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The Effects of Changes in EU Emission Trading Scheme on the Value of a Gas Fired Power Plant in Norway

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MSc in Business Analysis and Performance Management and MSc in Sustainable Development

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Bergen, spring 2011

Abstract

The purpose of this thesis was to study how changes in the carbon market influences the present value of the gas fired power plant operated by Naturkraft AS. The changes in focus here are the ones introduced in EU's Emission Trading Scheme after 2012, for instance higher prices on emissions and no more free emission allowances to the power generating industry.

Based on secondary data, I used a binomial real option's model that I programmed with Microsoft Visual Basic 6.5.

The principal conclusion was that the present value is only slightly affected by an increase in the price of emissions (3.8% reduction in PV in the highest price scenario), but much more so by the removal of free emission allowances (34% reduction in PV). Interestingly, I also found that the present value of the plant actually increases with higher prices of emission allowances given that the plant still receives free emission allowances.

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Nomenclature

Symbol	Meaning
$\frac{\mathbf{OWC_{2010}}}{\mathbf{sales_{2010}^{kWh}}}$	OWC as a percent of sales, equal to 11.22% in 2010
$\Delta_{\mathbf{i}}$	change in an underlying asset
$\mathbf{P_t}$	risk premium
$\mathbf{AV_n}$	abandonment value
$\mathbf{AV_{t,j}^{OWC}}$	end year/period abandonment value of operating working capital
$\mathbf{AV_{t/n}^{I\&\mathit{T}}}$	end year/period abandonment value of inventory and tools
$\mathbf{AV_{t/n}^{INT}}$	end year/period abandonment value of intangibles
$\mathbf{AV_{t/n}^{P\&\mathit{E}}}$	end year/period abandonment value of plant and equipment
$\mathbf{CF_{n,j}^{op}}$	operational cash flow
$\mathbf{CF_n}$	flexible cash flow from project
$\mathbf{CF_n^{SD}}$	total cash flow when having the option to shut down production
$\mathbf{CF_n^{fix}}$	net fixed cash flow
$\mathbf{CF_n^{var}}$	net variable cash flow
$\mathbf{ER_0^{NOK/Euro}}$	exchange rate between NOK and Euros (7.89 NOK/Euro on 8th May 2011)
$\mathbf{ER_0^{NOK/GBP}}$	current exchange rate of NOK per GBP (9.00 NOK/GBP 21th May 2011)
$\mathbf{EUAP_t^{Euro/EUA}}$	price of EUAs in Euro with the purching power of 2009
$\mathbf{EUAP_t^{NOK/kWh}}$	price of EUAs in NOK per kWh
$\mathbf{FOC_n}$	fixed operating costs as defined in Table 2, i. e. without depreciation, after tax
$\mathbf{FixAV_{t/n}}$	end year/period fixed abandonment value, from I&T, $P\&E$ and INT
$\mathbf{FixInv_{t/n}}$	fixed investments in I&T and $P\&E$
$\mathbf{FixTaxDed_n}$	fixed tax deductions from depreciation of all capital
$\mathbf{F_t}$	futures price, with transaction and deliverance at time t
$\mathbf{I\&}T_{\mathbf{t/n}}$	end year/period tax value of inventory and tools
$\mathbf{I_{ETS}^{change}}$	indicator variable assuming $(0 = \text{no change in ETS}, 1 = \text{change in ETS})$
$\mathbf{INT_{t/n}}$	end year/period tax value of intangibles
$\mathbf{OWC_{n,j}^{CF}}$	cash flow effect from change in operating working capital over a period
$\mathbf{P\&}E_{\mathbf{t/n}}$	end year/period tax value of plant and equipment
$\mathbf{P_t^{EUA}}$	price of EUAs on general form
$\mathbf{Q_n}$	volume produced
$\mathbf{REUA_t}$	annual number of EUAs received

$\bar{\mathbf{S}}$	long-run average price of the underlying asset
$\mathbf{S_{n,j}}$	price of underlying asset
$\mathbf{TaxDed_{n}^{I\&T}}$	tax deductions from depreciation of inventory and tools
$\mathbf{TaxDed_{t/n}^{INT}}$	tax deductions from depreciation of intangibles
$\mathbf{TaxDed_{t/n}^{P\&E}}$	tax deductions from depreciation of plant and equipment
$\mathbf{VOC_{n}}$	variable operating costs, as defined in Table 2, after tax
$\mathbf{V_{n,j}}$	value of project in sub period n with up moves j
$\mathbf{V_{n}^{plant}}$	value of the plant in each state in each period
$\mathbf{cm_{t}}$	contribution margin
$\mathbf{cost_{n,j_{ng}}^{NG}}$	cost of natural gas after tax
$\mathbf{cost_{t/n}^{UEUA}}$	cost of used EUAs after tax
$\mathbf{delta_{S}}$	number of shares bought at the start of the period
$\mathbf{elF_{t}}$	futures price for electrical power, with transaction and deliverance at time t
$\mathbf{elF_{t}^{Euros/MWh}}$	the futures price of electrical power denoted in euros per MWh
$\mathbf{elP_{n,j_{el}}}$	the price of electrical power per kWh
$\mathbf{elP_{t}^{kWh}}$	price of electrical power per kWh
$\mathbf{f_{j}}$	risk neutral denoting one out of four outcomes
$\mathbf{f_{u}}$ **and** $\mathbf{f_{d}}$	risk neutral probabilities for outcomes u and d
$\mathbf{highP_{t}^{EUA}}$	high EUA price scenario
$\mathbf{infl_{Euro}}$	inflation rate of Euro (2% equal to the target inflation rate of ECB)
$\mathbf{inv_{t/n}^{I\&T}}$	investments in of inventory and tools
$\mathbf{inv_{t/n}^{P\&E}}$	investments in of plant and equipment
$\mathbf{kWh_{t}}$	production per year, Naturkraft produces at capacity i. e. 3.5 TWh or nothing
$\mathbf{ln}\ S_{t,i}$	log of the price of an underlying asset
$\mathbf{lowP_{t}^{EUA}}$	low EUA price scenario
$\mathbf{medP_{t}^{EUA}}$	medium EUA price scenario
$\mathbf{ngF_{t}}$	futures price for natural gas, with transaction and deliverance at time t
$\mathbf{ngF_{t}^{NOK/kWh}}$	futures price of natural gas in NOK per chemical kWh
$\mathbf{ngF_{t}^{pence/BTU}}$	British Thermal Units equivalent to about 29.307 kWhs
$\mathbf{ngP_{n,j_{ng}}}$	cost of natural gas in NOK per kWh produced described in the binomial model
$\mathbf{ngP_{t}}$	the cost of natural gas per kWh produced
$\mathbf{r_{I\&T}^{depr}}$	rate of depreciation for inventory and tools (30%)
$\mathbf{r_{INT}^{depr}}$	rate of depreciation for intangibles (20%)

$\mathbf{r_{P\&E}^{change}}$	net change in plant and equipment (−3.6%)
$\mathbf{r_{P\&E}^{inv}}$	rate of depreciation for plant and equipment (4%)
$\mathbf{r_{P\&E}^{inv}}$	reinvestment rate for plant and equipment (0.4%)
$\mathbf{r_f}$	risk free interest rate after tax, logarithmic (2.68%)
$\mathbf{r_i}$	logarithmic return
$\mathbf{r^{tax}}$	the Norwegian nominal corporate tax rate, i.e. 28%
$\mathbf{sales_{n,j_{el}}^{el}}$	sales income after tax from electrical power
$\mathbf{sales_{t,jX}^{kWh}}$	annual income after tax from sale of electrical power
$\mathbf{sales_{t/n}^{REUA}}$	fixed cash flow from the potential sale of free EUAs
$\mathbf{tc_t^{NOK/kWh}}$	transport price of natural gas in NOK per chemical kWh
$\mathbf{tc_{2003}^{NOK/Sm^3}}$	the price of natural gas in NOK per Sm^3 equal to 0.125 nominally in 2003
$\mathbf{u_{el}^{j_{el}}}$ & $\mathbf{d_{el}^{n-j_{el}}}$	up an down moves for electrical power, calculated as in equation 3.1
$\mathbf{u_{ng}^{j_{ng}}}$ & $\mathbf{d_{ng}^{n-j_{ng}}}$	the up an down moves for natural gas, calculated as in equation 3.1
$\boldsymbol{\varepsilon_t}$	normally distributed with mean zero and standard deviation σ_ε at time t
$\boldsymbol{\rho_{X,Y}}$	coefficient of correlation between two underlying assets X and Y
$\boldsymbol{\sigma_i^2}$	variance in logarithmic returns
$\boldsymbol{\sigma_n}$	volatility for each sub periods
$\boldsymbol{\Delta t}$	time intervals, i. e. length of each period in the binomial model
$\mathbf{B}$	bank loan at the start of the period
$\mathbf{BTU}$	sales income after tax from electrical power
$\mathbf{E(S_t)}$	expected spot price at time t
$\mathbf{N}$	number of sub periods in total
$\mathbf{NoChP_t^{EUA/kWh}}$	EUA price scenario in which no changes are made to EU ETS
$\mathbf{OCA}$	end year/period tax value of operating current assets
$\mathbf{OCL}$	end year/period tax value of operating current liabilities
$\mathbf{OWC}$	end year/period tax value of operating working capital
$\mathbf{UEUA}$	number of used EUAs when producing 3.5 TWh (1 240 000 EUAs)
$\mathbf{a}$	approximated drift factor
$\mathbf{b}$	exponential weight on observation in an EWMA
$\mathbf{dz_X dz_Y}$	incremental correlation
$\mathbf{h}$	heat rate adjustment, adjusts from chemical energy to produced energy
$\mathbf{infl}$	logarithmic inflation target from the Central Bank of Norway (2.46%)
$\mathbf{j}$	indicates number of upward jumps in n steps
$\mathbf{n}$	indicates number of sub period

$\mathbf{u}, \mathbf{d}$	factors by which the value of the underlying asset move up or down
$\mathbf{v}(\ln \mathbf{S}_\mathbf{t})$	drift in the logarithm of the price of an underlying asset
$\boldsymbol{\delta}$	net dividends, assumed to be paid and reinvested continuously
$\boldsymbol{\kappa}$	mean reversion coefficient
$\boldsymbol{\sigma}$	annual volatility

1 Introduction

Under the United Nations Framework Convention on Climate Change (UNFCCC) in Rio de Janeiro in 1992, UN concluded that the world needs to reduce human greenhouse gas (GHG) emissions. This resulted in the ratification of the Kyoto protocol in 1997 by most of the western countries. The agreement specified regulations and entered into force on 16 February 2005 (UNFCCC Secretariat 2004).

Each Annex-I[1] country are allowed to emit a certain amount of GHGs of which units have been standardized and are referred to as Assigned Amount Units (AAU). To comply with the agreement, the parties of the agreement can either reduce their emissions or they can buy more AAUs from other parties. There are three mechanisms for trading. Firstly, the Clean Development Mechanism (CDM) allows a party to cause an additional emission reduction in a developing country and get saleable certified emission reduction (CER), i.e. credits, in return. Secondly, the Joint Implementation (JI) mechanism gives a party the right to earn emission reduction units (ERU) from a jointly implemented project that causes additional emission reduction in another country. Finally, the emission trading scheme (ETS) or the carbon market, allows the participants to trade emission allowances. One CER, ERU or AAU equals on metric ton of CO_2 equivalent. The Norwegian government will also use other means to fulfil their commitment to the Kyoto protocol, like investments in big research projects such as Carbon Capture Storage (CCS) technology, energy efficiency, renewable energy and so on (Statistics Norway 2009, 9).

1.1 The European Emission Trading Scheme

The EU ETS was introduced in 2005 and uses the cap and trade principle, i.e. there is a limit or “cap”, on the total amount of specified GHGs that can be emitted by the participants in the system. Also this system uses standardized emission allowances. One ton of CO_2 equivalent equals one EU allowance unit (EUA) and can be traded without restrictions within the EU ETS. The scheme now operates in the 27 EU countries plus Iceland, Lichtenstein and Norway. The latter joined on 28th of March 2008.

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[1] These countries include the industrial members of the OECD in 1992 and the countries in transition, including the Russian Federation, The Baltic States and Several Central and Eastern European States (UNFCCC Secretariat 2004).

From 2008 to 2012, about 40 percent of the emissions in Norway are covered by the EU ETS. The Norwegian government allocates about 15 million EUAs each year to its industry using a national allocation plan (NAP). About half of those are given away for free and the other half is auctioned. The most important reason for free allocation is probably fear of carbon leakage[2]. The oil and gas industry does not receive free allowances and the main rule is also not to grant any to companies established or expanded after 28 March 2008 (Ministry of the Environment 2008, 2-3).

1.1.1 The Auctioning of EUAs in the EU ETS After 2012

In what is called the third phase of the EU ETS (2013-2020), the Directorate-General (DG) for Climate Action states that a "*progressive move towards auctioning of allowances, will further enhance its effectiveness*". Moreover, there will no longer be any NAPs. Instead, the allocation of EUAs will be determined centrally in the EU. This will harmonize competition between countries and help preventing carbon leakage within EU. The DG for Climate Action also states that auctioning will be the main allocation method as of 2013 and that no allowances will be allocated for free to electricity production, "*with only limited and temporary options*" to deviate from this main rule (Directorate-General for Climate Action 2011). However, it is also decided that members of the EU ETS which want to establish their own auction platform, may do so, because regulation provides for "*adequate rules, as to the functioning of such auction platforms and the coordination with the common platform"*. The countries that have decided to opt out of the planned common platform for auctioning allowances for the third phase are Germany, Poland and the UK. The deadline for members to do this was 19th February 2011, so the rest of the member will use the centrally determined auctioning plan (Directorate-General for Climate Action 2011).

It was mentioned that only a few deviations will be allowed to the rule that states that power generators will no longer receive free allowances in the third phase. The member states that have this option to apply for a deviation from this rule are: Bulgaria, Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland and Romania. The free

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[2] The term "carbon leakage" is commonly seen in the Norwegian media, and refers to when a reduction in emissions in one place causes an increase in emissions somewhere else. Carbon leakage happens mainly for two reasons, one because businesses move their production elsewhere, and two, because they close down and other businesses start up in a different country to meet the now unmet demand for the relevant product.

allowances that these members might be allowed to give to their power generating industry will have to be phased out before 2020 (Directorate-General for Climate Action 2011).

1.1.2 The Price of EUAs in the EU ETS After 2012

In addition to the changes in the allocation plans, there will also be changes in the price of the EUAs. The cap and trade system automatically leads to higher prices when the total amount of allowed emissions is reduced. In March 2007, EU members sat themselves a set of demanding goals known as the “20-20-20” targets. The name refers to a reduction in EU GHG emissions of at least 20% relative below the 1990-levels, 20% energy consumption from renewable energy resources and a 20% reduction in primary energy use compared with projected levels, to be achieved by improving energy efficiency. The leaders of EU have also offered to increase the emission reductions from 20% to 30% if other major emitting countries commit to do their fair share under a global climate agreement. Also part of the 20-20-20-deal was a gradually reduced cap on emission allowances from 2013 towards 2020, which should lead to higher prices on EUAs. Whether or not the 20%-goals are actually reached will have a large effect on the price of EUAs. If for instance the energy efficiency improvement fails, a larger share of the reductions in emissions will have to be reached through the carbon market, causing a higher EUA price (Directorate-General for Climate Action 2010).

1.2 Motivation

I expect that the changes in the EU ETS in the third phase will have large consequences for the power generating industry. My interest is to analyze the consequences these changes may have on the profitability of a specific company. How will they adapt to changes? Modern gas fired power plants have far lower emissions than coal fired power plants and are often referred to as a transition technology from a carbon based economy to a sustainable economy based on renewable energy.

The management of Naturkraft AS, a gas fired power plant on the west coast of Norway, has stated that they are discriminated against because the NAP of Norway allocated less allowances to Naturkraft compared to similar companies in the EU ETS receives. When the NAP disappears after 2012, this discrimination will cease, but so will the free EUAs.

2 Research Question

The research question below was based on what I believed to be interesting, concerning the changes in the EU ETS in the third period. It is formulated as following:

What are the effects of the changes in EU ETS in the third phase relative to the second phase, on the present value of the gas fired power plant operated by

To answer the research question I will not take any normative stands regarding who should pay for emission reductions, the reality of global warming or the fairness of the EU ETS.

The changes referred to will be discussed in this thesis.

A gas fired power plant will be directly affected by changes in the carbon market through having to buy EUAs to a different price and through receiving a different amount of EUAs for free. This will enable me to work with hard numbers when discussing emissions. Studying a single plant as opposed to using a macro perspective has its advantages and drawbacks. It will be impossible to draw general conclusions to other parts of the economy, simply based on one project. It will in return give me extra insight on a firm level, and allow me to take into account the details which I would otherwise need to ignore. The choice is analogous to the choice between conducting a quantitative interview with perhaps thousands of respondents to a qualitative interview with only a few respondents.

The gas fired power plant operated by Naturkraft AS is located at Kårstø, a small industrial city along the west coast of Norway. Naturkraft is owned 50% by Statoil ASA and 50% by Statkraft AS. The latter is 100% owned by the Norwegian state while the former is 67% owned by the Norwegian state (Statoil ASA 2009). The plant was officially opened November 1, 2007 and has about 32 employees. The plant is built by Siemens and has a combined cycle turbine (Siemens u.d.). It has an installed effect of 430 MW, annual production capacity of about 3.5 TWh and an efficiency of about 59%. Each year the emissions amount to about 1.2 million tons of CO2 equivalents. The investment is of about NOK 2 billion (Naturkraft AS n.d.).

3 Theory

In the theory section of the thesis, as in the rest of the thesis, a strong emphasis will be put on real options theory. In the case of a gas fired power plant, it is important to recognice that the management is aymmetrically positioned to capitalize on upside outcomes, but can cut losses on downside outcomes. Thus a real options model seems to be the best fitting method of valuation. Monte Carlo simulations is perhaps the most commonly used real options valuation method, but one can also use discrete binomial methods, trinomial methods along with various continuous time models. I will use the binomial method due to its logical and surveyable structure. It also allows for extensive sensitivity analyses which I will need to conduct to answer my research question.

The most basic option theory and strategic analyses like Porter's Five Forces Framework and SWIMA are assumed to be known by the reader and will not be presented, although some of it is used.

3.1 Body of Literature Used in Thesis

Real option theory is similar to financial option theory in many aspects, to which McDonald (2006), Hull (2009) give a good presentation. Smith and Trigeorgis (2004) give a thourough understanding of what real options actually are, and a basic introduction to real options in isolation. While it is helpful to study single real options closely, Brosch (2008) stresses that real options must be evaluated in portfolios because the value of options depends on each other. He also provides a more advanced mathematical approach to real options, which is useful especially when using more than two or more correlated underlying assets. Real options on underlying assets with mean reverting prices is described well by Guimarães (2008) and Hahn and Dyer (2008), and finally Benninga (2008) shows many useful codes in Visual Basic. The literature above overlaps on many topics, but they all provide unique contributions to this thesis.

In the following sections I will quickly go through the basics of option theory while using more space on the theory which is especially relevant to my model.

3.2 Risk Neutral Valuation and Estimating Input Variables

The price can move either up or down in each time interval. One needs to know by how much and the likeliness of each outcome. Next the equations for both are presented, and the risk neutral valuation method is explained.

3.2.1 Estimating the Up and Down Moves

The up and down moves in the binomial model will be estimated as in equation 3.1.

equation 3.1 $\mathrm{u} = \mathrm{e}^{\sigma\sqrt{\Delta \mathrm{t}}}, \qquad \mathrm{d} = \mathrm{e}^{-\sigma\sqrt{\Delta \mathrm{t}}}$

Notes:

$\mathbf{u}, \mathbf{d}$	factors by which the value of the underlying asset move up or down
$\mathbf{\Delta t}$	time intervals, i. e. length of each period in the binomial model

The derivation of these can be found in appendix 7.1.2 along with their relations to the risk neutral probabilities. The up and down moves are here defined as independent of drift. This will be taken care of in the risk-neutral probabilities which are described next.

3.2.2 Risk Neutral Probabilities and Risk Neutral Valuation

The risk neutral probabilities are described in equation 3.2 and the derivation of these is based on the replicating portfolio technique which can be found in appendix 7.1.

equation 3.2 $\mathrm{f_u} = \frac{\mathrm{e}^{(\mathrm{r_f}-\delta)\times\Delta \mathrm{t}}-\mathrm{d}}{\mathrm{u}-\mathrm{d}}$ and $\mathrm{f_d} = 1 - \mathrm{f_u} = \frac{\mathrm{u}-\mathrm{e}^{(\mathrm{r_f}-\delta)\times\Delta \mathrm{t}}}{\mathrm{u}-\mathrm{d}}$

Notes:

$\mathbf{f_u}$**and** $\mathbf{f_d}$	risk neutral probabilities for outcomes u and d
$\boldsymbol{\delta}$	net dividends, assumed to be paid and reinvested continuously
$\mathbf{r_f}$	risk-free rate, logarithmic returns

In equation 3.3 below, (n, j) is used to reference the value in the node in sub period number n with j up-movements in n steps.

equation 3.3 $\mathrm{S_{n,j}} = \mathrm{S_0} \times \mathrm{u^j} \times \mathrm{d^{n-j}}$

Notes:

$S_{n,j}$	price of underlying asset
n	indicates number of sub period
j	indicates number of upward jumps in n steps

So (n, j) is simply a way to reference states in the system state space. Each time prices, cash flows or values etc. have (n, j) attached, its position in the system state space is revealed, but if only one time interval is discussed I will just use u or d to denote an up or down move outcome.

Now that the main parameters have been described the risk neutral valuation can be described. The risk neutral valuation process in a binomial multiplicative process is illustrated in Figure 3-1 below. It is clear that the value in period one relies on the values in period 2. This is why one normally starts by estimating the value in the last period first, and then work ones way backwards using equation 3.4 to the first period, hence the name valuation by backwards recursion (or just risk neutral valuation).

equation 3.4 $$V_{n,j} = e^{-r\times\Delta t} \times \left(V_{n+1,j+1} \times f_u + V_{n+1,j} \times f_d\right)$$

Notes:

$V_{n,j}$	value of project in sub period n with j up moves

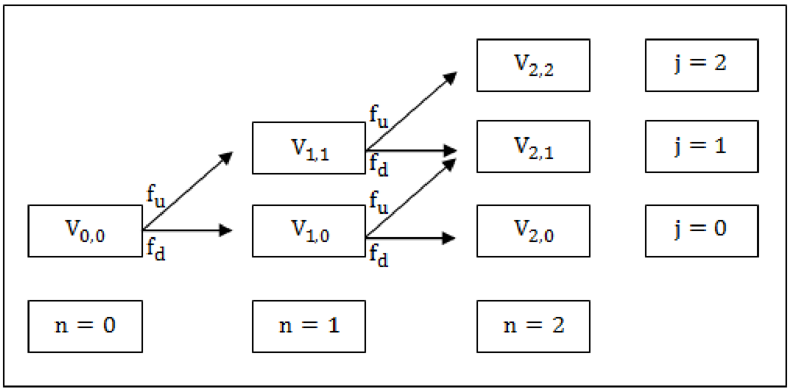

Figure 3-1 Referencing system

Risk-neutral valuation assumes unrestricted short sales and borrowing, arbitrage-free, frictionless and complete markets (Black and Scholes, 1973, cited in Brosch, 2008).

3.2.3 Two Correlated Underlying Assets

When two underlying assets are correlated one needs to evaluate them simultaneously in a binomial tree because the up and down moves for one asset will influence the moves in the other asset. The resulting three-dimensional binomial tree is visualized in Figure 3-2:

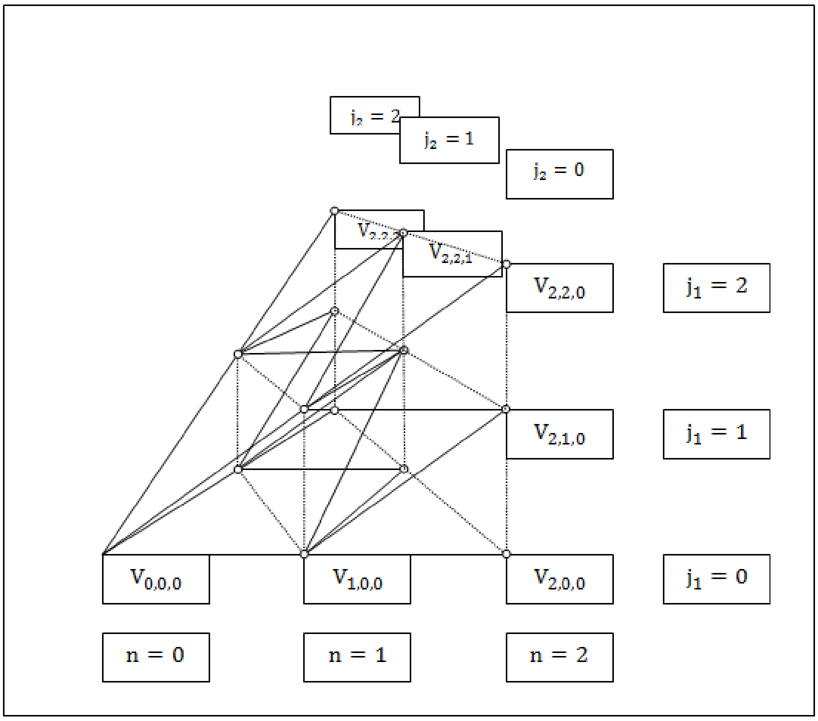

Figure 3-2 Recombining binomial tree with two correlated underlying assets

3.2.4 Referencing with Two Underlying Assets

When each underlying asset can move either up or down, then each state will have four possible outcomes in the next period, and the earlier reference system becomes insufficient.

I will continue to use n for periods or sub, and n for periods in the binomial tree. So that V_{n,j_x,j_y} refers to project value in period n with j_x and j_y upward moves in the underlying assets X and Y. However, sometimes it is not necessary to reference specific states in the system state space, but only how two subsequent states relates to each other (while their location in the system state space is irrelevant). If this is the case I can use j to indicate outcomes in general, but since each state has four possible successors I will denote each of them with footnotes 1, 2, 3 and 4. Which outcome each of those footnote numbers indicates is shown in equation 3.5.

equation 3.5
$$j = \begin{cases} 1 \text{ indicates both underlying assets moving up } (u_X \cap u_Y) \\ 2 \text{ indicates only first underlying asset moving up } (u_X \cap d_Y) \\ 3 \text{ indicates only second underlying asset moving up } (d_X \cap u_Y) \\ 4 \text{ indicates none of the underlying assets moving up } (d_X \cap d_Y) \end{cases}$$

If nothing else is specified the reader can assume for the rest of the thesis that footnotes 1 through 4 denotes the outcomes described in equation 3.5. For instance $f_1 = f_{u_X \cap u_Y}$ i.e. the risk-neutral probability of both of the underlying assets moving up by factor u. Since this is only a relative reference it doesn’t reference a specific state in the system state space. Should the need for doing that occur, I will go back to using (n, j_X, j_Y) which is the system used in Figure 3-2.

3.2.5 Risk-Neutral Probabilities with Two Underlying Assets

Each of the underlying assets’ up- and down factors can still be modelled using equation 3.1 and equation 3.1, and the backwards recursive method can still be used for valuation purposes. However, each node in the binomial tree will have four successors instead of two, each with a different risk-neutral probability. These probabilities can be derived using the same logic as with one underlying asset and were derived by Boyle et al. (1989 cited in Brosch 2008, 60):

equation 3.6
$$f_{u_X \cap u_Y} = \frac{1}{4}\left[1 + \rho_{X,Y} + \sqrt{\Delta t} \times \left(\frac{r_f - \frac{1}{2}\sigma_X^2}{\sigma_X} + \frac{r_f - \frac{1}{2}\sigma_Y^2}{\sigma_Y}\right)\right]$$

equation 3.7
$$f_{u_X \cap d_Y} = \frac{1}{4}\left[1 - \rho_{X,Y} + \sqrt{\Delta t} \times \left(\frac{r_f - \frac{1}{2}\sigma_X^2}{\sigma_X} - \frac{r_f - \frac{1}{2}\sigma_Y^2}{\sigma_Y}\right)\right]$$

equation 3.8
$$f_{d_X \cap u_Y} = \frac{1}{4}\left[1 - \rho_{X,Y} + \sqrt{\Delta t} \times \left(-\frac{r_f - \frac{1}{2}\sigma_X^2}{\sigma_X} + \frac{r_f - \frac{1}{2}\sigma_Y^2}{\sigma_Y}\right)\right]$$

equation 3.9
$$f_{d_X \cap d_Y} = \frac{1}{4}\left[1 + \rho_{X,Y} + \sqrt{\Delta t} \times \left(-\frac{r_f - \frac{1}{2}\sigma_X^2}{\sigma_X} - \frac{r_f - \frac{1}{2}\sigma_Y^2}{\sigma_Y}\right)\right]$$

Notes:

$\boldsymbol{\rho_{X,Y}}$	coefficient of correlation between two underlying assets X and Y
$\boldsymbol{\sigma_i^2}$	variance in logarithmic returns
$\mathbf{f_j}$	risk neutral denoting one out of four outcomes

3.2.6 Mean-Reverting Stochastic Processes

Mean-reversion means that the price of the underlying asset will tend to converge towards some long-run average price level, S_{norm}. In a mean reverting process, if the current price $S_t \neq S_{Norm}$, it will revert towards the normal level. Mean reversion can be modelled with trinomial trees, through adjusting the up and down moves, adjusting the risk-neutral probabilities or a combinations of these. This thesis will use a model that adjusts the risk-neutral probabilities and therefore refrain from presenting the other alternatives.

Mean reversion is most commonly found in commodities which are hard or costly to store. Otherwise, investors could have bought commodities when they were cheap and waited for them to revert back to their normal price and sell with a profit (Baron, et al. 2002). The price of assets which cannot be stored is simply determined by supply and demand. Higher-than-normal commodity prices reduce demand, encourage development of alternative products, and stimulate additional investments to increase the production of the commodity. This drives the price of the commodity back down, and vice versa for low prices.

3.2.6.1 Modelling Mean Reversion with One Underlying Asset

Nelson and Ramaswamy (1990, as cited in Hahn and Dyer 2008) model reversion by keeping the up and down moves fixed, but recalculates the probabilities in each node to incorporate mean reversion or local drift. In the GBM based model, the drift is constant and independent of underlying assets. In the the mean reversion model the drift needs to depend on time and the value of the underlying asset as in equation 3.10. In the Geometric Ornstein-Uhlenbeck process, given by equation 3.11, the drift depends on the current value of the underlying asset (S_t) and the long-run average price level $(\bar{S})$. Whenever the price of the underlying asset is above or below the mean value it will revert towards the mean value with "speed" κ. On logarithmic form I get equation 3.12, which is easier to work with, especially when applying Itô's lemma.

equation 3.10 $$dS_t = a(S,t)dt + \sigma dz$$

equation 3.11 $$dS_t = r_f + \kappa S_t(\bar{S} - S_t)dt + \sigma dz_t$$

equation 3.12 $$d\ln S_t = \left(r_f + \kappa(\ln\bar{S} - \ln S_t)\right)dt + \sigma dz_t$$

Notes:

$\bar{S}$	long-run average price of the underlying asset
κ	mean reversion coefficient

Using the result of Itô's lemma[3] on the process in equation 3.12 the result in equation 3.13 below follows. This expression is less messy than the one I would get if I didn't use the logarithmic form. Substituting $\mathrm{v}(\ln \mathrm{S_t}) = \mathrm{r_f} + \kappa(\ln \overline{\mathrm{S}} - \ln \mathrm{S_t}) - \frac{1}{2}\sigma^2$, I get the result in equation 3.13. The risk neutral probabilities of the up and down moves can be written on another form than earlier. With equation 7.108 it is possible to approximate the exponential functions when Δt is small. Again ignoring the higher powers, the approximation results of the exponential function in the equation for the risk-neutral probability is given in equation 3.14. Using these approximations the risk neutral probabilities can be presented as in equation 3.15. Hahn and Dyer (2008, 537) then use max- and min functions to make sure that the probability stays between zero and one, which can otherwise occur when mean reversion is involved. This censoring of the probabilities is shown in equation 3.16.

equation 3.13 $$\mathrm{d} \ln \mathrm{S_t} = \left(\mathrm{r_f} + \kappa(\ln \overline{\mathrm{S}} - \ln \mathrm{S_t}) - \frac{1}{2}\sigma^2\right)\mathrm{dt} + \sigma \mathrm{dz} = \mathrm{v}(\ln \mathrm{S_t})\mathrm{dt} + \sigma \mathrm{dz}$$

equation 3.14 $$\mathrm{u} = \mathrm{e}^{\sigma\sqrt{\Delta t}} \approx 1 + \sigma\sqrt{\Delta t}, \qquad \mathrm{d} \approx 1 - \sigma\sqrt{\Delta t}, \qquad \mathrm{a} = \mathrm{e}^{\mathrm{v(Y,t)}\Delta t} \approx 1 + \mathrm{v(Y,t)}\Delta t$$

equation 3.15 $$\mathrm{f_u} = \frac{\mathrm{a-d}}{\mathrm{u-d}} = \frac{(1+\mathrm{v(Y,t)}\Delta t)-(1-\sigma\sqrt{\Delta t})}{(1+\sigma\sqrt{\Delta t})-(1-\sigma\sqrt{\Delta t})} = \frac{\mathrm{v(Y,t)}\Delta t+\sigma\sqrt{\Delta t}}{2\sigma\sqrt{\Delta t}} = \frac{1}{2} + \sqrt{\Delta t}\frac{\mathrm{v(Y,t)}}{2\times\sigma}, \quad \mathrm{f_d} = 1 - \mathrm{f_u}$$

equation 3.16 $$\mathrm{f_u} = \max\left[0, \min\left[1, \left(\frac{1}{2} + \sqrt{\Delta t}\frac{\mathrm{v(Y,t)}}{2\times\sigma}\right)\right]\right]$$

Notes:

a	approximated drift factor
$\mathbf{v(\ln S_t)}$	drift in the logarithm of the price of an underlying asset

The censoring process in equation 3.16 causes slightly upward or downward biased values depending on the current price. However, the approximation converges rapidly and Hahn and Dyer (2008) argue that the values approximate within 1% when using quarter year time intervals $\Delta t = 3/12$.

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[3] For an informal derivation of Ito's lemma I refer the reader to read (Hull 2009). The book gives a good and easy-to-understand introduction to Ito's lemma.

3.2.6.2 Modelling Mean Reversion with Two Underlying Assets

Expanding the results so far to two underlying assets is straightforward. Any relation between the two assets is described in equation 3.19 with the incremental correlation between the two. I continue using the logarithmic prices.

equation 3.17 $$d \ln S_{t,X} = \left(\mathrm{r_f} + \kappa\left(\ln \overline{\mathrm{S}}_\mathrm{X} - \ln S_{t,X}\right)\right) \mathrm{dt} + \sigma_\mathrm{X} \mathrm{dz_X} = \mathrm{v}\left(\ln S_{t,X}\right)\mathrm{dt} + \sigma_\mathrm{X} \mathrm{dz_X}$$

equation 3.18 $$d \ln S_{t,Y} = \left(\mathrm{r_f} + \kappa\left(\ln \overline{\mathrm{S}}_\mathrm{Y} - \ln S_{t,Y}\right)\right) \mathrm{dt} + \sigma_\mathrm{Y} \mathrm{dz_Y} = \mathrm{v}\left(\ln S_{t,Y}\right)\mathrm{dt} + \sigma_\mathrm{Y} \mathrm{dz_Y}$$

equation 3.19 $$\mathrm{dz_X dz_Y} = \rho_\mathrm{XY} \mathrm{dt}$$

Notes:

$\ln S_{t,i}$	log of the price of an underlying asset
$\mathrm{dz_X dz_Y}$	incremental correlation

The rest of the derivation of risk-neutral probabilities is in many aspects similar to the one in section 3.2.5 (on page 17). The increments in the up and down moves (e.g. $\sigma\sqrt{\Delta t}$ in equation 3.1) and the risk neutral probabilities are given by the formulas below. To show that the equations used for risk-neutral probabilities are comparable to the ones used in section 3.2.5, equation 3.21 is shown on the same form as before.

equation 3.20 $$\Delta_\mathrm{X} = \sigma_\mathrm{X}\sqrt{\Delta \mathrm{t}}, \qquad \Delta_\mathrm{Y} = \sigma_\mathrm{Y}\sqrt{\Delta \mathrm{t}}$$

equation 3.21 $$\mathrm{f}_{\mathrm{u_X} \cap \mathrm{u_Y}} = \frac{\Delta_\mathrm{X}\Delta_\mathrm{Y} + \Delta_\mathrm{Y} \mathrm{v_X} \Delta \mathrm{t} + \Delta_\mathrm{X} \mathrm{v_Y} \Delta \mathrm{t} + \rho \sigma_\mathrm{X} \sigma_\mathrm{Y} \Delta \mathrm{t}}{4\Delta_\mathrm{X}\Delta_\mathrm{Y}} = \frac{1}{4}\left(1 + \rho + \sqrt{\Delta \mathrm{t}} \times \left(\frac{\mathrm{r_f} + \kappa\left(\overline{\mathrm{S_X}} - \mathrm{S_{t,X}}\right) - \frac{1}{2}\sigma_\mathrm{X}^2}{\sigma_\mathrm{X}} + \frac{\mathrm{r_f} + \kappa\left(\overline{\mathrm{S_Y}} - \mathrm{S_{t,Y}}\right) - \frac{1}{2}\sigma_\mathrm{Y}^2}{\sigma_\mathrm{Y}}\right)\right)$$

equation 3.22 $$\mathrm{f}_{\mathrm{u_X} \cap \mathrm{d_Y}} = \frac{\Delta_\mathrm{X}\Delta_\mathrm{Y} + \Delta_\mathrm{Y} \mathrm{v_X} \Delta \mathrm{t} - \Delta_\mathrm{X} \mathrm{v_Y} \Delta \mathrm{t} - \rho \sigma_\mathrm{X} \sigma_\mathrm{Y} \Delta \mathrm{t}}{4\Delta_\mathrm{X}\Delta_\mathrm{Y}}$$

equation 3.23 $$\mathrm{f}_{\mathrm{d_X} \cap \mathrm{u_Y}} = \frac{\Delta_\mathrm{X}\Delta_\mathrm{Y} - \Delta_\mathrm{Y} \mathrm{v_X} \Delta \mathrm{t} + \Delta_\mathrm{X} \mathrm{v_Y} \Delta \mathrm{t} - \rho \sigma_\mathrm{X} \sigma_\mathrm{Y} \Delta \mathrm{t}}{4\Delta_\mathrm{X}\Delta_\mathrm{Y}}$$

equation 3.24 $$\mathrm{f}_{\mathrm{d_X} \cap \mathrm{d_Y}} = \frac{\Delta_\mathrm{X}\Delta_\mathrm{Y} - \Delta_\mathrm{Y} \mathrm{v_X} \Delta \mathrm{t} - \Delta_\mathrm{X} \mathrm{v_Y} \Delta \mathrm{t} + \rho \sigma_\mathrm{X} \sigma_\mathrm{Y} \Delta \mathrm{t}}{4\Delta_\mathrm{X}\Delta_\mathrm{Y}}$$

Notes:

$\Delta_\mathbf{i}$	change in an underlying asset

3.2.6.3 Censoring Process for Two Underlying Assets

With four probabilities it is no longer possible to directly censor the probabilities like in equation 3.16 (on page 19). Instead Hahn and Dyer (2008) advocate using a method which involves decomposing the probabilities with Bayes' Rule into the marginal probabilities in equation 3.26 and the conditional probabilities in equation 3.27 through equation 3.30. Bayes' Rule is shown in equation 3.25.

equation 3.25 $$f_{u_X \cap u_Y} = f_{u_Y | u_X} \times f_{u_X}$$

equation 3.26 $$f_{u_X} = f_{u_X, u_Y} + f_{u_X, d_Y} = \frac{1}{2} + \frac{1}{2} \frac{v_X \Delta t}{\Delta_X}, \qquad f_{d_X} = f_{d_X, u_Y} + f_{d_X, d_Y} = \frac{1}{2} - \frac{1}{2} \frac{v_X \Delta t}{\Delta_X}$$

equation 3.27 $$f_{u_Y | u_X} = \frac{f_{u_X \cap u_Y}}{f_{u_X}} = \frac{\frac{\Delta_X \Delta_Y + \Delta_Y v_X \Delta t + \Delta_X v_Y \Delta t + \rho \sigma_X \sigma_Y \Delta t}{4 \Delta_X \Delta_Y}}{\frac{1}{2} + \frac{1}{2} \frac{v_X \Delta t}{\Delta_X}} = \frac{\Delta_X (\Delta_Y + \Delta t \times v_Y) + \Delta t (\Delta_Y \times v_X + \rho \sigma_X \sigma_Y)}{2 \Delta_Y (\Delta_X + \Delta t \times v_X)}$$

equation 3.28 $$f_{d_Y | u_X} = \frac{f_{u_X \cap d_Y}}{f_{u_X}} = \frac{\Delta_X (\Delta_Y - \Delta t \times v_Y) + \Delta t (\Delta_Y \times v_X - \rho \sigma_X \sigma_Y)}{2 \Delta_Y (\Delta_X + \Delta t \times v_X)}$$

equation 3.29 $$f_{u_Y | d_X} = \frac{f_{d_X \cap u_Y}}{f_{d_X}} = \frac{\Delta_X (\Delta_Y - \Delta t \times v_Y) - \Delta t (\Delta_Y \times v_X - \rho \sigma_X \sigma_Y)}{2 \Delta_Y (\Delta_X + \Delta t \times v_X)}$$

equation 3.30 $$f_{d_Y | d_X} = \frac{f_{d_X \cap d_Y}}{f_{d_X}} = \frac{\Delta_X (\Delta_Y + \Delta t \times v_Y) - \Delta t (\Delta_Y \times v_X + \rho \sigma_X \sigma_Y)}{2 \Delta_Y (\Delta_X + \Delta t \times v_X)}$$

The censoring procedure then goes as following. First the marginal probabilities and conditional probabilities for the outcomes are calculated and censored one by one like in equation 3.16. Next, the joint probabilities are recalculated with Bayes' Rule and censored. The recalculated joint probabilities should then be ready for use. A proof of the correct convergence using this method is shown in the appendix of Hahn and Dyer (2008, 547). In equation 3.27 extra details are included to show the equation's relation to equation 3.21 and equation 3.26.

3.2.6.4 Estimating Parameters for Mean Reversion

Using the logarithmic version of the Geometric Ornstein-Uhlenbeck model, a value for the parameter κ is required. Guimarães (2008) provides the recipee for estimating this parameter. The expression in equation 3.32 shows the change in the value of the logarithms of price in discrete time. To estimate the parameters of mean-reversion, run a regression on the dataset of log returns of prices using on the form in equation 3.33, and then estimate the parameters in equation 3.34.

equation 3.31 $$\mathrm{d}\ln S_t = \left(\kappa(\ln \bar{S} - \ln S_t) - \frac{1}{2}\sigma^2\right)\mathrm{d}t + \sigma \mathrm{d}z$$

equation 3.32 $$\ln S_t - \ln S_{t-\Delta t} = \ln \bar{S} \times (1 - e^{-\kappa\times\Delta t}) + \ln S_{t-1} \times (1 - e^{-\kappa\times\Delta t}) + \varepsilon_t$$

equation 3.33 $$\ln S_t - \ln S_{t-\Delta t} = \beta_0 + \beta_1 \times \ln S_{t-\Delta t} + \varepsilon_t$$

equation 3.34 $$\ln \bar{S} = -\frac{\beta_0}{\beta_1}, \qquad \kappa = -\ln(1 + \beta_1)$$

equation 3.35 $$\ln \bar{S} = \left(\frac{1}{\Delta t}\right) \times -\frac{\beta_0}{\beta_1}, \qquad \kappa = \left(\frac{1}{\Delta t}\right) \times -\ln(1 + \beta_1)$$

Notes:

$\boldsymbol{\varepsilon_t}$	normally distributed with mean zero and standard deviation σ_ε at time t

Finally, when working with such datasets, the results must be annualized by multiplying by the number of intervals per year $1/\Delta t$ as in equation 3.35.

3.3 Real Options Implications

3.3.1 Replicating Portfolio Applied on Real Options

Using exactly the same method to evaluate real options isn't always possible. Real options are different from financial options in that they aren't traded in arbitrage-free markets. They exist in imperfect markets with convenience yields and other sources of uncertainty. The position in an underlying asset would require a project equivalent in the market which value is correlated with the value of the underlying asset. When the underlying asset is a commodity, traded on an exchange with similar risk characteristics as the project as a financial instrument in a market, it will be possible to replicate it and use the no-arbitrage method. The typical examples will be gold, coal, oil and gas fields whose value of license is estimated using prices from exchanges. If the value of the real underlying asset isn't correlated with anything traded in financial markets, one would have to estimate the real asset's value as if it were traded in the market. So real options valuation is still applicable provided it is possible to find a reliable estimate of the market value of the real asset.

3.3.2 Portfolios of Real Options and Their Value

The value of real options on the same underlying asset cannot be estimated independently from each other. For example, the value of the option to abandon will interact and influence the value of the option to expand or shut down production. Therefore you will have to evaluate the value of a portfolio of real options. This means that when creating a binomial tree for the price development of the underlying assets you need to maximize value for the entire real options portfolio, to capture the actual real options value in each node.

3.4 Different Types of Flexibility

The reason for choosing real option to value something is usually that the management has opportunities or options to make adjustments or to take actions in response to changes in prices in the market. In the next two sections two such options will be described.

3.4.1 Option to Temporarily Shut Down Production

Smith og Trigeorgis (2004, 120) provides simple decisions rules for production decisions. Management may temporarily shut down production if the contribution margin from operation is negative, and by doing that removing the variable cash flow. The project may also have fixed inevitable cash flows that cannot be avoided by shutting down production. It can be intuitive to consider having the plant as an option to produce if the sales price (equivalent to the exercise price in a put option) is higher than variable cash flow (equivalent to the spot price). In each period, the cash flow can be described by equation 3.36. To value the project with risk-neutral valuation equation 3.37 can be applied in each state, when working towards present time from the end nodes. Zero switching cost is assumed.

equation 3.36 $$\mathrm{CF_n = CF_n^{fixed} + MAX[Q_n \times cm_t, 0]}$$

equation 3.37 $$\mathrm{V_n = CF_n + e^{-r_f \times \Delta t} \times \sum_{i=1}^{4} f_j \times V_{n+1,j}}$$

Notes:

$\mathbf{CF_n}$	flexible cash flow from project
$\mathbf{Q_n}$	volume produced
$\mathbf{cm_t}$	contribution margin
$\mathbf{CF_n^{fix}}$	net fixed cash flow

3.4.2 Option to Abandon Project

If prices develop unfavourably, and the value of abandoning the project is higher than the remaining operating value of keeping it, an economically rational manager would choose to abandon. The value of abandonment is equal to the maximum value of the salvage value and the value in its best alternative use. Even when the abandonment value is negative the option to abandon can be valuable if the present value of keeping the project is more negative.

I will first describe the valuation method with an example with one underlying asset only. Since I work backwards in the binomial tree, I will start with the situation in the last period. In Figure 3-1 the value of the investment project in the last period-states will equal the scrap value (the plant is abandoned when its lifetime is up) plus the operating profit for the last period as illustrated in equation 3.38:

equation 3.38 $\mathrm{CF}_{\mathrm{n,j}}^{\mathrm{plant}} = \mathrm{AV}_{\mathrm{n}} + \mathrm{CF}_{\mathrm{n,j}}^{\mathrm{op}}$

Notes:

$\mathbf{AV_n}$	abandonment value
$\mathbf{CF_{n,j}^{op}}$	operational cash flow

In earlier periods, the project will still receive the operational cash flow, but one cannot know for sure if the project is abandoned towards the end of the period, which will only happen if the abandonment value for that period is higher than the present value of continuing. In state $\mathrm{V_{n,j}}$ the plant will produce the operational cash flow $\mathrm{CF_{n,j}^{op}}$ plus the maximum value of abandoning today $\mathrm{AV_n}$ and the present value of continuing operations one more period. Since the present value of continuing operations one more period is not a cash flow in the current period, it is better to describe the valuation process in backwards recursive value calculations. The value of the investment project in period n can be described with equation 3.39. To find the present value of the investment project one will have to use this equation to find project value in all the states in the last period first, and work ones way towards the present, period by period.

equation 3.39 $\mathrm{V_{n,j}^{plant}} = \mathrm{CF_n^{op}} + \mathrm{MAX}\left[\mathrm{AV_n}, \mathrm{e}^{-\mathrm{r_f}\times\Delta\mathrm{t}} \times \left\{\mathrm{f_u} \times \mathrm{V_{n+1,j+1}} + \mathrm{f_d} \times \mathrm{V_{n+1,j}}\right\}\right]$

If I use two underlying assets instead of one, the value in each state can be calculated in exactly the same manner:

equation 3.40 $$V_{n,j}^{plant} = CF_n^{op} + MAX\left[AV_n, e^{-r_f \times \Delta t} \times \left\{\sum_{j=1}^{4} f_{n+1,j} \times V_{n+1,j}\right\}\right]$$

The value of the option to abandon is especially large in capital intensive industries or industries with a low degree of sunk cost.

3.5 Futures Prices and Expected Future Spot Prices

There are two main theories of the pricing of futures contracts. One is the theory of storage, which states that market players can offset risk in a forward contract by holding a short or long position in the underlying asset. Since risk hedged, the only compensation for taking such a position must be financing cost minus any net convenience yield. The price of such a futures contract can be described by equation 3.41 below (Hull 2009, 120). One of the assumptions of this theory is that it is possible to store the underlying asset and arbitrage free markets. The other theory is the theory of expectations, which is used when the underlying asset isn't storable. Here the forward price of a commodity price is the expected spot price during the delivery period plus an expected risk premium that compensates producers for bearing uncertainty of delivering against fixed prices. This is shown in equation 3.42 which can be rearranged into equation 3.43 (Huisman and Kilic 2011).

equation 3.41 $$F_t = S_0 \times e^{(r_f - \delta) \times t}$$

equation 3.42 $$F_t = E(S_t) + P_t$$

equation 3.43 $$E(S_t) = F_t - P_t$$

Notes:

$\mathbf{F_t}$	futures price, with transaction and deliverance at time t
$\mathbf{E(S_t)}$	expected spot price at time t
$\mathbf{P_t}$	risk premium

This suggests the expected spot price at time t, can be derived from the forward curve if one knows the value of P_t. The risk premium is, however, not directly observable and it can also change over time and over different time intervals. Inconveniently, the risk premium is often defined differently, but in my case, Figure 3-3 will show that when the risk premium is negative the futures price will be lower than the expected spot price which is called normal backwardation. When the risk premium is positive we say that the futures market is contango.

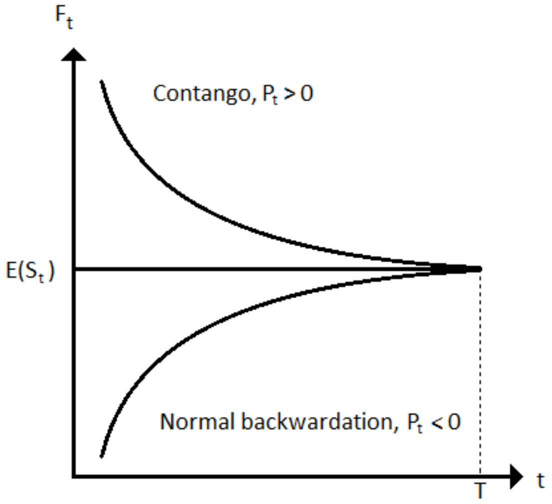

Figure 3-3 Backwardation and Contango (Botterud, Bhattacharayya and Ilic 2002, 5)

Sometimes for longer periods of time, the futures prices don't exist. It can also be difficult to find reliable estimates for the risk premium. In such cases one will have to use other methods for finding the expected future spot prices.

3.6 Estimating Volatility

To estimate the volatility one out of two methods are usually used. The first method is to calculate the implied volatility from the market prices of options traded on exchanges. The second method is to calculate the standard deviation from historic time series of logarithmic returns on spot prices, and use the historical standard deviation as an estimate for the future volatility.

3.6.1 Historical Volatility

Standard deviations are usually computed with equation 3.45. These standard deviations are again often based on daily, weekly or monthly log price changes, as in equation 3.44. When standard deviations are based on log price changes its normal to refer to them as volatilities. Finally the volatility based on sub periods is annualized with equation 3.46.

equation 3.44 $$r_t = \ln\left(\frac{\mathrm{price}_t}{\mathrm{price}_{t-\Delta t}}\right)$$

equation 3.45 $$\sigma_n = \sqrt{\left(\frac{1}{N-1}\right) \times \sum_{k=1}^{N}[(r-\bar{r})^2]}$$

equation 3.46 $$\sigma = \sigma_n \times \sqrt{\text{sub periods per year}}$$

Notes:

N	number of sub periods in total
σ_n	volatility for each sub periods
σ	annual volatility

3.6.1.2 Exponentially Weighted Moving Average

Volatility sometimes changes over time. Industries tend to be more volatile when they are first introduced to the market, but when they mature the volatility falls. Assuming that recent estimates of volatility are better predictors of future volatility than old ones, an exponentially weighted moving average (EWMA), which puts more weight on recent observations than on old ones, can be used. EWMA is calculated using equation 3.47 (McDonald 2006, 747):

equation 3.47 $$\sigma^2_{\mathrm{EWMA},t} = \sum_{i=1}^{n}\left[\frac{(1-b)\times b^{i-1}}{\sum_{j=1}^{n}(1-b)\times b^{j-1}}\right] \times r^2_{t-i} = \sum_{i=1}^{n}\left[\frac{(1-b)\times b^{i-1}}{1-b^n}\right] \times (r_i-\bar{r})^2$$

Notes:

b	exponential weight on observation in an EWMA
r_i	logarithmic return

The formula takes into account the n most recent sub periods. Because $\sum_{i=1}^{\infty}(1-b) \times b^{i-1} = 1$ and $\sum_{j=1}^{n}(1-b) \times b^{j-1} = 1-b^n$ the total sum of weights $\sum_{i=1}^{n}\left[\frac{(1-b)\times b^{i-1}}{1-b^n}\right] = 1$. The resulting variance is then used to estimate the annual volatility using equation 3.45 and equation 3.46.

3.6.2 Calculating Implied Volatility

Sometimes future volatility cannot be predicted by historical estimates at all. In such cases it is better to use implied volatility. This technique observes the market prices of options and calculates the implied volatility by solving the formulas for volatility. Usually some version of the Black-Scholes formula is used. To solve for volatility one needs the spot- and exercise price, risk-free rate, net convenience yield and time to maturity. After having plotted the inputs and the formula into a model one can use e.g. Goal Seek[4] to find the implied volatility.

3.6.2.1 Normal Problems with Implied Volatilities

The results often vary when calculating implied volatilities for different exercise prices, even for options with the same maturity. This is because of the many complications which the simple version of the Black-Scholes formula above doesn’t take into account. Such as seasonality, mean reversion, jump diffusion etc. Less troublesome assumptions are usually normally distributed log returns, constant convenience yield and risk-free rate, no transactions costs or taxes and no limits on short selling and borrowing (McDonald 2006).

Having gone through supporting theory it is time to start building the actual model, which will be done in the next section.

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[4] To use Goal Seek in Excel 2007, on the Data tab, in the Data Tools group, click What-If Analysis, and then click Goal Seek. This function is a simpler form of Solver, and can be used if you know the result that you want from a formula, but are unsure what input value the formula need’s to get that result.

4 Design – Building the Real Options' Model

To give an answer to my research question, I will perform a quantitative analysis on the project and estimate the present value. I will then analyse how this value changes when different price scenarios for EUAs are introduced.

The model will be built using a bottom-up approach as illustrated in Figure 4-1. Input variables will be defined, methods specified and assumptions clarified. Cash flows to labour, insurances etc. will first be explained on a detailed level and then embedded into consolidated fixed and variable cash flows on a higher level in the model. Towards the end of the section a final model will be constructed. The model can then be used for analytic purposes.

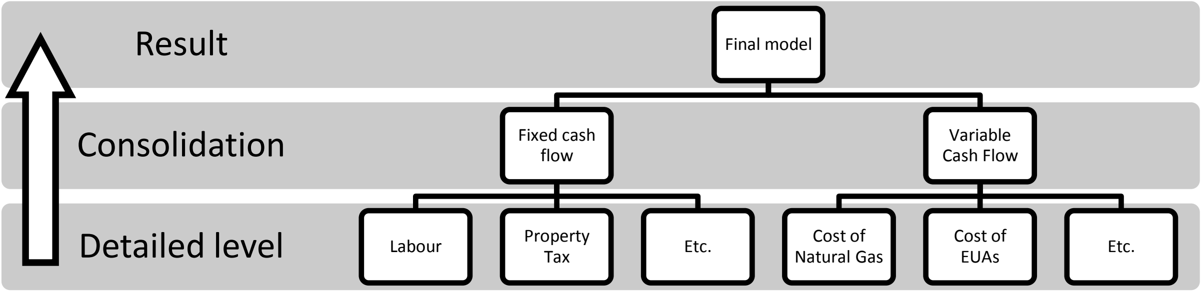

Figure 4-1 Bottom-up approach

This thesis will rely on secondary data only, i.e. data originally meant for other purposes than my thesis. This is cheaper for me, more time efficient and the required data is for the most part publicly available anyway. In some cases the data will have to be adapted to the specific needs of the analyses. Data will be gathered from sources such as annual reports, websites, government reports, books, academic articles, central banks, exchanges and other institutions.

4.1 Describing the Model

4.1.1 A Model Portfolio of Real Options versus the DCF method

To justify using real options instead of a DCF model imagine a power plant that at each point in time produces at full capacity regardless of market prices. The only income and costs are the sales income from electric power and the costs of natural gas. Since the plant is long electricity and short natural gas it can perfectly hedge its position by going short electricity forwards and long natural gas forwards. Systematic risk is then removed from the equation and the present value can be calculated with the following simplified equation:

equation 4.48 $$\mathrm{PV} = \sum_{0}^{\mathrm{T}} \left(\left(\mathrm{F}_{\mathrm{t}}^{\mathrm{EP}} - \mathrm{h} \times \mathrm{F}_{\mathrm{t}}^{\mathrm{NG}} \right) \times \mathrm{kWh}_{\mathrm{t}} \times \mathrm{e}^{-\mathrm{r}_{\mathrm{f}} \times \mathrm{t}} \right)$$

Notes:

$\mathbf{elF_t}$	futures price for electrical power, with transaction and deliverance at time t
$\mathbf{ngF_t}$	futures price for natural gas, with transaction and deliverance at time t
$\mathbf{kWh_t}$	production per year, Naturkraft produces at capacity i. e. 3.5 TWh or nothing
$\mathbf{h}$	heat rate adjustment, adjusts from chemical energy to produced energy

Using this equation would almost always result in a negative net present value, because in practice gas fired power plants are not obligated to produce at all times, they just have the option to do so. The option will only be exercised if the contribution margin income from selling electrical power is positive. A passively managed, base loading, power plant would never be profitable in today's highly volatile market because that strategy doesn't cut losses when prices turn out unfavourably.

The spread between the price of electricity and the cost of natural gas per kWh is often referred to as the spark spread, and gas fired power plants can be considered a string of spark spread options. Baron, et al. (2002, xxix) describes the valuation approach in the following way: *"Recognising this embedded optionality in power plants is indeed the additional, real-options profitability, as the profit at any time is the payoff to a spread option, max{0,* $S_t - h \times NG_t$*}"*. Geir Fuglseth, Director of Information of Naturkraft, confirms this in an email saying, "*When deciding whether to stop production or not, the references are always gas- and power prices at their respective exchanges*" [5]. As expected, the most important inputs in the valuation model are the price of natural gas and the price of electricity. In 2008 the prices of natural gas and electric power in the market made production unprofitable. With a negative contribution margin, Naturkraft temporarily shut down production and waited for prices to develop in their favour. By doing this, the management at Naturkraft reduced the negative effect of an unfortunate development in

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[5] The citation is translated from Norwegian to English. The Norwegian version: "*Referansen vil alltid være gass- og kraftprisene på respektive børser.".* This was sent to me as a response to an inquiry by the author.

prices by active management. Since equation 4.48 treats the plant as an obligation and not an option to produce, the method is insufficient. This is why I will primarily use real options in this thesis.

4.1.2 Allowed Real Options in the Model and Assumed Management Behaviour

Sick, G. (1995, as quoted in Brosch 2008, 8) defined a real option as "*the flexibility a manager has for making decisions about real assets.*". It is then necessary to make a few assumptions on behalf of the management/decision makers:

1. Their goal is to maximize the value of the company.
2. They are able to sort out which adjustments are the profit maximizing ones.
3. They are free to make all the adjustments they want, and they will make them if it maximizes the value of the company.

An unlimited number of ways to make decisions about real assets exist for managers with some creativity. I will only allow the management to temporarily shut down the production and liquidate if necessary, which are the real options I regard as the most important ones for a gas fired power plant. A real options' portfolio is necessary because the values of these real options depend on each other as discussed in section 3.3.2 (on page 22). Hence, each of the flexibility decision rules will be merged together into one portfolio decision rule that simultaneously takes into account both the option to abandon and the option to temporarily shut down. I will follow the previously stated decision rules in equation 3.36 (on page 23), and unite this with the decision rule for abandonment in equation 3.40 (on page 25).

How do I know that the management actually have the opportunity to follow these decision rules? In 2008 the market prices of natural gas and electrical power turned out in such a way that production wasn't profitable and the plant's production was stopped. However, in 2009 the price situation improved and the plant started producing again. Thus, the management's option to temporarily close down is justified both through statements and through actions. The opportunity to abandon the entire plant and liquidate has not been done before by Statoil or Statkraft as far as I know. Still, in a scenario where the present value of continuing operations is far lower than abandoning, I think it would be no big assumption to say that the management would choose the most lucrative option.

4.1.3 A Binomial Model with Two Correlated, Mean-Reverting Underlying Assets

As mentioned, the two most important input variables, electrical power and natural gas, will be used as underlying assets. The prices of these assets are assumed to be described with the geometric Ornstein Uhlenbeck process described in section 3.2.6.2 (on page 20) and I will use a binomial multiplicative model. In addition to the income from the sale of electrical power, the plant also has other costs, both fixed and variable. These are usually small relative to the cost of natural gas, which accounts for the lion's share of the cost per kWh. The entire model will be modelled with Microsoft Visual Basic 6.5.

4.1.4 Valuation Based on a Replicating Portfolio

The model is based on the possibility of replicating the value of gas fired power plant with a market portfolio with the same risk and payoff characteristics as the power plant. Since such a large proportion of the profitability of a gas fired power plant depends on the price of electrical power and natural gas, I will treat these two as the underlying assets in the model. These assets are constantly traded on exchanges in Europe, which means that one can easily take the long and short decisions in gas and power options needed to create a replicating portfolio. Of course, I will not actually estimate a replicating portfolio for each state in the binomial tree, but the theoretical possibility of doing that enables me to use risk-neutral probabilities and a risk-free discounting rate in backwards recursion method. The rationale behind this is explained in section 3.2.2 (on page 14).

4.1.5 Time Intervals in the Project Period

The model will cover the remaining lifetime of the plant. The plant was officially opened in late 2007, but chose not to produce in 2008. In 2009 it started up production again and has produced almost continuously since. Even though it didn't produce in 2008 I will assume that real depreciation occurred and that its lifespan of 25 years started rolling in the end of 2007, so that 2032 will be the last year of production.

Name of Period	End of year...	Valuation Method
The historic period	2007-2011, five years	DCF
Valuation period	2011-2032, 21 years	Real Options Valuation

Table 1 Project period

All the calculations will refer to total capital, nominally and after tax. The real options valuation covers a 21 year-period from the end of 2011 through the end of 2032. Each year in this period will be referred to with t. In which t = 0 refers to the end of 2011 and t = 21 refers to the end of 2032. In the binomial model I will split the valuation period into intervals of time. The length of each time interval corresponds to a sub period, and I will use Δt to reference the length of them in equations. The actual length of each period will be discussed after the model is fully developed.

4.2 Operating Costs Excluding Cost of Natural gas and Cost of EUAs

The costs of labour, maintenance, insurance etc. were found in annual reports from Naturkraft. At the time of writing, these reports cover the years from 2007 to 2010, while the real options valuation period begins in the start of year 2012. The gap from where the data inputs stop and real option model starts, will be filled with extrapolated data when needed. How this is done will be explained as I go through the various line items.

I will start by distinguishing between variable and fixed costs, which is important when deciding whether or not to shut down production. When making these classifications I will use simple logic and a personal judgement. Since the plant was shut down temporarily in 2008 but produced almost through the entire year of 2009, the variable costs for 2008 should be small compared to 2009, also some line items like property tax is naturally fixed and will be classified as such.

In Table 2 (below) some of the operational costs in the annual report are listed with their corresponding classifications. Those not included in the table (like the cost of natural gas and depreciation) will be dealt with later. Of course, I only have access to the costs from the 2010-annual report. To estimate the 2011 values I simply adjusted the fixed costs with inflation and the variable costs per kWh with inflation. I assume that the plant produces at capacity in 2011, i.e. 3.5 TWh.

Type of Operating Cost	Big Increase?	Cost driver level	Classification	2011 Costs after tax (MNOK)	Groups
Office cost	Yes	kWh	Variable	7.82	Variable Other Operating Costs (VOC)
Insurance	Yes	kWh	Variable	2.65	
Material cost	Yes	kWh	Variable	16.95	
Spare parts & consumables	Yes	kWh	Variable	16.36	
Technical maintenance	Yes	kWh	Variable	43.02	
Tot. Var op. cost ($\mathbf{VOC_0}$)				86.80	
Cost of labour[6]	No	Firm	Fixed	30.63	Fixed Other Operating Costs (FOC)
Property costs & tax	No	Firm	Fixed	17.09	
Consulting	Yes	Firm	Fixed	37.19	
Auditing, accounting and legal assistance	No	Firm	Fixed	1.62	
Travel and transport costs	No	Firm	Fixed	0.76	
Sales and marketing	No	Firm	Fixed	1.03	
"Other costs"	No	Firm	Fixed	0.35	
Tot. fixed op. cost ($\mathbf{FOC_0}$)				88.60	

Table 2 Classification of line items in the annual report

All the cost items in Table 2 are grouped into variable other operating costs and fixed other operating costs. These groups will be described in the model with the following equations:

equation 4.49 $$\mathrm{VOC_n} = \left(\frac{\mathrm{VOC_0}}{\mathrm{kWh}}\right)_{2011} \times \mathrm{kWh_t} \times \Delta\mathrm{t} \times \mathrm{e}^{\mathrm{infl} \times \mathrm{t}} \times (1 - \mathrm{r^{tax}})$$

equation 4.50 $$\mathrm{FOC_n} = \mathrm{FOC_0} \times \Delta\mathrm{t} \times \mathrm{e}^{\mathrm{infl} \times \mathrm{t}} \times (1 - \mathrm{r^{tax}})$$

Notes:

$\mathbf{VOC_n}$	variable operating costs, as defined in Table 2, after tax
$\mathbf{FOC_n}$	fixed operating costs as defined in Table 2, i. e. without depreciation, after tax
infl	logarithmic inflation target from the Central Bank of Norway (2.46%)
$\mathbf{r^{tax}}$	the Norwegian nominal corporate tax rate, i.e. 28%

---

[6] Interest cost on benefit obligations and return on pension plan is here excluded since these items aren't operational in nature.

4.3 Investments, Tax Deductions and Abandonment Values

Depreciation was not included in Table 2 because this line item needs special treatment. Depreciation is not a cash flow, but the tax deductions they cause are, and must be taken into account. At the same time I need to include the cash flow from reinvestments during the project period. In Norway tax-based depreciations are stipulated by The Norwegian Tax Administration, who uses of the reducing balance method shown in equation 4.51.

$$\text{equation 4.51} \qquad \begin{matrix}\text{tax based annual}\\ \text{depreciation}\end{matrix} = \begin{matrix}\text{depreciation}\\ \text{rate}\end{matrix} \times \begin{matrix}\text{book value at the}\\ \text{beginning of the year}\end{matrix}$$

This formula will be used in the model to estimate tax deductions on depreciation and to decide abandonment values, which are important when considering the opportunity to abandon project and sell off assets. The depreciation rate is different for different capital bases. The relevant depreciation rates are given in Table 3:

Groups	Description	Depreciation rate	Naturkraft's capital
Group a	Office machines	30%	Inventory and Tools
Group b	Goodwill	20%	Intangibles
Group h	Buildings and plant	4%	Plant and Equipment
No group	See section 4.3.3 for details	0%	Operational Working Capital

Table 3 Depreciation rates for capital in Norway (Norwegian Tax Administration 2011)

In Naturkraft the capital in the company is grouped into four main categories found in the column to the right in Table 3. To estimate net investments per period, it is necessary to first estimate how different capital bases relate to other factors in the model.

Section 4.3 shows how investments, tax deductions from depreciation of capital and abandonment values are modelled. Since Intangibles and Plant and Equipment are treated similarly as Inventory and Tools I choose to move these sections to appendix 7.2.

4.3.2 Inventory and Tools

Reinvestments in Inventory and Tools are assumed to perfectly counterbalance depreciations so that the value of the line item remains constant through the entire period, only adjusted with the general inflation as in equation 4.52.

Abandonment value of I&T is assumed to equal the tax value, so that it can also be described with equation 4.52. Assuming that investments follow general inflation, they can be described each year with equation 4.53. However in the model I need total investments over a period. I therefore use the definite integral of $\mathrm{inv}_t^{I\&T}$ between $(t-\Delta t)$ and t as shown in equation 4.54, in which n denotes periods and not years, i.e. the time interval from $(t-\Delta t)$ to t. Finally since reinvestments equal depreciation, I can simply describe tax deductions from depreciated I&T capital with equation 4.55.

equation 4.52 $$I\&T_t = AV_t^{I\&T} = I\&T_0 \times e^{\mathrm{infl}\times t}$$

equation 4.53 $$\mathrm{inv}_t^{I\&T} = I\&T_0 \times e^{\mathrm{infl}\times(t-1)} \times r_{I\&T}^{\mathrm{depr}}$$

equation 4.54 $$\mathrm{inv}_n^{I\&T} = \int_{t-\Delta t}^{t} \mathrm{inv}_t^{I\&T}\,dt = I\&T_0 \times r_{I\&T}^{\mathrm{depr}} \times \left(\frac{e^{\mathrm{infl}\times(t-1)}}{\mathrm{infl}}\right) \times \left(1-e^{-\mathrm{infl}\times\Delta t}\right)$$

equation 4.55 $$\mathrm{TaxDed}_n^{I\&T} = r^{\mathrm{tax}} \times \mathrm{inv}_n^{I\&T}$$

Notes:

$\mathbf{r_{I\&T}^{depr}}$	rate of depreciation for inventory and tools (30%)
$\mathbf{I\&T_{t/n}}$	end year/period tax value of inventory and tools
$\mathbf{TaxDed_{t/n}^{I\&T}}$	tax deductions from depreciation of inventory and tools
$\mathbf{AV_{t/n}^{I\&T}}$	end year/period abandonment value of inventory and tools
$\mathbf{inv_{t/n}^{I\&T}}$	investments in of inventory and tools

Using the formulas above it should be straightforward to estimate starting value, investments and tax deductions in 2011 for I&T:

equation 4.56 $$I\&T_0 = I\&T_{2010} \times e^{\mathrm{infl}} = \mathrm{MNOK}\ 3.53 \times e^{0.0246} = \mathrm{MNOK}\ 3.62$$

equation 4.57 $$\mathrm{inv}_0^{I\&T} = I\&T_{2010} \times r_{I\&T}^{\mathrm{depr}} = \mathrm{MNOK}\ 3.62 \times 0.30 = \mathrm{MNOK}\ 1.09$$

equation 4.58 $$\mathrm{TaxDed}_n^{I\&T} = r^{\mathrm{tax}} \times \mathrm{inv}_0^{I\&T} = 0.28 \times \mathrm{MNOK}\ 1.09 = \mathrm{MNOK}\ 0.31$$

4.3.3 Operating Working Capital

The nature of operating working capital (OWC) is somewhat different. First of all it will not depend solely on time, but also on sales. I define operating working capital in the same way as Koller, Goedhart and Wessels (2010, 139) do:

equation 4.59 $\mathrm{OWC} = \mathrm{OCA} - \mathrm{OCL}$

Notes:

OWC	end year/period tax value of operating working capital
OCA	end year/period tax value of operating current assets
OCL	end year/period tax value of operating current liabilities

Operating current assets are those assets necessary for the operation of business including working cash balances, trade accounts, receivables, and prepaid expenses, while operating current liabilities are the liabilities related to the ongoing operations of the firm, including accounts payable, accrued salaries, deferred revenue and income taxes payable.

4.3.3.2 Modelling Operating Working Capital

OWC is assumed to constitute a constant ratio of annual sales, which are calculated with equation 4.60. The common way to deal with OWC is to assume that it constitutes a constant portion of sales. That way specific inflation in the business is taken care of along with its dependency on ongoing business. If production stops, then receivables will be received and accounts payable will be paid. However, in a binomial model with recombining nodes, a technical problem occurs here. First of all, one cannot know from which previous state the current state was reached. Since the cash flow from OWC is equal to the change in OWC one will need to know the OWC level in both the current state and the previous state. So to correctly measure the cash flow effect from OWC one would have to use a non recombining lattice tree. This would cause the number of end nodes to increase by $2^{2\times \mathrm{n}}$ for each extra period instead of just n^2. Since this would seriously limit my possibility to add many periods I will instead model OWC in another way.

The choice to produce means that one will tie capital up in trade accounts, receivables and so on. This capital doesn’t carry any explicit interest cost but neither does it pay any interest income. Therefore the alternative cost of the capital is really the only cost. This is equal to the interest income Naturkraft could alternatively receive if they instead invested this capital in bonds. Therefore, I will model OWC as a production cost equal to the foregone interest.

I model sales per year in equation 4.60 and capital tied up in OWC in equation 4.61 below, i.e. as a constant portion of sales. I assume that the abandonment value of OWC is equal to the tax value and therefore equal to OWC in equation 4.61. The ratio$\left(\mathrm{OWC}_{2010}/\mathrm{sales}_{2010}^{\mathrm{kWh}}\right)$ in equation 4.61 is based on the ratio from 2010, which is the closest thing to a normal year of production available. The cash flow effect over one period is then modelled as in equation 4.62.

equation 4.60 $$\mathrm{sales}_{\mathrm{t,jX}}^{\mathrm{kWh}} = \mathrm{kWh}_{\mathrm{t}} \times (1 - \mathrm{r}^{\mathrm{tax}}) \times \mathrm{elP}_{\mathrm{t,jX}}^{\mathrm{kWh}}$$

equation 4.61 $$\mathrm{OWC}_{\mathrm{t,jX}} = \mathrm{AV}_{\mathrm{t,jX}}^{\mathrm{OWC}} = \left(\mathrm{OWC}_{2010}/\mathrm{sales}_{2010}^{\mathrm{kWh}}\right) \times \mathrm{sales}_{\mathrm{t,jX}}^{\mathrm{kWh}}$$

equation 4.62 $$\mathrm{OWC}_{\mathrm{n,jX}}^{\mathrm{CF}} = \mathrm{OWC}_{\mathrm{t,jX}} \times (\mathrm{e}^{\mathrm{r_f} \times \Delta \mathrm{t}} - 1)$$

Notes:

$\mathbf{sales}_{\mathbf{t,jX}}^{\mathbf{kWh}}$	annual income after tax from sale of electrical power
$\mathbf{elP}_{\mathbf{t}}^{\mathbf{kWh}}$	price of electrical power per kWh
$\mathbf{OWC}_{\mathbf{n,j}}^{\mathbf{CF}}$	cash flow effect from change in operating working capital over a period
$\mathbf{AV}_{\mathbf{t,j}}^{\mathbf{OWC}}$	end year/period abandonment value of operating working capital
$\dfrac{\mathbf{OWC}_{\mathbf{2010}}}{\mathbf{sales}_{\mathbf{2010}}^{\mathbf{kWh}}}$	OWC as a percent of sales, equal to 11.22% in 2010

I will model the cash flow effect from OWC as variable because it depends on whether or not the firm chooses to produce. The same goes for the abandonment value, which they will only receive from a period in which they produce. The OWC increased with about MNOK 56.78 from 2010 to 2011.

4.3.4 Summarizing Cash Flow from Tied Up Capital and Abandonment Values

Abandonment values, tax deductions and investments of I&T, P&E and INT are all independent of sales and can therefore be grouped together into consolidated line items.

equation 4.63 $$\mathrm{FixAV}_{\mathrm{t/n}} = \mathrm{AV}_{\mathrm{t/n}}^{\mathrm{I}\&T} + \mathrm{AV}_{\mathrm{t/n}}^{\mathrm{P}\&E} + \mathrm{AV}_{\mathrm{t/n}}^{\mathrm{INT}}$$

equation 4.64 $$\mathrm{FixInv}_{\mathrm{n}} = \mathrm{inv}_{\mathrm{n}}^{\mathrm{I}\&T} + \mathrm{inv}_{\mathrm{n}}^{\mathrm{P}\&E}$$

equation 4.65 $$\mathrm{FixTaxDed}_{\mathrm{n}} = \mathrm{TaxDed}_{\mathrm{n}}^{\mathrm{I}\&T} + \mathrm{TaxDed}_{\mathrm{n}}^{\mathrm{P}\&E} + \mathrm{TaxDed}_{\mathrm{n}}^{\mathrm{INT}}$$

Notes:

$\mathbf{FixTaxDed}_{\mathbf{t/n}}$	fixed tax deductions from depreciation of all capital

$\mathbf{FixAV_{t/n}}$	end year/period fixed abandonment value, from $\mathrm{I\&}T, P\&E\ and\ INT$
$\mathbf{FixInv_{t/n}}$	fixed investments in $\mathrm{I\&}T\ and\ P\&E$

I can also summarize some key starting year values:

equation 4.66 $$\mathrm{FixInv_0} = \mathrm{inv}_0^{\mathrm{I\&}T} + \mathrm{inv}_0^{\mathrm{P\&}E} = \mathrm{MNOK}\ 1.09 + \mathrm{MNOK}\ 6.45 = \mathrm{MNOK}\ 7.54$$

equation 4.67 $$\mathrm{FixTaxDed_0} = \mathrm{TaxDed}_0^{\mathrm{I\&}T} + \mathrm{TaxDed}_0^{\mathrm{P\&}E} + \mathrm{TaxDed}_0^{\mathrm{INT}} = \mathrm{MNOK}\ (0.31 + 17.41 + 9.72) = \mathrm{MNOK}\ 27.44$$

Operating working capital on the other hand is different from the others in that it depends on sales. It will therefore be treated independently and not consolidated with the other groups of capital.

4.4 European Unit Allowances

Each year Naturkraft has so far received a certain amount of free EUAs, which they can sell in the market or use in production (Climate and Pollution Agency 2011). They also have to buy extra EUAs in the market if they choose to produce since they use more than free EUAs. Since Naturkraft receives free EUAs regardless of whether or not they choose to produce, the cash flow from free EUAs is fixed, while the cost of EUAs in production is variable.

The price of EUAs is most likely correlated with the price of power and gas, and it would therefore be optimal to include it as an underlying asset in the option model, but because of the growing complexity of the model I have to treat this variable independently of the binomial process. It is very difficult to get good estimates on the volatility of this variable and including it as an underlying asset would in practice mean that I would look into thousands of scenarios in all the different states in the state space of the binomial tree. Instead I will only use three scenarios which in return will be meticulously explained and analysed in this section.

The price estimates in this section is heavily influenced by a report of a committee called Klimakur 2020 which is appointed by the Norwegian Pollution Control Authority. The report is based mainly on estimates from Point Carbon (an analytical company) and Statistics Norway (a government institution). These estimates are supplemented with knowledge and evaluated by the core group in Klimakur 2020.

In this section I will first present historical EUA prices, followed by the determinants of the price of EUAs. Future estimates from Point Carbon (PC) and Statistics Norway (SSB) will be discussed and summarized. Finally, I will model free EUAs and the cost from EUAs used in production.

4.4.1 Historical Prices

The EU ETS was established in 2005 to reach goals of emission reductions in a cost efficient way. So far the EU ETS has had two trading phases; the first was during the period 2005-2007, while the second and current phase is the period from 2008-2012. The first type of EUAs, traded only in the first phase, was not possible to save for later use, while the second type is possible to store and has been traded in both phases. Naturkraft used Norwegian allowances in the first phase, but changed to the second type EUAs in 2008. I will use the second type EUA in calculations in the model.

4.4.2 Determinants of Future Spot Prices

There are many determinants of the future spot price of EUAs. The 20-20-20-deal which was mentioned in section 1.1.2 (on page 11) is in many ways central. The price of EUAs will depend much on whether or not EU sticks to the self-imposed restrictions/goals in the deal. Other important factors are the economic and technologic development and the prices of oil, gas and coal. The price will also much depend on the deal that will replace the Kyoto Protocol and on how many countries that will be a part of that deal, whether or not alternative trading schemes are established elsewhere than in the EU, and if these will be connected to the EU ETS.

4.4.2.1 Development in the EU ETS in the Third Phase (2013-2020)

As mentioned in the introduction, the cap and trade system automatically leads to higher prices when the total amount of allowed emissions is reduced. This will happen annually in the years 2013-2020 and probably also after 2020. There are also some conditional changes involved. The Climate and Energy Package deal will be in effect regardless of what happens internationally, but the emission reduction-percentage will be increased from 20% to 30% if a "*satisfying*" climate agreement is negotiated in the years after 2012. The higher the ambitions of the EU the higher the prices of the EUAs will be. Also, whether or not the 20%-goals in the Climate and Energy Package deal are actually reached will be important for the EUA prices.

It has also been discussed to connect the EU ETS with other regional or national ETSs. If that happened it would lead to more cost efficient emission reductions. The most realistic candidates would be North America, Japan, Australia and New Zealand (Statistics Norway 2009, 15).

4.4.2.2 The Deal after the Current Kyoto Agreement Runs Out

How the next Kyoto agreement is designed and the number of participants is importance for the future spot price of EUAs. A comprehensive agreement including many countries will expand the carbon market.

4.4.2.3 Economic Growth and the Price of Electrical Power, Fossil Fuels and EUAs

The demand for fossil fuels is heavily influenced by the global economic growth. In a period of high economic expansion the demand for power will increase, which will lead to higher prices of electrical power. This will in return lead to increased demand (and hence prices) of fossil fuels and EUAs used to produce electrical power. Skilled analysts will need powerful software and expertise to make good estimates that take into account all the correlated prices and forces of competition which will affect the price of EUAs mentioned above. Highly detailed estimates are beyond the scope of this thesis, so I will rather use the estimates of PC and SSB. When Klimakur ordered these estimates they specifically asked for three scenarios. So both PC and SSB produced three scenarios each, but these will be consolidated towards the end, so that we don't end up with six scenarios in total but three.

4.4.3 Future Spot Price Estimates from Point Carbon

PC presents three scenarios for the expected future spot price of EUA:

Scenario number	Description
1	EU keeps its goal of 20% reductions in emissions within 2020 relative to the 1990-level. There are no connections with emission trading schemes of other countries.
2	EU increases its goal to 30% reductions in emissions within 2020 relative to the 1990-level. There are no connections with emission trading schemes of other countries.
3	EU increases its goal to 30% reductions in emissions within 2020 relative to the 1990-level. The EU ETS is connected to emission trading schemes in North America, Australia, Japan, Mexico and China.

The first two scenarios consider an isolated EU ETS with goals of 20% and 30% reductions in emissions. Whether or not the goals are actually reached is important. If they are, most of the costs related to emission reductions will be reached outside the carbon market, and the price of EUAs will be lower. Also, in the scenarios with an isolated EU, the EU power market has greater importance for the EUA price than if the market is linked with other markets. This is shown in Table 4. The price ranges in the table refer to whether or not the goals in the Climate and Energy Package deal are reached. If they're not reached, the price will be closer to the upper limit and vice versa. The prices are much more sensitive to the goals when the markets aren't linked.

In the third scenario, the ETS in China is assumed to be limited, but it is also assumed to be expanded within 2018. It is also assumed that the ETSs will be designed in the same way as in EU, i.e. the amount of quotas allocated will be somewhat close to the actual emissions in the early years but then gradually reduced.

	2012	2015	2020[7]	Scenario
EUA 20% goal, no linked markets	20	58	40-70	1
EUA 30% goal, no linked markets	19	50	50-95	2
EUA 30% goal, linked markets	18	25	30-50	3

Table 4 Price estimates from PC in Euro/EUA given with PPP of 2009[8]

PC concludes that the 30%-linked markets-scenario is the most likely scenario, but they will not back that claim up with a number in the publicly available version of the report. In the simulations that PC ran, the possibility of using CDM and JI credits were allowed. 1800 million tonnes of CO_2 equivalents were allowed using these credits (CERs and ERUs) in to 20%-scenario and in the 30%-scenario the use of credits were increased by 50% of the extra effort required to reach the goal. If the allowed use of such credits is increased the price of EUA will be lower.

4.4.4 Future Spot Price Estimates from SSB

SSB also outlined three different scenarios of what the price will be in 2012, 2015 and 2020. SSB focuses more on economic growth and the supply of credits (CERs and ERUs). In SSB's

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[7] The price ranges depends on the degree in which the goals of energy efficiency and renewable energy were reached. If the goals are reached the prices are expected to converge towards the lower limit and vice versa.

[8] (Statistics Norway 2009, 19)

base case scenario it is not assumed that EU will increase its 20% emission reduction goal to 30%. They do, however, assume that EU will in fact manage to reach the 20% goal. This can explain why their estimates are generally lower than those of PC.

	Outcomes	**2012**	**2015**	**2020**
Low price scenario	Prolonged financial crisis, great potential for energy efficiency improvement, low restrictions on credits	5	8	18
Base case scenario	Mixed outcomes	12	20	40
High price scenario	Short financial crisis, medium potential for energy efficiency improvement, high restrictions on credits	20	30	60

Table 5 Price estimates from SSB in Euro/EUA given with PPP of 2009[9]

There are several outcomes that can cause the low price scenario. One is a long-lasting financial crisis that causes low energy demands and emissions for a long period. Another explanation for the low price scenario is the allegedly great potential for increased energy efficiency in the new EU member states. More energy efficiency means less emission per energy unit and lower prices per EUA. Lastly, low restrictions on CERs and ERUs will also reduce the demand for EUAs.

In the base case scenario it is assumed that the economic growth level in the EU will reach normal levels again.

In the high price scenario there are high restrictions on the use of the CDM mechanism. The high price scenario is also explained with high economic growth and the possibility that EU increases the goal from 20% reduction in emissions to 30%.

4.4.5 Consolidating Estimates and a Summary

In this section the previous estimates from PC and SSB will be discussed and consolidated by Klimakur 2020.

4.4.5.1 Medium Price Scenario

The medium price scenario is primarily based on the 30%-linked markets scenario of PC, which is described as the most likely scenario even though this is the alternative with the lowest price. The price in 2020 expected to be 40 Euros which is also the base case scenario of SSB. The core group also expects a price of 18 Euros in 2012, which will increase smoothly

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[9] (Statistics Norway 2009, 20)

to 26 Euros in 2015 and 40 Euros in 2020. Even though PC and SSB focus on different determinants, they both agree that this is the most likely outcome.

4.4.5.2 Low Price Scenario

In the report from PC, which is referred to in the main report from Statistics Norway (2009), it is pointed out that the prices in 2020 can turn out to be 20 Euros if it is allowed to use more CERs and ERUs. In the estimates of SSB such low price estimates are explained with a lower level of ambition in EU or lower emissions than expected because of a prolonged financial crisis. SSB also points to a large potential for improvements in energy efficiency in Eastern Europe. Linked markets will, as before, still cause a lower price. The low price scenario is considered an unlikely outcome.

4.4.5.3 High Price Scenario

In the high price scenario a price of 60 Euros is given for 2020. This outcome can be a result of no linked markets, high restrictions on credits and that the goals in the Climate and Energy Package deal isn’t reached. The price for 2012 and 2015 is 25 Euros and 38 Euros. It is pointed out in the report that it’s not likely that all these high price determinants occur all at once. However, if they do the price can go even higher than in the high price scenario.

4.4.5.4 Summarized

The result that the core group in Klimakur 2020, decided to go with was given in Euro per EUA with the purchasing power parity of 2009. To get the nominal prices in NOK per EUA I adjust the numbers with the inflation target rate of ECB of 2% and multiply with today’s exchange rate:

equation 4.68 $$\mathrm{EUAP}_t^{\mathrm{NOK/kWh}} = \mathrm{EUAP}_t^{\mathrm{Euro/EUA}} \times \mathrm{ER}_0^{\mathrm{NOK/Euro}} \times (1 + \mathrm{infl}_{\mathrm{Euro}})^{(t+2)}$$

Notes:

$\mathbf{EUAP_t^{NOK/kWh}}$	price of EUAs in NOK per kWh
$\mathbf{EUAP_t^{Euro/EUA}}$	price of EUAs in Euro with the purching power of 2009
$\mathbf{ER_0^{NOK/Euro}}$	exchange rate between NOK and Euros (7.89 NOK/Euro on 8th May 2011)
$\mathbf{infl_{Euro}}$	inflation rate of Euro (2% equal to the target inflation rate of ECB)

The result of this process is given in Table 6.

		2012	2020	2030
Low Price	20%, linked markets, low restrictions on credits, prolonged financial crisis	152.81	223.81	312.55
Medium Price	30%-linked markets, medium restrictions on credits	171.91	447.61	792.24
High Price	20%-no linked markets, high restrictions on credits, failure to reach goal	238.77	671.42	1212.24

Table 6 Future expected nominal spot prices for quotas in NOK/EUAs

Statistics Norway (2009, 25) mentiones that PC states that the growth rate in price isn't constant, but they still assume that it is because they don't know when various events will occur. I will make the same assumption in my model. When drawing a straight line between the prices in 2012 and 2020 I get curves that can be described on the form in equation 4.69. The different scenarios is then described with equation 4.70 to equation 4.72.

equation 4.69 $$\mathrm{P}_t^{\mathrm{EUA}} = \beta_0 + \beta_1 \times \mathrm{t}$$

equation 4.70 $$\mathrm{lowP}_t^{\mathrm{EUA}} = 152.81 + 8.87 \times (\mathrm{t} - 1) = 143.94 + 8.87 \times \mathrm{t}$$

equation 4.71 $$\mathrm{medP}_t^{\mathrm{EUA}} = 171.92 + 34.46 \times (\mathrm{t} - 1) = 137.46 + 34.46 \times \mathrm{t}$$

equation 4.72 $$\mathrm{highP}_t^{\mathrm{EUA}} = 238.77 + 54.08 \times (\mathrm{t} - 1) = 184.69 + 54.08 \times \mathrm{t}$$

Notes:

$\mathbf{lowP_t^{EUA}}$	low EUA price scenario
$\mathbf{medP_t^{EUA}}$	medium EUA price scenario
$\mathbf{highP_t^{EUA}}$	high EUA price scenario
$\mathbf{P_t^{EUA}}$	price of EUAs on general form

In section 5 I will study how the value of the firm is affected by the different price scenarios.

4.4.6 EUAs as a Source of Fixed Income

The allocation of free allowances is based on company emissions in 1998 to 2001 as a default rule. And companies receive free allowances regardless of whether or not they actually need them in production in a specific year. This means that the biggest polluters receives the most allowances free of charge, and can earn the most from selling them if the prices of electricity and gas should turn out unfavourably. The income from free EUAs depends on the number of EUAs received per year and the price. In the second phase (2008-2012) of the EU ETS, Naturkraft was granted 1 617 340 EUAs or 323 468 EUAs per year (Climate and Pollution Agency 2011). In section 1.1.1 in the introduction it was described how the EU ETS will change after 2012. The main changes in the third period are given below in bullet points:

- There will no longer be any NAPs. Allocation will be determined centrally by the EU
- Auctioning, and not free allocation, will be the main method
- No free allowances will be given to electricity production

This means that because of the changes in the EU ETS, Naturkraft will in all likeliness lose their right to free allowances, and will go from receiving 328 468 EUAs per year to zero after 2012.

4.4.6.1 Modelling Income from Free EUAs

To describe the number of EUAs received each year with the changes in the third phase I will use equation 4.73[10]. This equation uses to the indicator variable, $\mathrm{I}_{\mathrm{ETS}}^{\mathrm{change}}$, to indicate the change in EU ETS. The variable is binary and will assume 0 to indicate an unchanged EU ETS in which Naturkraft continues to receive 323 468 EUAs annually. In the other scenario where the EU ETS is changed, $\mathrm{I}_{\mathrm{ETS}}^{\mathrm{change}}$ will be 1 and Naturkraft will still receive 323 468 EUAs for 2012 but 0 EUAS annually after 2012.

The cash flow after tax from free EUAs per year can then be described with equation 4.74. The same cash flow per period was found using the definite integral of the price function $\mathrm{P}_{\mathrm{t}}^{\mathrm{EUA}}$ between $\mathrm{t} - \Delta\mathrm{t}$ and t, multiplying with the EUAs received to find the corresponding cash flows per period. The result is shown in equation 4.75.

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[10] The IF-function is borrowed from Excel. For those not familiar with it, here is how it works:

$$IF(Logical\ test, \qquad Value\ if\ true, \qquad Value\ if\ false)$$

equation 4.73 $$\mathrm{REUA_t} = \mathrm{IF}\left(\mathrm{t} > 1,\ 323\,468\ \times \left(1 - \mathrm{I_{ETS}^{change}}\right),\ 323\,468\ \right)$$

equation 4.74 $$\mathrm{sales_t^{REUA}} = \mathrm{P_t^{EUA}} \times \mathrm{REUA_t} \times (1 - \mathrm{r^{tax}})$$

equation 4.75 $$\mathrm{sales_n^{REUA}} = \underbrace{\left(1 - \mathrm{I_{ETS}^{Change}}\right)\ \times \Delta \mathrm{t} \times 323\,468 \times \left(\beta_0 + \beta_1 \times \left(\mathrm{t} - \tfrac{1}{2} \times \Delta \mathrm{t}\right)\right) \times (1 - \mathrm{r^{tax}})}_{\text{Sales without change}} + \underbrace{\mathrm{IF}(\mathrm{n} = 1,\ 323\,468 \times (\beta_0 + \beta_1) \times (1 - \mathrm{r^{tax}}),\ 0)}_{\text{Sales during the first period with change}}$$

Notes:

$\mathbf{REUA_t}$	annual number of EUAs received (323 468)
$\mathbf{I_{ETS}^{change}}$	indicator variable assuming $(0 = \text{no change in ETS}, 1 = \text{change in ETS})$
$\mathbf{sales_{t/n}^{REUA}}$	fixed cash flow from the potential sale of free EUAs

4.4.7 An Additional Variable Operating Cost

As calculated in section 4.4.5.4 it is needed 1 240 000 EUAs per year if Naturkraft decides to produce. I assume that if Naturkraft first decides to produce they have to produce at a rate equal to capacity, i.e. 3.5 TWh per year. It is calculated in the same way as the other variable costs only that it follows the price estimates from section 4.4.5. The price estimate will be given on general from as in equation 4.69. The cost of EUAs per year is given in equation 4.76 and the cost per period is given in equation 4.77.

equation 4.76 $$\mathrm{cost_t^{UEUA}} = \mathrm{P_t^{EUA}} \times \mathrm{UEUA} \times (1 - \mathrm{r^{tax}})$$

equation 4.77 $$\mathrm{cost_n^{UEUA}} = \int_{\mathrm{t}-\Delta \mathrm{t}}^{\mathrm{t}} \mathrm{cost_t^{UEUA}}\, \mathrm{dt} = \mathrm{UEUA} \times (1 - \mathrm{r^{tax}}) \times \Delta \mathrm{t} \times \left(\beta_0 + \beta_1 \times \mathrm{t} - \tfrac{1}{2} \times \beta_1 \times \Delta \mathrm{t}\right)$$

Notes:

UEUA	number of used EUAs when producing 3.5 TWh (1 240 000 EUAs)
$\mathbf{cost_{t/n}^{UEUA}}$	cost of used EUAs after tax

From klif.no I find that the current[11] price of EUAs in NOK/EUA is 147.78. This means that the cost of used EUAs after tax in the starting year is then:

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[11] The site was visited on the 25th of February 2011 http://co2.klif.no/en/-HANDEL-/Kjop-et-bestemt-antall-kvoter/

equation 4.78 $\mathrm{cost}_0^{\mathrm{UEUA}} = \mathrm{P}_0^{\mathrm{EUA}} \times \mathrm{UEUA} \times (1 - \mathrm{r}^{\mathrm{tax}}) = 147.78\ \mathrm{NOK/EUA} \times 1.24 \times 10^6 \mathrm{EUAs} \times (1 - 0.28) = \mathrm{MNOK}\ 131.83$

4.5 Describing the Price of the Underlying Assets

I am now going to estimate the income from sale of electricity and the cost of buying natural gas. The price of electricity per kWh and the cost of natural gas per kWh are the underlying assets in the model used in this thesis. I can therefore not rely on annual reports. These costs aren't reflected in the annual reports of Naturkraft anyway, because Naturkraft has entered into a deal with its owners. The deal is called Tollingavtalen and makes sure that Naturkraft receives natural gas from its owners for free and gives them the produced gas free of charge in return (Naturkraft AS 2009; 2010). For making their production capacity available to its owners, Naturkraft receives an annual fee. However, when valuing the profitability of the plant, I have to use the perspective of the investors. And from the owners' point of view, I cannot consider the annual fee paid to Naturkraft as income, since it is paid for by the owners. Consequently I will "nullify" the deal in the annual reports when doing calculations. That means that I have to estimate income from the sale of electrical power and cost of natural gas independently from the annual report.

4.5.1 The Price of Electrical Power

Nord Pool Spot runs the largest market for electrical power in the world, and is owned by the Nordic transmission system operators. Nord Pool Spot calculates a system price, which is the reference price for derivatives at NASDAQ OMX Commodities (Nord Pool Spot AS 2011). The system price is determined by the intersection of the aggregated supply and demand curves representing all bids and offers in the entire Nordic region. The local prices in the regions sometimes deviate from the system price because of capacity constraints in the power grids, but the differences between the system price and the regional prices are usually small. Since the definition of the various geographical areas differs over time, Naturkraft's gas fired power plant could have belonged to several areas. Therefore the system price has to be used to describe trends over time.

4.5.1.1 Historical Prices

Historic records of the system price can be found in the archives of Nord Pool Spot. The price data cover the period from January 1996 to April 2011, and are reported as monthly averages of the daily spot system prices. The fact that I choose to use monthly data for both electricity and gas will be commented on in section 4.6.4 (on page 62). The prices are given in NOK/MWh, but are converted to NOK/kWh by dividing by with 1000. In Figure 4-3 the historical prices for the period May 2003 to December 2010 is shown. The data between January 1996 and February 2003 is not shown or used because I don't have price data for natural gas over this same period. These historical prices will be used to calculate volatility and mean reversion coefficients (Nord Pool Spot AS 2011).

4.5.1.2 Future Estimates of Mean Price

In section 3.2.6 (on page 17) the mean reversion processes was described. It was made clear that I will need to estimate the long-run mean value was denoted $\overline{\mathrm{S}}$ if I am to model mean reversion. I the model here $\overline{\mathrm{S}}_{\mathrm{t}}$ will depend on time because the price of electricity is expected to change in the future.

I will base future predictions of the spot price on the prices of futures. I then need to examine how the price of futures and the spot prices relate to each other. As was discussed in section 3.5 (on page 25) the futures price of non storable commodities is based on expected spot prices. If electricity is a non-storable commodity I should be able to use the result in equation 3.43 which stated that the expected spot price at time t was equal the futures price minus a risk premium. It can be argued that electricity is indirectly storable, because one could sell a MWh in a forward contract, buy some fossil fuel, and use that to produce the electricity at the maturity of the forward contract. So even though electricity isn't storable, commodities that can be converted into electricity are. However, in the NordPool area, such a high percentage of the electricity production comes from either hydro power plants[12] or nuclear plants that I choose to treat the futures market as best described with the expectations theory. This is also confirmed by allowing seasonal patterns and from various other sources such as Nord Pool AS (u.d.). What is left to do then to find the expected spot prices is to find a forward curve and estimate a risk premium curve.

Risk Premiums in NordPool's Trading Area

In an examination of the Nordic electricity market conducted by Botterud, Bhattacharayya and Ilic (2002) found that risk premiums (like I and unlike how they described it) for up to one year are positive in the electricity futures market. To see the result from their study see

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[12] 53% of total production in NordPool in 2007 was by hydro power (Huisman and Kilic 2011)

appendix 7.3. They explain the result by proposing that an overweight of consumers electricity want to hedge their products in the futures market, relative to the producers wanting to go long the same futures contracts. The drawbacks from their study is that it is relatively old, as they used data from September 1995 until the end of 2001. In such a new market, much can have changed since then, which I will discuss later. The results from their study also produce risk premiums which when annualized varies wildly without any clear trend (which is probably because of varying volatility due to seasonalities within a year). What I conclude from their study is that the annualized risk premium varies over time and that it was probably positive in the years covered by their data.

Huisman and Kilic (2011, 3) list a number of different studies all over the world, all intended to measure risk premiums in different electricity markets. Most of them find positive risk premiums, and all of them find that risk premiums (as annualized percentages) vary over time. Some of the markets in which some people find zero risk premiums, or even negative risk premiums in recent studies is actually in the NordPool area. This was the case for Carta and Villaplana (2008 as cited in Huisman and Kilic, 2011, 3) who found significant proof of backwardation in the the period January 2003 to January 2008.

Mork (2003) focuses on the expectations theory. He believes that the risk premium depends on whether hedging volume is larger on the buyer or the seller side. He uses the NordPool market to test this hypotesis. In 2000-2002 there were an unusual high a number of speculators with exchange memebership at NordPool. In Mork's (2003) test he sets out to test if the risk premium changed when the number of speculators changed. What he found was that the risk premium was much higher before 2000 than during 2000-2002. Actually he found no evidence of a risk premium different from zero in 2000-2002. What surprised him was that after 2002, when the speculators left (they were employed by Enron), the risk premium remained insignificantly different from zero.

In Huisman and Kilic's (2011) own study, they found different results in markets where the degree of indirect storability was different. They compared the Dutch market with NordPools market and found that time varying risk premiums was more prominent in the Dutch market, which power production is based mainly on gas fired power plants, than in the NordPool market. Their price data for the NordPool area was the closing prices for futures contracts on the first trading day of each month from 4 April 2005 through 1 December 2010. The regressen they ran was on the form in equation 4.79.

equation 4.79 $$\mathrm{F}_{\mathrm{t,T}} - \mathrm{S}_{\mathrm{T}} = \alpha_{\mathrm{p}} + \beta_{\mathrm{p}}\left(\mathrm{F}_{\mathrm{t,T}} - \mathrm{S}_{\mathrm{t}}\right) + \sigma_{\mathrm{p}}\epsilon_{\mathrm{p,t}}$$

The footnote t here refers to present time, while T is the maturity date. So $F_{t,T}$ is the futures price at time t with maturity at T. The expression $\mathrm{F}_{\mathrm{t,T}} - \mathrm{S}_{\mathrm{T}}$ on the left side of the equation should be a predictor for the risk premium. The study couldn't conclude that β_{p} was significantly different from zero in the NordPool area, but they found it to be so in the Dutch market. Their conclusion from the study was that markets with imperfect storability depend heavily on price expectations and that markets with perfect storability must include time varying risk premiums for their future prices to give predictive power of future expected spot prices.

So from going through both old and more recent studies, it may appear that the risk premium at NordPool used to be signficantly positive, but then changed to become insignificantly different from zero after 2000. Mork (2003) suggest that this is because that speculators have now become better aquianted with NordPool as a trading area, and that profits from speculation is competed away. Since I see no indication of the risk premium turning positive in the future I will here assume that the risk premium is equal to zero and threat the futures prices as unbiased predictors of expected spot prices. Thus, I will use equation 4.80 below when predicting future expected spot prices.

equation 4.80 $$\mathrm{E}(\mathrm{S}_{\mathrm{t}}) = \mathrm{F}_{\mathrm{t}}$$

Forward Curve in NordPool's Trading Area

To find forward prices I visited nasdaqomxcommodities.com[13]. These forwards are traded in Euros/MWh so to convert them to NOK/kWh I use the equation below:

equation 4.81 $$\mathrm{elF}_{\mathrm{t}} = \mathrm{elF}_{\mathrm{t}}^{\mathrm{NOK/kWh}} = \mathrm{elF}_{\mathrm{t}}^{\mathrm{Euros/MWh}} \times \mathrm{ER}_{0}^{\mathrm{NOK/Euro}} \times \frac{1000\ \mathrm{kWh}}{\mathrm{MWh}}$$

Notes:

$\mathbf{elF}_{\mathbf{t}}^{\mathbf{Euros/MWh}}$	the futures price of electrical power denoted in euros per MWh

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[13] Here I chose trading and market prices. Market: Nordic Electricity, Types: Year and Type: Forwards. I will then use then forward contracts with code ENOYR-(and the expiration year). Updated at 14th of February 2011

The prices I use are shown in table below:

Year	2012	2013	2014	2015	2016
Euro/MWh	48.95	45.65	45.15	46.80	48.78
NOK/kWh	0.3838	0.3579	0.3540	0.3669	0.3824

Figure 4-2 Forward prices for electrical power on nasdaqomxcommodities.com

These expected spot prices (also shown in Figure 4-4 on page 59 below) will be used later when estimating risk neutral probabilities with mean reversion. The longest maturity date of forward contracts is 2016, after this date I will assume that the price of electricity develops at the same rate as the general inflation. Since my research question doesn't require me to estimate the actual market value of the gas fired power plant, I find it unnecessary to devote more time and space here to make more detailed estimates of the future expected spot price. The mean values that I estimate here will be used as mean values towards which the actual price converges.

4.5.1.3 Modelling Sales Income from Electrical Power in the Binomial Model

To model price of electricity and sales income in the binomial model a starting price that will move up and down in the following periods. I find the historical prices from Nord Pool Spot (Jan-May) and the future prices from Nasdaq OMX Commodities (Jun-Dec). Since I want to avoid seasonalities effects I use an average of the prices in 2011 as the base year price for electrical power.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0.544	0.505	0.503	0.421	0.412	0.423	0.417	0.429	0.455	0.459	0.466	0.466

Table 7 Monthly prices of electricity through 2011 in NOK/kWh

Computing an average of the monthly prices I get $\mathrm{elP}_0 = 0.4583$. Using equation 3.3 from the theory section I model the price of electricity per kWh with equation 4.82. Since I assume that they will produce 3.5 TWh each year (or 0 TWh if they choose not to produce) the sales income can be described with equation 4.83.

equation 4.82 $$\mathrm{elP}_{\mathrm{n},\mathrm{j}_{\mathrm{el}}} = \mathrm{elP}_0 \times \mathrm{u}_{\mathrm{el}}^{\mathrm{j}_{\mathrm{el}}} \times \mathrm{d}_{\mathrm{el}}^{\mathrm{n}-\mathrm{j}_{\mathrm{el}}}$$

equation 4.83 $$\mathrm{sales}_{\mathrm{n},\mathrm{j}_{\mathrm{el}}}^{\mathrm{el}} = \mathrm{elP}_{\mathrm{n},\mathrm{j}_{\mathrm{el}}} \times \mathrm{kWh} \times \Delta\mathrm{t} \times (1 - \mathrm{tax})$$

Notes:

$\mathbf{elP}_{\mathbf{n},\mathbf{j}_{\mathbf{el}}}$	the price of electrical power per kWh
$\mathbf{u}_{\mathbf{el}}^{\mathbf{j}_{\mathbf{el}}}$ & $\mathbf{d}_{\mathbf{el}}^{\mathbf{n}-\mathbf{j}_{\mathbf{el}}}$	up an down moves for electrical power, calculated as in equation 3.1
$\mathbf{sales}_{\mathbf{n},\mathbf{j}_{\mathbf{el}}}^{\mathbf{el}}$	sales income after tax from electrical power

The sales income after tax for the starting year 2011 will be:

equation 4.84 $$\mathrm{sales}_{0,0}^{\mathrm{el}} = \mathrm{elP}_{0,0} \times \mathrm{kWh} \times \Delta\mathrm{t} \times (1 - \mathrm{tax}) = 0.4583\ \mathrm{NOK/kWh} \times 3.5 \times 10^9\ \mathrm{kWh/year} \times 1\ \mathrm{year} \times (1 - 0.28) = \mathrm{MNOK}\ 1\ 154.81$$

4.5.2 The Price and Cost of Natural Gas

Statoil is both co owner of Naturkraft and the biggest producer and exporter of natural gas in Norway. The gas that Statoil doesn’t sell to Europe it can sell to Naturkraft, which means that the price that Statoil alternatively could have gotten on exchanges in Europe is the alternative cost of selling it to Naturkraft instead.

In the 2009 annual report of Statoil it is stated: *“Derivatives related to natural gas and electricity are mainly OTC physical forward contracts and options, Nord Pool Spot forward contracts, and also NYMEX and ICE futures.”* (Statoil ASA 2010, 113). I will therefore use InterContinental Exchange (ICE) Futures in London to find both historical prices and future prices of natural gas.

4.5.2.1 *Historical Prices*

Historical spot price data for natural gas is not available for free, but I found free historical monthly future prices. I will approximate historical spot prices by using monthly future prices on the last day of trading for each monthly future. For example, from 1th November 2005 to 30th November 2005, futures for one specified unit of natural gas could be bought, which could be exercised the subsequent month. The price on 30th of November will then be a fairly good estimate for the spot price of gas during December.

The future prices from ICE Futures Europe's webpage [14] are denoted in pence per British Thermal Units (BTU) which I will to convert into NOK per kWh[15]. Since1 $\mathrm{BTU} = 29.307\ \mathrm{kWh}$ and $1\ \mathrm{GBP}^{16} = 100\ \mathrm{pence}$, I multiply the futures price with $\frac{\mathrm{BTU}}{29.307\ \mathrm{kWh}}$ and $\frac{\mathrm{GBP}}{100\ \mathrm{pence\ sterling}}$. Finally I multiply with the exchange rate between NOK and GBP to get the futures price in NOK per kWh, as in equation 4.85.

$$\text{equation 4.85} \qquad \mathrm{ngF}_t^{\mathrm{NOK/kWh}} = \mathrm{ngF}_t^{\mathrm{pence/BTU}} \times \frac{\mathrm{BTU}}{29.307\ \mathrm{kWh}} \times \frac{\mathrm{GBP}}{100\ \mathrm{pence\ sterling}} \times \mathrm{ER}_0^{\mathrm{NOK/GBP}}$$

Notes:

$\mathbf{ngF_t^{NOK/kWh}}$	futures price of natural gas in NOK per chemical kWh
$\mathbf{ngF_t^{pence/BTU}}$	British Thermal Units equivalent to about 29.307 kWhs
BTU	sales income after tax from electrical power
$\mathbf{ER_0^{NOK/GBP}}$	current exchange rate of NOK per GBP (9.00 NOK/GBP 21th May 2011)

However, to sell gas on the InterContinental Exchange, Statoil have to transport it there with pipelines. These costs will reduce the alternative cost of selling natural gas to Naturkraft, which is located in Norway and have a shorter distance to the refineries. The transport costs will here be assumed to be the only transaction cost (Enova 2003, 4). Gas used in the refinement facility at Kårstø, is brought ashore with several pipes Åsgård Transport and Statpipe. The rich gas is then refined to dry- and wet gas and transported to various locations in Europe or to Naturkraft. It costs nominally in 2003 between NOK 0.10 and NOK 0.15 per Sm^3 to transport one Sm^3 of natural gas from Norway to Europe with Europipe (Enova 2003, 23). This is almost explicitly capital costs, and it should therefore be ok to adjust this number by the general inflation to get the nominal cost in later years. I therefore adjust with $\mathrm{e}^{\mathrm{infl}\times(2011-2003+\mathrm{t})}$ to get the nominal prices per year after 2011.

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[14] To get the prices from the home screen of the webpage I choose Energy Indexes in the Report Center section. From here I get to make some choices in drop down lists. I choose Indices in Category, ICE Futures Europe in Market and UK and European Natural Gas in Report. Next, I specify a data time interval from May 2003 through December 2010. Several indexes will then show up; I will use the "UK NATURAL GAS INDEX (NBPI)".

[15] Notice that BTU and kWh here refers to the chemical energy stored in the gas, which is less than what Naturkraft actually manage to exploit.

[16] British pound

Again I want the cost to be given in NOK/kWh. I use the fact that $1\ \mathrm{Sm}^3 = 9.87\ \mathrm{kWh}$ in the conversion in equation 4.86. Europipe has a length of 660 kilometres and a capacity of 45.4 MSm^3 per day, while the pipe Statoil alternatively would have used (Europipe II) which has a length of 658 km and an available capacity of 64.8 MSm^3 per day. Assuming that the cost per Sm^3 is equal in the two pipes, and using an average of the cost 0.10 NOK and 0.15 NOK, I calculate the expected transport cost per kWh. To get the relevant price for Naturkraft, $\mathrm{tc}_t^{\mathrm{NOK/kWh}}$ as defined in equation 4.86 is then deducted from $\mathrm{ngF}_t^{\mathrm{NOK/kWh}}$ in equation 4.87 below.

equation 4.86 $$\mathrm{tc}_t^{\mathrm{NOK/kWh}} = \mathrm{tc}_{2003}^{\mathrm{NOK/Sm^3}} \times \frac{\mathrm{Sm^3}}{9.87\ \mathrm{kWh}} \times e^{\mathrm{infl}\times(2011-2003+t)}$$

Notes:

$\mathbf{tc}_{2003}^{\mathbf{NOK/Sm^3}}$	the price of natural gas in NOK per Sm^3 equal to 0.125 nominally in 2003
$\mathbf{tc}_t^{\mathbf{NOK/kWh}}$	transport price of natural gas in NOK per chemical kWh

It is also useful to denote the price of natural gas in NOK per kWh produced (and not kWhs of chemical energy). Since the market price on exchanges is denoted per kWh of chemical energy it will have to be adjusted. Naturkraft's plant can make use of about 58-59% of the chemical energy in the gas that they purchase (Naturkraft AS n.d.). I choose to use 59% as a base case scenario, and thus I multiply the relevant price estimate in equation 4.87 with 1/59%.

equation 4.87 $$\mathrm{ngP}_t = \left(\mathrm{ngF}_t^{\mathrm{NOK/kWh}} - \mathrm{tc}_t^{\mathrm{NOK/kWh}}\right) \times \frac{1}{0.59}$$

Notes:

$\mathbf{ngP}_t$	the cost of natural gas per kWh produced

The result of this process is summarized in Figure 4-3. It is clear from the figure that the relationship between the price of electrical power and the cost of natural gas was especially bad in 2008, and it is no wonder that the production was shut down in this year.

4.5.2.2 Future Estimate of Mean Price

I will use future prices of natural gas to estimate expected spot prices in the future. Future prices of natural gas exist six years into the future. I will use the future prices as an unbiased estimate for future expected spot price[17]. The results are illustrated in Figure 4-4 below.

After 2017 it is hard to get access to reliable estimates of future spot prices. Instead of making such estimations I am going to use a different method. Whenever the contribution margin in electricity production is high the profit will also be high. Large profits will soon be competed away if the environmental threat level is high and the plant has no sustainable advantages. Therefore I have performed external analysis, to see if this is the case. This is found in appendix 7.4. The analysis described a medium/high level of threat in the industry. Naturkraft have an internal strength related to transport costs, which has already been calculated when estimating the cost of natural gas. All of this count in favour that I can model the expected cost of natural gas as the price of electricity minus an equilibrium contribution margin. Watching Figure 4-3 it is obvious that spark spread has been unusually big in 2009. It makes sense that the future expected prices in Figure 4-4 converge towards a lower spark spread. Hence, I assume that the price converges against a level at which the contribution margin is in equilibrium. Moreover, I assume that the equilibrium spark spread will be equal to the one in 2016 adjusted for inflation in other years. In 2016 the price of electrical power is expected to be NOK 0.3814 per kWh and the cost of natural gas is expected to be NOK 0.3462. The contribution margin (excluding other variable costs) is then $\text{NOK } 0.3814 - \text{NOK } 0.3462 = \text{NOK } 0.0352$. Because of the other variable costs and the variable cost of EUAs the actual contribution margin will become negative. But because of the large volatility, it can still represent a long run equilibrium level giving a normal return on investments. The prices will fluctuate wildly and allow participants in the market to capitalize when prices fluctuate in their favour. The estimated prices for electricity derived from electricity and the equilibrium contribution margin are depicted in Figure 4-4 below.

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[17] To find these futures prices I visited theice.com chose “Market data” and then “Report Center”. I fill in the following info: Category: End of Day Report, Market: ICE Futures Europe, Report: ICE Futures Europe, Date: 31 March 2011, Contract: ICE UK Natural Gas Futures (Monthly). Since the prices are given per month, I will use an average of these per year. The conversion process to NOK/kWh is the same as before.

4.5.2.3 Modelling Cost of Natural Gas in the Binomial Model

To model cost of natural gas in the binomial model I need a starting cost value that will move up and down in the following periods. I will estimate this starting value with an average of the historic monthly futures prices from January through May, and the monthly futures from May to December, as a base year value. These historic and future values will be obtained in the same way before.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
58.48	56.01	53.49	60.14	59.92	57.55	58.38	59.65	59.18	63.81	68.28	70.98

Table 8 Monthly prices of natural gas through 2011 in pence/BTU

The average of these prices is then $60.49\ \mathrm{GBP/BTU}$, which when converted to NOK/kWh gives me $\mathrm{ngP}_0^{\mathrm{kWh}} = 0.2890\ \mathrm{NOK/kWh}$. Again using equation 3.3 from the theory section I will model the cost of natural gas in the binomial model as in equation 4.88. The volatilities necessary to calculate $\mathrm{u}_{\mathrm{ng}}^{\mathrm{j_{ng}}}$ and $\mathrm{d}_{\mathrm{ng}}^{\mathrm{n-j_{ng}}}$ will be estimated later.

equation 4.88 $$\mathrm{ngP}_{\mathrm{n,j_{ng}}} = \mathrm{ngP}_0^{\mathrm{kWh}} \times \mathrm{u}_{\mathrm{ng}}^{\mathrm{j_{ng}}} \times \mathrm{d}_{\mathrm{ng}}^{\mathrm{n-j_{ng}}}$$

equation 4.89 $$\mathrm{cost}_{\mathrm{n,j_{ng}}}^{\mathrm{NG}} = \mathrm{ngP}_{\mathrm{n,j_{ng}}} \times \mathrm{kWh} \times \Delta\mathrm{t} \times (1 - \mathrm{tax})$$

Notes:

$\mathbf{ngP_{n,j_{ng}}}$	cost of natural gas in NOK per kWh produced described in the binomial model
$\mathbf{u_{ng}^{j_{ng}}}$ & $\mathbf{d_{ng}^{n-j_{ng}}}$	the up an down moves for natural gas, calculated as in equation 3.1
$\mathbf{cost_{n,j_{ng}}^{NG}}$	cost of natural gas after tax

For example will the cost of natural gas in the starting year be:

equation 4.90 $$\mathrm{cost}_{0,0}^{\mathrm{NG}} = \mathrm{ngP}_{0,0} \times \mathrm{kWh} \times \Delta\mathrm{t} \times (1 - \mathrm{tax}) = 0.2890\ \mathrm{NOK/kWh} \times 3.5 \times 10^9\ \mathrm{kWh/year} \times 1\mathrm{year} \times (1 - 0.28) = \mathrm{MNOK}\ 728.20$$

4.5.3 A Note on Exchange Rates and Inflation Rates

The historic prices and futures on natural gas were originally denoted in GBP, while futures on electrical power were given in Euros. To find historic monthly and annual averages of NOK versus GBP and Euro, I visited the Web Pages of the Norwegian Central Bank and downloaded an Excel-file containing all the European currency exchange rates needed in my thesis (Norwegian Central Bank 2011).

Future estimates of exchange rates have been simplified significantly. I assume that the exchange rate for both Euros and GBP will be constant for the remaining lifetime of the plant. The future exchange rates in the model are all based on the rates on 21 May 2011, which were 7.84 NOK/Euro and 9.00 NOK/GBP. It isn't really necessary to estimate the future exchange rates, because the variance they cause will be baked into the volatilities that I will soon estimate in section 4.6.

4.5.4 Summarizing Price Data of Underlying Assets

The results from the two previous sections about electrical power and natural gas are illustrated below in figures. The historical prices are shown in Figure 4-3 below while future estimates are illustrated in Figure 4-4.

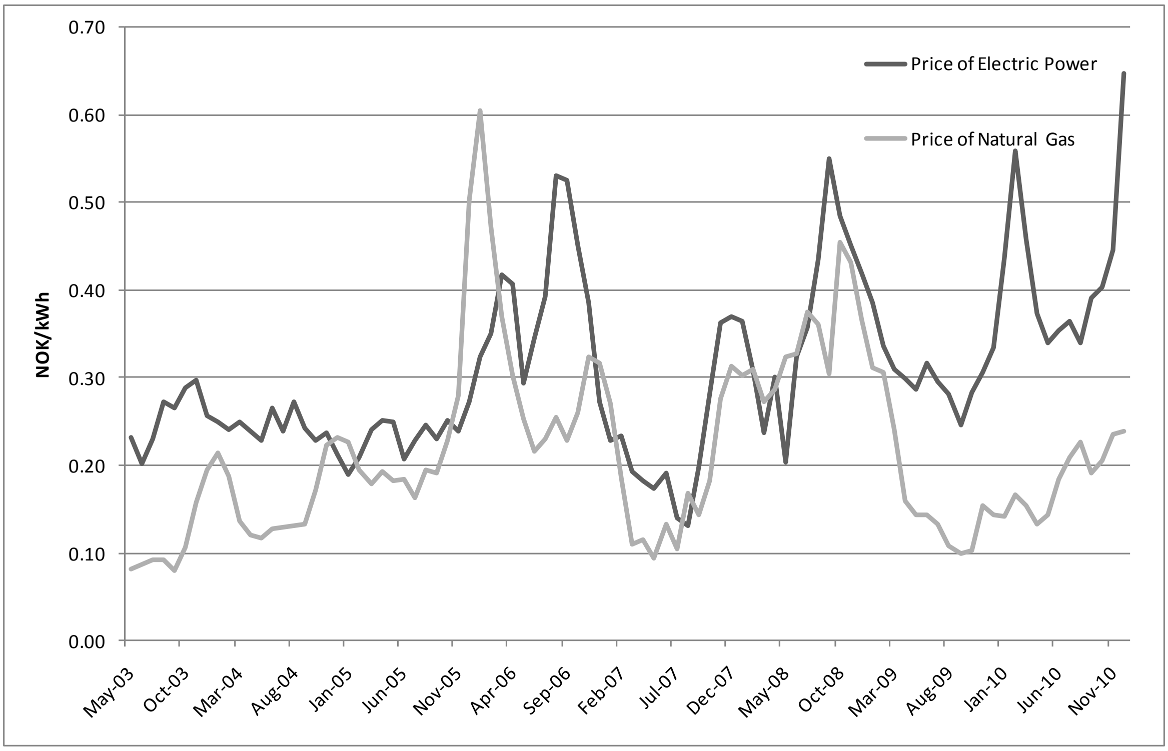

Figure 4-3 Historical cost of natural gas and price electrical power

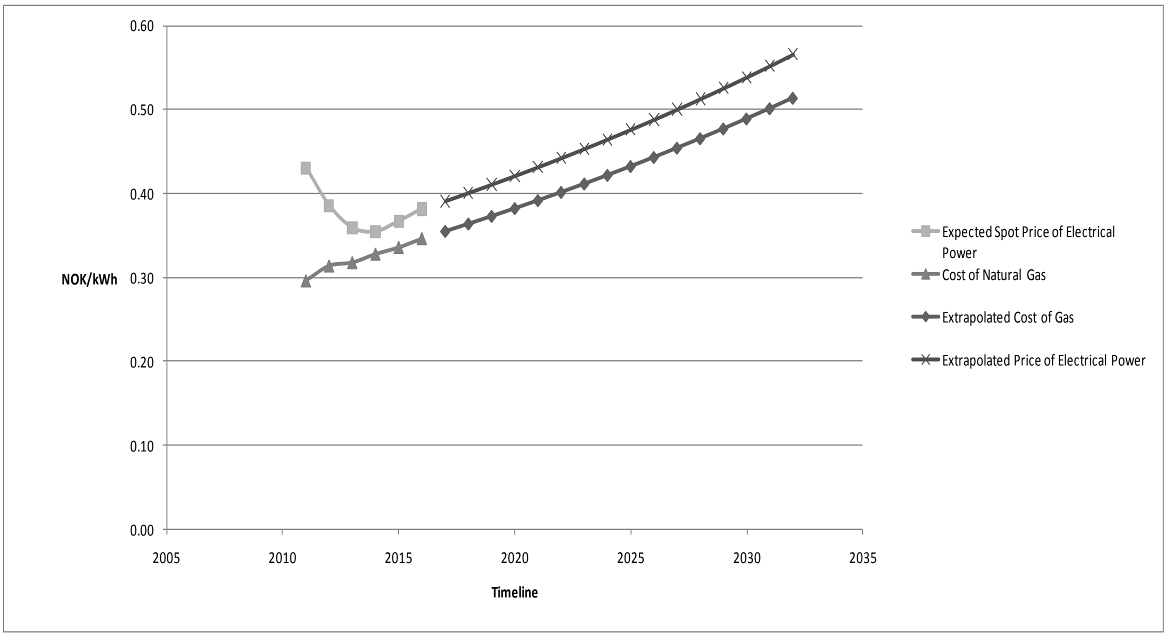

Figure 4-4 Future cost of natural gas and price electrical power derived by forwards

4.6 Volatility of the Underlying Assets

In section 3.6.1 procedures for estimating volatilities was described. Implied volatility was in many cases favoured because it relied on volatility expectations of the market contrary to historical volatility. I will therefore start of by examining implied volatility.

4.6.1 Implied Volatility

Calculating implied volatilities for the underlying assets turned out to be problematic. First of all the underlying assets, natural gas and electricity are traded in Euros and Pounds sterling which means that volatility from changes in the exchange rates aren’t embedded in the implied volatilities. In addition to this, the markets in question fail to fulfil the requirements of the basic versions of the Black-Scholes formula that was presented in section 3.6.2 (on page 27). The price of electrical power and natural gas both revert towards a mean which means that the volatility measure I would get would be smaller than the actual volatility. If I then adjust for mean reversion in the model I would adjust for mean reversion twice. With both underlying assets one will have to use imperfect replicating portfolios which imperfectly replicate the option value. The commodities are also described by seasonalities, so even if I predict the correct volatility for one season it might not be correct for the rest of the year. Moreover, natural gas usually carries large storage costs, which translates into a lower net convenience yield, which I don’t know the value of. It would also be difficult to estimate the convenience yield for either of the underlying assets again because of the non-storability characteristic.

Because of the many factors that complicate estimation of volatility implicitly with Black-Scholes, I will rather rely on historical volatility.

4.6.2 Historical Volatility

The volatility will be based on the monthly historical logarithmic returns of the underlying assets over the period May 2003 through December 2010. From Table 9 it's clear that the volatility of both underlying assets varies much over the period. Since the prices of both natural gas and electrical power were given per month, I will find the logarithmic returns per month with equation 3.44, and then calculate monthly standard deviations with Excel's built-in function, STDEV(). Finally, I convert this number into volatilities per year with equation 3.45. It is apparent that the volatility is high: 57.4% for electrical power and 69% for natural gas, but this is normal for these markets.

Year	Standard Deviation			
	Electric Power		Cost of Natural Gas	
2010	18.17%	63.0%	12.67%	43.9%
2009	9.40%	32.6%	19.51%	67.6%
2008	24.42%	84.6%	15.74%	54.5%
2007	21.89%	75.8%	32.56%	112.8%
2006	20.79%	72.0%	17.22%	59.6%
2005	10.70%	37.1%	20.26%	70.2%
2004	8.73%	30.2%	16.12%	55.9%
2003	12.58%	43.6%	18.76%	65.0%
Over total period	16.6%	57.4%	19.9%	69.0%
	(Monthly)	(Annually)	(Monthly)	(Annually)
MAX	24.42%	84.60%	32.56%	112.81%
MIN	8.73%	30.24%	12.67%	43.88%
Coefficient of correlation		17.35%		

Table 9 Volatility of natural gas and electrical power from June 2003 to December 2010

The volatility of gas and electricity in Table 9 is based on the cost of gas in NOK per kWh, i.e. natural gas was converted from price to cost with as described in section 4.5.2 (on page 53). This means that effects from currency exchange rates, energy efficiency and transaction cost are all incorporated into the volatilities.

In Table 9 annual volatilities of the price of electricity and cost of natural gas are 57.4% and 69.0%, and varied from 30.24% to 84.60% and from 112.81% to 43.88% for electrical power and natural gas, respectively. This indicates that the historic estimates of volatility are changing over time, and that an equally weighted average might not suffice. Believing that more recent volatility estimates are better predictors of future volatility, I decided to use the EWMA explained in the theory section 3.6.1.2 (on page 27). I will have to pick a value for b to use in equation 3.47. Since I want to put a relatively large weight on the most recent observations I will use b = 0.90. With observations from each month from May 2003 to December 2010, I get 91 observations[18]. I use equation 3.47 to find the EWMA of monthly variances, which I annualize by multiplying with (1/Δt). I then take the square root of the annualized variance to find the annualized volatility. The results are summarized in Table 10:

	Electric Power	Cost of Natural Gas
b	0.90	0.90
n	91	91
T	7.51	7.59
Δ_t	0.08	0.08
Variance / month	2.93%	2.28%
Variance / year	35.46%	27.30%
Volatility / year	59.55%	52.25%

Table 10 Volatility per year based on EWMA

The annual volatility is now lower than the one I got with an evenly weighted average. That is logical when looking at the development in volatility in Table 9 (on page 60), especially for natural gas. The downward trend in volatility is clear for gas, while not so much for electricity. I will use the volatilities found with EMWA in the model, i.e.:

$$\sigma_{el} = 59.55\%, \qquad \sigma_{ng} = 52.25\%$$

4.6.3 Coefficient of Correlation

While the monthly price data for natural gas covered the period from May 2003 until December 2010 the same data for electrical power covered only the period from January 1996 until March 2011. This means that the coefficient of correlation will have to be

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[18] I only got the price for May 2003, not the return, so the returns are from June 2003 only.

estimated on a period covered by both underlying assets, which is from May 2006 through December 2010. Using Excel's built-in Correl()-function I get a coefficient of correlation of 17.35%.

In section 4.1.1 (on page 29) it was suggested that the gas fired power plant can be considered a spark spread option. If electricity and gas had been perfectly correlated, the probability for them to create a large spark spread would be equal to zero. In the case where the prices are perfectly anti-correlated (=-100%) the probability for a large spark spread is equal to one. Therefore the value of the plant will increase with lower correlation.

4.6.4 A Note on Using Monthly Data

For both underlying assets there is price variation per hour, per day, week, month and quarter. The price data in this thesis is based on monthly data which will, because of mean reversion, ignore the variation that goes one in shorter time intervals. For example, the monthly averages I use in the model will not capture the variation in the price of power when it is lower on Sundays than on Mondays.

So if a power plant can be shut off and on with no extra cost, i.e. it happens instantly and no costly procedures are required, it can respond to all variation (like variation in price from second-to-second). This is of course never the case for gas fired power plants, but they have become less costly to shut off and on. In the nineties most combined-cycle power plants were planned as base load plants, just like nuclear power plants are today. However, as a response to the increased volatility in power and gas prices, they have increasingly been developed to be able respond to market developments.

The gas fired power plant in question was built by Siemens and is a combined cycle power plant which means it has one gas turbine and one steam turbine. Together the two turbines achieve a high efficiency, but the plant still has some start-up time. The plant is referred to as SCC5-4000F 1S and uses about eight hours to start up using a so called “hot start” (Hofmann and Emberger 2006). Smaller less powerful versions of the plant design have been specialized for easier start-up. SCC5-4000F 1S can be upgraded, but the plant still faces a significant start-up cost. That entails that the plant cannot automatically respond to daily or even weekly variations in price. It has to ignore short time intervals in which the contribution margin calls for another mode. This justifies the use of monthly data when calculating for instance the volatilities.

4.7 Estimating Parameters for Mean Reversion

Estimate the parameter κ, I will follow the recipe from section 3.2.6.4 (on page 21), and start by conducting a regression[19] on the same form as the expression in equation 3.33, using log returns from NOK per kWh as described above. After performing the regression on both electrical power and natural gas I get the values for β_0 and β_1, which are converted to the parameters I need using equation 3.34 and equation 3.35 (on page 22). For natural gas the log of the cost per kWh is used in the regression. The dataset was converted into NOK per kWh following the procedures described in section 4.5.1 and 4.5.2 (on page 48 and 53) before these parameters were estimated. The results are presented in Table 11:

	Electrical Power		**Cost of Natural Gas**	
	(Monthly)	(Annually)	(Monthly)	(Annually)
$\boldsymbol{\beta_0}$	-0.1325	-1.5905	-0.1780	-2.1363
$\boldsymbol{\beta_1}$	-0.1166	-1.3988	-0.1159	-1.3907
$\mathbf{\ln \bar{S}}$	-1.1371	-13.6447	-1.5362	-18.4342
$\boldsymbol{\kappa}$	0.1239	1.4873	0.1232	1.4781

Table 11 Parameters for mean reversion

From table 17 we see that the annualized mean reversion parameter κ is estimated to above 100% in both cases 149% and 148%. This means that prices will revert back to the mean price and past it when using time intervals of one year. I will, however, use shorter time intervals and therefore avoid this problem.

4.8 The Risk-Free Rate

When finding the capital cost one needs to take into account the cost of time, inflation and risk. I use nominal cash flows after tax to total capital in the model, and I will use a capital cost with the same properties. I assume that the investors are risk averse and diversified, which should be a fair assumption given that Statkraft and Statoil are government-owned

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[19] I used the built-in regression function in Excel 2007. On the Data tab, in the Analysis group, click Data Analysis, and then click Regression. In the Input Y Range I refer to the values for $\ln S_t - \ln S_{t-\Delta t}$, and in the Input X Range I refer to the $\ln S_{t-\Delta t}$ values.

companies (Statkraft AS n.d.), (Statoil ASA 2009)[20]. The risk adjustment will be made in the numerator with risk-neutral probabilities, so that I can discount with a risk-free capital cost in the denominator.

Optimally, one should discount each cash flow with a discount rate from a government bond with the same maturity, but for the sake of simplicity I will use a bond with a maturity of ten years from the Central Bank of Norway. The efficient interest on these bonds was 3.77% as of March 2011, which gives me the following risk-free interest rate after tax (Central bank of Norway 2011):

equation 4.91 $$\mathrm{r_f} = \ln\left(1 + 3.77\% \times (1 - 0.28)\right) - 1 = 2.68\%$$

Notes:

$\mathbf{r_f}$	risk free interest rate after tax, logarithmic (2.68%)

4.9 Summarizing Total Cash Flow, Parameters and Starting values

Now that all the necessary modelling is done, I summarize necessary parameters in Table 12, starting values in Table 13 and cash flows for 2011 in Table 14 below.[21]

Parameter	Assigned Annual Values	Found in Section	On Page
kWh	3.5 TWh	0	12
UEUA	1.2×10^6 EUAs	0	12
Total time period	21 years	4.1.5	32
$\mathbf{r_f}$ **(log after tax)**	2.68%	4.8	63
infl (log)	2.46%	4.2	33
$\boldsymbol{\sigma_{el}}$	59.55%	4.6.2	60
$\boldsymbol{\sigma_{ng}}$	52.25%	4.6.2	60
$\boldsymbol{\rho}$	17.35 %	4.6.3	61
$\boldsymbol{\kappa_{el}}$	148.73%	4.7	63
$\boldsymbol{\kappa_{ng}}$	147.81%	4.7	63
tax	28%	4.2	33

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[20] Statoil is a 67% owned by the government

[21] I don't really need Table 14, but to give the reader an idea of how the cash flows fit together in the real options model I thought that presenting the starting year could help.

$\mathbf{r}_{\mathbf{I}\&T}^{\mathbf{depr}}=\mathbf{r}_{\mathbf{I}\&T}^{\mathbf{inv}}$	30%	4.3.2	35
$\mathbf{r}_{\mathbf{P}\&E}^{\mathbf{depr}}$	4%	7.2.1	83
$\mathbf{r}_{\mathbf{P}\&E}^{\mathbf{inv}}$	0.4%	7.2.1	83
$\mathbf{r}_{\mathbf{INT}}^{\mathbf{depr}}$	20%	7.2.2	85
MVPE	30%	7.2.1	83
REUA2012	323 468 EUAs	4.4.6.1	46
$\mathbf{OWC_{2010}/sales_{2010}^{kWh}}$	11.22%	4.3.3.2	37

Table 12 Summarizing important parameters in the model

Name	**Starting Values**	**Found in Section**	**On Page**
Inventory & Tools ($\mathbf{I\&T_0}$)	3.62 MNOK	4.3.2	35
Plant & Equipment ($\mathbf{P\&E_0}$)	1 554.65 MNOK	7.2.1	83
Intangibles ($\mathbf{INT_0}$)	138.80 MNOK	7.2.2	85
$\mathbf{elP_0^{kWh}}$	0.4583 NOK	4.5.1.3	52
$\mathbf{npP_0^{kWh}}$	0.2890 NOK	4.5.2.3	57

Table 13 Summarizing starting values used as input in the model

	Classification	**MNOK 2011 after tax**	**Cash flow over periods is described with...**	**On page**
Sales of Electrical Power	Variable	1 154.81	equation 4.83	53
Cost of Natural Gas	Variable	-728.20	equation 4.89	57
Cash Flow from Used EUAs	Variable	-131.83	equation 4.77	47
CF OWC	Variable	-56.78	equation 4.62	38
Other Operating Costs	Variable	-86.80	equation 4.49	34
Contribution Margin	Variable	151.20		
Other Operating Costs	Fixed	-88.60	equation 4.50	34
Investments ($\mathbf{FixInv_0}$)	Fixed	-7.54	equation 4.64	38
Tax Deduction Depreciation	Fixed	27.44	equation 4.65	38
Cash Flow from Free EUAs	Fixed	34.42	equation 4.75	47
Total Cash Flow		48.08		

Table 14 Total cash flow of the gas fired power plant in base year

4.10 Modelling the Cash Flows in the Real Options Model

I have now shown how the cash flows of the gas fired power plant are calculated in periods, and it is now time to use them in the real option model to calculate project value. Real-option-wise all the fixed costs have been grouped together because they can be treated in a similar way. They will continue to occur until the plant is abandoned. The variable cash flows will occur until the production is shut down, and are also grouped into one total variable cash flow. This consolidation process is illustrated in Figure 4-1 on page 29. The consolidated cash flows are shown below:

equation 4.92 $\mathrm{CF}_{\mathrm{n}}^{\mathrm{fix}} = -\mathrm{FOC}_{\mathrm{n}} - \mathrm{FixInv}_{\mathrm{n}} + \mathrm{sales}_{\mathrm{n}}^{\mathrm{REUA}}$

equation 4.93 $\mathrm{CF}_{\mathrm{n}}^{\mathrm{var}} = \mathrm{sales}_{\mathrm{n,jX}}^{\mathrm{kWh}} - \mathrm{cost}_{\mathrm{n,jY}}^{\mathrm{ng}} - \mathrm{cost}_{\mathrm{n}}^{\mathrm{CEUA}} - \mathrm{VOC}_{\mathrm{n}} - \mathrm{OWC}_{\mathrm{n,jX}}^{\mathrm{ch}}$

Notes:

$\mathbf{CF}_{\mathbf{n}}^{\mathbf{fix}}$	net fixed cash flow
$\mathbf{CF}_{\mathbf{n}}^{\mathbf{var}}$	net variable cash flow

It should then be possible to create the portfolio decision rules.

4.10.2 The Portfolio Decision Rule

It is also assumed that the plant can temporarily shut down production, and the total cash flow from the plant, when having the option to shut down, is shown in equation 4.94. This cash flow is used when taking into account the option to abandon. The resulting value in each state in each period is given in equation 4.95. The rationale behind these equations was described in the theory section 3.4 and 3.4.2 (on page 23-25). Just like before, equation 4.95 will have to be used with the backwards recursive technique. Start estimating $\mathrm{V}_{\mathrm{n}}^{\mathrm{plant}}$ in all the states in the last period, and then work your way towards the present period one period at a time, to finally arrive at the present value. $\mathrm{V}_{0}^{\mathrm{plant}}$ should then equal the present value of the plant.

equation 4.94 $$\mathrm{CF}_n^{\mathrm{SD}} = \mathrm{CF}_n^{\mathrm{fix}} + \mathrm{MAX}[\mathrm{CF}_n^{\mathrm{var}}, 0]$$

equation 4.95 $$\mathrm{V}_n^{\mathrm{plant}} = \mathrm{CF}_n^{\mathrm{SD}} + \mathrm{MAX}\left[\mathrm{FixAV}_t + \mathrm{AV}_{(t,j)}^{\mathrm{OWC}}, e^{-r_f \times \Delta t} \times \left\{\sum_{j=1}^{4} f_{(n+1,j)} \times \mathrm{V}_{(n+1,j)}\right\}\right]$$

Notes:

$\mathbf{CF_n^{SD}}$	total cash flow when having the option to shut down production
$\mathbf{V_n^{plant}}$	value of the plant in each state in each period

To estimate the risk neutral probabilities in all the states in the model I use the formulas from theory section 3.2.6 (on page 17) and the parameters obtained in section 4.5, 4.6 and I can estimate risk neutral probabilities that take into account mean reversion in the model. The probabilities will be recalculated in each node in the binomial tree and censored. When the equation uses both t and n in the footnotes, it uses n to refer to the cash flow generated over the period but t to refer to the last year in the period.

4.10.3 Choosing the Number of Periods

The machine I am using has x64-Windows 7, with Pentium(R) Dual Core Processors of each 2.10 GHz and 4.00 GB installed physical memory. In my binomial model with recombining nodes, the number of end nodes is described with n^2. With limited computing capacity I have to reduce the number of sub periods. To see if the present value starts to converge towards some present value I start with only a few sub periods in the start and then add more and more until the computer becomes too slow.

In this test I disregarded the value of any financial assets, so that I can better focus on the operational value. Since I presented three EUA price scenarios in section 4.4.5.4 (on page 44), I will estimate the present value for each scenario and multiply each value with the probability for that scenario happening to arrive at the weighted present value. I don’t know the exact probability for each scenario, but I know that the medium scenario was the most likely one. As of now I will assume that the probabilities for the scenarios are 25%, 50% and 25% for the low price, the medium price and the high price scenarios, respectively. With no free EUAs allocated to the industry I estimate some PVs for different number of periods.

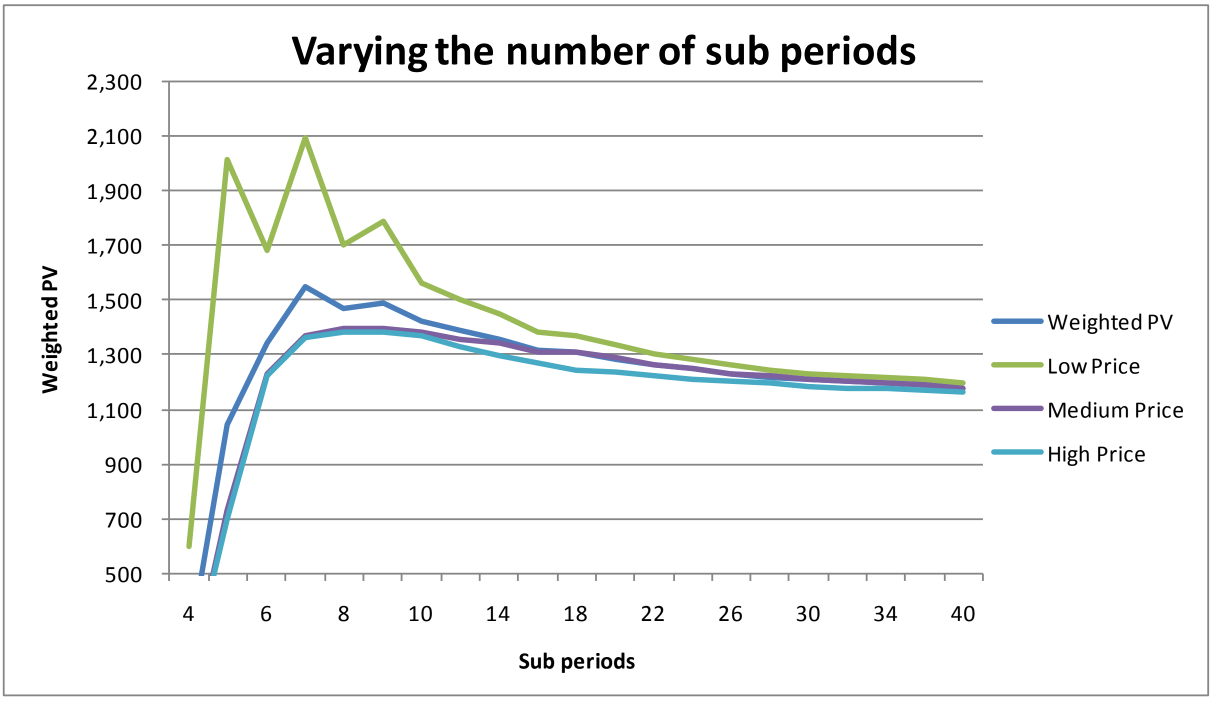

Figure 4-5 Convergence of PV when varying the number of sub periods in the model.

It is clear from the figure that the number of sub periods matters a lot when the number of sub periods is low. The weighted PV is actually as low as MNOK 227 when the number of sub periods is 4, but jumps to MNOK 1044 when using 5 sub periods. The weighted PV also varies some when the number of sub periods approaches 30, however much less than before. The weighted PV using some high numbers for n is shown below.

n	30	32	34	36	40
Weighted PV (MNOK)	1209	1200	1198	1191	1180

The weighted PV seems to converge towards somewhere around MNOK 1170 or so. The optimal length of time intervals would be one month since I used monthly price data. This would entail using 252 periods which would take up too much time. When using 30 sub periods the computing time for each PV in each scenario takes about 8 seconds, or about 24 seconds when computing the weighted PV. For more periods the computing time increases rapidly, which I will have to take into account when performing analyses.

5 Analysing Data

This thesis was going to examine the impact that changes in the third phase of EU ETS have on the present value of the plant relative to the second phase. As has been discussed, there are several changes, but the main ones are:

1. The price per EUA will change
2. The free EUAs will disappear after 2012

These changes can be examined one by one in isolation and simultaneously. In section 5.1.1, I will investigate what happens in each of the three price scenarios described in section 4.4.5.4 (on page 44) while assuming that the plant still receives its free allowances. After that, I will study the two effects simultaneously in section 5.1.2.

5.1 Varying Price Scenarios and Free EUAs

5.1.1 Varying Price Scenarios of EUAs

One change in the third phase of the EU ETS was a gradual reduction in the total amount of allowances in the EU ETS which will lead to higher EUA prices. If I am going to measure the impact of the changes in the third phase *relative* to the second phase, I need a scenario of what the price would have been if no changes had been introduced. Such a scenario isn't exactly something that I will find publicly available. Therefore I will create this scenario below and use it to compare with the other scenarios.

The price per EUA on the 25th of February 2011 was NOK 147.78. When adjusting this number with inflation to 2012 I get $\mathrm{NOK}\ 147.78 \times 1.025 = \mathrm{NOK}\ 151.5$. In section 4.4.5.4 (on page 44) it was assumed that in the low price scenario the price would increase with NOK 8.87 per year. I will assume that the price increase per year is the same in the "no change"-scenario as in the low price scenario. This means that the price can be described with the following equation:

equation 5.96 $$\mathrm{NoChP}_t^{\frac{\mathrm{EUA}}{\mathrm{kWh}}} = 151.5 + 8.87 \times t$$

Notes:

$\mathbf{NoChP_t^{EUA/kWh}}$	EUA price scenario in which no changes are made to EU ETS

I will now see what happens when the price of EUAs change. So far I assume that Naturkraft still receives its free allowances. I will use 30 sub periods.

Price Scenario	No Change	Low Price	Medium Price	High Price
PV (MNOK)	1596	1593	1662	2417
PV %	100%	99.8%	104.2%	151.5%

Table 15 Different PVs in different scenarios when keeping the free EUAs

It is apparent that the price of EUAs matters much for the value of the plant. Notice here that the present value is highest for the highest price scenario, and the lowest present value is found in the low price scenario. The present value is actually 51.5% higher in the high price scenario than in the No Change scenario. This effect is explained by free EUAs, which so far hasn't been removed. When the price of EUAs becomes high, the management will shut down production, but will still receive and sell free EUAs. The plant can only lose money from higher EUA prices when the contribution margin is initially positive. If it's negative and the price of EUA increases, they will only receive a larger fixed income from selling EUAs in the market (since they aren't buying EUAs in the production anyway it doesn't matter). This is also illustrated in the diagram below.

The point illustrated above has been made several times in the media to criticize the carbon market. Allocation of free EUAs can lead to adverse effects. What is to stop plant operators (especially those with obsolete equipment) from simply cutting down on maintenance, and rely solely on the sale of free EUAs? Such a strategy would lower their fixed costs while still receiving the fixed income from free EUAs. The risk of this happening is higher among inefficient and obsolete plants since they have less chance of producing with a profit at market prices. From a macro perspective, this is what the carbon market was designed to cause (high emitters to stop/reduce emissions), but some would, however, probably conclude that paying money to firms for doing nothing is a waste of taxpayers' money.

Free EUAs will, however, not be allocated to the power generating industry after 2012. This means that the adverse effects discussed above will be resolved. If the EU in addition decides to levy a duty on power imported from areas without a CO_2-cost, carbon leakage will probably also become a lot less relevant. In the next section I will examine all the changes in the ETS simultaneously.

5.1.2 Studying the Changes Simultaneously

I will now remove the free EUAs in every year except for 2012 and calculate PV in each EUA price scenario. The resulting present values are reproduced below.

Scenarios	No Change	Low Price	Medium Price	High Price
Full PV (MNOK)	1233	1232	1209	1186
Full PV %	100%	99.9%	98.1%	96.2%

Table 16 Price scenarios studied together with removed free EUAs using 30 periods.

Earlier the highest present value was actually the one in the high price scenario. With no free allowances after 2012, this is obviously no longer the case. The present values are now slightly reduced when the price of EUAs increase, so the adverse phenomenon of increased PV with higher EUA prices is gone. It is also clear that the present value is a lot less sensitive to EUA price changes than before, so most of the variation must have come from the free EUAs. That is natural since the variable cost of EUAs in production can more easily be avoided by temporarily shutting down production than the fixed income from free EUAs.

In Figure 5-1 below the numbers from the last two tables is summarized. It is clear from the figure that in the high price scenario a large part of the value lies in the present value of free EUAs.

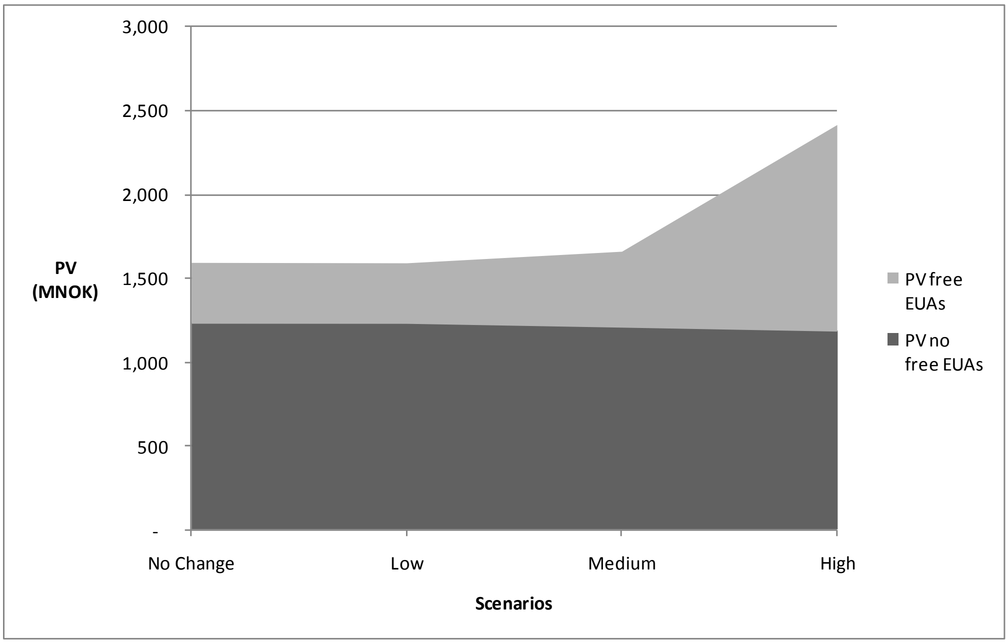

Figure 5-1 Decomposing the PV into PV of free EUAs and PV of plant with no free EUAs

It is clear that the dark grey area is less sensitive to price changes than the light grey area. This has been commented already, but it is also apparent that even though higher EUA prices will affect the price of the plant, they are nothing compared to the effect of removing the free EUAs. The weighted present value[22] of the plant drops from MNOK 1 834 when the plant receives the free EUAs to MNOK 1 209 when the plant loses them.

The present values presented here are low compared to the investment cost of about NOK two billion. They are probably also a bit unrealistic. If the prices of EUAs turn out as unfavourably as in the high price scenario, then other market prices of gas and electricity will probably change as well. As I mentioned earlier in section 4.4 (on page 39), the price of EUAs is probably correlated with the price of electricity and natural gas. So in the event of an extremely high price of EUAs, the price of electricity would probably increase as well while the price of natural gas would decrease because the price of EUAs is most likely correlated with the price of electricity and natural gas. Therefore, the PV’s sensitivity to changes in the price of EUAs is probably somewhat exaggerated here, but the main trends are still the same: When removing the free EUAs the effect of increasing the price of EUAs isn’t that drastic. From the no change scenario to the high price scenario the present value drops only to 96.2%.

5.2 Studying Results While Varying Volatilities in the Model

There are, however, many other sources of uncertainty in this model. Volatilities were based on historical values and abandonment values were assumed to be equal to a portion of tax values. In section 5.2 I will investigate if the results in 5.1 still hold when I change some important parameters in the model.

5.2.1 The Effect of Changing the Volatilities on the Weighted PV

To estimate volatilities I used the EWMA which is a method based on historical values. Unable to find the implied volatilities, there is really no guarantee that my estimates actually look like the actual expected future volatilities. Since volatility is perhaps the most important input variable in the real options model I am going to estimate what would happen to the weighted PV and the results in 5.1.1 if I change the volatilities for both underlying assets. Since the incremental effect on the weighted PV of changing the volatility in one underlying

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[22] Using the same weights as before, i.e. 25%, 50% and 25% for the low, medium and high price scenarios.

asset depends on the volatility value in the other underlying asset I will change them simultaneously:

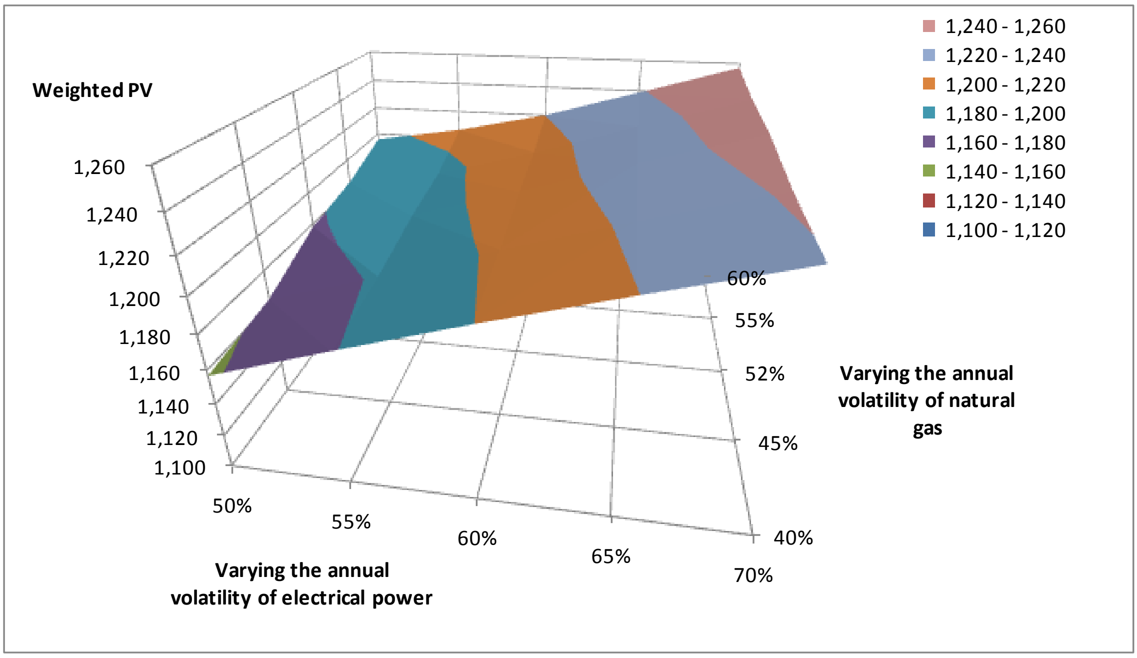

Figure 5-2 The effect on weighted PV of varying volatility in the model

The figure above measures the weighted PV (over the three EUA price scenarios). As expected the highest values are found in the areas where the volatilities are the highest. The thing to notice here is that all the weighted PVs are found within a range from 1150 to 1260, so even if the volatilities estimated based on historical values earlier don't exactly correspond to the expectations of the market the results wouldn't be radically different.

5.2.2 The Effect of Changing the Abandonment Value and Lifetime on the Weighted PV

Quite hastily I assumed that the abandonment value of inventory and tools and intangibles equalled the tax value, while the abandonment value of plant and equipment equalled 30% of tax value. I will now change the abandonment value of plant and equipment as a percentage of tax value. The results are shown below:

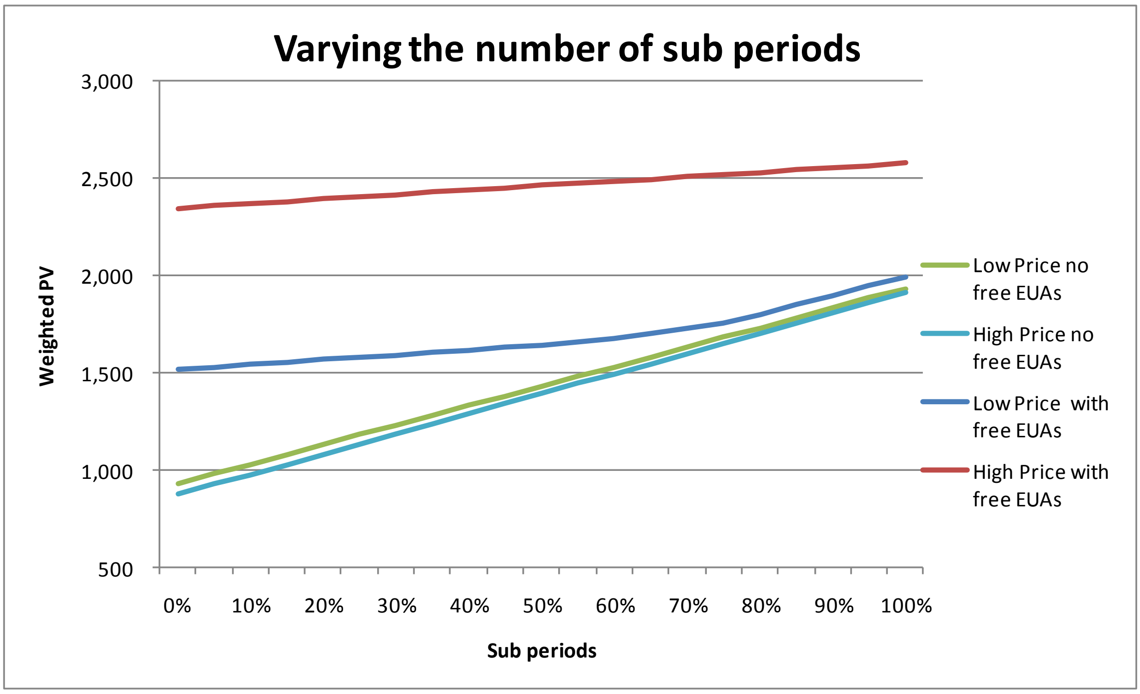

Figure 5-3 The results of changing the abandonment values using 30 periods

For the dark blue and red curve, I see that the same results still hold. The Present value is higher for the higher price scenario when the plant receives free EUAs. The two other curves show that the present value is slightly higher for low price scenario when the plant receives no free EUAs. This result holds for all abandonment values. It is also obvious here that the present value is much more volatile when they receive free EUAs.

5.3 Discoveries and Conclusion

The research question was about studying the changes in the third phase of the EU ETS and how they affected the present value of gas fired power plant.

In the first analysis in section 5.1.1 I studied the isolated effects on the present value, of varying the price of EUAs by introducing various scenarios. It was concluded that the present value is higher in the higher price scenarios. Actually, the present value is as much as 51.5% higher in the high price scenario as in the no change scenario. The present value is also very sensitive to changes in the price of EUAs when the power plant continues to receive free EUAs.

Later when I examined the price effects without the freely allocated EUAs in section 5.1.2, the adverse effect from the first analysis disappeared. Here the present value of the gas fired plant became smaller in the higher EUA price scenarios and the present value also became less volatile. A slight 3.8% in present value in the high price scenario relative to the no change scenario was the entire reduction. However, the most noticeable effect was a drop in the weighted present value from MNOK 1 834, when the plant received the free EUAs, to MNOK 1 209 when the plant loses them, i.e. a significant 34% reduction.

From section 5.2 it was concluded that the results from section 5.1 remains the same even when volatilities and abandonment values are changed. So even if some input variables in the model turn out to be incorrect the results should still hold.

All in all, the present value of gas fired power plants will be radically affected by the changes in the third phase of EU ETS. The increases in the price of EUAs are insignificant relative to the impact of losing free EUAs on the present value of the gas fired power plant.

5.4 Criticism and Further Research

5.4.1 Criticism

Many assumptions and choices have been made in the thesis to limit the complexity of the model. Some of them lead to inaccuracies with regard to estimated present values. Here are some of them.

5.4.1.1 Few Sub Periods

For the calculations to be more accurate, time periods should have been shorter. Some more periods could have been added, but the model would have been very slow to work with. I do believe the main conclusions will still hold, since I discovered in section 4.10.3 that the estimates seemed to converge towards some number not far from the ones I found.

Finally, it was stated in the thesis that the censoring process lead to convergence within 1% of actual value when the time periods was less than 3 months long. Seeing that my periods are longer than this, there is reason to believe that the censoring process may lead to biased results (I used 30 periods which corresponds to intervals of 0.7 years or 8.4 months).

5.4.1.2 Not Including EUAs as an Underlying Asset

Even though it was argued that the price of EUAs is probably correlated with the price of the other underlying asset it was not included to avoid creating a four dimensional binomial tree. This was simply because the model would again become too unwieldy and because I lacked info about the volatility and correlation.

5.4.1.3 Ignoring CER, JI, CCS

The Clean Development Mechanism allows a party to cause an additional emission reduction in a developing country and get saleable CERs in return. Secondly, the Joint Implementation mechanism gives a party the right to earn emission reduction units (ERU) from a jointly implemented project that causes additional emission reduction in another country. I decided to ignore both these possibilities in the thesis to keep things simple. A large corporation like Statoil ASA would probably had utilized such opportunities, and by doing that saved money compared to the scenarios I have outlined in the model. I have also ignored the possibility of successful results in the development of CCS technology. If successful it would lead to lower emission costs and thus lower sensitivity to the price of EUAs.

5.4.1.4 The Report from Klimakur 2020

This report is from 2009 and is probably a little outdated. Also, it sometimes gives away signs of unprofessionalism even though it is a governmental report. It sometimes fails to properly explain/justify decisions taken and small sections here and there don't make perfect sense. The main conclusions in the report were backed up sufficiently, but if I had access to another report or other estimates for the EUA prices I probably would have used that/those instead.

5.4.1.5 Ignoring Flexibility and Simplified Expected Spot Prices

In reality the management usually has an unlimited selection of real options. They can do much more than just abandoning or stopping production. These real options were ignored in my simplified model. The main results would probably be the same, but one cannot know for sure before testing.

Also the expected spot prices were simplified, this will cause the estimated present values to be unequal to market prices, but the conclusion should still be the same.

5.4.2 Further Research

In a few years it will be possible to get more reliable estimates on the volatility of the price returns of EUAs and also how it is correlated with other relevant prices like electricity and natural gas. It will then be interesting to create a more complete real options model which includes the price of EUAs as an underlying asset. Including more alternative courses of action for the management, such as the possibility of expanding capacity, would also be interesting. One could also try simulations, to see whether any conclusions change.

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7 Appendix

7.1 Deriving Risk Neutral Probabilities and The Up and Down Moves

7.1.1 Deriving Risk Neutral Probabilities

The value of an option can in principle be calculated by creating a replicating portfolio in the market with same payoff as the option in each outcome and thus also the same value. The replicating portfolio is consists of a position in the underlying asset, financed partly with a risk-free loan B. The price of the underlying asset can move up or down over the period. Any net dividends δ of owning the stock will add to the profits of owning the stock. The position in the underlying asset and the loan is found by solving equation 7.97 and equation 7.98 simultaneously, which gives the result in equation 7.99. The option value equals the replicating portfolio value, found as in equation 7.100.

equation 7.97 $\left(\mathrm{delta}_S \times dS_0 \times e^{\delta\times\Delta t}\right) + \left(B \times e^{r_f\times\Delta t}\right) = V_d$

equation 7.98 $\left(\mathrm{delta}_S \times uS_0 \times e^{\delta\times\Delta t}\right) + \left(B \times e^{r_f\times\Delta t}\right) = V_u$

equation 7.99 $\mathrm{delta}_S = e^{-\delta\times\Delta t} \times \frac{V_u - V_d}{S_0(u-d)}, \qquad B = e^{-r_f\times\Delta t} \times \frac{uV_d - dV_u}{u-d}$

equation 7.100 $V_0 = \mathrm{delta}_S \times S_0 + B$

Notes:

delta_S	number of shares bought at the start of the period
B	bank loan at the start of the period

The result is independent from attitudes towards risk. Other option values than V_0 in equation 7.100, would create an arbitrage opportunity and investors would instantly remove it by trading, hence the no arbitrage requirement. When I substitute delta_S and B from equation 7.99 into equation 7.100 and simplify, the result in equation 7.101 follows. To avoid arbitrage equation 7.102 must also hold.

equation 7.101 $$V_0 = e^{-r\times\Delta t} \times \left(V_u \times \frac{e^{(r_f-\delta)\times\Delta t}-d}{u-d} + V_d \times \frac{u-e^{(r_f-\delta)\times\Delta t}}{u-d}\right)$$

equation 7.102 $$u > e^{(r_f-\delta)\times\Delta t} > d$$

In equation 7.101 the terms $\frac{e^{(r_f-\delta)\times\Delta t}-d}{u-d}$ and $\frac{u-e^{(r_f-\delta)\times\Delta t}}{u-d}$ always sum to 1. Moreover, thanks to equation 7.102 both of the terms will always be positive. This means that they can be interpreted as probabilities for V_u and V_d, respectively. Since they are discounted at a risk-free rate they are interpreted as risk-free- or neutral probabilities, as defined in equation 3.2 (McDonald 2006):

equation 7.103 $$f_u = \frac{e^{(r_f-\delta)\times\Delta t}-d}{u-d} \text{ and } f_d = 1 - f_u = \frac{u-e^{(r_f-\delta)\times\Delta t}}{u-d}$$

7.1.2 Deriving the Up and Down Moves

In this section the up and down moves will be derived and matched with the risk-neutral probabilities. The derivation is partly based on Markov processes and the Geometric Brownian Motion (GBM) which is assumed to be known by the reader.

To derive the up and down moves I follow the recipe of Hull (2009, 249). The actual probability for the prices in a binomial model to move either up or down with factors u or d is now denoted p and (1-p). In the real world the value of the underlying asset by the end of a period is given by equation 7.104, which can be rearranged to equation 7.105.

equation 7.104 $$pS_0u + (1-p)S_0d = S_0e^{k\Delta t}$$

equation 7.105 $$p = \frac{e^{k\times\Delta t}-d}{u-d}$$

The volatility in lognormal returns in price is defined so that the standard deviation equals $\sigma\sqrt{\Delta t}$ per time interval. The variance is then $\sigma^2\Delta t$. To make the variance of the underlying asset in the binomial tree using real probabilities match the real variance, equation 7.106 must hold. From equation 7.105, p is then substituted into equation 7.106, which gives the result in equation 7.107. From here the kind of series expansion depicted in equation 7.108 is used, to find a solution to the equation given in equation 3.1 and equation 3.1, when ignoring terms in Δt^2 and higher powers of Δt.

equation 7.106 $$\mathrm{Var}(S) = E(S^2) - E(S)^2 = p \times u^2 + (1-p) \times d^2 - (pu + (1-p)d)^2 = \sigma^2 \Delta t$$

equation 7.107 $$e^{k\Delta t}(u+d) - ud - e^{2k\Delta t} = \sigma^2 \Delta t$$

equation 7.108 $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

equation 7.109 $$u = e^{\sigma\sqrt{\Delta t}}$$

equation 7.110 $$d = e^{-\sigma\sqrt{\Delta t}}$$

The same derivation can be done with risk-neutral probabilities, which would give the exact same result for u and d. The up and down moves are here defined as independent of the drift, which is taken care of in the risk-neutral probabilities. The relation between u, d, f_u and f_d used here, implicitly assumes that the GBM model sufficiently describes the movement in the price of the underlying assets.

7.2 Modelling of Plant and Equipment and Intangibles

7.2.1 Plant and Equipment

Plant and equipment (P&E) is the main investment of the whole project and will need only small annual reinvestments after the first initial investment, but will give relatively large tax deductions from depreciation. Average reinvestment rate in 2009 and 2010 was about $\left(\frac{5\ 885\ 839}{1\ 686\ 713\ 109} + \frac{7{,}487{,}762}{1{,}758{,}020{,}491}\right)/2 = 0.4\%$ of the balance value (Naturkraft AS 2010). To estimate the tax value I will use the depreciation from Table 3 and assume a constant reinvestment rate through the project period.

Rates	Symbol used	Geometric rate
Investment rate	$r_{P\&E}^{inv}$	0.4%
Depreciation rate	$r_{P\&E}^{depr}$	4.0%
Net change	$r_{P\&E}^{change}$	$0.4\% - 4.0\% = -3.6\%$

Table 17 Rates for plant and equipment

In the last row in Table 17 the net change in value is calculated, this is used to estimate the value of the line item in equation 7.111 below. Based on this value the reinvestments each year is calculated in equation 7.112. Reinvestments over periods are calculated with the definite integral of $\mathrm{inv}_t^{P\&E}$ between t and $t - \Delta t$ like in equation 7.113. In the same way I can calculate the accumulated tax deductions from depreciation over periods (in equation 7.114). Finally the abandonment value is assumed to be as low as 30% of the tax value (mostly sunk cost investments). However, they will get a tax deduction on the negative sales profit which is reflected in equation 7.115 as $\mathrm{P\&}E_{\mathrm{t}} \times (1 - 30\%) \times \mathrm{r}^{\mathrm{tax}}$.

$$\text{equation 7.111} \qquad \mathrm{P\&}E_{\mathrm{t}} = \mathrm{P\&}E_{0} \times \left(1 + \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)^{\mathrm{t}}$$

$$\text{equation 7.112} \qquad \mathrm{inv}_{\mathrm{t}}^{\mathrm{P\&}E} = \mathrm{P\&}E_{\mathrm{t-1}} \times \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{inv}}$$

$$\text{equation 7.113} \qquad \mathrm{inv}_{\mathrm{n}}^{\mathrm{P\&}E} = \int_{\mathrm{t-\Delta t}}^{\mathrm{t}} \mathrm{inv}_{\mathrm{t}}^{\mathrm{P\&}E} \mathrm{dt} = \mathrm{P\&}E_{0} \times \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{inv}} \times \frac{\left(1+\mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)^{\mathrm{t-1}}}{\ln\left(1+\mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)} \times \left(1 - \left(1 + \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)^{-\Delta\mathrm{t}}\right)$$

$$\text{equation 7.114} \qquad \mathrm{TaxDed}_{\mathrm{n}}^{\mathrm{P\&}E} = \mathrm{P\&}E_{0} \times \left(\mathrm{r}_{\mathrm{P\&}E}^{\mathrm{depr}} \times \mathrm{r}^{\mathrm{tax}}\right) \times \frac{\left(1+\mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)^{\mathrm{t-1}}}{\ln\left(1+\mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)} \times \left(1 - \left(1 + \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)^{-\Delta\mathrm{t}}\right)$$

$$\text{equation 7.115} \qquad \mathrm{AV}_{\mathrm{t}}^{\mathrm{P\&}E} = \mathrm{P\&}E_{\mathrm{t}} \times 30\% + \mathrm{P\&}E_{\mathrm{t}} \times (1 - 30\%) \times \mathrm{r}^{\mathrm{tax}}$$

Notes:

$\mathbf{r}_{\mathbf{P\&}E}^{\mathbf{inv}}$	rate of depreciation for plant and equipment (4%)
$\mathbf{r}_{\mathbf{P\&}E}^{\mathbf{inv}}$	reinvestment rate for plant and equipment (0.4%)
$\mathbf{r}_{\mathbf{P\&}E}^{\mathbf{change}}$	net change in plant and equipment (−3.6%)
$\mathbf{P\&}E_{\mathbf{t/n}}$	end year/period tax value of plant and equipment
$\mathbf{TaxDed}_{\mathbf{t/n}}^{\mathrm{P\&}E}$	tax deductions from depreciation of plant and equipment
$\mathbf{AV}_{\mathbf{t/n}}^{\mathrm{P\&}E}$	end year/period abandonment value of plant and equipment
$\mathbf{inv}_{\mathbf{t/n}}^{\mathrm{P\&}E}$	investments in of plant and equipment

Using these equations I can calculate starting value, investments and tax deductions of P&E in 2011:

equation 7.116 $\mathrm{P\&}E_0 = \mathrm{P\&}E_{2010} \times \left(1 + \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{change}}\right)^{\mathrm{t}} = \mathrm{MNOK}\ 1\,612.70 \times (1 - 0.036) = \mathrm{MNOK}\ 1\,554.65$

equation 7.117 $\mathrm{inv}_0^{\mathrm{P\&}E} = \mathrm{P\&}E_{2010} \times \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{inv}} = \mathrm{MNOK}\ 1\,612.70 \times 0.04 = \mathrm{MNOK}\ 6.45$

equation 7.118 $\mathrm{TaxDed}_0^{\mathrm{P\&}E} = \mathrm{P\&}E_{2010} \times \mathrm{r}_{\mathrm{P\&}E}^{\mathrm{depr}} \times \mathrm{r}^{\mathrm{tax}} = \mathrm{MNOK}\ 1\,554.65 \times 0.04 \times 0.28 = \mathrm{MNOK}\ 17.41$

7.2.2 Intangibles

Without any reinvestments intangibles (INT) are easily modelled. INT are related to privileges at and around the property and will be depreciated with rate $\mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}} = 20\%$ per year from Table 3.

The value of intangibles can then at any time be described with equation 7.119. The abandonment value is assumed to be 100% of the operating value:

equation 7.119 $\mathrm{INT}_{\mathrm{t}} = \mathrm{AV}_{\mathrm{t}}^{\mathrm{INT}} = \mathrm{INT}_0 \times \left(1 - \mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}}\right)^{\mathrm{t}}$

equation 7.120 $\mathrm{TaxDed}_{\mathrm{t}}^{\mathrm{INT}} = \mathrm{INT}_{\mathrm{t}-1} \times \mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}} \times \mathrm{r}^{\mathrm{tax}}$

equation 7.121 $\mathrm{TaxDed}_{\mathrm{n}}^{\mathrm{INT}} = \int_{\mathrm{t}-\Delta\mathrm{t}}^{\mathrm{t}} \mathrm{TaxDed}_{\mathrm{t}}^{\mathrm{INT}} \mathrm{dt} = \mathrm{INT}_0 \times \mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}} \times \mathrm{r}^{\mathrm{tax}} \times \frac{\left(1-\mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}}\right)^{\mathrm{t}-1}}{\ln\left(1-\mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}}\right)} \times \left(1 - \left(1 - \mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}}\right)^{-\Delta\mathrm{t}}\right)$

Notes:

$\mathbf{r}_{\mathbf{INT}}^{\mathbf{depr}}$	rate of depreciation for intangibles (20%)
$\mathbf{INT}_{\mathbf{t/n}}$	end year/period tax value of intangibles
$\mathbf{TaxDed}_{\mathbf{t/n}}^{\mathbf{INT}}$	tax deductions from depreciation of intangibles
$\mathbf{AV}_{\mathbf{t/n}}^{\mathbf{INT}}$	end year/period abandonment value of intangibles

With no reinvestments I can calculate starting value and tax deductions like this:

equation 7.122 $\mathrm{INT}_0 = \mathrm{INT}_{2010} \times \left(1 - \mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}}\right) = 173.50 \times (1 - 0.20) = \mathrm{MNOK}\ 138.80$

equation 7.123 $\mathrm{TaxDed}_0^{\mathrm{INT}} = \mathrm{INT}_{2010} \times \mathrm{r}_{\mathrm{INT}}^{\mathrm{depr}} \times \mathrm{r}^{\mathrm{tax}} = \mathrm{MNOK}\ 173.50 \times 0.20 \times 0.28 = \mathrm{MNOK}\ 9.72$

7.3 The Study by Botterud, Bhattacharayya and Ilic (2002)

In the study, Botterud, Bhattacharayya and Ilic (2002) used data from September 1995 until the end of 2001. For future data only the closing price on the last day of trading for each week was used. This study was based on equation 7.124 to describe the expected future spot price. It is assumed in the test that the speculators causing the results from the regression had unbiased predictions for expected future spot prices.

equation 7.124 $\mathrm{E(S_t) = F_t \times e^{r_p \times t}}$

Notes:

$\mathbf{r_p}$	risk premium as defined by Botterud, Bhattacharyya and Ilic (2002)

Solving for risk premium and using a time interval of one year, I get the result in equation 7.125 below. Also mentioned in the theory section, the future price will always converge towards the expected spot price when the current time approaches T. Knowing this, Botterud, Bhattacharayya and Ilic decided (for reasons unexplained) to approximate the spot price $\mathrm{S_T}$ with the price of futures with only one week maturity and maturity at T. Because of the short time interval of only one week, this approximation was very good. They also justify this approximation by plotting the weekly future price toghether with the spot price, which leads to almost no difference, and so, they perform the regression on the form given in equation 7.126.

equation 7.125 $\mathrm{r_p = \ln \frac{E_t(S_T)}{F_{t,T}}}$

equation 7.126 $\mathrm{\widehat{r_p} = \ln \frac{F_{T-1/52,T}}{F_{t,T}} = \ln F_{T-1/52,T} - \ln F_{t,T}}$

The regression analysis was used to calculate the risk premium for futures with 1 week, 4 weeks, 26 weeks and one year. Their results are summarized in Table 18 below.

	1 week	**4 weeks**	**26 weeks**	**52 weeks**
Sample size	326	323	300	275
Mean	-0.015	-0.035	-0.085	-0.183
St. Dev	0.101	0.187	0.432	0.399
p-value, z-test[23]	0.9968	0.9996	0.9997	1.0000
CFI[24]**, up-limit**	-0.001	-0.008	-0.020	-0.122

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[23] The z-test for $\mathrm{r_p} < 0$, given $\mathrm{r_p} = 0$ as null a hypothesis. The p-value refers the percent of the outcomes in which $\mathrm{r_p} < 0$.

[24] Confidence Interval

CFI, low limit	-0.030	-0.062	-0.149	-0.245

Table 18 Risk premium in the electricity market

The results suggest negative risk premiums for all the time intervals, shown in the "Mean"-row. In Figure 7-1 the annualized mean values for risk premium from Table 18 are shown in the curve. The diagram uses Δt along the x-axis and risk premium y-axis. The annualized values where simply created by dividing the mean values by their time interval $\left(r_p/\Delta t\right)$.

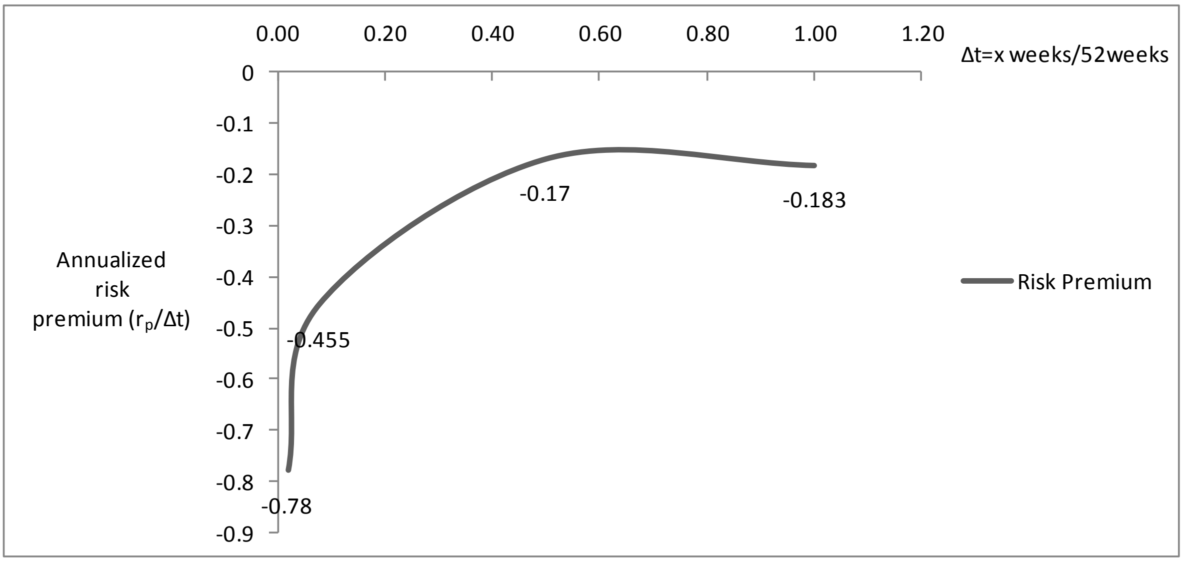

Figure 7-1 Annualized risk premium

In a simple world the graph should have been flat with a constant annualized value. This is clearly not the case, but the curve seems to converge towards a value around -0.17 to -0.18. Among several weaknesses of this test is that values for periods longer than 1 year don't exist, the test is old, and there is difficult to estimate any time varying risk premium.

7.4 An External Analysis

7.4.1 Threat of Entry and Rivalry

There are here no ways to differentiate the product. Moreover, no barriers to entry can be gained from proprietary technology or know-how in an industry that buys all its production capacity from the same manufacturers. Also, 307 TWh was traded on Nord Pool Spot in 2010 (Nord Pool Spot AS 2011), while Naturkraft only has a capacity of about 3.5 TWh per year (Naturkraft AS n.d.), so a single firm entering wouldn't change the price, and any cooperation to manipulate prices is unlikely. All of these elements increase the threat from both new entries and rivalry.

Out of 17 applicants for permission to establish gas- and coal fired power plants in Norway, 16 were granted permission while one application is still pending (The Norwegian Water Resources and Energy Administration 2011). In addition to this slow industry growth makes me conclude that the threat from both new entries and rivalry is high (International Energy Agency 2010).

7.4.2 The Threat of Suppliers and Buyers

One the one supply side the threat level is high; there are few suppliers of gas in Norway, Statoil extracts and sells most of it, which gives them some negotiation power, although it is a homogenous product. The suppliers can also threaten with forward vertical integration, which Naturkraft is a result of. Finally, single firms are considered small customers for the suppliers and the threat from entry was considered as high.

Regarding threat from suppliers, the threat level was considered low. Buyers include almost everyone in Scandinavia, and the product is traded Nord Pool Spot AS. The only threat from buyers is related to buyers integrating backwards, which some of them will do, if the sector starts earning large profits, but this threat is already reflected in the threat of entry. I'll therefore conclude that the threat of buyers is low.

7.4.3 Total Level of Environmental Threats

Weighing the threats up against each other I conclude that the environmental level of threat is high. This means that large profits in power generation quickly will be competed away by new entrants and rivalry. Profits beyond the cost of capital should not occur often or over longer periods of time.