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Hotelling competition with multi-purchasing

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Abstract: We analyze a Hotelling model where consumers either buy one out of two goods (single-purchase) or both (multi-purchase). The firms' pricing strategies turn out to be fundamentally different if some consumers multi-purchase compared to if all single-purchase. Prices are strategic complements under single-purchase, and increase with quality. In a multi-purchase regime, in contrast, prices are strategically independent because firms then act monopolistically by pricing the incremental benefit to marginal consumers. Furthermore, prices can decrease with quality due to overlapping characteristics. Higher preference heterogeneity increases prices and profits in equilibrium with single-purchase, but decreases them with multi-purchase.

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1 Introduction

An environment where some consumers buy several varieties of a good while others buy only one seems to be a reasonable description for a wide range of products. In particular this is the case for information goods; e.g., magazines and software programs. Readers may subscribe to more than one magazine, but they rarely buy more than one copy of the same issue. Some people install both Scientific Workplace and Mathematica on their computers, while others buy only one. However, people never (knowingly) buy more than one copy of the same software. Game platforms are another example; some people buy only Playstation3 or X-Box, while others buy both. Likewise, some people prefer to have both an iPhone and a conventional (smaller) mobile handset, while most people still just buy one type. The point is that while buying several different types of an information good enables consumers to enjoy a larger set of characteristics, the same is not true if he buys several units of the same information good; see Lancaster's (1966) characteristics representation of goods (c.f. also the discussion in Gabszewicz and Wauthy, 2003).

We use competition among magazines as an illustrative example, but our results are valid for the other examples given above as long as the Hotelling model fits. The readers' choices of single-purchase (Time Magazine or Newsweek) or multi-purchase (Time Magazine and Newsweek) depend on the prices and contents offered. At first glance, one might expect that better news coverage (which could be interpreted as higher quality) at Time and Newsweek makes multi-purchase more likely. We show that the opposite could be true. The reason is that while better news coverage clearly increases the magazines' attractiveness, it also makes it less imperative for news-hungry readers to buy both magazines. The latter effect tends to reduce the prices that the magazines can charge, possibly generating a hump-shaped relationship between equilibrium prices and news coverage under multi-purchase. We thus show that if the coverage is sufficiently good, it might be a dominant strategy for each magazines to sacrifice some sales and set such high prices that no-one will buy both magazines. Only if the readers have a strong interest in reading the same kind of story in both magazines (to get a "second opinion") will higher news coverage

unambiguously increase the likelihood of multi-purchase.

The key property of the multi-purchasing equilibrium is that it is a special type of monopoly regime. Rival's quality, not the rival's price, shapes demand, and prices are strategically independent even though they are determined by the quality levels at both magazines.

The starkly different properties of the purchase regimes are underscored by their comparative static properties. If the market is covered but consumers buy a single variant, equilibrium prices and profits are increasing in preference heterogeneity. By contrast, they are decreasing in preference heterogeneity under joint purchase.

These results have implications for management decisions, insofar as a market situation with some multi-purchasing may be very poorly approximated by a traditional model of single purchases. To take into account that some consumers are multi-purchasing is fundamental for pricing strategy decisions in the same way as it is crucial to understand whether goods are substitutes or complements (see Gentzkow, 2007, who analyzes competition between print and online newspapers).

Spatial differentiation à la Hotelling (1929) is a standard tool for analyzing media economics, see e.g. Anderson and Coate (2005), Gabszewicz et al. (2004), Liu *et al.* (2004), and Peitz and Valletti (2008).[2] The present paper is novel for the way the quality is introduced in the Hotelling framework. In particular, we assume that the greater is the difference between a magazine's profile and the reader's ideal type, the smaller his utility gain from an improved content quality in the magazine; Left-wing and Right-wing presentations of a Presidential scandal have different values to different readers (depending on readers' political views, for example). If we are zooming on game platforms, with a higher quality of Playstation3, the willingness to pay for the good increases more for Playstation-lovers than for X-Box-lovers. Such asymmetric gains from quality improvements seem reasonable also for the other examples mentioned above.[3]

[2]For a debate concerning the results of Liu *et al.* (2004), see Chou and Wu (2006) and Liu *et al.* (2006). For analysis of media market competition in non-Hotelling frameworks, see for instance Godes *et al.* (2009) and Kind *et al.* (2009).

[3]To our knowledge, the only paper that uses a somewhat similar formulation is Waterman (1989). In an extension in his analysis of the tradeoff between quality and variety in a Salop-

We show that our approach for modelling quality has the important implication that the higher is the quality of a good, the higher will its price be under single-purchase. This is in sharp contrast to standard results in Hotelling models, where prices are independent of whether firms provide high-quality or low-quality goods in a symmetric equilibrium with market coverage. This novelty of the present model may also be of interest in more traditional circumstances with a single discrete choice between the goods offered. The quality formulation in the present paper is somewhat reminiscent of the Mussa and Rosen (1978) formulation of vertical differentiation insofar as some consumers have higher willingness to pay for incremental quality: the horizontal taste differences also imply that those with a higher willingness to pay for one good's quality have a lower willingness to pay for the other's.

The present paper is also related to de Palma, Leruth, and Regibeau (1999), who analyze multi-purchase in a setting with Cournot competition and network effects (see also Ambrus and Reisinger, 2006), and to Gabszewicz and Wauthy (2003). The latter extends the Mussa and Rosen (1978) framework by allowing for multi-purchasing. Two firms sell vertically differentiated goods, and consumers may buy both variants. As in the present paper, consumers do not buy two units of the same good, and the outcome depends on the incremental utility gained by consumers from buying both products. In contrast to Gabszewicz and Wauthy, we allow for quality to interact with the distance-based utility, and analyze the incentives to invest in quality.

The equilibrium properties are also quite different from those in Gabszewicz and Wauthy. While they find no pure strategy equilibrium for some parameter values, we always have a pure strategy price equilibrium. In the Appendix we provide a detailed analysis of demand and reaction functions for our context, and derive more general properties which apply to duopoly differentiated products pricing games. These results hopefully prove useful for other applications, e.g. in spatial models where kinks in demand are quite natural. We therefore give results for generalizations of our model, and then illustrate. For example, we find that local monopoly equilibrium

framework, he allows quality to interact with transportation costs. He does not focus on the features of this formulation highlighted in the present paper.

cannot coexist with competitive equilibria, and there can be at most two competitive equilibria.

The rest of the paper is organized as follows. In Section 2 we describe the basic set-up of the model, and in Sections 3 and 4 we analyze competition under single-purchase and multi-purchase, respectively, with exogenous quality levels. The incentives to make quality investments are analyzed in Section 5, while Section 6 concludes and discusses some routes for future research. Some of the proofs are relegated to the Appendix, where we also offer a conceptual discussion of demand and reaction functions when we allow for both single-purchase and multi-purchase.

2 The model

Consider a model with two magazines, $i = 0,\ 1$, which provide news of interest for the readers (e.g. on foreign affairs or the state of the economy). We normalize the universe of possible news ($\mathbb{Q}$) to 1, and denote the news coverage of magazine i as $\mathbb{Q}_i \subseteq \mathbb{Q}$. The larger is the set $\mathbb{Q}_i$, the more attractive is the magazine for readers. Letting $q_i \in [0, 1]$ denote the measure of magazine $i's$ coverage, the magazines are thus vertically differentiated if $q_1 \neq q_2$. In the software example from the introduction, q_i could in the same vain be interpreted as a measure of the functionalities offered by program i.

The magazines are located at either end of a "Hotelling line" of length equal to 1. Magazine 0 is at the far left (point 0) and magazine 1 at the far right (point 1). Consumer tastes are uniformly distributed along the line, with the idea being that the magazines are horizontally differentiated in terms of the slant or spin they give to coverage, or indeed the way they present the news or tell the story. A consumer who is located at a distance x from point 0 receives utility equal to $R - tx$ from reading magazine 0 if the magazine has uncovered all possible news (q_0=1). Here R is interpreted as a reservation price, and t is the distance disutility parameter from not getting the most preferred type of product. Following the convention in the literature, we refer to this below as the "transportation costs". More generally, with a magazine price equal to p_0, consumer x's surplus from buying magazine 0

alone is given by

$$u_0 = (R - tx)\, q_0 - p_0. \tag{1}$$

The surplus from buying magazine 1 alone is similarly given by

$$u_1 = [R - t(1 - x)]\, q_1 - p_1. \tag{2}$$

Note that with risk neutral consumers, we might interpret q_i either as a measure of magazine i's news coverage or as the probability that the magazine contains a given main news story (like a Presidential scandal).[4] The values of q_0 and q_1 are assumed to be common knowledge under both interpretations, and might for instance depend on the number of journalists employed by each magazine.

The above describes preferences if consumers buy one magazine or the other, but we are also interested in the possibility of consuming both magazines. In Section 4 we describe the utility in the case of multi-purchase, where consumers possibly enjoy greater benefit by buying both magazines.

It is worth noting at this juncture that the formulations in (1) and (2) have an interest in their own right for the study of a single discrete choice between magazines. The formulation is novel for the way the "quality" variable is introduced, as it interacts with the distance-based utility.[5] In particular, the formulation implies that a greater news coverage at magazine 0 (higher q_0) is more valuable for a left-winger than for a right-winger, other things equal. As noted in the Introduction, this is reminiscent of the Mussa-Rosen (1978) formulation of vertical differentiation.

Aggregating the individual choices generates demands, $D_0(.)$ and $D_1(.)$. We assume away marginal production costs of magazines. Let the profit function of magazine i be given by

$$\pi_i = p_i D_i - C(q_i), \quad i = 0, 1, \tag{3}$$

[4] The latter interpretation of q_i works better for subscription than for newsstand sales. The reason for this is that it could be argued that the consumer can tell from the cover or rifling through the magazine whether there is a pertinent story if the decision to buy is made at the newsstand.

[5] A more standard way would set $u_0 = Rq_0 - tx - p_0$ etc: see Ziss (1993) for example.

where $C(q_i) \geq 0$ is the cost of investing in quality, with $C'(q_i) > 0$ and $C''(q_i) > 0$. We assume that $C(q_i)$ is sufficiently convex to ensure the existence of a stable, symmetric equilibrium. We shall though for the first part of the analysis consider the sub-games induced for given q_i's, in order to elucidate the differences between the market outcomes at which each consumer buys a single magazine (single-purchase) or else some consumers buy both magazines (multi-purchase).[6]

3 Single-purchase

Assume for now that each consumer buys one and only one of the magazines (single-purchase). We restrict attention to a range of parameter values which guarantee that all consumers are served and that both magazines are operative (market coverage and market-sharing). Below, we show that there is such an equilibrium if and only if:[7]

Assumption 1: $R \geq \frac{3}{2}t$

[6] In the recent two-sided markets literature (see the survey by Armstrong (2006), and the overview by Rochet and Tirole (2006)), these cases correspond to "single-homing" and "multi-homing."

[7] For higher t values than those obeying Assumption 1 there is a continuum of constrained monopoly equilibria where the market is fully covered yet each magazine does not wish to cut price and directly compete with its rival. The reader indifferent between the two magazines is also indifferent between buying and not. For still higher t values there is unconstrained local monopoly: recall $u_0 = (R - tx)\, q_0 - p_0$ so that 0's monopoly demand is $x = \left(R - \frac{p_0}{\mathbb{p}_0}\right) \frac{1}{t}$. Its monopoly price, $Rq_0/2$, implies that equilibrium $x = \frac{R}{2t}$. Thus for $x < \frac{1}{2}$; equivalently, $R < t$, we have a local monopoly. We do not dwell on these parameter ranges in the subsequent development of the model, though they are analyzed in some detail in the Appendix. Note though that demands are piecewise linear, and the kink is the "right" direction, i.e., downward, so that these monopoly segments in demand do not cause any equilibrium existence problems in the price sub-games, whatever parameters (conditional on assuming no joint purchases, which are dealt with below). Demand functions are linear, in 2 segments, shallow in the high-price "monopoly" region, and steeper in the lower price duopoly region. The kink gives rise to a marginal revenue discontinuity which is at the heart of the multiplicity noted above, and discussed at further length in the Appendix.

Solving $u_0 = u_1$ from (1) and (2) we find the location of the consumer who is indifferent between buying magazine 0 and magazine 1. This consumer's location is given by

$$\hat{x} = \frac{tq_0 + (R-t)(q_0 - q_1) - (p_0 - p_1)}{t(q_0 + q_1)}. \tag{4}$$

Demand for magazine 0 is thus $D_0 \equiv \hat{x}$, while demand for magazine 1 is $D_1 \equiv 1 - \hat{x}$.

For given q_0 and q_1, the magazines compete in prices, and setting $\partial \pi_i / \partial p_i = 0$ generates the price reaction function for Firm i[8]

$$p_i = \frac{p_j + (R-t)(q_i - q_j) + tq_i}{2}, \quad i, j = 0, 1 \text{ and } i \neq j. \tag{5}$$

Equation (5) makes it clear that prices are strategic complements: $\partial p_i / \partial p_j > 0$. The linear reaction function has the standard fifty-cents-on-the-dollar property familiar from Hotelling models. The price-quality interaction is quite novel though, as $\partial p_i / \partial q_j = -(R-t)/2 > 0$. The higher are the transportation costs, the less will the reaction function shift down when the rival's quality improves. This is due to the way quality enters the readers' utility function. In this regard, note that the traditional way of incorporating quality in Hotelling models is to let $u_i = Rq_i - t(|x - x_i|) - p_i$ (see e.g. Ziss, 1993). With that specification, $\partial p_i / \partial q_j = -R/2$, so that the shift is independent of t.

Solving the price reaction functions (for an interior solution, $\partial \pi_0 / \partial p_0 = \partial \pi_1 / \partial p_1 = 0$) implies that the outcome of the last stage is

$$p_i^* = \frac{R(q_i - q_j) + t(q_i + 2q_j)}{3}, \quad i, j = 0, 1 \text{ and } i \neq j. \tag{6}$$

From (6) we find, as expected, that the sub-game equilibrium price satisfies $dp_i^*/dq_i > 0$, which is consistent with the property noted above that the own reaction function shifts up more than the rival's shifts back. Note also that the price charged by magazine i is increasing in the consumers' reservation price, R, if i has an expected quality which is higher than that of its rival, j.

[8] Already the symmetric equilibrium and the rationale for A1 can be seen here: under symmetry, $p = tq$. This is the heart of the result that the duopoly region does cover the market: recall $u_0 = (R - tx) q_0 - p_0$ and so at $x = 1/2$ we have $(R - t/2) q - tq$ which is therefore positive iff $A1$ holds.

The relationship between p_i and q_j is less clear-cut; the "direct effect" of better quality in magazine j is to reduce p_i (see (5)). However, since magazine prices are strategic complements, the fact that $\partial p_j/\partial q_j > 0$ tends to make p_i an increasing function of q_j. We thus find an ambiguous relationship between p_i and q_j; $\frac{dp_i^*}{dq_j} = \frac{2}{3}\left(t - \frac{1}{2}R\right) \lesseqgtr 0$. If the magazine has sufficiently high market power (i.e., the transportation costs are so high that $t > \frac{1}{2}R$, but still satisfy $A1$), magazine i will increase its price if the rival's quality goes up.

Inserting (6) into (3) and (4) we obtain the sub-game equilibrium values:

$$D_i^* = \frac{R\left(q_i - q_j\right) + t\left(q_i + 2q_j\right)}{3t\left(q_i + q_j\right)} \text{ and} \tag{7}$$

$$\pi_i^* = \frac{\left[R\left(q_i - q_j\right) + t\left(q_i + 2q_j\right)\right]^2}{9t\left(q_i + q_j\right)} - C_i(q_i), \quad i, j = 0, 1 \text{ and } i \neq j. \tag{8}$$

From (6)-(8) it follows that the magazine with the higher quality has the higher demand, price and operating profits. It can further be verified that a higher quality of magazine i always reduces its rival's output and profitability.

It is now useful to characterize the equilibrium if the quality levels of the magazines are exogenously given by a common value q^S (we use superscript S for single-purchase). In this case the equilibrium common price (see (6)) is $p^S = q^S t$ and operating profits are $\pi^S = q^S t/2$. In summary:

Proposition 1: *Single-purchase. In a symmetric equilibrium with $q_i = q^S$ $(i = 0, 1)$, the magazines' operating profits are increasing in*

a) the heterogeneity of the readers ($d\pi^S/dt > 0$), and

b) in the quality levels ($d\pi^S/dq^S > 0$).

The result that equilibrium prices are increasing in t is standard (though it does not hold under multi-purchase, as we show below).[9] The intuition is simply that higher brand preference entails more inelastic demands, more market power, and higher prices. However, the quality result in Proposition 1 is in sharp contrast

[9] An alternative interpretation of t is that it measures the degree of product differentiation between the magazines. The larger t, the more differentiation there is, and so the more inelastic is demand. This induces higher equilibrium prices for any given q^S.

to standard results in symmetric Hotelling models, where prices and profits are independent of the quality of the goods.[10] To see why $dp^S/dq^S = t > 0$, note from equations (1) and (2) that $\frac{\partial}{\partial x}\left(\frac{\partial u_0}{\partial q_0}\right) = \frac{\partial}{\partial(1-x)}\left(\frac{\partial u_1}{\partial q_1}\right) = -t < 0$. This means that the larger is the difference between a given magazine's profile and the one preferred by a reader, the smaller is his utility gain from a greater news coverage in that magazine. If both magazines invest more in quality, the willingness to pay will thus increase most for the consumers in each magazine's own turf. An increase in q^S thereby implies that each magazine can charge higher prices; magazine 0 gains higher market power over consumers to the left of $x = 1/2$, while magazine 1 gains higher market power over consumers to the right of $x = 1/2$.

As noted above, quality is usually incorporated in Hotelling models by assuming that $u_i = Rq_i - t\left(|x - x_i|\right) - p_i$, implying that $\frac{\partial}{\partial x}\left(\frac{\partial u_i}{\partial q_i}\right) = 0$. With this specification, a symmetric increase in the quality would thus not enhance the magazines' market power over any of their consumers. This is why the equilibrium price is independent of whether firms provide high-quality or low-quality goods in standard symmetric Hotelling models.

This section provides a catalogue of results for the classic case of single-purchase. While some of them are standard, the way quality has been introduced leads to several differences. However, the main usefulness of the results above is to contrast them with what happens for multi-purchase. This we turn to next.

4 Multi-purchase

We shall now open up the possibility that at least some of the consumers buy both magazines. When they do so, they need to determine the value of buying a second one. Bear in mind that they naturally prefer the coverage of the magazine closer to their own position, and so will read that first. How much they gain from reading the other magazine depends on the degree of overlap in news coverage.

[10] This is the obverse facet of the result that profits are independent of (common) marginal costs. Basically, competition determines mark-ups independently of common costs: see the discussion in Armstrong (2006) for ramifications in the context of two-sided markets.

The interpretation of overlap differs a little according to the source of the quality of the magazines. If quality refers to the probability of carrying a particular news story, then, assuming that magazines' draws are independent, we can interpret $(1-q_0)\,q_1$ as the probability that the story is covered by magazine 1 but not by magazine 0, while $q_0 q_1$ is the probability that the story is covered by both magazines. Alternatively, if $\mathbb{Q}_i$ is interpreted as the fraction of the possible universe of stories carried by magazine i, then we can interpret $q_0 q_1$ as a measure of (expected) news overlap in the two magazines and $(1-q_0)\,q_1$ as a measure of non-overlapping news. Under both interpretations we let the value of a second opinion be $1-\beta$ per overlapped story.[11] The incremental benefit from first reading q_0 stories in magazine 0 and then q_1 stories in magazine 1 is thus $(1-q_0)\,q_1 + (1-\beta)\,q_0 q_1 = (1-\beta q_0)\,q_1$.

The case $\beta = 1$ corresponds to a zero extra value of reading stories based on the same underlying information in a second magazine. In this case there would clearly be no reason to buy both magazines if $q_0 = q_1 = 1$. However, if $\beta < 1$ the consumer finds it valuable *per se* to read both the left-wing and right-wing magazine even if the papers have the same news coverage. The case $\beta = 0$ means that all stories in the second outlet are fully valued, regardless of whether they have already been read. Of course, they are still subject to the disutility of not being of the optimal "spin."

We must distinguish between the case where everyone buys both magazines, and the case where only a share of the consumers do so. However, the former is quite trivially straightforward (as will become apparent from the analysis below: it involves pricing to make the most resistant consumer indifferent to adding the magazine, a form of monopoly pricing). We therefore deal with the latter case. Figure 1 illustrates one possible market outcome, where consumers located to the left of point A only read magazine 0, those between points A and B read magazine 0 first and then magazine 1. The consumers located between B and C likewise read magazine 1 first and then magazine 0, while those to the right of C only read magazine 1.

[11] Hence β represents the value "lost" to a magazine from having a story read elsewhere first.

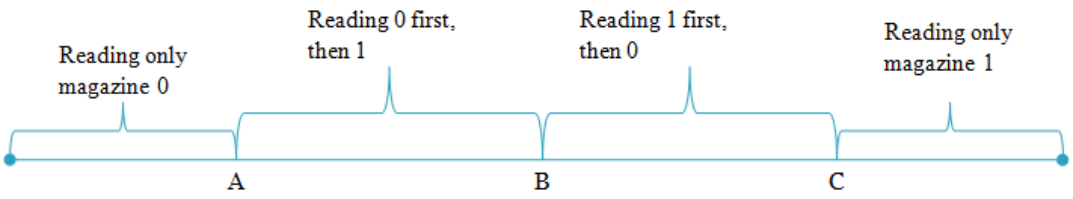

Figure 1: *Possible market outcome with multi-purchase.*

The utility of a consumer who reads magazine 0 first and then 1 equals

$$u_{01} = u_0 + \{[R - t(1-x)](1-\beta q_0) q_1 - p_1\}. \tag{9}$$

The first term on the right-hand side of (9) is the expected utility that the consumer gets from buying magazine 0. The second term is the additional utility that consumer obtains from also buying magazine 1.

Analogous to equation (9), we can write the expected utility of reading magazine 1 first and then 0 as

$$u_{10} = u_1 + \{(R - tx)(1-\beta q_1) q_0 - p_0\} \ . \tag{10}$$

With some degree of multi-purchase, demand for each magazine is by definition smaller than one ($D_i < 1$). Note that the consumer who is indifferent between reading magazine 1 first and then 0 and only reading magazine 1, is given by $u_{10} = u_1$ (location C in Figure 1). Clearly, for this consumer the price of magazine 1 is immaterial. Solving $u_{10} = u_1$ we thus find

$$x_C = \frac{1}{t}\left[R - \frac{p_0}{q_0(1-\beta q_1)}\right], \tag{11}$$

so that demand for magazine 0 depends on own price and the expected quality of the two magazines, and not on the price charged by the rival. This key property of the multi-purchase regime is not an artefact of the uniform reader distribution in the Hotelling model, but is more fundamental property. It stems from the nature of recognizing the demand as the incremental value, and that infra-marginal consumers are not indifferent between buying and not buying, nor between switching brands.[12]

[12]The property would not hold for example if the demand were specified as a "random choice" discrete utility model with i.i.d. idiosyncratic tastes, if choices were defined over all alternatives (including the joint one). However, it would seem more natural to define choices in the incremental manner done above, and then the property would hold still.

The above makes it clear that the multi-purchase equilibrium is a special type of monopoly regime. Rival quality – but not rival price – shapes demand. This property is what makes the regime particularly interesting – prices are strategically independent though they are determined by journalism quality at both papers. The strategic independence here stems directly from profit independence of rival price.[13]

Inserting (11) into equation (3) and solving $\partial\pi_0/\partial p_0 = 0$ we find $p_0 = \frac{Rq_0(1-\beta q_1)}{2}$ and $D_0 = \frac{R}{2t}$. For magazine 1 we likewise have $p_1 = \frac{Rq_1(1-\beta q_0)}{2}$ and $D_1 = \frac{R}{2t}$. Provided that $\frac{1}{2} < D_i = \frac{R}{2t} < 1$ (or $t < R < 2t$), the candidate equilibrium outcomes are thus given by:

$$p_i^* = \frac{Rq_i\left(1-\beta q_j\right)}{2}; \quad D_i^* = \frac{R}{2t}; \quad \pi_i^* = \frac{R^2 q_i\left(1-\beta q_j\right)}{4t} - C(q_i), \qquad i,j = 0,1 \text{ and } i \neq j. \tag{12}$$

The restriction that $t < R < 2t$ ensures that each magazine's output lies between one half and one; this is a necessary condition for there to be an equilibrium where some consumers (but not all) buy both magazines.[14] This clean condition is independent of the individual q_i's (subject to no firm wishing to deviate, as addressed below), since we cannot have multi-purchase of one magazine and not of the other.

The results that $dp_i^*/d\beta < 0$ and $d\pi_i^*/d\beta < 0$ are self-evident; a higher β reduces the value added by having a second source. This overlap effect is absent from single-purchase equilibria.

Under single-purchase, we found that the magazines' operating profits are strictly increasing in their expected quality levels and in the heterogeneity of the consumers. From (12) we find that the opposite may be true under multi-purchase:

Proposition 2: *Multi-purchase. In a symmetric equilibrium with* $q_i = q^M$

[13] Profit independence is sufficient but not necessary for strategic independence – consider the case of Cournot competition and exponential demands (and zero cost), where profits are not independent, but quantities are strategically independent.

[14] The outcome that a higher quality induces a higher price holds generally, while the equality of demands is a property of the uniform distribution in the Hotelling model. Suppose that the consumer density were $f(x)$. Then $\pi_0 = p_0 F(x_C)$ and $\frac{d\pi_0}{dp_0} = F(x_C) - p_0 f(x_C)\, tq_0\left(1-\beta q_0\right)$ and the candidate equilibrium price is $p_0 = \frac{F(x_C)}{f(x_C)} tq_0\left(1-\beta q_0\right)$. As long as $F(.)$ is log-concave, the RHS is decreasing in p_0 and the magazine with the higher quality again has the higher price.

$(i = 0, 1)$, *the magazines' operating profits are*

a) decreasing in the heterogeneity of the readers ($d\pi^M/dt < 0$)*, and*

b) hump-shaped functions of the expected quality levels if $\beta > 1/2$ (with $d\pi^M/dq^M > 0$ for $q^M < \frac{1}{2\beta}$ and $d\pi^M/dq^M < 0$ *for* $q^M > \frac{1}{2\beta}$).

Under single-purchase, when consumers become more heterogenous, each magazine's market power over its own consumers increases, resulting in higher prices and higher profits ($d\pi^S/dt > 0$). Under multi-purchase, on the other hand, greater consumer heterogeneity implies that each magazine will have a smaller market ($dD_i/dt < 0$) and thus lower profits ($d\pi^M/dt < 0$). The intuition for this result is the fundamental property outlined above that prices are strategically independent under multi-purchase, which in turn implies that prices are independent of t. The effect of greater consumer heterogeneity is consequently only to reduce the share of the population which is willing to pay for both magazines.

At the outset, the second part of Proposition 2 might seem even more surprising. To see the intuition for this result, note that there are two opposing effects for the magazines of an increase in q^M. The positive effect is that a higher quality level increases the consumers' willingness to pay for the magazines, as under single-purchase. The negative effect of a higher q is to make it less imperative for any of the consumers to buy both magazines, thereby tending to increase the competitive pressure between the media firms. This negative effect dominates if $q^M > \frac{1}{2\beta}$. Only if $\beta < 1/2$, so that consumers have a strong value from reading both magazines, will prices and profits be strictly increasing in q^M.

4.1 Exogenous quality levels: single-purchase vs. multi-purchase

In this sub-section we compare the multi-purchase and single-purchase outcomes from the perspectives of the media firms and the consumers, under the constraint that the magazines have the same (exogenous) quality levels. We further determine under which conditions single-purchase and multi-purchase equilibria actually exist. To limit the number of cases to consider, we assume that $\frac{3}{2}t \leq R \leq 2t$. This ensures

that there will be full market coverage under single-purchase (this requires that $\frac{3}{2}t \leq R$, c.f. Assumption 1) and that there might exist an equilibrium with multi-purchase (as shown above, a necessary condition for an outcome where some, but not all, consumers buy both goods is that $t \leq R \leq 2t$).

In the Appendix we prove the following:

Proposition 3: *Assume that* $\frac{3}{2}t \leq R \leq 2t$, *and that the expected quality levels of both magazines are equal to* q. *Compared to single-purchase, multi-purchase yields*

a) lower magazine prices ($p^M < p^S$) *and higher expected consumer surplus* ($CS^M > CS^S$) *and*

b) higher magazine profits if and only if $q < q^* \equiv \frac{R^2-2t^2}{\beta R^2}$.

Figure 2, where we have set $\beta = 1$, might be helpful to grasp the intuition for Proposition 3.[15] The left-hand side panel of the Figure shows that magazine prices are strictly increasing in q under single-purchase; a higher expected quality unambiguously allows the magazines to charge higher prices. This in turns implies that the magazines' operating profits are increasing in q under single-purchase, as shown by the right-hand side panel of the Figure. Under multi-purchase, on the other hand, magazine prices and profits are hump-shaped functions of q, as stated in Proposition 3. Note in particular that $p^M \to 0$ and $\pi^M \to 0$ as $q \to 1$. The intuition for this is that the additional benefit of buying a second magazine vanishes in this case. If magazine prices do not approach zero, readers to the left of $x = 1/2$ will thus buy only magazine 0 and those to the right of $x = 1/2$ will buy only magazine 1.[16] If $\beta < 1$, we always have $p^M > 0$ and $\pi^M > 0$. However, unless β is so small that $\frac{R^2-2t^2}{\beta R^2} > 1$, profits will necessarily be lower under multi-purchase than under single-purchase for sufficiently high values of q.

Despite the fact that magazine prices are lower under multi-purchase than under single-purchase, the second part of Proposition 3 shows that $\pi^M > \pi^S$ if q is sufficiently small ($q < q^*$). In the left-hand side panel of Figure 2 this is true if $q < 0.38$. The reason is simply that the price differences under the two regimes are then so

[15] The other parameter values in Figure 2 are $t = 1$ and $R = 1.8$.

[16] This is straightforward to see from the term in the bracket of equations (9) and (10).

small that the higher magazine sales under multi-purchase ($D^M > D^S = 1/2$) more than outweighs the lower profit margins. Note that if $\beta << 1$, we might have $q^* > 1$, in which case multi-purchase always generates the higher operating profits.

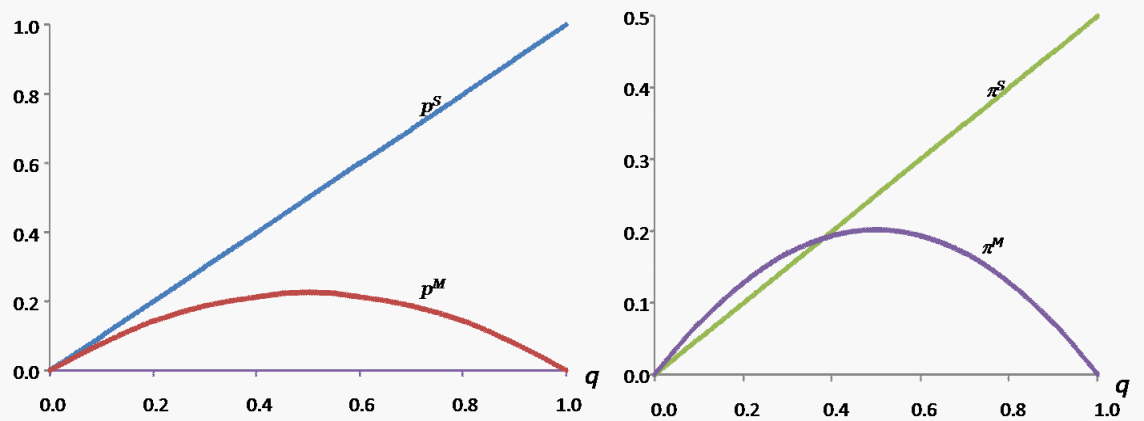

Figure 2: *Prices and profits under single-purchase and multi-purchase.*

Let us now analyze whether both single-purchase and multi-purchase constitute possible equilibria. For this purpose, let $q^{**} \equiv \left(4\sqrt{R\left(R-t\right)}+2t-3R\right)/R\beta$. It can be shown that $q^{**} > q^* \equiv \frac{R^2-2t^2}{\beta R^2}$. [17] We have (see Appendix):

Proposition 4: *Assume that* $\frac{3}{2}t \leq R \leq 2t$ and $q^{**} < 1$. *In this case there exists*

a) a unique equilibrium with multi-purchase for $q < q^*$,

b) multiple equilibria for $q \in (q^*, q^{**})$; *one with single-purchase and one with multi-purchase,*

c) a unique equilibrium with single-purchase for $q > q^{**}$.

Proposition 4 is illustrated in Figure 3, where we have set $\beta = 0.9$ (so that both p^M and π^M are strictly positive for all values of q). The existence of an equilibrium is shown by a solid curve, and non-existence of the candidate by a dotted curve.

Consistent with Proposition 3, the left-hand side panel shows that consumer surplus is always higher with multi-purchase, while the right-hand side panel shows

[17] To see that $q^{**} > q^*$, define $z \equiv \frac{R}{t}$ (with $\frac{3}{2} \leq z \leq 2$). We then have $q^{**} - q^* = \frac{2}{\beta z^2}\left(A - B\right)$, where $A \equiv 2z\sqrt{z\left(z-1\right)}$ and $B = \left(2z+1\right)\left(z-1\right)$. As both A and B are positive, it follows that $q^{**} - q^* > 0$ if $A > B$. This is true, since $A^2 - B^2 = 1 + 3z > 0$.

that magazine profits might be higher under single-purchase. However, for $q < q^*$ the media firms also prefer multi-purchase; a magazine which deviates from this equilibrium could charge a higher price and only sell to those consumers who do not buy the rival magazine, but that would excessively reduce sales. The quality of the magazines is simply too low to allow for a sufficiently high single-purchase price. This is different for $q > q^{**}$; single-purchase prices are then so high that each magazine prefers to sell only to its most "loyal" consumers, even if the rival should set the relatively low multi-purchase price and thus capture the larger share of the market. The magazines thereby unambiguously end up in the high price-high profit equilibrium. For $q \in (q^*, q^{**})$, though, it is unprofitable for either magazine to charge a high single-purchase price unless the rival does the same.

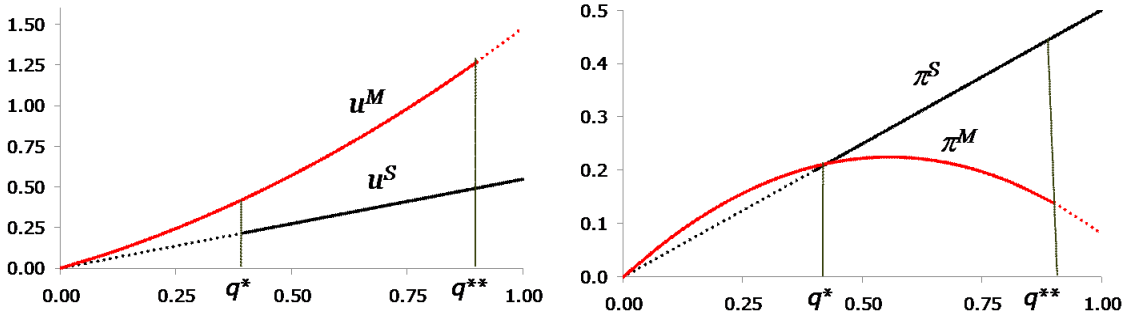

Figure 3: *Single-purchase vs. multi-purchase. Multiple equilibria.*

The discussion above provides an intuitive approach to finding the possible equilibria that may arise when we open up for multi-purchase. In the Appendix we offer a more formal and general analysis, and explain why we always have a pure strategy price equilibrium.

5 Investment incentives

In this final section we endogenize investments. We first derive the general first-order conditions for optimal investments under single-purchase and then under multi-purchase.

5.1 Investment incentives under single-purchase

The first-order condition for optimal investments in quality for magazine i under single-purchase is found by differentiating equation (8) with respect to q_i. This yields

$$\frac{\partial \pi_i^*}{\partial q_i} = p_i^* \frac{\partial D_i^*}{\partial q_i} + D_i^* \frac{\partial p_i^*}{\partial q_i} - C'(q_i) = 0, \qquad i = 0, 1, \tag{13}$$

where $\frac{\partial D_i^*}{\partial q_i} = \frac{(2R-t)q_j}{3t(q_0+q_1)^2} > 0$ and $\frac{\partial p_i^*}{\partial q_i} = \frac{R+t}{3} > 0$. By investing more in investigative journalism, the magazine thus expects to be able to increase its equilibrium output and to charge a higher price. These positive market responses are clearly increasing in the consumers' reservation price R (which puts an upper limit on the price that the magazines can charge). We further find the comparative static result:

Proposition 5: *Single-purchase. In a symmetric equilibrium with* $q_i = q^S$ $(i = 0, 1)$, *the media firms invest more in journalism the more heterogenous are the magazine readers* $(dq^S/dt > 0)$.

Proof:

Setting $q_0 = q_1 = q^S$ and inserting for (6) and (7) into (13) we find the first order condition when evaluated at a symmetric solution is:

$$\frac{4R + t}{12} = C'(q^S), \tag{14}$$

and hence $dq^S/dt = \frac{1}{12C''(q^S)} > 0$. *Q.E.D.*.

The reason why $dq^S/dt > 0$, is simply that the more heterogenous is the population of magazine readers, the higher is each magazine's market power on its own turf. An increase in t thus allows the magazines to set higher prices, making it more profitable to invest in journalism in order to increase output. Of course, in equilibrium the magazines still share the market equally, so that they actually gain no more output. But the higher q^S induced from a higher t is not a zero-sum game, since the equilibrium price, tq^S, is increasing in the common quality level.

5.2 Investment incentives under multi-purchase

To find optimal investments under multi-purchase, we use (12) to solve $\partial \pi_i / \partial q_i = 0$. By subsequently imposing symmetry, and setting $q_i = q^M$ for $i = 1, 2$, this yields the first order condition:

$$R^2 \frac{1 - \beta q^M}{4t} = C'(q) \tag{15}$$

From the comparative static properties of this expression, we can state:

Proposition 6: *Multi-purchase ($R < 2t$). In a symmetric equilibrium with $q_i = q^M$, the magazines' investments in quality are smaller*

a) the more heterogenous are the magazine consumers ($dq^M/dt < 0$) and

b) the weaker are the consumers' preferences for being informed by both magazines ($dq^M/d\beta < 0$).

Proof:

$$\frac{dq^M}{dt} = -\frac{\left(1 - \beta q^M\right) R^2}{R^2 \beta t + 4t^2 C''(q^M)} < 0 \text{ and } \frac{dq^M}{d\beta} = -\frac{q^M R^2}{R^2 \beta t + 4t C''(q^M)} < 0. \; Q.E.D.$$

Note that the relationship between the heterogeneity of the consumers and the investment incentives is the opposite in this case compared to single-purchase. The reason why $dq^M/dt < 0$, is that the larger is t, the smaller is the size of the market for each magazine (recall that $D_i = R/2t$). The gain from investing more in quality to increase the magazine price is therefore strictly decreasing in t under multi-purchase.

6 Conclusions

In this paper we analyze a Hotelling model where the consumers are not restricted to buy only one variety. When some consumers multi-purchase, this changes firms' pricing strategies. Under single-purchase, prices and operating profits are strictly increasing in quality levels. Under multi-purchase, in contrast, prices and profits can be hump-shaped functions of the quality levels. If the quality levels of both goods are sufficiently high, the additional benefit of buying the second variant might vanish.

Other things equal, competition will then press prices down towards marginal costs. However, in this case it is a dominant strategy for the firms to set such high prices that no-one will buy more than one of the varieties.

One topic for further research is to analyze multi-purchase in a two-sided market structure. Many information goods, such as online newspapers, are financed by advertising. Since these goods are offered for free in order to attract more customers (and thus increase advertising revenue), the degree of multi-purchasing (termed "multi-homing" in this context) is by its very nature high. It should also be noted that a scoop published by an online newspaper typically becomes available from rival outlets within minutes. As a consequence, the willingness to pay for a second online newspaper will presumably be small. This may help explain the observation that online newspapers rarely charge readers.

Finally, we have not addressed here the possible endogenous choice of locations, and instead we have situated **the goods** at the ends of the Hotelling line. This question is a topic for our further research: it remains to be seen whether firms will avoid the lower prices associated to multiple purchases by locating apart, or whether it is possible that they will capitalize on the non-overlapping parts of their qualities and serve the market from its mid-point (i.e., minimum differentiation).

7 Appendix

7.1 Discussion of demand and reaction functions

Finding the equilibria for this model is somewhat elaborate because of the various kinks in demand. What we find is rather particular: there are either two equilibria or one (along with a possibility of a continuum of local monopoly equilibria that preclude any other equilibrium). Gabszewicz and Wauthy (2003) find for a vertical differentiation model with the option of multi-purchase that there is also the additional possibility of no equilibrium. This is not true in our set-up, and we want to explain why. In doing so, we will establish various properties of the reaction functions which are instrumental in describing the equilibrium. The properties, and the

techniques we use, pertain to several other duopoly problems which exhibit kinks in demand, e.g., in spatial models where kinks in demand are quite natural. (e.g., Anderson, 1988, Anderson and Neven, 1991, Peitz and Valletti, 2008). We therefore give a detailed presentation about how to find the reaction functions and the implications for the nature of equilibria. We work through the details for the current example, but the techniques and short-cuts have a wider applicability.

7.1.1 Finding the reaction functions

The duopoly problem involves best-reply price choices where different price pairs correspond to different demand segments. Typically, price choices can be bounded below by constant marginal cost (here zero) and some maximum (reservation) price at which no consumer will buy. In the present case, the maximal price is Rq_i, $i = 1, 2$, which is the maximum the most dedicated consumer (the one located at the firm location) will pay. The strategy space is then a rectangle (a compact and convex set).

Next, divide this strategy space into the constituent regimes corresponding to the demand regimes (e.g., local monopoly and single-purchase, etc.) We then find the *conditional reaction functions,* which are the profit maximizing prices conditional upon being in a particular demand regime. Assuming (as we do henceforth) that each demand regime entails a strictly (-1)-concave demand, these conditional reaction functions are simply the solution to the first order condition, because profits are then quasi-concave over the demand regime.[18]

When the conditional reaction function lies within its corresponding regime in the joint price space, the conditional reaction function represents a local maximum in profit. If the conditional reaction function solution lies above the relevant regime in the price space (i.e., at a higher price), then profit is increasing in own price throughout the region. This follows from quasi-concavity of profit. Conversely, if the conditional reaction function lies below its price-space region, profits are falling throughout the regime.

[18] In the present problem, demands are piecewise linear and so conditional profits are quadratic functions.

We can now deal simply with the boundaries between regimes in the price space. First, if profits rise towards a boundary from both above and below, then the boundary is a local maximum to profit. This situation corresponds to a downward kink in demand (i.e., steeper demand for lower prices). Second, if profits rise in both directions away from the boundary, the boundary is ruled out as being part of the reaction function since it is a local minimum. This corresponds to an upward kink in the demand function (and a corresponding jump up from negative to positive marginal revenue).

The full solution is either a higher or a lower price, and this is the indication that profits will need to be evaluated to find the solution. Last, if profits rise towards a boundary and continue rising once it is passed, the solution is not on the boundary. This can occur for both types of kink noted above. Either marginal revenue each side of the kink is negative, or it is positive. In the latter case, profits rise as price falls, while profits rise as price rises in the former case.

The upshot is that the conditional reaction functions indicate whether profits are increasing, decreasing, or locally maximized within a region. This is illustrated in Figure 4 below for the case at hand. Note that profits are always increasing from the boundary towards the interior of the price space, because pricing at marginal cost yields zero profit, and pricing at the reservation price yields zero profits as long as almost all consumers do not buy at that price (as is true here and most usually).

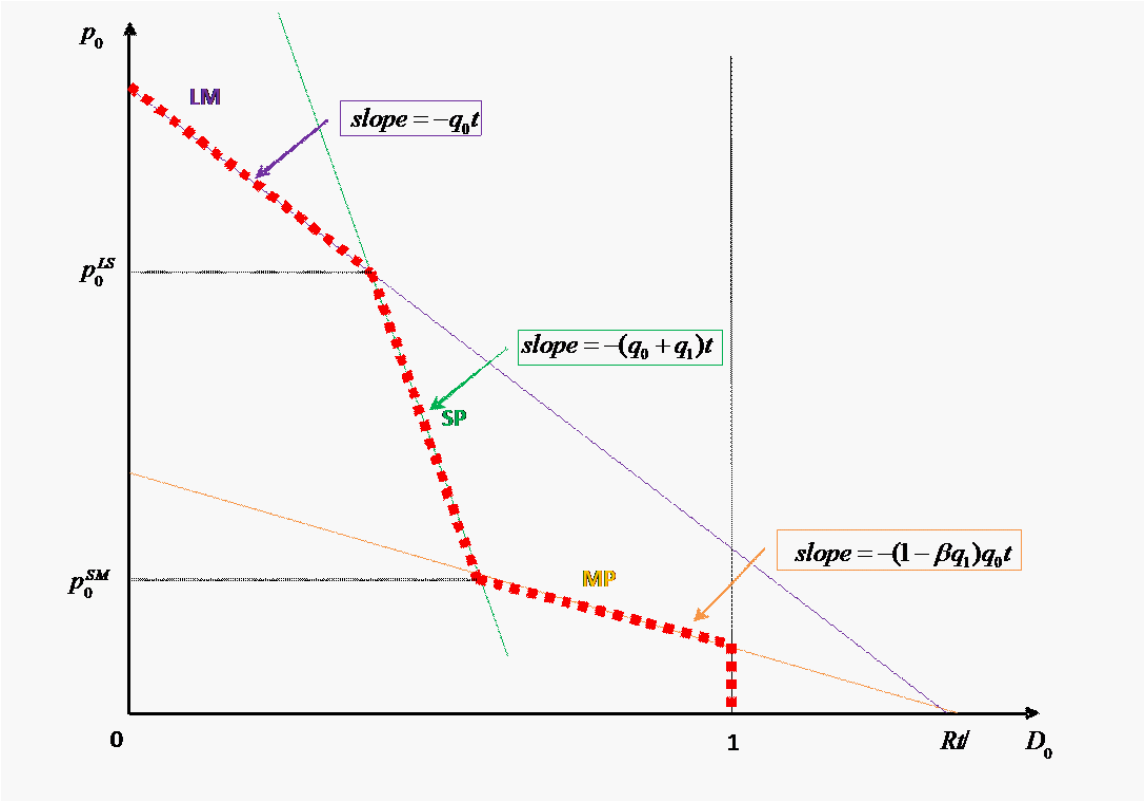

Figure 4: *Conditional reaction functions, general case.*

Local maxima are then determined by the direction of profit increases. A unique global maximum is indicated by profit increases toward it from all points below and above. Note that this may be a boundary (corresponding to the second type of demand kink noted above), and this will occur if there is no interior conditional reaction function crossed for the rival price considered. There remains the case of multiple local maxima, and these need to be directly compared (although there may still be short-cuts to choosing which is operative, as per the analysis below).

The reaction functions already enable us to give some characterizations of equilibrium. We focus here on the properties of the present game, which are nonetheless shared with several other contexts. First, if the reaction functions are continuous, there is at least one equilibrium (since they must cross). Second, if the only jumps are upward, then there always exists an equilibrium if firms are symmetric (in the present case, if $q_0 = q_1$). This is because the reaction function must then cross the 45-degree line ($p_0 = p_1$). However, notice that without symmetry, and if the reac-

tion function slopes down over some of its traverse (as it does here), it may *a priori* be possible that one reaction function goes through the discontinuity in the other, and so jeopardizes equilibrium existence. Nonetheless, in the current problem, and others of its like, this cannot happen.

The reason is as follows (and this property is shared by other models with similar properties). For high enough (joint) prices, there is a natural monopoly regime. The boundary of this regime (in the joint price space) is downward-sloping, and occurs where prices are such that the market is fully covered and the indifferent consumer at the market boundary between firms is also indifferent between buying and not. Call this the Local Monopoly (LM) boundary. Below that regime, reaction functions slope up, and any discontinuities are upward jumps.

Then there are two cases. Either the reaction functions have already crossed (at least once) before reaching the local monopoly boundary, or they have not. If they have not, then they must cross on the boundary or above it. The reason is that the reaction function follows the boundary *down* after touching it, and is then independent of the rival's price (in the interior of the local monopoly regime). There is then either a continuum of local monopoly equilibria on the boundary, or else a single one in the interior of the local monopoly region (with some consumers not buying). This means there must be an equilibrium (involving local monopoly) if there is no "competitive" equilibrium. The converse is also true: if there is a competitive equilibrium then there is no local monopoly equilibrium. To see this, suppose then that the reaction functions have already crossed. When they reach the boundary, they move down it, and then strike out independently. This means that they cannot cross again.

There is a further property of note in the present problem (also shared with other problems). First, if the reaction functions have positive slope below one in the competitive regions, and no jumps, there is at most one competitive equilibrium, and, by the results above, there is only one equilibrium. Second, if there is a single jump up, and still the reaction functions have positive slope below one in the competitive regions, there are at most two equilibria in the competitive regions.[19]

[19] With k such jumps, there can be at most $k+1$ competitive equilibria.

By the results above, there is no other equilibrium.

In summary, under the conditions given, there is always at least one equilibrium. If there is an equilibrium with each firm a strict local monopoly, then there is no other equilibrium. There are at most two competitive equilibrium, and if there is such, there can be no local monopoly equilibrium. Finally, there can be a continuum of "touching" local monopoly equilibria on the local monopoly boundary, in which case there is no other equilibrium.

7.1.2 Application to the specific example

We now analyze the firms' demand and reaction functions in more detail. There are at most 3 interior segments to the individual magazines' demand functions.

There are two "monopoly" segments to demand. For high prices (of both firms), each magazine is a local monopoly. Then inverse demand for Magazine 0 is given by setting the single magazine utility (1) to zero as

$$p_0 = q_0 \left(R - t\hat{x} \right), \tag{16}$$

where $\hat{x}$ is here and below the number of copies of Magazine 0 sold.

The other "monopoly" region is for low prices, when some readers buy both magazines. They buy 0 as long as its incremental value is positive; from (11), 0's inverse demand is

$$p_0 = q_0 \left(1 - \beta q_1 \right) \left(R - t\hat{x} \right). \tag{17}$$

Comparing to (16), (17) is lower, with flatter slope. Both demands emanate from the same horizontal intercept: when $p_0 = 0$, $\hat{x} = R/t$. We will suppose for the discussion below that this exceeds 1 (i.e., $R \geq t$), which is the case throughout the paper. This implies that demand will be capped at 1 (everyone buys) at a price above zero.

The last segment is the competitive segment imposed by the single-purchase regime. From (4),

$$p_0 = tq_1 + R \left(q_0 - q_1 \right) + p_1 - t \left(q_0 + q_1 \right) \hat{x}, \tag{18}$$

which is steeper than both of the other monopoly segments above. This segment moves out parallel as rival price p_1 rises, while the other segments stay put.

Now superimpose the 3 segments on the same diagram along with the vertical segment at 1: see Figure 4. Where they intersect is where regimes shift. The critical values are calculated below, and are given on the Figure: the demand function is shown in red dots. The inverse demand function is thus given by the flattest segment, (16), until this hits (18) at a price

$$p_0^{LS} = \left(2R - t - \frac{p_1}{q_1}\right) q_0 \tag{19}$$

It then follows the steepest segment, (18), until it hits the flatter segment, (17), at

$$p_0^{SM} = \left(2R - t - \frac{p_1}{q_1}\right) \frac{q_0\,(1 - \beta q_1)}{\beta q_0 + 1}, \tag{20}$$

which it then follows till it reaches the market constraint (unit demand). Of course, depending on the value of p_1, the single-purchase segment may dominate one or both of the others over the relevant range. The two kinks in the demand, one up and one down, generate two different types of behavior in the reaction function.

The reaction function diagram is usefully broken up into 3 regions, corresponding to the 3 segments above. From (16) and the analogous condition for Magazine 1, Local Monopoly for both transpires if 0's monopoly demand, $\left(R - \frac{p_0}{q_0}\right)\frac{1}{t}$ plus 1's demand, $\left(R - \frac{p_1}{q_1}\right)\frac{1}{t}$, sum to no more than 1. This means $\left(2R - \frac{p_0}{q_0} - \frac{p_1}{q_1}\right) \leq t$. When the inequality is weak, the market is not fully covered. On the boundary of this regime, the locus $\left(2R - \frac{p_0}{q_0} - \frac{p_1}{q_1}\right) = t$ (the Local Monopoly boundary), demands sum to 1 but there is a consumer with zero surplus. This is the region in the top right of Figure 5.

At the other extreme, there is joint purchase by some customers if the two magazines' demands sum to more than 1. (If each is 1, there is joint purchase by all readers.) From (17), Magazine 0's demand is $\left(R - \frac{p_0}{q_0(1-\beta q_1)}\right)\frac{1}{t}$, and similarly 1's demand is $\left(R - \frac{p_1}{q_1(1-\beta q_0)}\right)\frac{1}{t}$, so the condition is $\left(2R - \frac{p_0}{q_0(1-\beta q_1)} - \frac{p_1}{q_1(1-\beta q_0)}\right) > t$, which is the region in the bottom left around the origin in Figure 5. In between these regions lies the single-purchase region. Its boundaries correspond to the kinks in the demand curve.

We know from the earlier text what the conditional reaction functions must look like, conditional on being in a particular region. That is, we can find the reaction function corresponding to each demand segment, as if that linear demand constituted the actual demand, and intersect it with the region of applicability. As noted in the preceding sub-section, that is not sufficient to find the reaction function, since magazines may deviate to another conditional reaction function, or indeed to the higher boundary. This can only happen if another conditional reaction function (or boundary) lies vertically above or below.

The conditional reaction functions and the derivation of the reaction function are shown in Figure 5. Recall that a deviation from a region to its own boundary is not profitable since such point was already viable (and revealed not preferred) on the region's demand segment. Second, the lower boundary cannot constitute a most profitable deviation since the demand kink there is upward, corresponding to an upward jump in marginal revenue.

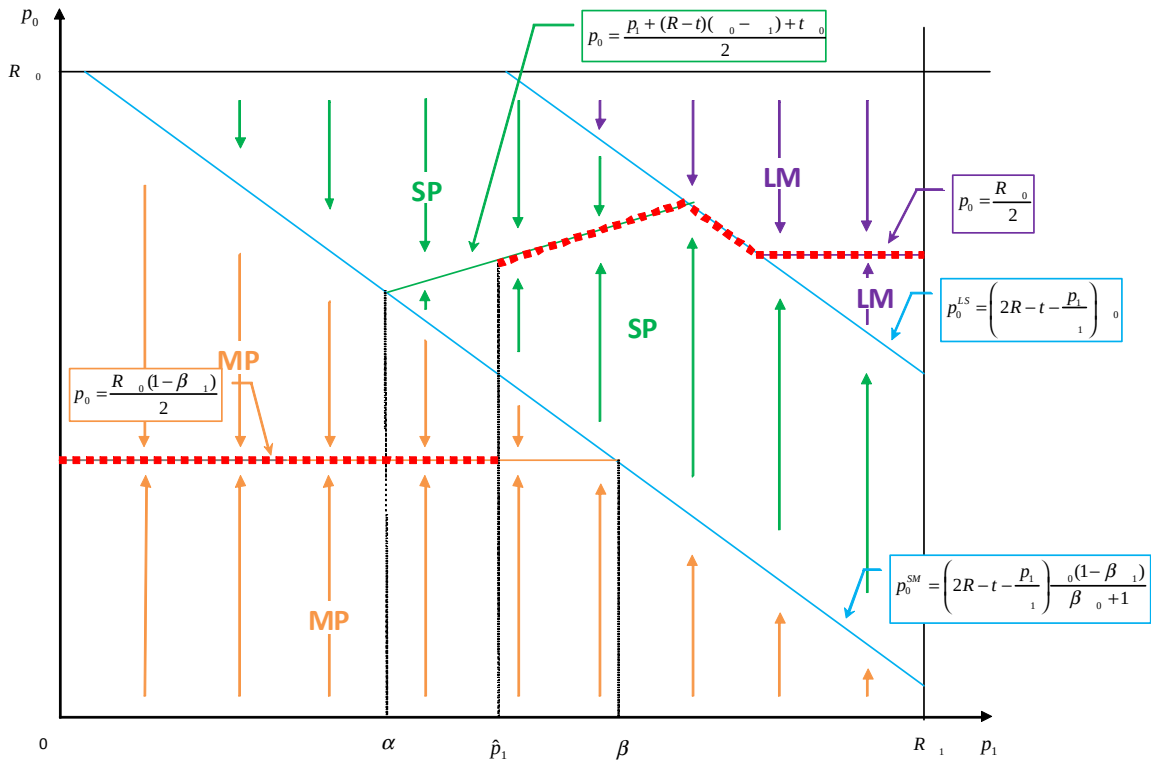

Figure 5: *Conditional reaction functions.*

The conditional reaction function for Magazine 0 in the joint purchase region is flat right across the region. The next region out is single-purchase, which comprises a stripe on top of the joint purchase region; the reaction function is upward sloping (slope 1/2) across this region. The final conditional reaction function is the flat one in the Local Monopoly region.

Any price p_1 left of the point α in Figure 5 entails a unique local maximum, which is therefore a global maximum, on the lowest conditional reaction function. For any price p_1 above the point β, there is again a unique local maximum, which is therefore global. It is on the middle conditional reaction function (the single-purchase one) until this conditional reaction function reaches the Local Monopoly boundary. The local maximum (hence the global maximum and the reaction function) then follow the Local Monopoly boundary down until it reaches the highest of the conditional reaction functions, the local monopoly one, which is then followed to the highest possible p_1.

Between the points α and β there are two conditional reaction functions operative, and so two local maxima. It is straightforward to argue that there is a jump up in the reaction function from the lower to the middle conditional reaction function at some point between α and β. Note that at point α the global maximum is on the lower conditional reaction function: the higher conditional reaction function, having just begun, represents an inflection point at α. Likewise, at point β the global maximum is on the higher conditional reaction function because the lower conditional reaction function represents an inflection point. By profit continuity along the conditional reaction functions, there is a switch between conditional reaction functions where they have equal profits. Notice that profit on the lower conditional reaction function is constant as a function of p_1. However, along the higher conditional reaction function, profit is increasing with p_1. Therefore there is a unique rival price, $\hat{p}_1$, where profits are equal, as shown in Figure 5, and the reaction function follows the single-purchase conditional reaction function beyond that.

We summarize this in Figure 6, where we illustrate the three types of competitive equilibria. In the first panel there are two equilibria (one single-purchase and one multi-purchase). In the second panel there is a unique multi-purchase equilibrium,

while in the third panel there is a unique single-purchase equilibrium.

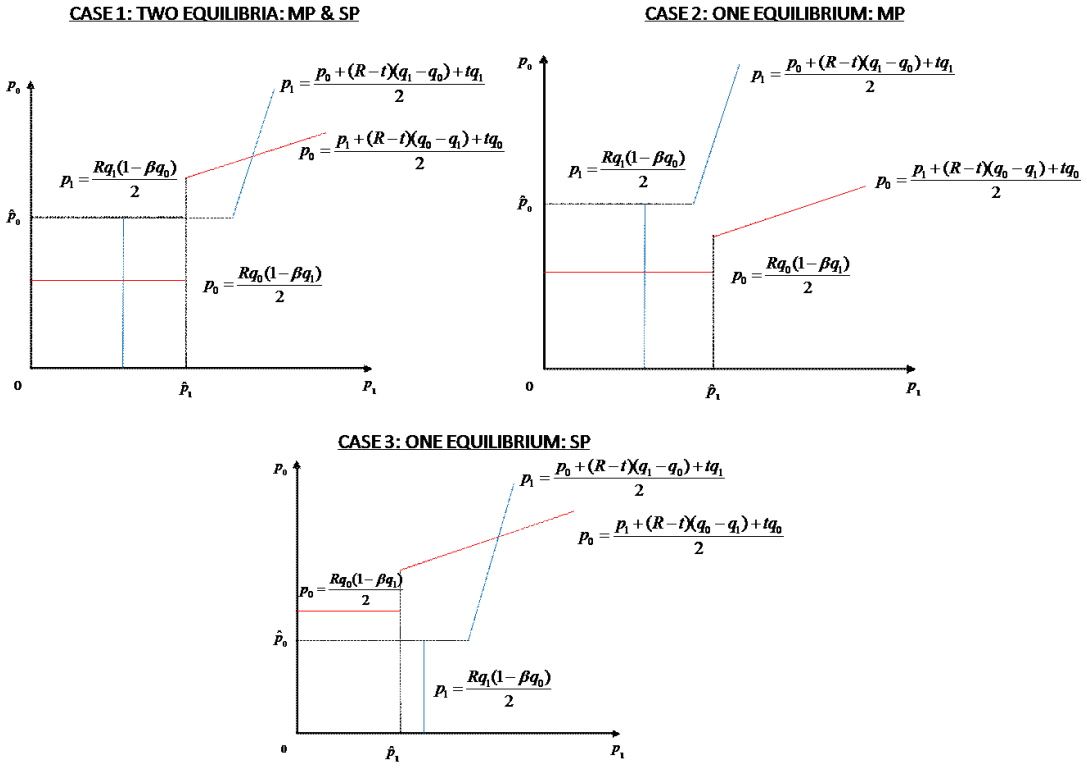

Figure 6: *Competitive equilibrium types.*

7.2 *Proof of Proposition 3:*

Inserting $q_i = q_j = q$ into (6) and (8) we find $p^S = qt$ and $\pi^S = qt/2 - C(q)$, while (12) yields $p^M = Rq(1-\beta q)/2$ and $\pi^M = \frac{R^2 q(1-\beta q)}{4t} - C(q)$. This implies that

$$p^S - p^M = \frac{2t - R\,(1-q\beta)}{2} q > 0$$

for all relevant values of β and q. We further have

$$\pi^S - \pi^M = q\frac{2t^2 - R^2\,(1-q\beta)}{4t} > 0 \text{ for } q > q^* = \frac{R^2 - 2t^2}{\beta R^2}.$$

The consumers who buy only one magazine are clearly better off under multi-purchase, since $p^S > p^M$. To show that the same is true for those who read both magazines, it suffices to show that the utility of the consumer located at $x = 1/2$

is higher if he buys both magazines under multi-purchase ($u_{ij}^M(x=1/2)$) than if he buys only one magazine under single-purchase ($u_i^S(x=1/2)$). This is true, since

$$u_{ij}^M(x=1/2) - u_i^S(x=1/2) = \frac{qt\,(1+q\beta)}{2} > 0.$$

Q.E.D.

7.3 *Proof of Proposition 4:*

If both magazines price according to single-purchase, we have $\pi^S = qt/2$. Suppose that magazine i deviates (superscript D), and sets the price that maximizes profits if he also sells to some of the consumers who buy the rival magazine. This optimal price is independent of the price charged by the rival - cf. the discussion leading to equation (11)) - such that $p_i^D = \frac{Rq(1-\beta q)}{2}$ and $\pi_i^D = \frac{R^2 q(1-\beta q)}{4t} - C(q)$. Since $\pi_i^D = \pi^M$, it follows that magazine i deviates from single-purchase if and only if $\pi^M > \pi^S$, in which case also the rival will do the same. This proves Proposition 4a).

To prove Propositions 4b) and 4c), suppose that magazine i believes that the rival sets the multi-purchase price; $p_j = Rq\,(1-\beta q)\,/2$. Will it be optimal for magazine i to charge a higher price, and accept that he will not sell to any of the readers who buys magazine j? The location of the reader who is indifferent between the two magazines is then given by $u_0 = u_1$. Inserting for $p_j = Rq\,(1-\beta q)\,/2$ this yields

$$D_i = \frac{2\,(qt - p_i) + qR\,(1-q\beta)}{4qt}.$$

Solving $\partial \pi_i / \partial p_i = 0$ we find

$$p_i = \frac{R\,(1-q\beta) + 2t}{4} q \text{ and } \pi_i = q\frac{(2t + R\,(1-q\beta))^2}{32t} - C(q).$$

Since

$$\pi_i > \pi^M \text{ for } q > q^{**} = \frac{4\sqrt{R\,(R-t)} + 2t - 3R}{R\beta},$$

it is thus optimal for firm i to deviate from multi-purchase and sell only to those who do not buy the rival magazine if and only if $q > q^{**}$. If $q > q^{**}$ it follows that both the magazines will have incentives to set single-purchase prices. However, for this to be an equilibrium, it must also be true that the readers will actually not buy

both papers at these prices. To check out that this holds, we insert for $p_i = p_j = p^S$ into equations (1) and (9) for $x = 1/2$ to find:

$$u_i^{p=p^S} = \frac{2R-3t}{2}q$$
$$u_{ij}^{p=p^S} = \frac{2(2R-3t)-q\beta(2R-t)}{2}q.$$

If $u_i^{p=p^S} > u_{ij}^{p=p^S}$ the reader located at $x = 1/2$ will only buy one of the magazines at $p = p^S$. This requires that $q > q^{***} \equiv \frac{2R-3t}{\beta(2R-t)}$ (such that the single-purchase prices are sufficiently high). Calculating the difference between q^{**} and q^{***} we obtain

$$q^{**} - q^{***} = 2\frac{2\sqrt{R(R-t)}(2R-t)-(R-t)(4R-t)}{R\beta(2R-t)}. \tag{21}$$

The denominator in (21) is always positive. It can further be shown that the numerator is positive if $R^2t(R-t)(R(8R-5t)+t^2) > 0$, which is always the case for $t < R < 2t$. The readers will consequently not buy both magazines if $p = p^S$ and $q > q^{**}$. *Q.E.D.*

8 References

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