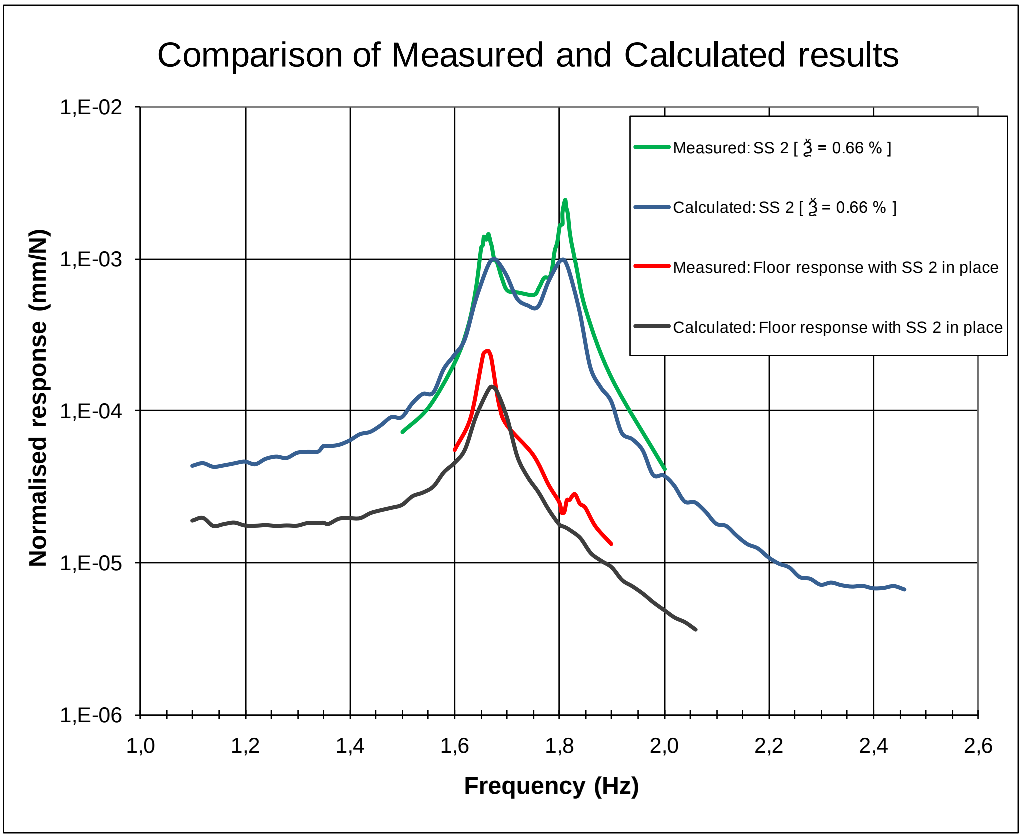

Figure 4.14. Comparison of experimentally measured results [2] and calculated results from Ruaumoko for secondary system 2. The x-axis is excitation frequency and the y-axis is the displacement divided by the excitation force.

5. Response Analysis

In this analysis there will be performed both harmonic excitation tests and earthquake excitation tests.

For the harmonic excitation the first test is an analysis of the primary structure without the secondary structure attached. The dynamic response functions for the various floors are generated.

The second harmonic test includes the secondary structure. The damping ratios for the secondary structures during the harmonic tests are constant. The secondary structures have damping ratios of 0.8 % and 5.0 %, see Table 5.1. The response of the secondary structure is analyzed as well as the response of the floor at the location of the secondary structure. In addition to the analysis of the secondary structure and the floor at the SS location, a ratio of response between the secondary structures and the primary structure is shown. This ratio is interesting because it illustrates the amplification factors for the different secondary structures.

The seismic coefficients and the expected accelerations of the primary structure are the first things that are looked at during the earthquake excitation. The next chapter is a look at the floor response at the location of the secondary structure, as was done for the harmonic excitation.

The third chapter discusses the response of the secondary structure in detail. The amplifications of the total accelerations from the ground up to the fourth floor and up to the secondary structure are shown. The comparison between the responses from the floor response spectra method and when the secondary structure is analyzed jointly with the primary structure is made. The fourth chapter investigates the effect of the damping ratio for the secondary systems. In this chapter, two imaginary secondary systems (iSS1 and iSS2) have been made in order to have more data to verify the different trends. The damping ratios investigated are shown in Table 5.1.

Table 5.1. The damping ratios and natural frequencies for the primary structures and the secondary structures used in the harmonic excitation tests and the earthquake excitation tests.

Primary Structure - Damping ratio and Natural frequency							
		Harmonic Excitation		Earthquake Excitation			
Mode	Nat. Freq. (Hz)	CDR (%)		CDR (%)			
5	1.6750	0.80		0.80			
Secondary Structure - Damping ratios and Natural frequencies							
		Harmonic Excitation		Earthquake Excitation			
System	Natural Frequency (Hz)	CDR1 (%)	CDR2 (%)	CDR1 (%) [2]	CDR2 (%)	CDR3 (%)	CDR4 (%)
SS1	1.2024	0.80	5.00	0.42	0.08	2.00	5.00
SS2	1.8092	0.80	5.00	0.66	0.08	2.00	5.00
SS3	1.3513	0.80	5.00	0.07	0.08	2.00	5.00
SS6	1.6684	0.80	5.00	0.08	0.08	2.00	5.00
SS7	1.6999	0.80	5.00	5.77	0.08	2.00	5.00
SS8/9	1.9577	0.80	5.00	2.28	0.08	2.00	5.00
iSS1	1.5630	-	-	-	0.08	2.00	5.00
iSS2	2.2250	-	-	-	-	2.28	-

5.1 Harmonic Excitation

The harmonic loads on the Cardington building is four external sinusoidal forces acting in the east-west direction. The forces have a maximum amplitude of 4 kN and are applied at the four corners of the roof. The structure is excited for 30 seconds and the excitation frequency range is from 1.1 Hz to 2.5 Hz. This frequency range includes mode shape 5 of the primary structure and all of the secondary structures mode shapes. To create the response functions needed, the maximum displacement in the steady-state response (15 to 30 sec) is plotted for every frequency.

This thesis will be comparing the six secondary systems stated in Table 4.4, but with constant damping ratios. The damping ratios of the secondary systems are 0.8 % in the first test and 5 % in the second test. The damping ratio for the primary system is 0.8 % in both tests.

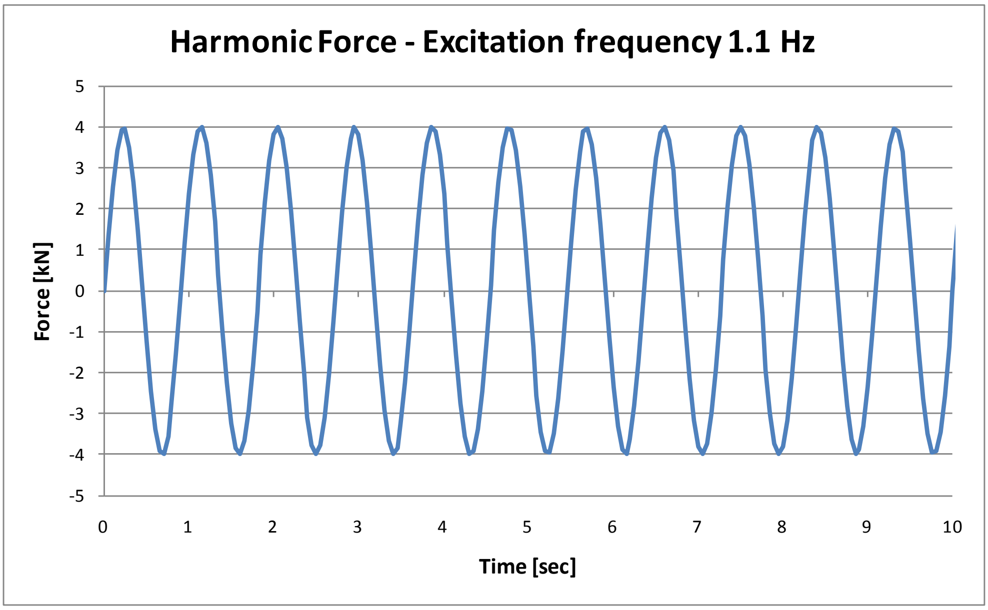

Figure 5.1. The harmonic excitation frequency, 1.10 Hz, from 0-10 seconds.

5.1.1 Primary structure without the secondary structures

The primary structure without the secondary system attached is only excited in the east-west direction; hence the fifth mode shape will be dominant. The maximum displacements in all the corner-columns have been measured, as well as the floor displacement at the location of the secondary system. The average (maximum) displacement in the corner-columns in each floor has been calculated and it is confirmed that the structure moves like the fifth mode shape, see Figure 5.2 and Figure 5.3.

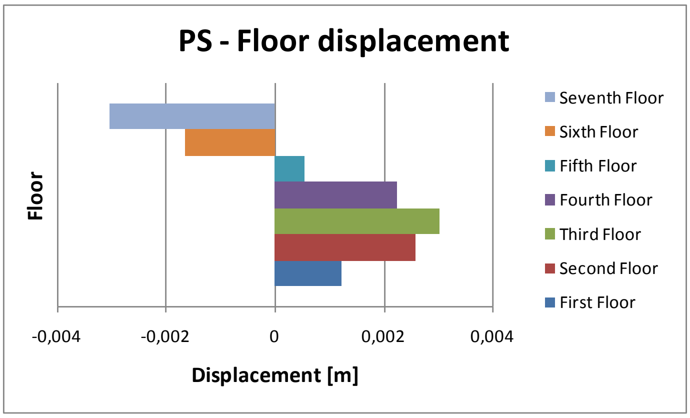

Figure 5.2. Average floor displacement of the primary system (Measured at the four corner-columns of each floor).

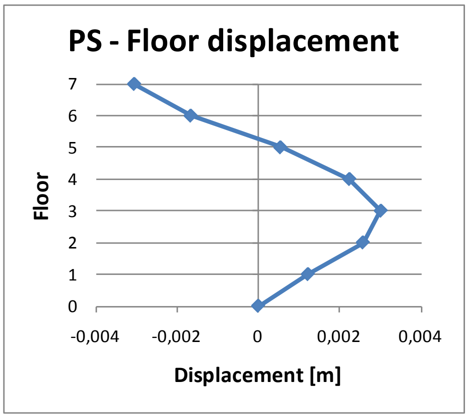

Figure 5.3. Average floor displacement (Measured at the four corner-columns of each floor).

The fifth mode shape was defined by a static analysis in chapter 4.2.2 and is modeled below.

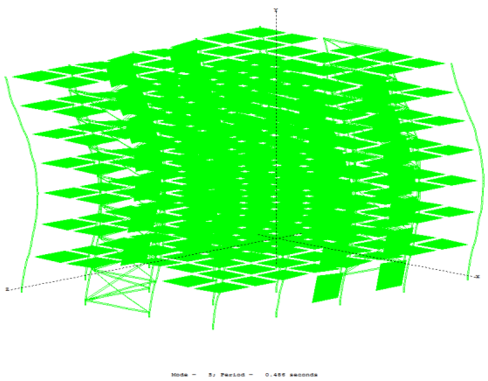

Figure 5.4. The 5th mode shape.

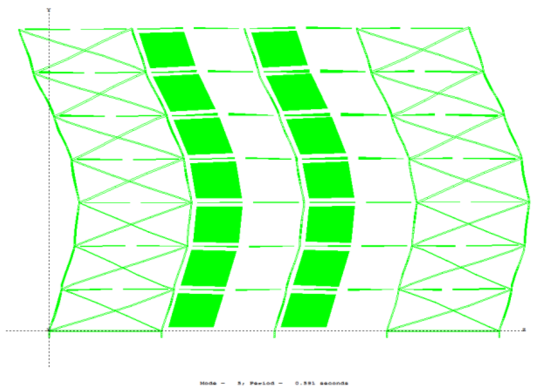

Figure 5.5. The 5th mode shape (east-west plane).

The primary systems maximum displacement during each excitation frequency is calculated. The frequency excitation starts at 1.10 Hz and goes up to 2.50 Hz by a step size of 0.02 Hz. The response function of the primary structure at the location of the secondary system is shown in Figure 5.7 and the response of each floor is shown in Figure 5.6.

From Figure 5.6 it is seen that the fifth floor has the lowest displacement at the natural frequency, and the seventh and the fourth floor have the largest displacement at the natural frequency. This is consistent with the fifth mode shape. The response function shows only mode shape five because the neighboring two east-west modes have natural frequencies equal to 0.57 Hz and 2.82 Hz. However, there is a rotational mode shape at 2.05 Hz (and one at 0.71 Hz), but it does not seem to affect the response of the primary system.

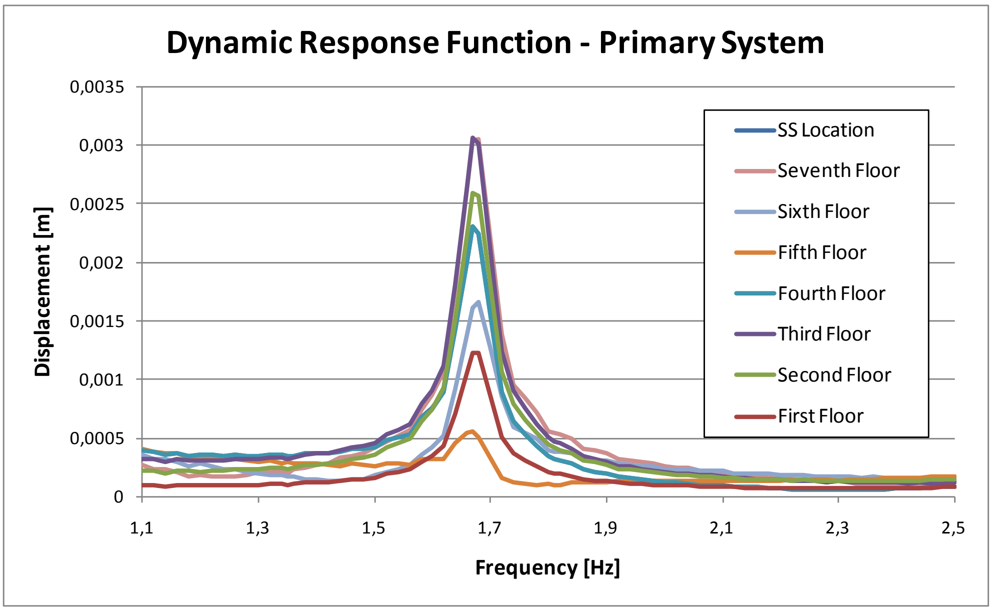

Figure 5.6. Primary System: All floors: Dynamic response functions for displacement. 0.8 % Damping. (Measured at the four corner-columns of each floor).

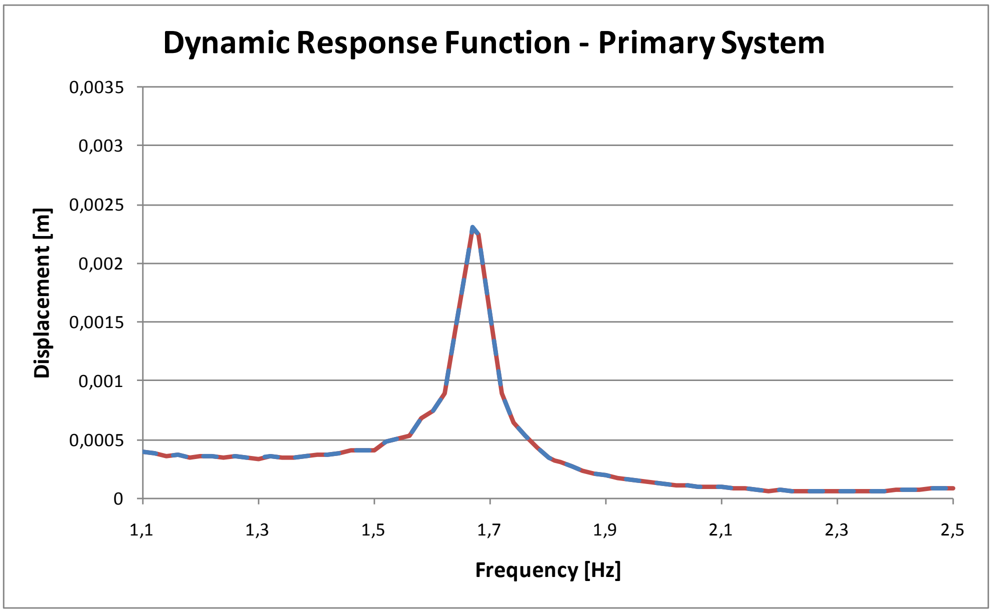

Figure 5.7. SS location: Dynamic response function for displacement. The red curve is the displacement of the fourth floor. The dashed blue curve is the displacement of the node at the location of the SS.

The response function of the node at the location of the secondary system is shown together with the average value of the fourth floor is shown in Figure 5.7. It is seen that the difference is minimal, i.e. the floor can be considered rigid.

5.1.2 Primary structure with the secondary structures

The secondary structure is now put in place, and the harmonic excitation is the same as for the primary system. All six secondary systems that have measured results (1, 2, 3, 6, 7 and 8/9), taken from Table 4.4, are tested. The tests have been performed with constant damping ratios for the secondary systems. In the first test the secondary systems have 0.8 % damping and in the second test they have 5.0 % damping. This has been done because when the systems all have the same damping ratio; it is easier to compare them to each other. Additional secondary systems have been made when needed.

Table 5.2. Damping ratio and natural frequency of the primary system in both tests.

Primary Structure			
Mode	Direction of vibration	Natural Frequency (Hz)	Critical Damping Ratio (%)
5	z - East-West	1.6750	0.80

Table 5.3. Damping ratio and natural frequency of the secondary systems in test 1.

Secondary Structure - Test 1			
System	Direction of vibration	Natural Frequency (Hz)	Critical Damping Ratio (%)
1	z - East-West	1.2024	0.8
2	z - East-West	1.8092	0.8
3	z - East-West	1.3513	0.8
6	z - East-West	1.6684	0.8
7	z - East-West	1.6999	0.8
8 / 9	z - East-West	1.9577	0.8

Table 5.4. Damping ratio and natural frequency of the secondary systems in test 2.

Secondary Structure - Test 2			
System	Direction of vibration	Natural Frequency (Hz)	Critical Damping Ratio (%)
1	z - East-West	1.2024	5.0
2	z - East-West	1.8092	5.0
3	z - East-West	1.3513	5.0
6	z - East-West	1.6684	5.0
7	z - East-West	1.6999	5.0
8 / 9	z - East-West	1.9577	5.0

5.1.2.1 Damping ratio equal to 0.8 %

Response function for the secondary systems

The first analysis is performed with a damping ratio of 0.8 % for the secondary systems. The primary system always has a damping ratio of 0.8 %.

Figure 5.8 shows the displacement response function for the secondary systems, along with the response function for the location of the SS. “PS alone” means that the PS is excited by itself without the SS in place and the floor-node at the location of the SS is measured. “SS 1” means that SS 1 is mounted on the PS and the response of SS 1 is measured.

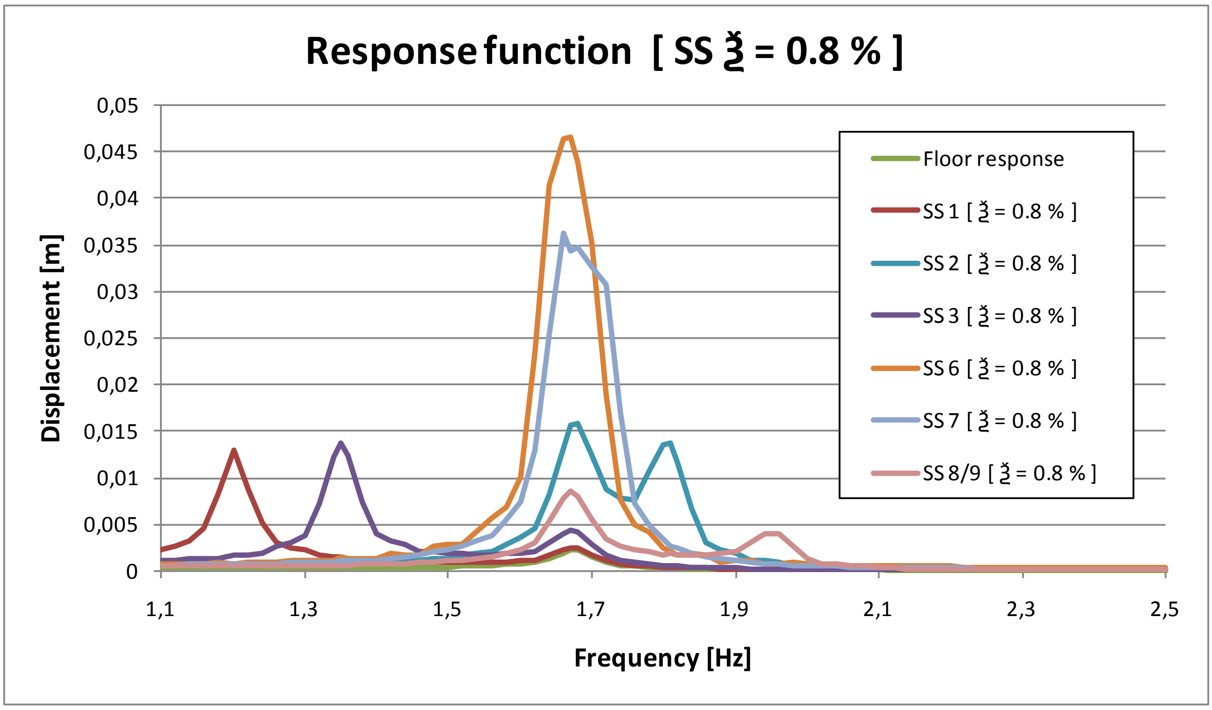

Figure 5.8. Displacement response function for secondary systems with critical damping ratio equal to 0.8 %.

From the response function it is seen that the displacement of the secondary system is largest when the natural frequency of the SS and the PS are the same. The peak response of the secondary systems happens at the natural frequency of either the SS or the PS. For SS no. 2, 6, 7 and 8/9 the maximum response occurs at the natural frequency of the PS. For SS no. 1 and 3 the maximum response occurs at the natural frequency of the SS. The displacement gets larger when the natural frequency of the secondary system approaches the natural frequency of the primary system.

The large magnification of response seen for systems no. 6 and 7 is also partly because the secondary system becomes a *Dynamic Vibration Absorber*, a phenomenon discussed in chapter 2.6.4. The short version is that the secondary system with its natural frequency equal to the natural frequency of the primary system will absorb the response of the primary system, making the response of the primary system less, see Figure 5.9, but the response of the secondary system larger.

Displacement of the floor at the location of the secondary system

The mass of the secondary systems are small compared to the overall weight of the primary system. The heaviest system, SS 8/9 with 5 sandbags, has a mass of approximately 5.500E+03 kg. The modal mass of mode shape 5 for the primary system is 2.039E+06 kg, which makes the mass of the secondary system equal to 0.27 % of the mass of the primary system. The total mass of primary system is 3.89E+06 kg, which means that the modal mass of mode 5 is about half the total mass. The modal mass gets smaller for every mode, so the modal mass is as expected. Figure 5.9 shows the displacement response function of the floor at the location of the secondary systems with 0.8 % damping.

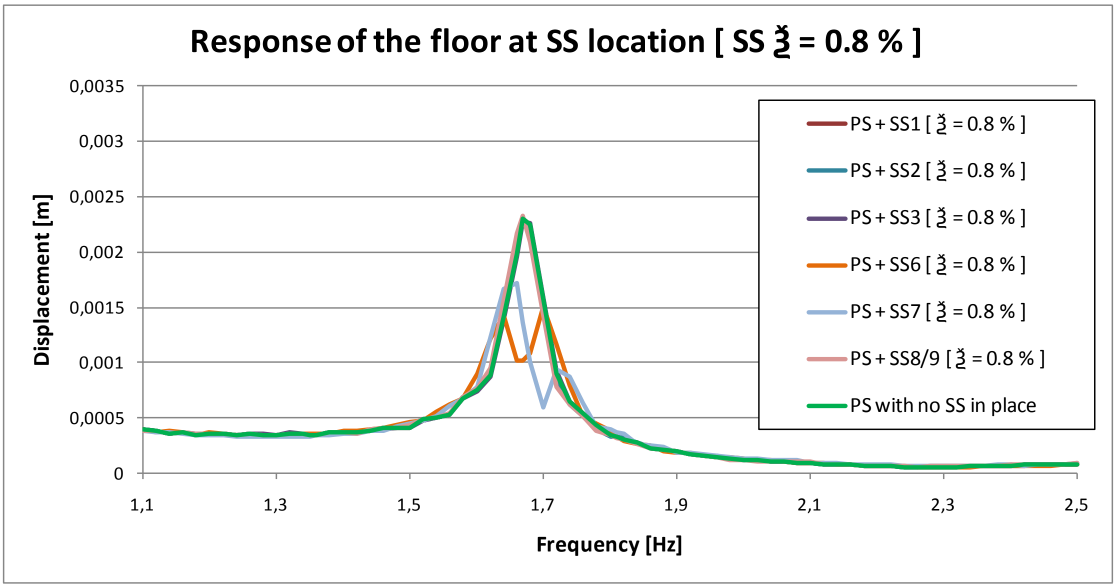

Figure 5.9. Displacement response function for the node where the secondary system is located (the floor) for 0.8 % damping.

The displacements at the location of the secondary systems are more or less identical to the displacement caused by the primary system alone, except for SS 6 and 7. One would think that the mass of secondary system 6 and 7, 0.16 % and 0.22 % of PS modal mass, is too small to have any influence on the response of the primary system. Secondary system 8/9 has more mass than system 6 and 7, but it does not affect the motion of the primary system. The difference between SS 8/9 and SS 6 and 7 is that the latter two has their natural frequency close to the natural frequency of the primary system.

Table 5.5. Modal mass ratio for secondary systems.

Modal mass ratio for secondary systems			
Primary systems modal mass for mode shape 5 = 2.039E+06 kg			
System	Direction of vibration	Natural Frequency (Hz)	Modal mass ratio [%]
1	z - East-West	1.2024	0.054
2	z - East-West	1.8092	0.054
3	z - East-West	1.3513	0.108
6	z - East-West	1.6684	0.162
7	z - East-West	1.6999	0.216
8 / 9	z - East-West	1.9577	0.270

From Figure 5.10 it is seen that secondary system 6, which has the largest displacement, acts as a vibration absorber (Chapter 2.6.4). Because the natural frequency of the secondary systems is equal to the natural frequency of the primary system, it acts as an absorber. This gives two peak values, i.e., the system now has two different natural frequencies. The phenomenon has been explained in chapter 2.6.4. A vibration absorber is a good thing for the primary system, but for the secondary system, in this test, it gives a larger displacement than all the other secondary systems.

The curves in Figure 5.10 represent the displacement of the node at the location of the secondary system, i.e., the fourth floor. The effect from the secondary system can be seen throughout the floors. Figure 5.11 shows the response function for the top floor. The displacement of the primary system drops by 34 % at the fourth floor and by 35 % at the top floor.

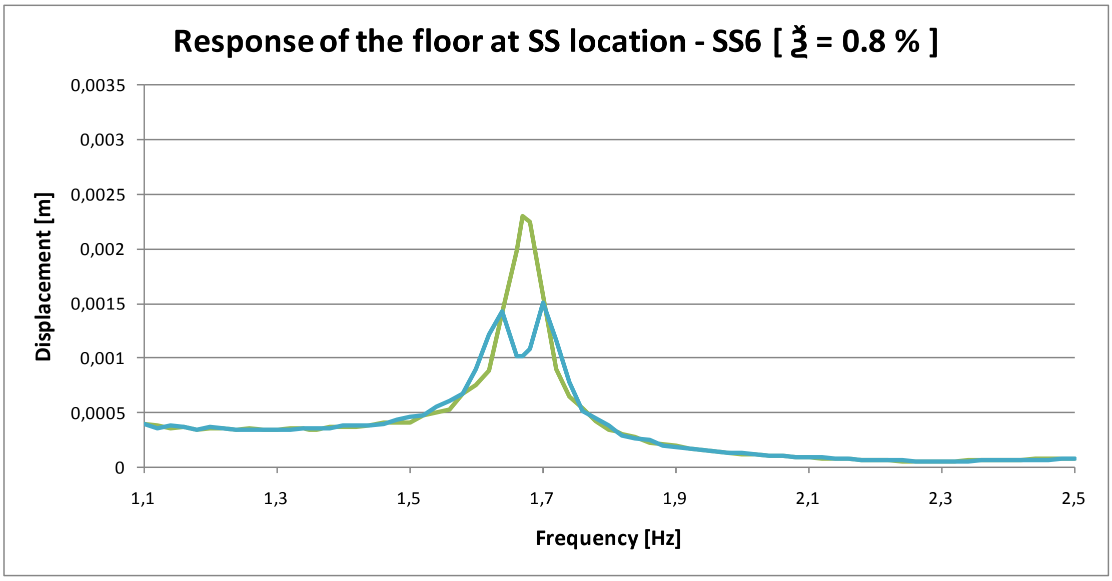

Figure 5.10 Displacement response function. The green curve represents the floor response at the location of the SS for the primary system without the SS. The blue curve represents the floor response of the primary system with SS6 in place.

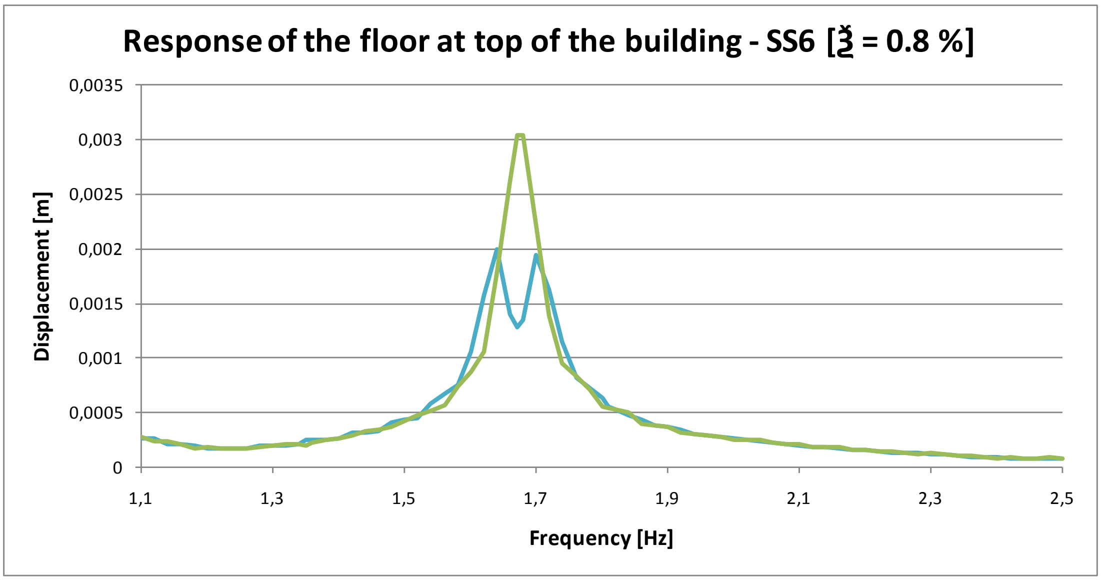

Figure 5.11. Displacement response function at top floor location. The green curve represents the top floor response of the primary system without the SS. The blue curve represents the top floor response of the primary system with SS6 in place.

The same thing can be seen on the response function created by secondary system 7. The natural frequency for secondary system 7 is slightly higher (1.69 Hz) than the natural frequency of the primary system (1.67 Hz), hence the curve has a “V shape” that is slightly off center.

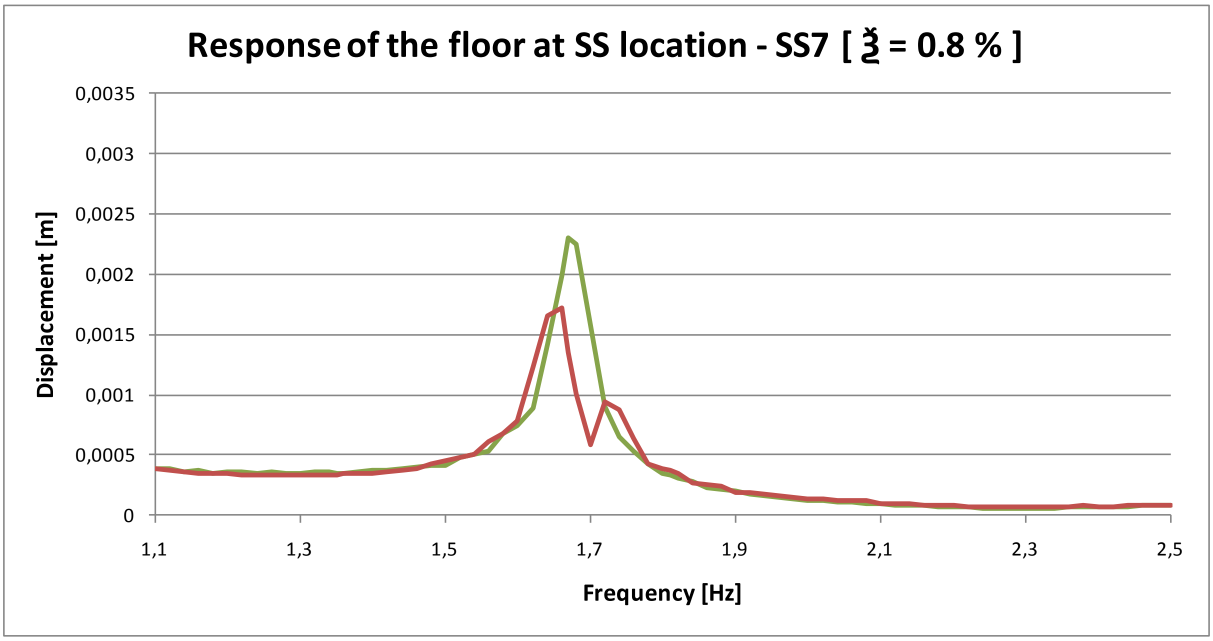

Figure 5.12. Displacement response function. The green curve represents the floor response at the location of the SS for the primary system without the SS. The red curve represents the top floor response of the primary system with SS7 in place.

5.1.2.2 Damping ratio equal to 5.0 %, with comparison to 0.8 % damping ratio

Response function for the secondary systems

The second analysis is performed with a damping ratio of 5.0 % for the secondary system. The primary system still has a damping ratio of 0.8 %. The displacement response function is shown below.

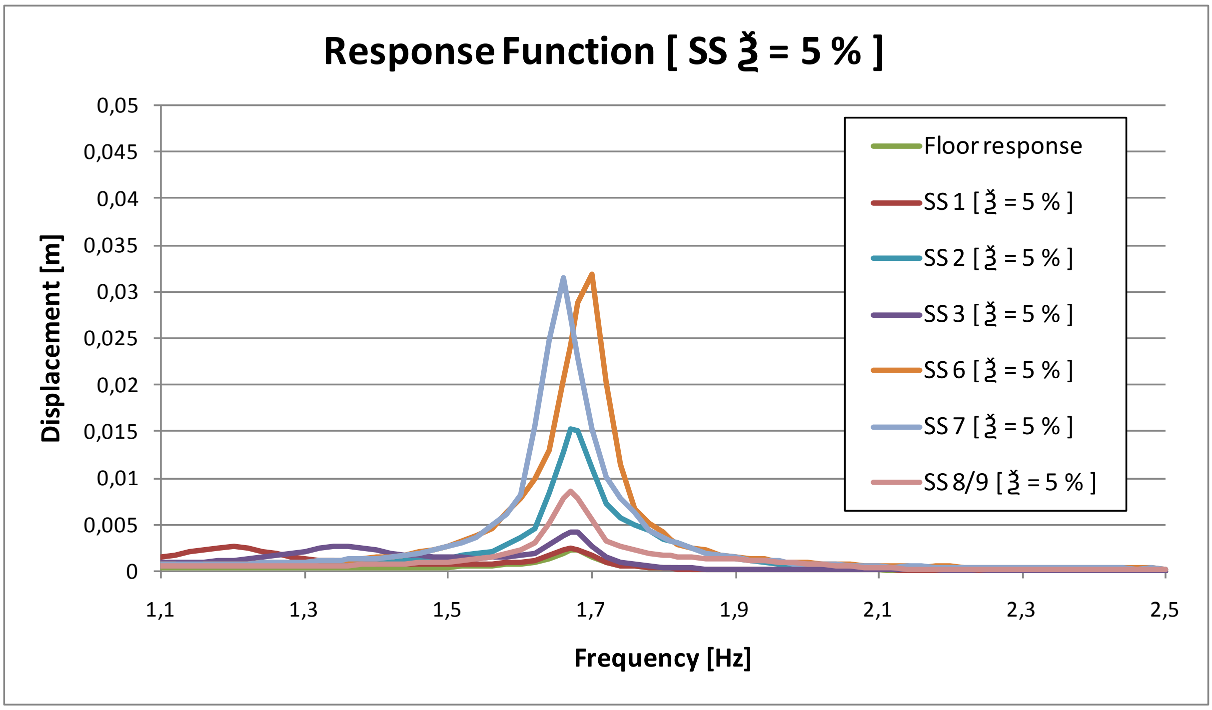

Figure 5.13. Displacement response function for secondary systems with 5.0 % damping.

The secondary systems all have a damping ratio of 5.0 %. This makes the response at the natural frequencies much less visible. For system 2 and 8/9 there are no visible peak response, but there is a change in the decent of the curve. How big this change is, is shown later on in chapter 5.1.3. The reason why secondary systems 6 and 7 have not got their peak values at their natural frequencies or at the natural frequency of the primary system is because of a phenomenon called *dynamic vibration absorber*, which has been mentioned several times earlier (chapter 2.6.4).

The differences made by the damping ratios are shown in the figures on the next page.

The differences between the peak responses are considerably big at the natural frequency of the secondary system, which is expected. A sharp, thin curve means that the damping ratio is small, and a flat, wide curve means that the damping ratio is big. This observation can be used to estimate the damping ratio of systems by applying the *Half-power bandwidth method* explained in chapter 2.6.1.1.

The peak response at the natural frequency of the primary system is more or less the same for the two damping ratios for secondary system 1, 2, 3 and 8/9, but for the secondary systems that have natural frequencies close to the natural frequency of the primary structure (SS 6 and 7), the response is not the same.

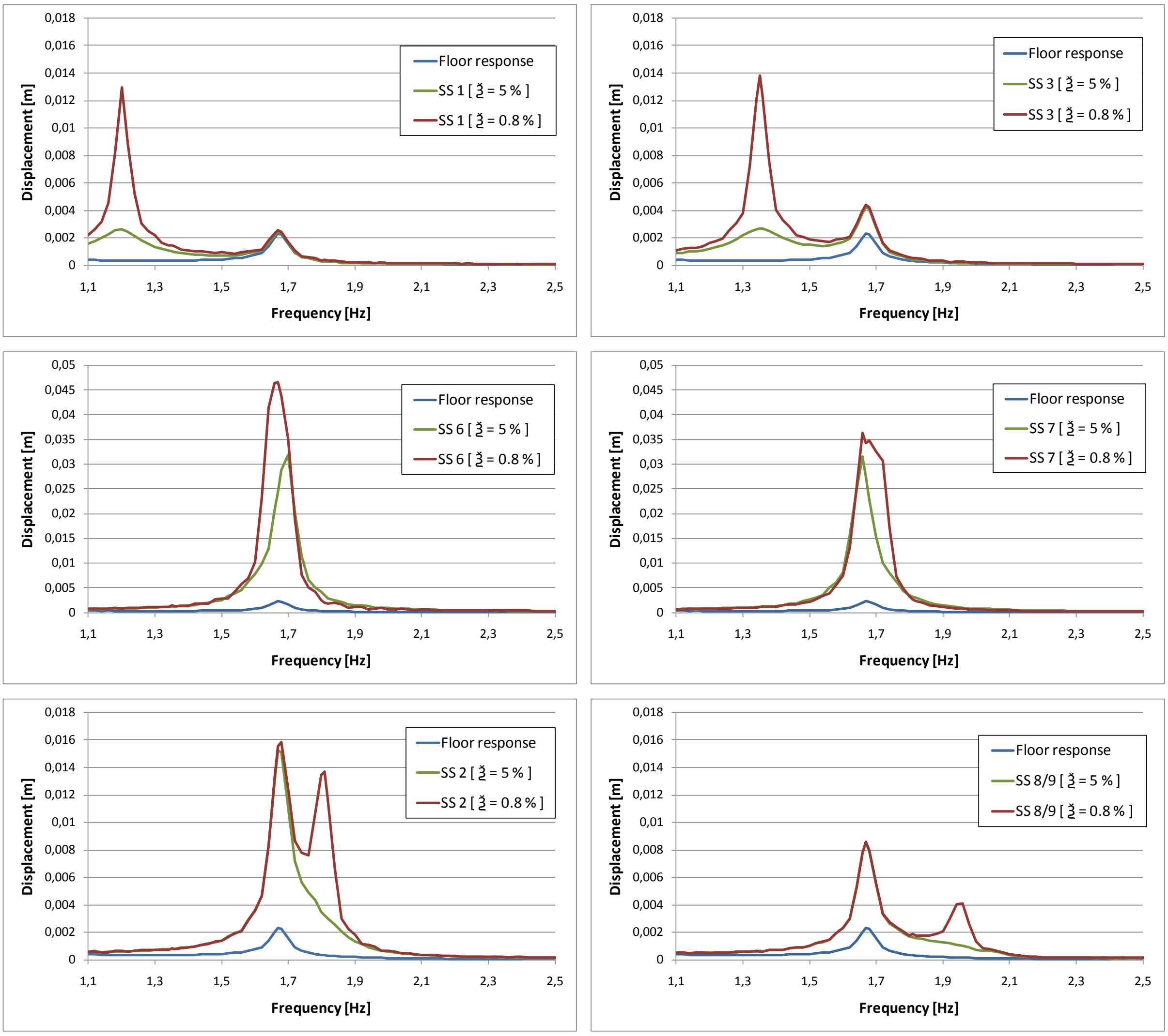

Figure 5.14. Displacement response function comparison for secondary systems with 0.8 % and 5.0 % damping. The blue curve is the displacement response of the floor at the location of the SS. The red curve is the displacement response for SS with 0.8 % damping. The green curve is the displacement response for SS with 5.0 % damping.

Displacement of the floor at the location of the secondary system

The secondary system with 5.0 % damping acts like a vibration absorber like the system with 0.8 % damping, but the primary system shows only one peak amplitude.

In the test with the damping ratio of the secondary system set to 0.8 %, the response was the same on both peaks because it had the same damping ratio as the primary system. For the test with the 5.0 % damping ratio the mode shape corresponding to the secondary system is damped with 5.0 %, but all other mode shapes have 0.8 %. When the damping ratio is as high as 5.0 %, the response for the secondary system at its natural frequency is too small to make a difference. This can be seen in Figure 5.15 and Figure 5.16.

The natural frequency for secondary system 6 is lower than the natural frequency for the primary system, i.e., the first peak value that was seen in the 0.8 % damping test is gone. For secondary system 7 the natural frequency is bigger than the natural frequency of the primary system, i.e., the second peak value seen from the 0.8 % damping test is gone. The same thing happens to the displacement response spectra in Figure 5.13.

The change in the natural frequency for secondary system 6 and the primary system is from both having natural frequencies equal to 1.67 Hz, to having 1.645 Hz and 1.696 Hz, respectively.

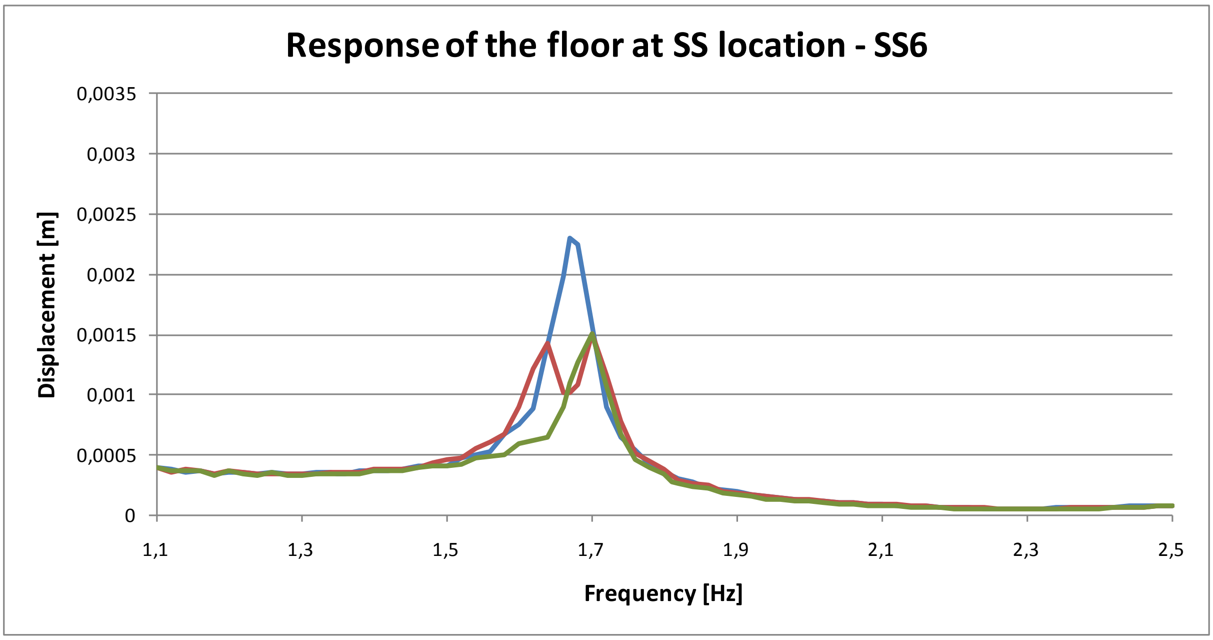

Figure 5.15. Displacement response function comparison at SS location for 5.0 and 0.8 % damping. The blue curve is the floor response of the PS at the location of the SS. The red curve is the displacement response for the floor with SS6 in place with 0.8 % damping. The green curve is the displacement response for the floor with SS6 in place with 5.0 % damping.

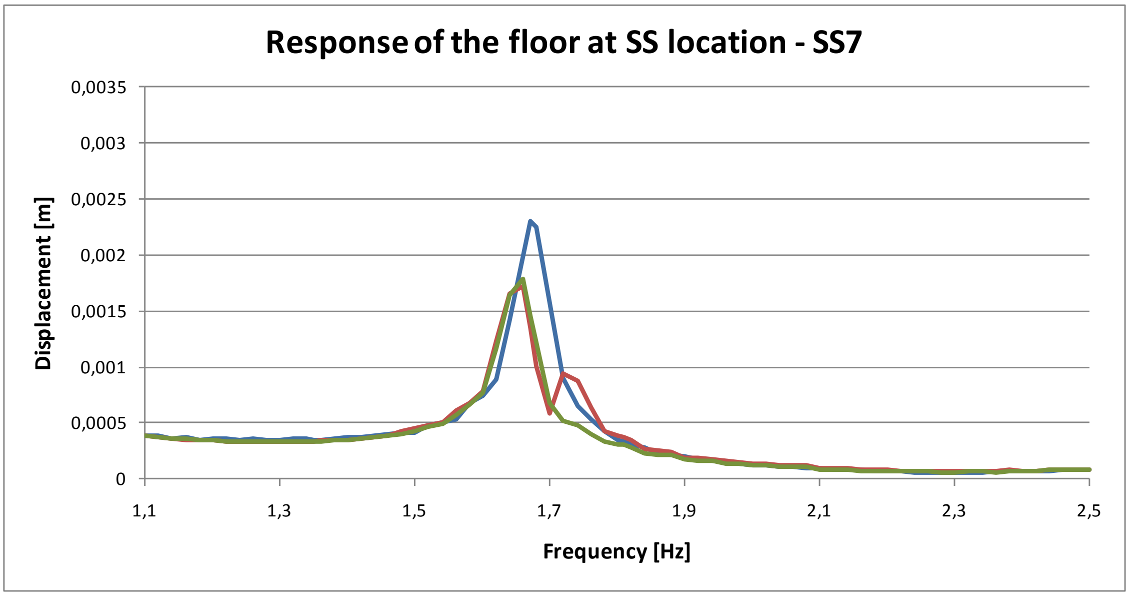

Figure 5.16. Displacement response function comparison at SS location for 5.0 and 0.8 % damping. The blue curve is the floor response of the PS at the location of the SS. The red curve is the displacement response for the floor with SS7 in place with 0.8 % damping. The green curve is the displacement response for the floor with SS7 in place with 5.0 % damping.

5.1.2.3 Damping ratio equal to 0.08 %, with comparison to 0.8 % and 5.0 % damping ratio

Displacement of the node at the location of the secondary system

A third run is performed with the damping ratio of secondary system 6 equal to 0.08 %. As expected, the peak that was damped out by the 5.0 % damping, is now much higher than for the two other damping ratios, but the second peak is about the same for all ratios. This is because the model uses modal damping, which dampens each mode with a specified damping ratio. The mode for the secondary system, first peak, has a damping ratio of 0.08 % in this third run and the second peak belongs to the primary system which always has 0.8 % damping.

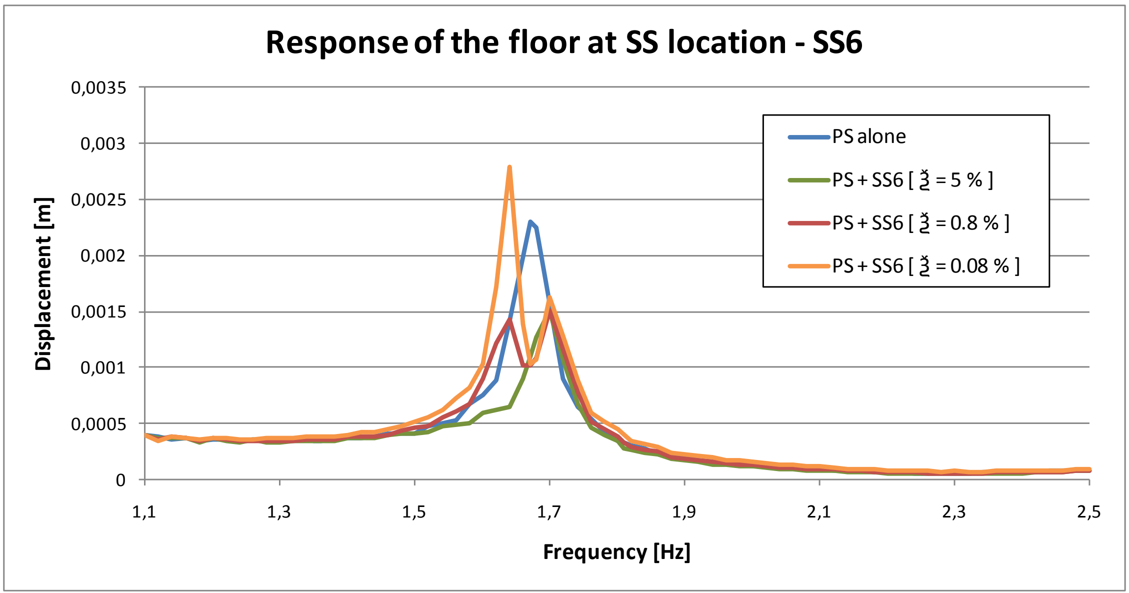

Figure 5.17. Displacement response function comparison at SS no. 6 location for 5.0, 0.8 and 0.08 % damping.

5.1.3 Ratio of response between secondary and primary system

The ratio of the response is interesting because it illustrates the amplification factors for the secondary systems and it can be used to make a design response spectra for secondary systems mounted on the Cardington Building.

In this chapter the ratio of response between the secondary systems and the primary system is investigated. This ratio is calculated by dividing the response of the secondary systems with the floor response of the primary system at the location of the secondary system.

From Figure 5.15 and 5.16 it is seen that the floor response of the primary system is different when secondary system 6 and secondary system 7 are attached. This means that the ratio generated by the primary systems floor response with the secondary systems in place, referred to as “SS / PS with SS in place”, is different from the ratio generated by the primary systems floor response without the secondary systems in place, referred to as “SS / PS without SS in place”. The two ways of generating the ratio of response are both shown in these chapters.

5.1.3.1 Damping ratio equal to 0.8 %

Ratio of response: SS / PS without SS in place

The secondary systems with the maximum displacement in the response function, SS 6 and 7 in Figure 5.8, have the lowest ratio of response for the graph with *no SS present*, with the exception of secondary system 8/9. It can be seen that the ratio of response gets larger when the natural frequency of the secondary system goes towards the natural frequency of the primary system, but when the natural frequency gets too close, the ratio drops. This happens because the primary system has its peak value at 1.67 Hz.

The ratio of response curve can be used to make the beginning of a design response spectra for secondary systems mounted on the Cardington Building. In order to make a functional design response function, more secondary systems are needed, as well as a larger frequency range.

Ratio of response: SS / PS with SS in place

Figure 5.19 shows the ratio when the displacement of the secondary systems is divided by the primary systems displacement when the different secondary systems are attached. The secondary systems with natural frequencies away from the natural frequency of the primary system have the same peak as in Figure 5.18, but secondary systems 6 and 7 now has the highest amplitude. This difference occurs because the displacement of these two systems is different at the natural frequency of the primary system. They act as dynamic vibration absorbers. The displacement of the primary system has been shown in Figure 5.10.

The difference in the peak values for the two ways of portraying the ratio is about 50 % higher for the secondary systems that are divided by their own floor response. This last way of portraying the ratio is the “real” graph, but when engineers test the secondary systems, they only have the floor response of the primary system when it is alone, therefore making the “SS / PS without SS in place” graph the one used in designing.

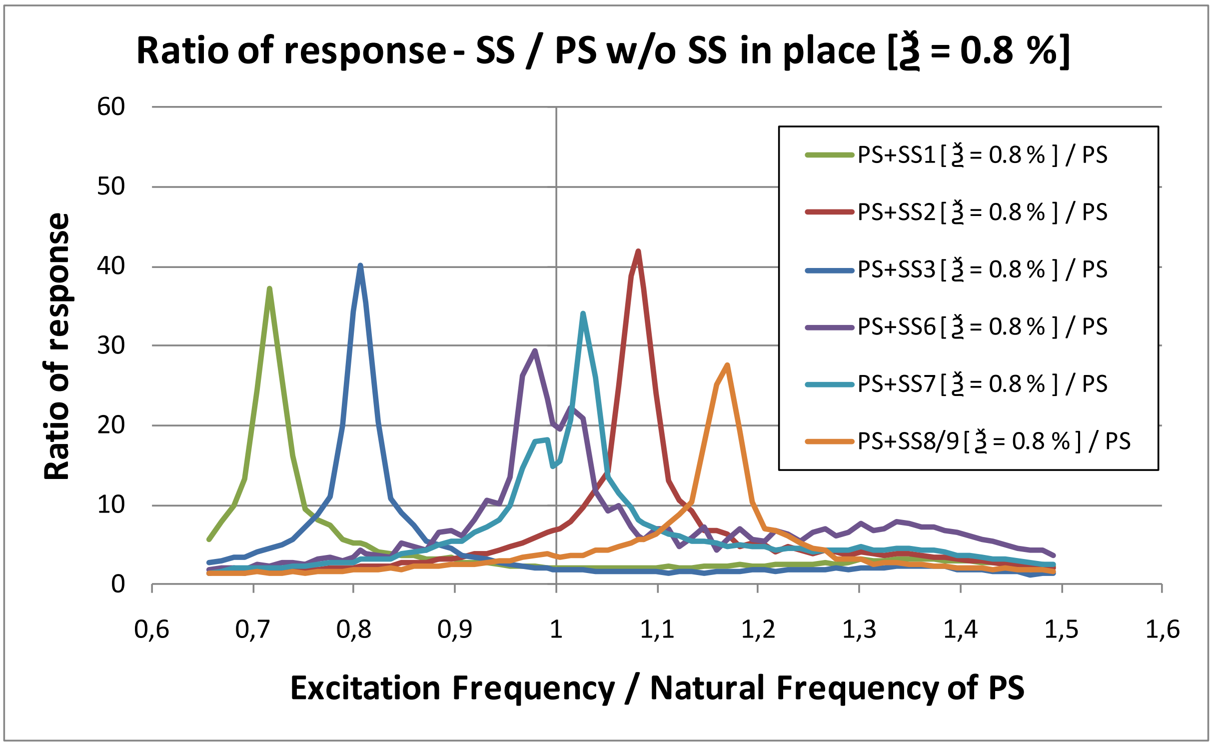

Figure 5.18. Ratio, 0.8 % damping: The displacement of the secondary systems divided by the displacement floor response of the primary system when there are no secondary systems attached.

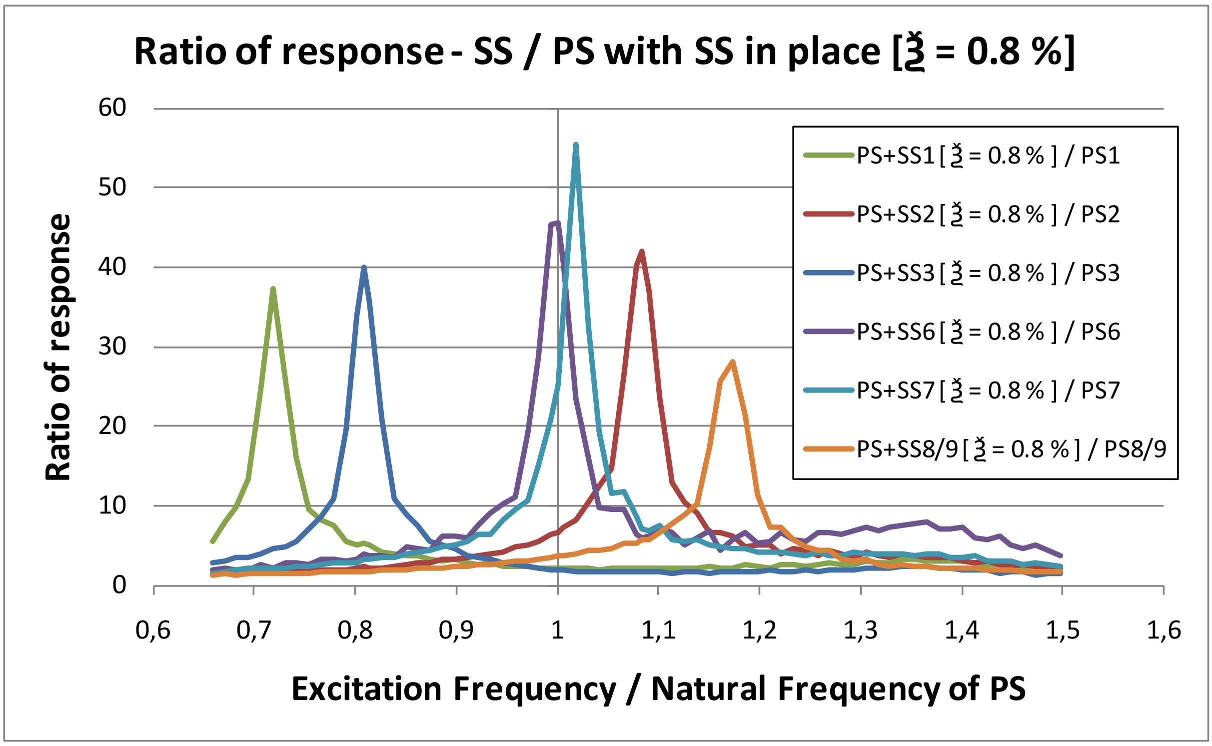

Figure 5.19. Ratio, 0.8 % damping: The displacement of the secondary systems divided by the displacement floor response of the primary system with the secondary systems attached.

5.1.3.2 Damping ratio equal to 5.0 %

Ratio of response: SS / PS without SS in place

Secondary system 6 and 7 has the highest ratio for 5.0 % damping, unlike the “SS / PS without SS in place” figure for 0.8 % damping. Secondary system 6 and 7 has a spiked curve, but the other secondary systems have a curve that is much more flat. This can be explained by the fact that secondary systems 6 and 7 have got their natural frequency shifted. Because of the vibration absorber effect, the peak for the secondary system is shifted away from the primary system’s peak value frequency.

Ratio of response: SS / PS without SS in place

By dividing the secondary system response with the response of the floor with the secondary system in place, gives us the same results as for the ratio with 0.8 % damping. The only difference is in the curve for secondary systems 6 and 7. The peak values for the two systems are about 2.25 times larger than the other systems, but for 0.8 % damping, secondary systems 6 and 7 are only 1.5 times larger.

The difference can be explained by thinking that the primary system, with 0.8 % damping, gives the secondary systems with 5.0 % damping a push. The primary system thereby reduces the effect of the damping on the secondary system. This can also be seen in the earthquake tests performed in chapter 5.2.3 (The effect of the damping ratio for secondary systems with natural frequencies inside the resonant frequency range of the primary system).

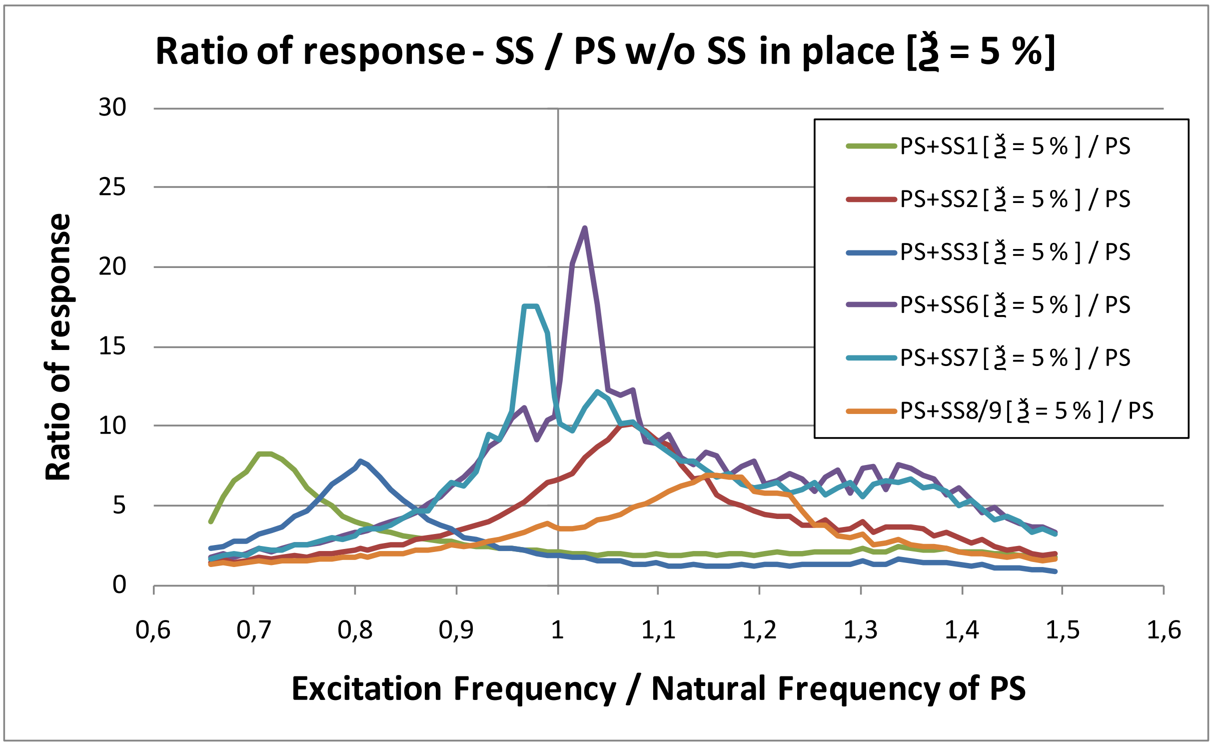

Figure 5.20. Ratio of response, 5 % damping: The displacement of the SS divided by the displacement floor response of the PS without the SS present.

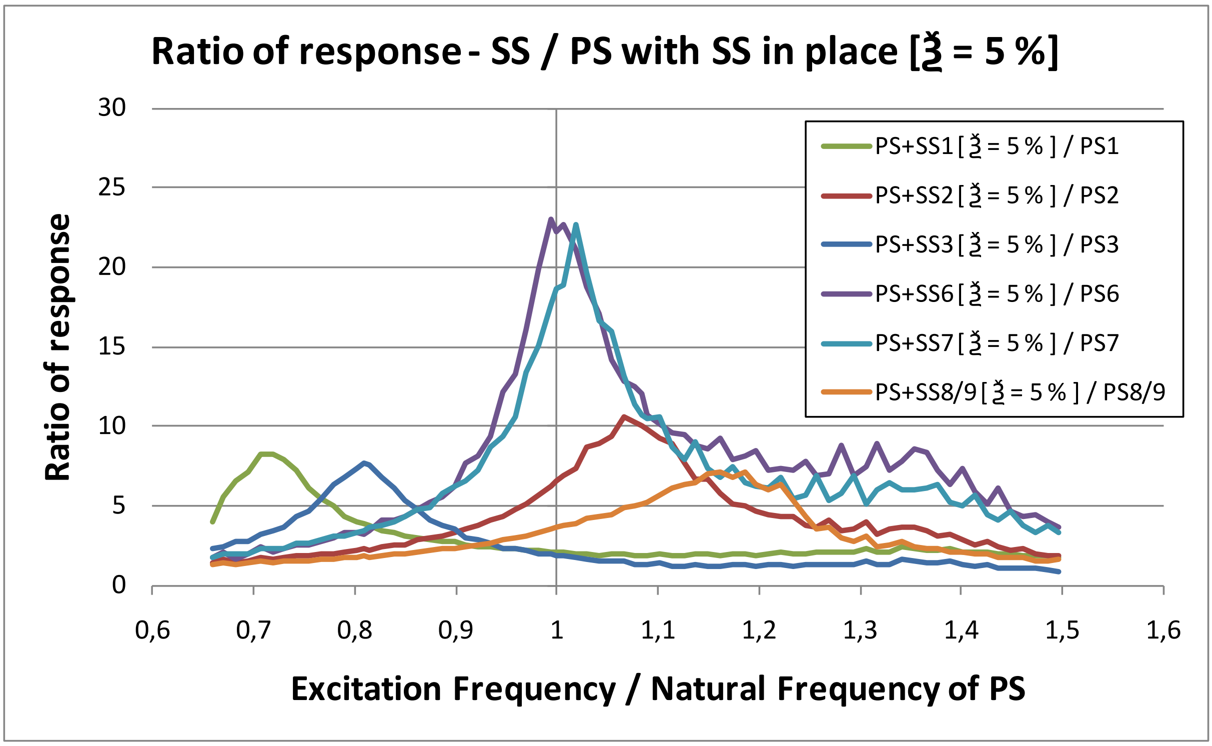

Figure 5.21. Ratio of response, 5 % damping: The displacement of the SS divided by the displacement floor response of the PS with the SS present.

5.1.3.3 Comparison between 0.8 % and 5.0 % damping for the ratio of response where the floor response of the primary system is generated without the secondary systems in place

The ratio of response for the secondary systems, with natural frequencies far away from the natural frequency of the primary system, is not affected at the primary systems natural frequency. The amplification for secondary system 1, 2 and 3 is about 40 times bigger than the primary structure for 0.8 % damping, while it is about 8 times bigger for 5.0 % damping for these secondary systems.

Secondary system 8/9 looks the same as the above secondary systems, but the amplification is not as great. The secondary system with 0.8 % damping has a peak amplitude that is 28 times higher than the primary system's peak amplitude and the 5.0 % damping system is about 7 times higher.

Secondary systems 6 and 7 have two peak values each, but the amplitude for the secondary systems with 5.0 % damping is very low because of the high damping ratio.

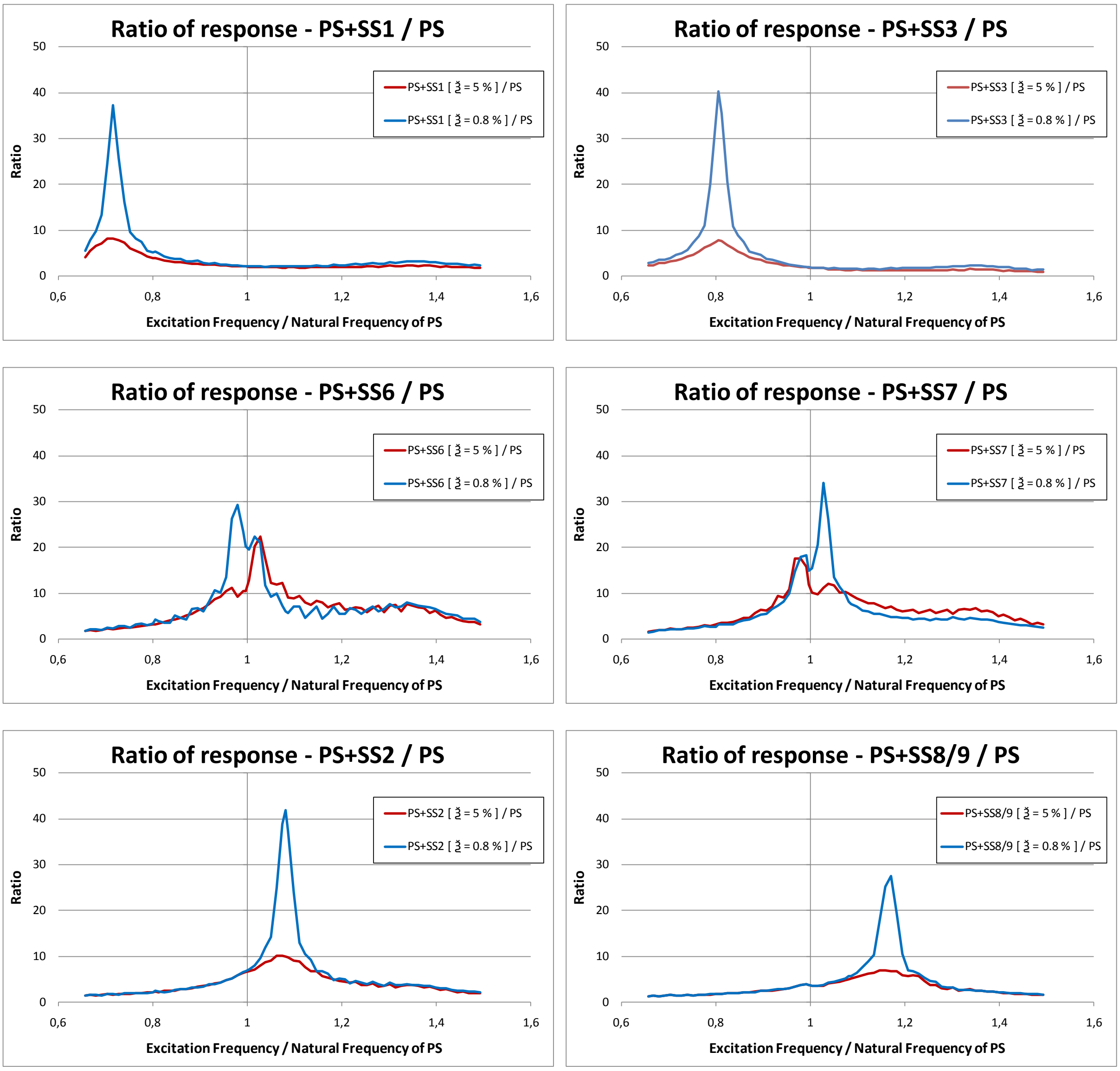

Figure 5.22. Ratio of response. The displacement of the secondary system divided by the displacement floor response of the primary system without the secondary systems in place.

5.2 Earthquake Excitation

The earthquakes used in this analysis come from earthquakes that have been recorded on the County Building in Selfoss, Iceland. The records consist of two earthquakes, one from June 2000 with a magnitude of 6.5 and one in May 2008 with a magnitude of 6.3. The epicenter occurred 14.1 km and 8 km from the County Building. There exist no data recordings from earthquakes on the Cardington Building, but due to the comparison made with the harmonic excitation, the structure is assumed to behave correctly. The earthquakes from 2000 and 2008 both have recordings in the east-west direction and in the north-south direction. The earthquakes are therefore split into 4 earthquakes, which are all applied separately and in the east-west direction.

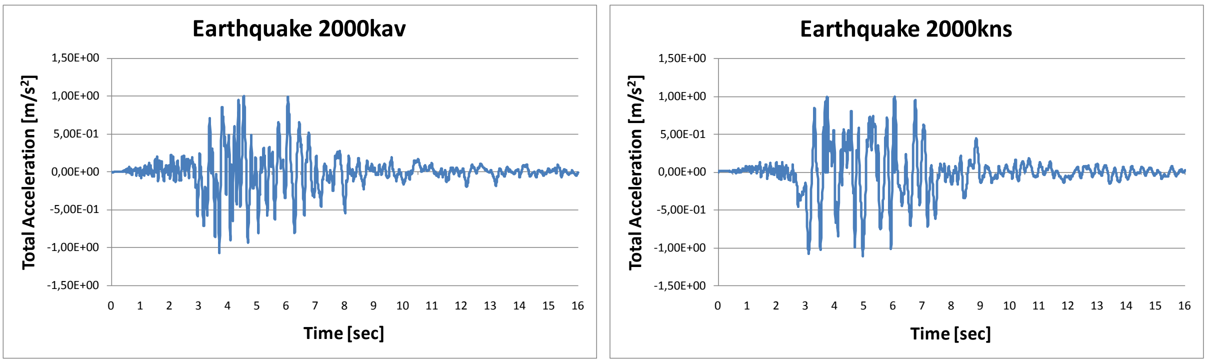

Figure 5.23. Total acceleration time series for the Selfoss 2000 earthquakes. These are the total accelerations used as input accelerations for the tests.

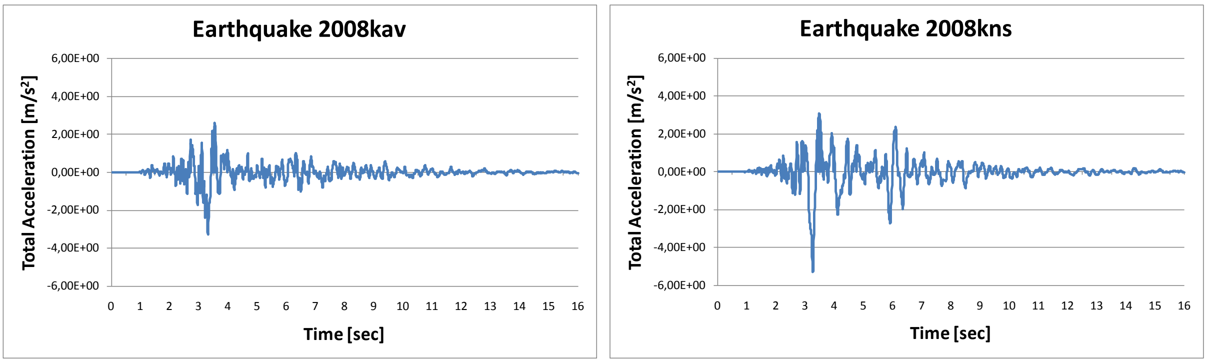

Figure 5.24. Total acceleration time series for the Selfoss 2008 earthquakes. These are the total accelerations used as input accelerations for the tests.

The damping ratios for the secondary systems are the same as the ones in Table 4.4, because in the earthquake tests the systems are only compared to their selves. A decay test has been performed in chapter 4.3 to check the damping ratios of the secondary systems and the primary system.

Standards, like Eurocode 8, states that for very important and / or dangerous non-structural elements the floor response spectra method has to be used. These earthquake tests plan to confirm that the floor response of a node in the building can be extracted and act as a ground motion on an isolated secondary system. The motion of the secondary system when it is isolated and when it is mounted on the primary system should be the same for the Eurocode 8 statement to be true.

5.2.1 Acceleration response spectra for the primary system

The acceleration response spectra for all four earthquakes are found below. These spectrums give an indicator on how big the accelerations are expected to be at the different periods. The period of the primary structure, the Cardington Building, is $1/1.167\ Hz = 0.6\ seconds$. This period is shown in the two following figures, thus marking the maximum response acceleration values expected for the primary system.

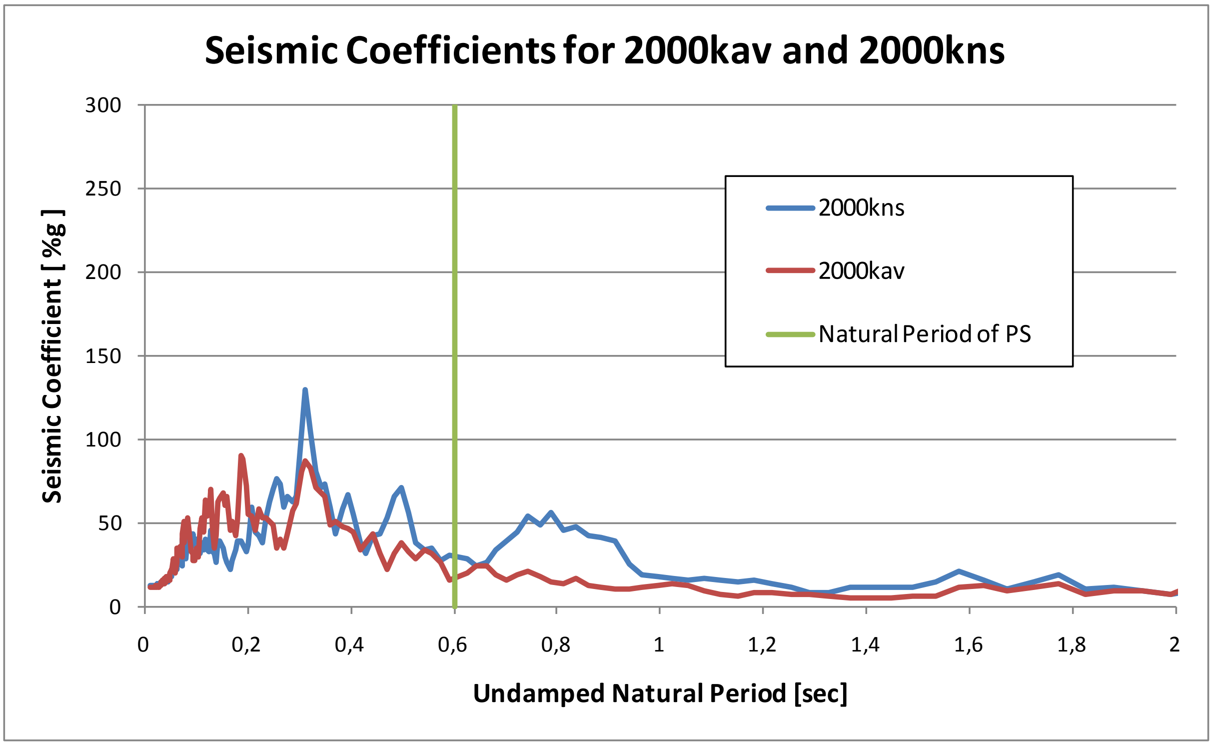

Figure 5.25. Acceleration response spectra for earthquake 2000kav and 2000kns.

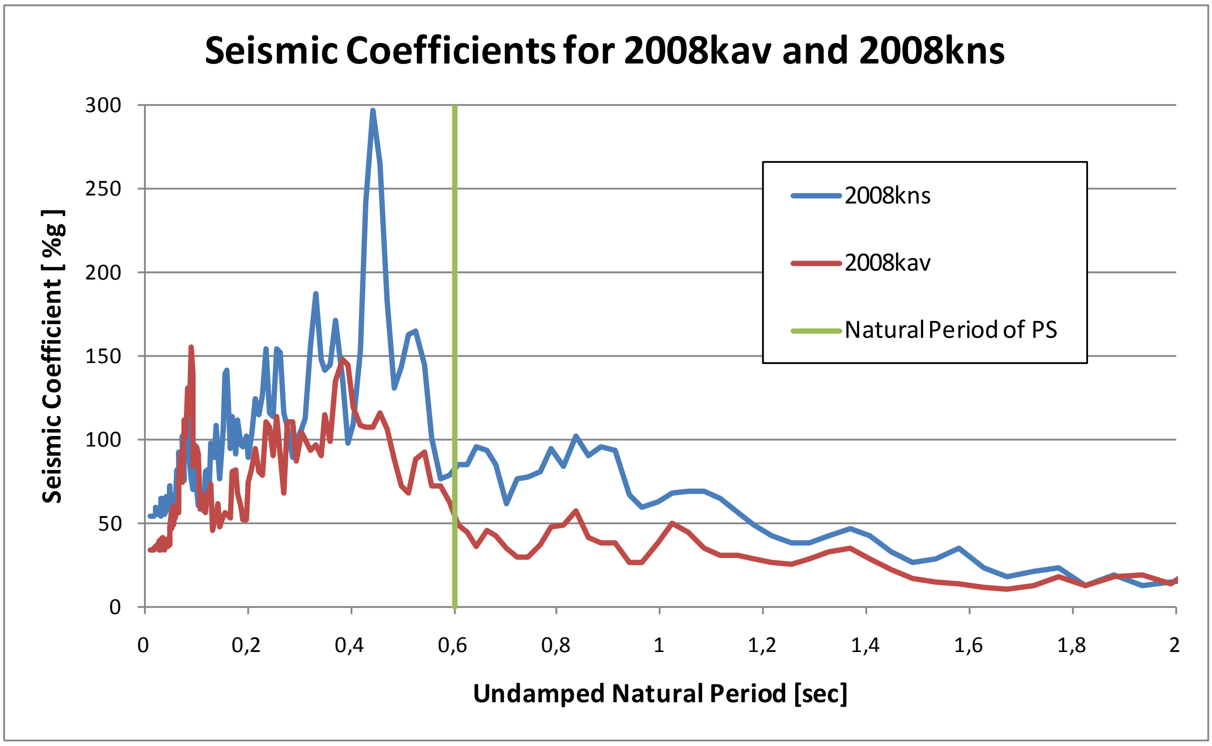

Figure 5.26. Acceleration response spectra for earthquake 2008kav and 2008kns.

The response spectral acceleration for the four earthquakes, at the natural frequency of the primary system, can be found from Figure 5.25 and Figure 5.26, and they are as follows:

Table 5.6. Seismic Coefficient and Relative Acceleration expected.

Primary Structure		
Earthquake	Seismic Coefficient (% g)	Relative Acceleration (m/s^2)
2000kav	16.75	1.6
2000kns	30.24	2.9
2008kav	55.00	5.4
2008kns	82.00	8.0

These values are the maximum accelerations expected for the primary system. The calculated accelerations from the FE model are shown in Figure 5.27. The natural period of the primary system is a favorable period when exposed to the earthquakes used in these tests. The low periods, below 0.5 seconds, have a much higher seismic coefficient. For earthquake 2008kns it is as high as 300 % g at 0.45 seconds.

According to the mode shape from the FE model, Figure 4.9, the maximum accelerations are expected to occur at the 3rd and 7th floor. This is true for all earthquakes except for 2008kav, where the 4th floor has the highest accelerations. The calculated accelerations are 1.38, 1.35, 1.09 and 1.06 times higher than the expected values for 2000kav, 2000kns, 2008kav and 2008kns, respectively. For the earthquakes that occurred in 2008 the expected values compare very well to the expected accelerations. These are the earthquakes with the highest seismic coefficients. The earthquakes that occurred in 2000, the ones with the lowest seismic coefficients, do not compare as well as the 2008 earthquakes to the expected accelerations.

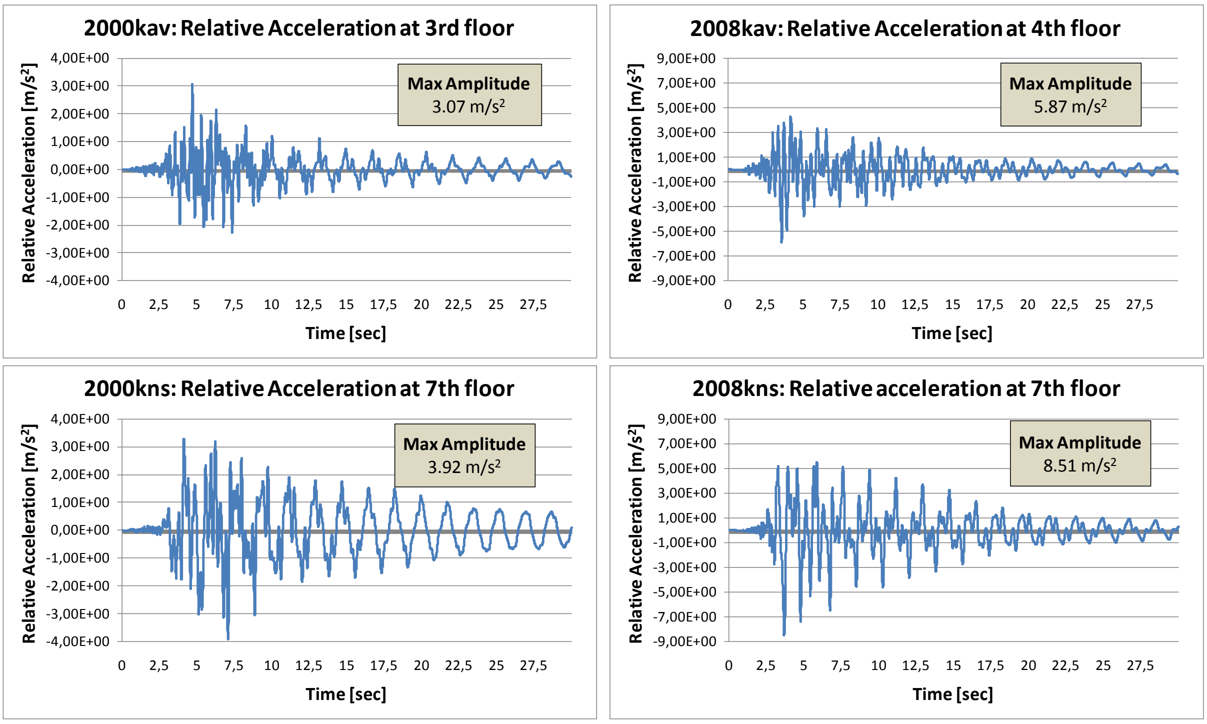

Figure 5.27. Maximum relative acceleration of the Cardington Building.

5.2.2 Floor response at the location of the secondary structure

From Figure 5.9 it is seen that secondary systems 6 and 7 affects the response of the floor-node at the secondary system location during harmonic excitation. This should also be visible when the earthquakes are applied. The four figures below show the acceleration time series for secondary system 1, 2, 3 and 8/9 during earthquake “2008kns”.

The differences in acceleration between the two responses are minimal. Earthquake 2008kns is representative for all four earthquakes; see Appendix B: “Earthquake floor response at the location of the secondary structure”. “PS with SS1” refers to the response of the node at the secondary system location is measured with secondary system 1 present. “PS alone” refers to the response of the floor node is measured without the influence of a secondary system.

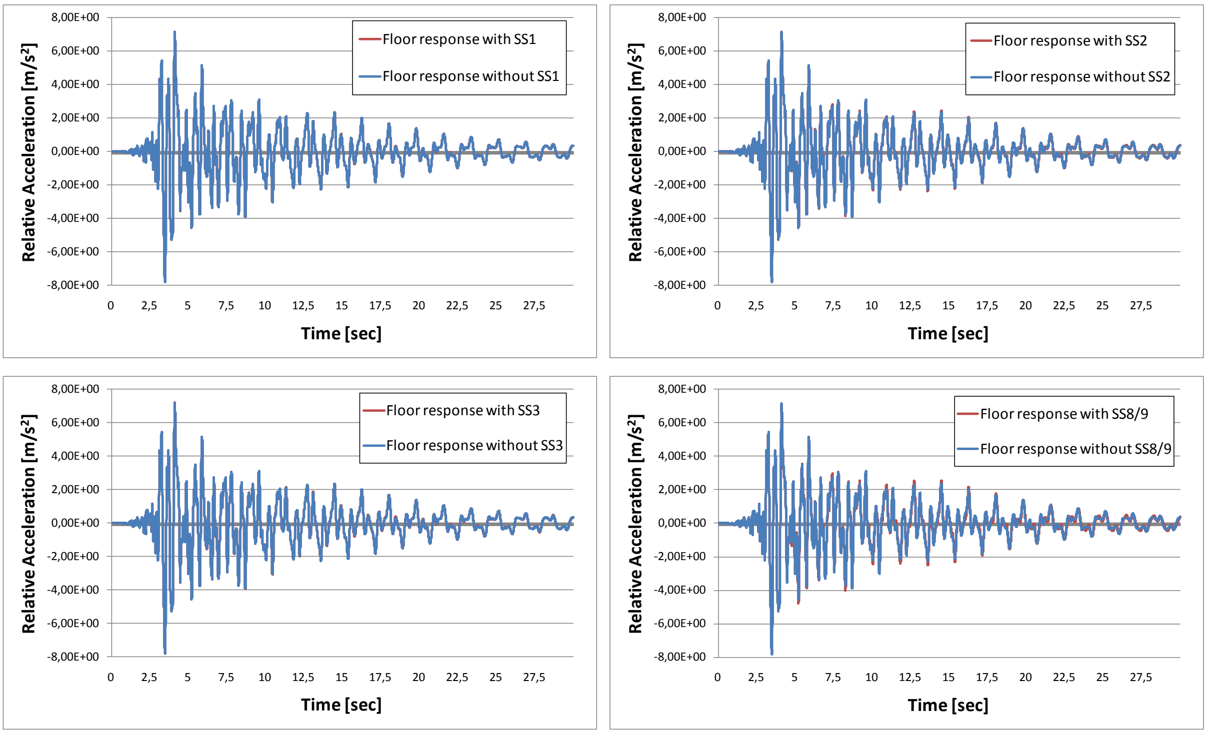

Figure 5.28. 2008kns: Relative acceleration time series comparison between the floor responses at the location of the secondary system with the secondary system in place (red curve) and without the secondary system in place (blue curve).

In the graphs above, the blue curves are the only visible ones. This is because the motion of the floor with and without the secondary system in place is exactly the same. The same thing was seen for the harmonic excitation; see Figure 5.9.

The two figures below show the relative acceleration time series for secondary system 6 and 7 during earthquake 2008kns. These were the two systems that had an effect on the response of the primary system during the harmonic excitation. During the earthquake excitation, the secondary systems do not have a big effect on the motion, but there is a small change in the acceleration of secondary system 6 after twelve seconds. The harmonic excitation showed that the response of the primary system only got affected around the natural frequency (Figure 5.15). This means that when the earthquake goes through the natural frequency area, the acceleration is affected in a different way than for the primary system by itself.

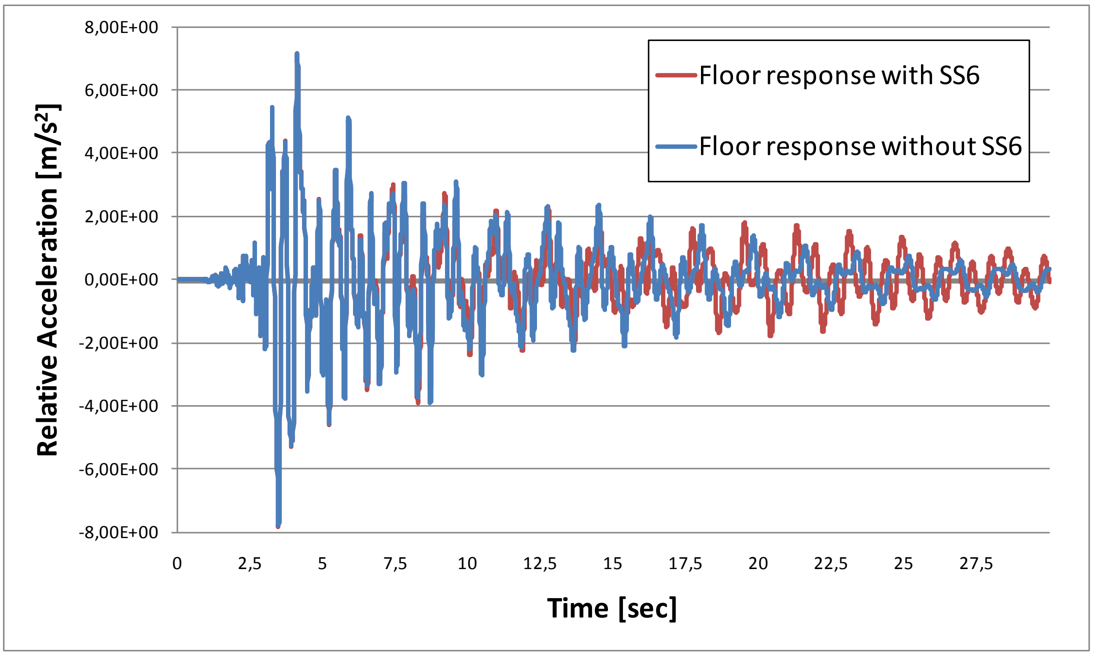

Figure 5.29. 2008kns: Relative acceleration time series comparison between the floor responses at the location of the SS with SS6 in place and without SS6 in place.

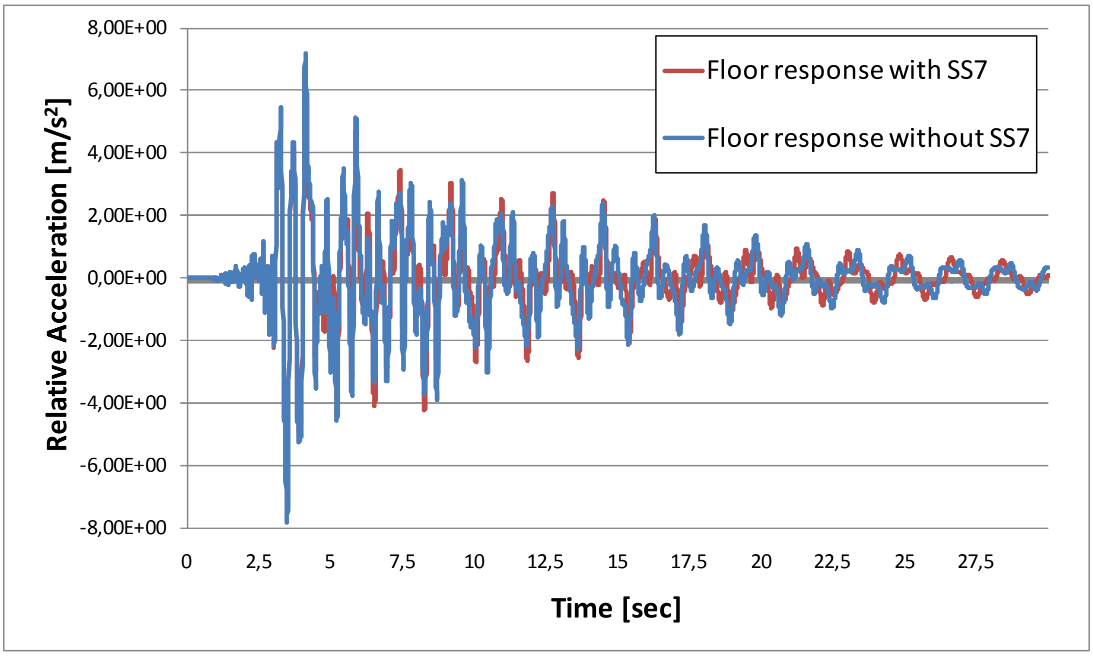

Figure 5.30. 2008kns: Relative acceleration time series comparison between the floor responses at the location of the SS with SS7 in place and without SS7 in place.

5.2.3 Response of the secondary structure

In order to get the results needed for the floor response spectra method, a couple of tests have to be performed. Firstly, the primary system alone has to be excited by the four earthquakes separately. The total acceleration time series at the node of where the secondary system is located has to be extracted. The secondary systems are the same systems used by Snæbjörnsson et al. [2] in 2000.

Table 5.7. The secondary systems natural frequencies and CDR.

Secondary Structure			
System	Direction of vibration	Natural Frequency (Hz)	Critical Damping Ratio (%)
1	z - East-West	1.2024	0.42
2	z - East-West	1.8092	0.66
3	z - East-West	1.3513	0.07
6	z - East-West	1.6684	0.08
7	z - East-West	1.6999	5.77
8 / 9	z - East-West	1.9577	2.28

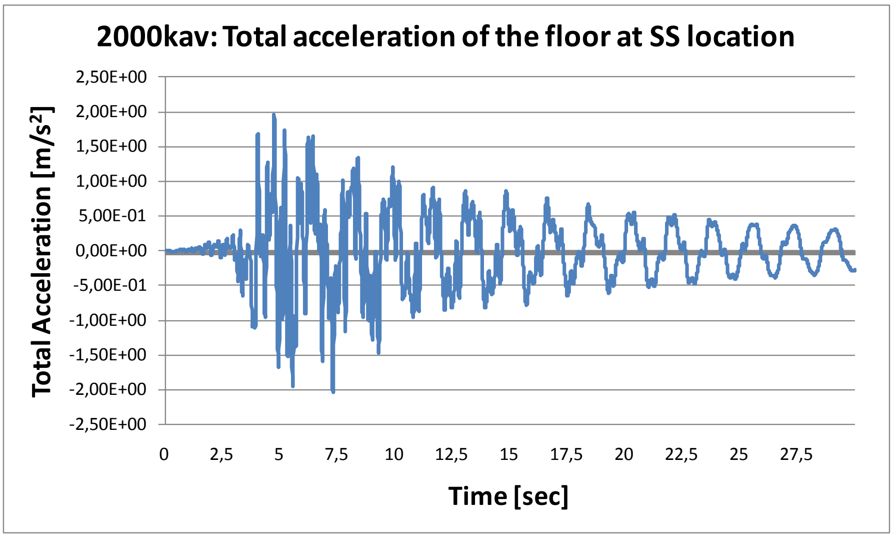

Figure 5.31. Total acceleration time series of the floor at SS location during earthquake 2000kav.

This acceleration acts as a ground motion on isolated versions of the secondary systems. Figure 5.32 illustrates the basic idea. The acceleration time series that are created from the isolated tests are to be compared to the time series made when the secondary system is attached to the primary system.

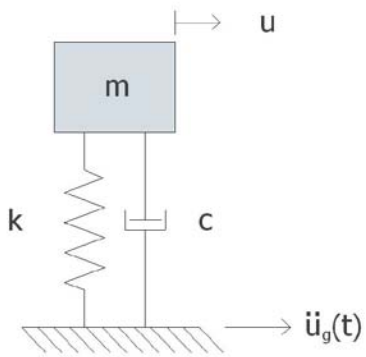

Figure 5.32. Ground motion on a SDF system.

5.2.3.1 Amplification of the total acceleration from the ground to the secondary system

The total accelerations measured at the fourth floor in the building are expected to be higher than the total accelerations from the input earthquakes. The accelerations are amplified for each floor.

The amplification for the Cardington Building is shown below. Earthquake 2008kns has a peak amplitude of 5.28 m/s^2, see Figure 5.33, but this spike is not representative for the rest of the response. One spike does not make much difference on the response of the building because the spike does not exist long enough for the building to react and move accordingly.

By excluding this peak amplitude, the accelerations from the earthquake gets amplified by a factor of 1.80 and 1.62 between the ground and the fourth floor for earthquake 2000kns and 2008kns, respectively. The amplification factor between the fourth floor and the secondary system is 2.92 for both earthquakes.

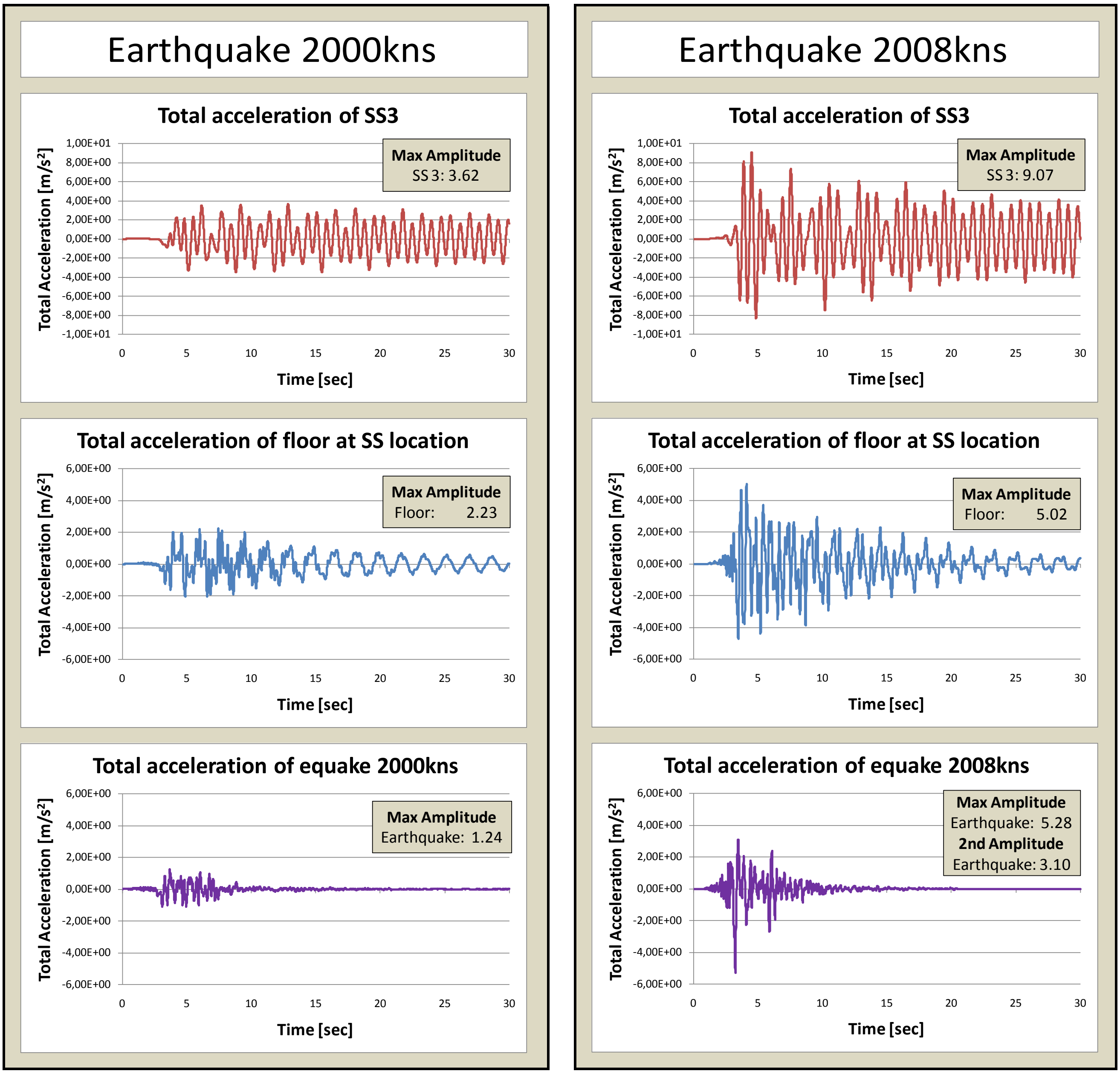

Figure 5.33. Comparison of total acceleration between the ground, the fourth floor and secondary system 3 for the north-south component of the main earthquakes of June 2000 and May 2008.

5.2.3.2 Floor response spectra method compared to SS analyzed jointly with PS

The relative acceleration time series for the separated secondary systems are compared with the time series when the secondary system is mounted on the primary system. For the floor response spectra method to be true, the calculated accelerations have to be more or less equal to each other.

The ratio between the natural frequency of the secondary system and natural frequency of the primary system is written in the caption of each figure (fss/fps). For secondary systems 2 and 8/9 (nat. freq. higher than nat. freq. of PS) the two methods are equal to each other for all earthquakes.

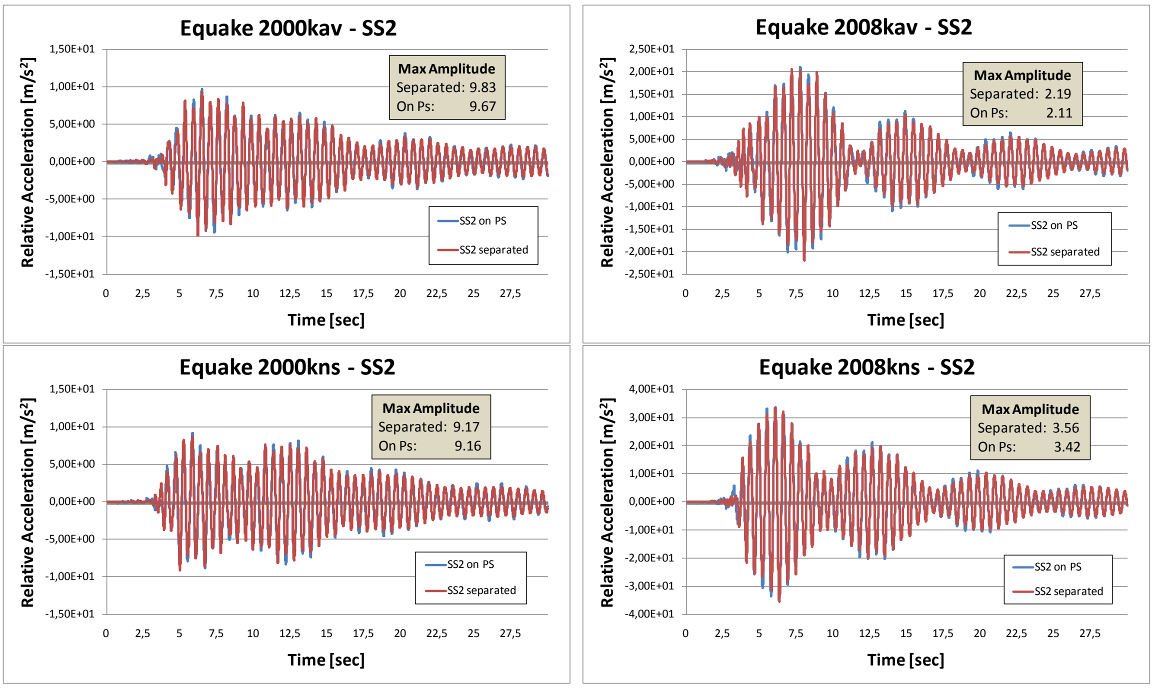

Figure 5.34. Comparison of relative acceleration between SS2 analyzed separately (red curve) and SS2 analyzed jointly (blue curve), fss/fps = 1.080.

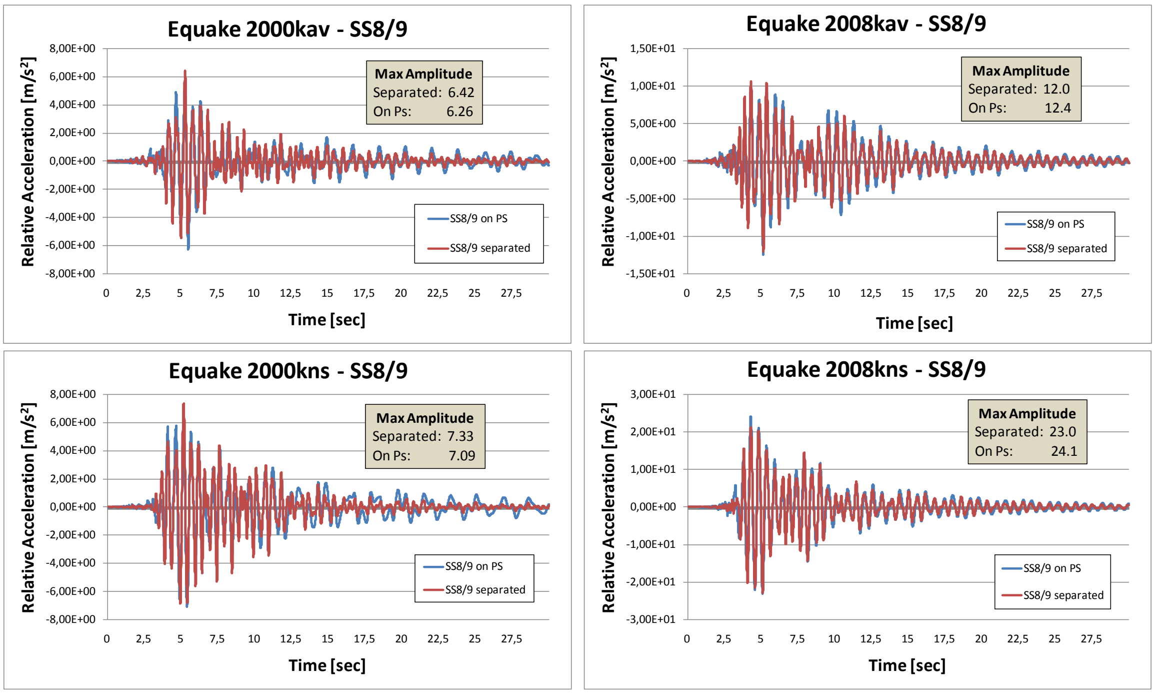

Figure 5.35. Comparison of relative acceleration between SS8/9 analyzed separately (red curve) and SS8/9 analyzed jointly (blue curve), fss/fps = 1.169.

The secondary systems, shown in Figure 5.34 & 5.35, experience a beating effect. This happens because the transient response and steady-state response are out of phase with each other; see Figure 2.6 in chapter 2.5.1.

Secondary systems 1 and 3 (nat. freq. lower than nat. freq. of PS) have a more random curve. The maximum amplitudes for the SS that are separated have on average higher amplitudes.

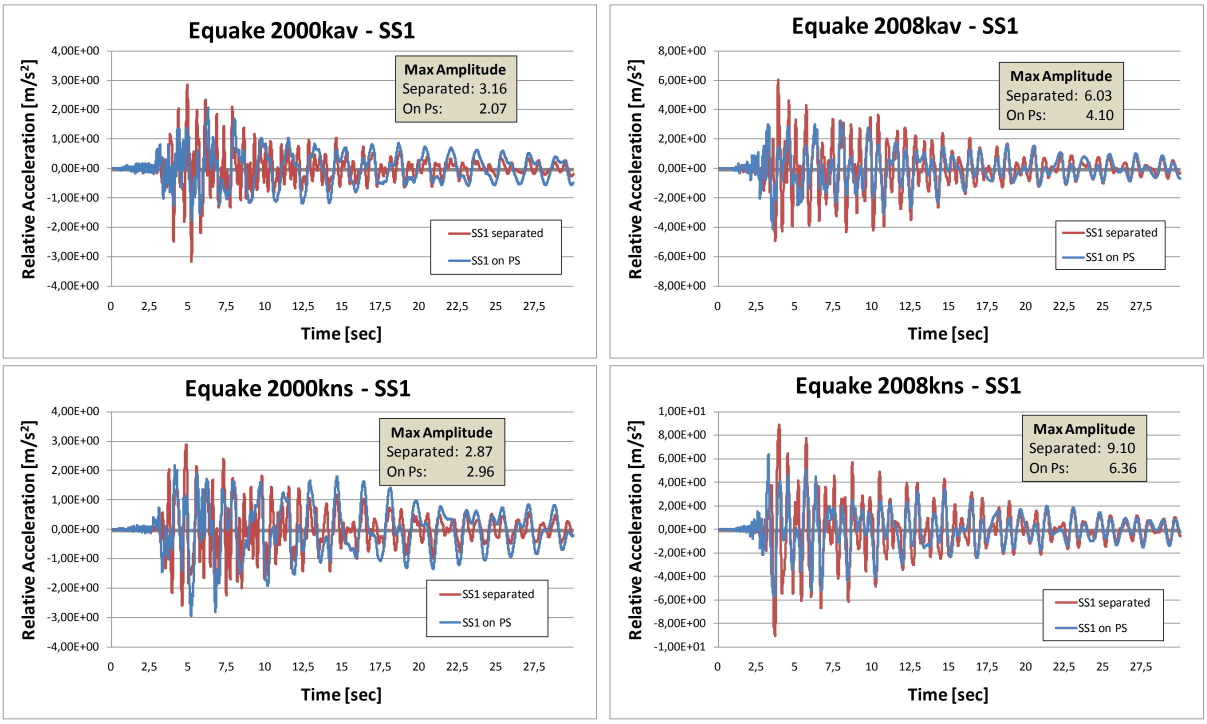

Figure 5.36. Comparison of relative acceleration between SS1 analyzed separately (red curve) and SS1 analyzed jointly (blue curve), fss/fps = 0.718.

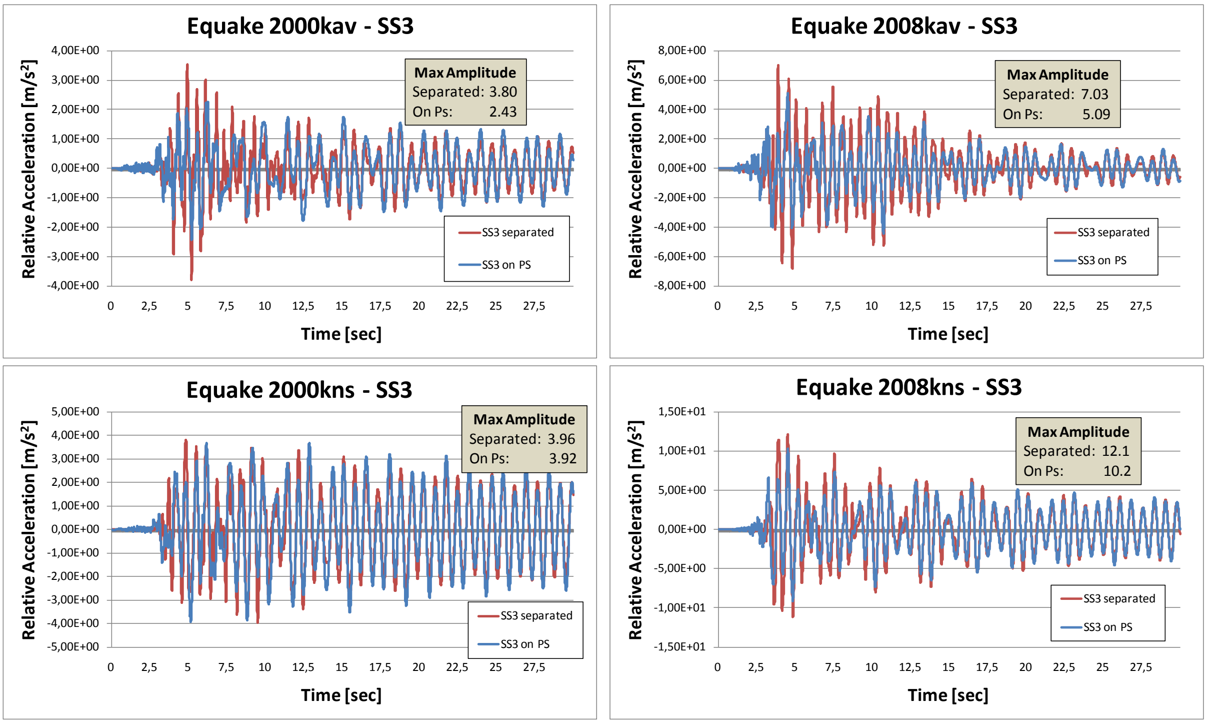

Figure 5.37. Comparison of relative acceleration between SS3 analyzed separately (red curve) and SS3 analyzed jointly (blue curve), fss/fps = 0.807.

The two last secondary systems, no. 6 and 7 (nat. freq. inside the nat. freq. range of the PS), have relative acceleration responses that have big differences in the amplitudes.

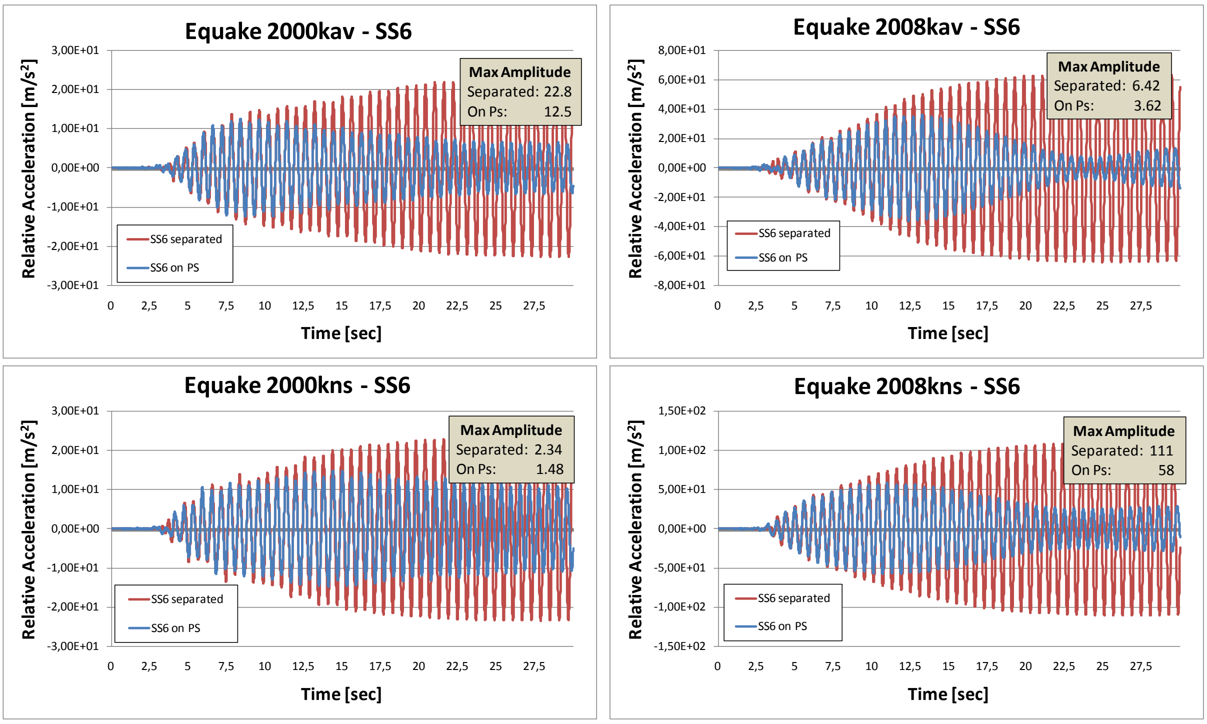

Figure 5.38. Comparison of relative acceleration between SS6 analyzed separately (red curve) and SS6 analyzed jointly (blue curve), fss/fps = 0.996.

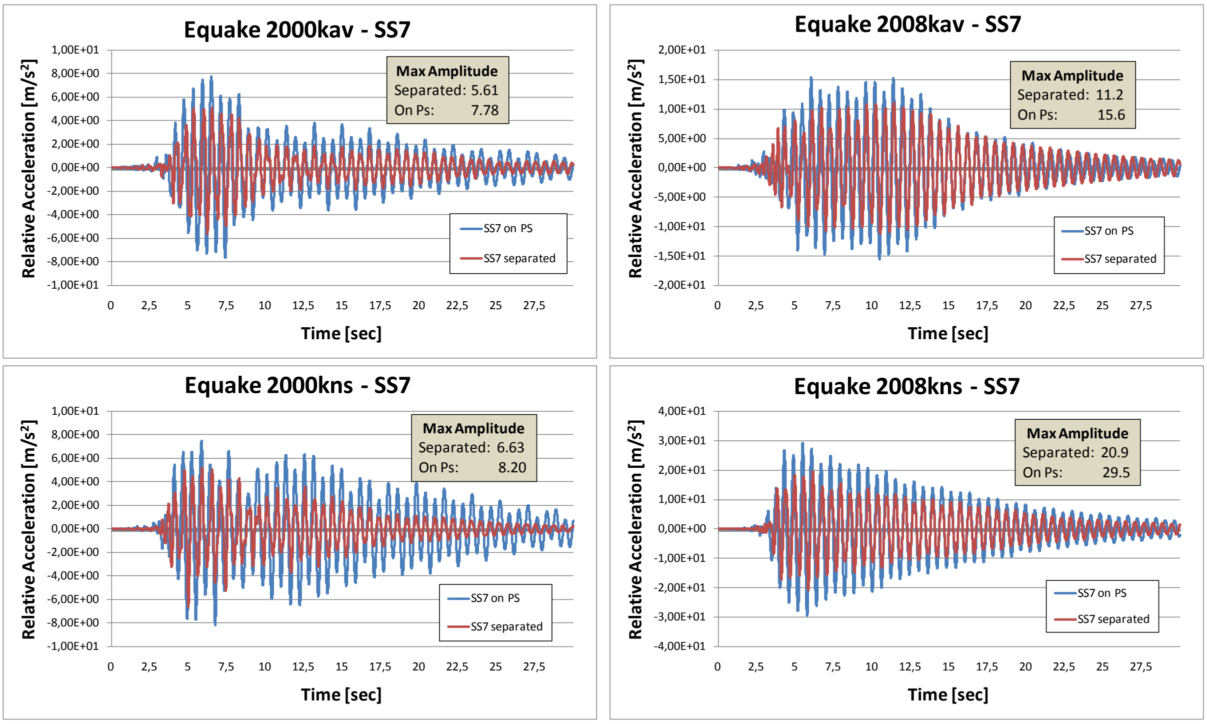

Figure 5.39. Comparison of relative acceleration between SS7 analyzed separately (red curve) and SS7 analyzed jointly (blue curve), fss/fps = 1.015.

There seems to be a pattern for the type of response for the secondary systems. The system with the natural frequency near the natural frequency of the primary system (1.675 Hz), SS 2 (1.809 Hz), seems to get a distinct beating curve and the difference in the amplitude is minimal.

The secondary systems with their natural frequency far away from the natural frequency of the primary system, SS 1 (1.2024 Hz), SS 3 (1.351 Hz) and SS 8/9 (1.958 Hz), get a more random curve and the amplitude of when the SS is separated tends to be larger.

The secondary systems with natural frequencies at the natural frequency of the primary system, SS 6 (1.668 Hz) and SS 7 (1.699 Hz), get some beating motion, but the amplitudes between the two methods for calculating the response are different.

The six secondary systems presented in these tests are not enough to verify the trend. In order to do so, two imaginary secondary systems are made with natural frequencies equal to 1.563 Hz and 2.225 Hz. Imaginary system 1 (iSS1) is close to the natural frequency of the primary system (1.675 Hz) and imaginary system 2 (iSS2) is far away. The relative acceleration time series for the imaginary systems are shown below. The extra secondary systems confirm that the beating effect is a phenomenon that can be seen close to the natural frequency of the primary system.

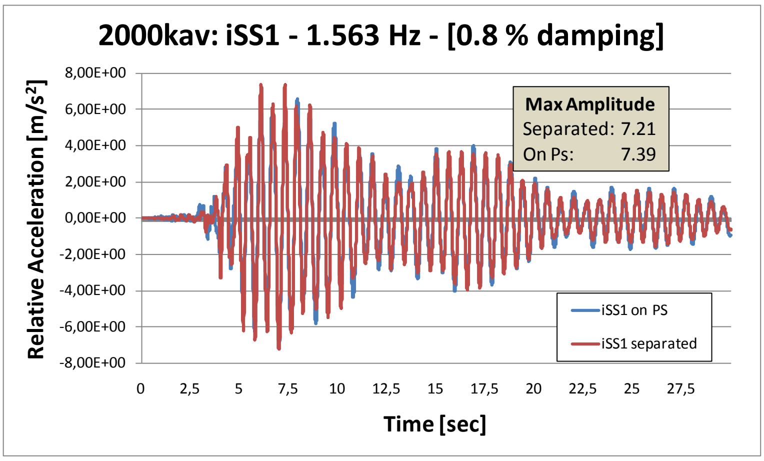

Figure 5.40. Comparison of relative acceleration between iSS1 analyzed separately (red curve) and iSS1 analyzed jointly (blue curve), fss/fps = 0.933.

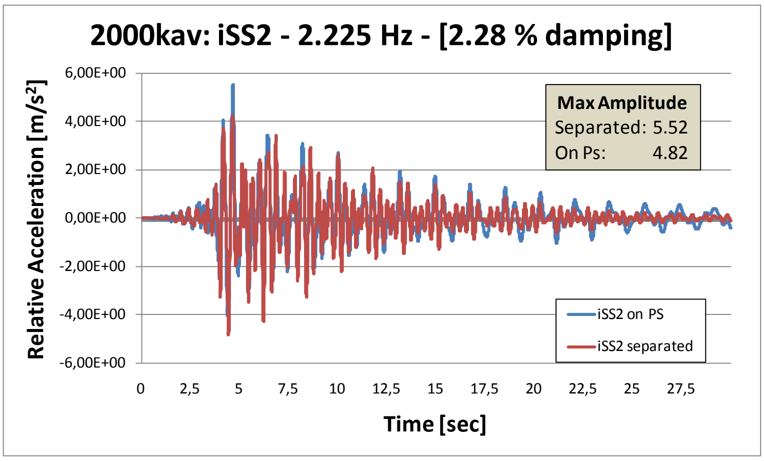

Figure 5.41. Comparison of relative acceleration between iSS2 analyzed separately (red curve) and iSS2 analyzed jointly (blue curve), fss/fps = 1.328.

The differences in the response for the secondary systems with natural frequencies close to the natural frequency of the primary system (1,675 Hz), SS2, SS8/9 and iSS1, are minimal. The differences are shown in Table 5.8. The average differences are 1.02, 1.00 and 0.98 for SS2, SS8/9 and iSS1, respectively.

The secondary systems with natural frequencies far away from the natural frequency of the primary system, SS1, SS3 and iSS2, have noticeable differences between the two methods, and the peak amplitude of the SS when it is separated tends to be bigger. The difference during earthquake 2000kns is minimal, but during earthquake 2000kav the difference is ≈1.50. The average differences are 1.35, 1.29 and 1.15 for SS1, SS3 and iSS2, respectively.

Secondary systems 6 and 7, which have their natural frequencies inside the resonant frequency range of the PS, show a big difference between the two methods, but these are also the systems with the lowest damping ratio. The effect of the damping ratio is discussed in the next chapters. The average differences in the amplitudes are 1.77 and 0.74 for SS6 and SS7, respectively.

Table 5.8. Maximum Amplitude Difference between SS analyzed separately and SS analyzed jointly.

Maximum Amplitude Difference: SS analyzed separated divided by SS analyzed on the PS							
System No.	Natural frequency (Hz)	Damping ratio (%)	2000kav	2000kns	2008kav	2008kns	Average Difference
			Difference*	Difference*	Difference*	Difference*	
SS1	1.2024	0.42	1.53	0.97**	1.47	1.43	1.35
SS2	1.8092	0.66	1.02	1.00	1.04	1.04	1.02
SS3	1.3513	0.07	1.56	1.01	1.38	1.19	1.29
SS6	1.6684	0.08	1.82	1.58	1.77	1.91	1.77
SS7	1.6999	5.77	0.72	0.81	0.72	0.71	0.74
SS8/9	1.9577	2.28	1.03	1.03	0.97	0.95	1.00
iSS1	1.5630	0.80	0.98	-	-	-	0.98
iSS2	2.2250	2.28	1.15	-	-	-	1.15

* = Amplitude of SS analyzed separated / Amplitude of SS analyzed when attached to the PS

** A difference of less than 1 means the amplitude of "SS on PS" is bigger than "SS separated"

The effect of the damping ratio for secondary systems

One would expect that a change in the damping ratio only would affect the acceleration amplitudes and the same reduction/amplification of the accelerations should be seen for both the floor response method and the method where the SS is attached to the PS. It is seen for the secondary systems with natural frequencies inside the resonant frequency range of the primary system that the damping ratio seems to govern which method of measuring the response that has the largest amplitudes. This is an interesting occurrence that is explored further in this chapter.

Secondary system 6 has the largest average difference in amplitude, but it also has the lowest damping ratio (0.08 %). Secondary system 7 has the highest damping ratio (5.77 %), and the amplitude of the "SS on PS" tends to be larger than for the "SS separated. These two "coincidences" indicates that the damping ratio acts differently for the two methods.

In order to find out what effect the damping ratio has on the response, new damping ratios has been applied to the secondary systems. The new damping ratios are shown in Table 5.9.

Table 5.9. New damping ratios for the secondary systems.

Secondary Structure				
Secondary System	Natural frequency (Hz)	Damping ratio 1 (%)	Damping ratio 2 (%)	Damping ratio 3 (%)
1	1.2024	(0.42)*	2.00	5.00
2	1.8092	0.08	2.00	5.00
3	1.3513	(0.07)*	2.00	5.00
6	1.6684	(0.08)*	2.00	5.00
7	1.6999	0.08	2.00	(5.77)*
8/9	1.9577	0.08	(2.28)*	5.00
* () means that the damping ratio has been tested before				

The effect of the damping ratio for secondary systems with natural frequencies inside the resonant frequency range of the primary system

The responses to the new damping ratios for secondary systems 6 and 7 are shown in the figures that follow.

Damping ratio equal to 0.08 % - low damping ratio

The response of SS6 with 0.08 % has already been shown, see Figure 5.38. The response of SS7 with the same damping ratio is shown below, and the same thing that happened to SS6, happens to SS7. The response of when the secondary system is separated is larger and the shape is not the same.

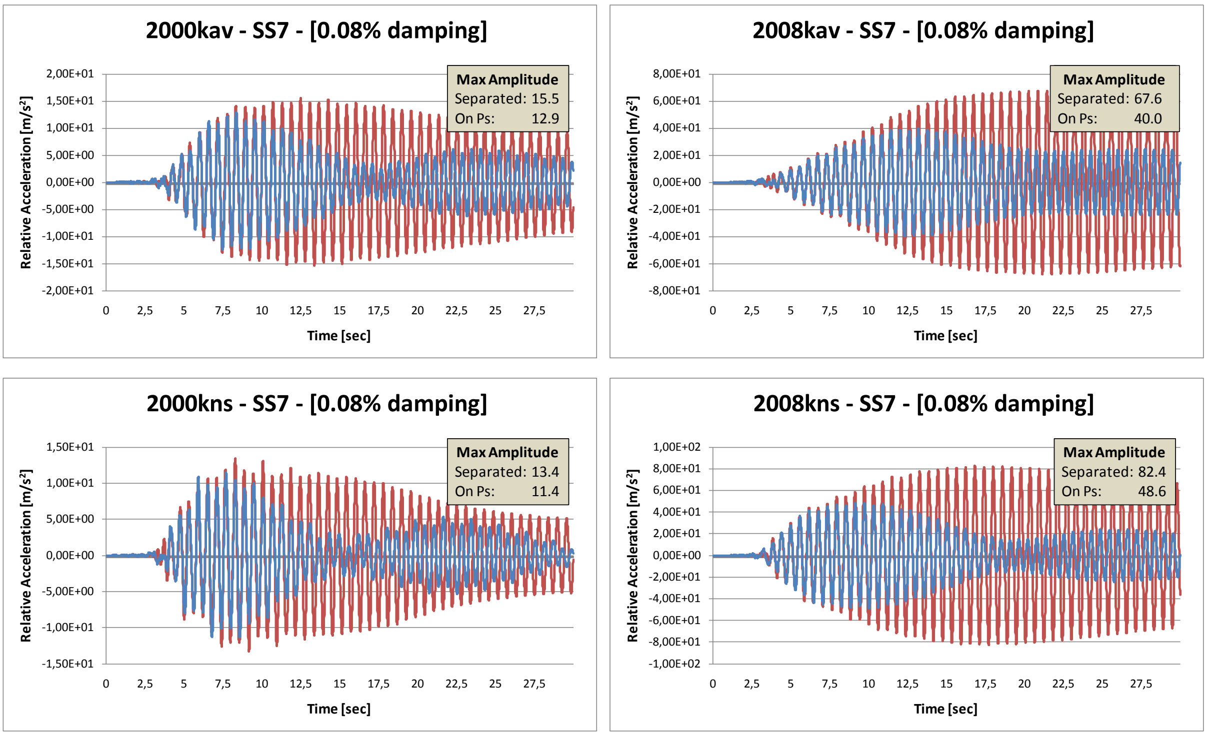

Figure 5.42. Comparison of relative acceleration between SS7 analyzed separately (red curve) and SS7 analyzed jointly (blue curve) - [0.08 % damping], fss/fps = 1.015.

Damping ratio equal to 2.0 % - medium damping ratio

Both secondary systems 6 and 7 are now excited with a damping ratio of 2.0 %. The maximum amplitudes are very similar to each other. The shaped are varying, but in general they are similar.

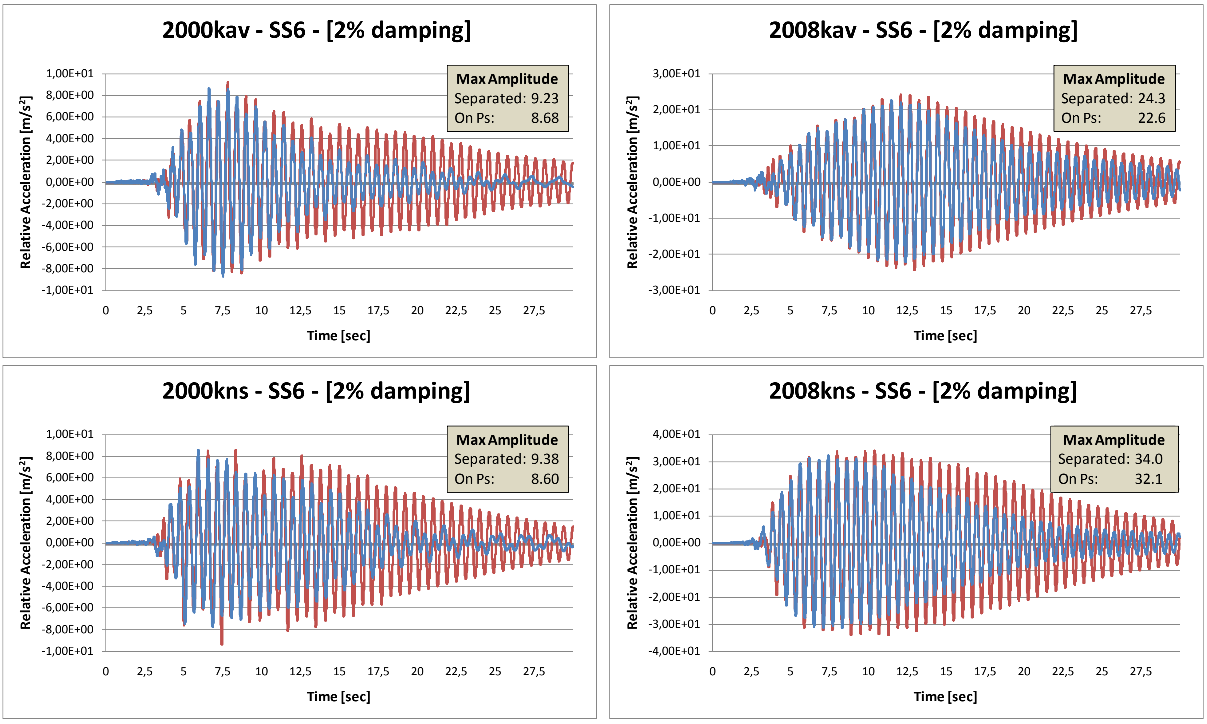

Figure 5.43. Comparison of relative acceleration between SS6 analyzed separately (red curve) and SS6 analyzed jointly (blue curve) - [2.0 % damping], fss/fps = 0.996.

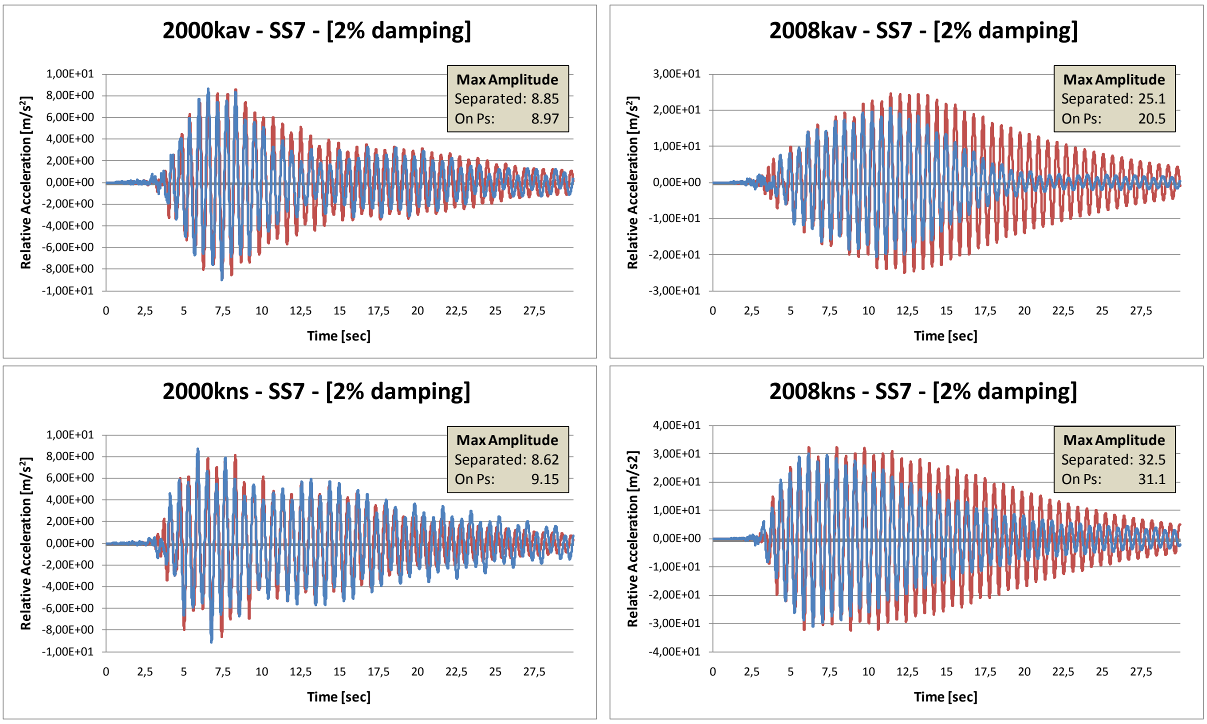

Figure 5.44. Comparison of relative acceleration between SS7 analyzed separately (red curve) and SS7 analyzed jointly (blue curve) - [2.0 % damping], fss/fps = 1.015.

Damping ratio equal to 5.0 % - high damping ratio

Secondary system 7 has been tested for a damping ratio of 5.77 %, which is shown in Figure 5.39. A damping ratio of 5.0 % is applied to secondary system 6 and the results are shown below. The amplitudes of the secondary system when analyzed jointly (SS on PS) are larger for this damping ratio. This was also seen in the test for secondary system 7.

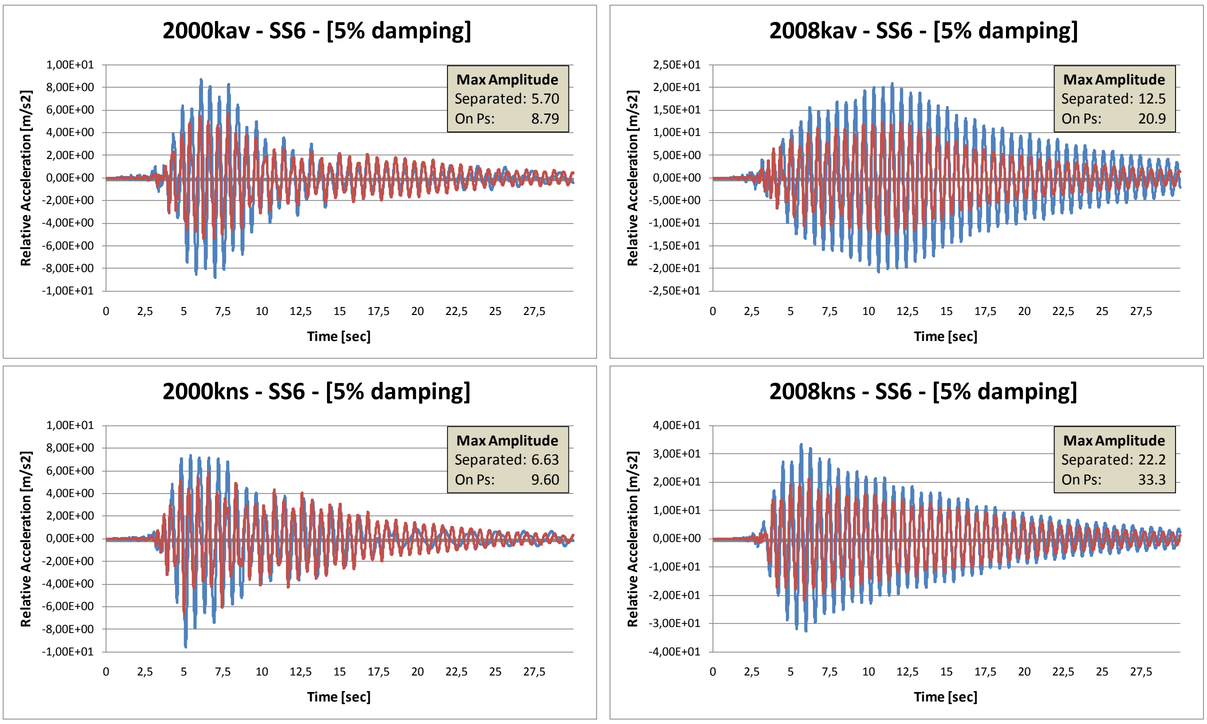

Figure 5.45. Comparison of relative acceleration between SS6 analyzed separately (red curve) and SS6 analyzed jointly (blue curve) - [5.0 % damping], fss/fps = 0.996.

Summary for the different damping ratios

The following statements are only valid for secondary systems with nat. freq. inside the resonant frequency range of the PS.

After these four new tests it is safe to say that the damping ratio affects the accuracy of the analyses using a decoupled SS and floor response excitation. If the damping ratio is low (0.08 %), the two methods responses of the secondary system do not have the same shape. The response made from the floor excitation has much higher amplitude (≈40 %) and does not have the same damped curve.

The amplitudes for the medium damping ratio (2.0 %) are more or less the same, but the average amplitudes made from the floor excitation are higher.

For the secondary systems with high damping (5.0 %), the amplitudes of the secondary system when analyzed jointly are higher than the amplitudes for the floor excitation. This makes the floor response method the less safe method for this damping ratio.

From the above observations it is clear that the primary system affects the secondary system with its damping ratio (0.8 %). When the damping ratio of the SS is much lower than the PS damping ratio, the PS in effect heightens the damping ratio of the SS when it is attached to the PS. When the damping ratio is much higher than the PS damping ratio, the PS in effect reduces the damping for the SS when it is attached to the PS. When the damping ratio is similar, the primary system has a small effect.

The effect of the damping ratio for secondary systems with natural frequencies outside the resonant frequency range of the primary system

The damping ratio for secondary systems with natural frequencies outside the resonant frequency range of the primary system does not have the same effect on the response as the ones with natural frequencies inside the resonant frequency range. The only effect seen from changing the damping ratio is that the amplitudes are higher for the systems with lower damping. The maximum amplitudes are similar and the amplitude of the secondary system when it is separated is larger. More tests with different earthquakes and secondary systems can be seen in Appendix B: "The effect of the damping ratio for SS during earthquakes".

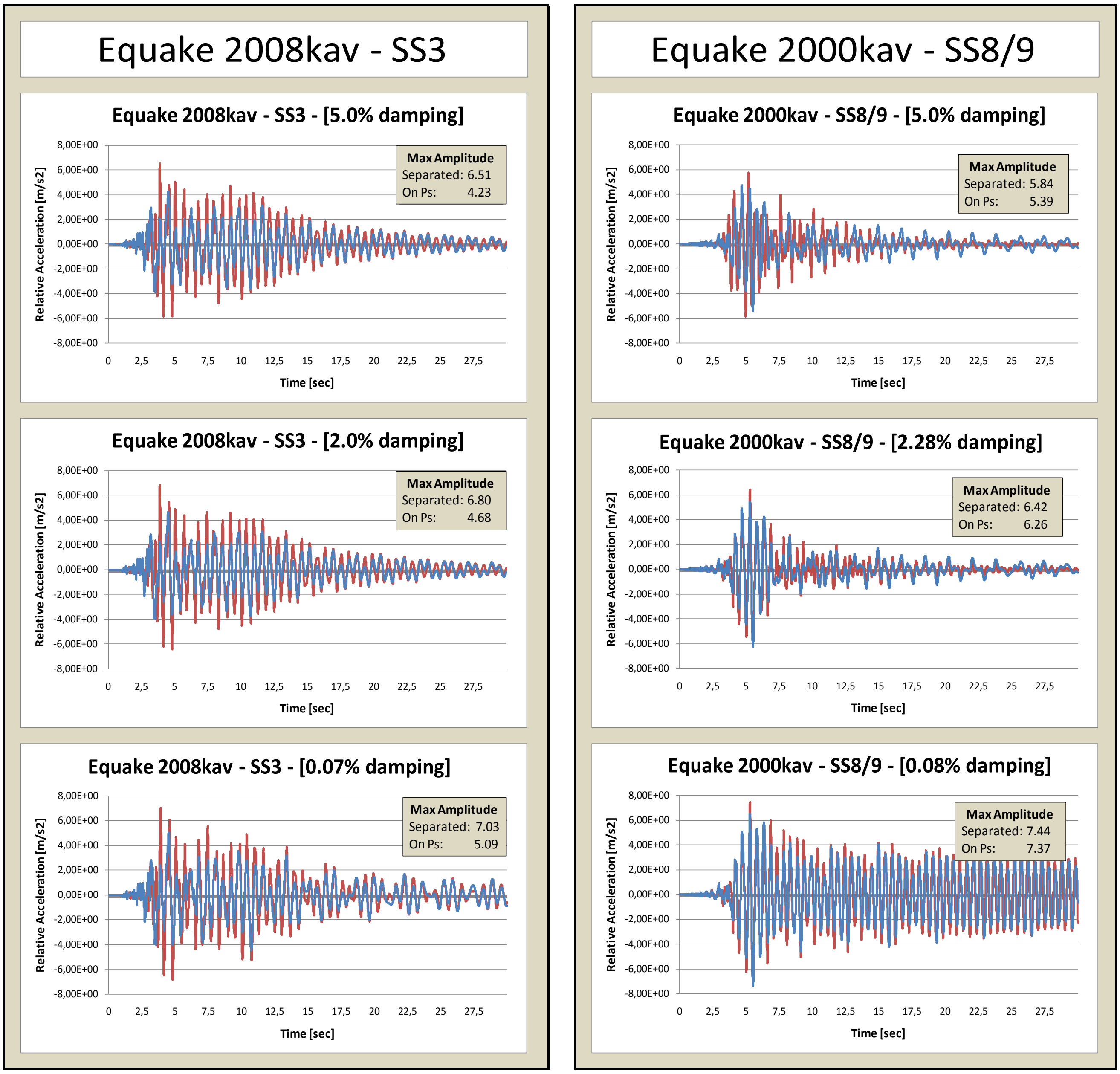

Figure 5.46. Comparison of relative acceleration between SS (8/9 and 3) analyzed separately (red curve) and SS (8/9 and 3) analyzed jointly (blue curve], fss3/fps = 0.807, fss8/9/fps = 1.169.

Summary of the difference in amplitudes for the two methods

Table 5.10 shows the average difference between the maximum amplitudes for the two methods of measuring. The differences in amplitudes for the secondary systems tested by J.T. Snæbjörnsson et al. [2] in 2000 with the original damping ratio and with the additional damping ratios are shown, as well as the two imaginary secondary systems.

Table 5.10. Maximum amplitude difference between the secondary system analyzed separately (floor response method) and secondary system analyzed jointly (SS on PS).

Maximum Amplitude Difference: SS analyzed separated divided by SS analyzed on the PS						
System No.	Damping ratio (%)	2000kav	2000kns	2008kav	2008kns	Average Difference
		Difference*	Difference*	Difference*	Difference*	
SS1 (1.2024 Hz)	0.42	1.53	0.97**	1.47	1.43	1.35
	2.00	1.54	-	-	-	1.54
	5.00	1.55	-	-	-	1.55
SS2 (1.8092 Hz)	0.08	0.98	-	-	-	0.98
	0.66	1.02	1.00	1.04	1.04	1.02
	2.00	1.06	-	-	-	1.06
	5.00	1.08	-	-	-	1.08
SS3 (1.3513 Hz)	0.07	1.56	1.01	1.38	1.19	1.29
	2.00	-	-	1.45	-	1.45
	5.00	-	-	1.54	-	1.54
SS6 (1.6684 Hz)	0.08	1.82	1.58	1.77	1.91	1.77
	2.00	1.06	1.09	1.08	1.06	1.07
	5.00	0.65	0.69	0.60	0.67	0.65
SS7 (1.6999 Hz)	0.08	1.20	1.18	1.69	1.70	1.44
	2.00	0.97	0.94	1.18	1.05	1.04
	5.77	0.72	0.81	0.72	0.71	0.74
SS8/9 (1.9577 Hz)	0.08	1.01	-	-	-	1.01
	2.28	1.03	1.03	0.97	0.95	1.00
	5.00	1.08	-	-	-	1.08
iSS1 (1.5630 Hz)	0.80	0.98	-	-	-	0.98
	2.00	1.06	-	-	-	1.06
	5.00	1.03	-	-	-	1.03
iSS2 (2.2250 Hz)	2.28	1.15	-	-	-	1.15

* = Amplitude of SS analyzed separated / Amplitude of SS analyzed when attached to the PS

** A difference of less than 1 means the amplitude of "SS on PS" is bigger than "SS separated"

Summary of the damping ratios effect

The figure below shows the different effect the damping ratio has on the floor response method and the method when the secondary system is attached to the primary system. The data plotted is the data found in Table 5.10. The x-axis is the natural frequency of the secondary system divided by the natural frequency of the primary system, i.e., the value where the natural frequencies are the same is 1. The y-axis is the ratio of difference between the two methods, i.e., the maximum (average) acceleration of the floor response spectrum divided by the maximum (average) acceleration of the analysis when the secondary system is attached to the primary system. When the values are negative it means that the floor response method gives lower amplitudes than the actual amplitudes.

The figure illustrates what has been said in the subchapters earlier in this chapter. When the natural frequency of the secondary system is far below the resonant frequency range of the primary system the amplitudes generated by the floor response are about 1.5 times higher. When the natural frequencies are just outside the resonant frequency range the differences are very small, and when the natural frequencies are inside the resonant frequency range the differences are big for 0.08 % and 5 % damping ratios and the floor response method gives lower amplitudes than the actual amplitudes for the 5 % damping ratio.

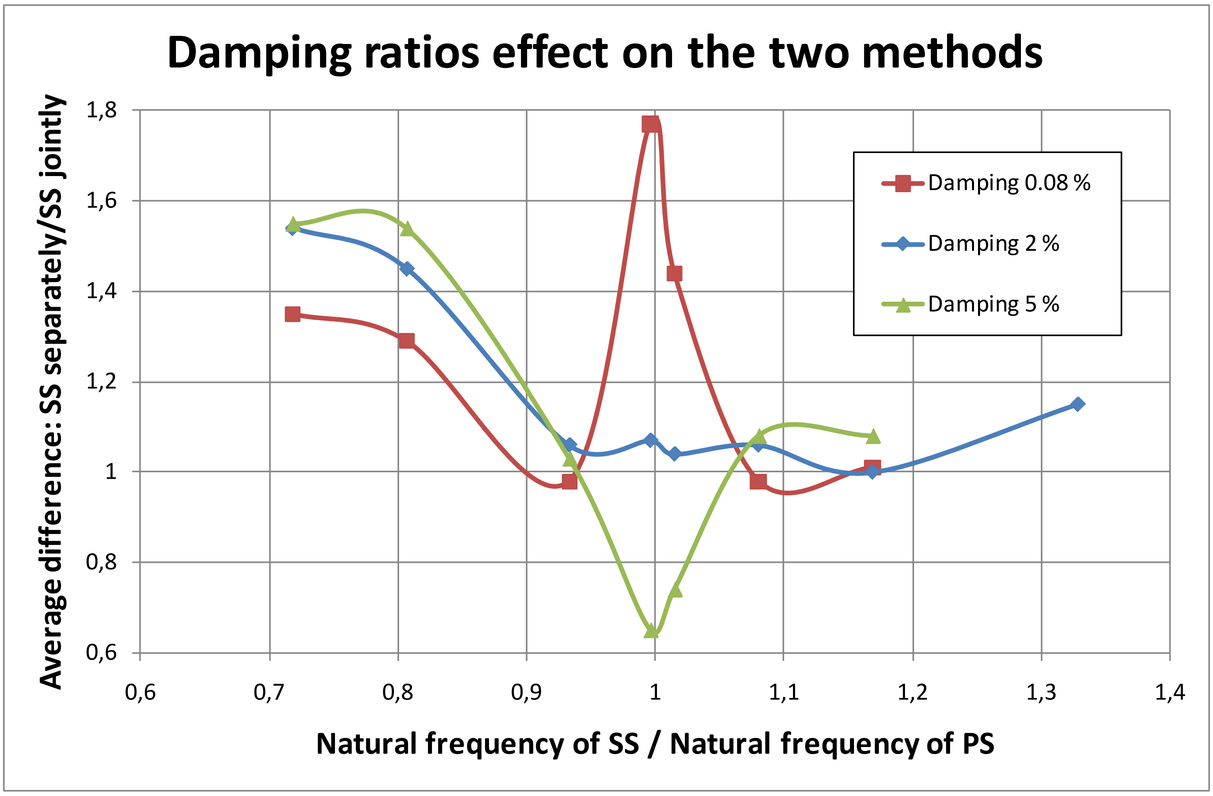

Figure 5.47. The effect the damping ratios have on the difference in the acceleration for the floor response method and the method where the secondary system is attached to the primary system. Average difference is "Amplitude of the floor response method / Amplitude of the secondary system analyzed jointly with PS".

6. Codal provisions for design of non-structural elements

Over the past 40 years there have been performed a lot of research on developing methods to analyze the non-structural elements during seismic excitation. However, these methods were mainly focused on the safety of critical equipment in, for example, power plants. In the later years it has been shown that the non-structural elements in conventional buildings also should be taken into consideration in the earthquake design process.

The non-structural elements are often neglected since they are usually added after the building is erected and not always predefined. Also, the focus of the engineers has traditionally been to prevent structural failure.

One example demonstrating the importance of considering the performance of non-structural elements comes from the Northridge earthquake [13] that occurred on January 21, 1994. It had a moment magnitude of 6.7 and caused more than $40 billion in damage. Amongst the damaged buildings were several hospitals (for example, Olive View, Holy Cross Hospital, etc.). All of these hospitals had to be shut down because of damage to non-structural elements. As a result, a law was made that stated that all acute care units and emergency rooms had to be in earthquake-proof structures by January 1, 2005.

6.1 Eurocode 8

Eurocode 8 is the European building code for design of structures for earthquake resistance and is developed by the European Committee for Standardization (CEN) [14]. It has been made mandatory for European public works since March 2010, although it will co-exist with the old standards in many countries for a period.

Eurocode 8 [15] states that for very important and / or dangerous non-structural elements the floor response spectra method is to be used.

> *"For non-structural elements of great importance or of a particularly dangerous nature, the seismic analysis shall be based on a realistic model of the relevant structures and on the use of appropriate response spectra derived from the response of the supporting structural elements of the main seismic resisting system."*

For all other cases the elements have to resist a design seismic load, F_a, as follows:

$$F_a = S_a \, W_a \, \gamma_a \; / \; q_a \qquad \textbf{(6.1)}$$

where F_a is the horizontal (or vertical) seismic force, W_a is the mass of the element, S_a is the spectral acceleration, γ_a is the importance factor of the element and q_a is the behavior factor of the element.

The spectral acceleration is given by the following equation:

$$S_a = \alpha\, S\, [\, 3\,(1 + z/H)/\left(\, 1 + (1 - T_a/T_1)^2\, \right) - 0.5\,] \tag{6.2}$$

where α is the ratio of the design ground acceleration on type A ground, a_g, to the acceleration of gravity, S is the soil factor, T_a is the fundamental natural period of the non-structural element, T_1 is the fundamental natural period of the primary structure, z is the height of the non-structural element above rigid foundation and H is the height of the primary structure above rigid foundation. Eurocode 8 uses a standard damping ratio of 5.0 % for all the equations.

To get the seismic coefficient, the α variable is taken out of the equation. The soil factor is assumed to be 1 (stiff soil), the height of the non-structural element above rigid foundation is 16.025 m and the height of the primary structure above rigid foundation is 25.26 m, thus the seismic coefficient is:

$$C_a = S\left[\, 3\,(1 + z/H)/\left(\, 1 + (1 - T_a/T_1)^2\, \right) - 0.5\, \right] = 4.86\, /\left(\, 1 + (1 - T_a/T_1)^2\, \right) - 0.5 \tag{6.3}$$

6.1.1 Seismic Coefficient comparison between Eurocode 8 and FE model

The curve made by the seismic coefficient equation is shown below. The seismic coefficient generated by equation **6.3** for each secondary system is marked with a colored box. From the curve it is seen that a secondary system with a high natural period, (low natural frequency), gets a lower coefficient than a secondary system with a low natural period (high natural frequency).

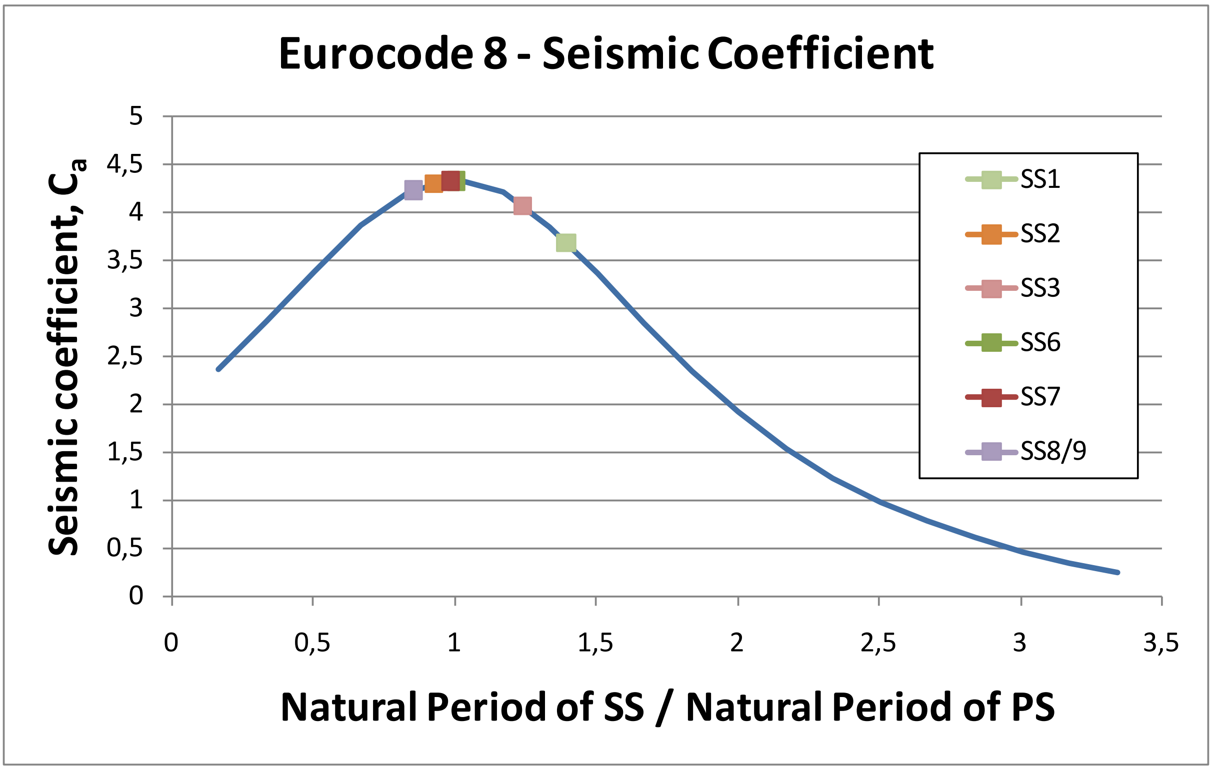

Figure 6.1. Seismic Coefficients generated by Eurocode 8 (Equation 6.3). Colored boxes mark the secondary systems seismic coefficient.

The actual seismic coefficients for 2000kav, i.e., the ratio between the max total acceleration of the secondary system analyzed jointly with the primary system divided by the max total acceleration of the ground, are shown together with the seismic coefficient calculated from the Eurocode 8 (Equation **6.3.**) in Table 6.1. The seismic coefficients generated by the analysis where the secondary systems are analyzed separately (floor response) are shown in the same table. The same results are also shown in Figure 6.2.

Earthquake 2000kav is representative for all four earthquakes. The same tables and figures for the three other earthquakes are found in Appendix B: Seismic Coefficient comparison.

The differences between the Eurocode 8 equation and the actual seismic coefficients are relatively large; especially for secondary system no. 6 and 7. The seismic coefficients from the Eurocode 8 equation are larger for SS1 and SS3 and lower for SS6 and SS7. This can be seen for all four earthquakes. The differences between the floor response (SS analyzed separately) and Eurocode 8 are much lower than for the actual seismic coefficient (SS analyzed jointly).

The damping ratios for all the secondary systems are 5 %, because Eurocode 8 uses a 5 % damping ratio as standard. From Figure 5.47 it is seen that this damping ratio makes the relative acceleration of the analysis where the secondary system is attached to the primary system bigger for secondary system no. 6 and 7 than for the analysis where the secondary system is analyzed separately. From Table 6.1 it is seen that the total acceleration response of the “SS analyzed jointly” is larger than the “SS analyzed separately” for all secondary systems.

Eurocode 8 states that for very important and / or dangerous non-structural elements the floor response spectra method is to be used. This method corresponds well to equation **6.3**, but the response is lower than the actual response for all the secondary systems. The biggest differences between the responses occur when the secondary systems have natural frequencies inside the resonant frequency range of the primary systems (SS6 and SS7). The other secondary systems show a relatively good comparison between the responses.

Table 6.1. Comparison of seismic coefficients for earthquake 2000kav between the actual seismic coefficients, the seismic coefficients generated when the floor response method is used and the seismic coefficients calculated from the Eurocode 8 standard (Equation 6.3).

Earthquake 2000kav				
Secondary System	Ta/T1	Max SS total accel. / Max ground total accel.		Eurocode 8
		SS analyzed jointly	SS analyzed seprately	C_a - Equation 6.3
SS1 - 5 % damping	1.393	1.38	1.35	3.68
SS2 - 5 % damping	0.926	5.81	5.29	4.30
SS3 - 5 % damping	1.240	2.03	1.98	4.07
SS6 - 5 % damping	1.004	8.54	4.96	4.33
SS7 - 5 % damping	0.985	7.31	4.86	4.33
SS8/9 - 5 % damping	0.856	4.62	4.23	4.23

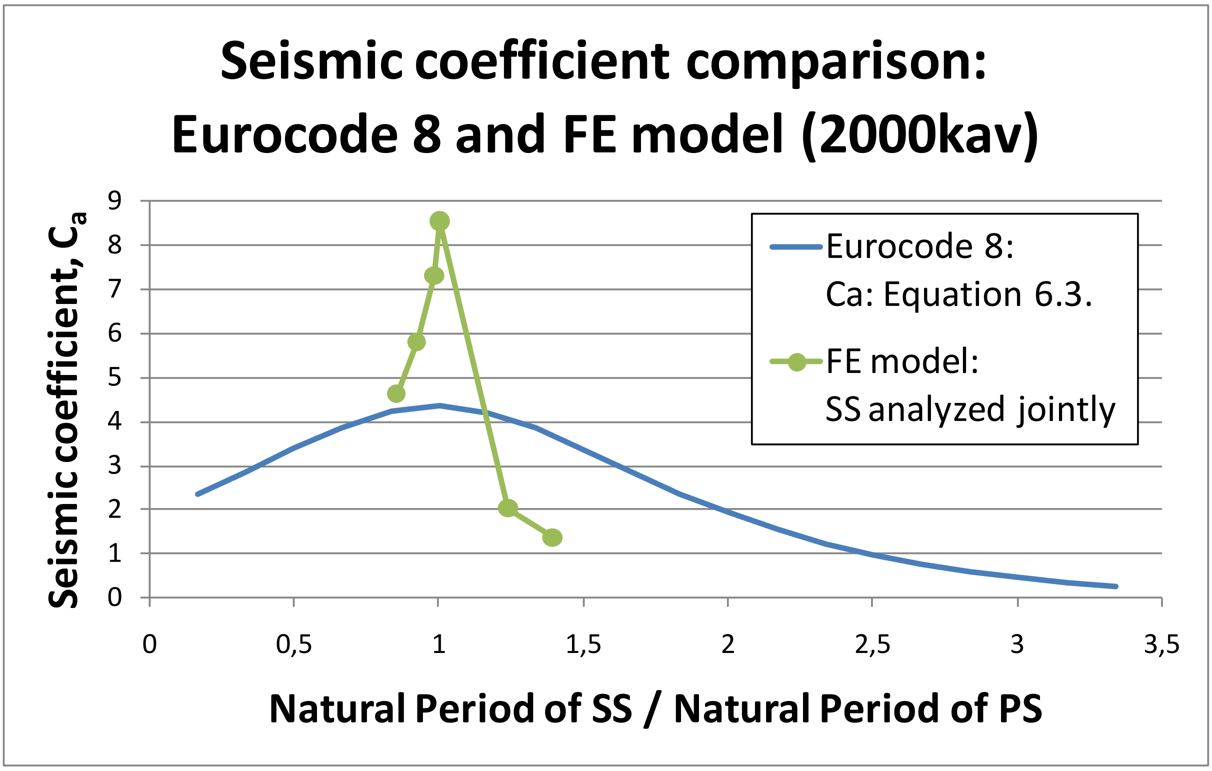

Figure 6.2. Comparison of seismic coefficients for earthquake 2000kav between the actual seismic coefficients and the seismic coefficients calculated from the Eurocode 8 standard (Equation 6.3). Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Green curve means that the secondary system is analyzed jointly with the primary system.

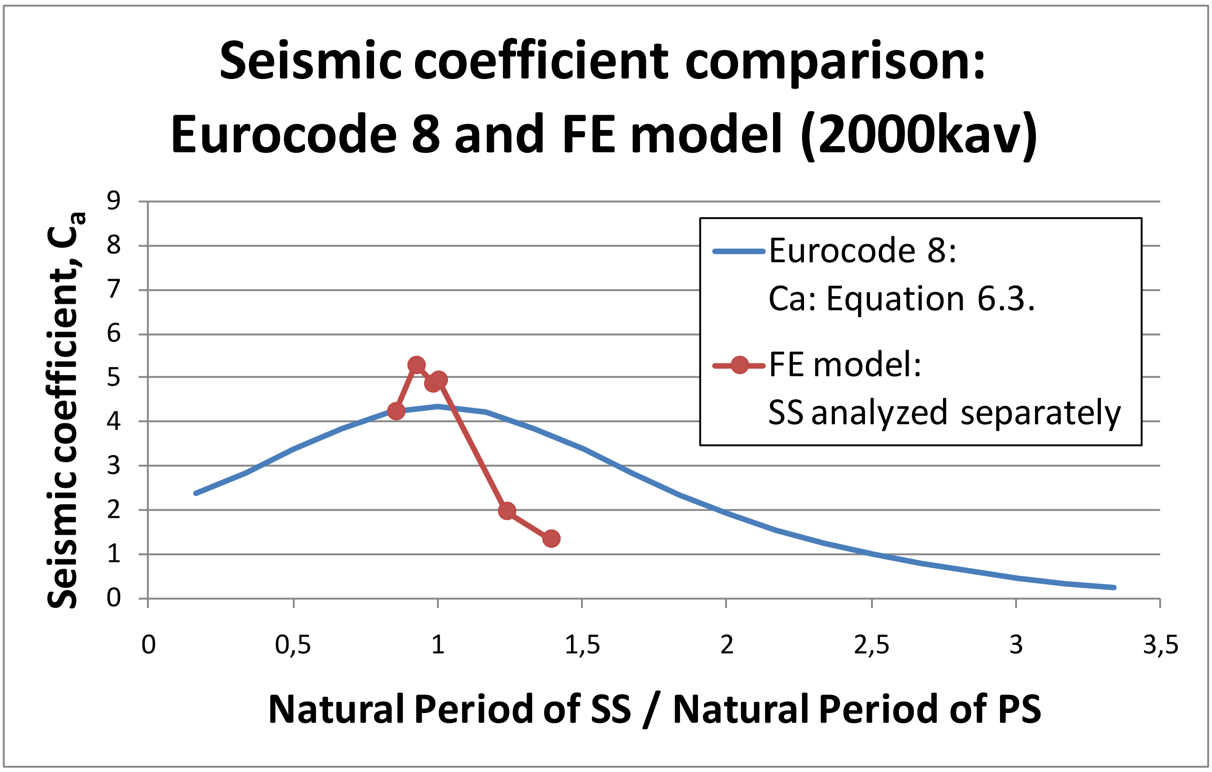

Figure 6.3. Comparison of seismic coefficients for earthquake 2000kav between the seismic coefficients generated when the floor response method is used and the seismic coefficients calculated from the Eurocode 8 standard (Equation 6.3). Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Red curve means that the secondary system is analyzed separately (floor response method).

7. Conclusion

In this thesis a numerical FE model of the Cardington Building has been made, along with FE models for the various secondary systems. Analyses of the FE models have been performed, both separately and jointly, during harmonic excitations and earthquake excitations. Also, the seismic coefficients obtained from the earthquake excitations have been compared with the seismic coefficients calculated by equations from the Eurocode 8 standard.

From the harmonic excitation tests it is seen that the secondary systems with their natural frequencies inside the resonant frequency range of the primary system get a much higher displacement than the ones outside the resonant frequency range, but in exchange the displacement of the primary system is reduced. For secondary system 6 the displacement of the primary system drops by 34 % at the fourth floor and by 35 % at the top floor (Figure 5.10 and Figure 5.11). This is true for secondary systems with both 0.8 % damping and 5.0 % damping.

In the earthquake excitation tests a floor response spectra method, represented in this thesis by analyzing the secondary systems separately, has been compared to the responses generated when the secondary system is analyzed jointly with the primary system. There are big differences in amplitudes between the two methods of analyzing the response, but the floor response gives, for the most part, a larger response. This makes the floor response a more safe method to use, but in some cases it gives amplitudes that are approximately 30 % higher, Table 5.10 (not counting SS6 and SS7).

The natural frequency of the secondary system does make a difference in the response between the two ways of measuring. A natural frequency far away from the resonant frequency range of the primary system tends to give the secondary systems (SS1, SS3 and iSS2) a large difference in amplitude between the two methods, but always on the safe side, which in this case means that the floor response analysis gives higher amplitudes. This same pattern seems to be true for all damping ratios.

The secondary systems with natural frequencies just outside the resonant frequency range of the primary system (SS2, SS8/9 and iSS1) show a very good comparison between the two methods in both the amplitudes and the shape of the response. The differences do not vary much with a change in the damping ratio.

The big differences in the response, both in amplitude and in shape, happens when the secondary system has a natural frequency that is inside the resonant frequency range of the primary system (SS6 and SS7). For a low damping ratio (0.08 %) the response of the secondary system when analyzed separately has much higher amplitudes (≈40 %) than given by the analysis where the secondary system is mounted on the primary system.

If the secondary system has at least a medium damping ratio (2.0 %) the differences in the amplitudes are not great, and the response of the two methods compare well to each other.

The secondary systems with a high damping ratio (5.0 %) will make the amplitudes of the floor response lower than the ones measured from the analysis where the secondary system is mounted on the primary system. This means that the real response is higher than the one obtained from the floor response method. Standards, like Eurocode 8, normally use a damping ratio of 5.0 %, which makes this a notable effect.

It is clear that the primary system affects the secondary system with its damping ratio (0.8 %) when the secondary system has a natural frequency that is inside the resonant frequency range of the primary system. When the damping ratio of the secondary system is much lower than the damping ratio of the primary system, the primary system in effect heightens the damping ratio of the secondary system when it is attached to the primary system. When the damping ratio is much higher than the damping ratio of the primary system, the primary system in effect reduces the damping for the secondary system when it is attached to the primary system. For the secondary systems with natural frequencies that are outside the resonant frequency range of the primary system, the damping ratio of the primary system does not seem to have any effect on the secondary system's response.

In spite of the various shortcomings of the floor response approach discussed above it is found to give a reasonably good description of the overall response of the secondary system.

The seismic coefficients calculated by the Eurocode 8 equation correspond well to the floor response method's seismic coefficients, but the actual seismic coefficients (secondary system analyzed jointly with the primary system) did not correspond as well, especially for secondary system no. 6 and 7. These are the two secondary systems with natural frequencies inside the natural frequency range of the primary system. The actual seismic coefficient for secondary system 6 during earthquake 2000kav is twice as high as the seismic coefficient calculated by the equation from the Eurocode 8 standard (Figure 6.2).

The Eurocode 8 standard states that the equation should only to be used for secondary systems that are not very important and / or dangerous. This makes the error less severe, but the equation should be representative none the less. The seismic coefficient at the natural frequency of the primary system is too low.

8. References

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Appendix A:

Theoretical Background

Single Degree of Freedom

Simple structures can be modeled as a single-degree-of-freedom (SDOF) system. A water tank with a massless tower is the most common example, along with the idealized one-story frame. The structure is defined by a system that consist of a mass, spring, damper and force (Figure A.1).

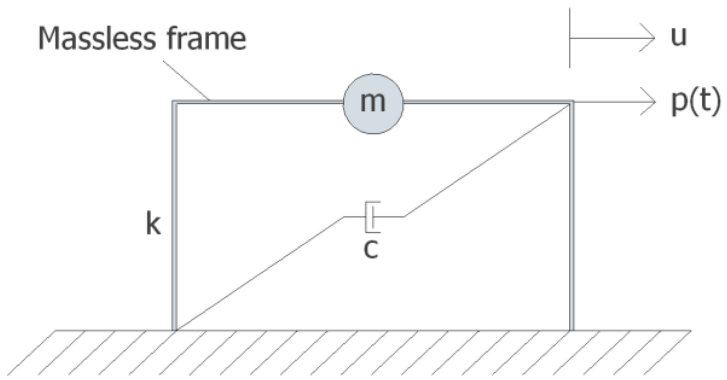

Figure A.1. Idealized one-story frame.

Normally, the first goal is to find the response of the system through the equation of motion. One way of establishing the equation of motion is by Newton's Second Law, which states that force is equal to the mass (m) times acceleration ($\ddot{u}$).

$$F = m\,\ddot{u} \tag{A.1}$$

By drawing a free-body diagram of the system (Figure A.2) and applying Newton's Second Law, the equation of motion can be found.

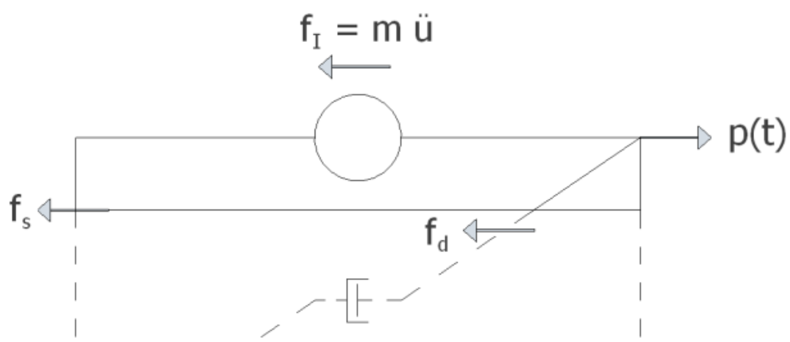

Figure A.2. Free-body diagram of the one-story frame.

The forces acting on the system are:

$$F = p(t) - f_s - f_d \quad \textbf{(A.2)}$$

where f_s is the spring force and f_d is the damping force. The spring force and damping force can be rewritten:

$$f_s = k\,u \quad \textbf{(A.3)}$$

$$f_d = c\,\dot{u} \quad \textbf{(A.4)}$$

where k is the stiffness, c is the damping, u is displacement and $\dot{u}$ is velocity. Substituting equation **(A.1)** into **(A.2)** and using the definitions from equation **(A.3)** and **(A.4)**:

$$m\,\ddot{u} + c\,\dot{u} + k\,u = p(t) \quad \textbf{(A.5)}$$

This is called the *equation of motion*.

Stiffness

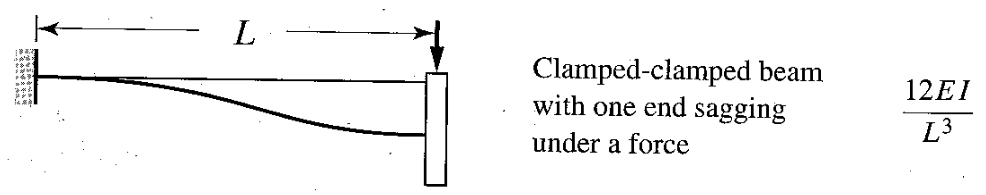

Figure A.3. Equivalent Spring Constants ([4],page 38).

The stiffness, k, can be determined, if the top beam is rigid, in the following way:

$$k = \frac{12\,E\,I}{L^3} \quad \textbf{(A.6)}$$

where E is the Young's modulus (modulus of elasticity), I is the moment of inertia (second moment of area) and L is the length of the element.

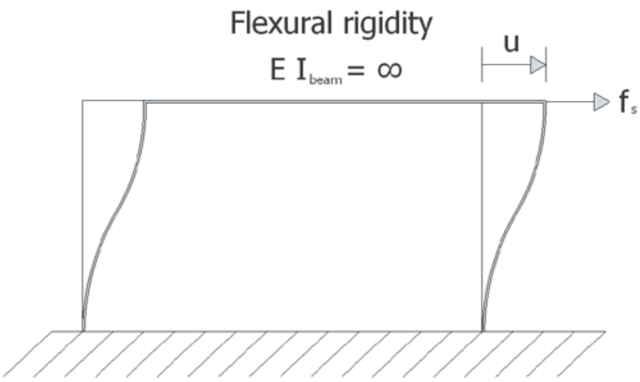

Figure A.4. Flexural rigidity ([7], page 9).

Multiple degree of freedom

The SDOF system can be expanded into a two degree of freedom system which is usually expressed in a matrix form ([7], page 345-348):

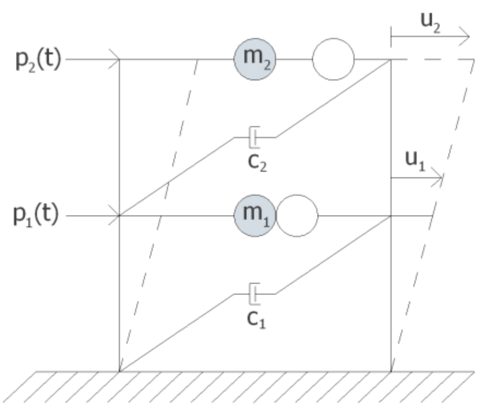

Figure A.5. Two-story frame.

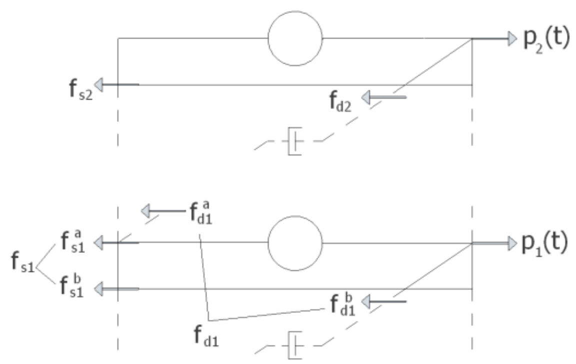

Figure A.6. Free-body diagram of the two-story frame.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} f_{d1} \\ f_{d2} \end{bmatrix} + \begin{bmatrix} f_{s1} \\ f_{s2} \end{bmatrix} = \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} \tag{A.7}$$

or, in matrix notation:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}} + \boldsymbol{f_d} + \boldsymbol{f_s} = \boldsymbol{p}(t) \tag{A.8}$$

where $\boldsymbol{m}$ is the lumped mass matrix. The next step is to find the damping matrix, $\boldsymbol{f_d}$, and the stiffness matrix, $\boldsymbol{f_s}$. The force f_{s1} is made up of contributions from both the first and second floor:

$$f_{s1} = f_{s1}^a + f_{s1}^b = k_1\,u_1 + k_2\,(u_1 - u_2) \tag{A.9}$$

The force f_{s2} is made up of contributions only from the second floor:

$$f_{s2} = k_2\,(u_2 - u_1) \tag{A.10}$$

This gives the following force-displacement relation:

$$\boldsymbol{f_s} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{k}\,\boldsymbol{u} \tag{A.11}$$

The damping force, $\boldsymbol{f_d}$, can be treated in the same way, which gives:

$$\boldsymbol{f_d} = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \boldsymbol{c}\,\dot{\boldsymbol{u}} \tag{A.12}$$

Substituting equation (**A.11**) and (**A.12**) into (**A.7**):

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} p_1(t) \\ p_2(t) \end{bmatrix} \tag{A.13}$$

or in matrix notation:

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}} + \boldsymbol{c}\,\dot{\boldsymbol{u}} + \boldsymbol{k}\,\boldsymbol{u} = \boldsymbol{p}(t) \tag{A.14}$$

The equation of motion looks the same as for the SDOF, but it is now in matrix form and the equation is *coupled* with regard to the damping and stiffness.

To include more DOF's, the same methodology can be applied again.

Free vibration

To find the natural frequencies of the system, one must look at the structure when it is undergoing free vibration. Free vibration means that if the structure is pulled out of its equilibrium position it can vibrate without any external dynamic forcing ($\boldsymbol{p}(t) = 0$). A convenient start is to look at undamped vibrations, which means that **c** = **0** for the time being.

The free vibration of a linear undamped multiple degree of freedom system is then ([7], page 39):

$$\boldsymbol{m}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{k}\,\boldsymbol{u}(t) = \boldsymbol{0} \tag{A.15}$$

in which $\boldsymbol{0}$ is a zero vector. A solution to the free vibration can be described as:

$$\boldsymbol{u}(t) = q_n(t)\,\phi_n \qquad (\, n \leq \text{Number of degrees of freedom} = \text{N}\,) \tag{A.16}$$

where ϕ_n are the natural modes of the system, and $q_n(t)$ is a harmonic function:

$$q_n(t) = a_n \cos\omega_n t + b_n \sin\omega_n t \tag{A.17}$$

where a_n and b_n are constants that can be determined from the initial conditions and ω_n are the natural frequencies of the system.

This can also be written as:

$$q_n(t) = c_n\, e^{i\omega_n t} \tag{A.18}$$

Combining equation (3) and (4) gives:

$$\boldsymbol{u}(t) = \phi_n\,(a_n \cos\omega_n t + b_n \sin\omega_n t) \tag{A.19}$$

To substitute this equation back into the free vibration equation (2), the second time derivative of $\boldsymbol{u}(t)$ is also needed.

First derivative:

$$\dot{\boldsymbol{u}}(t) = \phi_n\,(-\omega_n\, a_n \sin\omega_n t + \omega_n\, b_n \cos\omega_n t) \tag{A.20}$$

Second derivative:

$$\ddot{\boldsymbol{u}}(t) = \phi_n \left(-\omega_n{}^2\, a_n \, \cos\omega_n t \, - \, \omega_n{}^2\, b_n \, \sin\omega_n t\right) \qquad \textbf{(A.21)}$$

Isolating the expression " $-\omega_n{}^2$ ":

$$\ddot{\boldsymbol{u}}(t) = \phi_n \left(-\omega_n{}^2\right) \left(a_n \, \cos\omega_n t + \, b_n \, \sin\omega_n t\right) \qquad \textbf{(A.22)}$$

This equation can now be written as:

$$\ddot{\boldsymbol{u}}(t) = \phi_n \left(-\omega_n{}^2\right) q_n(t) \qquad \textbf{(A.23)}$$

Substituting equation (**A.16**) and (**A.23**) into equation (**A.15**) gives:

$$\left[-\omega_n{}^2 \, \boldsymbol{m} \; \phi_n + \, \boldsymbol{k} \, \phi_n \, \right] q_n(t) = \boldsymbol{0} \qquad \textbf{(A.24)}$$

This equation can be satisfied in two ways. One way is saying that $q_n(t) = 0$, but that implies $\boldsymbol{u}(t) = \boldsymbol{0}$, which means that there is no motion. This is called a trivial solution, and does not give a useful result. Instead, let the equation $-\omega_n{}^2 \, \boldsymbol{m} \; \phi_n + \boldsymbol{k} \, \phi_n$ equal to 0. In other words, the following equation must be satisfied:

$$\boldsymbol{k} \, \phi_n = \omega_n{}^2 \, \boldsymbol{m} \; \phi_n \qquad \left(\boldsymbol{k} \; = \omega_n{}^2 \, \boldsymbol{m} \right) \qquad \textbf{(A.25)}$$

This is called the *matrix eigenvalue problem*. The equation can be written as follows:

$$\left[\boldsymbol{k} \; - \omega_n{}^2 \, \boldsymbol{m} \, \right] \phi_n = \boldsymbol{0} \qquad \textbf{(A.26)}$$

Again there exists two different ways to satisfy the equation, but setting $\phi_n = 0$ gives a trivial solution, i.e., no motion. The equation has a nontrivial solution if:

$$det \left[\boldsymbol{k} \; - \omega_n{}^2 \, \boldsymbol{m} \, \right] = \boldsymbol{0} \qquad \textbf{(A.27)}$$

From this equation, the natural frequencies are calculated, and to get the natural mode shapes, ϕ_n, the known natural frequencies are inserted into equation (**A.26**) and solved for ϕ_n. However, this does not give the absolute values for the vectors ϕ_n, but only the shape of the vector. This means that it can be scaled. A popular way of scaling is called normalization of modes, which is discussed later.

Orthogonality of Modes

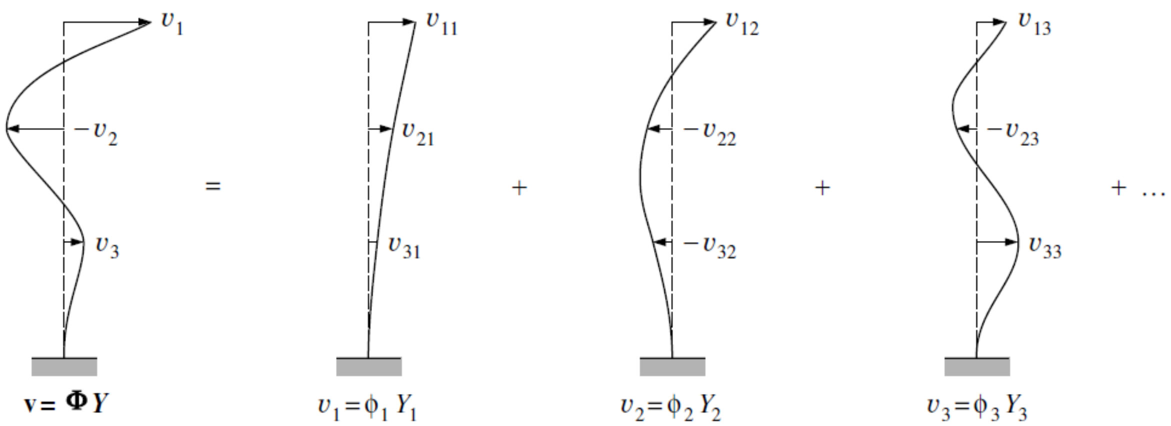

Figure A.7. Representing deflections as sum of modal components ([3], page 220)

The displacement for any structure can be split up into modal contributions based on the normal mode shapes of the structure. The amplitudes of the normal modes can then be superimposed to get the total displacement. For one of the modal components (u_n), the displacements are given by multiplying the mode shape vector (ϕ_n) with a time dependent modal amplitude (Y_n) ([3], page 220-223):

$$u_n = \phi_n \, \mathrm{Y}_n \qquad \textbf{(A.28)}$$

The sum of all the displacements from the modal components gives the total displacement:

$$\boldsymbol{u} = \phi_1 \, \mathrm{Y}_1 + \phi_2 \, \mathrm{Y}_2 + \cdots + \phi_n \, \mathrm{Y}_n = \sum_{n=1}^{N} \phi_n \, \mathrm{Y}_n \qquad \textbf{(A.29)}$$

or, in matrix notation:

$$\boldsymbol{u} = \boldsymbol{\Phi} \, \mathbf{Y} \qquad \text{where } \boldsymbol{\Phi} = \begin{bmatrix} \phi_{11} & \cdots & \phi_{1N} \\ \vdots & \ddots & \vdots \\ \phi_{N1} & \cdots & \phi_{NN} \end{bmatrix} = modal\ matrix \qquad \textbf{(A.30)}$$

To include this into the equation of motion, the velocity, $\dot{\boldsymbol{u}}$ and acceleration vector $\ddot{\boldsymbol{u}}$ is needed**.** This is done by taking the second time derivative of $\boldsymbol{u}$ with respect to time, which has been shown before. The $modal\ matrix$ $(\boldsymbol{\Phi})$ is not a function of time and thus does not change:

$$\dot{\boldsymbol{u}} = \boldsymbol{\Phi} \, \dot{\mathbf{Y}} \qquad \text{and} \qquad \ddot{\boldsymbol{u}} = \boldsymbol{\Phi} \, \ddot{\mathbf{Y}} \qquad \textbf{(A.31)}$$

Inserting equation (**A.30**) and (**A.31**) into the equation of motion gives:

$$\boldsymbol{m}\,\Phi\,\ddot{\mathbf{Y}}(t) + \boldsymbol{k}\,\Phi\,\mathbf{Y}(t) = \boldsymbol{p}(t) \quad \textbf{(A.32)}$$

The next step is to premultiply the above equation with the transpose of ϕ_n (ϕ_n^T). Premultiplying means that all elements are multiplied with the same number/variable (when $A + B = C$ then $2A + 2B = 2C$). Premultiplying the above equation:

$$\phi_n^T\,\boldsymbol{m}\,\Phi\,\ddot{\mathbf{Y}}(t) + \phi_n^T\,\boldsymbol{k}\,\Phi\,\mathbf{Y}(t) = \phi_n^T\,\boldsymbol{p}(t) \quad \textbf{(A.33)}$$

The natural modes can be shown to satisfy the following orthogonality conditions, when $\omega_n \neq \omega_r$:

$$\phi_n^T\,\boldsymbol{k}\,\phi_r = 0 \text{ and } \phi_n^T\,\boldsymbol{m}\,\phi_r = 0 \quad \textbf{(A.34)}$$

This orthogonality implies that the square matrices for the stiffness and mass are diagonal, i.e.:

$$\phi_n^T\,\boldsymbol{k}\,\phi_n \neq 0 \text{ and } \phi_n^T\,\boldsymbol{m}\,\phi_n \neq 0 \quad \textbf{(A.35)}$$

By expanding equation (**A.33**), and using equation (**A.34**) and (**A.35**), all terms except the nth term will vanish:

$$\phi_n^T\,\boldsymbol{m}\,\phi_n\,\ddot{\mathrm{Y}}_\mathrm{n}(t) + \phi_n^T\,\boldsymbol{k}\,\phi_n\,\mathrm{Y}_\mathrm{n}(t) = \phi_n^T\,\boldsymbol{p}(t) \quad \textbf{(A.36)}$$

The diagonal elements are:

$$K_n = \phi_n^T\,\boldsymbol{k}\,\phi_n \text{ and } M_n = \phi_n^T\,\boldsymbol{m}\,\phi_n \quad \textbf{(A.37)}$$

The relationship between these two equations is shown by using equation (**A.25**):

$$K_n = \phi_n^T\,\boldsymbol{k}\,\phi_n = {\omega_n}^2\,M_n \quad \textbf{(A.38)}$$

The force is defined in the same way:

$$P_n(t) = \phi_n^T\,\boldsymbol{p}(t) \quad \textbf{(A.39)}$$

To include damping at this point, it is assumed that C_n can be expressed with the same orthogonality condition as K_n and M_n (see equation (15)):

$$\phi_n^T\, \boldsymbol{c}\, \phi_r = 0 \quad (\,\omega_n \neq \omega_r\,) \qquad \text{and} \qquad C_n = \phi_n^T\, \boldsymbol{c}\, \phi_n \tag{A.40}$$

This is the most common approach to define damping, called classical damping, which gives the same natural modes as without the damping. The classical damping matrix is usually applied when a building has its damping mechanisms spread evenly in the building, i.e., a relatively symmetric building without too many abnormalities. An example of a classical damping is Rayleigh damping. This consists of a mass- and stiffness-proportional damping:

$$\boldsymbol{c} = \alpha\, \boldsymbol{m} + \beta\, \boldsymbol{k} \tag{A.41}$$

where α and β are constants.

The equation of motion, including damping, is now *uncoupled*. The equation for the 2DOF system mentioned earlier would be:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} \ddot{Y}_1(t) \\ \ddot{Y}_2(t) \end{bmatrix} + \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\begin{bmatrix} \dot{Y}_1(t) \\ \dot{Y}_2(t) \end{bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\begin{bmatrix} Y_1(t) \\ Y_2(t) \end{bmatrix} = \begin{bmatrix} P_1(t) \\ P_2(t) \end{bmatrix} \tag{A.42}$$

or on a more compact form:

$$M_n\, \ddot{Y}_n(t) + C_n\, \dot{Y}_n(t) + K_n\, Y_n(t) = P_n(t) \tag{A.43}$$

Normally, this equation is divided by the generalized mass since it is more convenient and physically reasonable to express the equation with the damping ratio, because it is a variable that can be measured experimentally or estimated with good precision.

$$\ddot{Y}_n(t) + 2\,\xi_n\, \omega_n\, \dot{Y}_n(t) + {\omega_n}^2\, Y_n(t) = \frac{P_n(t)}{M_n} \tag{A.44}$$

where ξ_n is the critical damping ratio, defined as:

$$\xi_n = \frac{C_n}{2\, M_n\, \omega_n} \tag{A.45}$$

If ξ_n is equal to 1, we have a state of critical damping, which explains the name.

The equation of motion is *uncoupled*, which means that it can be expressed as n SDOF systems. To obtain the total response of the MDOF system, the n SDOF equations are solved and their effects are superimposed.

Normalization of modes

As mentioned earlier, the *matrix eigenvalue problem* only determines the natural modes within a multiplication factor. This means that the modes can be scaled to our liking, which is called normalization. Sometimes it is most convenient to scale it so the largest element is equal to one or the top floor of a building is equal to one. In theoretical usage and in computer programs it is most beneficial to scale it so that the mass (M_n) have unit values ([7], page 409):

$$M_n = \phi_n^T \, \boldsymbol{m} \, \phi_n = 1 \qquad \boldsymbol{M} = \boldsymbol{\Phi}^{\mathrm{T}} \, \boldsymbol{m} \, \boldsymbol{\Phi} = \mathbf{I} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix} \qquad \textbf{(A.46)}$$

where $\mathbf{I}$ is an identity matrix, i.e., a diagonal matrix with only ones. With the mass normalized, the stiffness becomes:

$$K_n = \phi_n^T \, \boldsymbol{k} \, \phi_n = {\omega_n}^2 \, M_n = {\omega_n}^2 \qquad \mathbf{K} = \boldsymbol{\Phi}^{\mathrm{T}} \, \boldsymbol{k} \, \boldsymbol{\Phi} = \Omega^2 = \begin{bmatrix} {\omega_1}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & {\omega_N}^2 \end{bmatrix} \qquad \textbf{(A.47)}$$

where Ω is the *spectral matrix***.**

Forced vib

Since the MDOF system has been uncoupled, the same equations that are valid for the SDOF system are also valid for the MDOF system. Therefore the derivation can be done for a SDOF system.

Harmonic excitation

The system is subjected to a harmonically varying load p(t). The load is of sine-wave form with an amplitude p_0 and circular frequency ω ([3], page 33-35).

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = p_0 \sin \omega\, t \qquad \textbf{(A.48)}$$

To simplify the solution process, the undamped system is investigated first.

$$m\,\ddot{u}(t) + k\,u(t) = p_0 \sin \omega\, t \qquad \textbf{(A.49)}$$

The general solution consists of both a *complementary* and a *particular solution*. It can be shown that the *complementary solution* is [3]:

$$u_c(t) = A \cos \omega_n t + B \sin \omega_n t \qquad \textbf{(A.50)}$$

The *complementary solution* is the solution of the homogenous equation, i.e., the solution to the free vibration. A and B are constants that define the amplitude of the cos and sine functions. The values of them depend on the initial conditions.

The *general solution* consists of both a *complementary* and a *particular solution*. The *particular solution* is the solution that depends of the form of excitation, in this case, the harmonic excitation. The *particular solution* is:

$$u_p(t) = C \sin \omega\, t \quad \text{and} \quad \ddot{u}_p(t) = -\,\omega^2\, C \sin \omega\, t \qquad \textbf{(A.51)}$$

where C is the amplitude.

The above equations are substituted into equation (**A.49**):

$$-\,\omega^2\, m\, C \sin \omega\, t + k\, C \sin \omega\, t = p_0 \sin \omega\, t \qquad \textbf{(A.52)}$$

To find out what C is, the equation is first divided by $\sin \omega\, t$:

$$-\,\omega^2\, m\, C + k\, C = p_0 \qquad \textbf{(A.53)}$$

To isolate C, the equation is divided by k and the relationship $k/m = {\omega_n}^2$ is used:

$$-\,\frac{\omega^2}{{\omega_n}^2}\, C + C = \frac{p_0}{k} \qquad \textbf{(A.54)}$$

Rearranging the equation and noting that $\frac{\omega^2}{{\omega_n}^2} = \beta^2$:

$$C = \frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right] \qquad \textbf{(A.55)}$$

Now both the *complimentary solution* and the *particular solution* have been found, and the *general solution* becomes:

$$u(t) = u_c(t) + u_p(t) = A\ \cos \omega_n\, t +\ B\ \sin \omega_n\, t + \frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\ \sin \omega\, t \qquad \textbf{(A.56)}$$

As said earlier, both A and B depend on the initial conditions. If the system starts at rest, i.e., $u(0) = 0$ and $\dot{u}(0) = 0$, the constant A is calculated by setting $t = 0$, and B is found by taking the first derivative and then setting $t = 0$:

$$A = 0 \qquad \text{and} \qquad B = -\frac{p_0}{k}\left[\frac{\beta}{1-\beta^2}\right] \qquad \textbf{(A.57)}$$

By doing so the *general solution* response becomes:

$$u(t) = \frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\ (\sin \omega\, t - \beta\, \sin \omega_n\, t) \qquad \textbf{(A.58)}$$

where $p_0/k = u_{st}$ is the displacement the system would get if only affected by the static load p_0. $[1/1-\beta^2]$ is the magnification factor representing an amplification effect based on the applied excitation. $\frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\ \sin \omega\, t$ is called the steady-state response, and $\frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\ \beta\, \sin \omega_n\, t$ is called the transient response. The latter one will disappear when damping is included. This means that very often in practical cases with damping the dominating response in the first second(s), if it is big enough to matter, is based on the transient response, and later only on the steady-state response.

If the initial conditions are not equal to zero, the *general solution* becomes:

$$u(t) = u(0)\cos\omega_n t + \left(\frac{\dot{u}(0)}{\omega_n} - \frac{p_0}{k}\left[\frac{\beta}{1-\beta^2}\right]\right)\sin\omega_n t + \frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\sin\omega t \quad \textbf{(A.59)}$$

where:

$$Transient\ response = u(0)\cos\omega_n t + \left(\frac{\dot{u}(0)}{\omega_n} - \frac{p_0}{k}\left[\frac{\beta}{1-\beta^2}\right]\right)\sin\omega_n t \quad \textbf{(A.60)}$$

$$Steady\ state\ response = \frac{p_0}{k}\left[\frac{1}{1-\beta^2}\right]\sin\omega t \quad \textbf{(A.61)}$$

A normal way of portraying the influence of dynamic excitation is by defining the *response ratio* at rest:

$$R(t) = \frac{u(t)}{u_{st}} = \frac{u(t)}{p_0/k} = \left[\frac{1}{1-\beta^2}\right](\sin\omega t - \beta\sin\omega_n t) \quad \textbf{(A.62)}$$

Both the transient and steady state response is graphed in the figure below. One thing that is worth mentioning is that the two responses in the case below are out of phase with each other. This gives the total response a beating effect.

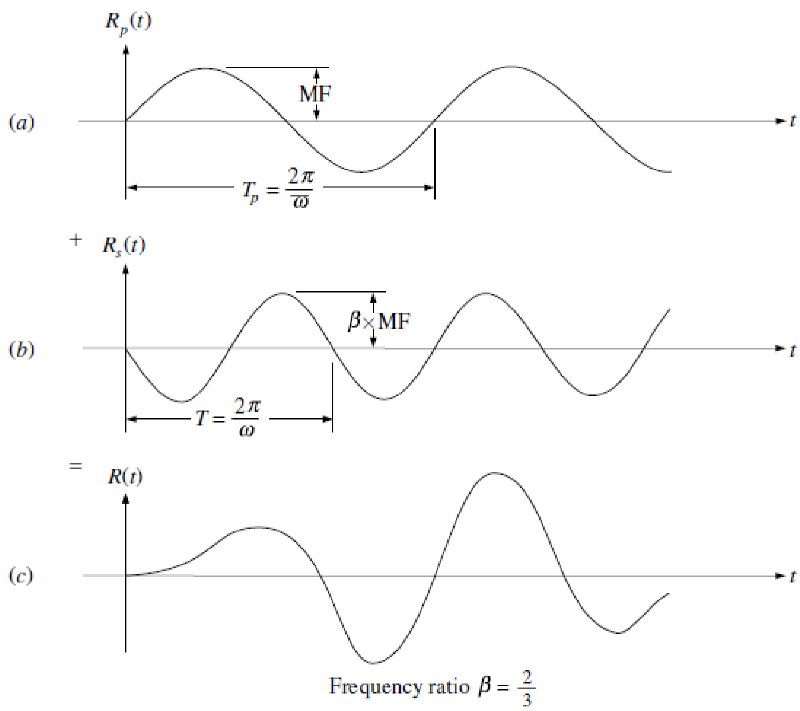

Figure A.8. a) steady state response; b) transient response; c) total response ([3], page 35)

With viscous damping

It can be shown ([3], page 36-37) that the viscously damped total response is:

$$u(t) = [\, A \, \cos \omega_D \, t + \, B \, \sin \omega_D \, t \,] \, e^{-\xi \omega t} + \frac{p_0}{k} \left[\frac{1}{(1-\beta^2)^2 + (2\,\xi\,\beta)^2} \right] [\, (1 - \beta^2) \sin \omega \, t - 2 \, \xi \, \beta \cos \omega \, t \,] \qquad \textbf{(A.63)}$$

where $\omega_D = \omega \sqrt{1 - \xi^2}$ and $e^{-\xi \omega t}$ governs the rate of decay in the transient response. If the transient response (first term on the right hand side) and the steady state response (last term) is graphed, it is clear that the transient response disappears relatively rapidly within the first four periods.

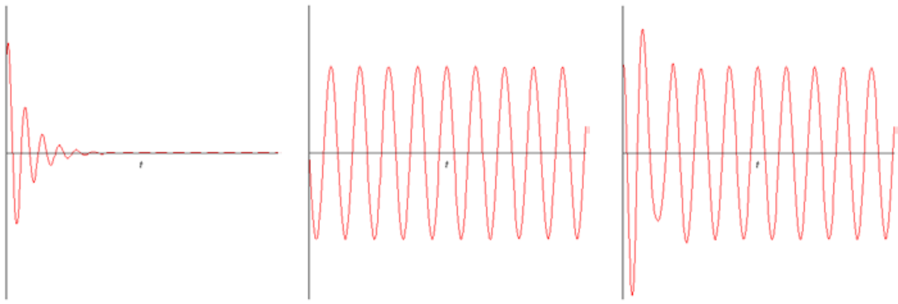

Figure A.9. Left) Transient response; Middle) Steady State response; Right) Total response.

Damping

Evaluation of viscous-damping ratio

Buildings, as well as most structures are underdamped, i.e., $\xi \ll 1$. When a structure is underdamped the free vibration decays out with time as the structure slowly losses its energy until the oscillations stop.

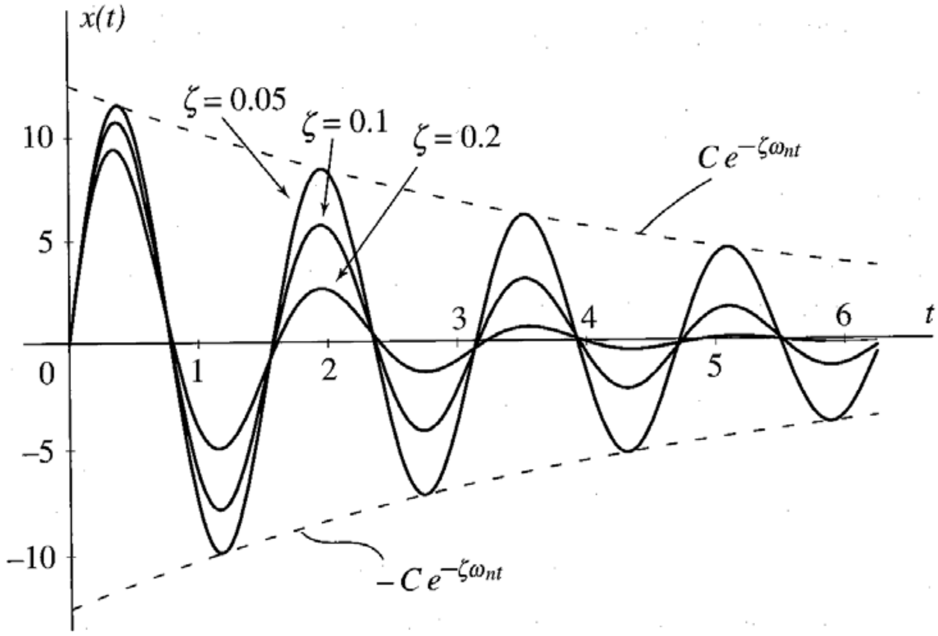

Figure A.10. Response of an underdamped system ([4],page 91).

There are many ways of determining the damping ratio of a system. Two of the most common ones are the Half-power bandwidth method ([7], page 83) and the Free-Vibration Decay method ([5], page 35).

Half-power bandwidth method

The Half-power bandwidth method uses the resonant amplitude, or rather $1/\sqrt{2}$ times the resonant amplitude, to calculate the damping ratio.

The equation is written as follows: $\xi = \omega_b - \omega_a / 2\omega_n$, where ω_a and ω_b are the forcing frequencies on either side of the resonant frequency, ω_n, at which the amplitude is $1/\sqrt{2}$ times the resonant amplitude. (Anil K. Chopra [7], *Dynamics of Structures*, 2007, page 83)

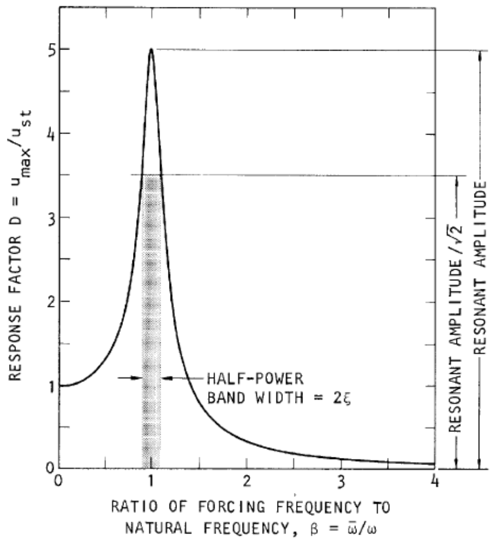

Figure A.11. Definition of half-power bandwidth ([8], page 34).

The Free-Vibration Decay Method

The Free-Vibration Decay Method is the simplest and most used method for measuring damping. To use this method, one has to excite a system and then suddenly stop the excitation. The next step is to evaluate the free vibration response. The rate of decay of oscillation is the interesting factor. By using two peak amplitude values and the number of oscillations between them, one can determine the damping ratio of the system by using the equations below ([5], page 35):

The logarithmic decrement, Λ, is:

$$\Lambda = \ln \frac{x_I}{x_{II}} \tag{A.64}$$

where x_I and x_{II} are the peak amplitudes shown in Figure A.12.

The rate of decay is exponential, which makes:

$$x_I = X\, e^{-\xi\omega t} \quad \text{and} \quad x_{II} = X\, e^{-\xi\omega(t+\tau_d)} \tag{A.65}$$

where τ_d is the period of the damped oscillation.

Substituting the equations for x_I and x_{II} into the original equation gives:

$$\Lambda = \ln \frac{X\, e^{-\xi\omega t}}{X\, e^{-\xi\omega(t+\tau_d)}} = \xi\omega\tau_d \tag{A.66}$$

and since:

$$\tau_d = \frac{2\,\pi}{\omega_d} = \frac{2\,\pi}{\omega\sqrt{1-\xi^2}} \qquad \text{then:} \qquad \Lambda = \frac{2\,\pi\,\xi}{\sqrt{1-\xi^2}} = \ln\frac{x_I}{x_{II}} \tag{A.67}$$

For the underdamped scenario it is preferred to evaluate two peak amplitudes that are many cycles apart. By expanding the equation with number of cycles, N, and knowing that $\Lambda \cong 2\,\pi\,\xi$ for small values of ξ, the damping ratio becomes:

$$\xi = \frac{1}{2\,\pi}\ln\left(\frac{x_i}{x_j}\right)\frac{1}{N} \tag{A.68}$$

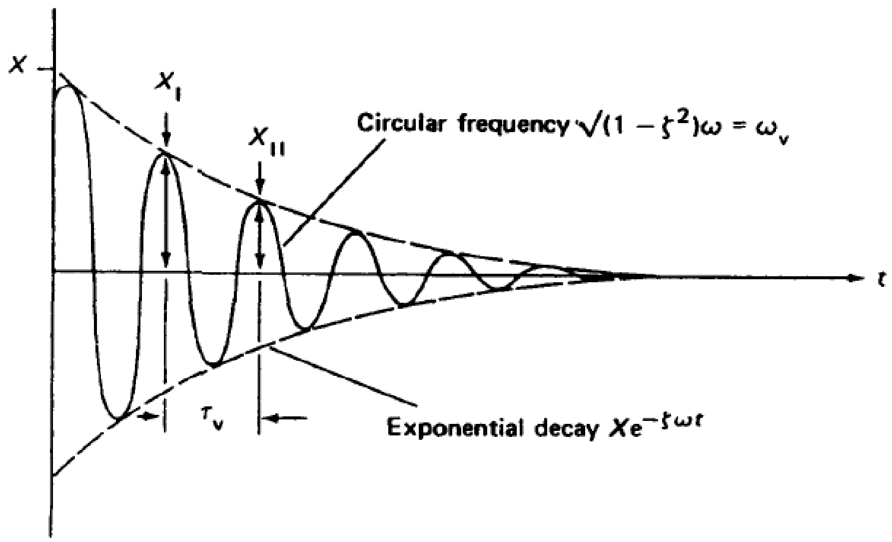

Figure A.12. Vibration Decay ([5], page 35)

Rayleigh Damping

Rayleigh damping is the traditional damping model used in step-by-step or time-history programs. The reason why it is so popular is that the Rayleigh damping only uses two variables and all matrices are already computed from the analysis. The matrices used are the mass matrix and the stiffness matrix ([6], page 4).

$$C = \alpha M + \beta K \tag{A.69}$$

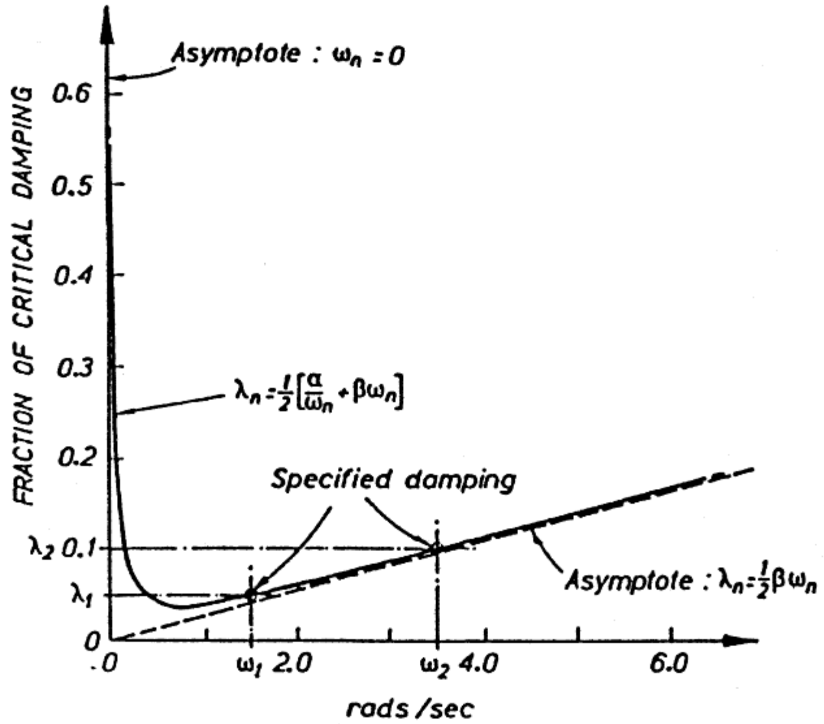

Figure A.13. Rayleigh Damping Model. ([6], page 3)

Caughey Damping

The Caughey Damping Model is used when one has to have the same damping ratio on a great number of modes. The damping matrix can be expressed as follows ([6], page 6):

$$\boldsymbol{C} = \boldsymbol{M} \sum_{b=0}^{N-1+q} \boldsymbol{a_b} \; [\boldsymbol{M}^{-1}\boldsymbol{K}]^{b} \qquad \textbf{(A.70)}$$

where q is any integer and N is the number of nodes at which damping is specified.

The original equation of the Caughey Damping Model is awkward to use for any computation. Wilson and Penzien have developed a more simple way of obtaining the same damping model. It uses the properties of orthogonality of the mode shapes, which is done in the chapter: Orthogonality of modes. It uses the generalized mass of the ith mode:

$$m_i = \phi_i^T \, \boldsymbol{M} \, \phi_i \qquad \textbf{(A.71)}$$

The same way the generalized damping is computed:

$$c_i = 2\, \lambda_i \, m_i \, \omega_i = \phi_i^T \, \boldsymbol{C} \, \phi_i \qquad \textbf{(A.72)}$$

where λ is the *critical* damping ratio.

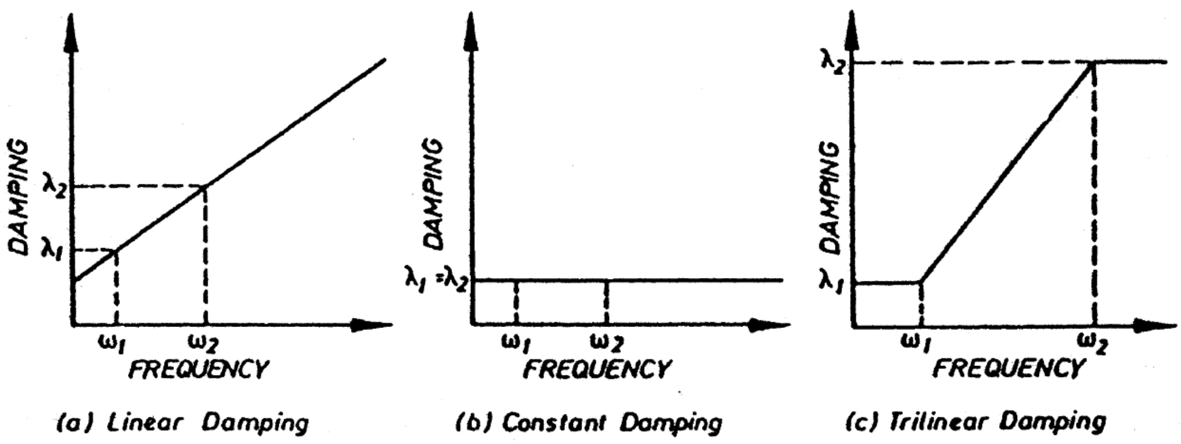

Figure A.14. Caughey Damping Model: Linear or Tri-linear Damping Model. ([6], page 6)

Effect of a Secondary System: Dynamic Vibration Absorber

When a single mass, without damping, is excited by a harmonic force with a frequency equal to the natural frequency of the main mass, the response is infinite (resonance). Now a second mass is introduced to the system, an absorber mass, which is tuned to have the same natural frequency as the main mass. By doing so, the response of the main mass at its natural frequency becomes zero. This happens because the absorber mass changes the 1DOF into a 2DOF which has two resonance frequencies, neither of which equals the original resonance frequency of the main mass. The absorber mass *absorbs* the motion of the main mass [9].

The response at the two new natural frequencies is also infinite, which may not be a problem when a machine is running at working frequency, but it may cause problems during startup and shutdown. When damping is introduced, the response is no longer infinite at the natural frequencies, but the response of the main mass is no longer zero at the original resonance frequency [9].

This phenomenon occurs in the results of this report, and an example of the occurrence is shown in Figure 2.13. The green curve represents the response of the Primary System (Main mass) when it is excited with a frequency equal to its natural frequency. The blue curve represents the response of the Primary System with a Secondary System attached.

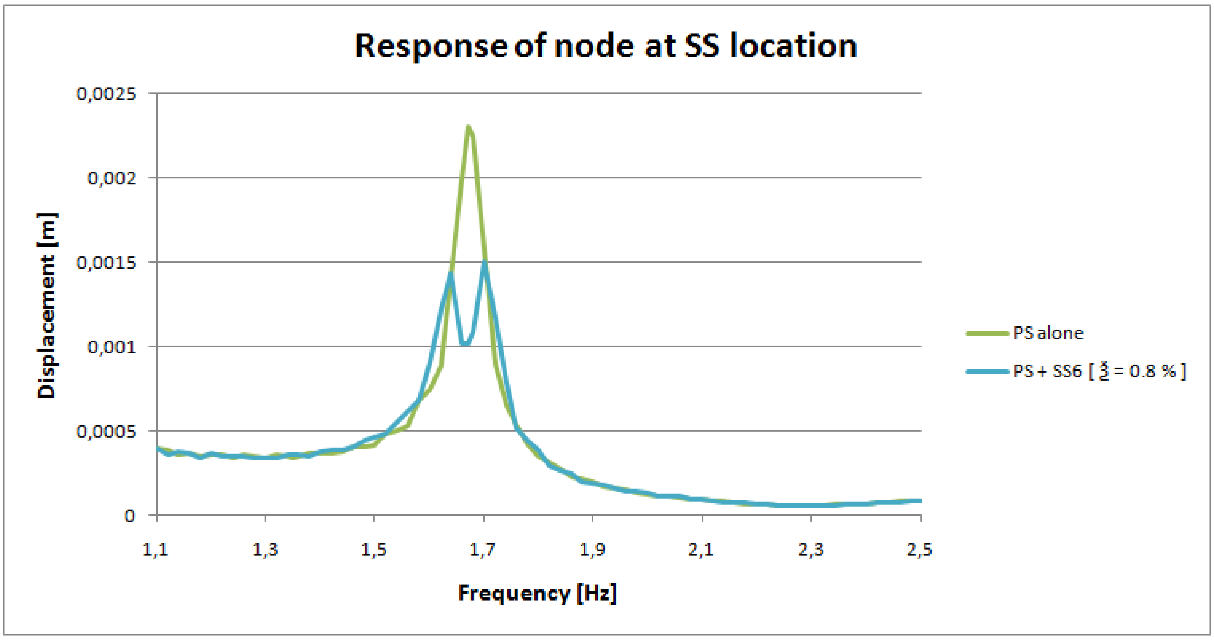

Figure A.15. A response spectrum showing the damping effect when the SS system is attached.

Solutions of the equation of motion

There are various ways of obtaining the solution to the equation of motion, e.g., Duhamel's Integral, Frequency-Response Method, Central Difference Method and Direct Integration (Newmark's Method).

Duhamel's Integral is often preferred for arbitrary loading, see Appendix A: Duhamel's integral.

Direct integration of the equation of motion is the method used by the FEM program, Ruaumoko.

Duhamel's integral

The above equations are only valid for harmonic excitation. If we wish to expose the structure to any other excitation scenarios, we have to develop new equations. One way is by using Duhamel's integral. This integral is valid for any arbitrary excitation and is therefore very useful when dealing with for example ground motion. Duhamel's integral is basically the sum of infinitesimal responses.

One way to express Duhamel's integral is by looking at a unit-impulse ($f = 1$) at time equal zero, i.e., a Dirac delta function $\delta(t)$.

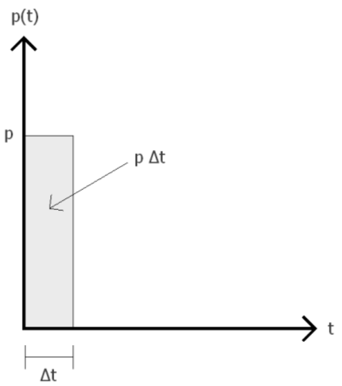

Figure A.16. Unit impulse.

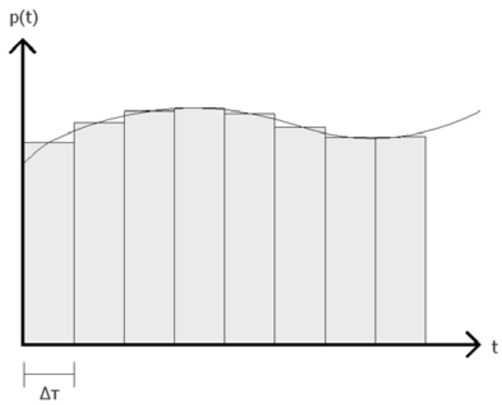

Figure A.17. Duhamel's Integral.

The equation of motion for a damped SDF system:

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = 0 \qquad \textbf{(A.73)}$$

By assuming that the system is under damped and that $\Delta t \ll \omega_n$, the solution becomes:

$$u(t) = e^{-\xi\omega_n t}\left(u_o \cos\omega_D\, t + \frac{\dot{u}_o + \xi\omega_n u_o}{\omega_D} \sin\omega_D\, t \right) \qquad \textbf{(A.74)}$$

The mass is in its equilibrium position at start, i.e., at rest initial conditions:

$$u = \dot{u} = 0 \qquad \text{for } t < 0 \text{ and } t = 0^- \qquad \textbf{(A.75)}$$

The impulse load is defined as:

$$f = 1 = p\,\Delta\text{t} = m\,\dot{u}_2 - m\,\dot{u}_1 = m\,\dot{u}(t=0) - m\,\dot{u}(t=0^-) = \dot{u}_o \tag{A.76}$$

From this the boundary conditions are:

$$u(t=0) = u_o = 0 \qquad \text{and} \qquad \dot{u}(t=0) = \dot{u}_o = \frac{1}{m} \qquad \textbf{(A.77) (A.78)}$$

Substituting the boundary conditions back into equation (**A.74**) gives:

$$u(t) = g(t) = \frac{1}{m\,\omega_D}\,e^{-\xi\omega_n t}\sin\omega_D\,t \tag{A.79}$$

where g(t) is the *unit-impulse function response*.

To make the function valid for arbitrary excitation, we must expand the above equation. If the impulse happened at $t = \tau$ instead of $t = 0$, i.e., $p(t) = \delta(t-\tau)$, the impulse response would be:

$$u(t-\tau) = \frac{1}{m\,\omega_D}\,e^{-\xi\omega_n(t-\tau)}\sin\omega_D\,(t-\tau) \qquad \text{when } t \geq \tau \tag{A.80}$$

It is shown that the response can be calculated at a certain point by the above equation. By making a series of impulse responses and superposing them, we get the solution to an arbitrary load. It can be shown that an arbitrary load can be broken down into a series of impulses:

$$p(t) \approx \sum p(\tau)\,\Delta\tau\,\delta(t-\tau) \tag{A.81}$$

Because we have a linear system the same can be said about the response:

$$u(t) \approx \sum p(\tau)\,\Delta\tau\,u(t-\tau) \tag{A.82}$$

By letting $\Delta\tau \rightarrow 0$ and taking the integral instead of the sum, we get the exact solution:

$$u(t) = \int_0^t p(\tau)\,u(t-\tau)\,d\tau \tag{A.83}$$

Substituting equation (**A.80**) into the above equation gives the general expression of *Duhamel's integral*:

$$u(t) = \frac{1}{m\,\omega_D}\int_0^t p(\tau)\,e^{-\xi\omega_n(t-\tau)}\sin[\omega_D\,(t-\tau)]\;d\tau \tag{A.84}$$

Direct integration of the equation of motion

In 1959, N. M. Newmark developed several methods for solving differential equations numerically by integration. The two most well know methods are the average acceleration and the linear acceleration. Newmark's methods are time-stepping methods. ([6], page 46-47)

The Newmark Constant Average Acceleration method is often used in computer programs and Ruaumoko, the FEM program used in this thesis, is no exception. The method is a step-by-step integration of the equation of motion. The acceleration is assumed to be constant during the time step, i.e., time t and time $t + \Delta t$. This gives the following relation between the accelerations:

$$\ddot{u} = \frac{\ddot{u}(t)+\ddot{u}(t+\Delta t)}{2} \quad \textbf{(A.85)}$$

Integrating with respect to time over the time-step Δt gives the velocity and displacement. By rearranging and letting the increment in the displacement be the variable, the increment in the acceleration is:

$$\Delta\ddot{u} = \ddot{u}(t + \Delta t) - \ddot{u}(t) = \frac{4\,\Delta u}{(\Delta t)^2} - \frac{4\,\dot{u}(t)}{\Delta t} - 2\,\ddot{u}(t) \quad \textbf{(A.86)}$$

and the increment in the velocity is:

$$\Delta\dot{u} = \dot{u}(t + \Delta t) - \dot{u}(t) = \frac{2\,\Delta u}{\Delta t} - 2\,\dot{u}(t) \quad \textbf{(A.87)}$$

Substituting the acceleration and velocity into the equation of motion at time $t + \Delta t$ gives:

$$m\,[\,\ddot{u}(t) + \Delta\ddot{u}\,] + \;c\,[\,\dot{u}(t) + \Delta\dot{u}\,] + \;k\,[\,u(t) + \Delta u\,] = p(t + \Delta t) \quad \textbf{(A.88)}$$

The stiffness term can be rewritten as:

$$k(t + \Delta t)[\,u(t) + \Delta u\,] = k(t)[\,u(t)\,] + k_t[\,\Delta u\,] = F_{Elastic}(t) + k_t[\,\Delta u\,] \quad \textbf{(A.89)}$$

where $k(t)$ is the secant stiffness matrix at time t, the elastic forces are the nodal equivalent of the member forces at time t and k_t is the current tangent stiffness matrix.

The damping term may also be rewritten similarly to the stiffness term:

$$c(t + \Delta t)[\,\dot{u}(t) + \Delta\dot{u}\,] = c(t)[\,\dot{u}(t)\,] + c_t[\,\Delta\dot{u}\,] = F_{Damping}(t) + c_t[\,\Delta\dot{u}\,] \quad \textbf{(A.90)}$$

where the damping forces are at time t and the matrix c_t is the current tangent damping matrix.

Substituting the two above equations into the equation of motion gives:

$$m\,[\,\Delta\ddot{u}\,] + c_t[\,\Delta\dot{u}\,] + k_t[\,\Delta u\,] = p(t+\Delta t) - m\,[\,\ddot{u}(t)\,] - F_{Damping}(t) - F_{Elastic}(t) \qquad \textbf{(A.91)}$$

Inserting equation (**A.86**) and (**A.87**) into equation (**A.91**) gives:

$$\left[\frac{4}{(\Delta t)^2}\,m + \frac{2}{\Delta t}c_t + k_t\right][\,\Delta u\,] = p(t+\Delta t) + m\left[\ddot{u}(t) + \frac{4}{\Delta t}\,\dot{u}(t)\right] + 2\,c_t[\,\dot{u}(t)\,] - F_{Damping} - F_{Elastic} \qquad \textbf{(A.92)}$$

If the damping matrix is constant (does not change with time), the above equation can be simplified to the following:

$$\left[\frac{4}{(\Delta t)^2}\,m + \frac{2}{\Delta t}c_t + k_t\right][\,\Delta u\,] = p(t+\Delta t) + m\left[\ddot{u}(t) + \frac{4}{\Delta t}\,\dot{u}(t)\right] + F_{Damping} - F_{Elastic} \qquad \textbf{(A.93)}$$

From this equation the incremental displacement can be calculated, hence the displacement, velocity and acceleration vectors, along with the member forces, can be updated. When the damping matrix and the stiffness matrix are updated, the whole process can be repeated to find the next incremental displacement.

The Newmark Constant Average Acceleration method is unconditionally stable. However, the time step should be very small (0.01-0.02 sec) to avoid getting any errors in the accuracy.

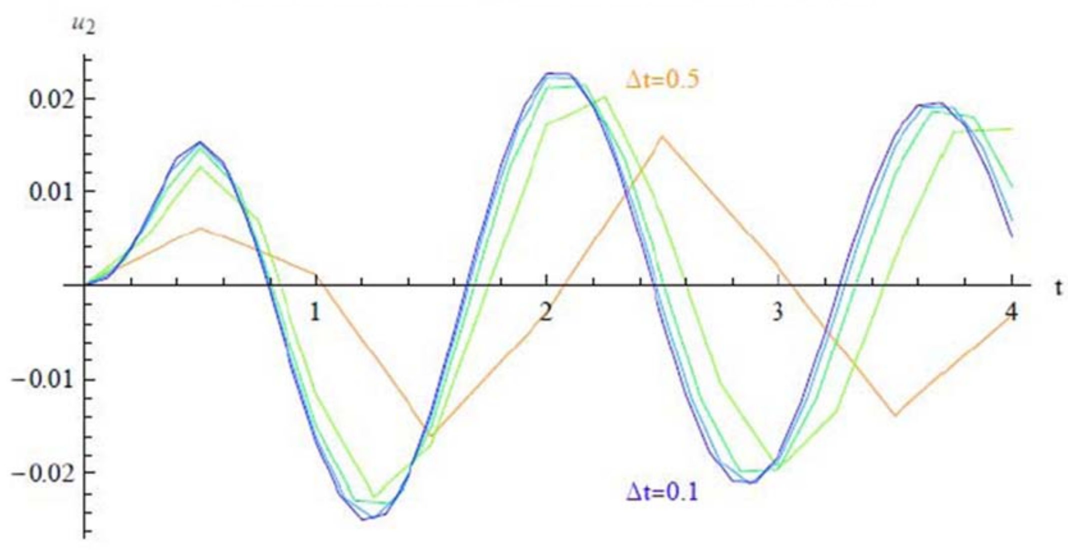

Figure A.18. Newmark Constant Average Acceleration method - Time steps.

Ground vibration

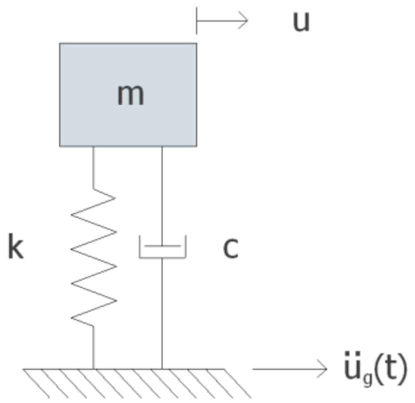

Figure A.19. Ground vibration $\ddot{u}_g$.

The equation of motion for a structure subjected to an earthquake ground motion is ([7], page 203):

$$m\,\ddot{u} + c\,\dot{u} + k\,u = -\,m\,\ddot{u}_g(t) \qquad \textbf{(A.94)}$$

or divided by the mass:

$$\ddot{u} + 2\,\xi\,\omega\;\dot{u} + \omega^2\,u = -\,\ddot{u}_g(t) \qquad \textbf{(A.95)}$$

The solution to the equation above can be solved by using Duhamel's integral (**A.84**):

$$u(t) = -\,\frac{1}{\omega_D}\int_0^t \ddot{u}_g(\tau)\;e^{-\xi\omega_n(t-\tau)}\;\sin[\omega_D\,(t-\tau)]\;d\tau \qquad \textbf{(A.96)}$$

Response spectrum

A response spectrum is basically a plot of the peak response or peak steady-state response of a structure with varying natural frequency. In a system where the excitation is harmonic, the steady-state response is usually the preferred response, i.e., the transient response is not of interest. If a system undergoes an earthquake load, the peak (transient) response is the one desired.

The response spectrum is a very well known concept in earthquake engineering. It was already initiated in 1932 by M. A. Biot [10]. It is especially useful when you are in a seismic area. Given the earthquake ground motion, the response spectrum is made by exposing systems with different natural frequencies to the same ground motion. The damping ratio is fixed, but by running multiple response spectra with different damping ratios you cover the relevant damping values.

A response spectrum has peak amplitudes, which indicates that the excitation frequency hits the natural frequency of one of the modes.

The figure below is an example of a response spectrum. This specific spectrum shows the displacement plotted against frequency. The spectrum belongs to secondary system 3, which has been calculated in the analysis part of this thesis (Chapter 5).

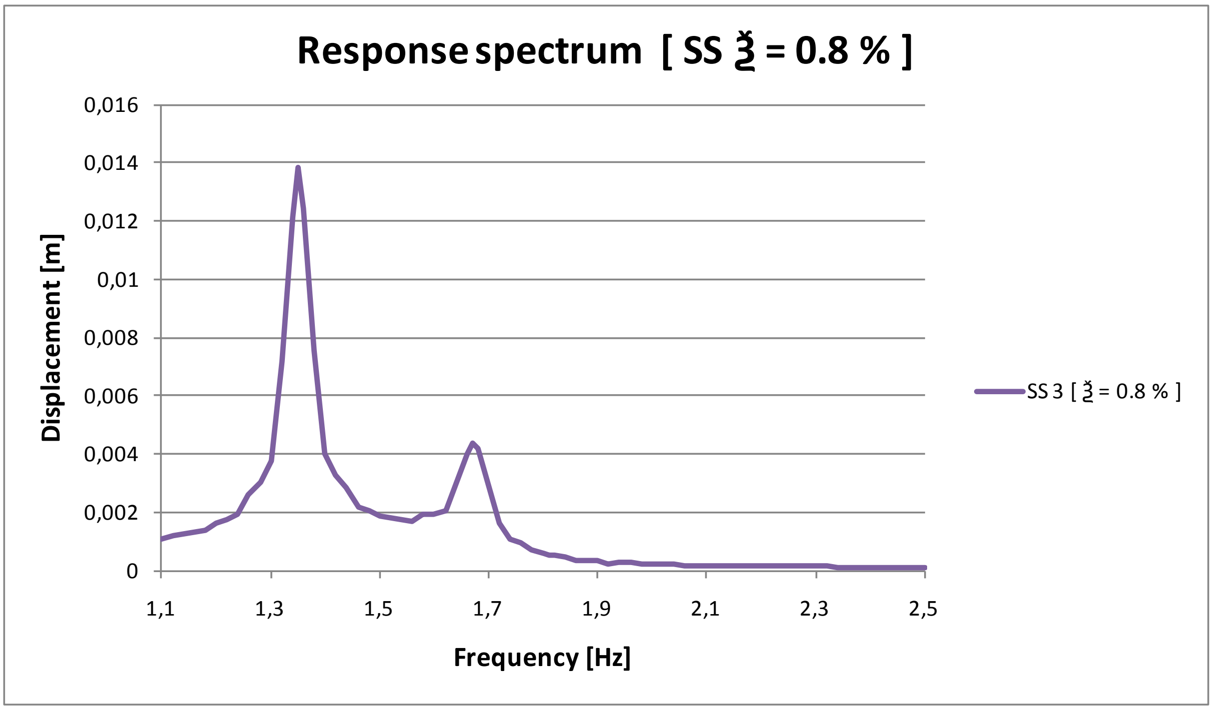

Figure A.20. Response spectrum.

Appendix B:

Results

Earthquake floor response at the location of the secondary structure

"PS with SS1" means that the response of the node at the secondary system location is measured with secondary system 1 present. "PS alone" means that the response of the floor node is measured without the influence of a secondary system.

Earthquake 2000kav (floor response):

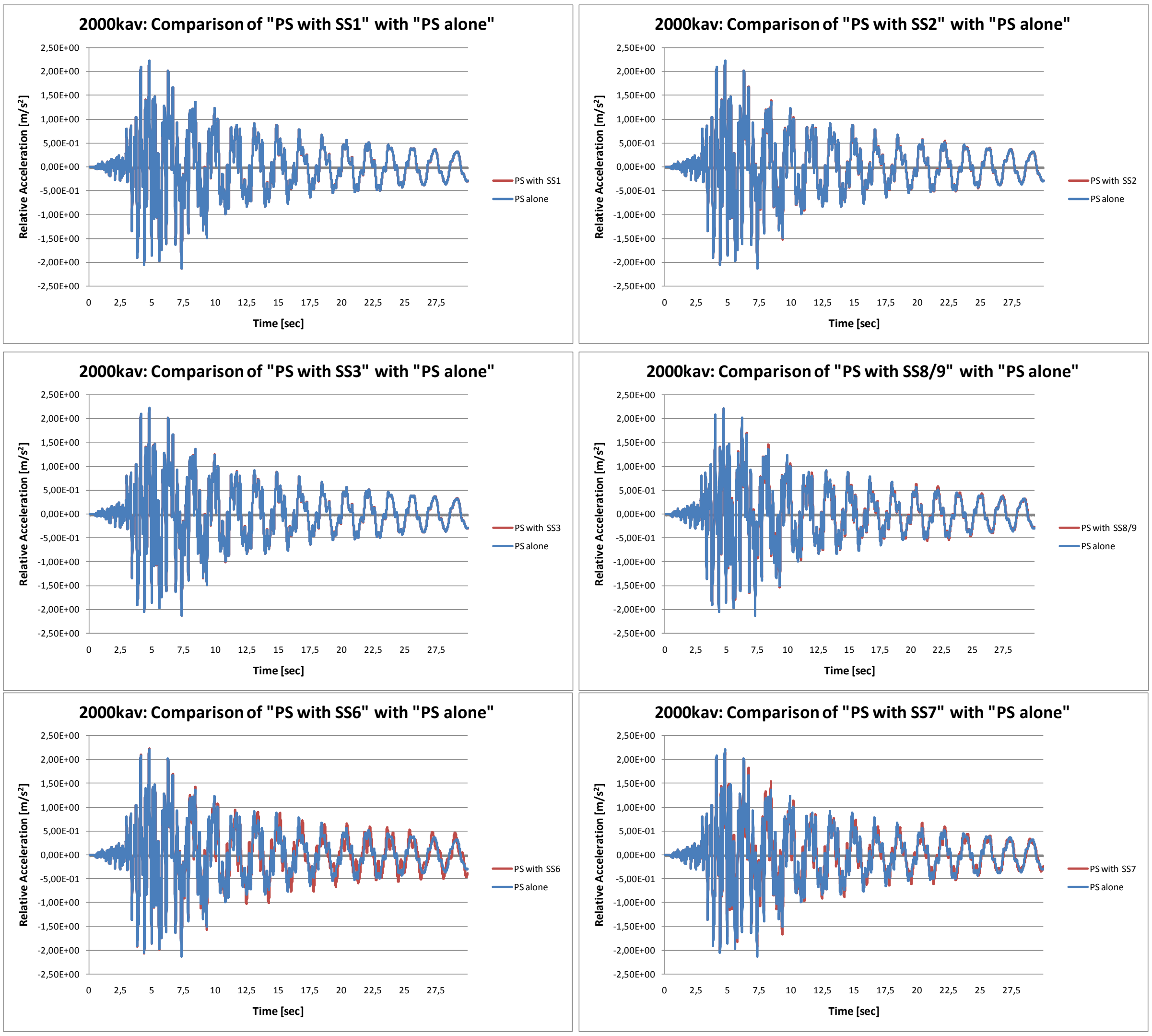

Figure B.1. 2000kav: Relative acceleration time series comparison between PS with SS in place and PS alone.

In the graphs above, where only the blue curve is visible, the motion of the floor with and without the secondary system in place is exactly the same.

Earthquake 2000kns (floor response):

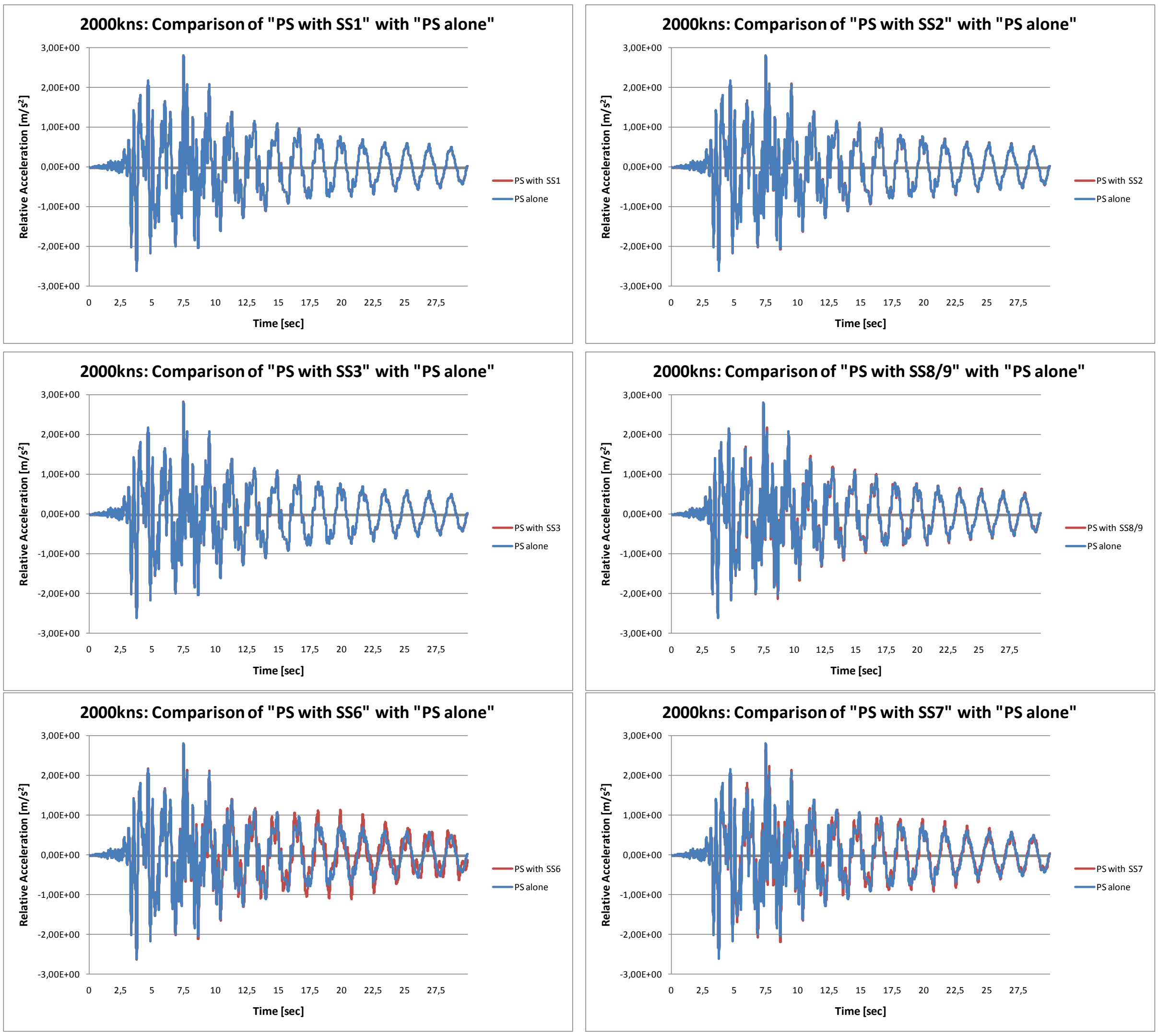

Figure B.2. 2000kns: Relative acceleration time series comparison between PS with SS in place and PS alone.

Earthquake 2008kav (floor response):

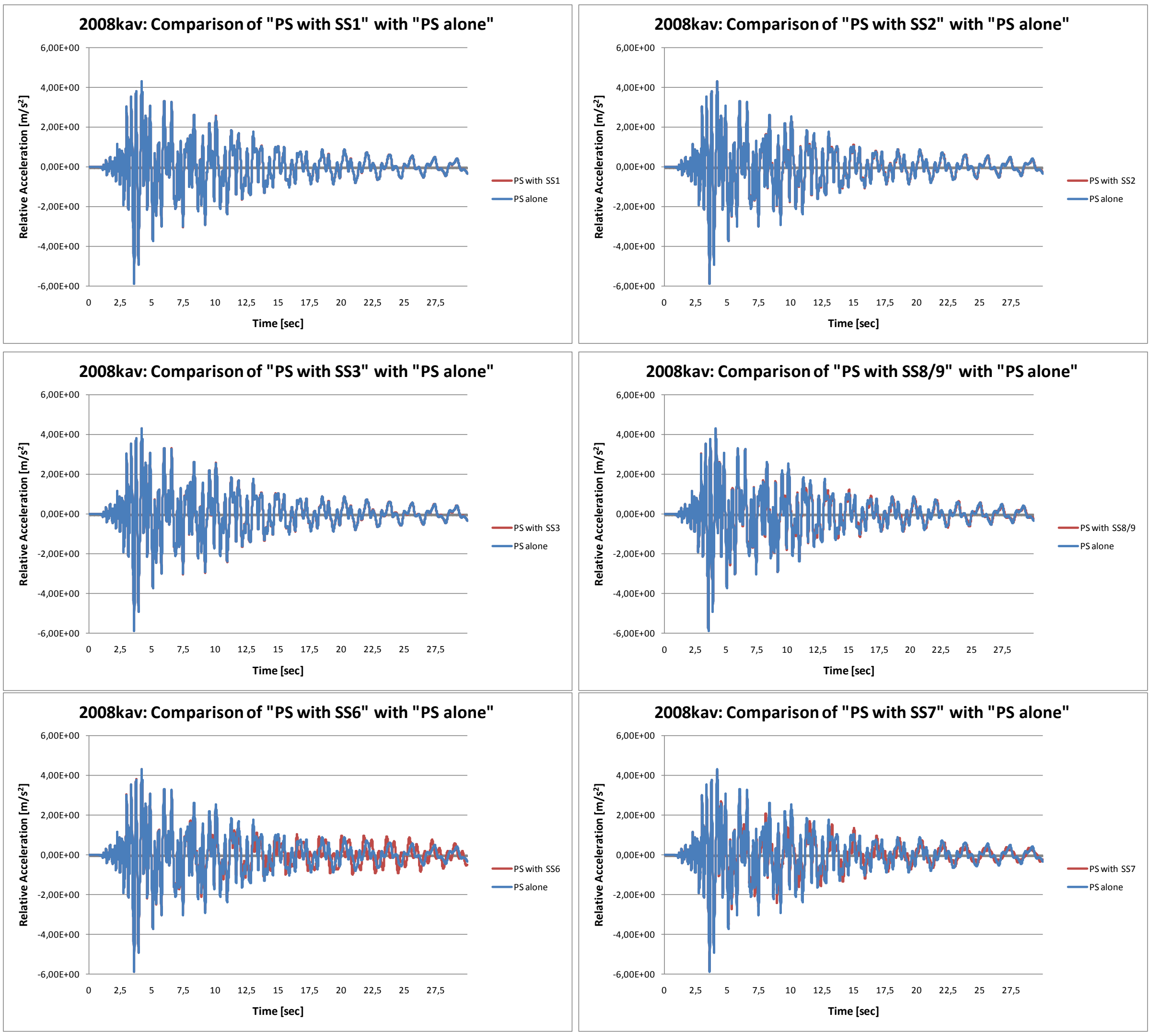

Figure B.3. 2000kav: Relative acceleration time series comparison between PS with SS in place and PS alone.

The effect of the damping ratio for SS during earthquakes

Secondary system 2 during earthquake 2000kav and secondary system 1 during earthquake 2000kav are shown below.

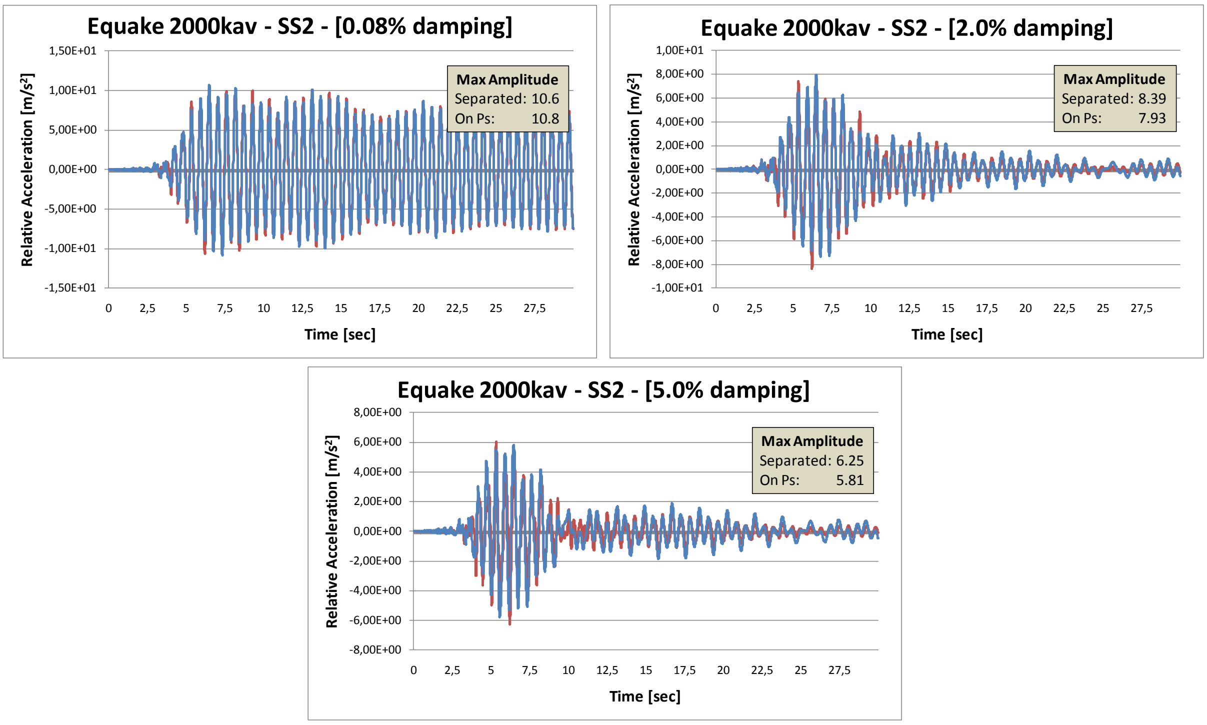

Figure B.4. Comparison of relative acceleration between SS analyzed separately (red curve) and SS analyzed jointly (blue curve).

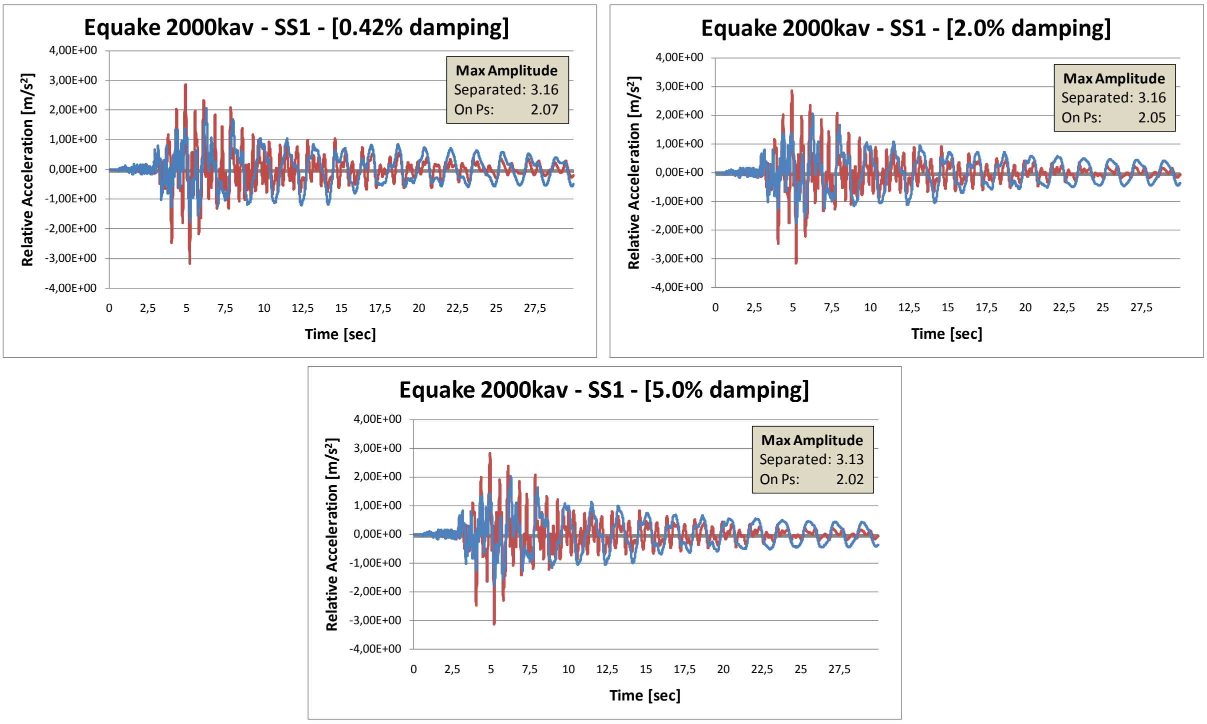

Figure B.5.Comparison of relative acceleration between SS analyzed separately (red curve) and SS analyzed jointly (blue curve).

Seismic Coefficient comparison

The seismic coefficient comparisons for earthquake 2000kns, 2008kav and 2008kns are shown below. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation **6.3**). Green curve means that the secondary system is analyzed jointly with the primary system. Red curve means that the secondary system is analyzed separately (floor response method).

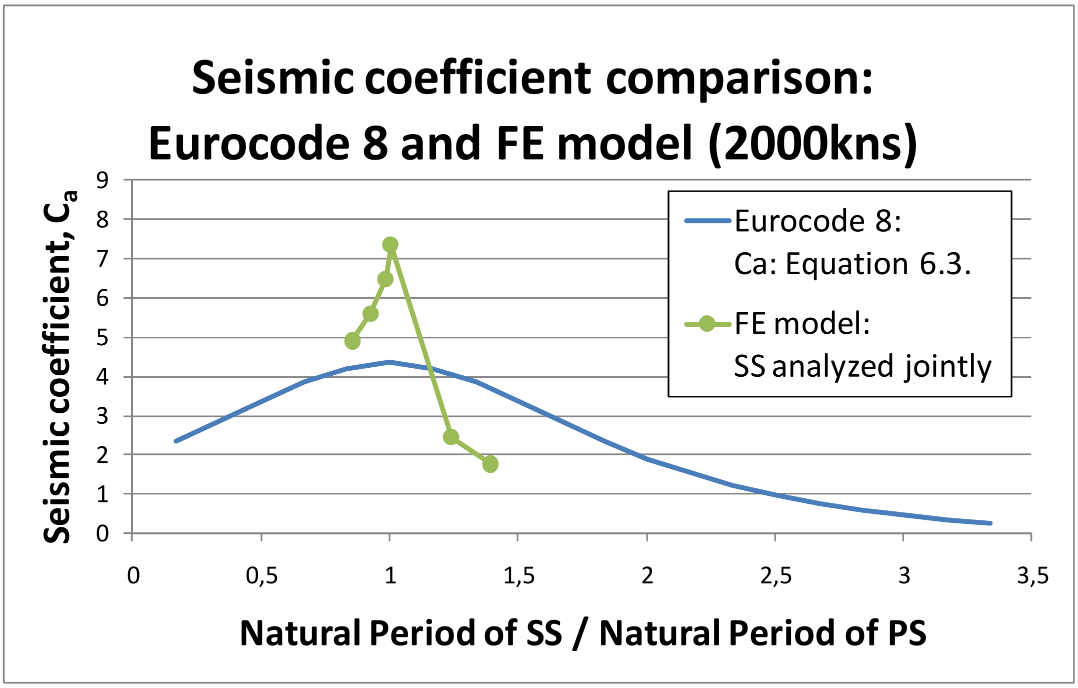

Figure B.6. Seismic coefficient comparison for earthquake 2000kns. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Green curve means that the secondary system is analyzed jointly with the primary system.

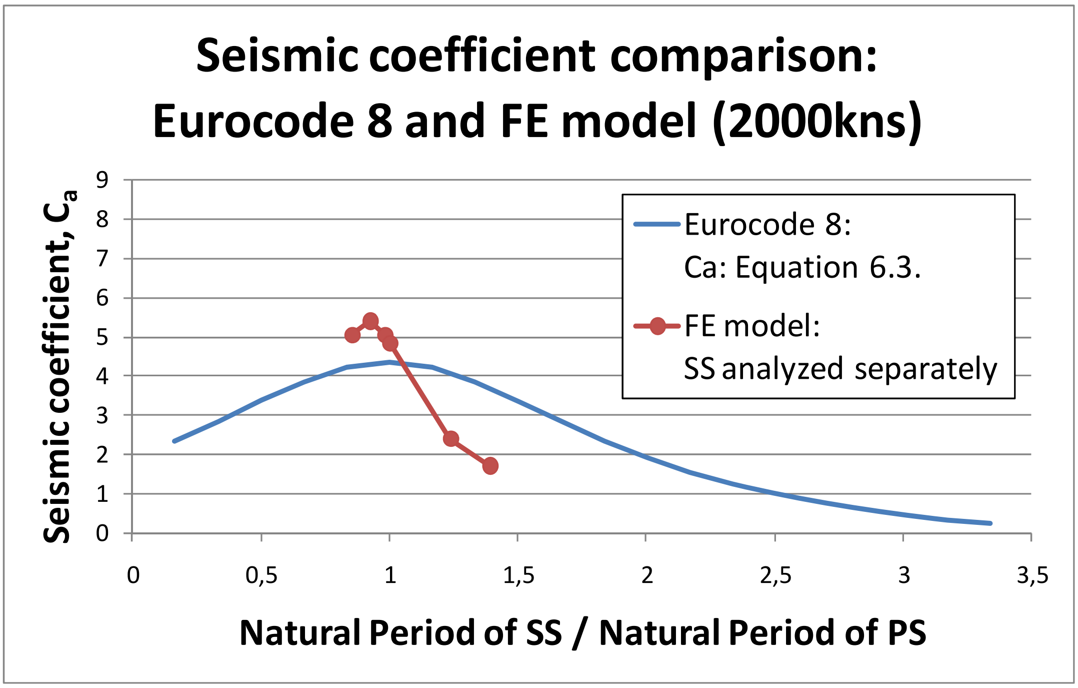

Figure B.7. Seismic coefficient comparison for earthquake 2000kns. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Red curve means that the secondary system is analyzed separately (floor response method).

The seismic coefficient comparison for earthquake 2008kav is shown below.

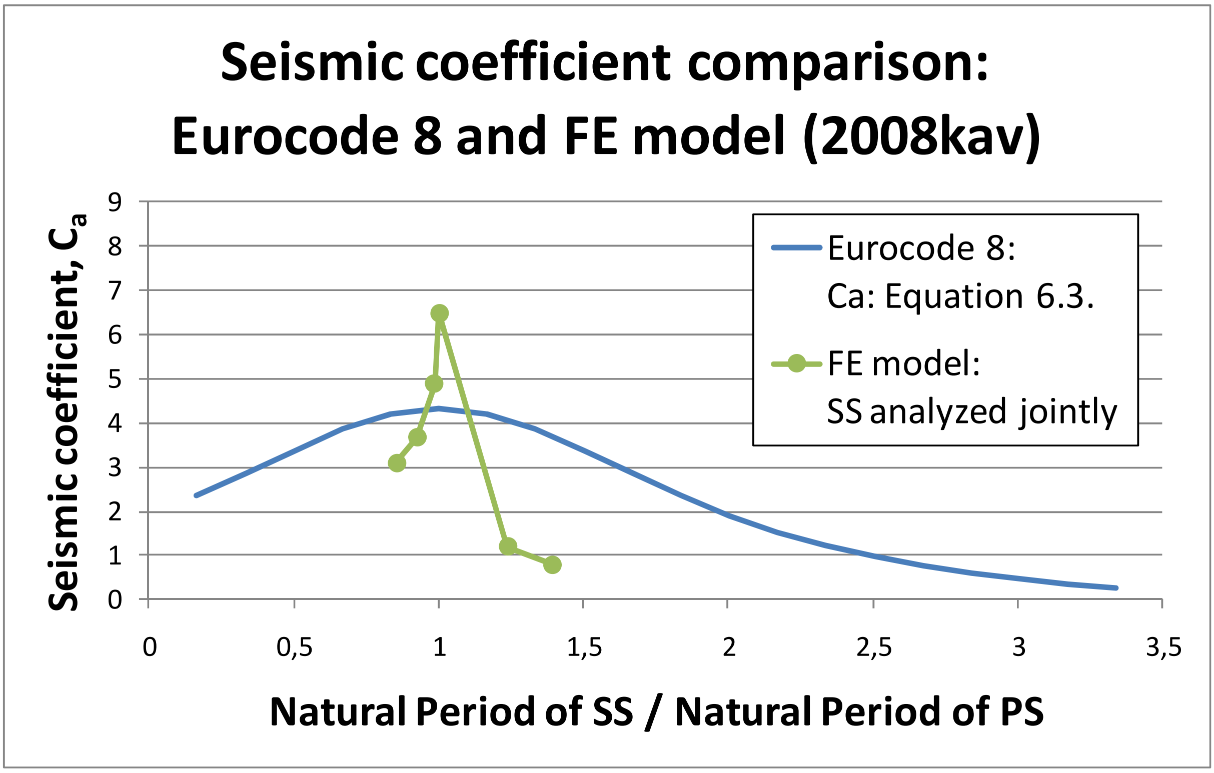

Figure B.8. Seismic coefficient comparison for earthquake 2008kav. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Green curve means that the secondary system is analyzed jointly with the primary system.

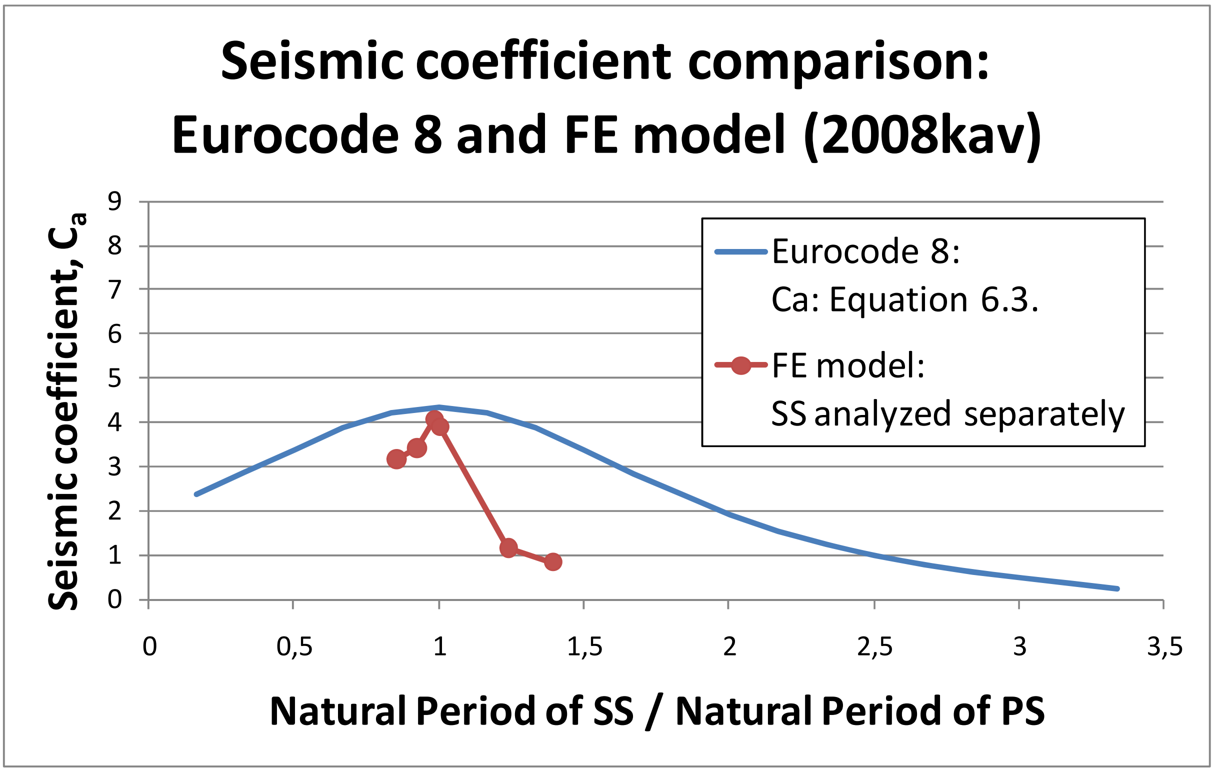

Figure B.9. Seismic coefficient comparison for earthquake 2008kav. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Red curve means that the secondary system is analyzed separately (floor response method).

The seismic coefficient comparison for earthquake 2008kns is shown below.

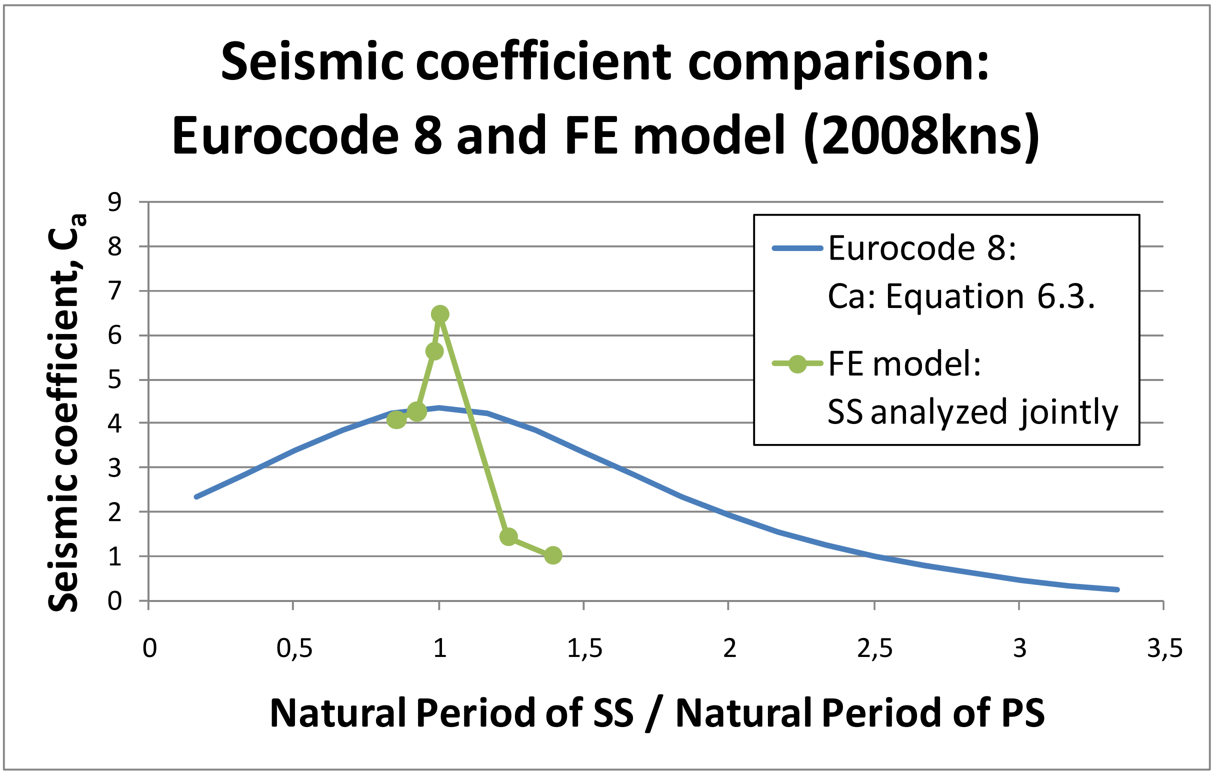

Figure B.10. Seismic coefficient comparison for earthquake 2008kns. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Green curve means that the secondary system is analyzed jointly with the primary system.

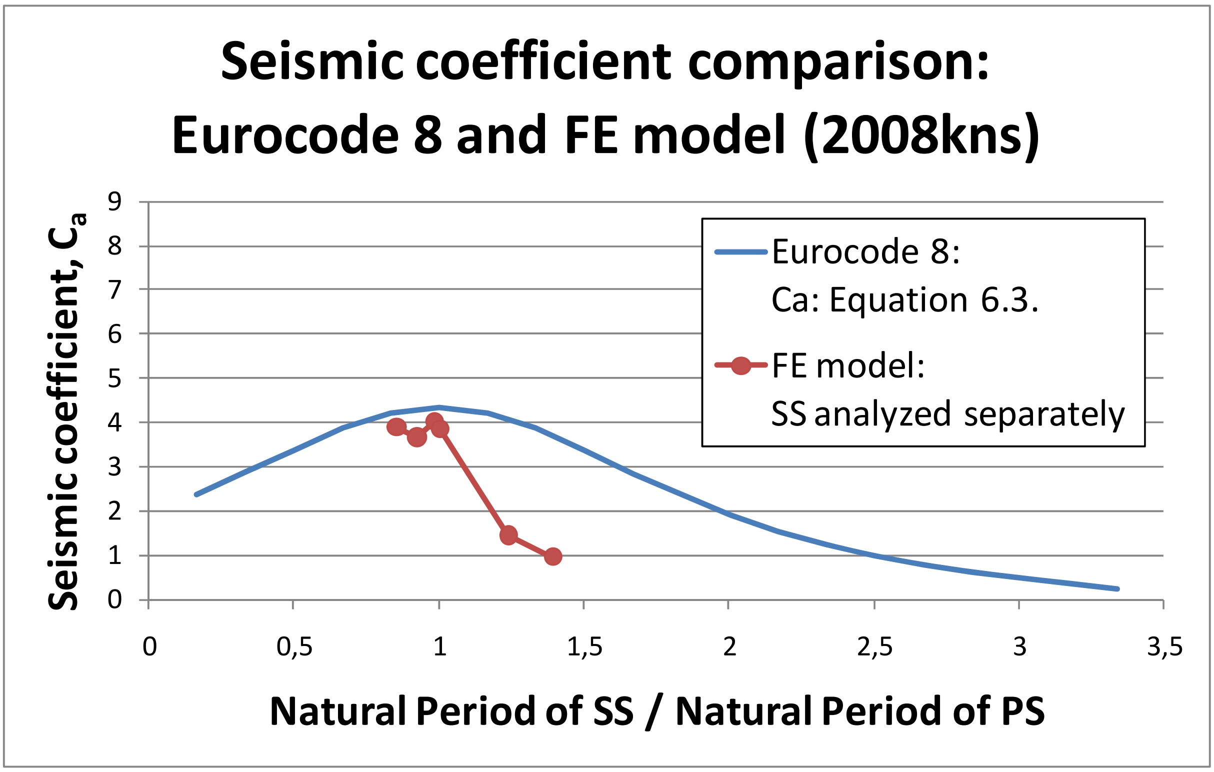

Figure B.11. Seismic coefficient comparison for earthquake 2008kns. Blue curve is the seismic coefficient calculated by the Eurocode 8 standard (Equation 6.3). Red curve means that the secondary system is analyzed separately (floor response method).

Ruaumoko Model

Example of a FE model for the primary system

```
Cardington Building, Primary system, Earthquake 2000kav
2 0 0 4 3 0 0 0                                      ! Control Parameters
DEFAULT                                              ! Earthquake Directions
478 577 16 12 10 0 9.81 0.0081 0.0069 0.005 30.0     ! Structural control Parameters
0  1  0 20 0.1 0.1 0.1 0                             ! Output Parameters
0 0 0.001 0.0 0.0 0.0 0.0 0.0 0.0                    ! Iteration Parameters
1 0.8 2 0.8 3 0.8 4 0.8 5 0.8 6 0.8 7 0.8 8 0.8 9 0.8 10 0.8   ! User Specified Modal Damping Parameters

NODES 0
1    0      -0.55   0      1  1  1  1  1  1  0  3   ! Base
under Columns at Ground Floor
2    7.5    -0.55   0      1  1  1  1  1  1  0  3
3    15     -0.55   0      1  1  1  1  1  1  0  3
4    22.5   -0.55   0      1  1  1  1  1  1  0  3
5    0      -0.55   7.5    1  1  1  1  1  1  0  3
6    7.5    -0.55   7.5    1  1  1  1  1  1  0  3
7    15     -0.55   7.5    1  1  1  1  1  1  0  3
8    22.5   -0.55   7.5    1  1  1  1  1  1  0  3
9    0      -0.55   15     1  1  1  1  1  1  0  3
10   7.5    -0.55   15     1  1  1  1  1  1  0  3
11   15     -0.55   15     1  1  1  1  1  1  0  3
12   22.5   -0.55   15     1  1  1  1  1  1  0  3
13   0      -0.55   22.5   1  1  1  1  1  1  0  3
14   7.5    -0.55   22.5   1  1  1  1  1  1  0  3
15   15     -0.55   22.5   1  1  1  1  1  1  0  3
16   22.5   -0.55   22.5   1  1  1  1  1  1  0  3
17   0      -0.55   30     1  1  1  1  1  1  0  3
18   7.5    -0.55   30     1  1  1  1  1  1  0  3
19   15     -0.55   30     1  1  1  1  1  1  0  3
20   22.5   -0.55   30     1  1  1  1  1  1  0  3
21   0      0       0      0  0  0  0  0  0  0  3   !
Columns at Ground Floor
22   7.5    0       0      0  0  0  0  0  0  0  3
23   15     0       0      0  0  0  0  0  0  0  3
24   22.5   0       0      0  0  0  0  0  0  0  3
25   0      0       7.5    0  0  0  0  0  0  0  3
26   7.5    0       7.5    0  0  0  0  0  0  0  3
27   15     0       7.5    0  0  0  0  0  0  0  3
28   22.5   0       7.5    0  0  0  0  0  0  0  3
29   0      0       15     0  0  0  0  0  0  0  3
30   7.5    0       15     0  0  0  0  0  0  0  3
31   15     0       15     0  0  0  0  0  0  0  3
32   22.5   0       15     0  0  0  0  0  0  0  3
33   0      0       22.5   0  0  0  0  0  0  0  3
34   7.5    0       22.5   0  0  0  0  0  0  0  3
35   15     0       22.5   0  0  0  0  0  0  0  3
36   22.5   0       22.5   0  0  0  0  0  0  0  3
37   0      0       30     0  0  0  0  0  0  0  3
38   7.5    0       30     0  0  0  0  0  0  0  3
39   15     0       30     0  0  0  0  0  0  0  3
40   22.5   0       30     0  0  0  0  0  0  0  3
41   0      3.625   0      0  0  0  0  0  0  0  0   !
Columns  at Floor 1
42   7.5    3.625   0      0  0  0  0  0  0  0  3
43   15     3.625   0      0  0  0  0  0  0  0  3
44   22.5   3.625   0      0  0  0  0  0  0  0  0
45   0      3.625   7.5    0  0  0  0  0  0  0  3
46   7.5    3.625   7.5    0  0  0  0  0  0  0  3
47   15     3.625   7.5    0  0  0  0  0  0  0  3
48   22.5   3.625   7.5    0  0  0  0  0  0  0  3
49   0      3.625   15     0  0  0  0  0  0  0  3
50   7.5    3.625   15     0  0  0  0  0  0  0  3
```

51	15	3.625	15	0	0	0	0	0	0	0	3	
52	22.5	3.625	15	0	0	0	0	0	0	0	3	
53	0	3.625	22.5	0	0	0	0	0	0	0	3	
54	7.5	3.625	22.5	0	0	0	0	0	0	0	3	
55	15	3.625	22.5	0	0	0	0	0	0	0	3	
56	22.5	3.625	22.5	0	0	0	0	0	0	0	3	
57	0	3.625	30	0	0	0	0	0	0	0	0	
58	7.5	3.625	30	0	0	0	0	0	0	0	3	
59	15	3.625	30	0	0	0	0	0	0	0	3	
60	22.5	3.625	30	0	0	0	0	0	0	0	0	
61	0	7.375	0	0	0	0	0	0	0	0	0	!
Columns at Floor 2												
62	7.5	7.375	0	0	0	0	0	0	0	0	3	
63	15	7.375	0	0	0	0	0	0	0	0	3	
64	22.5	7.375	0	0	0	0	0	0	0	0	0	
65	0	7.375	7.5	0	0	0	0	0	0	0	3	
66	7.5	7.375	7.5	0	0	0	0	0	0	0	3	
67	15	7.375	7.5	0	0	0	0	0	0	0	3	
68	22.5	7.375	7.5	0	0	0	0	0	0	0	3	
69	0	7.375	15	0	0	0	0	0	0	0	3	
70	7.5	7.375	15	0	0	0	0	0	0	0	3	
71	15	7.375	15	0	0	0	0	0	0	0	3	
72	22.5	7.375	15	0	0	0	0	0	0	0	3	
73	0	7.375	22.5	0	0	0	0	0	0	0	3	
74	7.5	7.375	22.5	0	0	0	0	0	0	0	3	
75	15	7.375	22.5	0	0	0	0	0	0	0	3	
76	22.5	7.375	22.5	0	0	0	0	0	0	0	3	
77	0	7.375	30	0	0	0	0	0	0	0	0	
78	7.5	7.375	30	0	0	0	0	0	0	0	3	
79	15	7.375	30	0	0	0	0	0	0	0	3	
80	22.5	7.375	30	0	0	0	0	0	0	0	0	
81	0	11.125	0	0	0	0	0	0	0	0	0	!
Columns at Floor 3												
82	7.5	11.125	0	0	0	0	0	0	0	0	3	
83	15	11.125	0	0	0	0	0	0	0	0	3	
84	22.5	11.125	0	0	0	0	0	0	0	0	0	
85	0	11.125	7.5	0	0	0	0	0	0	0	3	
86	7.5	11.125	7.5	0	0	0	0	0	0	0	3	
87	15	11.125	7.5	0	0	0	0	0	0	0	3	
88	22.5	11.125	7.5	0	0	0	0	0	0	0	3	
89	0	11.125	15	0	0	0	0	0	0	0	3	
90	7.5	11.125	15	0	0	0	0	0	0	0	3	
91	15	11.125	15	0	0	0	0	0	0	0	3	
92	22.5	11.125	15	0	0	0	0	0	0	0	3	
93	0	11.125	22.5	0	0	0	0	0	0	0	3	
94	7.5	11.125	22.5	0	0	0	0	0	0	0	3	
95	15	11.125	22.5	0	0	0	0	0	0	0	3	
96	22.5	11.125	22.5	0	0	0	0	0	0	0	3	
97	0	11.125	30	0	0	0	0	0	0	0	0	
98	7.5	11.125	30	0	0	0	0	0	0	0	3	
99	15	11.125	30	0	0	0	0	0	0	0	3	
100	22.5	11.125	30	0	0	0	0	0	0	0	0	
101	0	14.875	0	0	0	0	0	0	0	0	0	!
Columns at Floor 4												
102	7.5	14.875	0	0	0	0	0	0	0	0	3	
103	15	14.875	0	0	0	0	0	0	0	0	3	
104	22.5	14.875	0	0	0	0	0	0	0	0	0	
105	0	14.875	7.5	0	0	0	0	0	0	0	3	
106	7.5	14.875	7.5	0	0	0	0	0	0	0	3	
107	15	14.875	7.5	0	0	0	0	0	0	0	3	
108	22.5	14.875	7.5	0	0	0	0	0	0	0	3	
109	0	14.875	15	0	0	0	0	0	0	0	3	
110	7.5	14.875	15	0	0	0	0	0	0	0	3	
111	15	14.875	15	0	0	0	0	0	0	0	3	
112	22.5	14.875	15	0	0	0	0	0	0	0	3	
113	0	14.875	22.5	0	0	0	0	0	0	0	3	
114	7.5	14.875	22.5	0	0	0	0	0	0	0	3	
115	15	14.875	22.5	0	0	0	0	0	0	0	3	
116	22.5	14.875	22.5	0	0	0	0	0	0	0	3	
117	0	14.875	30	0	0	0	0	0	0	0	0	
118	7.5	14.875	30	0	0	0	0	0	0	0	3	
119	15	14.875	30	0	0	0	0	0	0	0	3	

120	22.5	14.875	30	0	0	0	0	0	0	0	0	
121	0	18.625	0	0	0	0	0	0	0	0	0	!
Columns at Floor 5												
122	7.5	18.625	0	0	0	0	0	0	0	0	3	
123	15	18.625	0	0	0	0	0	0	0	0	3	
124	22.5	18.625	0	0	0	0	0	0	0	0	0	
125	0	18.625	7.5	0	0	0	0	0	0	0	3	
126	7.5	18.625	7.5	0	0	0	0	0	0	0	3	
127	15	18.625	7.5	0	0	0	0	0	0	0	3	
128	22.5	18.625	7.5	0	0	0	0	0	0	0	3	
129	0	18.625	15	0	0	0	0	0	0	0	3	
130	7.5	18.625	15	0	0	0	0	0	0	0	3	
131	15	18.625	15	0	0	0	0	0	0	0	3	
132	22.5	18.625	15	0	0	0	0	0	0	0	3	
133	0	18.625	22.5	0	0	0	0	0	0	0	3	
134	7.5	18.625	22.5	0	0	0	0	0	0	0	3	
135	15	18.625	22.5	0	0	0	0	0	0	0	3	
136	22.5	18.625	22.5	0	0	0	0	0	0	0	3	
137	0	18.625	30	0	0	0	0	0	0	0	0	
138	7.5	18.625	30	0	0	0	0	0	0	0	3	
139	15	18.625	30	0	0	0	0	0	0	0	3	
140	22.5	18.625	30	0	0	0	0	0	0	0	0	
141	0	22.375	0	0	0	0	0	0	0	0	0	!
Columns at Floor 6												
142	7.5	22.375	0	0	0	0	0	0	0	0	3	
143	15	22.375	0	0	0	0	0	0	0	0	3	
144	22.5	22.375	0	0	0	0	0	0	0	0	0	
145	0	22.375	7.5	0	0	0	0	0	0	0	3	
146	7.5	22.375	7.5	0	0	0	0	0	0	0	3	
147	15	22.375	7.5	0	0	0	0	0	0	0	3	
148	22.5	22.375	7.5	0	0	0	0	0	0	0	3	
149	0	22.375	15	0	0	0	0	0	0	0	3	
150	7.5	22.375	15	0	0	0	0	0	0	0	3	
151	15	22.375	15	0	0	0	0	0	0	0	3	
152	22.5	22.375	15	0	0	0	0	0	0	0	3	
153	0	22.375	22.5	0	0	0	0	0	0	0	3	
154	7.5	22.375	22.5	0	0	0	0	0	0	0	3	
155	15	22.375	22.5	0	0	0	0	0	0	0	3	
156	22.5	22.375	22.5	0	0	0	0	0	0	0	3	
157	0	22.375	30	0	0	0	0	0	0	0	0	
158	7.5	22.375	30	0	0	0	0	0	0	0	3	
159	15	22.375	30	0	0	0	0	0	0	0	3	
160	22.5	22.375	30	0	0	0	0	0	0	0	0	
161	0	26.125	0	0	0	0	0	0	0	0	0	!
Columns at Floor 7												
162	7.5	26.125	0	0	0	0	0	0	0	0	3	
163	15	26.125	0	0	0	0	0	0	0	0	3	
164	22.5	26.125	0	0	0	0	0	0	0	0	0	
165	0	26.125	7.5	0	0	0	0	0	0	0	3	
166	7.5	26.125	7.5	0	0	0	0	0	0	0	3	
167	15	26.125	7.5	0	0	0	0	0	0	0	3	
168	22.5	26.125	7.5	0	0	0	0	0	0	0	3	
169	0	26.125	15	0	0	0	0	0	0	0	3	
170	7.5	26.125	15	0	0	0	0	0	0	0	3	
171	15	26.125	15	0	0	0	0	0	0	0	3	
172	22.5	26.125	15	0	0	0	0	0	0	0	3	
173	0	26.125	22.5	0	0	0	0	0	0	0	3	
174	7.5	26.125	22.5	0	0	0	0	0	0	0	3	
175	15	26.125	22.5	0	0	0	0	0	0	0	3	
176	22.5	26.125	22.5	0	0	0	0	0	0	0	3	
177	0	26.125	30	0	0	0	0	0	0	0	0	
178	7.5	26.125	30	0	0	0	0	0	0	0	3	
179	15	26.125	30	0	0	0	0	0	0	0	3	
180	22.5	26.125	30	0	0	0	0	0	0	0	0	
181	3.75	3.625	0	0	0	0	0	0	0	0	3	! Floor 1
182	11.25	3.625	0	0	0	0	0	0	0	0	3	
183	18.75	3.625	0	0	0	0	0	0	0	0	3	
184	0	3.625	3.75	0	0	0	0	0	0	0	3	
185	3.75	3.625	3.75	0	0	0	0	0	0	0	3	
186	7.5	3.625	3.75	0	0	0	0	0	0	0	3	
187	11.25	3.625	3.75	0	0	0	0	0	0	0	3	
188	15	3.625	3.75	0	0	0	0	0	0	0	3	

189	18.75	3.625	3.75	0	0	0	0	0	0	0	3	
190	22.5	3.625	3.75	0	0	0	0	0	0	0	3	
191	3.75	3.625	7.5	0	0	0	0	0	0	0	3	
192	11.25	3.625	7.5	0	0	0	0	0	0	0	3	
193	18.75	3.625	7.5	0	0	0	0	0	0	0	3	
194	0	3.625	11.25	0	0	0	0	0	0	0	3	
195	3.75	3.625	11.25	0	0	0	0	0	0	0	3	
196	7.5	3.625	11.25	0	0	0	0	0	0	0	3	
197	11.25	3.625	11.25	0	0	0	0	0	0	0	3	
198	15	3.625	11.25	0	0	0	0	0	0	0	3	
199	18.75	3.625	11.25	0	0	0	0	0	0	0	3	
200	22.5	3.625	11.25	0	0	0	0	0	0	0	3	
201	3.75	3.625	15	0	0	0	0	0	0	0	3	
202	11.25	3.625	15	0	0	0	0	0	0	0	3	
203	18.75	3.625	15	0	0	0	0	0	0	0	3	
204	0	3.625	18.75	0	0	0	0	0	0	0	3	
205	3.75	3.625	18.75	0	0	0	0	0	0	0	3	
206	7.5	3.625	18.75	0	0	0	0	0	0	0	3	
207	11.25	3.625	18.75	0	0	0	0	0	0	0	3	
208	15	3.625	18.75	0	0	0	0	0	0	0	3	
209	18.75	3.625	18.75	0	0	0	0	0	0	0	3	
210	22.5	3.625	18.75	0	0	0	0	0	0	0	3	
211	3.75	3.625	22.5	0	0	0	0	0	0	0	3	
212	11.25	3.625	22.5	0	0	0	0	0	0	0	3	
213	18.75	3.625	22.5	0	0	0	0	0	0	0	3	
214	0	3.625	25.675	0	0	0	0	0	0	0	3	
215	3.75	3.625	25.675	0	0	0	0	0	0	0	3	
216	7.5	3.625	25.675	0	0	0	0	0	0	0	3	
217	11.25	3.625	25.675	0	0	0	0	0	0	0	3	
218	15	3.625	25.675	0	0	0	0	0	0	0	3	
219	18.75	3.625	25.675	0	0	0	0	0	0	0	3	
220	22.5	3.625	25.675	0	0	0	0	0	0	0	3	
221	3.75	3.625	30	0	0	0	0	0	0	0	3	
222	18.75	3.625	30	0	0	0	0	0	0	0	3	
223	3.75	7.375	0	0	0	0	0	0	0	0	3	! Floor 2
224	11.25	7.375	0	0	0	0	0	0	0	0	3	
225	18.75	7.375	0	0	0	0	0	0	0	0	3	
226	0	7.375	3.75	0	0	0	0	0	0	0	3	
227	3.75	7.375	3.75	0	0	0	0	0	0	0	3	
228	7.5	7.375	3.75	0	0	0	0	0	0	0	3	
229	11.25	7.375	3.75	0	0	0	0	0	0	0	3	
230	15	7.375	3.75	0	0	0	0	0	0	0	3	
231	18.75	7.375	3.75	0	0	0	0	0	0	0	3	
232	22.5	7.375	3.75	0	0	0	0	0	0	0	3	
233	3.75	7.375	7.5	0	0	0	0	0	0	0	3	
234	11.25	7.375	7.5	0	0	0	0	0	0	0	3	
235	18.75	7.375	7.5	0	0	0	0	0	0	0	3	
236	0	7.375	11.25	0	0	0	0	0	0	0	3	
237	3.75	7.375	11.25	0	0	0	0	0	0	0	3	
238	7.5	7.375	11.25	0	0	0	0	0	0	0	3	
239	11.25	7.375	11.25	0	0	0	0	0	0	0	3	
240	15	7.375	11.25	0	0	0	0	0	0	0	3	
241	18.75	7.375	11.25	0	0	0	0	0	0	0	3	
242	22.5	7.375	11.25	0	0	0	0	0	0	0	3	
243	3.75	7.375	15	0	0	0	0	0	0	0	3	
244	11.25	7.375	15	0	0	0	0	0	0	0	3	
245	18.75	7.375	15	0	0	0	0	0	0	0	3	
246	0	7.375	18.75	0	0	0	0	0	0	0	3	
247	3.75	7.375	18.75	0	0	0	0	0	0	0	3	
248	7.5	7.375	18.75	0	0	0	0	0	0	0	3	
249	11.25	7.375	18.75	0	0	0	0	0	0	0	3	
250	15	7.375	18.75	0	0	0	0	0	0	0	3	
251	18.75	7.375	18.75	0	0	0	0	0	0	0	3	
252	22.5	7.375	18.75	0	0	0	0	0	0	0	3	
253	3.75	7.375	22.5	0	0	0	0	0	0	0	3	
254	11.25	7.375	22.5	0	0	0	0	0	0	0	3	
255	18.75	7.375	22.5	0	0	0	0	0	0	0	3	
256	0	7.375	25.675	0	0	0	0	0	0	0	3	
257	3.75	7.375	25.675	0	0	0	0	0	0	0	3	
258	7.5	7.375	25.675	0	0	0	0	0	0	0	3	
259	11.25	7.375	25.675	0	0	0	0	0	0	0	3	
260	15	7.375	25.675	0	0	0	0	0	0	0	3	

261	18.75	7.375	25.675	0	0	0	0	0	0	0	3	
262	22.5	7.375	25.675	0	0	0	0	0	0	0	3	
263	3.75	7.375	30	0	0	0	0	0	0	0	3	
264	18.75	7.375	30	0	0	0	0	0	0	0	3	
265	3.75	11.125	0	0	0	0	0	0	0	0	3	! Floor 3
266	11.25	11.125	0	0	0	0	0	0	0	0	3	
267	18.75	11.125	0	0	0	0	0	0	0	0	3	
268	0	11.125	3.75	0	0	0	0	0	0	0	3	
269	3.75	11.125	3.75	0	0	0	0	0	0	0	3	
270	7.5	11.125	3.75	0	0	0	0	0	0	0	3	
271	11.25	11.125	3.75	0	0	0	0	0	0	0	3	
272	15	11.125	3.75	0	0	0	0	0	0	0	3	
273	18.75	11.125	3.75	0	0	0	0	0	0	0	3	
274	22.5	11.125	3.75	0	0	0	0	0	0	0	3	
275	3.75	11.125	7.5	0	0	0	0	0	0	0	3	
276	11.25	11.125	7.5	0	0	0	0	0	0	0	3	
277	18.75	11.125	7.5	0	0	0	0	0	0	0	3	
278	0	11.125	11.25	0	0	0	0	0	0	0	3	
279	3.75	11.125	11.25	0	0	0	0	0	0	0	3	
280	7.5	11.125	11.25	0	0	0	0	0	0	0	3	
281	11.25	11.125	11.25	0	0	0	0	0	0	0	3	
282	15	11.125	11.25	0	0	0	0	0	0	0	3	
283	18.75	11.125	11.25	0	0	0	0	0	0	0	3	
284	22.5	11.125	11.25	0	0	0	0	0	0	0	3	
285	3.75	11.125	15	0	0	0	0	0	0	0	3	
286	11.25	11.125	15	0	0	0	0	0	0	0	3	
287	18.75	11.125	15	0	0	0	0	0	0	0	3	
288	0	11.125	18.75	0	0	0	0	0	0	0	3	
289	3.75	11.125	18.75	0	0	0	0	0	0	0	3	
290	7.5	11.125	18.75	0	0	0	0	0	0	0	3	
291	11.25	11.125	18.75	0	0	0	0	0	0	0	3	
292	15	11.125	18.75	0	0	0	0	0	0	0	3	
293	18.75	11.125	18.75	0	0	0	0	0	0	0	3	
294	22.5	11.125	18.75	0	0	0	0	0	0	0	3	
295	3.75	11.125	22.5	0	0	0	0	0	0	0	3	
296	11.25	11.125	22.5	0	0	0	0	0	0	0	3	
297	18.75	11.125	22.5	0	0	0	0	0	0	0	3	
298	0	11.125	25.675	0	0	0	0	0	0	0	3	
299	3.75	11.125	25.675	0	0	0	0	0	0	0	3	
300	7.5	11.125	25.675	0	0	0	0	0	0	0	3	
301	11.25	11.125	25.675	0	0	0	0	0	0	0	3	
302	15	11.125	25.675	0	0	0	0	0	0	0	3	
303	18.75	11.125	25.675	0	0	0	0	0	0	0	3	
304	22.5	11.125	25.675	0	0	0	0	0	0	0	3	
305	3.75	11.125	30	0	0	0	0	0	0	0	3	
306	18.75	11.125	30	0	0	0	0	0	0	0	3	
307	3.75	14.875	0	0	0	0	0	0	0	0	3	! Floor 4
308	11.25	14.875	0	0	0	0	0	0	0	0	3	
309	18.75	14.875	0	0	0	0	0	0	0	0	3	
310	0	14.875	3.75	0	0	0	0	0	0	0	3	
311	3.75	14.875	3.75	0	0	0	0	0	0	0	3	
312	7.5	14.875	3.75	0	0	0	0	0	0	0	3	
313	11.25	14.875	3.75	0	0	0	0	0	0	0	3	
314	15	14.875	3.75	0	0	0	0	0	0	0	3	
315	18.75	14.875	3.75	0	0	0	0	0	0	0	3	
316	22.5	14.875	3.75	0	0	0	0	0	0	0	3	
317	3.75	14.875	7.5	0	0	0	0	0	0	0	3	
318	11.25	14.875	7.5	0	0	0	0	0	0	0	3	
319	18.75	14.875	7.5	0	0	0	0	0	0	0	3	
320	0	14.875	11.25	0	0	0	0	0	0	0	3	
321	3.75	14.875	11.25	0	0	0	0	0	0	0	3	
322	7.5	14.875	11.25	0	0	0	0	0	0	0	3	
323	11.25	14.875	11.25	0	0	0	0	0	0	0	3	
324	15	14.875	11.25	0	0	0	0	0	0	0	3	
325	18.75	14.875	11.25	0	0	0	0	0	0	0	3	
326	22.5	14.875	11.25	0	0	0	0	0	0	0	3	
327	3.75	14.875	15	0	0	0	0	0	0	0	3	
328	11.25	14.875	15	0	0	0	0	0	0	0	3	
329	18.75	14.875	15	0	0	0	0	0	0	0	3	
330	0	14.875	18.75	0	0	0	0	0	0	0	3	
331	3.75	14.875	18.75	0	0	0	0	0	0	0	3	
332	7.5	14.875	18.75	0	0	0	0	0	0	0	3	

333	11.25	14.875	18.75	0	0	0	0	0	0	0	3	
334	15	14.875	18.75	0	0	0	0	0	0	0	3	
335	18.75	14.875	18.75	0	0	0	0	0	0	0	3	
336	22.5	14.875	18.75	0	0	0	0	0	0	0	3	
337	3.75	14.875	22.5	0	0	0	0	0	0	0	3	
338	11.25	14.875	22.5	0	0	0	0	0	0	0	3	
339	18.75	14.875	22.5	0	0	0	0	0	0	0	3	
340	0	14.875	25.675	0	0	0	0	0	0	0	3	
341	3.75	14.875	25.675	0	0	0	0	0	0	0	3	
342	7.5	14.875	25.675	0	0	0	0	0	0	0	3	
343	11.25	14.875	25.675	0	0	0	0	0	0	0	3	
344	15	14.875	25.675	0	0	0	0	0	0	0	3	
345	18.75	14.875	25.675	0	0	0	0	0	0	0	0	
346	22.5	14.875	25.675	0	0	0	0	0	0	0	3	
347	3.75	14.875	30	0	0	0	0	0	0	0	3	
348	18.75	14.875	30	0	0	0	0	0	0	0	3	
349	3.75	18.625	0	0	0	0	0	0	0	0	3	! Floor 5
350	11.25	18.625	0	0	0	0	0	0	0	0	3	
351	18.75	18.625	0	0	0	0	0	0	0	0	3	
352	0	18.625	3.75	0	0	0	0	0	0	0	3	
353	3.75	18.625	3.75	0	0	0	0	0	0	0	3	
354	7.5	18.625	3.75	0	0	0	0	0	0	0	3	
355	11.25	18.625	3.75	0	0	0	0	0	0	0	3	
356	15	18.625	3.75	0	0	0	0	0	0	0	3	
357	18.75	18.625	3.75	0	0	0	0	0	0	0	3	
358	22.5	18.625	3.75	0	0	0	0	0	0	0	3	
359	3.75	18.625	7.5	0	0	0	0	0	0	0	3	
360	11.25	18.625	7.5	0	0	0	0	0	0	0	3	
361	18.75	18.625	7.5	0	0	0	0	0	0	0	3	
362	0	18.625	11.25	0	0	0	0	0	0	0	3	
363	3.75	18.625	11.25	0	0	0	0	0	0	0	3	
364	7.5	18.625	11.25	0	0	0	0	0	0	0	3	
365	11.25	18.625	11.25	0	0	0	0	0	0	0	3	
366	15	18.625	11.25	0	0	0	0	0	0	0	3	
367	18.75	18.625	11.25	0	0	0	0	0	0	0	3	
368	22.5	18.625	11.25	0	0	0	0	0	0	0	3	
369	3.75	18.625	15	0	0	0	0	0	0	0	3	
370	11.25	18.625	15	0	0	0	0	0	0	0	3	
371	18.75	18.625	15	0	0	0	0	0	0	0	3	
372	0	18.625	18.75	0	0	0	0	0	0	0	3	
373	3.75	18.625	18.75	0	0	0	0	0	0	0	3	
374	7.5	18.625	18.75	0	0	0	0	0	0	0	3	
375	11.25	18.625	18.75	0	0	0	0	0	0	0	3	
376	15	18.625	18.75	0	0	0	0	0	0	0	3	
377	18.75	18.625	18.75	0	0	0	0	0	0	0	3	
378	22.5	18.625	18.75	0	0	0	0	0	0	0	3	
379	3.75	18.625	22.5	0	0	0	0	0	0	0	3	
380	11.25	18.625	22.5	0	0	0	0	0	0	0	3	
381	18.75	18.625	22.5	0	0	0	0	0	0	0	3	
382	0	18.625	25.675	0	0	0	0	0	0	0	3	
383	3.75	18.625	25.675	0	0	0	0	0	0	0	3	
384	7.5	18.625	25.675	0	0	0	0	0	0	0	3	
385	11.25	18.625	25.675	0	0	0	0	0	0	0	3	
386	15	18.625	25.675	0	0	0	0	0	0	0	3	
387	18.75	18.625	25.675	0	0	0	0	0	0	0	3	
388	22.5	18.625	25.675	0	0	0	0	0	0	0	3	
389	3.75	18.625	30	0	0	0	0	0	0	0	3	
390	18.75	18.625	30	0	0	0	0	0	0	0	3	
391	3.75	22.375	0	0	0	0	0	0	0	0	3	! Floor 6
392	11.25	22.375	0	0	0	0	0	0	0	0	3	
393	18.75	22.375	0	0	0	0	0	0	0	0	3	
394	0	22.375	3.75	0	0	0	0	0	0	0	3	
395	3.75	22.375	3.75	0	0	0	0	0	0	0	3	
396	7.5	22.375	3.75	0	0	0	0	0	0	0	3	
397	11.25	22.375	3.75	0	0	0	0	0	0	0	3	
398	15	22.375	3.75	0	0	0	0	0	0	0	3	
399	18.75	22.375	3.75	0	0	0	0	0	0	0	3	
400	22.5	22.375	3.75	0	0	0	0	0	0	0	3	
401	3.75	22.375	7.5	0	0	0	0	0	0	0	3	
402	11.25	22.375	7.5	0	0	0	0	0	0	0	3	
403	18.75	22.375	7.5	0	0	0	0	0	0	0	3	
404	0	22.375	11.25	0	0	0	0	0	0	0	3	

405	3.75	22.375	11.25	0	0	0	0	0	0	0	3	
406	7.5	22.375	11.25	0	0	0	0	0	0	0	3	
407	11.25	22.375	11.25	0	0	0	0	0	0	0	3	
408	15	22.375	11.25	0	0	0	0	0	0	0	3	
409	18.75	22.375	11.25	0	0	0	0	0	0	0	3	
410	22.5	22.375	11.25	0	0	0	0	0	0	0	3	
411	3.75	22.375	15	0	0	0	0	0	0	0	3	
412	11.25	22.375	15	0	0	0	0	0	0	0	3	
413	18.75	22.375	15	0	0	0	0	0	0	0	3	
414	0	22.375	18.75	0	0	0	0	0	0	0	3	
415	3.75	22.375	18.75	0	0	0	0	0	0	0	3	
416	7.5	22.375	18.75	0	0	0	0	0	0	0	3	
417	11.25	22.375	18.75	0	0	0	0	0	0	0	3	
418	15	22.375	18.75	0	0	0	0	0	0	0	3	
419	18.75	22.375	18.75	0	0	0	0	0	0	0	3	
420	22.5	22.375	18.75	0	0	0	0	0	0	0	3	
421	3.75	22.375	22.5	0	0	0	0	0	0	0	3	
422	11.25	22.375	22.5	0	0	0	0	0	0	0	3	
423	18.75	22.375	22.5	0	0	0	0	0	0	0	3	
424	0	22.375	25.675	0	0	0	0	0	0	0	3	
425	3.75	22.375	25.675	0	0	0	0	0	0	0	3	
426	7.5	22.375	25.675	0	0	0	0	0	0	0	3	
427	11.25	22.375	25.675	0	0	0	0	0	0	0	3	
428	15	22.375	25.675	0	0	0	0	0	0	0	3	
429	18.75	22.375	25.675	0	0	0	0	0	0	0	3	
430	22.5	22.375	25.675	0	0	0	0	0	0	0	3	
431	3.75	22.375	30	0	0	0	0	0	0	0	3	
432	18.75	22.375	30	0	0	0	0	0	0	0	3	
433	3.75	26.125	0	0	0	0	0	0	0	0	3	! Floor 7
434	11.25	26.125	0	0	0	0	0	0	0	0	3	
435	18.75	26.125	0	0	0	0	0	0	0	0	3	
436	0	26.125	3.75	0	0	0	0	0	0	0	3	
437	3.75	26.125	3.75	0	0	0	0	0	0	0	3	
438	7.5	26.125	3.75	0	0	0	0	0	0	0	3	
439	11.25	26.125	3.75	0	0	0	0	0	0	0	3	
440	15	26.125	3.75	0	0	0	0	0	0	0	3	
441	18.75	26.125	3.75	0	0	0	0	0	0	0	3	
442	22.5	26.125	3.75	0	0	0	0	0	0	0	3	
443	3.75	26.125	7.5	0	0	0	0	0	0	0	3	
444	11.25	26.125	7.5	0	0	0	0	0	0	0	3	
445	18.75	26.125	7.5	0	0	0	0	0	0	0	3	
446	0	26.125	11.25	0	0	0	0	0	0	0	3	
447	3.75	26.125	11.25	0	0	0	0	0	0	0	3	
448	7.5	26.125	11.25	0	0	0	0	0	0	0	3	
449	11.25	26.125	11.25	0	0	0	0	0	0	0	3	
450	15	26.125	11.25	0	0	0	0	0	0	0	3	
451	18.75	26.125	11.25	0	0	0	0	0	0	0	3	
452	22.5	26.125	11.25	0	0	0	0	0	0	0	3	
453	3.75	26.125	15	0	0	0	0	0	0	0	3	
454	11.25	26.125	15	0	0	0	0	0	0	0	3	
455	18.75	26.125	15	0	0	0	0	0	0	0	3	
456	0	26.125	18.75	0	0	0	0	0	0	0	3	
457	3.75	26.125	18.75	0	0	0	0	0	0	0	3	
458	7.5	26.125	18.75	0	0	0	0	0	0	0	3	
459	11.25	26.125	18.75	0	0	0	0	0	0	0	3	
460	15	26.125	18.75	0	0	0	0	0	0	0	3	
461	18.75	26.125	18.75	0	0	0	0	0	0	0	3	
462	22.5	26.125	18.75	0	0	0	0	0	0	0	3	
463	3.75	26.125	22.5	0	0	0	0	0	0	0	3	
464	11.25	26.125	22.5	0	0	0	0	0	0	0	3	
465	18.75	26.125	22.5	0	0	0	0	0	0	0	3	
466	0	26.125	25.675	0	0	0	0	0	0	0	3	
467	3.75	26.125	25.675	0	0	0	0	0	0	0	3	
468	7.5	26.125	25.675	0	0	0	0	0	0	0	3	
469	11.25	26.125	25.675	0	0	0	0	0	0	0	3	
470	15	26.125	25.675	0	0	0	0	0	0	0	3	
471	18.75	26.125	25.675	0	0	0	0	0	0	0	3	
472	22.5	26.125	25.675	0	0	0	0	0	0	0	3	
473	3.75	26.125	30	0	0	0	0	0	0	0	3	
474	18.75	26.125	30	0	0	0	0	0	0	0	3	
475	0	0	18.75	1	1	1	1	1	1	0	3	
476	0	0	11.25	1	1	1	1	1	1	0	3	

477	22.5	0	18.75	1	1	1	1	1	1	0	3
478	22.5	0	11.25	1	1	1	1	1	1	0	3

ELEMENTS 3

1	8	1	21	0	0	x	! Base elements
2	8	2	22	0	0	x	
3	8	3	23	0	0	x	
4	8	4	24	0	0	x	
5	8	5	25	0	0	x	
6	7	6	26	0	0	x	
7	7	7	27	0	0	x	
8	8	8	28	0	0	x	
9	8	9	29	0	0	x	
10	7	10	30	0	0	x	
11	7	11	31	0	0	x	
12	8	12	32	0	0	x	
13	8	13	33	0	0	x	
14	7	14	34	0	0	x	
15	7	15	35	0	0	x	
16	8	16	36	0	0	x	
17	8	17	37	0	0	x	
18	8	18	38	0	0	x	
19	8	19	39	0	0	x	
20	8	20	40	0	0	x	
21	2	21	41	0	0	x	! Column elements at Ground Floor
22	5	22	42	0	0	x	
23	5	23	43	0	0	x	
24	2	24	44	0	0	x	
25	2	25	45	0	0	x	
26	1	26	46	0	0	x	
27	1	27	47	0	0	x	
28	2	28	48	0	0	x	
29	2	29	49	0	0	x	
30	1	30	50	0	0	x	
31	1	31	51	0	0	x	
32	2	32	52	0	0	x	
33	2	33	53	0	0	x	
34	1	34	54	0	0	x	
35	1	35	55	0	0	x	
36	2	36	56	0	0	x	
37	2	37	57	0	0	x	
38	5	38	58	0	0	x	
39	5	39	59	0	0	x	
40	2	40	60	0	0	x	
41	2	41	61	0	0	x	! Columns elements at Floor 1
42	5	42	62	0	0	x	
43	5	43	63	0	0	x	
44	2	44	64	0	0	x	
45	2	45	65	0	0	x	
46	1	46	66	0	0	x	
47	1	47	67	0	0	x	
48	2	48	68	0	0	x	
49	2	49	69	0	0	x	
50	1	50	70	0	0	x	
51	1	51	71	0	0	x	
52	2	52	72	0	0	x	
53	2	53	73	0	0	x	
54	1	54	74	0	0	x	
55	1	55	75	0	0	x	
56	2	56	76	0	0	x	
57	2	57	77	0	0	x	
58	5	58	78	0	0	x	
59	5	59	79	0	0	x	
60	2	60	80	0	0	x	
61	2	61	81	0	0	x	! Columns elements at Floor 2
62	5	62	82	0	0	x	
63	5	63	83	0	0	x	
64	2	64	84	0	0	x	

65	2	65	85	0	0	x	
66	1	66	86	0	0	x	
67	1	67	87	0	0	x	
68	2	68	88	0	0	x	
69	2	69	89	0	0	x	
70	1	70	90	0	0	x	
71	1	71	91	0	0	x	
72	2	72	92	0	0	x	
73	2	73	93	0	0	x	
74	1	74	94	0	0	x	
75	1	75	95	0	0	x	
76	2	76	96	0	0	x	
77	2	77	97	0	0	x	
78	5	78	98	0	0	x	
79	5	79	99	0	0	x	
80	2	80	100	0	0	x	
81	4	81	101	0	0	x	! Columns elements at Floor 3
82	6	82	102	0	0	x	
83	6	83	103	0	0	x	
84	4	84	104	0	0	x	
85	4	85	105	0	0	x	
86	3	86	106	0	0	x	
87	3	87	107	0	0	x	
88	4	88	108	0	0	x	
89	4	89	109	0	0	x	
90	3	90	110	0	0	x	
91	3	91	111	0	0	x	
92	4	92	112	0	0	x	
93	4	93	113	0	0	x	
94	3	94	114	0	0	x	
95	3	95	115	0	0	x	
96	4	96	116	0	0	x	
97	4	97	117	0	0	x	
98	6	98	118	0	0	x	
99	6	99	119	0	0	x	
100	4	100	120	0	0	x	
101	4	101	121	0	0	x	! Columns elements at Floor 4
102	6	102	122	0	0	x	
103	6	103	123	0	0	x	
104	4	104	124	0	0	x	
105	4	105	125	0	0	x	
106	3	106	126	0	0	x	
107	3	107	127	0	0	x	
108	4	108	128	0	0	x	
109	4	109	129	0	0	x	
110	3	110	130	0	0	x	
111	3	111	131	0	0	x	
112	4	112	132	0	0	x	
113	4	113	133	0	0	x	
114	3	114	134	0	0	x	
115	3	115	135	0	0	x	
116	4	116	136	0	0	x	
117	4	117	137	0	0	x	
118	6	118	138	0	0	x	
119	6	119	139	0	0	x	
120	4	120	140	0	0	x	
121	4	121	141	0	0	x	! Columns elements at Floor 5
122	6	122	142	0	0	x	
123	6	123	143	0	0	x	
124	4	124	144	0	0	x	
125	4	125	145	0	0	x	
126	3	126	146	0	0	x	
127	3	127	147	0	0	x	
128	4	128	148	0	0	x	
129	4	129	149	0	0	x	
130	3	130	150	0	0	x	
131	3	131	151	0	0	x	
132	4	132	152	0	0	x	
133	4	133	153	0	0	x	
134	3	134	154	0	0	x	
135	3	135	155	0	0	x	
136	4	136	156	0	0	x	

137	4	137	157	0	0	x	
138	6	138	158	0	0	x	
139	6	139	159	0	0	x	
140	4	140	160	0	0	x	
141	4	141	161	0	0	x	! Columns elements at Floor 6
142	6	142	162	0	0	x	
143	6	143	163	0	0	x	
144	4	144	164	0	0	x	
145	4	145	165	0	0	x	
146	3	146	166	0	0	x	
147	3	147	167	0	0	x	
148	4	148	168	0	0	x	
149	4	149	169	0	0	x	
150	3	150	170	0	0	x	
151	3	151	171	0	0	x	
152	4	152	172	0	0	x	
153	4	153	173	0	0	x	
154	3	154	174	0	0	x	
155	3	155	175	0	0	x	
156	4	156	176	0	0	x	
157	4	157	177	0	0	x	
158	6	158	178	0	0	x	
159	6	159	179	0	0	x	
160	4	160	180	0	0	x	
161	9	41	184	185	181	0	! 1 Floor
162	9	181	185	186	42	0	
163	9	182	187	188	43	0	
164	9	43	188	189	183	0	
165	9	183	189	190	44	0	
166	9	184	45	191	185	0	
167	9	185	191	46	186	0	
168	9	186	46	192	187	0	
169	9	187	192	47	188	0	
170	9	188	47	193	189	0	
171	9	189	193	48	190	0	
172	9	45	194	195	191	0	
173	9	191	195	196	46	0	
174	9	46	196	197	192	0	
175	9	192	197	198	47	0	
176	9	47	198	199	193	0	
177	9	193	199	200	48	0	
178	9	194	49	201	195	0	
179	9	195	201	50	196	0	
180	9	196	50	202	197	0	
181	9	197	202	51	198	0	
182	9	198	51	203	199	0	
183	9	199	203	52	200	0	
184	9	49	204	205	201	0	
185	9	201	205	206	50	0	
186	9	50	206	207	202	0	
187	9	202	207	208	51	0	
188	9	51	208	209	203	0	
189	9	203	209	210	52	0	
190	9	204	53	211	205	0	
191	9	205	211	54	206	0	
192	9	206	54	212	207	0	
193	9	207	212	55	208	0	
194	9	208	55	213	209	0	
195	9	209	213	56	210	0	
196	9	53	214	215	211	0	
197	9	211	215	216	54	0	
198	9	54	216	217	212	0	
199	9	212	217	218	55	0	
200	9	55	218	219	213	0	
201	9	213	219	220	56	0	
202	9	214	57	221	215	0	
203	9	215	221	58	216	0	
204	9	218	59	222	219	0	
205	9	219	222	60	220	0	
206	9	61	226	227	223	0	! 2 Floor
207	9	223	227	228	62	0	
208	9	224	229	230	63	0	

209	9	63	230	231	225	0	
210	9	225	231	232	64	0	
211	9	226	65	233	227	0	
212	9	227	233	66	228	0	
213	9	228	66	234	229	0	
214	9	229	234	67	230	0	
215	9	230	67	235	231	0	
216	9	231	235	68	232	0	
217	9	65	236	237	233	0	
218	9	233	237	238	66	0	
219	9	66	238	239	234	0	
220	9	234	239	240	67	0	
221	9	67	240	241	235	0	
222	9	235	241	242	68	0	
223	9	236	69	243	237	0	
224	9	237	243	70	238	0	
225	9	238	70	244	239	0	
226	9	239	244	71	240	0	
227	9	240	71	245	241	0	
228	9	241	245	72	242	0	
229	9	69	246	247	243	0	
230	9	243	247	248	70	0	
231	9	70	248	249	244	0	
232	9	244	249	250	71	0	
233	9	71	250	251	245	0	
234	9	245	251	252	72	0	
235	9	246	73	253	247	0	
236	9	247	253	74	248	0	
237	9	248	74	254	249	0	
238	9	249	254	75	250	0	
239	9	250	75	255	251	0	
240	9	251	255	76	252	0	
241	9	73	256	257	253	0	
242	9	253	257	258	74	0	
243	9	74	258	259	254	0	
244	9	254	259	260	75	0	
245	9	75	260	261	255	0	
246	9	255	261	262	76	0	
247	9	256	77	263	257	0	
248	9	257	263	78	258	0	
249	9	260	79	264	261	0	
250	9	261	264	80	262	0	
251	9	81	268	269	265	0	! 3 Floor
252	9	265	269	270	82	0	
253	9	266	271	272	83	0	
254	9	83	272	273	267	0	
255	9	267	273	274	84	0	
256	9	268	85	275	269	0	
257	9	269	275	86	270	0	
258	9	270	86	276	271	0	
259	9	271	276	87	272	0	
260	9	272	87	277	273	0	
261	9	273	277	88	274	0	
262	9	85	278	279	275	0	
263	9	275	279	280	86	0	
264	9	86	280	281	276	0	
265	9	276	281	282	87	0	
266	9	87	282	283	277	0	
267	9	277	283	284	88	0	
268	9	278	89	285	279	0	
269	9	279	285	90	280	0	
270	9	280	90	286	281	0	
271	9	281	286	91	282	0	
272	9	282	91	287	283	0	
273	9	283	287	92	284	0	
274	9	89	288	289	285	0	
275	9	285	289	290	90	0	
276	9	90	290	291	286	0	
277	9	286	291	292	91	0	
278	9	91	292	293	287	0	
279	9	287	293	294	92	0	
280	9	288	93	295	289	0	

281	9	289	295	94	290	0	
282	9	290	94	296	291	0	
283	9	291	296	95	292	0	
284	9	292	95	297	293	0	
285	9	293	297	96	294	0	
286	9	93	298	299	295	0	
287	9	295	299	300	94	0	
288	9	94	300	301	296	0	
289	9	296	301	302	95	0	
290	9	95	302	303	297	0	
291	9	297	303	304	96	0	
292	9	298	97	305	299	0	
293	9	299	305	98	300	0	
294	9	302	99	306	303	0	
295	9	303	306	100	304	0	
296	9	101	310	311	307	0	! 4 Floor
297	9	307	311	312	102	0	
298	9	308	313	314	103	0	
299	9	103	314	315	309	0	
300	9	309	315	316	104	0	
301	9	310	105	317	311	0	
302	9	311	317	106	312	0	
303	9	312	106	318	313	0	
304	9	313	318	107	314	0	
305	9	314	107	319	315	0	
306	9	315	319	108	316	0	
307	9	105	320	321	317	0	
308	9	317	321	322	106	0	
309	9	106	322	323	318	0	
310	9	318	323	324	107	0	
311	9	107	324	325	319	0	
312	9	319	325	326	108	0	
313	9	320	109	327	321	0	
314	9	321	327	110	322	0	
315	9	322	110	328	323	0	
316	9	323	328	111	324	0	
317	9	324	111	329	325	0	
318	9	325	329	112	326	0	
319	9	109	330	331	327	0	
320	9	327	331	332	110	0	
321	9	110	332	333	328	0	
322	9	328	333	334	111	0	
323	9	111	334	335	329	0	
324	9	329	335	336	112	0	
325	9	330	113	337	331	0	
326	9	331	337	114	332	0	
327	9	332	114	338	333	0	
328	9	333	338	115	334	0	
329	9	334	115	339	335	0	
330	9	335	339	116	336	0	
331	9	113	340	341	337	0	
332	9	337	341	342	114	0	
333	9	114	342	343	338	0	
334	9	338	343	344	115	0	
335	9	115	344	345	339	0	
336	9	339	345	346	116	0	
337	9	340	117	347	341	0	
338	9	341	347	118	342	0	
339	9	344	119	348	345	0	
340	9	345	348	120	346	0	
341	9	121	352	353	349	0	! 5 Floor
342	9	349	353	354	122	0	
343	9	350	355	356	123	0	
344	9	123	356	357	351	0	
345	9	351	357	358	124	0	
346	9	352	125	359	353	0	
347	9	353	359	126	354	0	
348	9	354	126	360	355	0	
349	9	355	360	127	356	0	
350	9	356	127	361	357	0	
351	9	357	361	128	358	0	
352	9	125	362	363	359	0	

353	9	359	363	364	126	0	
354	9	126	364	365	360	0	
355	9	360	365	366	127	0	
356	9	127	366	367	361	0	
357	9	361	367	368	128	0	
358	9	362	129	369	363	0	
359	9	363	369	130	364	0	
360	9	364	130	370	365	0	
361	9	365	370	131	366	0	
362	9	366	131	371	367	0	
363	9	367	371	132	368	0	
364	9	129	372	373	369	0	
365	9	369	373	374	130	0	
366	9	130	374	375	370	0	
367	9	370	375	376	131	0	
368	9	131	376	377	371	0	
369	9	371	377	378	132	0	
370	9	372	133	379	373	0	
371	9	373	379	134	374	0	
372	9	374	134	380	375	0	
373	9	375	380	135	376	0	
374	9	376	135	381	377	0	
375	9	377	381	136	378	0	
376	9	133	382	383	379	0	
377	9	379	383	384	134	0	
378	9	134	384	385	380	0	
379	9	380	385	386	135	0	
380	9	135	386	387	381	0	
381	9	381	387	388	136	0	
382	9	382	137	389	383	0	
383	9	383	389	138	384	0	
384	9	386	139	390	387	0	
385	9	387	390	140	388	0	
386	9	141	394	395	391	0	! 6 Floor
387	9	391	395	396	142	0	
388	9	392	397	398	143	0	
389	9	143	398	399	393	0	
390	9	393	399	400	144	0	
391	9	394	145	401	395	0	
392	9	395	401	146	396	0	
393	9	396	146	402	397	0	
394	9	397	402	147	398	0	
395	9	398	147	403	399	0	
396	9	399	403	148	400	0	
397	9	145	404	405	401	0	
398	9	401	405	406	146	0	
399	9	146	406	407	402	0	
400	9	402	407	408	147	0	
401	9	147	408	409	403	0	
402	9	403	409	410	148	0	
403	9	404	149	411	405	0	
404	9	405	411	150	406	0	
405	9	406	150	412	407	0	
406	9	407	412	151	408	0	
407	9	408	151	413	409	0	
408	9	409	413	152	410	0	
409	9	149	414	415	411	0	
410	9	411	415	416	150	0	
411	9	150	416	417	412	0	
412	9	412	417	418	151	0	
413	9	151	418	419	413	0	
414	9	413	419	420	152	0	
415	9	414	153	421	415	0	
416	9	415	421	154	416	0	
417	9	416	154	422	417	0	
418	9	417	422	155	418	0	
419	9	418	155	423	419	0	
420	9	419	423	156	420	0	
421	9	153	424	425	421	0	
422	9	421	425	426	154	0	
423	9	154	426	427	422	0	
424	9	422	427	428	155	0	

425	9	155	428	429	423	0	
426	9	423	429	430	156	0	
427	9	424	157	431	425	0	
428	9	425	431	158	426	0	
429	9	428	159	432	429	0	
430	9	429	432	160	430	0	
431	9	161	436	437	433	0	! 7 Floor
432	9	433	437	438	162	0	
433	9	434	439	440	163	0	
434	9	163	440	441	435	0	
435	9	435	441	442	164	0	
436	9	436	165	443	437	0	
437	9	437	443	166	438	0	
438	9	438	166	444	439	0	
439	9	439	444	167	440	0	
440	9	440	167	445	441	0	
441	9	441	445	168	442	0	
442	9	165	446	447	443	0	
443	9	443	447	448	166	0	
444	9	166	448	449	444	0	
445	9	444	449	450	167	0	
446	9	167	450	451	445	0	
447	9	445	451	452	168	0	
448	9	446	169	453	447	0	
449	9	447	453	170	448	0	
450	9	448	170	454	449	0	
451	9	449	454	171	450	0	
452	9	450	171	455	451	0	
453	9	451	455	172	452	0	
454	9	169	456	457	453	0	
455	9	453	457	458	170	0	
456	9	170	458	459	454	0	
457	9	454	459	460	171	0	
458	9	171	460	461	455	0	
459	9	455	461	462	172	0	
460	9	456	173	463	457	0	
461	9	457	463	174	458	0	
462	9	458	174	464	459	0	
463	9	459	464	175	460	0	
464	9	460	175	465	461	0	
465	9	461	465	176	462	0	
466	9	173	466	467	463	0	
467	9	463	467	468	174	0	
468	9	174	468	469	464	0	
469	9	464	469	470	175	0	
470	9	175	470	471	465	0	
471	9	465	471	472	176	0	
472	9	466	177	473	467	0	
473	9	467	473	178	468	0	
474	9	470	179	474	471	0	
475	9	471	474	180	472	0	
476	12	34	58	0	0	-x	! Inside cross bracing at Stairs
477	12	38	54	0	0	x	
478	13	54	78	0	0	-x	
479	13	58	74	0	0	x	
480	13	74	98	0	0	-x	
481	13	78	94	0	0	x	
482	14	94	118	0	0	-x	
483	14	98	114	0	0	x	
484	14	114	138	0	0	-x	
485	14	118	134	0	0	x	
486	15	134	158	0	0	-x	
487	15	138	154	0	0	x	
488	15	154	178	0	0	-x	
489	15	158	174	0	0	x	
490	12	23	47	0	0	-x	! Inside cross bracing opposite Stairs
491	12	27	43	0	0	x	
492	13	43	67	0	0	-x	
493	13	47	63	0	0	x	
494	13	63	87	0	0	-x	
495	13	67	83	0	0	x	
496	14	83	107	0	0	-x	

497	14	87	103	0	0	x	
498	14	103	127	0	0	-x	
499	14	107	123	0	0	x	
500	15	123	147	0	0	-x	
501	15	127	143	0	0	x	
502	15	143	167	0	0	-x	
503	15	147	163	0	0	x	
504	12	38	59	0	0	z	! Outside cross bracing at Stairs
505	12	39	58	0	0	-z	
506	13	58	79	0	0	z	
507	13	59	78	0	0	-z	
508	13	78	99	0	0	z	
509	13	79	98	0	0	-z	
510	14	98	119	0	0	z	
511	14	99	118	0	0	-z	
512	14	118	139	0	0	z	
513	14	119	138	0	0	-z	
514	15	138	159	0	0	z	
515	15	139	158	0	0	-z	
516	15	158	179	0	0	z	
517	15	159	178	0	0	-z	
518	12	22	43	0	0	z	! Outside cross bracing opposite Stairs
519	12	23	42	0	0	-z	
520	13	42	63	0	0	z	
521	13	43	62	0	0	-z	
522	13	62	83	0	0	z	
523	13	63	82	0	0	-z	
524	14	82	103	0	0	z	
525	14	83	102	0	0	-z	
526	14	102	123	0	0	z	
527	14	103	122	0	0	-z	
528	15	122	143	0	0	z	
529	15	123	142	0	0	-z	
530	15	142	163	0	0	z	
531	15	143	162	0	0	-z	
532	10	34	38	0	0	-x	! Beams at Stairs
533	10	38	39	0	0	z	
534	11	58	59	0	0	z	
535	11	78	79	0	0	z	
536	11	98	99	0	0	z	
537	11	118	119	0	0	z	
538	11	138	139	0	0	z	
539	11	158	159	0	0	z	
540	11	178	179	0	0	z	
541	10	23	27	0	0	-x	! Beams opposite Stairs
542	10	22	23	0	0	z	
543	11	42	43	0	0	z	
544	11	62	63	0	0	z	
545	11	82	83	0	0	z	
546	11	102	103	0	0	z	
547	11	122	123	0	0	z	
548	11	142	143	0	0	z	
549	11	162	163	0	0	z	
550	16	165	145	404	446	0	
551	16	145	125	362	404	0	
552	16	125	105	320	362	0	
553	16	105	85	278	320	0	
554	16	85	65	236	278	0	
555	16	65	45	194	236	0	
556	16	45	25	476	194	0	
557	16	169	149	414	456	0	
558	16	149	129	372	414	0	
559	16	129	109	330	372	0	
560	16	109	89	288	330	0	
561	16	89	69	246	288	0	
562	16	69	49	204	246	0	
563	16	49	29	475	204	0	
564	16	462	420	152	172	0	
565	16	420	378	132	152	0	
566	16	378	336	112	132	0	
567	16	336	294	92	112	0	
568	16	294	252	72	92	0	

```
569     16      252     210     52      72      0
570     16      210     477     32      52      0
571     16      452     410     148     168     0
572     16      410     368     128     148     0
573     16      368     326     108     128     0
574     16      326     284     88      108     0
575     16      284     242     68      88      0
576     16      242     200     48      68      0
577     16      200     478     28      48      0



PROPS
1       BEAM                                                            ! Inner Columns Ground Floor --> 3rd
floor
1       4       0       0       0       0       0
40.00E+9 1.74E+10 0.16  0.004266666     0.002133333     0.002133333     0       0       0       0
        3924
0

2       BEAM                                                            ! Outer Columns z axis Ground Floor -->
3rd floor
1       4       0       0       0       0       0
40.00E+9 1.74E+10 0.1   0.001854166     0.001333333     0.000520833     0       0       0       0
        2452.5
0

3       BEAM                                                            ! Inner Columns 3rd floor --> 7th floor

1       4       0       0       0       0       0
30.00E+9 1.30E+10 0.16  0.004266666     0.002133333     0.002133333     0       0       0       0
        3924
0

4       BEAM                                                            ! Outer Columns z axis 3rd floor --> 7th
floor
1       4       0       0       0       0       0
30.00E+9 1.30E+10 0.1   0.001854166     0.001333333     0.000520833     0       0       0       0
        2452.5
0

5       BEAM                                                            ! Outer Columns x axis Ground Floor -->
3rd floor
1       4       0       0       0       0       0
40.00E+9 1.74E+10 0.1   0.001854166     0.000520833     0.001333333     0       0       0       0
        2452.5
0

6       BEAM                                                            ! Outer Columns x axis 3rd floor --> 7th
floor
1       4       0       0       0       0       0
30.00E+9 1.30E+10 0.1   0.001854166     0.000520833     0.001333333     0       0       0       0
        2452.5
0

7       BEAM                                                            ! Inner Base
1       4       0       0       0       0       0
40.00E+9 1.74E+10 4     2.666666666     1.333333333     1.333333333     0       0       0       0
        98100
0
```

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8         BEAM                                                                 ! Outer Base
1         4         0         0         0         0         0
40.00E+9  1.74E+10  1.44      0.3456    0.1728    0.1728    0         0         0         0         35316
0


9         QUADRILATERAL                                                        ! Floor 1st floor --> 7th floor
0         0         30.00E+9  0.15      0.25      24525


10        BEAM                                                                 ! Beams Ground Floor
1         4         0         0         0         0         0
40.00E+9  1.74E+10  0.1625    0.023731771         0.022885417         0.000846354         0         0         0         0
          3985.3125
0


11        BEAM                                                                 ! Beams 1st floor --> 7th floor
1         4         0         0         0         0         0
30.00E+9  1.30E+10  0.1625    0.013067708         0.012221354         0.000846354         0         0         0         0
          3985.3125
0


12        BEAM                                                                 ! Cross Bracing bottom        (steel)
1         4         3         3         0         0         0
56.0E+9   24E+9     0.0020    6.683E-06 6.670E-06 16.70E-09 0         0         0         0         153.036
0


13        BEAM                                                                 ! Cross Bracing nearly bottom     (steel)
1         4         3         3         0         0         0
56.0E+9   24E+9     0.0015    2.825E-06 2.810E-06 12.50E-09 0         0         0         0         114.777
0


14        BEAM                                                                 ! Cross Bracing middle        (steel)
1         4         3         3         0         0         0
56.0E+9   24E+9     0.0010    0.842E-06 0.833E-06 8.33E-09  0         0         0         0         76.518
0


15        BEAM                                                                 ! Cross Bracing top  (steel)
1         4         3         3         0         0         0
56.0E+9   24E+9     0.0005    0.418E-06 0.417E-06 1.04E-09  0         0         0         0         38.259
0


16        QUADRILATERAL                                              ! Panels that do not contribute significantly to the stiffness of
the structure
1         0         16.00E+07 0.15      0.10      19620



WEIGHTS
1         0         0         0         0         0         0          ! Base under Columns at Ground Floor
45        10791     10791     10791     0         0         0          ! Sandbags Floor 1
46        21582     21582     21582     0         0         0
47        21582     21582     21582     0         0         0
48        10791     10791     10791     0         0         0
49        21582     21582     21582     0         0         0
50        43164     43164     43164     0         0         0
51        43164     43164     43164     0         0         0
52        21582     21582     21582     0         0         0
53        10791     10791     10791     0         0         0
54        21582     21582     21582     0         0         0
55        21582     21582     21582     0         0         0
56        10791     10791     10791     0         0         0
65        10791     10791     10791     0         0         0          ! Sandbags Floor 2
66        21582     21582     21582     0         0         0
67        21582     21582     21582     0         0         0
68        10791     10791     10791     0         0         0
```

69	21582	21582	21582	0	0	0	
70	43164	43164	43164	0	0	0	
71	43164	43164	43164	0	0	0	
72	21582	21582	21582	0	0	0	
73	10791	10791	10791	0	0	0	
74	21582	21582	21582	0	0	0	
75	21582	21582	21582	0	0	0	
76	10791	10791	10791	0	0	0	
85	10791	10791	10791	0	0	0	! Sandbags Floor 3
86	21582	21582	21582	0	0	0	
87	21582	21582	21582	0	0	0	
88	10791	10791	10791	0	0	0	
89	21582	21582	21582	0	0	0	
90	43164	43164	43164	0	0	0	
91	43164	43164	43164	0	0	0	
92	21582	21582	21582	0	0	0	
93	10791	10791	10791	0	0	0	
94	21582	21582	21582	0	0	0	
95	21582	21582	21582	0	0	0	
96	10791	10791	10791	0	0	0	
105	10791	10791	10791	0	0	0	! Sandbags Floor 4
106	21582	21582	21582	0	0	0	
107	21582	21582	21582	0	0	0	
108	10791	10791	10791	0	0	0	
109	21582	21582	21582	0	0	0	
110	43164	43164	43164	0	0	0	
111	43164	43164	43164	0	0	0	
112	21582	21582	21582	0	0	0	
113	10791	10791	10791	0	0	0	
114	21582	21582	21582	0	0	0	
115	21582	21582	21582	0	0	0	
116	10791	10791	10791	0	0	0	
125	10791	10791	10791	0	0	0	! Sandbags Floor 5
126	21582	21582	21582	0	0	0	
127	21582	21582	21582	0	0	0	
128	10791	10791	10791	0	0	0	
129	21582	21582	21582	0	0	0	
130	43164	43164	43164	0	0	0	
131	43164	43164	43164	0	0	0	
132	21582	21582	21582	0	0	0	
133	10791	10791	10791	0	0	0	
134	21582	21582	21582	0	0	0	
135	21582	21582	21582	0	0	0	
136	10791	10791	10791	0	0	0	
145	10791	10791	10791	0	0	0	! Sandbags Floor 6
146	21582	21582	21582	0	0	0	
147	21582	21582	21582	0	0	0	
148	10791	10791	10791	0	0	0	
149	21582	21582	21582	0	0	0	
150	43164	43164	43164	0	0	0	
151	43164	43164	43164	0	0	0	
152	21582	21582	21582	0	0	0	
153	10791	10791	10791	0	0	0	
154	21582	21582	21582	0	0	0	
155	21582	21582	21582	0	0	0	
156	10791	10791	10791	0	0	0	
165	10791	10791	10791	0	0	0	! Sandbags Floor 7
166	21582	21582	21582	0	0	0	
167	21582	21582	21582	0	0	0	
168	10791	10791	10791	0	0	0	
169	21582	21582	21582	0	0	0	
170	43164	43164	43164	0	0	0	
171	43164	43164	43164	0	0	0	
172	21582	21582	21582	0	0	0	
173	10791	10791	10791	0	0	0	
174	21582	21582	21582	0	0	0	
175	21582	21582	21582	0	0	0	
176	10791	10791	10791	0	0	0	
191	21582	21582	21582	0	0	0	! Sandbags Floor 1
192	21582	21582	21582	0	0	0	
193	21582	21582	21582	0	0	0	
194	21582	21582	21582	0	0	0	

195	43164	43164	43164	0	0	0	
196	43164	43164	43164	0	0	0	
197	43164	43164	43164	0	0	0	
198	43164	43164	43164	0	0	0	
199	43164	43164	43164	0	0	0	
200	21582	21582	21582	0	0	0	
201	43164	43164	43164	0	0	0	
202	43164	43164	43164	0	0	0	
203	43164	43164	43164	0	0	0	
204	21582	21582	21582	0	0	0	
205	43164	43164	43164	0	0	0	
206	43164	43164	43164	0	0	0	
207	43164	43164	43164	0	0	0	
208	43164	43164	43164	0	0	0	
209	43164	43164	43164	0	0	0	
210	21582	21582	21582	0	0	0	
211	21582	21582	21582	0	0	0	
212	21582	21582	21582	0	0	0	
213	21582	21582	21582	0	0	0	
216	57143	57143	57143	0	0	0	! Stair floor 1
217	57143	57143	57143	0	0	0	! Stair floor 1
233	21582	21582	21582	0	0	0	! Sandbags Floor 2
234	21582	21582	21582	0	0	0	
235	21582	21582	21582	0	0	0	
236	21582	21582	21582	0	0	0	
237	43164	43164	43164	0	0	0	
238	43164	43164	43164	0	0	0	
239	43164	43164	43164	0	0	0	
240	43164	43164	43164	0	0	0	
241	43164	43164	43164	0	0	0	
242	21582	21582	21582	0	0	0	
243	43164	43164	43164	0	0	0	
244	43164	43164	43164	0	0	0	
245	43164	43164	43164	0	0	0	
246	21582	21582	21582	0	0	0	
247	43164	43164	43164	0	0	0	
248	43164	43164	43164	0	0	0	
249	43164	43164	43164	0	0	0	
250	43164	43164	43164	0	0	0	
251	43164	43164	43164	0	0	0	
252	21582	21582	21582	0	0	0	
253	21582	21582	21582	0	0	0	
254	21582	21582	21582	0	0	0	
255	21582	21582	21582	0	0	0	
258	57143	57143	57143	0	0	0	! Stair floor 2
259	57143	57143	57143	0	0	0	! Stair floor 2
275	21582	21582	21582	0	0	0	! Sandbags Floor 3
276	21582	21582	21582	0	0	0	
277	21582	21582	21582	0	0	0	
278	21582	21582	21582	0	0	0	
279	43164	43164	43164	0	0	0	
280	43164	43164	43164	0	0	0	
281	43164	43164	43164	0	0	0	
282	43164	43164	43164	0	0	0	
283	43164	43164	43164	0	0	0	
284	21582	21582	21582	0	0	0	
285	43164	43164	43164	0	0	0	
286	43164	43164	43164	0	0	0	
287	43164	43164	43164	0	0	0	
288	21582	21582	21582	0	0	0	
289	43164	43164	43164	0	0	0	
290	43164	43164	43164	0	0	0	
291	43164	43164	43164	0	0	0	
292	43164	43164	43164	0	0	0	
293	43164	43164	43164	0	0	0	
294	21582	21582	21582	0	0	0	
295	21582	21582	21582	0	0	0	
296	21582	21582	21582	0	0	0	
297	21582	21582	21582	0	0	0	
300	57143	57143	57143	0	0	0	! Stair floor 3
301	57143	57143	57143	0	0	0	! Stair floor 3
317	21582	21582	21582	0	0	0	! Sandbags Floor 4

318	21582	21582	21582	0	0	0	
319	21582	21582	21582	0	0	0	
320	21582	21582	21582	0	0	0	
321	43164	43164	43164	0	0	0	
322	43164	43164	43164	0	0	0	
323	43164	43164	43164	0	0	0	
324	43164	43164	43164	0	0	0	
325	43164	43164	43164	0	0	0	
326	21582	21582	21582	0	0	0	
327	43164	43164	43164	0	0	0	
328	43164	43164	43164	0	0	0	
329	43164	43164	43164	0	0	0	
330	21582	21582	21582	0	0	0	
331	43164	43164	43164	0	0	0	
332	43164	43164	43164	0	0	0	
333	43164	43164	43164	0	0	0	
334	43164	43164	43164	0	0	0	
335	43164	43164	43164	0	0	0	
336	21582	21582	21582	0	0	0	
337	21582	21582	21582	0	0	0	
338	21582	21582	21582	0	0	0	
339	21582	21582	21582	0	0	0	
342	57143	57143	57143	0	0	0	! Stair floor 4
343	57143	57143	57143	0	0	0	! Stair floor 4
359	21582	21582	21582	0	0	0	! Sandbags Floor 5
360	21582	21582	21582	0	0	0	
361	21582	21582	21582	0	0	0	
362	21582	21582	21582	0	0	0	
363	43164	43164	43164	0	0	0	
364	43164	43164	43164	0	0	0	
365	43164	43164	43164	0	0	0	
366	43164	43164	43164	0	0	0	
367	43164	43164	43164	0	0	0	
368	21582	21582	21582	0	0	0	
369	43164	43164	43164	0	0	0	
370	43164	43164	43164	0	0	0	
371	43164	43164	43164	0	0	0	
372	21582	21582	21582	0	0	0	
373	43164	43164	43164	0	0	0	
374	43164	43164	43164	0	0	0	
375	43164	43164	43164	0	0	0	
376	43164	43164	43164	0	0	0	
377	43164	43164	43164	0	0	0	
378	21582	21582	21582	0	0	0	
379	21582	21582	21582	0	0	0	
380	21582	21582	21582	0	0	0	
381	21582	21582	21582	0	0	0	
384	57143	57143	57143	0	0	0	! Stair floor 5
385	57143	57143	57143	0	0	0	! Stair floor 5
401	21582	21582	21582	0	0	0	! Sandbags Floor 6
402	21582	21582	21582	0	0	0	
403	21582	21582	21582	0	0	0	
404	21582	21582	21582	0	0	0	
405	43164	43164	43164	0	0	0	
406	43164	43164	43164	0	0	0	
407	43164	43164	43164	0	0	0	
408	43164	43164	43164	0	0	0	
409	43164	43164	43164	0	0	0	
410	21582	21582	21582	0	0	0	
411	43164	43164	43164	0	0	0	
412	43164	43164	43164	0	0	0	
413	43164	43164	43164	0	0	0	
414	21582	21582	21582	0	0	0	
415	43164	43164	43164	0	0	0	
416	43164	43164	43164	0	0	0	
417	43164	43164	43164	0	0	0	
418	43164	43164	43164	0	0	0	
419	43164	43164	43164	0	0	0	
420	21582	21582	21582	0	0	0	
421	21582	21582	21582	0	0	0	
422	21582	21582	21582	0	0	0	
423	21582	21582	21582	0	0	0	

426	57143	57143	57143	0	0	0	! Stair floor 6
427	57143	57143	57143	0	0	0	! Stair floor 6
443	21582	21582	21582	0	0	0	! Sandbags Floor 7
444	21582	21582	21582	0	0	0	
445	21582	21582	21582	0	0	0	
446	21582	21582	21582	0	0	0	
447	43164	43164	43164	0	0	0	
448	43164	43164	43164	0	0	0	
449	43164	43164	43164	0	0	0	
450	43164	43164	43164	0	0	0	
451	43164	43164	43164	0	0	0	
452	21582	21582	21582	0	0	0	
453	43164	43164	43164	0	0	0	
454	43164	43164	43164	0	0	0	
455	43164	43164	43164	0	0	0	
456	21582	21582	21582	0	0	0	
457	43164	43164	43164	0	0	0	
458	43164	43164	43164	0	0	0	
459	43164	43164	43164	0	0	0	
460	43164	43164	43164	0	0	0	
461	43164	43164	43164	0	0	0	
462	21582	21582	21582	0	0	0	
463	21582	21582	21582	0	0	0	
464	21582	21582	21582	0	0	0	
465	21582	21582	21582	0	0	0	
468	57143	57143	57143	0	0	0	! Stair floor 7
469	57143	57143	57143	0	0	0	! Stair floor 7
478	0	0	0	0	0	0	! Last Node

LOADS

1	0	0	0	0	0	0	! First Node
478	0	0	0	0	0	0	! Last Node

EQUAKE null.eqs ! x direction earthquake
4 1 0.005 981.0 -1

EQUAKE null.eqs ! y direction earthquake
4 1 0.005 981.0 -1

EQUAKE SR2000KAV.eqs ! z direction earthquake
4 1 0.005 981.0 -1

Example of a FE model for the secondary system

```
Secondary system 1, decoupled, Floor response
2 0 0 4 3 0 0 0                                          ! Control Parameters
DEFAULT                                                  ! Earthquake Parameters
2 1 1 12 10 0 9.81 0.0081 0.0069 0.005 30.0              ! Structural Parameters
0  1  0 20.0 0.1 0.1 0.1 0                               ! Output Parameters
0 0 0.001 0.0 0.0 0.0 0.0 0.0 0.0                        ! Iteration Parameters
1 0.42 2 0.8 3 0.8 4 0.8 5 0.8 6 0.8 7 0.8 8 0.8 9 0.8 10 0.8   ! User Specified Modal Damping Parameters

NODES 0
1      0      0      0      1      1      1      1      1      1      0      ! Node 4th floor
2      0      1.15   0      1      1      0      1      1      1      0      ! Node SS

ELEMENTS
1      1      1      2      0      0      x      ! SS

PROPS
1      BEAM
1      4      0      0      0      0      0
3.00E+09 1.30E+09 1   7.95724E-06   2.65E-06 5.30E-06 0   0   0   0   0
0

WEIGHTS
1      0      0      0      0      0      0
2      10791  10791  10791  0      0      0      ! SS

LOADS
1      0      0      0      0      0      0      ! 4th floor
2      0      0      0      0      0      0      ! SS

EQUAKE null.eqs                          ! x direction earthquake
4 1 0.005 981.0 -1

EQUAKE null.eqs                          ! y direction earthquake
4 1 0.005 981.0 -1

EQUAKE ACC2000KAVTOT.eqc                 ! z direction earthquake
1 1 0.005 98100 -1 0 0 1
```