DYNAMIC CAPITAL STRUCTURE WITH CALLABLE DEBT AND DEBT RENEGOTIATIONS

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Abstract. We consider a dynamic model of the capital structure of a firm with callable debt that takes into account that equity holders and debt holders have a common interest in restructuring the firm's capital structure in order to avoid bankruptcy costs. Far away from the bankruptcy threat the equity holders use the call feature of the debt to replace the existing debt in order to increase the tax advantage to debt. When the bankruptcy threat is imminent, the equity holders propose a restructuring of the existing debt in order to avoid bankruptcy. This proposal makes both debt holders and equity holders better off and re-optimize the firm's capital structure. Both the lower and upper restructuring boundaries are derived endogenously by the equity holders' incentive compatibility constraints. Our way of renegotiating the debt when the bankruptcy treat is imminent is different from the way the coupons of the debt is renegotiated in the the strategic debt service models of, e.g., Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997). In our model the entire debt (principal as well as all future coupon rates) is restructured. It is not just the current coupon payment which is fine tuned.

An important part of the debt renegotiation is to derive endogenously the value of debt and equity if the debt restructuring proposal is rejected since this determines the relative bargaining power between the two parties. However, since these values are off the equilibrium path, they have to be derived by an iterative procedure.

Our model offers a rational explanation for violations of the absolute priority rule. In equilibrium the debt holders do accept a restructuring proposal from the equity holders which leaves some value to the equity holders even though the debt holders do not get their full principal back. The reason why the debt holders do accept such a proposal is that the alternative if they reject the equity holders' proposal is not necessarily an immediate liquidation of the firm. In most cases the equity holders would continue to pay the existing coupons until the conditions become even worse before eventually withholding the coupons and de facto forcing the firm into bankruptcy. Since the value of the debt in this alternative situation is lower than the value the debt holders get if they accept the equity holders' proposal, they are willing to accept the proposal even though the equity holders also get a piece of the pie.

We also find that the firm's objective function is fairly flat over a large area so the capital structure of the firm can vary a lot without any significant costs or losses to the firm's stake holders.

We investigate how firm value, equity value, debt value, par coupon rates, leverage, and yield spreads change in a static comparative analysis. Our results show that optimal leverage is inversely related to both growth options and earnings risk.

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1. Introduction

Most capital structure models ignore the fact that equity holders and debt holders have a common interest in restructuring the firm's capital structure in order to avoid bankruptcy costs when the bankruptcy threat becomes imminent. The empirical evidence concerning firms in financial distress show that most often debt holders and equity holders come to an agreement of how to restructure the firm's capital structure in such a way that the firm can continue operation either voluntarily before the firm enters chapter 11 or as part of the chapter 11 process (Weiss 1990, Gilson, John, and Lang 1990, Morse and Shaw 1988). In this paper we present a model which incorporates this type of debt renegotiations into a dynamic capital structure model. Basically, we extend the dynamic capital structure model of Goldstein, Ju, and Leland (2001) to include debt renegotiations at the lower boundary. In our model the capital structure of the firm is re-optimized whenever a lower boundary is hit, and the existing debt and equity holders negotiate how to split the firm value between them. Hence, in our model the firm is in fact never liquidated as opposed to the Goldstein-Ju-Leland model where the firm is liquidated definitively the first time the lower boundary is hit. We find that by introducing debt renegotiations the tax advantage to debt is significantly increased and that, in equilibrium, the debt holders rationally accept deviations from the absolute priority rule. In addition, we find that the optimal leverage is inversely related to both the the growth of the firm's earnings and its risk.

Our model falls in the category of structural credit risk models, where all relevant data of the firm are common knowledge to all investors. In this type of model there are no asymmetric information issues or agency problems. The interior optimal solution for the capital structure comes from counterbalancing tax advantages to debt with debt restructuring costs and costs of financial distress. We work with a very simple capital structure consisting of equity and a single class of callable perpetual debt.

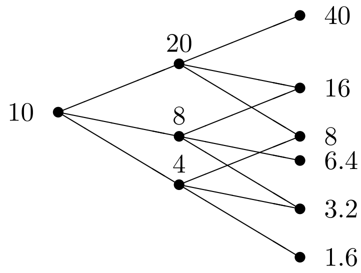
The model is set up as a dynamic capital structure model with earnings before interest and tax payments (EBIT) as the only governing state variable. When EBIT hits an upper boundary, the capital structure of the firm is re-optimized. This is implemented by calling the current outstanding debt and issuing new debt with higher principal and coupon. When EBIT hits a lower boundary, the capital structure of the firm is also re-optimized. This is implemented by canceling all existing debt and equity in the firm and issuing new debt and equity so that the optimal capital structure is reestablished. The old debt and equity holders negotiate how to split the proceeds from issuing the new debt and equity between them. Both the lower and upper boundaries are determined by the equity holders' incentive constraints. Hence, it is common knowledge that the equity holders are the ones who determine when to call the debt and when to renegotiate the debt. Since the call feature of the debt is explicitly stated in the debt contract, the debt holders have no legal right to refuse to get the principal of the debt back (plus possibly a call premium) and forgo all future coupon payments (at the upper boundary). However, during the debt renegotiation phase (at the lower boundary) the debt holders have the right to reject any restructuring proposals from the equity holders. If the restructuring proposal from the equity holders is rejected by the debt holders, the equity holders face two alternatives: (i) they can continue to service the debt by paying the original coupons and possibly make a new restructuring proposal later on or (ii) they can withhold the coupons, which forces the debt holders to declare the firm bankrupt. Hence, it makes no sense for the equity holders to make a debt restructuring proposal unless they know that it will be accepted by the debt holders. The delicate issue here is to figure out what the optimal alternative for the equity holders would be if the debt holders reject the proposal and what the corresponding values of debt and equity are. The problem is that these values are off the equilibrium path and, therefore, not derived as part of the solution. We will, however, propose an iterative procedure that can extract these off-the-equilibrium-path debt and equity values.[1] The equity holders' restructuring proposal is constructed such that both debt and equity holders get the off-the-equilibrium-path values they would have had if the debt holders had rejected the proposal. In addition, the proceeds (excess of the sum of the off-the-equilibrium-path values of debt and equity) from issuing new debt and equity are split between the existing debt and equity holders with a fraction γ to the equity holders and the rest to the debt holders.[2] Obviously, this debt restructuring proposal will never be rejected by the debt holders. Between the points in time when the capital structure of the firm is re-optimized, the capital structure of the firm will vary as the realized earnings of the firm fluctuate stochastically and thereby change the value of debt and equity. Hence, the firm can change credit rating class both down and up in the time period from one capital structure re-optimization date to the next re-optimization date. However, the capital structure (i.e., the fraction of debt and equity) that the firm re-optimize to at each re-optimization date (i.e., at the date when one of the boundaries is hit) is always the same, since the model is stationary. The specific characteristics of the firm determine the optimal initial capital structure that the firm always returns to at each re-optimization date and the boundaries that trigger the dates of when the capital structure is re-optimized. We will see how all the parameters of the firm characteristics influence these decision variables.

As it is the case with the other dynamic capital structure models we are aware of (Kane, Marcus, and McDonald 1985, Fischer, Heinkel, and Zechner 1989a, Goldstein, Ju, and Leland 2001), our model has a very useful scaling property basically saying that if we have solved our model with EBIT initiated at the level one, we can just re-scale all the results to get the solution for other initial EBIT values. Whenever one of the boundaries is hit, the firm's capital structure is re-optimized. Hence, the scaling property gives that (besides the scaling factor) everything repeats itself whenever one of the boundaries is hit. This feature of our model makes it tractable to solve the model all though numerical solution methods are necessary. Especially finding the off-the-equilibrium-path values is somewhat numerically demanding.

We are not the first to consider the problems related to the debt and equity holders' incentive to avoid paying the bankruptcy costs when the threat of bankruptcy becomes imminent. Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) were the first to look at what has later been termed strategic debt service. In these models the objective is, for each coupon paying date, to find the following two coupon payment levels: (i) the lowest coupon payment the debt holders would accept from the equity holders and still not declare the firm bankrupt when the equity holders possess all the bargaining power and also (ii) the highest coupon payment the equity holders would accept to pay to the debt holders instead of not paying any coupon at all and face a bankruptcy call when the debt holders have all the bargaining power. However, in these types of models the firm's capital structure is never re-optimized in a truly dynamic sense. A period of financial distress is likely followed by another period of financial distress. When firms enter financial distress, the outcome is often an agreement about a total restructuring of the capital structure, not just an agreement about reducing the coupon payment of this period (or the next period). We model a game of debt renegotiation where both principal and coupon of the existing debt is renegotiated as a consequence of the firm's financial distress situation in order to reestablish the firm's optimal capital structure. This effectively brings the firm out of the financial distress situation and avoids the threatening bankruptcy costs.

To illustrate this key point of our paper in a discrete setting, consider the following trinomial model of a firm's earnings process:

[1]This iterative procedure gives a solution to the bargaining game which is similar in spirit to the finite number of sequential offers refinement of the Nash equilibria in the Rubinstein bargaining game (Rubinstein 1982, Rubinstein 1987).
[2]This simple split mimics a reduced form version of the standard Nash bargaining game solution concept.



These numbers are the per period earnings from the firm. We assume that the process continues in a multiplicative way each period in such a way that if, e.g., earnings of 20 are realized after the first period, the possible earnings in the succeeding period are 40, 16, and 8 and so on. We assume that the total value of the optimally levered firm is 100 at date zero when the earnings are 10 and that the optimal initial capital structure is 50% callable debt with a per period total coupon of 5 and 50% equity. That is, the initial value of the debt is 50 and the initial value of the equity is also 50.[3] If the earnings increase to 20 after the first period it is optimal to call the outstanding debt and to issue new debt in order to re-optimize the firm's capital structure. Because of the multiplicative structure of the model the new optimal capital structure will again be a leverage of 50% and the total coupon will be 10. If the earnings decrease to 4 after the first period the equity holders optimally withhold paying the coupons. In this case the debt holders take over the firm, i.e. the earnings generating process. Again because of the multiplicative structure of the model, the firm has a value of 40 after the debt holders have re-optimized its capital structure. Hence, de-facto debt holders receive 40 minus the bankruptcy costs. If the earnings decrease to 8 after the first period we assume that the equity holders continue to pay the coupons of 5 to the debt holders. Again because of the multiplicative structure of the model the firm's assets would have had a value of 80, *if* the firm's capital structure was optimal, i.e., a total coupon of 4 and a leverage of 50%. However, because of the higher than optimal coupons, the debt would be slightly more valuable and the equity would be less valuable. To pick some round numbers we assume that the debt has a value of 45 and the equity has a value of 32. In this case the equity holders could re-optimize the capital structure of the firm by retiring 20% of the outstanding debt. However, they would have to pay the debt holders 10 in order to do so.[4] This would increase the total firm value from 77 to 80. The coupons would be reduced to 4 and the capital structure would be back to the optimal 50% leverage. However, it would not be optimal for the equity holders to retire the old debt because they would have to pay 10 to get a value increase of 8. This is the same argument as in Leland (1994, Section VIII). Hence, in this case the capital structure will remain in-optimal and await further changes in the earnings. This is the standard dynamic capital structure story (Fischer, Heinkel, and Zechner 1989a, Goldstein, Ju, and Leland 2001).

Introducing strategic debt service in our example would change the equity holders behavior in the case where the earnings are reduced to 8 after the first period. If we assume that the bankruptcy costs are 50% the equity holders could reduce the coupon payments to below 4 without risking that the debt holders would declare the firm bankrupt. If the debt holders did declare bankruptcy they would only receive 40 after the bankruptcy costs have been paid. 40 is also the value of newly issued debt with a

[3]For this illustrative example we have picked some nice round numbers for the firm value, the optimal capital structure, and the optimal coupon size of the debt. However, by selecting the four values for the riskless interest rate, the call premium of the debt, and the risk-neutral probabilities of the different future states of the model in the right way it should be clear that these numbers can be justified.

[4]Imagine that the debt has been issued as five bonds each with a coupon of one and that each bond is hold by a separate individual. If the equity holders offer anything less than 10 to any of the individual debt holders, even though the current market price is only 9, they would not sell, since the debt holders know that by holding on to their bond it would increase to a value of 10 as soon as the capital structure is re-optimized, which would happen as soon as one of the five outstanding bonds are retired.

coupon of 4 when the earnings is 8. But the debt holder's original claim is worth more than 40 since it still has a higher principal and a higher contract coupon. Hence, the debt holders would still be better off by not declaring bankruptcy if the equity holders reduced the coupon to 4. This is the strategic debt service story (Anderson and Sundaresan 1996, Mella-Barral and Perraudin 1997).

Alternatively, the capital structure of the firm could be re-optimized permanently, which we believe is much more common in the real world than period-by-period coupon squeezing. The value of the firm with a re-optimized capital structure is 80 and hence debt and equity holders do have a common interest in re-optimizing the capital structure. The question, however, is how the debt and equity holders should split this value. Inspired by the strategic debt service models one could argue that by threatening to withhold the coupons, the equity holders can get away with only paying the debt holders 40 and keeping the rest of the 80 for themselves. However, since this is a bargaining game the other extreme namely that the debt holders require all 80 in order not to declare bankruptcy is equally plausible. Hence, any split in which the equity holders receive 40γ and the debt holders receive $80 - 40\gamma$, for $\gamma \in [0, 1]$, can be justified. Had the bankruptcy costs been 40% instead of 50%, the equity holders cannot squeeze the debt holders down to less than 48. In this case only splits in which the equity holders receive 32γ and the debt holders receive $80 - 32\gamma$, for $\gamma \in [0, 1]$, are possible.[5] The point of our paper is that, if the capital structure of the firm should be permanently re-optimized, the above arguments are incomplete. The debt holders would realize that the equity holders' threat of withholding the coupons is non-credible. If the equity holders were faced with the alternative of withholding the coupons or to continue paying the coupons in full, they would choose to continue paying the coupons since this gives them a value of 32 whereas withholding the coupons would give them a value of zero. Hence, if the debt holders and the equity holders cannot come to an agreement of re-optimizing the firms capital structure, the alternative is *not* that the the firm goes bankrupt, but rather it is that the equity holders continue to pay the original coupons and that the firm survives. That is, the only thing the debt and equity holders need to bargain about is the gain in total firm value from re-optimizing the capital structure, i.e., the difference between a firm value of 80 and a firm value of 77. Hence, the possible value splits can be reduced to the ones in which the equity holders receive $32 + 3\gamma$ and the debt holders receive $45 + 3(1 - \gamma)$, for $\gamma \in [0, 1]$. Note that this is independent of the size of the bankruptcy costs. Hence, the strategic debt service argument can both under- and overestimate the fraction of the split that the debt holders must have as a minimum in a capital structure renegotiation game. This looks like a minor extension of the strategic debt service model but it is complicated by the fact that the value of the firm, if the equity holders continue to pay the existing coupons, also includes the value of renegotiating the split of a gain from a re-optimization of the capital structure at a different earnings value. This fact has been ignored in our illustrative example since we just took the firm values of 77 from the model before any renegotiations were allowed. Our full model do take that into account and at the same time endogenously, consistently, and simultaneously determine the firm value, the optimal capital structure, the optimal coupon size of the debt, and the optimal actions of the equity holders including when to propose a debt restructuring and what to propose in a continuous-time continuous-state-space setting.

In the literature there are a lot of other good explanations for why firms choose a specific capital structure. There may be many plausible reasons why it is unrealistic to assume a symmetric information structure as we have done in our model. The asymmetric information literature is full of explanations

[5] For this simple example we have ignored the fact that changing the bankruptcy costs would change the initial price of the debt and therefore the initial optimal capital structure, the optimal coupon size, etc. However, this is only for illustrative purpose.

different from our explanation. Our explanation basically says that the firm's capital structure is determined by counterbalancing the tax advantage to debt with costs of debt restructuring or costs of financial distress whereas the asymmetric information literature includes agency costs and signaling issues. Myers and Majluf (1984) argue that agency costs (cf. also Jensen and Meckling 1976) can lead to the so-called pecking order theory of different capital structure choices. This theory, however, is a consequence of giving the manager of the firm the wrong incentives. If the manager's incentives can be realigned, both equity holders and debt holders will *ex ante* be better off (Dybvig and Zender 1991). In asymmetric information models the firm's capital structure can also be used to credibly signal the value of new investment projects and other variables that the insiders of the firm have better information about than do the outside investors. An example of this type of model is Brennan and Kraus (1987). Going beyond asymmetric information models also behavioral finance models and bounded rationality models can give new explanations and insight to why firms' capital structure is determined the way it is. However, we think that it is important that we fully understand the simplest model of symmetric information where the firm's capital structure is purely driven by the trade off between the tax advantage to debt and the costs of restructuring the debt and possible bankruptcy costs. Not until we have exhausted the implications of these assumptions should we try to add another layer of complexity to the model in order to see if this new layer gives a better explanation of the phenomena we observe in practice.

Within the class of symmetric information dynamic capital structure models our model gives a number of insights of which we will mention some here.

Goldstein, Ju, and Leland (2001) find that their dynamic capital structure model gives much lower leverage ratios than static capital structure models, *ceteris paribus*. By adding debt renegotiations to the model we find that leverage ratios increase relative to the results of Goldstein, Ju, and Leland (2001) approximately back to the level of the static capital structure models such as Leland (1994). Moreover, the introduction of debt renegotiations increases the tax advantage to debt by 50% relative to a dynamic capital structure model with no debt renegotiation for realistic parameter values.

A very useful insight from Kane, Marcus, and McDonald (1985) and Fischer, Heinkel, and Zechner (1989a) is that by no-arbitrage the unlevered firm value does not exist as a traded security and therefore we do not have any drift restrictions on this process. Our analysis supports this insight all though we reinterpret the equilibrium argument in the Fischer-Heinkel-Zechner model. Goldstein, Ju, and Leland (2001) are not willing to accept this idea that the unlevered firm value does not exist as a traded security, and use Microsoft, which has a leverage of practically zero, as a counterexample. However, for high growth firms like Microsoft our model predicts a capital structure of almost no debt and, hence, for these types of firms the optimally levered firm value and the unlevered firm values are almost the same. So the counterexample of Goldstein, Ju, and Leland (2001) does not really have any consequence.

Our model gives a simple explanation of the violation of the absolute priority rule for firms in financial distress, which is a very well documented empirical observation (Weiss 1990, Eberhart, Moore, and Roenfeldt 1990, Betker 1995). Basically, on the equilibrium path it is perfectly rational for the debt holders to accept a restructuring proposal from the equity holders which leaves some value to the equity holders even though the debt holders do not get their full principal back. The reason why the debt holders do accept such a proposal is that the alternative if they reject the equity holders' proposal is not necessarily an immediate liquidation of the firm. In most cases the equity holders would continue to pay the existing coupons until the conditions become even worse before eventually withholding the coupons and de facto forcing the firm into bankruptcy. Since the value of the debt in this alternative situation is lower than the value the debt holders get if they accept the equity holders' proposal, they are willing to accept the proposal even though the equity holders also get a piece of the pie.

Finally, our model can be used to question the remarkably small empirically observed bankruptcy costs that are reported in many studies (Warner 1977, Weiss 1990). Suppose the bankruptcy costs are estimated as the proceeds from selling the liquidated firm's assets or the sum of the prices of the firm's debt and equity after recovering from chapter 11 minus the sum of the firm's debt and equity prices just prior to entering chapter 11. In a symmetric information model like ours, the time as well as all direct and indirects costs of bankruptcy are perfectly anticipated by the investors and therefore already incorporated in the pre chapter 11 prices of the firm's debt and equity. The post chapter 11 prices still incorporate all indirect bankruptcy costs such as the loss of skilled employees and the loss of customer as well as supplier confidence, etc. since these costs are borne by the new firm. The direct bankruptcy costs are borne by the old debt and equity holders and are therefore not included in the post chapter 11 prices. Hence, this method of estimating the bankruptcy costs only gives an estimate of the true direct bankruptcy costs.

The paper is organized as follows. In section 2 we set up the framework of our EBIT based model. We then formulate a benchmark model with no possibilities for debt term renegotiations in section 3. In section 4 we introduce the first simple type of debt renegotiations as an *ex ante* determined split of the firm value. We attempt to improve on this simple type of debt renegotiation in section 5, where we try to design the debt restructuring proposal in such a way that the debt holders would never reject the offer. However, this proposal requires knowledge of the values of debt and equity if the debt holders do reject the debt restructuring offer. Unfortunately, these off-the-equilibrium-path values cannot be derived from the usual solution method. In section 6 we present an iterative solution method that is capable of deriving these off-the-equilibrium-path values. In section 7 we make some comparative statics of our model. In section 8 we discuss Fischer, Heinkel, and Zechner (1989a) and Goldstein, Ju, and Leland (2001) in relation to our model. Finally, we conclude in section 9. Appendices A and B contains the finer details of our fixed point solution method.

2. Dynamic Capital Structure Models

We model a firm run by equity holders, which has issued a single class of callable perpetual corporate debt with a fixed instantaneous coupon rate, C. The call feature of debt allows equity holders to better exploit the tax advantage to debt by increasing the amount of outstanding debt (and thereby the coupon payment rate) when earnings increase. For tractability, the capital structure is limited to a single class of debt. That is, we only allow the amount of outstanding debt to be increased by calling all of the existing debt and issuing new debt.[6] Debt is called at a premium and there is a cost of issuing new debt which is proportional to the principal.

When earnings decrease, equity holders and debt holders have a common interest in restructuring the debt to avoid bankruptcy costs. A key issue is how to distribute the value of the firm between equity holders and debt holders when this restructuring occurs.

The dynamic adjustment of the capital structure through an infinite series of calls and renegotiations of debt imply that the firm will, in fact, never go bankrupt. Compared to most of the earlier (static) models in the literature, such as the models by Black and Cox (1976), Leland (1994), and François and Morellec (2002) where the debt and equity values are known when the bankruptcy and call boundaries have been hit, the dynamic adjustment in our model leads to a fixed-point problem when solving for the initial values of debt and equity. That is, we have no exogenously given boundary conditions. The

[6]Otherwise, the incentive of the equity holders to sequentially increase the outstanding debt by issuing new debt with the same seniority and thereby diluting the existing debt will lead to a market break down de facto making it impossible for the firm to borrow at all.

boundary conditions needed to solve for the initial values of debt and equity depend on the values of debt and equity after the restructuring. But these in turn depend on the optimal capital structure chosen after the restructuring. This fixed-point problem has already been studied in Kane, Marcus, and McDonald (1985), Fischer, Heinkel, and Zechner (1989a), and Goldstein, Ju, and Leland (2001), but as we will see, their fixed-point argument does not give the off-the-equilibrium-path values of debt and equity at the lower boundary, which are needed to determine the relative bargaining power between the debt holders and the equity holders in the renegotiation phase.

The firm is governed by an exogenously given underlying state variable, ξ, which is the firm's instantaneous earnings before interest and tax payments (EBIT). The equity holders receive the remaining earnings from the firm after the coupons have been paid to the debt holders and corporate taxes have been paid to the tax authorities.[7] Following Goldstein, Ju, and Leland (2001), we call ξ the EBIT process and assume that it follows a geometric Brownian motion under the pricing measure $\mathbb{Q}$, i.e.[8]

$$d\xi_t = \xi_t \hat{\mu} dt + \xi_t \sigma dW_t, \tag{1}$$

with a given starting point, ξ_0. Here $\hat{\mu}$ and σ are constants parameterizing drift and volatility and W is a standard Brownian motion under the measure, $\mathbb{Q}$. We can think of the origin of the EBIT process, ξ, as the cash flow process generated by a production technology initially owned by an entrepreneur. The entrepreneur has the option to create a firm (at a certain cost) based on the EBIT process by issuing equity and (callable perpetual) debt.

Moreover, assume that the riskless interest rate, r, in the economy is constant. Since there are taxes on interest income, however, this is not the discount rate used for pricing under the pricing measure $\mathbb{Q}$. Let τ_i denote the tax rate on interest income in the hands of investors. The discount rate is then $(1-\tau_i)r$. This reflects an assumption that not only is interest income taxed at the rate of τ_i, but there is also a tax subsidy at the rate of τ_i associated with interest expenses. Hence, thinking in terms of dynamic replication of contingent claims, the effective interest rate paid on the money market account used for borrowing in the replicating portfolio is $(1-\tau_i)r$. That is, the price of the replicating portfolio is computed using the after-tax riskless rate. We assume throughout that $\hat{\mu} < (1-\tau_i)r$, since otherwise the cash flows generated from the EBIT process will not have a finite market value.

The market value corresponding to the EBIT process is, at any given date, the total value of the optimal mix of debt and equity that can be issued based on this EBIT process less the costs of obtaining it (Kane, Marcus, and McDonald 1985). This value will, of course, reflect the advantages and drawbacks of issuing debt including corporate tax savings as well as personal interest and dividend tax payments, and potential bankruptcy costs (Kane, Marcus, and McDonald 1984). This value process is similar to the unlevered firm value process, A, in Fischer, Heinkel, and Zechner (1989a). Note that we do not assume that any of these two processes (neither the value of the unlevered firm, A, nor the EBIT process, ξ) are price processes of traded securities. Hence, we do not have any no-arbitrage restrictions that give us the drift of A and ξ under the pricing measure. However, we do assume that there exist a unique pricing measure denoted $\mathbb{Q}$. That is, we assume that there exists *traded* securities such that any new

[7] In general, tax payments can be negative if the coupon rate is higher than the EBIT. In real life this does not lead to a symmetric tax refund. To mimic this friction the tax refunds will be reduced to a fraction, ϵ, of the original tax refund in our model.

[8] Since we are working with an infinite time horizon we do not want to use the term 'equivalent martingale measure' for $\mathbb{Q}$ because with an infinite time horizon the usual Girsanov measure transformation using the drift does not give an equivalent measure. As long as we do not say anything about the EBIT process behavior under the physical measure and how the existence of $\mathbb{Q}$ is related to no arbitrage under the physical measure we are on safe ground. Hence, we just take equation (1) as a definition.

claim that we may want to introduce can be dynamically replicated by already existing traded securities and therefore priced.[9]

In section 8 we will return to how the drift, $\hat{\mu}$, of the EBIT process under the pricing measure can be determined implicitly if we have simultaneous observations of the EBIT process, ξ, and some value process of a traded security based on the EBIT process, e.g. the value process of the equity of the firm.

In our model we consider different ways of restructuring the firm's debt. In each case the conditions determining when the restructuring occurs are determined by the so-called restructuring policy. The restructuring policy is parameterized by two boundaries, the renegotiation (or bankruptcy) boundary, $\underline{\xi}$, and the call of debt boundary, $\bar{\xi}$. That is, when ξ reaches the lower boundary, $\underline{\xi}$, the debt is renegotiated (or the firm is declared bankrupt) and when ξ reaches the upper boundary, $\bar{\xi}$, the debt is called. Obviously, $\underline{\xi} < \xi_0 < \bar{\xi}$. These boundaries will later be derived endogenously by incentive compatibility constraints, but for now they are exogenously given.

The claims on the EBIT process we consider, e.g. debt and equity, will be time-homogeneous claims, in the sense that they do not have a fixed maturity. The payoffs depend only on the current level of ξ and the level of ξ when the debt and equity was issued. Therefore, we denote the price at any given date t when the EBIT process is ξ_t of debt and equity issued at some date $s \leq t$ when the EBIT process was ξ_s, provided that the EBIT process $\{\xi_u\}_{u\in[s,t]}$ in the time period $[s,t)$ has stayed inside the interval $(\underline{\xi},\bar{\xi})(=(d\xi_s,u\xi_s))$ as $D(\xi_t;\xi_s)$ and $E(\xi_t;\xi_s)$.

In Appendix A we analyze the debt and equity price functions and derive (cf. equation (34) in Appendix A) that both debt and equity are positive homogeneous of degree one in (ξ_t,ξ_s). That is,

$$D(\lambda\xi_t;\lambda\xi_s) = \lambda D(\xi_t;\xi_s) \tag{2}$$

and

$$E(\lambda\xi_t;\lambda\xi_s) = \lambda E(\xi_t;\xi_s), \tag{3}$$

for any $\xi_t \in [\underline{\xi},\bar{\xi}])(=[d\xi_s,u\xi_s])$ and $\lambda \in \mathbb{R}_+$.[10] Note moreover, that this homogeneity property implies that the restructuring policy $(\underline{\xi},\bar{\xi})$ for each new issue of debt can be written as $(d\xi_s,u\xi_s)$ for some fixed constants d and u.

Furthermore, for notational simplicity note that the initial values of debt and equity at the date when the debt is issued can be written as

$$D(\xi_s;\xi_s) = \xi_s D(1;1) = D\xi_s$$

and

$$E(\xi_s;\xi_s) = \xi_s E(1;1) = E\xi_s,$$

where D and E are constants determined as $D = D(1;1)$ and $E = E(1;1)$. Here we have used the positive homogeneity property from equations (2) and (3).

Debt is issued at par, i.e. the principal of the debt issued at date s with a coupon rate $c^*\xi_s$ (cf. Part 1 of Conjecture A.1 in Appendix A) is $D(\xi_s;\xi_s)$. The debt is callable at a premium, λ, at any given later date $t \geq s$, i.e. the debt can be called (by the equity holders) at any given date t by paying the debt

[9]The EBIT process, ξ, and the value process, A, of the unlevered firm are not, in general, in the set of traded securities that can be used to dynamically replicate new (derivative) securities which are functions of the EBIT process.

[10]Note that we have assumed that the EBIT process $\{\xi_u\}_{u\in[s,t]}$ in the time period $[s,t)$ has stayed inside the interval $(d\xi_s,u\xi_s)$. Hence, if $\xi_t \in \{d\xi_s,u\xi_s\}$, we know that it is the first time since date s that it hits one of the boundaries. Thus, we can still use the result from equation (34).

holders $(1+\lambda)D(\xi_s;\xi_s)$.[11] To issue debt there are costs proportional to the par value of the debt. We denote the proportional factor k.[12] That is, the total proceeds to the entrepreneur at date s when the EBIT process is ξ_s for issuing both perpetual debt with a coupon rate $c^*\xi_s$ and equity is

$$A(\xi_s) = E(\xi_s;\xi_s) + (1-k)D(\xi_s;\xi_s) = \xi_s\big(E(1;1) + (1-k)D(1;1)\big) = A\xi_s, \tag{4}$$

where A is a constant defined as

$$A = E(1;1) + (1-k)D(1;1) = E + (1-k)D.$$

Moreover, if the firm is declared bankrupt, a proportion, α, of the proceeds of the sale of the assets of the firm (i.e. the EBIT generating process, ξ) is lost in bankruptcy costs.[13] Note that the assets of the firm is sold off as a going concern (as an acquisition), i.e. the firm is acquired by an entrepreneur who again can optimally lever the firm.

De facto the firm is run by the equity holders in the sense that they decide (i) when to call the debt and (ii) at each instant in time whether to pay the coupons to the debt holders or not. That is, it is the incentives of the equity holders which endogenously determine the restructuring policy. However, both equity holders and debt holders anticipate these incentives as soon as the coupon rate of the debt is fixed so in that sense the restructuring policy is common knowledge when the debt is issued.

The key issue is what happens at the lower restructuring boundary. We will work through three different assumptions of boundary behavior in order to compare the results of the different assumptions and to more accurately fit our model within the existing literature.

3. No Renegotiations of Debt

Let us first set up a benchmark case in which there are no possibilities for renegotiating the debt terms. For simplicity, assume that debt is issued at date zero when the EBIT process is initiated at ξ_0. When the EBIT process, ξ, hits $u\xi_0$ the old debt is called (retired) at a premium, λ, and new debt is issued with higher par value in order to take advantage of the higher level of the EBIT process by increasing the tax shield. That is, we have the following values of debt and equity at the call of debt boundary, $u\xi_0$,

$$\begin{aligned} D(u\xi_0;\xi_0) &= (1+\lambda)D(\xi_0;\xi_0) \\ &= (1+\lambda)D\xi_0 \end{aligned} \tag{5}$$

and

$$\begin{aligned} E(u\xi_0;\xi_0) &= E(u\xi_0;u\xi_0) + (1-k)D(u\xi_0;u\xi_0) - (1+\lambda)D(\xi_0;\xi_0) \\ &= \big(Au - (1+\lambda)D\big)\xi_0. \end{aligned} \tag{6}$$

These equations are usually termed the value matching conditions at the upper boundary $u\xi_0$. Moreover, the equity holders find it optimal to call the debt when the following condition at $u\xi_0$ is fulfilled

$$E_1(u\xi_0;\xi_0) = A. \tag{7}$$

[11]Fischer, Heinkel, and Zechner (1989b) study the *ex ante* optimal size of the call premium.
[12]These costs removes the incentive of the equity holders to restructure the debt continuously in the case where there is no call premium of the debt, i.e. $\lambda = 0$.
[13]If $\alpha < \lambda$ the equity holders might (at the upper boundary) be better off withholding the coupons and thereby forcing the firm into bankruptcy than calling the debt since the bankruptcy costs are lower than the call premium of the debt. However, the way our model is setup we ignore such strategic behavior.

Here E_1 denotes the partial derivative of the equity price function $(\xi, \xi_0) \mapsto E(\xi, \xi_0)$ with respect to the first variable, ξ.[14] This condition is usually termed the smooth pasting condition at the upper boundary $u\xi_0$.[15]

When the governing state variable, ξ, hits $d\xi_0$ the equity holders withhold the coupon rate to the debt holders who immediately declare the firm bankrupt. The assets of the firm, i.e. the EBIT process, ξ, is acquired as a going concern by a new entrepreneur who again can lever the firm optimally.[16] A fraction, α, of the proceeds covers the bankruptcy costs. The rest of the proceeds goes first to the debt holders to cover their original principal and finally to the equity holders. This is in accordance with the absolute priority rule. However, because the equity holders have limited liability, in most bankruptcy cases, the debt holders will not be able to get their full principal back. This happens when the proceeds from the sales less the bankruptcy costs are smaller than the original debt principal. Hence, we have the following value matching conditions at $d\xi_0$

$$\begin{aligned} D(d\xi_0;\xi_0) &= \min\Big\{(1-\alpha)\big(E(d\xi_0;d\xi_0)+(1-k)D(d\xi_0;d\xi_0)\big), D(\xi_0;\xi_0)\Big\} \\ &= \min\big\{(1-\alpha)Ad, D\big\}\xi_0 \end{aligned} \tag{8}$$

and

$$\begin{aligned} E(d\xi_0;\xi_0) &= \max\Big\{(1-\alpha)\big(E(d\xi_0;d\xi_0)+(1-k)D(d\xi_0;d\xi_0)\big) - D(\xi_0;\xi_0), 0\Big\} \\ &= \max\big\{(1-\alpha)Ad - D, 0\big\}\xi_0. \end{aligned} \tag{9}$$

Moreover, the equity holders find it optimal to declare bankruptcy (by withholding the coupons to the debt holders) when the following smooth pasting condition at $d\xi_0$ is fulfilled

$$E_1(d\xi_0;\xi_0) = (1-\alpha)A1_{\{(1-\alpha)Ad\geq D\}}. \tag{10}$$

In Appendix B we verify that there exists a fixed-point solution for the prices of debt and equity. From this solution we derive the optimal capital structure of the firm.

In figure 1 we have depicted the fixed-point solutions for debt and equity for the set of base case parameters considered in section 7. The figure shows the value of debt and equity separately as functions of the current EBIT value, ξ. The debt was issued with an optimal coupon rate of $c^* = .63$ at date zero when the initial EBIT value, ξ_0, was one. The lower limit on the horizontal axis is the optimal bankruptcy boundary, $d = .21$, and the upper limit is the optimal call of debt boundary, $u = 2.63$, given the debt

[14] This comes from differentiating on both sides of the value matching condition for equity, equation (6), with respect to the running EBIT value, ξ, and evaluating it for $\xi = u\xi_0$. To be exact, the left-hand side of equation (7) is

$$\frac{\partial}{\partial \xi} E(\xi;\xi_0)\Big|_{\xi=u\xi_0} = E_1(u\xi_0;\xi_0)$$

and the right hand side is

$$\begin{aligned} \frac{\partial}{\partial \xi}\left(E(\xi;\xi)+(1-k)D(\xi;\xi)-(1+\lambda)D(\xi_0;\xi_0)\right)\Big|_{\xi=u\xi_0} &= \frac{\partial}{\partial \xi}\Big(\xi\big(E(1;1)+(1-k)D(1;1)\big)-(1+\lambda)D(\xi_0;\xi_0)\Big)\Big|_{\xi=u\xi_0} \\ &= \frac{\partial}{\partial \xi}\big(A\xi-(1+\lambda)D(\xi_0;\xi_0)\big)\Big|_{\xi=u\xi_0} \\ &= A. \end{aligned}$$

Cf., e.g., Dixit (1993) for more about this subject.

[15] This (or similar) smooth pasting or high contact condition is used throughout the literature (Merton 1973, Leland 1994, Mella-Barral 1999). Merton (1973, footnote 60) is the only one giving an argument for the validity of this condition. Cf. Dixit (1991), Dixit (1993), Brekke and Øksendal (1991), and Brekke and Øksendal (1994) for explanations of what type of optimality this condition leads to.

[16] Note that compared to most of the other models in the literature, cf. e.g. the models by Leland (1994) and Goldstein, Ju, and Leland (2001), we allow the firm to continue operation and to be optimally levered again by the (possible) new owner instead of introducing an *ad hoc* liquidation value of the assets of the firm.

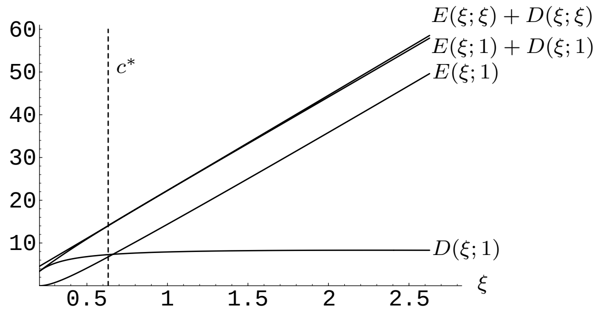


FIGURE 1. Debt and equity values both with fixed coupon rate, c^*, (fixed optimally when the debt is initially issued at date zero when $\xi_0 = 1$) and with a coupon rate which is continuously optimally determined to the given EBIT level, ξ, as a function of the EBIT level, ξ, for the model with no possibilities of renegotiations of the debt terms in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, and $\epsilon = 50\%$.

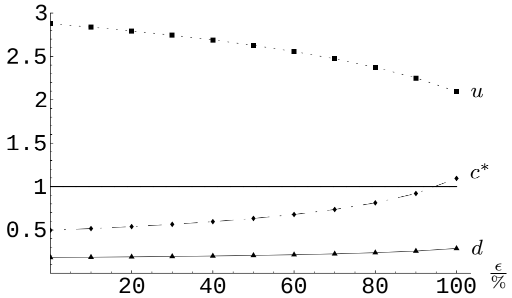
that has been issued at date zero. The optimal coupon rate is also depicted. Notice that the equity value at the lower boundary smooth pastes horizontally to zero and that the debt value is below its par value ($D(1;1)$) indicating that the debt holders do not get their full par value when the firm goes bankrupt. Note also how the debt value function is literally horizontal for high values of EBIT. In figure 1 we have also depicted the fixed-point solutions for the sum of the values of debt and equity, $E(\xi;1) + D(\xi;1)$, in order to compare it with the value, $E(\xi;\xi) + D(\xi;\xi)$, of an artificial firm which at any instant in time has the optimal capital structure. Notice that within the range determined by the debt restructuring policy the difference in value between these two are fairly small. By no-arbitrage the difference is actually determined by the costs of obtaining the optimal capital structure. Hence, at the upper boundary it is the issuing costs of the new debt, kDu, and at the lower boundary it is bankruptcy costs and the issuing costs of new debt, $\alpha Ad + kDd$.

In figure 2 we have depicted various results of the model as a function of the fractional reduction, ϵ, of the tax rate when EBT is negative. In figure 2(a) we see the debt restructuring policy, (d, u), and the optimal coupon rate, c^*. Note that as the possibilities for getting tax refunds when EBT is negative get better, the higher the optimal coupon rate is, and the more often it is optimal for the firm to re-optimize its capital structure, i.e., the interval $[d, u]$ shrinks. In figure 2(b) we see how the initial values of debt, D, and equity, E, change as the possibilities of getting tax refunds change. In this figure we have also depicted the total value to the original entrepreneur, $E + (1-k)D$, and the key figure, TAD, measuring the tax advantage to debt, which is defined as

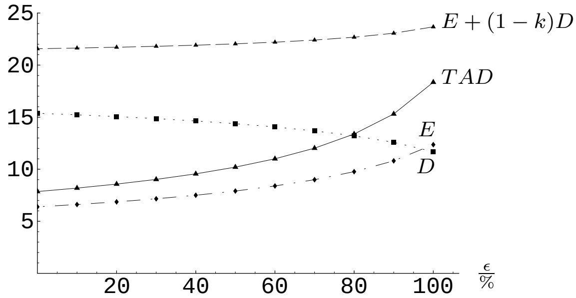
$$TAD = \frac{E + (1-k)D}{\frac{1-\tau_e}{(1-\tau_i)r - \hat{\mu}}} - 1.$$

This number gives the increase in percentage terms of the whole firm value relative to an (in-optimally) 100% equity financed firm (Goldstein, Ju, and Leland 2001). Surprisingly enough, even though TAD more than doubles from around 8% to 18% as the possibilities for getting tax refunds when EBT is negative increase from no refund to full refund, the total firm value to the original entrepreneur is almost constant.[17] In Goldstein, Ju, and Leland (2001) they report a tax advantage to debt of around 8% for $\epsilon = .5$, whereas we have a tax advantage to debt of around 10%. The reason why our tax advantage to

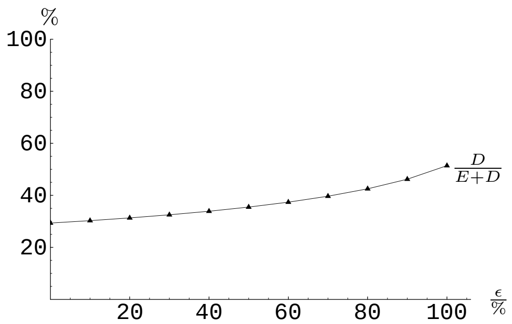
[17]Strictly speaking it is not correct to depict TAD in the same figure as D, E, and $E+(1-k)D$ since TAD is in percentage terms whereas the others are in EBIT units.



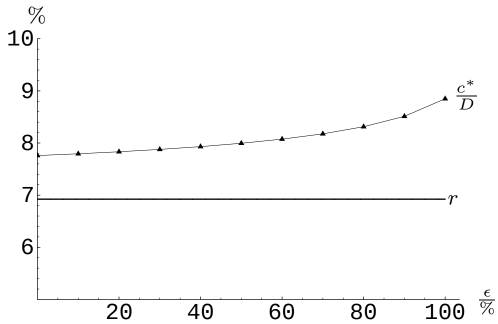
(a) Upper and lower restructuring boundaries



(b) Debt and equity values



(c) Leverage



(d) Optimal coupon rate and yield spread

FIGURE 2. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the fractional reduction, ϵ, of the tax rate when EBT is negative for the model with no possibilities of renegotiations of the debt terms in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, and $k = 3\%$.

debt is higher is because we allow for further tax advantages to debt even after a bankruptcy in that we allow for the firm, ξ, to be acquired as a going concern by a new entrepreneur, cf. equations (8) and (9).

In figure 2(c) the initial leverage ratio in percentage terms (the capital structure of the firm) is depicted as a function of the possibilities for getting tax refunds when EBT is negative. It should be noted that this is only depicting the initial (and optimal) capital structure of the firm. As it can be seen from figure 1, the capital structure varies dynamically in a fairly large range (15%–100%) in the base case until it is optimal again to re-optimize the capital structure. Finally, figure 2(d) depicts the initial yield of the debt issued measured as the coupon rate relative to the par value of the debt. Subtracting the before tax riskless interest rate, r, from this number gives the yield spread of the issued (risky) debt. Notice that the initial yield spread varies from around 80 basis points to 180 basis points as the possibilities for getting tax refunds when EBT is negative increase from no refund to full refund. But again this is only the initial yield spread. Yield spreads observed at other dates than the issuing dates can (in the base case) vary from around 70 basis points when the debt is close to being called to over 900 basis points when the bankruptcy threat is imminent.

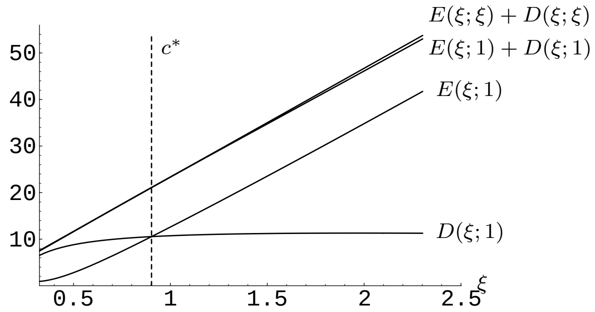


FIGURE 3. Debt and equity values both with fixed coupon rate, c^*, (fixed optimally when the debt is initially issued at date zero when $\xi_0 = 1$) and with a coupon rate which is continuously optimally determined to the given EBIT level, ξ, as a function of the EBIT level, ξ, using the first attempt to model renegotiations in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, $\eta = 12.5\%$, and $\epsilon = 50\%$.

4. Renegotiation of Debt: A First Attempt

After having solved the debt equity valuation problem in the benchmark case with no possibilities for renegotiations of the debt terms, we investigate other types of behavior at the lower boundary of the EBIT process. We assume that debt is issued at date zero when the EBIT process is initiated at ξ_0. The call feature of the debt is exactly identical to the case with no renegotiation of debt in section 3. Hence, the upper value matching conditions from equations (5) and (6) and the corresponding smooth pasting condition from equation (7) are identical. However, the behavior on the lower boundary, $\underline{\xi} = d\xi_0$, is changed.

When the governing state variable, ξ, hits $d\xi_0$ the old debt is renegotiated in the following way: the old debt is retired and new debt with lower par value and coupon rate is issued in order to avoid bankruptcy. An *a priori* fixed fraction $\eta \in [0,1]$ of the total proceeds goes to the equity holders and the rest goes to the original debt holders.[18] That is, we have the following value matching conditions at $d\xi_0$

$$\begin{aligned} D(d\xi_0;\xi_0) &= (1-\eta)\big(E(d\xi_0;d\xi_0) + (1-k)D(d\xi_0;d\xi_0)\big) \\ &= (1-\eta)Ad\xi_0 \end{aligned} \tag{11}$$

and

$$\begin{aligned} E(d\xi_0;\xi_0) &= \eta\big(E(d\xi_0;d\xi_0) + (1-k)D(d\xi_0;d\xi_0)\big) \\ &= \eta A d\xi_0. \end{aligned} \tag{12}$$

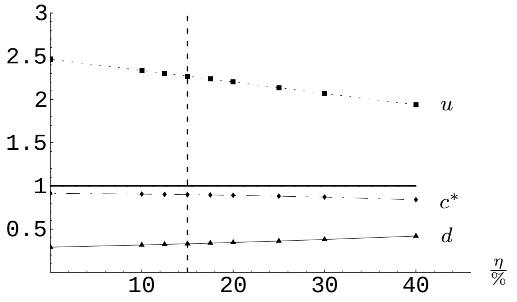
Moreover, the equity holders find it optimal to initiate the renegotiation of the debt when the following smooth pasting condition at $d\xi_0$ is fulfilled

$$E_1(d\xi_0;\xi_0) = \eta A. \tag{13}$$

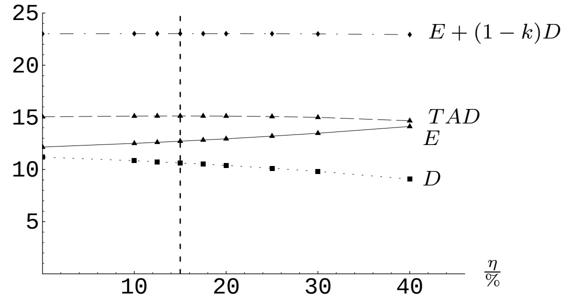
Finding the fixed-point solution for the debt and equity values and verifying Conjecture A.1 in Appendix A is done exactly as in the previous section (section 3).

In figure 3 we have depicted the fixed-point solutions for debt and equity in the case where there are renegotiations at the lower boundary. We have again used the set of base case parameters considered in section 7. In addition we have set $\eta = 12.5\%$. Comparing the renegotiation case with the no-renegotiation

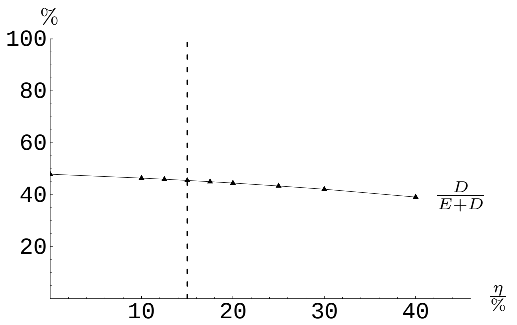
[18]The economic interpretation of the constant η is that it is a way of distributing bargaining power between the debt holders and the equity holders in the phase of the debt renegotiations.



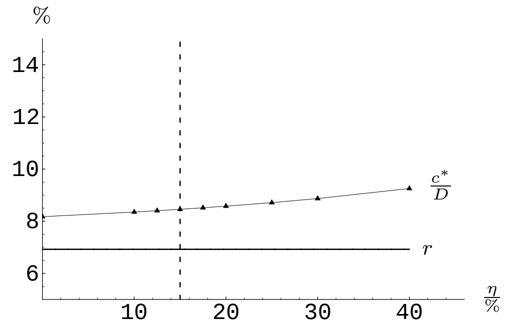
(a) Upper and lower restructuring boundaries



(b) Debt and equity values



(c) Leverage



(d) Optimal coupon rate and yield spread

FIGURE 4. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as function of the fraction of bargaining power the equity holders have, η, using the first attempt to model renegotiations in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, and $\epsilon = 50\%$.

case from figure 1 we see that the equity holders actually do have some value at the lower boundary and that the optimal coupon rate, c^*, has increased by close to 50%. Moreover, the lower boundary itself is also increased by around 50% from .21 to .32 and the upper boundary has decreased from around 2.63 to 2.30.

In figure 4 we have depicted various results of the model as function of the fraction of bargaining power the equity holders have, η. In figure 4(a) we see the debt restructuring policy, (d, u), and the optimal coupon rate, c^*. Note that the higher the fraction of bargaining power the equity holders have the smaller the interval $[d, u]$. In figure 4(b) we see how the initial values of debt, D, and equity, E, change as the fraction of bargaining power the equity holders have change. We have also depicted the total proceeds to the original entrepreneur, $E + (1-k)D$. If we assume that debt holders and equity holders can commit themselves to a given fixed distribution of bargaining power, η, then *ex ante* both parties will be best off if $\eta = 15\%$ since this value of η maximizes $E + (1-k)D$. This level of η is indicated by the dotted vertical line in all four sub-figures of figure 4. It is interesting to note that it is *ex ante* optimal to let the equity holders have such a large part of the restructured firm value less the costs of obtaining the optimal capital structure. A lower η value would, *ceteris paribus*, increase the debt value because the debt holders get more at each renegotiation. Therefore, the tax shield is better exploited. On the other hand, the equity holders get less. Since it is the equity holders who decide when to propose the restructure, a lower η will postpone a potential restructuring for both low and high EBIT values. This latter effect reduces firm value because the firm's capital structure is not re-optimized to exploited the tax shield so often. Hence, the tax shield is exploited less efficiently. These two effects, a static effect demanding a low η and a dynamic effect demanding a high η, counterbalances each other at an η level of .15. In figure 4(c) the initial leverage ratio in percentage terms is depicted as function of the fraction of bargaining power the equity holders have. As it is seen, the leverage falls as the fraction of bargaining power the equity holders have increase. Finally figure 4(d) depicts the initial yield of the debt, which have a tendency to increase as the fraction of bargaining power the equity holders have increase.

The way this renegotiation is setup is quite simple: together the equity holders and the debt holders will be better off by re-optimizing the capital structure of the firm to the optimal one based on the current value of EBIT. However, there are costs of obtaining this since they have to issue new debt. Still, if the current EBIT value is far enough away from the EBIT value at the date when the current capital structure was determined the benefits (in form of increased market value of the whole firm) will outweigh the costs of obtaining the optimal capital structure. But this simple renegotiation does not contribute much to the question of how this gain from the debt restructuring should be split between the original debt holders and equity holders. As it is, we just imagine that both equity holders and debt holders have already put all their old claims on the firm into one pot and forgotten how much each of them have contributed to the pot. After the re-optimization of the capital structure they start negotiating about how to split the pot between them. This is not a very realistic or satisfactory way of thinking of the debt renegotiation.

We would like to think of the outcome of this debt renegotiation as an agreement between the debt holders and the equity holders made voluntarily in the sense that both parties can reject the renegotiation. As we have set up the model, the initiative to propose the restructuring is with the equity holders since the smooth pasting condition is based on the equity value. That is, we can interpret the renegotiation as a one-shot take-it-or-leave-it offer to the debt holders proposed by the equity holders. Since it is the equity holders who time when they make this offer it will of course be beneficial for the equity holders—otherwise they would not have done it. So the equity holders would never like to reject their own debt restructuring proposal. The question is whether the debt holders would like to accept the proposal or not. That of course depends on what would happen if they reject the debt restructuring proposal. As a first attempt we simply assume that the threat of the equity holders is that if the debt holders do not accept their restructuring proposal, then the equity holders would withhold the coupon rate which again would trigger an immediate declaration of bankruptcy by the debt holders. If this is a credible threat by the equity holders, the debt holders would accept the restructuring proposal whenever $\eta \leq \alpha$. (If $\eta > \alpha$ the debt holders could get more by declaring the firm bankrupt than by accepting the equity holders' debt restructuring proposal unless the proposal is made for a very high value of ξ.) Hence, as a first attempt, we assume that η, which reflects the distribution of bargaining power between debt holders and equity holders, is in the interval $[0, \alpha]$.

5. Renegotiation of Debt: A Second Attempt

The problem with our first attempt to model the renegotiation game is that it is not obvious that the threat of the equity holders of withholding the coupons if the debt holders reject their restructuring proposal is a credible threat. In fact, it almost never is, because if the debt holders declare the firm bankrupt the equity holders would in most cases get nothing (unless the proposal is made for a fairly high value of ξ so that there would be something left for the equity holders after the bankruptcy costs have been paid and the debt holders have gotten their full principal back). In most cases, it would be better for the equity holders to continue paying the original coupon rate (or maybe try to reduce them marginally as in the strategic debt service models (Anderson and Sundaresan 1996, Mella-Barral and Perraudin 1997))[19] and thereby avoiding bankruptcy if their restructuring proposal is rejected by the debt holders. We would still like to model the renegotiation procedure as if the equity holders make a one-shot take-it-or-leave-it offer to the debt holders. Moreover, the proposal should be made so that it is always accepted by the debt holders.

We assume that if the debt restructuring proposal is rejected, either the equity holders will continue to pay the existing coupon rate or they will withhold the coupon rate, which leads to an immediate declaration of bankruptcy. First let us analyze the situation at the lower boundary, $d\xi_0$, which is the point where we assume that the equity holders make their restructuring proposal. If the equity holders continue to pay the original coupon rate (after a rejection) the equity value is

$$\underline{E}^c \equiv E(d\xi_0; \xi_0).$$

On the other hand, if the equity holders withhold the original coupon rate the firm is declared bankrupt and the equity value is

$$\underline{E}^b \equiv \max\big\{(1-\alpha)Ad - D, 0\big\}\xi_0.$$

It is the equity holders who determine whether to continue to pay the original coupon rate or to de facto declare bankruptcy if the restructuring proposal is rejected. Hence, the equity value is the maximum of the two alternatives, i.e.

$$\underline{E}^{nr} \equiv \max\{\underline{E}^c, \underline{E}^b\}.$$

The corresponding value of the debt in case of a rejection of the equity holders' restructuring proposal depends on the choice of the equity holders. If the equity holders continue to pay the original coupon rate the value of the debt is

$$\underline{D}^c \equiv D(d\xi_0; \xi_0)$$

in which case the debt holders are not in a position to force bankruptcy. On the other hand, if the equity holders do not pay the original coupon rate, the debt holders immediately declare bankruptcy. The bankruptcy value of debt is

$$\underline{D}^b \equiv \min\big\{(1-\alpha)Ad, D\big\}\xi_0.$$

The choice of whether to declare bankruptcy or to continue to pay the original coupon rate, after the debt holders have rejected the restructuring proposal, is in the hands of the equity holders. Therefore, the value of the debt is

$$\underline{D}^{nr} \equiv \underline{D}^c 1_{\{\underline{E}^c \geq \underline{E}^b\}} + \underline{D}^b 1_{\{\underline{E}^c < \underline{E}^b\}}.$$

Knowing the value of debt and equity if a debt restructuring offer is rejected, we can compute the gain from the debt restructuring as the value of the optimally levered firm less the costs of obtaining the optimal leverage minus the value the firm would have had if the restructuring proposal was rejected as

$$\underline{R} = Ad\xi_0 - (\underline{E}^{nr} + \underline{D}^{nr}). \tag{14}$$

We now assume that this debt restructuring gain, $\underline{R}$, is distributed between the equity holders and the debt holders so that the equity holders receives the *a priori* fixed fraction $\gamma \in [0, 1]$ of this gain and the

[19] It is not this type of strategic debt service that we pursuit in this model. As already argued in the introduction of the paper we consider this type of behavior unrealistic as it is not what we observe in practice. Our model is build to capture the incentives to restructure the entire debt with new principal and new coupon rate so that the equity holders can make a fresh start.

debt holders receive the remaining fraction $1-\gamma$.[20] Hence, the new value matching conditions at the lower boundary, $d\xi_0$, are

$$D(d\xi_0;\xi_0) = (1-\gamma)\utilde{R} + \utilde{D}^{nr} \tag{15}$$

and

$$E(d\xi_0;\xi_0) = \gamma\utilde{R} + \utilde{E}^{nr}. \tag{16}$$

The idea behind this new attempt is that it should ensure that both equity holders and debt holders will voluntarily accept the restructuring proposal since both parties are promised a value which is at least as high as the value they would have had if the debt restructuring proposal was rejected. The idea is that if $\utilde{R}$ is non-negative each party get a positive fraction of the gain in addition to their own rejection value. Unfortunately, the equations (15) and (16) does not give a unique distribution of the values of debt and equity in the case where, if the restructuring proposal is rejected, it is optimal to continue paying at the existing coupon rate. That is, if we assume $\utilde{E}^c \geq \utilde{E}^b$ then

$$\utilde{D}^{nr} = \utilde{D}^c = D(d\xi_0;\xi_0) \qquad \text{and} \qquad \utilde{E}^{nr} = \utilde{E}^c = E(d\xi_0;\xi_0),$$

which from equation (14) imply that

$$\utilde{R} = Ad\xi_0 - (\utilde{E}^c + \utilde{D}^c).$$

But since

$$\utilde{E}^c + \utilde{D}^c = E(d\xi_0;\xi_0) + D(d\xi_0;\xi_0) = Ad\xi_0$$

at the lower restructuring boundary, we have that

$$\utilde{R} = 0. \tag{17}$$

However, if $\utilde{R} = 0$ then equations (15) and (16) are vacuous. Hence, we cannot say anything about how much of $Ad\xi_0$ to give to the equity holders and how much to give to the debt holders. That is, in the language of the model in the previous section (section 4), η can be anything in the interval $[0,1]$. The indeterminacy of the distribution between debt holders and equity holders come from the following circularity. Because it is common knowledge when the equity holders make their restructuring proposal and also what they will propose, the debt and equity values at the boundaries just prior to when the restructuring proposal is made are by no-arbitrage determined by how much each party will get after the restructuring offer is accepted. But the debt restructuring proposal itself is determined by the values of debt and equity if the debt restructuring proposal is not accepted, which in this case is the values of debt and equity if we continue with the original coupon rate. These values, however, should include that the equity holders will have incentive to make a new debt restructuring proposal in, say, a few seconds. That is, these values must be equal to the values of debt and equity if the debt restructuring proposal is accepted. This is why we get $\utilde{R} = 0$. Expressed in another way, our problem is that we are not able to determine the off-the-equilibrium-path values of debt and equity individually.

In the case where $\utilde{E}^c < \utilde{E}^b$, the model is identical to the model derived in the previous section (section 4) with $\eta = \gamma\alpha$.

Hence, our second attempt did not bring us any new aspects to how to endogenously determine the distribution of Ad between debt holders and equity holders. However, it have highlighted that the problem

[20]Again the economic interpretation of γ is that it is a way of modeling the distribution of bargaining power between debt holders and equity holders in the phase of the debt renegotiation.

is to determine the off-the-equilibrium-path values of debt and equity individually when $\xi_0 = 1$. That is, $E(d; 1)$ and $D(d; 1)$.

6. Renegotiation of Debt: The Iterative Approach

One way to get the off-the-equilibrium-path values of debt and equity individually is by the following iterative procedure. Imagine that there are a commonly known finite number, $n \in \mathbb{N}$, of options to propose a restructuring of the firm's debt. Whenever the equity holders make a debt restructuring proposal this number is reduced by one. When there are no more restructuring options left (when $n = 0$) the only possibility at the lower boundary is to declare bankruptcy. This case with no more restructuring options left is exactly our benchmark case, which we have studied extensively in section 3. If we augment our notation of the value of debt and equity, the incentive compatible restructuring policy parameters, and the optimal coupon rate parameter with the remaining number of restructuring proposals, we have in section 3 derived the functions $D(\xi_t; \xi_s, 0)$ and $E(\xi_t; \xi_s, 0)$ giving us the value of debt and equity when the current value of the EBIT process is ξ_t and the coupon rate of the existing debt is $c_0^* \xi_s$. In section 3 we have also derived that the incentive compatible restructuring policy is $(d_0 \xi_s, u_0 \xi_s)$ and that the optimal coupon rate to be used at every future restructuring is c_0^* multiplied by the value of the EBIT process at the date when the new debt is issued.

With this information at hand we have the off-the-equilibrium-path values of debt and equity if there is one restructuring option left. This is so, because as soon as the equity holders have made the debt restructuring proposal the last restructuring options is exercised and there are no more options left. Hence, the off-the-equilibrium-path values of debt and equity, i.e., the debt and equity values if the debt restructuring proposal is rejected are already known from the benchmark case. Thus, we are now able to derive the value functions of debt and equity, $D(\cdot; \cdot, 1)$, $E(\cdot; \cdot, 1)$, the new incentive compatible restructuring policy parameters d_1 and u_1, and the new optimal coupon rate parameter, c_1^*. With this information at hand we have the off-the-equilibrium-path values of debt and equity if there are two restructuring options left.

The idea is to continue this iterative procedure until the debt and equity values, the incentive compatible restructuring policy parameters, and the optimal coupon rate parameter converge. In each iteration we face a problem very similar to the one solved in sections 3 and 4. That is, we have to use our conjecture-verification argument from appendices A and B in each iteration. To be precise, assume that we have the values of debt and equity, $D(\xi_t; \xi_s, n-1)$ and $E(\xi_t; \xi_s, n-1)$, when the current value of the EBIT process is ξ_t and the coupon rate of the existing debt is $c_{n-1}^* \xi_s$ and there are $n-1$ restructuring options left. Our problem is now to find the value functions of debt and equity, $D(\cdot; \cdot, n)$ and $E(\cdot; \cdot, n)$, the new incentive compatible restructuring policy parameters, d_n and u_n, and the new optimal coupon rate parameter, c_n^*, when there are n restructuring options left. To solve that problem we first have to specify the new value matching and smooth pasting conditions.

The upper value matching and smooth pasting conditions are the same as used in all the previous cases. That is, we have the following value matching conditions for debt and equity at the call of debt boundary, $u_n \xi_0$,

$$\begin{aligned} D(u_n \xi_0; \xi_0, n) &= (1+\lambda) D(\xi_0; \xi_0, n) \\ &= (1+\lambda) D_n \xi_0 \end{aligned} \tag{18}$$

and

$$\begin{aligned}E(u_n\xi_0;\xi_0,n) &= E(u_n\xi_0;u_n\xi_0,n)+(1-k)D(u_n\xi_0;u_n\xi_0,n)-(1+\lambda)D(\xi_0;\xi_0,n)\\ &= \big(A_n u_n-(1+\lambda)D_n\big)\xi_0.\end{aligned} \tag{19}$$

Moreover, the equity holders find it optimal to call the debt when the following smooth pasting condition at $u\xi_0$ is fulfilled

$$E_1(u_n\xi_0;\xi_0,n)=A_n. \tag{20}$$

At the lower boundary we will use the ideas from the model in the previous section (section 5). That is, we first have to calculate the gain from restructuring. If the equity holders choose to continue to pay the existing coupon rate (after a rejection) their equity value will be

$$\underline{E}_n^c \equiv E(d_n\xi_0;\frac{c_n^*}{c_{n-1}^*}\xi_0,n-1)=E(d_n;\frac{c_n^*}{c_{n-1}^*},n-1)\xi_0.$$

As soon as the equity holders have proposed the restructuring, one renegotiation option is lost. Therefore, in order to calculate the value of continuing with the existing coupon rate we should use the already calculated equity value function for $n-1$ remaining renegotiation options. Moreover, the existing coupon rate is $C_n^*=c_n^*\xi_0$. So the question is, at which EBIT level, ξ, would it had been optimal (with only $n-1$ renegotiations options left) to choose the same coupon rate, C_n^*. Since the optimal coupon rate parameter (with only $n-1$ renegotiations options left) is c_{n-1}^* the equity value of continuing with the existing coupon rate must be calculated as if the debt has been issued when the EBIT value, ξ, was $\frac{c_n^*}{c_{n-1}^*}\xi_0$ because that would have given an optimal coupon rate of[21]

$$c_{n-1}^*\frac{c_n^*}{c_{n-1}^*}\xi_0=c_n^*\xi_0=C_n^*.$$

Similarly, because one of the renegotiation options is lost, the bankruptcy value for the equity holders is

$$\begin{aligned}\underline{E}_n^b &\equiv \max\big\{(1-\alpha)\big(E(d_n\xi_0;d_n\xi_0,n-1)+(1-k)D(d_n\xi_0;d_n\xi_0,n-1)\big)-D(\xi_0;\xi_0,n),0\big\}\\ &= \max\big\{(1-\alpha)A_{n-1}d_n-D_n,0\big\}\xi_0.\end{aligned}$$

Since it is still the equity holders who determine whether to continue to pay the original coupon rate or to de facto declare bankruptcy, if the restructuring proposal is rejected, the equity value is the maximum of the two alternatives, i.e.

$$\underline{E}_n^{nr}\equiv\max\{\underline{E}_n^c,\underline{E}_n^b\}.$$

The corresponding value of the debt in case of a rejection of the equity holders' restructuring proposal depends on the choice of the equity holders. If the equity holders continue to pay the coupon rate the debt holders have a claim with value

$$\underline{D}_n^c\equiv D(d_n\xi_0;\frac{c_n^*}{c_{n-1}^*}\xi_0,n-1)=D(d_n;\frac{c_n^*}{c_{n-1}^*},n-1)\xi_0$$

[21]This procedure of re-adjusting the initial level of the EBIT, ξ_0, in order to get the coupon rate right with one renegotiation option less left does not necessarily give that the debt will have the same principal with $n-1$ renegotiation options left as was the case with n renegotiation options left even though the coupon rate is right. In fact, the principal will only be of the right size if $D(1;1,n)=D(1;\frac{c_n^*}{c_{n-1}^*},n-1)$. However, this is only a minor inexactitude of our numerical procedure for two reasons. (i) The principal will only play a role if either (a) the debt is called or (b) if the firm is liquidated and the acquisition value of the firm net of bankruptcy costs is higher than the principal. When the firm is in financial distress neither (a) nor (b) is the case and therefore the value of the debt is fairly insensitive to the exact size of the principal of the debt. (ii) When n goes to infinity our numerical calculations show that $D(1;1,n)-D(1;\frac{c_n^*}{c_{n-1}^*},n-1)$ converge to zero. Hence, in the limit the principal will be of the right size, cf. figure 5.

in which case the debt holders are not in a position to force bankruptcy. On the other hand, if the equity holders do not pay the coupons, the debt holders declare bankruptcy. The bankruptcy value of debt is

$$\begin{aligned}\underline{D}_n^b &\equiv \min\big\{(1-\alpha)\big(E(d_n\xi_0; d_n\xi_0, n-1) + (1-k)D(d_n\xi_0; d_n\xi_0, n-1)\big), D(\xi_0; \xi_0, n)\big\} \\ &= \min\big\{(1-\alpha)A_{n-1}d_n, D_n\big\}\xi_0.\end{aligned}$$

Since the choice of whether to declare bankruptcy or to continue to pay the original coupon rate, after the debt holders have rejected the restructuring proposal, is in the hands of the equity holders, the value of the debt is

$$\underline{D}_n^{nr} = \underline{D}_n^c 1_{\{\underline{E}_n^c \geq \underline{E}_n^b\}} + \underline{D}_n^b 1_{\{\underline{E}_n^c < \underline{E}_n^b\}}.$$

We can now compute the gain from the debt renegotiation as the value of the optimally levered firm less the costs of obtaining the optimal leverage minus the value the firm would have had if the restructuring proposal was rejected as

$$\underline{R}_n = A_{n-1}d_n\xi_0 - (\underline{E}_n^{nr} + \underline{D}_n^{nr}). \tag{21}$$

Again we distribute this gain from the debt restructuring between the equity holders and the debt holders so that the equity holders receives the *a priori* fixed fraction γ and the debt holders receive the remaining fraction $1-\gamma$. Hence, the new value matching conditions at the lower boundary, $d_n\xi_0$, are

$$D(d_n\xi_0; \xi_0, n) = (1-\gamma)\underline{R}_n + \underline{D}_n^{nr} \tag{22}$$

and

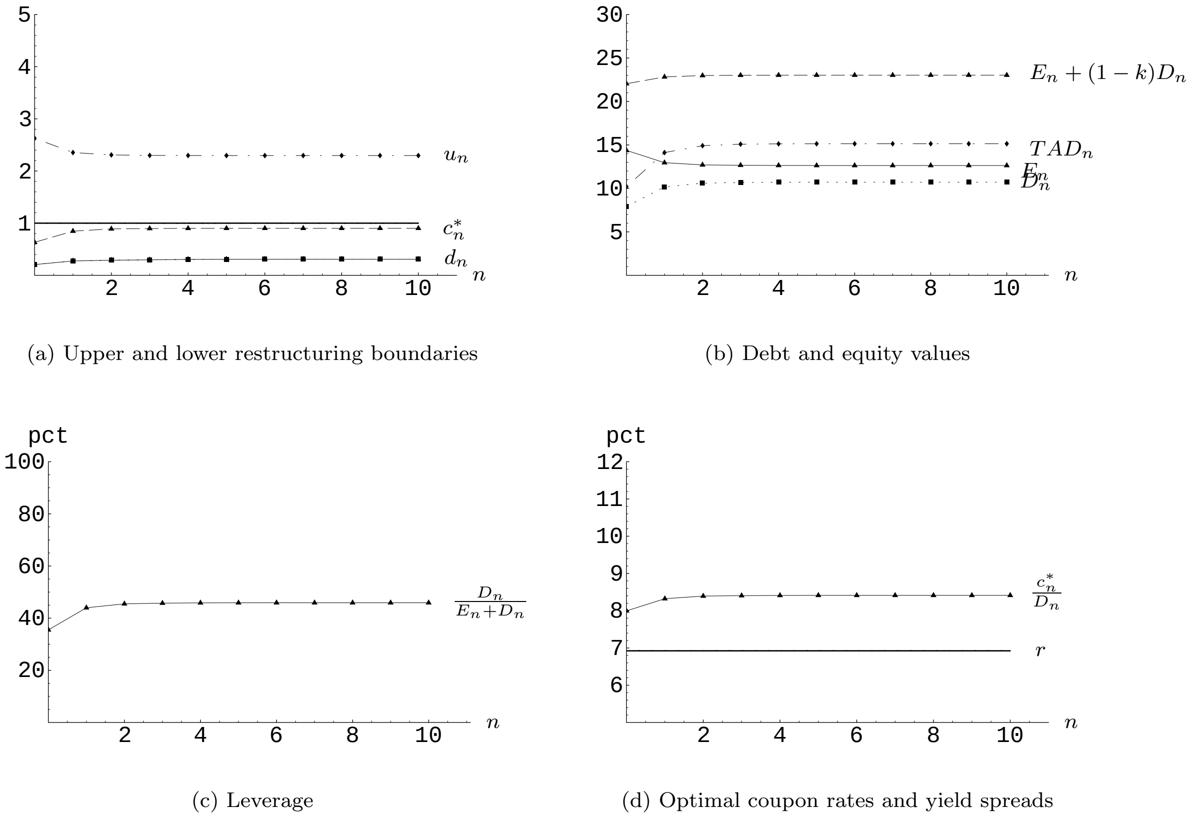
$$E(d_n\xi_0; \xi_0, n) = \gamma\underline{R}_n + \underline{E}_n^{nr}. \tag{23}$$

The new smooth pasting condition is obtained by differentiating with respect to ξ and evaluating for $\xi = d_n\xi_0$ on both sides of equation (23), cf. footnote 14,

$$E_1(d\xi_0; \xi_0, n) = \frac{\partial}{\partial\xi}\Big(\gamma\underline{R}_n + \underline{E}_n^{nr}\Big)\Big|_{\xi=d_n\xi_0}. \tag{24}$$

This gives us the necessary equations to solve for the debt and equity values for n remaining restructuring options by the same procedure as outlined in sections 2 and 3. Of course, we have to go through the whole procedure of conjecturing and checking the conjecture as outlined in appendices A and B, cf. Conjecture A.1, for each additional renegotiation option added, n. Starting with the values from section 3 for zero remaining renegotiations options this inductive procedure should eventually lead to a convergent solution as n goes to infinity, which takes into account the debt holders' right to reject any restructuring proposal and continue with the original coupon rate. Numerical experiments shows that the results stabilize quite quickly (after approximately 10-20 iterations depending on the parameters and the accuracy required). In the base case the convergence can be followed in figure 5. In spirit this solution procedure is similar to the finite number of sequential offers refinement of the Nash equilibria used in the Rubinstein bargaining game (Rubinstein 1982, Rubinstein 1987). In a standard Rubinstein sequential offer bargaining game with no limit on the number of sequential offers any split of the value is a Nash equilibrium. Put popularly, as long as there is no rule for what will happen, if the players cannot come to an agreement on a split of the value, they will continue to negotiate forever. If a final stage is introduced with an exogenously given fixed split of the value (in our case this is liquidation of the firm) then there is only one Nash equilibrium for any bargaining game with a finite number of sequential offers until the final stage is reached.[22] Our iterative solution procedure is finding this limiting Nash

[22] This is one mechanism to formalize what will happen if the players cannot come to an agreement.



(a) Upper and lower restructuring boundaries

(b) Debt and equity values

(c) Leverage

(d) Optimal coupon rates and yield spreads

FIGURE 5. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the number of restructuring options, n, for the iterative model in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, $\gamma = 50\%$, and $\epsilon = 50\%$.

equilibrium as the number of remaining sequential offers goes to infinity. This solution procedure is also proposed (without any relation to Rubinstein bargaining or other economic arguments) by Øksendal and Sulem (2001, Chapter 4) as a way to approximate the solution to an impulse control problem by solving a series of iterated optimal stopping time problems more or less the same way as we do in our numerical procedure. Moreover, Øksendal and Sulem (2001) show (under standard regularity conditions) that the proposed solution procedure do lead to a solution that gives the optimal value function of the problem, i.e. the optimal firm value as well as the optimal values of debt and equity, etc. The procedure do also give optimal controls, i.e. an optimal debt coupon rate and an optimal restructuring policy, but as it is common for optimal control problems, the proposed optimal controls are not unique. That is, there might be other debt coupon rates and corresponding restructuring policies which can lead to the same optimal firm value and debt and equity values.

Firstly, in figure 5 we study the convergence to the fix point as the number of restructuring options, n, goes to infinity. Figure 5(a) shows the upper and lower restructuring boundaries as a function of the number of remaining restructuring options. The levels of the boundaries quickly stabilize around their fixed points. The same is true for the debt and equity values and their derived key figures TAD, total firm value, leverage, and yield spread in figures 5(b), 5(c), and 5(d). More restructuring options allow the

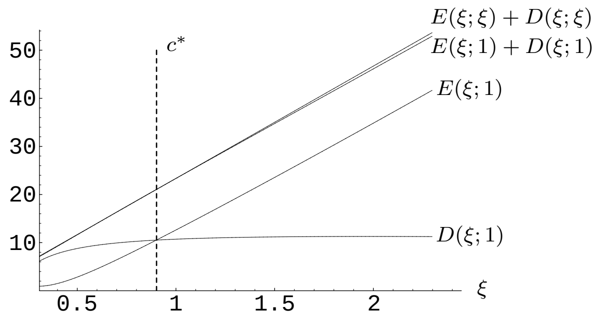


FIGURE 6. Debt and equity values both with fixed coupon rate, c^*, (fixed optimally when the debt is initially issued at date zero when $\xi_0 = 1$) and with a coupon rate which is continuously optimally determined to the given EBIT level, ξ, as a function of the EBIT level, ξ, for the iterative model in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, $\gamma = 50\%$, and $\epsilon = 50\%$.

firm to better exploit of the tax shield and to avoid bankruptcy. This increases the tax advantage to debt as seen from the TAD graph in figure 5(b) and narrows the interval $[d, u]$ in figure 5(a) as the number of restructuring options increase. This in turn increases the total firm value marginally (cf. figure 5(b)) while shifting the capital structure towards more debt (cf. figure 5(c)). This explains why the equity value is lower and why the debt value is higher when there are more options to renegotiate (cf. figure 5(b)). Note, however, that the restructuring boundaries as well as the values of debt and equity and their derived key figures, TAD, total firm value, leverage, and yield spread, for zero remaining renegotiation options, $n = 0$, are significantly different from the fixed point values derived when there are many remaining renegotiation options. Hence, the conclusions derived from a model with the debt renegotiation option at the lower boundary are significantly different from similar conclusions derived from a model ignoring the debt renegotiation option at the lower boundary as e.g., Fischer, Heinkel, and Zechner (1989a) and Goldstein, Ju, and Leland (2001).

In figure 6 we have depicted the fixed-point solutions for debt and equity for the iterative model after convergence ($n = 10$) for the set of base case parameters considered in section 7. Here and throughout we denote the convergent solutions by omitting n in the notation. The solutions look very similar to the solutions we had for our simple renegotiation of debt model from section 4, cf. figure 3. At the lower boundary, $\xi = .31$ the equity holders have a value of around .94 and the debt holders have a value of around 6.17. That the equity holders get 13.2% of the total value at the lower boundary indicates that the alternative, if the equity holder's restructuring proposal had been rejected by the debt holders, would have been to continue with the existing debt. Because if the alternative for the equity holders would have been to withhold the coupons then the equity holders should only have had $\gamma\alpha = 12.5\%$ of the total value, cf. equation (23). Note the relatively small difference between $E(\xi;\xi) + D(\xi;\xi)$ and $E(\xi;1) + D(\xi;1)$, cf. figure 6 at the fixed point. This substantiates our earlier claim that in the case where $\underline{E}^c \geq \underline{E}^b$ then the gain to leverage, $\underline{R}$, is zero at each *single* restructuring, cf. equation (17). In fact, at the fixed point, if $\underline{E}^c \geq \underline{E}^b$ then the difference at the boundaries between $E(\xi;\xi) + D(\xi;\xi)$ and $E(\xi;1) + D(\xi;1)$ is exactly the issuing costs of new debt, i.e., kdD at the lower boundary and kuD at the upper boundary.

Moreover, the fact that the equity holders get as much as 13.2% of the total proceeds in the debt renegotiation looks *ex post* like a serious violation of the absolute priority rule in the sense that the equity holders get a significant fraction of the value even though the debt holders have not gotten their full principal back—in this case the debt holders only get 58% of their principal back. However, the debt

The risk neutral drift of the EBIT process	$\hat{\mu}$	2%
The volatility of the EBIT process	σ	25%
The after-tax riskless interest rate	$r(1-\tau_i)$	4.5%
The tax rate on interest payments	τ_i	35%
The effective tax rate on dividends	τ_e	50%
The debt call premium	λ	5%
The bankruptcy costs	α	25%
The restructuring costs or more precisely the issuing costs of new debt	k	3%
The distribution of bargaining power between debt and equity holders	γ	50%
The fractional reduction of the tax rate when EBT is negative	ϵ	50%

TABLE 1. Base case parameter values.

holders are still getting a better deal than if they had rejected the equity holders' restructuring offer. Because in that case, the equity holders would have continued paying the coupon rate a little longer before withholding the coupons. From figure 1 we see that the value of the debt in that case is around 4.85 at $\xi = .31$ which is clearly lower than 6.17. When there are no restructuring options left ($n = 0$) in our model the absolute priority rule is not violated.[23]

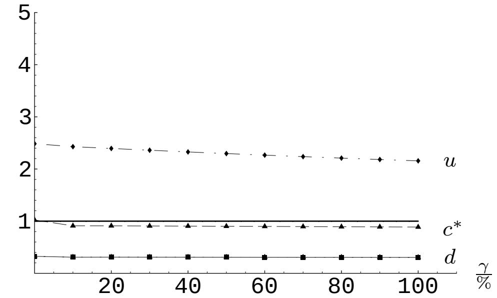
7. Comparative statics and Numerical results for the Iterative Approach.

In this section we study the numerical solutions of our iterative model. Since we do not have a formula for the debt and equity values we can write down and take derivatives of etc., we will have to show all the comparative statics by graphs. Most of the results presented in this section are quite intuitive and as we would expect them to be given our knowledge about the models in the earlier literature. However, we do not think that this makes these results obsolete because it is our only way of demonstrating how our model behaves. We will be quite brief and only comment on the surprising and interesting results. Our base case parameters for the comparative statics are given in table 1

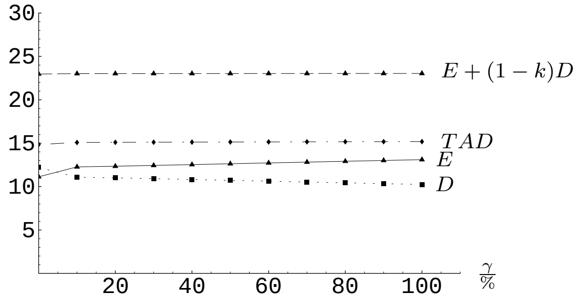
In figure 7 we study the effect of changing the bargaining power distribution, γ. The equity value increases as more and more bargaining power is distributed to the equity holders. At the same time the leverage goes down marginally and the yield spread is increased. The results are very similar to our first attempt to model renegotiations of debt in section 4, cf. figure 4. Note that η corresponds to $\gamma\alpha$ so the scaling on the horizontal axis is different. At first it seems plausible that the equity value increases and that leverage goes down as more and more bargaining power is distributed to the equity holders. But at the fixed-point solution the gain to leverage in each iteration, R, which is what the debt holders and equity holders do negotiate about, is zero. So in that sense it should not matter how the bargaining power is distributed between debt and equity holders. The reason why it does matter can only be the influence from the off-the-equilibrium-path values of debt and equity for n small.

In figure 8 we study the effect of changing the EBIT volatility, σ. The equity value increases, leverage and TAD go down, and the yield spread increases as the volatility increases. From figure 8(b) it can be seen that the model with renegotiations of debt has the usual asset substitution incentive for the equity holders. That is, after the debt has been issued the equity holders have incentive to try to increase the

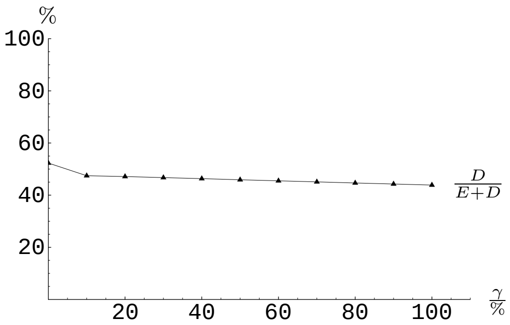
[23]Mella-Barral (1999) considers a different model where it is also optimal for the debt holders to deviate from the absolute priority rule by offering the equity holders a small share of the proceeds at liquidation in order to speed up the liquidation process. Both models adds aspects to the whole game of chapter 7 versus chapter 11 in the US bankruptcy code. Cf., e.g., Weiss (1990), Gilson, John, and Lang (1990), and Morse and Shaw (1988) for empirical studies of US bankruptcy code chapter 11 versus chapter 7.



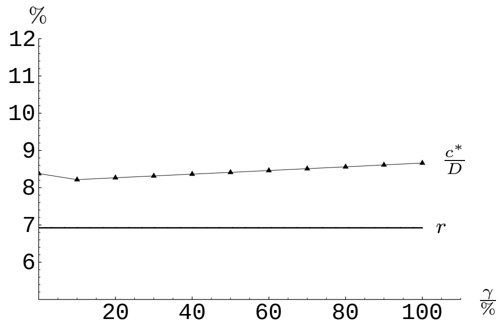
(a) Upper and lower restructuring boundaries



(b) Debt and equity values



(c) Leverage

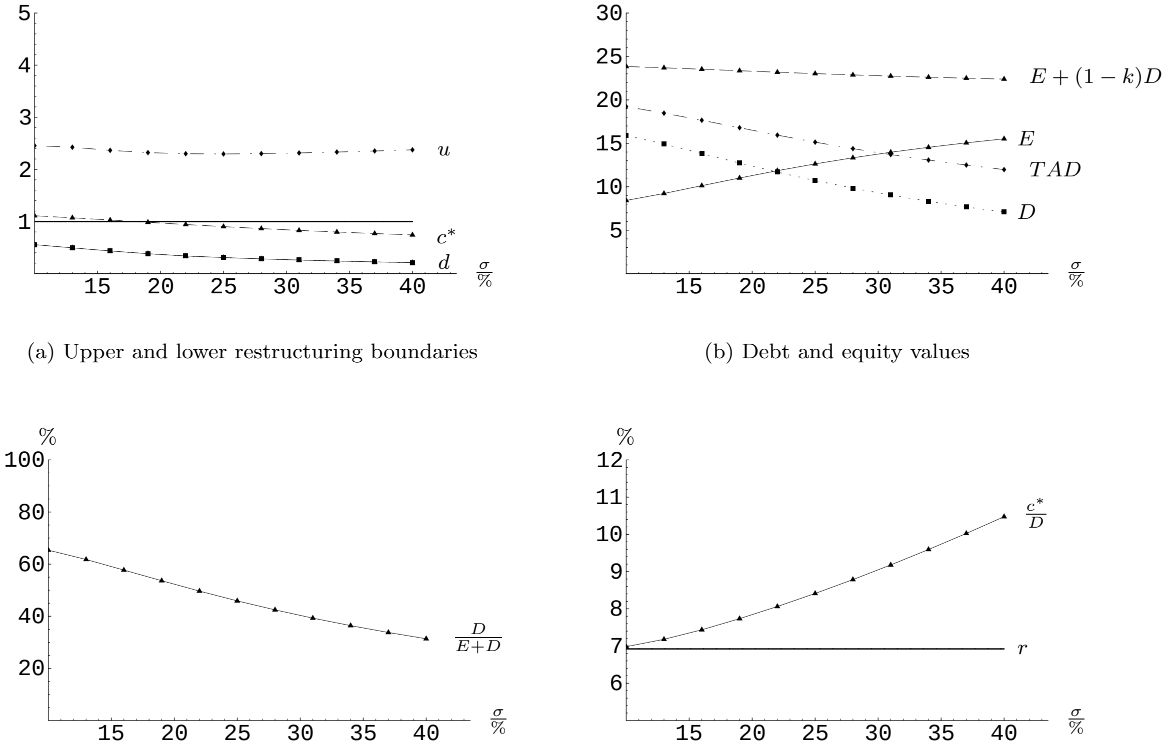


(d) Optimal coupon rate and yield spread

FIGURE 7. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the way the bargaining power is distributed between the debt and equity holders, γ, for the iterative model in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, and $\epsilon = 50\%$.

volatility of the underlying EBIT process (e.g., by the way they select new projects for the firm and retire old projects) since this activity will increase the value of the equity. However, compared to the usual static model there is a dynamic effect which limits the equity holders incentive to increase the volatility in the following way: after the debt has been issued the equity holders do take into account that this activity of increasing the EBIT volatility at the same time decreases the debt value as well as the whole firm value in all future debt calls and renegotiations. With the given base case parameters the advantage for the equity holders to increase the volatility of the EBIT process still remains as the results show, because this dynamic effect is already capitalized into the current equity price. However, because of this dynamic effect the incentive is limited (but not eliminated) compared to a static capital structure model. The reason for this is that increasing the EBIT volatility results (with the given base case parameters) in a smaller initial total firm value, which is what is important for the equity holders in all future restructurings.

In figure 9 we study the effect of changing the costs of issuing new debt. The boundaries widens so that the debt is restructured less frequently when the costs increase. The influence on debt and equity values, leverage and yield spreads are fairly small even though TAD decreases from 20% to 10%.



(a) Upper and lower restructuring boundaries

(b) Debt and equity values

(c) Leverage

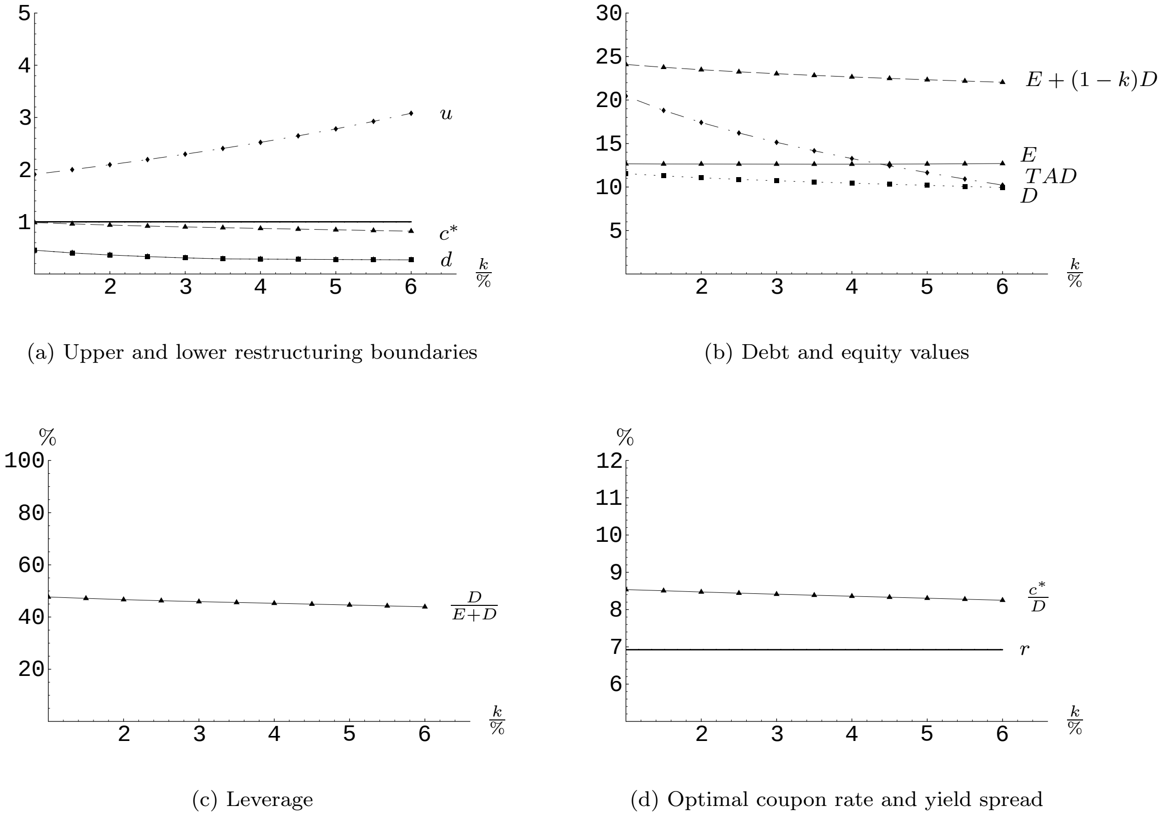
(d) Optimal coupon rate and yield spread

FIGURE 8. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the EBIT volatility, σ, for the iterative model in the base case: $\hat{\mu} = 2\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, $\gamma = 50\%$, and $\epsilon = 50\%$.

In figure 10 we study the effect of changing the bankruptcy costs factor. Since the firm actually never go bankrupt—it is just a far out in the future threat that when all the n renegotiation options have been exercised then eventually the firm will go bankrupt—it should have a very limited influence on the equilibrium results. Indeed the results in figure 10 substantiate this claim.

In figure 11 we study the effect of changing the possibilities for tax refunds when EBT is negative, ϵ. It is not surprising that total firm value increases and that leverage increases as the possibilities for tax refunds increases. However, we were surprised by the magnitude of the change. Note also that both TAD and the yield spread increases dramatically as the possibilities for tax refunds increases. Especially figure 11(a) tells us that the standard assumption in the literature, cf., e.g., the models of Kane, Marcus, and McDonald (1985), Fischer, Heinkel, and Zechner (1989a), Leland (1994), and Goldstein, Ju, and Leland (2001), of full tax refunds when EBT is negative (the case with $\epsilon = 1$) is highly unrealistic in the sense that the optimal initial coupon rate is set so high that the firm will always have negative EBT since $c^* > \bar{\xi}$. Actually, this is the case as soon as ϵ is higher than .93. In a model with no renegotiations this effect is not nearly as dramatic as can be seen by comparing with figure 2.

Finally, in figure 12 we study the effect of changing the risk neutral drift of the EBIT process, $\hat{\mu}$. It is not surprising that the total firm value increases as the risk neutral drift approaches the after-tax



(a) Upper and lower restructuring boundaries

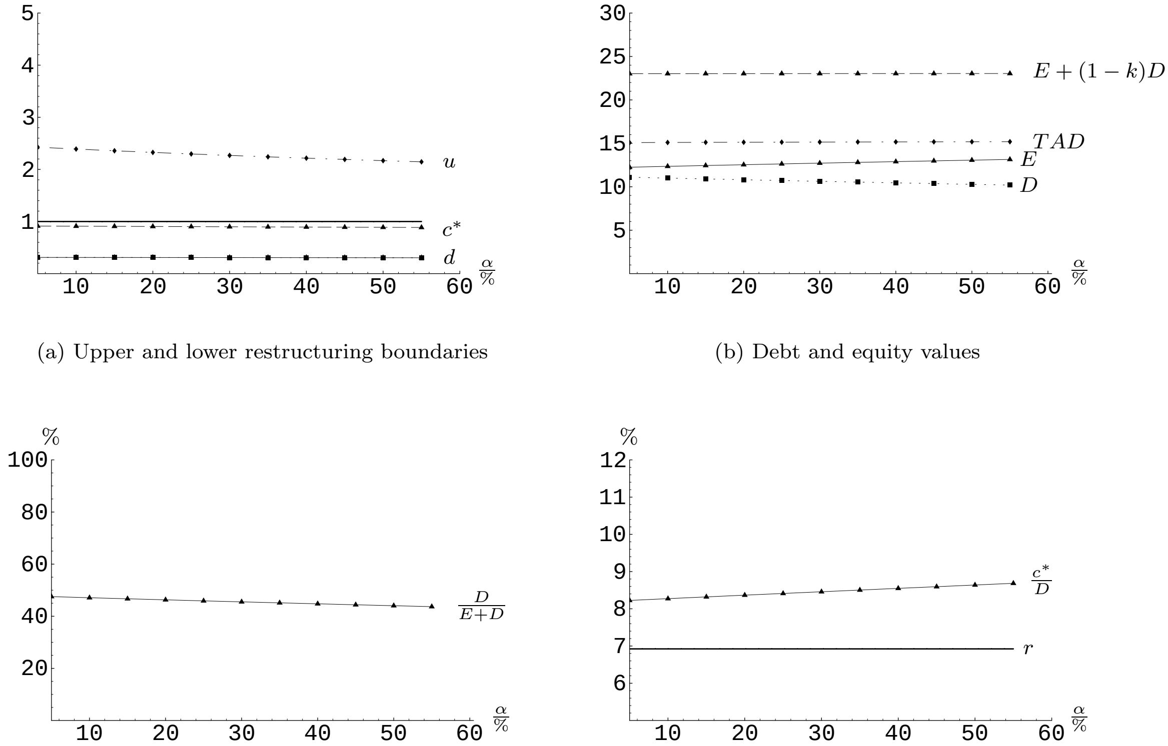
(b) Debt and equity values

(c) Leverage

(d) Optimal coupon rate and yield spread

FIGURE 9. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the costs of restructuring (issuing costs of new debt) factor, k, for the iterative model in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1 - \tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $\gamma = 50\%$, and $\epsilon = 50\%$.

riskless interest rate. This is the usual Microsoft/dot-com stories. These firms have dramatically high price-earnings ratio. Therefore, for high growth firms a small change in expected earnings has a dramatic effect on the firm value and therefore also on the equity value. However, notice how the optimal capital structure contains less and less debt as the risk neutral drift approaches the after-tax riskless interest rate. This happens because of the limited tax refunds when EBT is negative. If we allow for full tax refunds when EBT is negative, i.e. $\epsilon = 100\%$, our numerical results show that with $\hat{\mu} = 4\%$ and all other parameters as in the base case then the optimal leverage is around 60% and that the TAD shoots up to above 75% while the optimal coupon rate exceeds ten times the current earnings. Hence, the reduction of the leverage is purely tax driven: even thought the firm's EBIT has great potential it is not optimal (because of the limited tax refunds) to take on more debt than such that the current earnings can cover the coupon rates. Given this observation it is clear that the yield spread go down since the debt value is not increasing with the firm value as the risk neutral drift approaches the after-tax riskless interest rate. De facto the debt becomes more or less riskless as the leverage decrease. Hence, our model predicts that firms with low growth expectations (e.g., utility based firms) will have high optimal leverage and fairly high yield spreads whereas firms with high growth expectations will have very low optimal leverage and also very low yield spreads.



(a) Upper and lower restructuring boundaries

(b) Debt and equity values

(c) Leverage

(d) Optimal coupon rate and yield spread

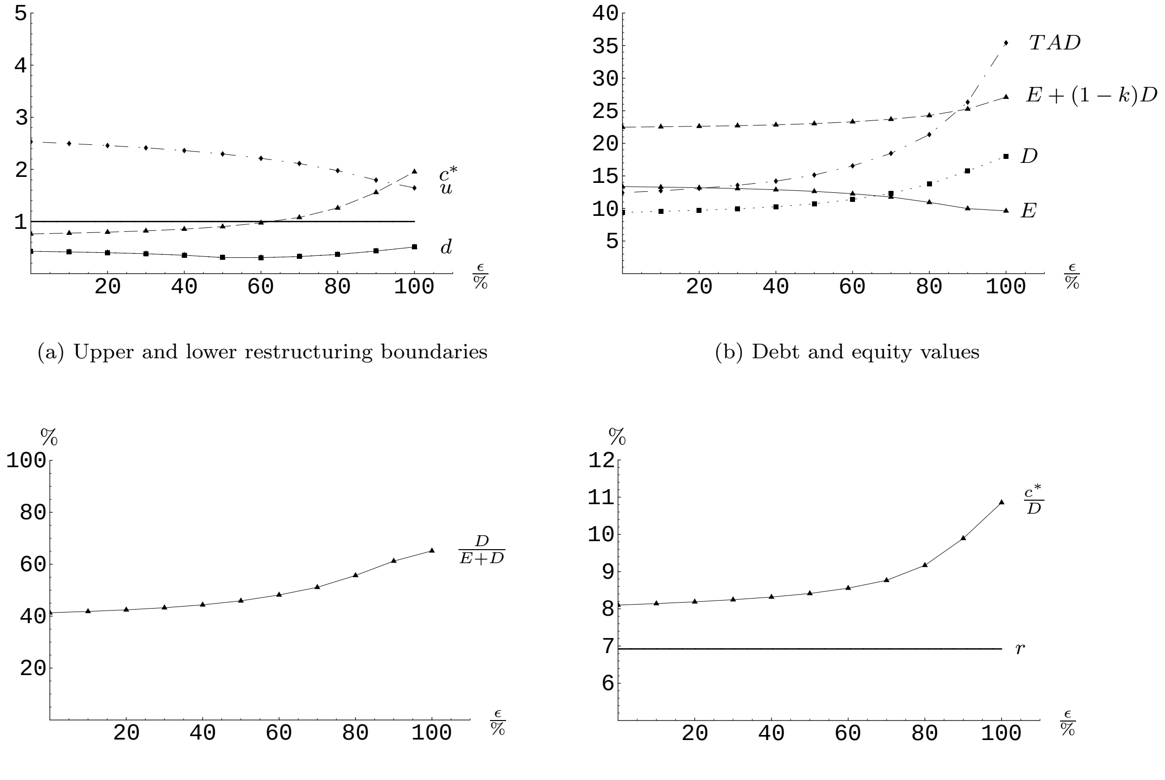
FIGURE 10. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the bankruptcy costs factor, α, for the iterative model in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $k = 3\%$, $\gamma = 50\%$, and $\epsilon = 50\%$.

8. The Risk Neutral Drift of the EBIT Process

In section 2 when we started modeling the EBIT process our starting point was an exogenously given drift, $\hat{\mu}$, of the EBIT process under the pricing measure, $\mathbb{Q}$. It is an interesting empirical question how this drift can be estimated. As we saw in section 7 one of the implications of our model is that the optimal leverage, the optimal coupon rate, the yield spread, etc. are highly dependent on the value of $\hat{\mu}$, cf. figure 12. Therefore, figure 12 gives us more or less the answer to the question of how to determine $\hat{\mu}$: from simultaneous observations of the equity value (right after the debt has been issued) and the value of the EBIT process a unique value for the risk neutral drift of the EBIT process, $\hat{\mu}$, can be backed out. That is, for a given $\hat{\mu}$, we can go through the whole derivation in sections 2–6 of the iterative model to find the value function of equity, $E(\cdot;1)$ as function of ξ. Hence, we can define the function, $PE(\cdot)$, which gives the price earnings ratio (PE ratio) of the firm as function of $\hat{\mu}$

$$PE(\hat{\mu}) = \frac{E(\xi;\xi)}{\xi} = E.$$

This function is well-defined because the equity value is homogeneous of degree one jointly in the two variables, cf. equation (3). Moreover, figure 12 indicates that PE is an increasing function of $\hat{\mu}$. This implies that PE has an inverse. Hence, given that we can observe the firm's initial PE ratio, $\hat{\mu}$ can be



(a) Upper and lower restructuring boundaries

(b) Debt and equity values

(c) Leverage

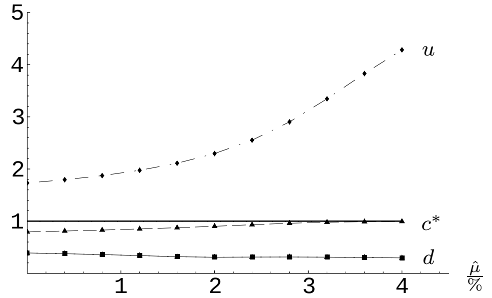
(d) Optimal coupon rate and yield spread

FIGURE 11. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the fractional reduction, ϵ, of the tax rate when EBT is negative for the iterative model in the base case: $\hat{\mu} = 2\%$, $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, and $\gamma = 50\%$.

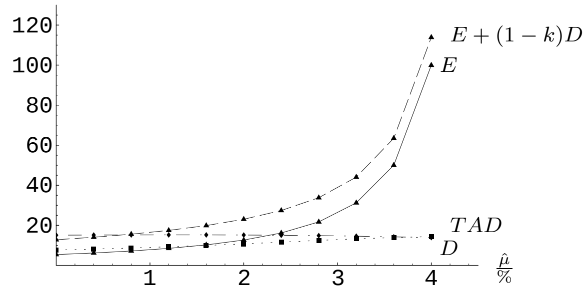
determined. This answer to the question of how to determine $\hat{\mu}$ is similar in spirit of how to back out the (implicit) volatility of the underlying security if one has simultaneous observations of the prices of an option and the underlying security in a Black-Scholes model, and hence, we term this the *implicit* risk neutral drift of the EBIT process. This argument, however, requires that the volatility, σ, of the EBIT process is already known. If both $\hat{\mu}$ and σ are unknown, then we need simultaneous observations of both the value of the EBIT process, the value of equity, and the value of debt right after the debt has been issued. (Uniqueness of the solution in this case is yet unsolved.)

Another solution to the problem of how to find $\hat{\mu}$ is to determine it in equilibrium. Apparently, this is what Fischer, Heinkel, and Zechner (1989a) claim to do. However, we are not able to get to the same conclusion by following their arguments. Fischer, Heinkel, and Zechner (1989a) start by specifying exogenously the value process, A, of the unlevered firm *including the value of their leverage potential* but excluding the costs of obtaining the leverage. Moreover, they assume that this value process is a geometric Brownian motion of the form

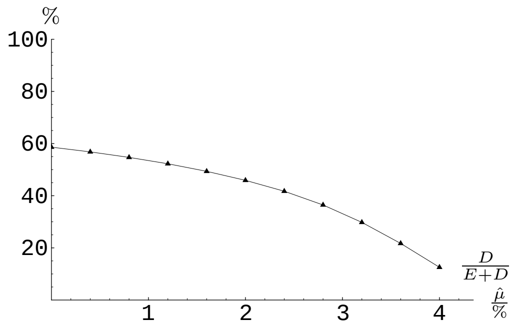
$$dA_t = A_t\hat{\mu}dt + A_t\sigma dW_t \tag{25}$$



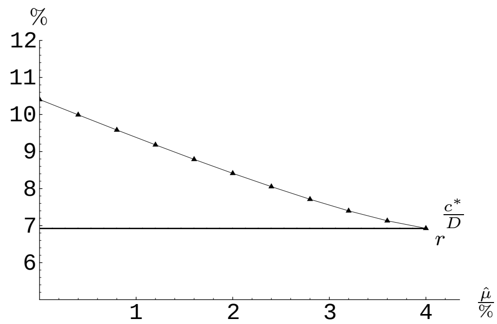
(a) Upper and lower restructuring boundaries



(b) Debt and equity values



(c) Leverage



(d) Optimal coupon rate and yield spread

FIGURE 12. Upper and lower restructuring boundaries, initial (when $\xi = 1$) debt and equity values, initial leverage, initial optimal coupon rates, and initial optimal yield spreads as a function of the risk neutral drift of the EBIT process, $\hat{\mu}$, for the iterative model in the base case: $\sigma = 25\%$, $r(1-\tau_i) = 4.5\%$, $\tau_i = 35\%$, $\tau_e = 50\%$, $\lambda = 5\%$, $\alpha = 25\%$, $k = 3\%$, $\gamma = 50\%$, and $\epsilon = 50\%$.

under an exogenously given and unique pricing measure, denoted $\mathbb{Q}$. That is , they assume that there exists *traded* securities such that any new claim that we may want to introduce can be dynamically replicated by already existing traded securities and therefore priced.[24] This is the same arguments as we used in section 2. With A as the governing state variable, Fischer, Heinkel, and Zechner (1989a) derive, using the same method as we have used in sections 2–3 (with ξ as the governing state variable), the value of debt and equity as functions of the current value of the unlevered firm process. Fischer, Heinkel, and Zechner (1989a) does not include renegotiations of the debt in their model, they only work with liquidations at the lower boundary (similar to what Goldstein, Ju, and Leland (2001) do in their model and what we do in sections 2–3).

Given the dynamics of A as specified in equation (25) and the assumption that A at the same time represents the price at which one should be able to acquire the firm before paying the costs of optimally lever the firm, we get the following requirement

$$A_t = E(A_t; A_t) + (1-k)D(A_t; A_t), \tag{26}$$

[24]Note that the value, A, of the unlevered firm is not, in general, one of the traded securities and can therefore not be used in the dynamically replicating argument to price new (derivative) securities which are functions of the value of the unlevered firm.

where the notation is adapted from section 2. That is, the value of the unlevered firm must at all times be the value of the debt and equity of the (newly) optimally levered firm less the costs of obtaining the optimal leverage. By homogeneity, equation (26) can be reduced to

$$1 = E + (1-k)D, \tag{27}$$

which Fischer, Heinkel, and Zechner (1989a) call a no-arbitrage condition on their model. They use this equation to determine the risk neutral drift, $\hat{\mu}$, of the unlevered firm value process. In that they claim (and we have no reason to doubt this claim as it will become clear soon) that there is only one $\hat{\mu}$ which fulfills equation (27).

Comparing the Fischer-Heinkel-Zechner model to the EBIT based model we have developed in sections 2–3 we have also defined the value process A as

$$A_t = E(\xi_t;\xi_t) + (1-k)D(\xi_t;\xi_t) = \kappa\xi_t.$$

That is, there exists a constant, κ, such that[25]

$$A_t = \kappa\xi_t.$$

Hence, if the unlevered firm value process, A, is a geometric Brownian motion so is the EBIT process, ξ, and vice versa. Moreover, the risk neutral drift, $\hat{\mu}$, (and the volatility, σ) of the unlevered firm value process, A, and the EBIT process, ξ, are the same.

So how is it possible that in the EBIT based model developed in sections 2–3 there are no restrictions on $\hat{\mu}$ whereas in the unlevered firm value based model of Fischer, Heinkel, and Zechner (1989a) $\hat{\mu}$ is uniquely determined? The answer is simple: In the development of the unlevered firm value based model, the after-tax payout rates on debt and equity, cf. table 2 in Appendix A, has to be specified as linear functions of (A, C) instead of (ξ, C). That is, in the unlevered firm value based model the payout rate to, say, equity must be specified as $\delta_A A + b_A C$ whereas in the EBIT based model it is specified as $\delta_\xi\xi + b_\xi C$. δ_ξ and b_ξ can be found in table 2 in Appendix A. In order to make the two models equivalent, we must specify *the same* payout rate for the unlevered firm value based model as for the EBIT based model. Thus, we must have $\delta_A A + b_A C = \delta_\xi\xi + b_\xi C$, for all C and all A and ξ such that $A = \kappa\xi$. This imply that $b_A = b_\xi$ and

$$\delta_A = \delta_\xi\frac{1}{\kappa}.$$

Hence, the ratio, κ, between A and ξ must be specified exogenously before we can specify the payout rate of equity in order to make the unlevered firm value based model equivalent to the EBIT based model. But from the ratio, κ, $\hat{\mu}$ can be backed out implicitly by the same argument as with the PE ratio. Hence, specifying κ exogenously is the same as specifying $\hat{\mu}$ exogenously. This is the reason why there is only one $\hat{\mu}$ in the unlevered firm value based model which is consistent, i.e. which fulfills equation (27).

Goldstein, Ju, and Leland (2001) also uses an endogenous drift, $\hat{\mu}$, which they claim depend on the coupon rate when the tax advantage to debt is measured. To illustrate how $\hat{\mu}$ can depend on the debt issued consider the following example from Goldstein, Ju, and Leland (2001, p. 495). Suppose that the current level of EBIT is $\xi_0 = 100$ and that the PE ratio is 20. Then the total value of all future earnings is $V_0 = 100 \times 20 = 2000$. Furthermore, let the corporate tax rate be $\tau_c = .35$. Thus the corporate tax payout rate is $\tau_c(\xi - C) = 0.35(100 - C)$, for a given coupon rate C. Suppose that the firm pays out 35

[25]Actually, $\kappa = E(1,1) + (1-k)D(1,1) = E + (1-k)D$. In section 2 we called the constant κ for A, but for this section it will be too confusing to have both A as a geometric Brownian motion and also a scaling constant.

as a dividend rate to the equity holders. This implies that the total payout from the firm is

$$\begin{aligned}\text{total initial payout rate} &= \text{interest} + \text{corporate tax} + \text{equity dividend}\\ &= C + .35(100 - C) + 35 = 70 + .65C,\end{aligned}$$

thus,

$$V_0\big((1-\tau_i)r - \hat{\mu}\big) = \xi_0 = 70 + .65C. \tag{28}$$

Hence, from equation (28) we can back out $\hat{\mu}$ as a function of the coupon rate, C. However, in our opinion, there is a problem in equation (28). First it is assumed that $\xi_0 = 100$ but in equation (28) one gets $\xi_0 = 70 + .65C$. The missing $30 - .65C$ stems from the assumption that the firm pays the amount 35 in dividend to the equity holders. Hence, there is remaining free cash in the firm. The dividend policy is not irrelevant in Goldstein, Ju, and Leland (2001), as well as in our model, the dividend paid out to the equity holders is residually determined as all of the remaining cash in the firm. Otherwise one needs to specify how the remaining free cash flow is used (investments or disinvestments in the EBIT generating process) and take that into consideration, as well as how dividends affects the process for EBIT, in the initial optimization problem.

9. Conclusion

In this paper we have analyzed a dynamic capital structure model which includes debt renegotiations when the bankruptcy threat becomes imminent. We have extended the Goldstein-Ju-Leland model in two ways: (i) instead of liquidating the firm and thereby definitively losing the tax shield, we assume that the firm is acquired as a going concern by another entrepreneur in case of bankruptcy. Moreover, (ii) we allow for voluntary debt renegotiations. These two extensions both work in the same direction. In our base case example the tax advantage to debt in the Goldstein-Ju-Leland model is around 8% and a leverage is around 35%. (i) extends the tax advantage to debt to 10%, and (ii) extends it further up to 15%. At the same time leverage is increased by (i) to 40% and by (ii) further up to around 50%. This figure for leverage is similar to the figure for a static capital structure model with the same parameter values. The point where the equity holders make their take-it-or-leave-it restructuring offer to the debt holders is determined by standard smooth pasting conditions such that the usual incentive compatibility constraints for the equity holders are fulfilled. In the base case example the equity holders get around 13% of the firm value, whereas the old debt holders get the rest. In extreme cases (for $\hat{\mu} = 4\%$) the equity holders can get as much as 60% of the value. In both cases the debt holders get less than their full principal back. Even though this looks like an *ex post* violation of the absolute priority rule, the debt holders are still better off by accepting this restructuring offer from the equity holders than by rejecting it. The reason is that if they reject the offer, the equity holders would continue to pay the existing coupon rate for some time before eventually withholding the coupons when the earnings of the firm are even lower. This alternative has a lower value to the debt holders than the value of accepting the restructuring proposal right away.

The numerically demanding part of our analysis is to calculate the off-the-equilibrium-path values of debt and equity. It turns out that these values are very important in order to understand how the equity holders should optimally time their take-it-or-leave-it restructuring offer. Moreover, these values endogenously determine how to split the firm value between the existing debt and equity holders. At the fixed-point solution the exogenously given bargaining power distribution, γ, only plays a minor role in the sense that the gain to restructuring, R, is zero in each iteration. Hence, there is no gain to exogenously distribute. The firm value split is fully determined by the endogenously given off-the-equilibrium-path values. To emphasize this point we will go through the following simple example: suppose we assume that the firm value split (η) is exogenously given in what we could call a reduced form model (a model of the type described in section 4). If we set the firm value split in the reduced form model equal to the firm value split our iterative model determined endogenously, we do not get the same lower and upper boundaries as optimal for the reduced form model as we do for our iterative model.[26] The reason is that in the iterative model the split is not a fixed number, η. The iterative model takes into account that if the equity holders want the debt holders to accept their restructuring proposal for another (off-the-equilibrium-path) value of d, they might very well have to offer them a bigger share of the value.

Our results indicate that empirically we should observe huge variations in leverage between the dates when firms realign their capital structure either by renegotiating their debt terms or by issuing new securities. Even for fairly small transactions costs as in our examples the earnings of the firm should have changed by a factor 2.5 before it is optimal for the firm to realign its capital structure. In the mean time between realignment dates the fluctuations in earnings result in fluctuations in debt and equity values that makes the leverage vary from around 21% to 87% in our base case example. Moreover, for the same reasons the object function of the firm is very flat over a large area, so even though our model can find a unique optimal capital structure for the firm, it is not that important for the total firm value that the firm targets it capital structure at that exact level. Furthermore, other issues outside our model even of small magnitude could change the optimum quite a bit because of the fairly flat objective function.

One of the things our model cannot explain is why some firms end up continuing operation after a debt restructuring while others end up being liquidated. In our model the firm is always more valuable alive than liquidated and, hence, it is natural that it should never be liquidated. We see two ways that the model can be extended to include this feature. Either we can introduce an exogenously given probability that the debt and equity holders cannot reach an agreement of how to restructure the firm, or we can introduce a second state variable measuring the value of the firm's assets. In this last model the firm can be liquidated simply because the firm's assets are more valuable than the present value of the EBIT process optimally levered. This can, e.g., be the case if the assets can be used for some other purpose than generating the current EBIT. A first attempt in this direction is Flor (2002).

Our model cannot capture strategic debt service of the type where it is simply the current coupon rate which is reduced, whereas the capital structure stays fixed as it is modeled in Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997). However, it is fairly easy to extend our model to incorporate both types of debt renegotiations simply by introducing an extra boundary condition above d but below one, say s. As long as $\xi \in (s\xi_0, u\xi_0)$, the debt is serviced with the full coupon rate, $c^*\xi_0$, but when $\xi \in (d\xi_0, s\xi_0]$ the debt is only serviced according to the strategic debt service models. Finally, when ξ eventually hits either $d\xi_0$ or $u\xi_0$, the firm's capital structure is re-optimized and the game starts all over again.

Finally, our model does not include investment decisions. The EBIT process is exogenously given. It is not possible within the model to retain some of the earnings to increase the scale of the EBIT process or to increase the dividend payments by decreasing the scale of the EBIT process. As the model is, all earnings must be paid out to debt holders, equity holders, and the tax authorities.

[26]For the base case parameters the values are very close but not identical. But for more extreme parameter values the differences can be substantial. For example, for $\hat{\mu} = 4\%$ the optimal d and u are .30 and 4.28 for the iterative model, whereas for the reduced form model with the same split (60% of the firm value goes to the equity holders) the optimal d and u are .18 and 3.71.

Appendix A. Fixed Point Solution to the Pricing of Debt and Equity

To set up some notation, we quickly reiterate parts of the standard theory. First, consider a very simple claim paying one unit of account when ξ hits the lower boundary, $\underline{\xi}$, but only if the lower boundary has been hit before the upper boundary, $\bar{\xi}$. The price, denoted $\underline{P}$, of this claim as a function of the current level of EBIT, ξ, can be derived as

$$\underline{P}(\xi) = \frac{-\bar{\xi}^{x_2}\xi^{x_1} + \bar{\xi}^{x_1}\xi^{x_2}}{\Sigma}, \qquad \xi \in [\underline{\xi}, \bar{\xi}],$$

where

$$x_1 = \frac{(\frac{1}{2}\sigma^2 - \hat{\mu}) + \sqrt{(\hat{\mu} - \frac{1}{2}\sigma^2)^2 + 2(1-\tau_i)r\sigma^2}}{\sigma^2},$$

$$x_2 = \frac{(\frac{1}{2}\sigma^2 - \hat{\mu}) - \sqrt{(\hat{\mu} - \frac{1}{2}\sigma^2)^2 + 2(1-\tau_i)r\sigma^2}}{\sigma^2},$$

and

$$\Sigma = \bar{\xi}^{x_1}\underline{\xi}^{x_2} - \underline{\xi}^{x_1}\bar{\xi}^{x_2}.$$

This follows from the fact that $\underline{P}$ solves the linear ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\xi^2\underline{P}''(\xi) + \hat{\mu}\xi\underline{P}'(\xi) - (1-\tau_i)r\underline{P}(\xi) = 0, \tag{29}$$

with the boundary conditions

$$\underline{P}(\underline{\xi}) = 1 \qquad \text{and} \qquad \underline{P}(\bar{\xi}) = 0.$$

Similarly, interchanging the boundary conditions, we find the price of a claim paying one unit of account when ξ hits the upper boundary, $\bar{\xi}$, but only if the upper boundary has been hit before the lower boundary, $\underline{\xi}$. The price, denoted $\bar{P}$, of this claim as a function of the current level of EBIT, ξ, can be derived as

$$\bar{P}(\xi) = \frac{\underline{\xi}^{x_2}\xi^{x_1} - \underline{\xi}^{x_1}\xi^{x_2}}{\Sigma}, \qquad \xi \in [\underline{\xi}, \bar{\xi}].$$

We also need a claim that pays a dividend stream at the rate $\delta\xi_s + b$ at any date $s \in [t, \infty)$. The date t value of this claim can easily be derived (either using risk neutral expectations or Gordon's formula) as

$$\Delta\xi_t + B,$$

where

$$\Delta = \frac{\delta}{(1-\tau_i)r - \hat{\mu}} \qquad \text{and} \qquad B = \frac{b}{(1-\tau_i)r}.$$

With these three claims priced we are able to price all claims of interest in our model. Consider the claim which pays off a dividend stream at the rate $\delta\xi_t + b$ at any date, t, until one of the boundaries has been hit. When it hits one of the boundaries, it pays out a final lump-sum payment. If the lower boundary, $\underline{\xi}$, has been hit first, it pays out $\underline{F}$, and if the upper boundary, $\bar{\xi}$, has been hit first, it pays out $\bar{F}$. The price, denoted $F(\cdot;\delta, b, \underline{\xi}, \underline{F}, \bar{\xi}, \bar{F})$, of this claim as a function of the current level of EBIT, ξ, can easily be derived as

$$F(\xi;\delta, b, \underline{\xi}, \underline{F}, \bar{\xi}, \bar{F}) = \Delta\xi + B + (\underline{F} - \Delta\underline{\xi} - B)\underline{P}(\xi) + (\bar{F} - \Delta\bar{\xi} - B)\bar{P}(\xi), \qquad \xi \in [\underline{\xi}, \bar{\xi}]. \tag{30}$$

Equation (30) has the following easy interpretation: The date t value of getting the dividend stream at the rate $\delta\xi_s + b$ at any date $s \in [t, \infty)$ is $\Delta\xi_t + B$. Eventually ξ will hit either $\underline{\xi}$ or $\bar{\xi}$. If, e.g., $\underline{\xi}$ has been hit first, the claim pays out $\underline{F}$ and the rest of the dividend stream (with a value of $\Delta\underline{\xi} + B$) is forgone. The net present value of this back at date t is $(\underline{F} - \Delta\underline{\xi} - B)\underline{P}(\xi_t)$. The same argument applies for the upper boundary, $\bar{\xi}$.

Plugging in the definitions of $\underline{P}$ and $\bar{P}$ reveals[27]

$$
\begin{aligned}
F(\xi;\delta,b,\underline{\xi},\underline{F},\bar{\xi},\bar{F}) = {} & \frac{1}{\Sigma}\big((\bar{F} - \Delta\bar{\xi} - B)\underline{\xi}^{x_2} - (\underline{F} - \Delta\underline{\xi} - B)\bar{\xi}^{x_2}\big)\xi^{x_1} \\
& + \frac{1}{\Sigma}\big((\underline{F} - \Delta\underline{\xi} - B)\bar{\xi}^{x_1} - (\bar{F} - \Delta\bar{\xi} - B)\underline{\xi}^{x_1}\big)\xi^{x_2} + \Delta\xi + B, \qquad \xi \in [\underline{\xi},\bar{\xi}].
\end{aligned} \tag{33}
$$

A very useful property of the price function, F, is the following positive homogeneity of degree one property

$$
F(\lambda\xi;\delta,\lambda b,\lambda\underline{\xi},\lambda\underline{F},\lambda\bar{\xi},\lambda\bar{F}) = \lambda F(\xi;\delta,b,\underline{\xi},\underline{F},\bar{\xi},\bar{F}), \tag{34}
$$

for any $\xi \in [\underline{\xi},\bar{\xi}]$ and $\lambda \in \mathbb{R}_+$. This is very easily checked by directly applying equation (33).[28]

The economic intuition behind this homogeneity property is quite trivial: if the unit of account was changed, e.g., from \$ to €, on all inputs, then also the value will change accordingly from being measured in \$ to being measured in €. This result heavily relies on the scaling invariant feature of the geometric Brownian motion and that everything except for monetary units is specified in rates.

With this machinery in place we return to the problem of finding the optimal dynamic capital structure of the firm and the values of debt and equity. Basically, debt and equity are claims of the form F just derived. We just have to find δ, b, $\underline{\xi}$, $\underline{F}$, $\bar{\xi}$, and $\bar{F}$ for both debt and equity.

In order to find the after-tax payout rate, $\delta\xi + b$, on debt and equity we need to know the tax rules. Coupon payments paid out to the debt holders are expenses for the firm, i.e. they are subtracted from the EBIT (giving earnings before taxes, EBT) before the firm pays corporate tax. Hence, the total tax paid on coupons is the personal interest tax paid by the debt holders. For dividends the firm must first pay tax on its EBT before dividends can be paid out to the equity holders. On top of that the equity holders pay dividend tax on dividends paid out. Hence, the after-tax payout rate, $\delta\xi + b$, on debt is specified as

$$
\delta\xi + b = (1 - \tau_i)C
$$

[27]Alternatively, equation (33) can be derived by observing that F is the solution to the inhomogeneous ODE (suppressing δ, b, $\underline{\xi}$, $\underline{F}$, $\bar{\xi}$, and $\bar{F}$ in the notation for F)

$$
\frac{1}{2}\sigma^2\xi^2 F''(\xi) + \hat{\mu}\xi F'(\xi) - (1-\tau_i)rF(\xi) + \delta\xi + b = 0, \tag{31}
$$

with the boundary conditions

$$
F(\underline{\xi}) = \underline{F} \qquad \text{and} \qquad F(\bar{\xi}) = \bar{F}.
$$

Recall that the general solution to the ODE (31) is

$$
F(\xi;\delta,b,\underline{\xi},\underline{F},\bar{\xi},\bar{F}) = k_1\xi^{x_1} + k_2\xi^{x_2} + \frac{\delta\xi}{(1-\tau_i)r - \hat{\mu}} + \frac{b}{(1-\tau_i)r}, \qquad \xi \in [\underline{\xi},\bar{\xi}], \tag{32}
$$

where k_1 and k_2 are determined by the boundary conditions. Simple manipulations reveal that

$$
k_1 = \frac{1}{\Sigma}\big((\bar{F} - \Delta\bar{\xi} - B)\underline{\xi}^{x_2} - (\underline{F} - \Delta\underline{\xi} - B)\bar{\xi}^{x_2}\big)
$$

and

$$
k_2 = \frac{1}{\Sigma}\big((\underline{F} - \Delta\underline{\xi} - B)\bar{\xi}^{x_1} - (\bar{F} - \Delta\bar{\xi} - B)\underline{\xi}^{x_1}\big).
$$

[28]That it is true for any $\xi \in (\underline{\xi},\bar{\xi})$ follows directly. That it is also true for $\xi \in \{\underline{\xi},\bar{\xi}\}$ must be checked separately. It comes from the fact that $F(\underline{\xi}) = \underline{F}$ and $F(\bar{\xi}) = \bar{F}$. The reason why, e.g., $F(\bar{\xi}) = \bar{F}$ is because if $\xi = \bar{\xi}$ then the upper boundary is hit immediately and therefore (i) there is no waiting time until one of the two boundaries will be hit and (ii) there is zero probability that the lower boundary will be hit before the upper boundary.

	δ	b
D	0	$(1-\tau_i)C$
$E_{\xi \geq C}$	$1-\tau_e$	$-(1-\tau_e)C$
$E_{\xi < C}$	$1-\epsilon\tau_e$	$-(1-\epsilon\tau_e)C$
$D+E_{\xi \geq C}$	$1-\tau_e$	$(\tau_e-\tau_i)C$
$D+E_{\xi < C}$	$1-\epsilon\tau_e$	$(\epsilon\tau_e-\tau_i)C$

TABLE 2. Values for δ and b for debt (D) and equity (E) both when EBT is positive and when it is negative.

and for equity it is specified as

$$\delta\xi + b = \begin{cases} (1-\tau_e)(\xi - C), & \text{if } \xi \geq C, \\ (1-\epsilon\tau_e)(\xi - C), & \text{if } \xi < C. \end{cases}$$

Here τ_e denotes the effective tax rate on dividends. That is,

$$\tau_e = \tau_c + (1-\tau_c)\tau_d,$$

where τ_c denotes the corporate tax rate, and τ_d denotes the personal dividend tax rate. ϵ denotes the fraction that the effective tax rate is reduced to when earnings after interest payments, $\xi - C$, are negative, cf. footnote 7. The values assigned to δ and b for debt and equity are summarized in table 2. By adding the payout rates of debt and equity we see that there is a tax advantage to debt if and only if the effective tax rate on dividends is higher than the personal tax on interest payments, i.e. $\tau_e > \tau_i$, when EBT is positive, i.e. $\xi \geq C$. However, for $\xi < C$ it is possible that there can be a tax disadvantage to debt. This happens when $\epsilon\tau_e < \tau_i$. For the rest of the paper we assume that $\tau_e > \tau_i$ such that there is a tax advantage to debt for positive EBT. This implies that the optimal capital structure of the firm will include some debt.

For a given (optimal) coupon rate, C^*, on the debt, and a given restructuring policy, $(\underline{\xi}, \bar{\xi})$ (determined endogenously in the model when the debt was issued, e.g. at date zero when the EBIT process was ξ_0), we must specify the value of debt and equity when one of the restructuring boundaries has been hit. However, contrary to a static capital structure model these values are not known since when the restructuring boundaries have been hit, the (possible new) owner will again issue debt and equity and continue operation. To establish some notation we denote these values (the notation is obvious) as $\underline{D}$, $\bar{D}$, $\underline{E}$, and $\bar{E}$. Furthermore, denote the earliest date after the debt issue when a restructuring boundary has been hit as τ. Since the situation at date τ, when the (possible new) owner of the whole firm reissues new debt, is exactly identical to the situation at date zero when the original entrepreneur issued the original debt—except that the EBIT process is now ξ_τ instead of ξ_0—we conjecture that the coupon rate of the newly issued debt will be $\frac{\xi_\tau}{\xi_0}C^*$ at date τ, the new restructuring policy will be $(\frac{\xi_\tau}{\xi_0}\underline{\xi}, \frac{\xi_\tau}{\xi_0}\bar{\xi})$, and the new boundary values will be $\frac{\xi_\tau}{\xi_0}\underline{D}$, $\frac{\xi_\tau}{\xi_0}\bar{D}$, $\frac{\xi_\tau}{\xi_0}\underline{E}$, and $\frac{\xi_\tau}{\xi_0}\bar{E}$. In fact, we will later prove that this conjecture is correct.

To be formal,

Conjecture A.1. (1) *The optimal coupon rate C^* determined just prior to issuing the debt at a given date s when the EBIT process is ξ_s can be written as*

$$C^* = c^*\xi_s,$$

for a given constant c^*.

(2) *The incentive compatible restructuring policy* $(\underline{\xi}, \bar{\xi})$, *which is common knowledge as soon as the coupon rate* C^* *is fixed at a given date* s *when the EBIT process is* ξ_s, *can be written as*

$$\underline{\xi} = d\xi_s$$

and

$$\bar{\xi} = u\xi_s,$$

for given constants $d \in (0,1)$ *and* $u \in (1,\infty)$.

(3) *The values of debt and equity when one of the boundaries (induced by the commonly known restructuring policy fixed by the optimally determined coupon rate at a given date* s *when the EBIT process is* ξ_s*) has been hit can be written as*

$$\underline{D} = \underline{d}\xi_s,$$
$$\bar{D} = \bar{d}\xi_s,$$
$$\underline{E} = \underline{e}\xi_s,$$

and

$$\bar{E} = \bar{e}\xi_s,$$

for given constants $\underline{d}$, $\bar{d}$, $\underline{e}$, *and* $\bar{e}$.

Given Conjecture A.1 we can derive (and denote) the price at any given date t when the EBIT process is ξ_t of debt and equity issued at some date $s \le t$ when the EBIT process was ξ_s, provided that the EBIT process $\{\xi_u\}_{u\in[s,t]}$ in the time period $[s,t)$ has stayed inside the interval $(d\xi_s, u\xi_s)$ as

$$D(\xi_t;\xi_s) = F(\xi_t; 0, (1-\tau_i)c^*\xi_s, d\xi_s, \underline{d}\xi_s, u\xi_s, \bar{d}\xi_s)$$

and

$$E(\xi_t;\xi_s) = F(\xi_t; 1-\tau_e, -(1-\tau_e)c^*\xi_s, d\xi_s, \underline{e}\xi_s, u\xi_s, \bar{e}\xi_s). \tag{35}$$

To be exact, the equity value, E, has only the form (35) if $c^* \le d$. This is due to the asymmetric tax regime the equity holders face. They have to pay at the tax rate τ_e when the firm's earnings (net of coupon payments) are positive, but they can only deduct at the tax rate $\epsilon\tau_e$ when the firm's earnings (net of coupon payments) are negative. Hence, if $c^* \in (d,u)$, then the equity value has to be pieced together in the following way so that it is one-time continuously differentiable

$$E(\xi_t;\xi_s) = \begin{cases} F(\xi_t; 1-\epsilon\tau_e, -(1-\epsilon\tau_e)c^*\xi_s, d\xi_s, \underline{e}\xi_s, c^*\xi_s, e^*\xi_s), & \text{if } \xi_t < c^*\xi_s, \\ F(\xi_t; 1-\tau_e, -(1-\tau_e)c^*\xi_s, c^*\xi_s, e^*\xi_s, u\xi_s, \bar{e}\xi_s), & \text{if } \xi_t \ge c^*\xi_s. \end{cases} \tag{36}$$

Here, e^* is the equity value when $\xi_t = c^*$ and $\xi_s = 1$. This value is determined by requiring the equity value function, E, to be continuously differentiable in the first variable at the point $\xi_t = c^*\xi_s$.[29] Finally,

[29]Formally, we have to add the condition that the equity value at the point where the two parts are pieced together in a one-time continuously differentiable way can be written as $e^*\xi_s$ to our Conjecture A.1. That is, we have to add to part 3 of Conjecture A.1 that the value of equity, at the point where $\xi_t = c^*\xi_s$, can be written as

$$E(c^*\xi_s;\xi_s) = e^*\xi_s$$

for a given constant e^*.

if $c^* \geq u$, the firm always has negative earnings net of coupon payments, so E has the form

$$E(\xi_t;\xi_s) = F(\xi_t; 1-\epsilon\tau_e, -(1-\epsilon\tau_e)c^*\xi_s, d\xi_s, \underline{e}\xi_s, u\xi_s, \bar{e}\xi_s).$$

Appendix B. Verification of Conjecture A.1

In this appendix we verify that there exists solutions to debt and equity that fulfills Conjecture A.1. Equations (5), (6), (8), and (9) immediately verify part 3 of Conjecture A.1 and that

$$\begin{aligned}
\underline{d} &= \min\{(1-\alpha)Ad, D\},\\
\bar{d} &= (1+\lambda)D,\\
\underline{e} &= \max\{(1-\alpha)Ad - D, 0\},
\end{aligned}$$

and

$$\bar{e} = Au - (1+\lambda)D.$$

For given c^*, d, and u, the two constants D and E can be found by solving for the initial debt and equity values for $\xi_t = \xi_s = 1$. That is, using the expression of F from equation (30) we have the following two equations in two unknowns[30]

$$\begin{aligned}
D = {} & \Delta_D\big(1 - d\underline{P}(1) - u\bar{P}(1)\big) + B_D\big(1 - \underline{P}(1) - \bar{P}(1)\big)\\
& + \min\{(1-\alpha)Ad, D\}\underline{P}(1) + (1+\lambda)D\bar{P}(1)
\end{aligned}$$

and

$$\begin{aligned}
E = {} & \Delta_E\big(1 - d\underline{P}(1) - u\bar{P}(1)\big) + B_E\big(1 - \underline{P}(1) - \bar{P}(1)\big)\\
& + \max\{(1-\alpha)Ad - D, 0\}\underline{P}(1) + \big(Au - (1+\lambda)D\big)\bar{P}(1).
\end{aligned}$$

Here,

$$\Delta_D = 0, \qquad \Delta_E = \frac{(1-\tau_e)}{(1-\tau_i)r - \hat{\mu}}, \qquad B_D = \frac{c^*}{r}, \qquad \text{and} \qquad B_E = \frac{-(1-\tau_e)c^*}{(1-\tau_i)r}.$$

For a given c^*, we can find d and u by the two smooth pasting conditions, equations (7) and (10). Unfortunately, these two equations in two unknowns can only be solved numerically. However, by Euler's theorem E_1 is positive homogeneous of degree zero because E itself is positive homogeneous of degree one, cf. equation (3). That is,

$$E_1(\lambda\xi_t;\lambda\xi_s) = E_1(\xi_t;\xi_s),$$

for any $\xi_t \in [d\xi_s, u\xi_s]$ and $\lambda \in \mathbb{R}_+$. Hence, equations (7) and (10) are identical independent of the actual level of ξ_0. Therefore, the solutions d and u are also independent of the actual level of ξ_0. That is, we have verified part 2 of Conjecture A.1.[31]

Finally, the optimal coupon rate of the debt, which is determined just prior to the debt issue at date zero, is given by

$$C^* = \operatorname*{argmax}_{C\in\mathbb{R}_+} A(\xi_0).$$

[30] If $c^* \in (d,u)$, there are three unknowns: D, E, and e^*. The third equation comes from differentiability of the equity function, E, at $\xi_t = c^*\xi_s$, cf. equation (36).

[31] In fact, this positive homogeneity property of degree zero of E_1 can also be used to verify that e^* is independent of the actual level of ξ_0. This verifies the missing conjecture in the case where $c^* \in (d,u)$, cf. footnote 29.

That is so because it is the original entrepreneur who determines the coupon rate of the perpetual debt that he or she would like to issue. Naturally, the entrepreneur sets the coupon rate in order to maximize his or her own value. Moreover, rewriting C as $c\xi_0$ gives exactly the same optimization problem since ξ_0 is positive

$$C^* = \operatorname*{argmax}_{C\in\mathbb{R}_+} A(\xi_0) = \xi_0\big(\operatorname*{argmax}_{c\in\mathbb{R}_+} \frac{A(\xi_0)}{\xi_0}\big).$$

Plugging in the definition of $A(\xi_0)$ from equation (4) gives

$$\begin{aligned}
c^* &= \operatorname*{argmax}_{c\in\mathbb{R}_+} \frac{E(\xi_0;\xi_0) + (1-k)D(\xi_0;\xi_0)}{\xi_0} \\
&= \operatorname*{argmax}_{c\in\mathbb{R}_+} \Bigg(\frac{F(\xi_0; 1-\tau_e, -(1-\tau_e)c\xi_0, d(c)\xi_0, \underline{e}(c)\xi_0, u(c)\xi_0, \bar{e}(c)\xi_0)}{\xi_0} \\
&\qquad + (1-k)\frac{F(\xi_0; 0, (1-\tau_i)c\xi_0, d(c)\xi_0, \underline{d}(c)\xi_0, u(c)\xi_0, \bar{d}(c)\xi_0)}{\xi_0} \Bigg) \\
&= \operatorname*{argmax}_{c\in\mathbb{R}_+} \big(F(1; 1-\tau_e, -(1-\tau_e)c, d(c), \underline{e}(c), u(c), \bar{e}(c)) \\
&\qquad + (1-k)F(1; 0, (1-\tau_i)c, d(c), \underline{d}(c), u(c), \bar{d}(c)) \big).
\end{aligned}$$

Note that we have emphasized the dependence of d, u, $\underline{d}$, $\bar{d}$, $\underline{e}$, and $\bar{e}$ on the given coupon rate parameter, c, cf. part 2 and 3 of Conjecture A.1. Finally, notice that the optimal coupon rate parameter, c^*, in the above optimization is independent of the initial level ξ_0 so we have verified the last part (part 1) in Conjecture A.1. That is, we have now verified that there exists a fixed-point solution to our debt and equity valuation problem giving us solutions to the value of debt and equity fulfilling Conjecture A.1. Because of our conjecture-verification method of finding the fixed-point solution, we cannot rule out that there might be other fixed-point solutions which do not fulfill Conjecture A.1.

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