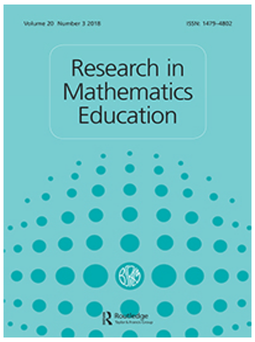

Research in Mathematics Education

ISSN: 1479-4802 (Print) 1754-0178 (Online) Journal homepage: https://www.tandfonline.com/loi/rrme20

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To cite this article: Tuula Koljonen, Andreas Ryve & Kirsti Hemmi (2018) Analysing the nature of potentially constructed mathematics classrooms in Finnish teacher guides – the case of Finland, Research in Mathematics Education, 20:3, 295-311, DOI: 10.1080/14794802.2018.1542338

To link to this article: https://doi.org/10.1080/14794802.2018.1542338

Published online: 13 Dec 2018.

RESEARCH IN MATHEMATICS EDUCATION
2018, VOL. 20, NO. 3, 295–311
https://doi.org/10.1080/14794802.2018.1542338

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Analysing the nature of potentially constructed mathematics classrooms in Finnish teacher guides – the case of Finland

Tuula Koljonen[a], Andreas Ryve[a,b] and Kirsti Hemmi[c]

[a]Department of School of Education, Culture and Communication, Malardalen University, Västerås, Sweden; [b]Faculty of Education, Østfold University College, Halden, Norway; [c]Mathematics and science education, Åbo Akademi University, Vasa, Finland

ABSTRACT

Given that curriculum materials serve as cultural artefacts, this study addresses the need for more research on curriculum materials in different contexts. Most studies concerning curriculum materials have been conducted in US and, therefore little is known about the nature of materials in other cultural-educational contexts. The aim of this paper is to identify the underlying cultural norms of potentially constructed classrooms, by analysing recurrent activities in the most commonly used Finnish teacher guides at primary-school level. We identified three norms embedded in them: (1) creating opportunities for learning through a variety of activities and communication; (2) keeping the class gathered around a specific mathematical topic; and (3) concurrent active involvement of teachers and students. The results add to knowledge about both teacher guides and the Finnish educational context. Moreover, it adds to the growing body of methodologies, as our analytical approach is novel in the context of textual analysis.

ARTICLE HISTORY
Received 28 April 2016
Accepted 24 October 2018

KEYWORDS
Cultural norms; Finland; mathematics teacher guides; potentially constructed classroom; text analysis

Introduction

Curriculum materials (CMs), such as commercially produced textbooks and teacher guides, are widely acknowledged teacher tools in mathematics education. Research suggests the importance of CMs in the teaching of mathematics, concluding that mathematics textbooks have a profound influence on the learning opportunities presented to students as they largely shape what, and how, mathematics is to be taught and learned (e.g. Ball & Cohen, 1996; Charalambous, Delaney, Hsu, & Mesa, 2010; Fan, Zhu, & Miao, 2013; Jablonka & Johansson, 2010; Stein & Kim, 2009; Stein, Remillard, & Smith, 2007; Johansson, 2006). It is also argued that CMs present ideas about teaching and learning mathematics that reflect cultural values (e.g. Haggarty & Pepin, 2002; Pepin, Gueudet, & Trouche, 2013). An extensive amount of research has focused on textbooks, whereas teacher guides (TGs) have received less attention. This is reasonable, since in many countries teachers plan and conduct instruction that is close to the students' textbook (Stein et al., 2007). However, in Finland, over 90 per cent of primary-school teachers

CONTACT Tuula Koljonen tuula.koljonen@mdh.se

use and appreciate TGs for planning and conducting instruction (Joutsenlahti & Vainionpää, 2010; Pehkonen, Ahtee, & Lavonen, 2007). The teachers are also satisfied with TGs, and consider them as very helpful in differentiating their teaching (e.g. Heinonen, 2005). Interestingly, in Finland, since the 1980s TGs have been produced by collaborative teams of teachers, teacher educators and other experts (Niemi, Toom, & Kallioniemi, 2012). In addition, Finnish curriculum programmes and TGs have started being used in other countries such as Sweden and Italy. Despite this, few studies have focused on Finnish mathematics textbooks (e.g. Törnroos, 2005; Wikman & Horsley, 2012) or TGs (Hemmi, Krzywacki, & Liljekvist, 2018; Koljonen, 2014). Therefore, by examining Finnish TGs we will contribute to international research on TGs as well as add to our knowledge about the features in the Finnish mathematics education tradition.

Our approach rests on the assumption that cultural educational values and norms are embedded in the text of TGs (Hemmi et al., 2018; Neuman, Hemmi, Ryve, & Wiberg, 2014; Pepin et al., 2013). Moreover, our earlier studies show that the Finnish TGs are very similar to each other in structure and content (Hemmi et al., 2018). We have also found the same recurrent activities in the TGs, as emphasised by Finnish teacher educators as important for effective mathematics teaching (Hemmi & Ryve, 2015). Therefore, in this study we assume that (1) the cultural norms of Finnish teaching affect how CMs are designed, and (2) in turn, by studying CMs we may capture essential features and values of the potentially constructed mathematics classroom as reflected in the material. By potentially constructed mathematics classrooms, we refer to implicit and explicit ways that TGs construct mathematics classrooms. The relation between these and real and factual classrooms is complex and is not easy to predict. Yet, we view TGs as a potential intermediary between the intentions in the intended curriculum and the level of teachers and classroom activities in the implemented curriculum (e.g. Johansson, 2006; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

This article presents an analysis of the recurrent activities observed in nine Finnish TGs[1] (Grades 1–6), and our research question is:

> What kind of potentially constructed mathematics classroom do the recurrent activities of Finnish teacher guides mediate?

Thus, the article will contribute to the existing knowledge base on curriculum materials, by providing the field with a broader and a more varied image about a cultural educational context other than the US, more precisely about the Finnish TGs in general and, in particular, the Finnish educational context. More significant, this study contributes to the international field by developing a method to capture the underlying cultural norms of potentially constructed mathematics classrooms through curriculum studies.

Text analysis of curriculum materials

Fan et al. (2013) showed that the analysis of a single textbook or textbook series most often focuses on either mathematical content or topic, or the role of textbooks, and with a focus on similarities and differences within and between countries, whilst only few studies focus on cultures and values. Exceptions to this are studies by Haggarty and Pepin (2002) and Pepin et al. (2013). Haggarty and Pepin examined how the topic of angles in textbooks at the lower secondary level was structured, and what was included in the books, in three educational contexts: England, France and Germany. They identified differences between the textbooks and traced the differences to the educational traditions in each country. Pepin et al. (2013) compared mathematics curriculum documents, commonly used textbooks, and teaching practices with respect to educational traditions in France and Norway. In line with Haggarty and Pepin they found a similar relationship, related to cultural and historical traditions. Further, the Swedish Research Council recently mapped the research on CMs in mathematics education, showing that 66 per cent of this research relates exclusively to the US context whereas only 11 per cent relates exclusively to other context (Vetenskapsrådet, 2015).

Several researchers have focused on the way in which a text, in general, positions the reader. For instance, Herbel-Eisenmann (2007) studied how a unit in a standard-based middle school mathematics textbook in the US positioned the students by using a discourse analytic framework to investigate the language construction as an expression of the textbook's voice. She found that the language positioned the textbook as the mathematical authority and simultaneously positioned the student as a doer of the scripted instruction represented by the textbook.

The way in which text positions the reader, whereby the teacher is clearly the assumed reader, has lately attracted some attention within TG research. Remillard (2012) discusses this in terms of the notions *speaking to* (i.e. TGs communicate the central ideas in the curriculum and provide for flexible customisation) and *talking through* (i.e. TGs focus on what the teacher should do). Further, Remillard proposes *forms of address,* as an appropriate analytical lens, as a means through which the TG addresses the teacher. Forms of address comprise five categories: (1) *structure*, referring to how the material is organised and what it contains; (2) *look*, referring to the purely visual appearance of the material; (3) *voice*, referring to how the authors communicate with teachers; (4) *medium*, referring to means of communication, e.g. radio, newspaper or TG; and (5) *genre*, referring to what kind of literary text it is, e.g. fiction or pedagogical. In Koljonen (2014) we analysed the TGs as a *whole* and accounted for all five categories, which helped us categorise the Finnish TGs based on what kind of support they offer Finnish teachers. The analysis showed that the TGs offer rich and varied resources for teachers in their everyday work in designing and enacting mathematics teaching and learning in practice. Meanwhile, in Hemmi et al. (2018) we analysed sample lessons of topics representing different mathematical areas and central themes in Finnish TGs. Besides a homogeneous structure and content, the analysis also showed that the TGs communicate their content both as *speaking to* teachers and as *talking through* them.

Another area of interest within text analysis is found in the cross-national or cross-cultural studies, which have gained recognition and attention through large-scale international comparisons. Remillard, Van Steenbrugge, and Bergqvist (2016) conducted a cross-cultural analysis of TGs' suggested lessons in three distinct school systems: the US, Flanders in Belgium, and Sweden. They used an analytical tool based on educative aspects (material that includes support for student' learning as well as for teacher learning and teaching; cf., e.g. Ball & Cohen, 1996; Davis & Krajcik, 2005) concerning what kind of guidance the TGs offered teachers, by investigating the authors' written communication. Their findings reflected strong similarities between the materials within a region, which in turn reflected the cultural traditions and educational priorities in each system. As in the previous study, when examining textbooks from three countries (Cyprus, Ireland, and

Taiwan) Charalambous et al. (2010) found more differences in the textbooks across the countries rather than within them, and suggested that textbooks should thus be regarded as one possible factor contributing to the quality of instruction and thereby as having an effect on students' learning opportunities.

The theoretical foundation of studying cultural norms

The idea of norms or cultural norms becomes an essential concept when studying the potentially constructed classroom in TGs. We understand cultural norms as the regularities of the practice and the social interaction established by a group regarding what is perceived as acceptable or desirable. These norms are then the shared rules regarding the expectations of behaviour, rather than actual behaviour. And it is the values of a culture that shape norms, and involve a taken-as-shared idea of what constitutes an appropriate and desirable mathematics classroom (cf. Hiebert et al., 2003). So, CMs as a cultural system-specific artefact (Pepin et al., 2013) both create and maintain these cultural values, and thus legitimize and reflect the different cultural educational values of countries (cf. Haggarty & Pepin, 2002). In other words, CMs reflect the specific character of the teaching and learning activities that are potentially realised in classrooms. CMs also represent the link between the intended and implemented curriculum (Valverde et al., 2002) and, as such, reflect the whole system of ideas, beliefs and norms.

Inspired by Clarke et al. (2007), we use the notions of form and function to reveal the embedded cultural norms within the recurrent activities in the TGs of three dominating mathematics textbook series in Finland as further described below.

Methodology

Metodological approach

In Koljonen (2014), a pattern of recurrent activities was revealed. Altogether, six recurrent activities were found within each lesson suggestion: (1) teacher-led classroom instruction; (2) mental calculation; (3) problem-solving; (4) games and playing activities; (5) homework; and (6) students working in the textbook. All these activities were clearly illustrated on the lesson page spreads of the TGs with distinct headings, and were located in the same place within a textbook series. These activities were not included in the students' textbook. In this study, we focus on the first five activities; we do not include students' work in the textbook in this study, as the student-textbook relationship is beyond the method of document analysis.

The examined guides covered almost 87 per cent of the Finnish market in 2008 (Joutsenlahti & Vainionpää, 2010). Laskutaito[2] (Numeracy, Nu) has been the most popular CM in Finland since the 1990s (Joutsenlahti & Vainionpää, 2010); meanwhile, Tuhattaituri (Jack-of-all-trades, JT) recently gained popularity and is now one of the two most commonly used textbook series together with Nu, the two covering slightly below 83 per cent of the market. We also examine Uusi Matikkamatka (New Math Journey, NMJ), as we wanted to add a newer product to our analysis in order to be able to predict possible trends and analyse the stability of the cultural norms across time. We examined three TGs from each series – Grades 1, 3 and 6 – which follow the current Finnish national curricular standards (FNBE, 2004). For Grades 1 and 6 we selected the autumn versions, and for Grade 3 the spring version; this meant that there were two years (4 semesters) between the analysed guides. This allowed us to narrow the number of guides included, and simultaneously be able to grasp the picture throughout the whole primary-school level. Hence, we analysed the recurrent activities in the three most commonly used Finnish textbook series from autumn 2012 to spring 2014, and revisited them during the second half of 2015 and again in December 2017 while preparing for this analysis. Thus, the units of analysis correspond to the headings on the TGs' lesson pages. We analysed one to two headings per recurrent activity and chapter, randomly chosen, within each textbook series. In total, 15–21 activities per heading within each series were analysed.

Method of analysis

The purpose of the analysis was to access the underlying structures and norms of the constructed Finnish classroom. Following Clarke et al. (2007), we analysed the recurrent activities to characterise the didactical aspects of the TGs' constructed classroom. Clarke et al. used these notions as a framework for analysing the observable and explicit events in video-recorded classroom lessons. Accordingly, the events – i.e. the recognisable and familiar activities – were the components of a lesson. These components had the capacity to identify recurrent pedagogical elements in a teacher's classroom and their regularity in the sequencing of these elements. Hence, we interpret the recurrent activity types in the analysed TGs as the potential events presented to teachers for the purpose of being used in teaching. In doing so, we had to operationalise form and function in a slightly different way than Clarke and colleagues did in order to be able to use this analytical tool on data supplied by texts. The didactic questions *what*, *how* and *why* are related to the form and function of teaching, as they highlight the content, implementation and expected outcomes of the education for students' learning. The activities were analysed based on how the text presented its didactic aspects: visual features, social participation, intention, action, expected outcome, and inferred meaning (see Table 1 below). In this way, we were able to capture the didactic aspects of the recurring activities.

In some cases, form and function were explicitly expressed or motivated by the authors, but in other cases they were implicit. We have taken into account the context in which the activities were presented when analysing and interpreting them. The texts – meaning the sentences, expressions, and their meaning on both the information pages and the chapter pages – were therefore relevant when analysing the recurrent activities on the lesson pages. First we analysed each of the randomly chosen recurrent activities in terms of both their form and function, through the implicit and explicit statements. Especially important at this stage was meaning-making, as well as noting whether there was a uniform form but also whether they were aligned with the lesson goals. Then, each individually analysed activity within an activity type was joined into a merged summary of that specific activity type. This procedure resulted in five merged summaries, one for each recurrent activity type: teacher-led classroom instruction; mental calculation; problem-solving; games and playing activities; and homework.

Table 2 shows one of these merged summaries: Games and Playing Activities. Especially important here was to incorporate all the key features that emerged and their equivalents within each type of activity. Finally, in all five merged summaries we searched for commonalities between the different recurrent activities and noted three equivalent key features: (ii), (iii) and (v) (see column of inferred meaning in Table 2).

Table 1. The analytical tool of form and function.

Main category	Subcategory	Description of the category
FORM	A. Visual features	Description of how the recurrent activities in the TGs are visualised/actualized or what they exhibit. How and where are the recurrent activities presented? Are they located in the same place each time? How is the activity designed?
	B. Social participants	Description of how the actors are involved and the manner in which they participate. Which students are involved in the recurrent activities and how? What do the teachers do and how are they involved?
FUNCTION	C. Intention	Description of why the recurrent activity might occur or how it should be implemented. The text could include words such as input, purpose, aim, or objectives to refer this subcategory.
	D. Action	Description of how the recurrent activity might occur or how it should be implemented.
	E. Expected outcome	Description of the expected result of the recurrent activity and what it might/should lead to. What should the content and layout of the activity lead to? That is, how are they related to students' learning? The text could indicate the outcome/result, product, or effect of the activity.
	F. Inferred meaning	Description of how the recurrent activity is relevant to students' learning. Draw conclusions about the function by reasoning. These could be some kind of derived senses or meanings, which could be either conditional, concrete, or indirect.

Table 2. The merged summary of Games and Playing Activities (Abbreviated GPs).

Form		Function			
Visual features	Social participants	Intention	Action	Outcome	Inferred meaning
A heading conveying "Games and Playing Activities" (GPs) is sited on suggested lesson pages and located in the same place within a series (see Figure 1). Each lesson includes 1–8 GPs related to the lesson goals. *"The TG offers ideas for activating exercises, games and playing activities, etc., for instance, that suit working in pairs and groups."* (Koivisto, Salonen, Sintonen, Uus-Leponiemi, & Ilmavirta, 2007, p. 5) → (ii) (iii) (v)	*" … the child needs help to be actively engaged in a range of experience-based learning experiences. Just counting is not enough. TG therefore has many suggestions for different oral and activating exercises."* (Lilli et al., 2011, p. 4) → (ii) (iii) (v) *"the teacher [who] select activities for the class"* (e.g. Asikainen et al., 2012, p. 4) → (v) *"The teacher is to guide pupils to practice tasks according to each pupil's skills and needs."* (Rikala et al., 2008, p. 5) → (ii) (v) *"Working together [in pairs and groups] is fruitful, especially … for example in routine training with the help of games."* (Salonen et al., 2004, p. 4) → (ii) (iii) (v)	The lesson goal is *"to amplify the understanding of the concept of fractions and the fact that a fractional number in which the numerator and denominator are the same always means a whole."* (Lilli, Putkonen, Sinnemäki, & Mikkonen, 2012, p. 66) and corresponding to Figure 4. *"Tips provide activating ideas for teaching math. The suggestions of GP activities will motivate pupils to study math."* (Asikainen, Nyrhinen, Rokka, & Vehmas, 2012, p. 4) → (i) *"Exercises, games and play train the objectives of the lesson or previously learned moments."* (Lilli, Putkonen, Sinnemäki, & Mikkonen, 2011, p. 7) → (iv) *"Activating lessons reflect either the lessons learned or represents the basics of the future, learning math, in the form of games and play."* (Asikainen et al., 2012, p. 4) → (iv)	*"It is worthwhile to bravely try different working methods during math lessons."* (Koivisto, Salonen, Sintonen, Uus-Leponiemi, & Ilmavirta, 2007, p. 5) → (ii) (v) 1 out of 6 offered GPs: *"**1. Memory Stick Cards** (Appendix 5b). The cards in the attachment are cut off. Cards are spread on the desk. Pupils can collect pairs (for example, 10 mm and 1 cm), either alone or in pairs, when the cards are facing up or, as in the memory game, facing down."* (Asikainen, Nyrhinen, Rokka, & Vehmas, 2012, p. 208) → (ii) (v) GPs are quite explicitly described, as shown in the sequence play below. *"**A counting game**. Pupils sit in a circle. There might be two circles instead in a larger class. The numbers 1–10 are counted going around the circle. After reaching 10, the counting starts again from the beginning, number 1. The pupil who says 10 sits down. The game continues until only one pupil is standing. The game is to be repeated by counting the numbers backwards from 10 to 1. Then the pupil who says 1 sits down."* (Rikala, Sintonen, & Uus-Leponiemi, 2008, p. 17). → (ii) (iii) (v)	*"The focus areas in teaching mathematics besides computational skills are also, e.g. collaborative skills, communication skills, games, and playing activities"* (Lilli, Ranta, Putkonen, & Sinnemäki, 2011, p. 7) → (ii) (iii) *" … through different senses and by being actively engaged … the child will experience learning."* (Salonen, Sintonen, & Uus-Leponiemi, 2004, p. 4) → (ii) (v) *"Pupils can participate in songs by clapping and playing, making it easier to learn and memorize things."* (Haapaniemi, Mörsky, Tikkanen, Vehmas, & Voima, 2012, p. 5) → (i) (v)	Our analysis shows at least five key features: (i) motivating and stimulating students' learning **(ii) offering variety in teaching** **(iii) offering all students the opportunity to communicate mathematics** (iv) creating a common ground for further teaching **(v) activating teachers and students** → The bold key features are the same features that emerged within all the five activity types.

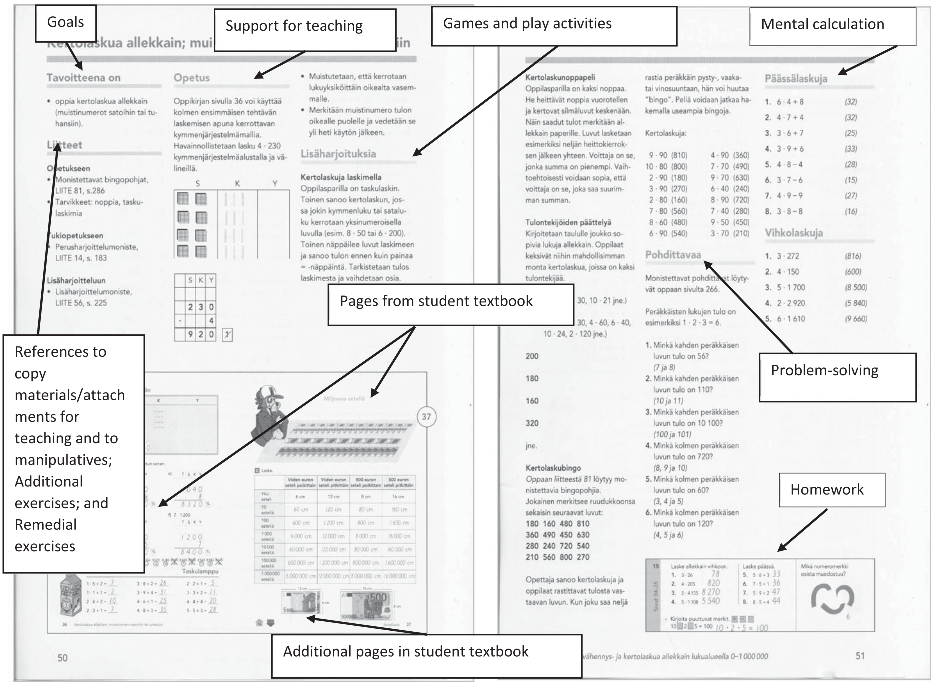


Figure 1. An example of a suggestion for a mathematics lesson in a teacher guide (Lilli et al., 2012, pp. 50–51, reproduced with permission).

In sum, form and function were used to analyse the recurrent activities explicitly and implicitly in the text (i.e. sentences, expressions and their meaning), to access the underlying structures and norms of the constructed Finnish classroom. The analysis of form and function was initially conducted in the original language of the TGs (Finnish) by the first author and later verified by the third author as they both master the language; thereafter, they discussed the findings and their interpretations with the second author of the article, seeking agreement (cf. internal reliability).

Results and analysis

We found three intertwined and overlapping features within all activities: (1) creating opportunities for learning through a variety of activities and communication; (2) keeping the class gathered around a specific mathematical topic; and (3) concurrent active involvement of teachers and students. These key features represent the joint pattern of the embedded underlying cultural norms, mediating the image of the constructed mathematics classroom. These features further mediate the picture of advocating whole-class teaching around a specific goal or topic. Below, we will show the three parts of the cultural norm with some illustrative examples.[3]

Norm 1: creating opportunities for learning through a variety of activities and communication

The first identified norm concerns how opportunities for learning are created through various activities and communication. In Figure 1, we present a picture of a typical lesson page spread from one of the TGs (Lilli et al., 2012, pp. 50–51), displaying how the recurrent activities are visualised. TGs offer students variety through these recurrent activities, both between and within lessons. The explicit learning goals are realised by the *form*, as either the learning content, core content, or learning goals. The goals are visible at the top left in Figure 1. These goals display the focus of what students should have the opportunity to learn during the lessons, and thus represent the embedded intentions. These intentions constitute the *function*, offering the rationale for implementing these activities. All five recurrent activities – teacher-led instruction; mental calculation; problem-solving; games and playing activities; and homework – are related to these goals. These offer a variety in opportunities to learn, both between the activities and sometimes also within them.

Besides different opportunities to learn in accordance with students' cognitive levels, offered through task differentiation, ready-made differentiation is offered through both form and function. For example, the form of problem-solving tasks offers two cognitive levels: (1) less challenging, found in the guides; and (2) more challenging, found in the students' textbooks. The form of the less challenging problem-solving tasks often entails two problems on slightly different cognitive levels.

The purpose (*function*) is that all students should be able to work with at least one of the two problems, optimising all students' opportunity to learn. The function offers opportunities for both rote learning (see example in the upper right corner of Figure 2, below) and applied mathematics. The excerpt below illustrates an example of a less challenging problem, demanding applied mathematics (Asikainen et al., 2012, p. 164):

(1) The shortest side of the rectangle is 2 cm. Calculate the longer side of the rectangle when its perimeter is 26 cm. (**11 cm**)
(2) The shortest side of the triangle is 2 cm shorter than the longest side of the triangle. The longest side is 1 cm longer than the third side of the triangle. Calculate the three sides of the triangle when its perimeter is 21 cm. (**6 cm, 7 cm and 8 cm**)

The *function* of Task 1 requires students to apply their mathematical knowledge of geometrical properties. It demands a higher cognitive level than calculating area by simply applying a formula, since the students have to interpret the relationships between the

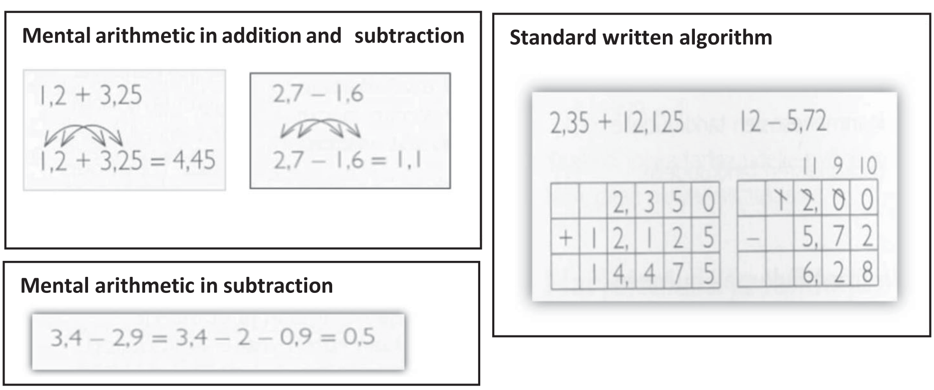


Figure 2. Strategies for computing decimal numbers in addition and subtraction (p. 78, reproduced with permission).

sides by calling into play their conceptual knowledge about both perimeter and parallelity. The *function* of Task 2 is cognitively demanding on an even, but higher level, requiring an interpretation of how the sides of a triangle are related to each other in relation to the triangle's perimeter. More opportunities to learn are offered in the students' textbooks, where the more challenging problems (2) are found, requiring more student effort.

Opportunities to learn through oral communication are highlighted in the guides. In Figure 2, we show an excerpt from Koivisto et al. (2007), where oral communication in the whole class is central:

This excerpt displays some of the eight suggestions[4] on the lesson pages, which comprise a part of the support for teachers in and for their instruction. The *form* presents how students might compute decimal numbers in addition and subtraction using both mental arithmetic strategies (split method and jump method) as well as standard written algorithms. This implies the *function* of the activity by stressing that the visible alternatives are to be discussed during this teacher-led classroom instruction in a whole-class setting. This indicates communication as a means to learn mathematics, whereby students should share their thinking but also contribute to and discuss alternative approaches. The importance of having more than one strategy is stressed in the text, especially when guided to subtract from a smaller integer (cf. jump method). Moreover, the intention (*function*) is also to prepare teachers for students' different ways of thinking when elaborating with different decimal numbers and, by extension, to prepare teachers to orchestrate the instruction as a whole-class lesson session (related to Norm 3). Communication as an opportunity to learn is realised foremost as a means or tool for learning mathematics, and only occasionally as a goal in the students' textbook.

Norm 2: keeping the class gathered around a specific mathematical topic

The second identified norm concerns keeping the lessons around a specific mathematical topic and within the same content area. This is realised through mathematical tasks for instruction and homework.

Mathematical tasks for instruction offer teachers guidance for teaching, with great emphasis on communication as a means for learning (cf. Norm 1). Further, keeping students within the same topic and content area enables communication around a specific topic. Below we show an example of a mathematical task for instruction in which oral interaction with the students is required for the whole-class activity. This has the consequence that all students participate in communicating about the same mathematical topic, even though they can be on different cognitive levels:

> We write numbers 4 and 3 on the board. Perform addition and subtraction of these two numbers. We write on the board: $4 + 3 = 7$ and $4 - 3 = 1$. Now we introduce and use the concepts of sum and difference. (Lilli et al., 2011, p. 90)

The teacher's and the students' social participation and active involvement are recognised in the *form*, in which the use of personal pronouns like "we" includes both the teacher's and the students' interaction during this activity. Regarding the *function*, teacher-led instruction is clearly related to the lesson goals but only implicitly to the students' learning. In this case the students should learn the concepts of sum and difference, but should also understand the inverse operations of addition and subtraction.

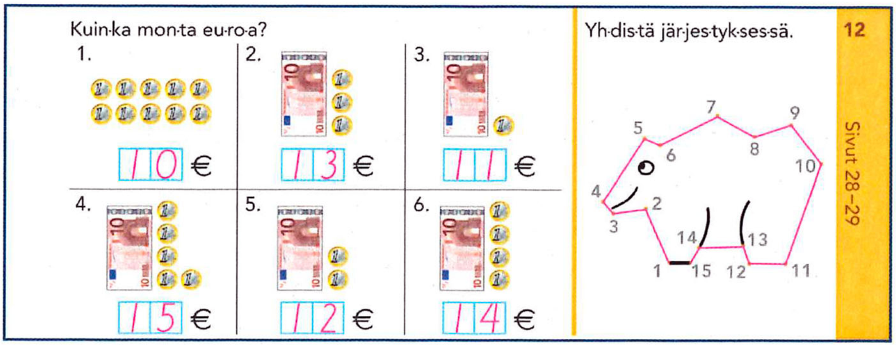


Figure 3. A typical homework assignment. (Lilli et al., 2011, p. 41, reproduced with permission).

Homework helps keep the students gathered around a specific mathematical topic. To illustrate a typical homework assignment, we select one from Lilli et al., (2011) (Figure 3):

The *form* of a homework assignment is typically a number of small tasks, focusing on the central aspects of the previous lessons and always connected to the lesson goals. The lesson goal here was to practice quantities from 10 to 15. Homework is presented on a suitable cognitive level, allowing most students to manage it without help. While homework is explicitly related to a lesson and its specific learning goals, it will also help students keep up with the next lesson, which will be a continuation of the last one. However, the various series stress the specific *function* of homework differently. The NMJ series emphasises that homework is connected to the objectives of the lessons and, thus, is related to learning. Others, such as the Nu series, emphasise cognitive learning, whereby repetition is a way to acquire knowledge. This indicates that the overall *function* of homework is for the students to practice elements related to the goals, so they will be better prepared for new lessons and elements. We therefore regard homework as an extension of learning.

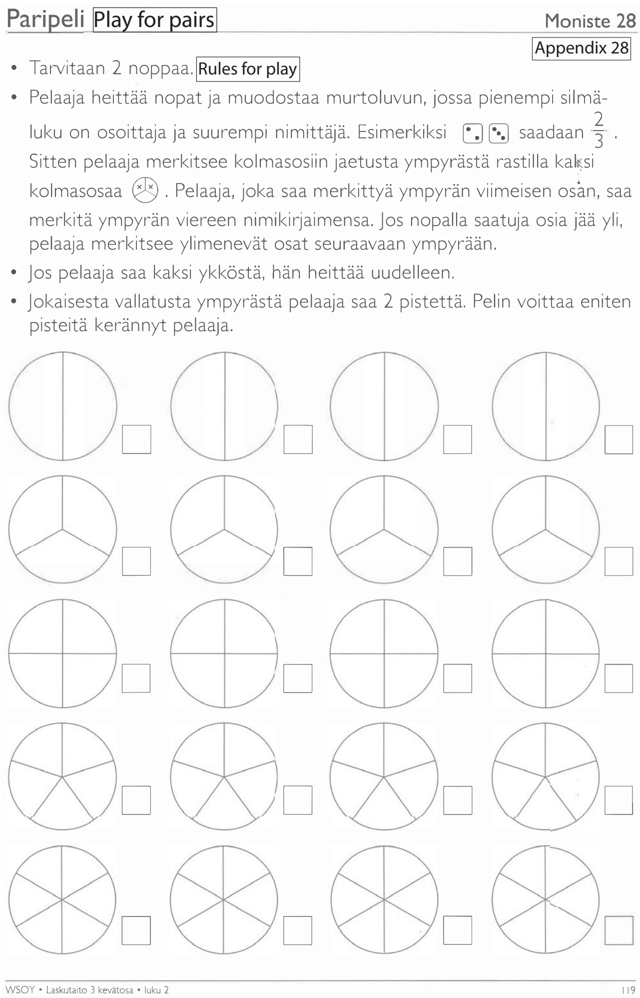
Norm 3: concurrent active involvement of teachers and students

The third identified norm concerns teachers' and students' concurrent active involvement during lessons. How the guides activate teachers and students is exemplified through two recurrent activities: mental calculation; and games and playing activities.

Mental calculation is realised through small mathematical tasks, often no more than three to six per lesson. The example below, presented in Asikainen et al. (2012, p. 142), serves as a typical example of what is called a mental calculation in the Finnish guides and is meant as an activity for all students:

(1) 3 • 40 cm (**120 cm**)
(2) You have a 135 cm long wood plank and **40 cm** is sawed off from it. How much of the wood plank do you have left? (**95 cm**)
(3) 4 • 10 + 3 • 9 (**67**)

As seen in the *form* of this mental calculation, it comprises two routine tasks meant to enhance students' speed and precision through practicing procedural skills; and another, referred to as a smaller textual word problem, in which students practice applied



Paripeli Play for pairs

Moniste 28

Appendix 28

- Tarvitaan 2 noppaa. Rules for play
- Pelaaja heittää nopat ja muodostaa murtoluvun, jossa pienempi silmäluku on osoittaja ja suurempi nimittäjä. Esimerkiksi saadaan $\frac{2}{3}$. Sitten pelaaja merkitsee kolmasosiin jaetusta ympyrästä rastilla kaksi kolmasosaa. Pelaaja, joka saa merkittyä ympyrän viimeisen osan, saa merkitä ympyrän viereen nimikirjaimensa. Jos nopalla saatuja osia jää yli, pelaaja merkitsee ylimenevät osat seuraavaan ympyrään.
- Jos pelaaja saa kaksi ykköstä, hän heittää uudelleen.
- Jokaisesta vallatusta ympyrästä pelaaja saa 2 pistettä. Pelin voittaa eniten pisteitä kerännyt pelaaja.

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Figure 4. A game about fractions played in pairs. (Salonen et al., 2004, p. 119, reproduced with permission).

mathematics by connecting between mathematics and everyday contexts. It is specified at the beginning, on the introductory pages, that teachers should "choose and read out loud the mental calculation tasks. It is preferable to write the calculations on the whiteboard … " (Lilli et al., 2011, p. 25). Even though these tasks are mainly performed individually by the students, this shows that teachers have a role in orchestrating the activity, which is carried out at the whiteboard and further discussed in a whole-class setting. At the *function* level, mental calculation is task-differentiated (cf. the problem-solving example, Norm 1) and therefore meant for all students, yet is "related either to the objectives of the current lesson or to the repetition of the previously learned topics" (Asikainen et al., 2012, p. 4).

Games and playing activities also activate both teachers and students. For example, the fraction game (Figure 4), which is one of six suggestions for this specific lesson, is to be played in pairs and connected to the mathematical topic of the lesson. The *form* is visualised on the lesson page as follows: "The game is played in pairs. One pair needs Handout

28 and two dice. The information needed is in the appendix" (Salonen et al., 2004, p. 67). This game is presented as a follow-up activity after a whole-class introduction and is displayed at the centre of Figure 1, where four different games and playing activities with different group working methods are offered: two pair activities; one individual activity; and one whole-class activity. The explicit *function* of the game is to create common ground and strengthen the understanding of fractions as well as the concepts of numerator and denominator. However, the functions differ both within and between the various guides as well as the different activities. For instance, Rikala et al. (2008) stresses that "Working together is fruitful, for example through games, as they practice both problem-solving as well as routine tasks" (p. 4), whereas the NMJ series highlights games and playing activities as motivating students' mathematical learning.

While the functions can differ, teachers are still always active since they present the activities to the students, as the activities are only found in the guides and not in the students' textbooks. For example, teachers are active when they decide to include a certain game, and they then have to copy, hand out and explain it to the students, or read out loud the specific tasks (cf. mental calculation, above). Teaching requires active teachers who balance between the offered activities, whereas students' active participation is mainly stimulated through the different recurrent activities with their inherent working methods as well as the embedded task differentiation (see Norm 1). Thus, the recurrent activities' embedded differentiation offers students the opportunity to participate on a cognitive level that is appropriate for them.

To sum up the results, our analysis shows that the recurrent activities in the TGs convey three shared norms through which a potentially constructed classroom can be understood: (1) creating opportunities for learning through a variety of activities connected to communication; (2) keeping the class gathered around a specific mathematical topic; and (3) concurrent active involvement of teachers and students. These specific educational norms embedded within TGs can be traced to the country specific educational traditions (cf. Hemmi & Ryve, 2015) and they provide the international field with knowledge about what values the Finnish CMs in general are based on. By looking closely at the revealed cultural norms, we can identify some overarching assumptions about the Finnish classroom. The key features, found within all the analysed activities, represent the underlying structures and cultural norms of the TGs, requiring teachers to lead and to keep the students gathered around the foregrounded mathematical ideas being taught. It is very clear that offering a variety of activities and task differentiation constitutes an important factor in and for teaching, since it is assumed that students learn and acquire knowledge differently. Given the differences among students, the range of assigned tasks offers a relatively large dissemination. The students are thus kept within the same mathematical topic and content area with the support of, e.g. the mathematical tasks for instruction as well as the communication as a means. Communication led by the teacher and between students is also highlighted as important for students' development of conceptual understanding. Problem-solving, games and playing activities are meant to be teacher-led in whole-class instruction, with teachers and students participating concurrently. In sum, the results of our analysis of recurrent activities provide evidence of an image of the potentially constructed Finnish classroom as promoting whole-class teaching.

Concluding discussion

This investigation aimed to identify what kind of potentially constructed mathematics classroom is mediated by the recurrent activities of Finnish teacher guides (TGs) at primary-school level. In this study, TGs are regarded as cultural specific artefacts used by teachers when designing their teaching, whereby TGs reflect the cultural educational values of a specific context (cf. Haggarty & Pepin, 2002; Pepin et al., 2013). The Finnish TGs are quite homogeneously constructed indicating an existence of possible cultural script (Hemmi et al., 2018; Koljonen, 2014). Although we know that Finnish teachers utilise the TGs to a great extent when planning their teaching practice (Joutsenlahti & Vainionpää, 2010) we cannot state anything about actual classrooms based on this study. Hence, the results show only norms of potentially constructed mathematics classrooms.

As such, the results contribute to two strands of research. Firstly, the knowledge of Finnish traditions and educational priorities as reflected by the analysis of the embedded norms, contributes to what is regarded as important in the Finnish educational context (Remillard et al., 2016). Secondly, the results are also interesting since the focus on potentially constructed classrooms adds to international research on how to conceptualise and analyse teacher guides. That is, within international research on curriculum materials and teacher guides researchers have been focusing on aspects such as mathematical content (Haggarty & Pepin, 2002), how students are positioned (Herbel-Eisenmann, 2007) and how teachers are positioned in relation to the text (Remillard, 2012) to mention but a few while our focus on potentially constructed classrooms is under-examined.

Addressing the methodology of a systematic examination of a set of recurrent activities by form and function, we conclude that this might be a productive approach for mapping the underlying cultural norms. We regard this as our most significant as well as our operative contribution to the international field as we have developed and used a novel analytical approach in order to capture the underlying cultural norms in texts. While most methodological approaches involving classrooms' cultural norms entail collecting video data on teaching practices (e.g. Clarke et al., 2007; Stigler & Hiebert, 1999) or interviewing teacher educators (e.g. Hemmi & Ryve, 2015) as well as studying feedback discussions between student teachers and mentors (e.g. Corey, Peterson, Lewis, & Bukarau, 2010), we focused on the TGs as a tool for analysing the underlying norms of classroom practice.

TGs are produced within certain educational traditions and are shaped by national perspectives on education as well as on the specific school subjects (cf. Andrews, 2007). When investigating other countries' materials concerns taken-for-granted and hidden aspects of the materials and the teaching (cf. Hiebert et al., 2003). Yet, with no explicit evidence that teachers use TGs for planning instruction, it would be hard to draw conclusions about cultural norms solely based on TGs' texts. For example, the investigated JT series caused us trouble at first because of its minimal amount of general information and rationale. However, since the structure and recurrent activities provided a quite similar pattern, in line with the other two investigated materials, this did not affect the analysis of the cultural norms in our case. Based on our findings, we conclude that the methodology of a systematic examination of a set of dominating TGs, by form and function might be a productive approach for mapping the underlying cultural norms of potentially constructed classrooms. Further research is needed if we want to say something about the relation between the findings from the analyses of the TGs and the actual mathematics classrooms.

Notes

1. In Finland, comprehensive school is from Grades 1 to 9, and comprises primary school (Grades 1–6) and lower secondary school (Grades 7–9). Children start school, beginning with Grade 1, the year they turn seven.
2. As the textbook series, to some degree, reflect the textbooks' underlying assumptions and ideas through their distinctive titles, the first author of this article translated the Finnish titles into English.
3. All the excerpts from the teacher guides are translated from Finnish into English by the first author of this article.
4. The commas in the numbers of this excerpt are the Finnish convention for denoting a decimal, equivalent to the English convention of using a full stop.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Andrews, P. (2007). Negotiating meaning in cross-national studies of mathematics teaching: kissing frogs to find princes. *Comparative Education*, *43*(4), 489–509. doi:10.1080/03050060701611888

Asikainen, K., Nyrhinen, K., Rokka, P., & Vehmas, P. (2012). *Tuhattaituri 3b*. Helsinki: Otava.

Asikainen, K., Nyrhinen, K., Rokka, P., & Vehmas, P. (2012). *Tuhattaituri 6a*. Helsinki: Otava.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is-or might be-the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher*, *25*(9), 6–8, 14. doi:10.3102/0013189X025009006

Charalambous, C.Y., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning*, *12*(2), 117–151. doi:10.1080/10986060903460070

Clarke, D., Mesiti, C., O'Keefe, C., Xu, L. H., Jablonka, E., Mok, I. A. C., & Shimizu, Y. (2007). Addressing the challenge of legitimate international comparisons of classroom practice. *International Journal of Educational Research*, *46*(5), 280–293. doi:10.1016/j.ijer.2007.10.009

Corey, D. L., Peterson, B. E., Lewis, B. M., & Bukarau, J. (2010). Are there any places that students use their heads? Principles of high-quality Japanese mathematics instruction. *Journal for Research in Mathematics Education*, *41*(5), 438–478.

Davis, E. A., & Krajcik, J. S. (2005). Designing educative curriculum materials to promote teacher learning. *Educational Researcher*, *34*(3), 3–14. doi:10.3102/0013189X034003003

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. *ZDM*, *45*(5), 633–646. doi:10.1007/s11858-013- 0539-x

FNBE. (2004). *National core curriculum for basic education 2004*. Helsinki: Finnish National Board of Education. http://www.oph.fi/english/curricula_and_qualifications/basic_education

Haapaniemi, S., Mörsky, S., Tikkanen, A., Vehmas, P., & Voima, J. (2012). *Tuhattaituri 1a*. Helsinki: Otava. Original work published 2006.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks in England, France and Germany: Some challenges for England. *Research in Mathematics Education*, *4*(1), 127–144. doi:10.1080/14794800008520106

Heinonen, J.-P. (2005). *Opetussuunnitelmat vai oppimateriaalit. Peruskoulun opettajien käsityksiä opetussuunnitelmien ja oppimateriaalien merkityksestä opetuksessa* [Curricula or educational materials. Elementary school teachers' conceptions of curriculum and teaching materials for educational purposes]. (*Diss.*, The University of Helsinki. 2005:257). Helsinki: DARK.

Hemmi, K., Krzywacki, H., & Liljekvist, T. (2018). Investigating Finnish teacher guides as a resource for mathematics teaching. *Scandinavian Journal of Educational Research*, *62*, 911–928. doi:10.1080/00313831.2017.1307278

Hemmi, K., & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education*, *18*(6), 501–521. doi:10.1007/s10857-014-9293-4

Herbel-Eisenmann, B. (2007). From intended curriculum to written curriculum: Examining the 'voice' of a mathematics textbook. *Journal of Research in mathematics Education*, *38*(4), 344–369. doi:10.2307/30034878

Hiebert, J., Gallimore, R., Gamier, H., Bogard Giwin, K., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study*. Washington, DC: National Center for Educational Statistics.

Jablonka, E., & Johansson, M. (2010). Using texts and tasks: Swedish studies on mathematics textbooks. In B. Sriraman, C. Bergsten, S. Goodchild, G. Palsdottir, B. Dahl, B. D. Söndergaard, & L. Haapasalo (Eds.), *The first sourcebook on Nordic research in mathematics education* (pp. 363–372). Charlotte, NC: Information Age Publishing.

Johansson, M. (2006). *Teaching mathematics with textbooks: A classroom and curricular perspective* (*Diss*., Luleå University of Technology, 2006:35). Luleå: Luleå University of Technology.

Joutsenlahti, J., & Vainionpää, J. (2010). Oppimateriaali matematiikan opetuksessa ja osaamisessa. [Learning materials in the teaching and learning of mathematics]. In E.K. Niemi & J. Metsämuuronen (Eds.), *Miten matematiikan taidot kehittyvät? Matematiikan oppimistulokset peruskoulun viidennen vuosiluokan jälkeen vuonna 2008* (pp. 137–145) [*How do pupils' mathematical skills develop? The learning outcomes at the end of the fifth grade in compulsory school for* 2008] (pp. 137–145). Helsinki: Opetushallitus [The Finnish National Board of Education].

Koivisto, M., Salonen, M., Sintonen, A-M., Uus-Leponiemi, T., & Ilmavirta, R. (2007). *Laskutaito 6, syysosa*. Helsinki: WSOY.

Koljonen, T. (2014). *Finnish teacher guides in mathematics: Resources for primary school teachers in designing teaching* (Lic., Mälardalen University, 2014:178). Västerås: Mälardalen University.

Lilli, M., Putkonen, H., Sinnemäki, J., & Mikkonen, V. (2011). *Matikkamatka 1, syksy*. Vantaa: Tammi.

Lilli, M., Ranta, P., Putkonen, H., & Sinnemäki, J. (2011). *Matikkamatka 6, syksy*. Vantaa: Tammi.

Lilli, M., Putkonen, H., Sinnemäki, J., & Mikkonen, V. (2012). *Uusi Matikkamatka 3, kevät*. Helsinki: Sanoma Pro Oy.

Neuman, J., Hemmi, K., Ryvke, A., & Wiberg, M. (2014). Mathematics textbooks' impact on classroom instruction: Examining the views of 278 Swedish teachers. In H. Sifverberg, T. Kärki, & M. S. Hannula (Eds.), *Nordic research in mathematics education – Proceedings of NORMA14* (pp. 215–225). Turku: Finnish Research Association for Subject Didactics.

Niemi, H., Toom, A., & Kallioniemi, A. (Eds.) (2012). *Miracle of education: The principles and practice of teaching and learning in Finnish schools*. Rotterdam: Sense Publishers.

Pehkonen, E., Ahtee, M., & Lavonen, J. (Eds.) (2007). *How Finns learn mathematics and science*. Rotterdam: Sense Publishers.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Re-sourcing teachers' work and interactions: A collective perspective on resources, their use and transformation. *ZDM*, *45*(7), 929–943. doi:10.1007/s11858-013-0526-2

Remillard, J. (2012). Modes of engagement: Understanding teachers' transactions with mathematics curriculum resources. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources* (pp. 105–122). Dordrecht: Springer.

Remillard, J. T., Van Steenbrugge, H., & Bergqvist, T. (2016). *A cross-cultural analysis of the voice of six teacher's guides from three cultural contexts*. Paper presented at the AERA annual meeting, Washington, DC.

Rikala, S., Sintonen, A-M., & Uus-Leponiemi, T. (2008). *Laskutaito 1, syysosa*. Helsinki: WSOY.

Salonen, M., Sintonen, A-M., & Uus-Leponiemi, T. (2004). *Laskutaito 3, kevätosa* (9th edition). Helsinki: WSOY.

Stein, M. K., & Kim, G. (2009). The role of mathematics curriculum materials in large-scale urban reform. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers* *at work: Connecting curriculum materials and classroom instruction* (pp. 37–55). New York, NY: Routledge.

Stein, M. K., Remillard, J. T., & Smith, M. S. (2007). How curriculum influence student learning. In F. K. Lester Jr (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 319–369). Charlotte, NC: Information Ace.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: The Free Press.

Törnroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation*, *31*(4), 315–327. doi:10.1016/j.stueduc.2005.11.005

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. London: Kluwer Academic Publishers.

Vetenskapsrådet. (2015). *Kartläggning av forskning om formativ bedömning, klassrumsundervisning och läromedel i matematik* [Mapping research into formative assessment, classroom teaching and mathematics curriculum programs]. Delrapport från vetenskapsrådet (SKOLFORSK-projektet). [Report from the Swedish Research Council (SKOLFORSK-project).]. Stockholm: Vetenskapsrådet. [Swedish Research Council].

Wikman, T., & Horsley, M. (2012). Down and up: Textbook research in Australia and Finland. *IARTEM e-Journal*, *5*(1), 45–53.