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Discussion paper

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On the Competitive Effect of Informative Advertising*

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Abstract

This paper analyses the competitive effects of informative advertising. The seminal work by Grossman and Shapiro (1984) show that informative advertising results in lower prices and that firms may benefit from advertising restrictions. A crucial assumption in their model is that captive (partially informed) consumers are not price responsive. Replicating their model in a Hotelling duopoly version, we show that results are in fact reversed if we allow for captive consumers to respond to prices. We then use general demand functions and derive exact conditions for the competitive effect to prevail. A main result is that the procompetitive effect depends on the nature of competition and the relative price elasticities of the monopoly and the competitive demand segments.

Keywords: Informative Advertising; Price Competition; Product differentiation

JEL Classification: D83; L13; M37

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1 Introduction

Informative advertising, as opposed to persuasive advertising, is generally perceived to promote competition (Bagwell, 2007). When a firm advertises, consumers receive (at low costs) information about products, prices, etc. This information is claimed to make the firm's demand curve more price elastic and competition more intense, resulting in lower prices and profits.[1] In this paper, we challenge the robustness of the pro-competitive effect of informative advertising.

Butters (1977) offers a first formal analysis of informative advertising in a multi-firm setting. Firms produce homogeneous products (at constant unit costs) and compete in terms of prices and advertising. Advertising is distributed randomly and informs consumers about a firm's (product's) existence and price, resulting in the following three segments of consumers: (i) *uniformed* consumers who receive no ads, and therefore do not buy any of the products; (ii) *captive* consumers who receive an ad from only one firm and buy this product provided that the price is below their reservation price; and (iii) *selective* consumers who receive ads from more than one firm and buy the product with the lowest price.[2]

Grossman and Shapiro (1984), henceforth GS, extend the work by Butters (1977) to horizontally differentiated products using a Salop-type model. In this setting advertising informs not just about existence and price, but also about the firm's *location* (or the product's characteristics). Thus, selective consumers do not necessarily choose the product with lowest price,

[1] It is also argued that informative advertising can faciliate entry, as it provides a means through which a new entrant can inform potential buyers (Bagwell, 2007). However, several papers have also looked at strategic incentives for the incumbent to use (informative) advertising to deter (or accommodate) entry; see, e.g., Schmalensee (1983) and Ishigaki (2000) for homogeneous products and Fudenberg and Tirole (1984) and Boyer and Moreaux (1999) for differentiated products. See also Brekke and Straume (2009) of an application to pharmaceutical markets. In this paper we do not address the issue of advertising and entry.

[2] Butters (1977) shows that firms adopt mixed strategies in any Nash equilibrium when the number of firms is finite. However, in the limit case where the number of firms becomes sufficiently large, firms charge prices above marginal costs but earn zero profits in expectation (due to advertising costs). Thus, this is an equilibrium model of monopolistic competition with informative advertising.

but balance price differences against travelling costs and buy the product that yields the higher net utility. A striking result from their analysis is that informative advertising triggers competition and leads to *lower* prices.[3] In the same line GS show that profits may well *decrease* when advertising becomes *less* costly. While a more efficient advertising technology increases firms' incentives to advertise, more advertising *triggers* price competition. The net effect on profits depends on the strength of the direct cost effect relative to the strategic price effect. GS show that the latter can dominate, suggesting that firms may benefit from advertising restrictions as they soften price competition.

A crucial assumption in GS is that the demand from captive consumers (who only know about one of the products) is *perfectly price inelastic*.[4] The reservation price is assumed to be sufficiently high, such that *all* captive consumers buy (one unit of) the product they are informed about irrespective of the price. Consequently, only demand from selective consumers (informed about more than one product) is elastic with respect to prices. Thus, advertising will *by assumption* lead to lower demand elasticity as it implies that the competitive segment becomes larger.

We find this assumption quite restrictive. In the current paper, we therefore revisit the GS model by allowing for demand from captive consumers to be price elastic. In the first part we replicate their model by using the familiar Hotelling version (Tirole, 1988: 292-4). In the second part we generalise this model by using general demand and advertising cost functions. In both parts we first derive the price equilibrium for given levels of information (advertising). Afterwards, we endogenise the degree of information by allowing for this to be a choice variable for the firms, as in the informative advertising models, and derive the symmetric price-advertising equilibrium.

In the Hotelling setting we show that the pro-competitive effect of informative advertising is in fact *reversed* once we allow for demand in the

[3] Using a random utility, non-localized competition model, Christou and Vettas (2008) also find that higher advertising levels are associated with lower prices.

[4] This assumption is indeed made by most papers, see, e.g., Butters (1977), Meurer and Stahl (1994), Ishigaki (2000), Christou and Vettas (2008), Simbanegavi (2009). See also Tirole (1988: 292-4).

monopoly segment to respond to prices. Informative advertising now leads to *higher* prices. The reason is that partially informed consumers have (on average) higher transport (mismatch) costs, and therefore are more price responsive than fully informed consumers. We also show that a more costly advertising technology reduces prices and profits, implying that firms never benefit from advertising restrictions. Thus, the pro-competitive effect of informative advertising is highly sensitive to the extreme assumption of price inelastic demand in the monopoly segment; an assumption that might be unreasonable considering markets for differentiated products.

In a generalised version of the basic model, where we abstract from the Hotelling framework, we derive general conditions for the competitive effect of informative advertising to prevail. A main result is that the competitive effect depends on the nature of competition and the relative price elasticities of the monopoly and the competitive demand segments created by informative advertising. More precisely, we show that informative advertising leads to lower (higher) prices if and only if prices are strategic complements and partially informed consumers are less (more) price elastic than the fully informed consumers. These results confirm our findings in the specialised Hotelling version of the informative advertising model.

Our paper is not the first to report a positive relationship between informative advertising and prices. This relationship is present in Soberman (2004), Brekke and Kuhn (2006) and Hamilton (2009) who all relax the assumption of perfect price inelasticity in the monopoly demand segment.[5] The main contributions of the current paper is to investigate the competitive effects of informative advertising in great detail and derive more general conditions for the competitive effects. First, we consider the case of exogenous and potentially asymmetric information levels. Here we show that more consumers informed about the *own* product affects *own* pricing only through the price response by the rival (strategic complements), while more consumers

[5] Soberman (2004) is a short note that only focuses on the effect on prices, ignoring effects on demand and profits. Brekke and Kuhn (2006) is an application to the pharmaceutical market, and is not focusing on competitive effects in general. Hamilton (2009) is mainly concerned with the welfare properties, i.e., whether informative advertising is over- or undersupplied, though competitive effects are mentioned.

informed about the *rival* product have a direct effect on *own* pricing (lower prices to mitigate loss in market shares). Moreover, we show that the firm with more informed consumers sets a lower (higher) price if demand in the monopoly segment is more (less) price elastic than the demand in the competitive segment. However, irrespective of the price effects, the firm with more informed consumers always obtains a higher profit.

Second, we derive the comparative statics with respect to advertising technology, product differentiation and consumers' valuation of the products (reservation price). Higher consumer valuation is always beneficial for the firms when the monopoly demand segment is responding to prices. A higher degree of product differentiation leads to higher prices, but the effects on advertising and profits depends on whether or not the partially informed consumers are price sensitive or not. Similarly, a more efficient advertising technology always boosts advertising incentives, but the effects on prices and profits depend again on the price responsiveness in the monopoly demand segment.

Finally, we propose a more general demand system in order to investigate more general conditions for the existence of the pro-competitive effect of informative advertising. Here we show that the effect depends on the nature of competition and the relative price elasticities of the two segments created by informative advertising, as explained above.

Other related papers include the following. Simbanegavi (2009) uses a duopoly version of the Salop model to study the incentives for semicollusion (on either price or advertising). This paper, too, recognises that consumers in the monopoly segment may be responsive to prices, but in the equilibrium analysis attention is restricted to the case with price inelastic demand from captive consumers. Christou and Vettas (2008) address the competitive effects of informative advertising but on the basis of a very different modelling approach. They use a random utility model, where each consumer's gross valuation of a product is randomly drawn from some distribution and observed only after the receipt of an ad. They study the equilibrium properties both under non-localized and localized (GS model) competition and show the correspondence between the two models. A main finding is that pure strategy equilibrium might fail to exists as firms may find it profitable to deviate to a high price serving only captive consumers. This feature is also present in our paper, and we carefully derive the existence condition, which is not done in the previous studies. Christou and Vettas (2008) show that when the number of firms increases, advertising becomes lower, while the effect on prices is ambiguous. Thus, there might be a positive relationship between advertising and prices as the number of firms increases, but not for a given number of firms. In Meurer and Stahl (1994) consumers observe prices while firms decide whether to inform them about product characteristics. In Bester and Petrakis (1995) consumers know that two firms exists and the price of the product in their region (local market), but only learn the price from the other firm once they have received an ad. However, none of these studies have scrutinized the common, but surely not innocent, assumption of perfectly price inelastic demand in the monopoly segment. The present paper seeks to shed more light on this issue.

The rest of the paper is organised as follows. In section 2 we present the Hotelling duopoly version of the GS model. In section 3 we apply general demand (and advertising cost) functions in the duopoly framework. In section 4 we conclude the paper.

2 A Hotelling Duopoly Model

We start by replicating the duopoly version of Grossman and Shapiro (1984), henceforth GS, as presented in Tirole (1988: 292-4). Consider a market with two firms, indexed by $i = 1, 2$, offering one product each at price p_i. The firms (or products) are located at either end of the unit interval $S = [0, 1]$, where $z_1 = 0$ and $z_2 = 1$ are the locations of firm (product) 1 and 2, respectively.

In this market there is a uniform distribution of consumers on the interval S with mass 1. Each consumer demands one unit of either product or no product at all. The utility to an arbitrary consumer $x \in S$ of consuming product i is given by

$$u_i = v - p_i - t\,|x - z_i|\,, \tag{1}$$

where v is the gross consumption benefit (or reservation price), and t is the travelling cost per unit distance between the consumer's location x and the location of product (or firm) i.

Consumers are ex ante uninformed about the products available in the market. To generate demand, each firm must advertise its product to the consumers.[6] We let $a_i \in (0, 1)$ be the advertising level of product i. Advertising is assumed to contain true information about product characteristics (location) and price. In our model and similar to GS and Butters (1977), a_i is equivalent to the share of consumers who obtain information about product i.

Demand of firm i, with potential size a_i, can then be decomposed into two parts: (i) a fraction $1 - a_j$ of *captive* consumers who are informed only about product i; and (ii) a fraction a_j of *selective* consumers who are informed about both products. The residual fraction $(1 - a_i)(1 - a_j)$ of consumers remain uninformed and do not demand either product. We refer to the first segment as the *monopoly* segment (of firm i), and the second segment as the *competitive* segment (for both firms).

Consumers informed about both products trade off relative prices and distances, and choose the product that provides the higher net utility. The consumer who is exactly indifferent between product 1 and 2, i.e., for whom $u_1(\widehat{x}) = u_2(\widehat{x})$, is located at

$$\widehat{x} = \begin{cases} 1 & if \quad p_1 \leq p_2 - t \\ \frac{1}{2} - \frac{p_1 - p_2}{2t} & if \quad p_1 \in (p_2 - t, p_2 + t) \\ 0 & if \quad p_1 \geq p_2 + t \end{cases} . \tag{2}$$

All (fully informed) consumers to the left of $\widehat{x}$ demand product 1, while the residual fraction demand product 2. In the subsequent analysis, we assume existence of a competitive segment, which requires the following two conditions to be fulfilled (in equilibrium): (i) $\widehat{x} \in (0, 1) \Leftrightarrow t > |p_1 - p_2|$, and (ii) $u_i(\widehat{x}) > 0 \Leftrightarrow v - \frac{t}{2} > \frac{p_1 + p_2}{2}$. Thus, the transport cost (t) must be sufficiently high relative to the price difference, and the average net benefit

[6] As in GS we abstract from consumer search for products.

cannot be lower than the average price level. Below we report the exact conditions in each part of the analysis.

Consumers only informed about product i, demand this product provided that consumption yields non-negative utility. The consumer who is exactly indifferent between buying or not buying product i, i.e. for whom, $u_i(\widetilde{x}_i) = 0$ is located at:

$$\widetilde{x}_i = \begin{cases} 1 - z_i & if \quad p_i \leq v - t \\ \left|\frac{v-p_i}{t} - z_i\right| & if \quad p_i \in (v-t, v) \\ z_i - 0 & if \quad p_i \geq v \end{cases} . \tag{3}$$

Thus, if the reservation price (transport cost) is sufficiently high (low) relative to the price, then all partially informed consumers will buy product i. However, if the gross surplus for the most distant consumer, $v - t$, is sufficiently low, then consumers will trade-off the benefit against the costs, and some (those located farthest away from the firm) decide not to buy the product. In the extreme case of a very low v, no consumer is willing to buy the product, but this case is ruled out by the assumption of a competitive segment. Notably, GS focus solely on the first case with a *perfectly price inelastic* monopoly demand segment. In the following, we will allow for partially informed consumers to respond to price.

The demand for product 1 and 2 can now be written as:

$$D_1 = \int_0^{\widetilde{x}_1} a_1(1-a_2)\,ds + \int_0^{\widehat{x}} a_1 a_2 ds = a_1(1-a_2)\,\widetilde{x}_1 + a_1 a_2 \widehat{x}, \tag{4}$$

$$D_2 = \int_{\widetilde{x}_2}^{1} a_2(1-a_1)\,ds + \int_{\widehat{x}}^{1} a_1 a_2 ds = a_2(1-a_1)(1-\widetilde{x}_2) + a_1 a_2 (1-\widehat{x}). \tag{5}$$

where the first term (in both equations) is the demand from partially informed consumers, corresponding to the firm's monopoly segment, whereas the second term is the competitive segment shared by the firms. Notice that the assumption of a competitive segment implies that $\widetilde{x}_2 < \widehat{x} < \widetilde{x}_1$.

2.1 Price equilibrium with exogenous information

Let us start by assuming that the degree of product information among consumers is exogenous, i.e., $a_i \in (0,1)$, $i = 1,2$. The gross profit to firm i is given by

$$V_i = (p_i - c)\, D_i, \tag{6}$$

where c is a constant marginal production cost. Without loss of generality, we let $c = 0$ in the following analysis. The firms set price in order to maximise (gross) profits. Prices are set simultaneously and independently.

Price inelastic monopoly demand

Maximising (6) with respect to price, assuming that $v - t \geq p_i$, and solving the corresponding set of first-order conditions, yields the following price equilibrium:[7]

$$p_i^A = t\left(\frac{4a_i + 2a_j - 3a_i a_j}{3a_i a_j}\right), \quad i,j = 1,2 \text{ and } i \neq j. \tag{7}$$

Inserting (7) into (2), (4) and (6), we obtain

$$\widehat{x}^A = \frac{1}{2} - \frac{a_1 - a_2}{3a_1 a_2}, \tag{8}$$

$$D_i^A = \frac{4a_i + 2a_j - 3a_i a_j}{6}, \tag{9}$$

$$V_i^A = t\frac{(4a_i + 2a_j - 3a_i a_j)^2}{18 a_i a_j}. \tag{10}$$

The price equilibrium defined by (7) constitutes an equilibrium if and only if the following assumptions are satisfied:[8]

$$\widetilde{x}_i^A = 1 \quad \Leftrightarrow \quad v > t\left(\frac{4a_i + 2a_j}{3a_i a_j}\right). \tag{11}$$

[7] The second order conditions are always fulfilled. Furthermore, the Jacobian is strictly positive, i.e., $J = \frac{3}{4t^2}a_i^2 a_j^2 > 0$, so we have a unique and stable equilibrium.

[8] It is readily checked that the conditions in (12) and (11) also guarantee $u_i\left(\widehat{x}^A\right) > 0$.

$$\widehat{x}^A \in (0,1) \Leftrightarrow \left|\frac{a_i - a_j}{a_i a_j}\right| < \frac{3}{2}, \tag{12}$$

In addition, we need to ensure that each firm cannot profitably deviate by charging such a high price that only the monopoly segment is served. The maximum price firm i can charge is $p_i^D = v - t$ (with a covered market), which yields the following deviation profits: $V_i^D = (v-t)\, a_i (1-a_j)$. Thus, existence of the price equilibrium in (7) requires that $V_i^A \left(p_i^A, p_i^A\right) \geq V_i^D \left(p_i^D, p_j^A\right)$, which is always true if

$$v \leq t\left(1 + \frac{(4a_i + 2a_j - 3a_i a_j)^2}{18 a_i^2 a_j (1-a_j)}\right). \tag{13}$$

Assuming the restrictions in (11)-(13) to hold, we can now investigate the effect of information on the equilibrium outcomes.

The impact of more consumers informed of own product (a_i) and rival product (a_j) on equilibrium outcomes is obtained by taking the partial derivatives of (7)-(10), yielding the following results:

Proposition 1 *In a Hotelling duopoly model with imperfect information and a price inelastic monopoly segment, more information regarding own product (a_i) and rival product (a_j) has the following effects:*

$$\begin{aligned}
\frac{\partial p_i^A}{\partial a_i} &< 0, \frac{\partial p_i^A}{\partial a_j} < 0, \frac{\partial \widehat{x}^A}{\partial a_1} < 0, \frac{\partial \widehat{x}^A}{\partial a_2} > 0, \\
\frac{\partial D_i^A}{\partial a_i} &> 0, \frac{\partial D_i^A}{\partial a_j} \gtrless 0, \frac{\partial V_i^A}{\partial a_i} \gtrless 0, \frac{\partial V_i^A}{\partial a_j} < 0.
\end{aligned}$$

A proof is provided in the Appendix.

Thus, more information about either of the products leads to lower equilibrium prices. In the limit case $a_i = a_j = 1$, then $p_i^A = t$, the standard outcome under full information (with $c = 0$). A greater number of informed consumers implies a larger competitive segment, which in turn triggers price competition. This result is consistent with GS. Here, we show that the pro-competitive effect is robust to asymmetric levels of product information (within the boundaries defined above).

Considering demand, information has a direct effect and an indirect effect via changes in relative prices. A larger share of consumers informed about product i increases both the monopoly segment (of firm i) and the competitive segment (shared by the firms). The market share in the competitive segment is negatively affected by own information. However, the direct effect dominates, yielding a positive net demand effect of own information. A larger share of consumers informed about the rival product has a negative direct demand effect, as consumers are shifted from firm i's monopoly segment to the competitive segment. However, since p_i decreases in a_j by more than p_j, firm i captures a larger market share in the competitive segment, resulting in an ambiguous net demand effect.

Finally, the effect of consumer information about the own product on (gross) profit is ambiguous: a_i, reduces price but increases demand. However, more information about the rival product reduces own profits: a higher a_j reduces price and potentially demand of firm i.

A comparison across firms yields

$$p_i^A - p_j^A = \frac{2t\left(a_i - a_j\right)}{3a_i a_j},\ D_i^A - D_j^A = \frac{a_i - a_j}{3}$$
$$V_i^A - V_j^A = \frac{2t\left(a_i + a_j - a_i a_j\right)\left(a_i - a_j\right)}{3a_i a_j},$$

from which we obtain the following.

Proposition 2 *In a Hotelling duopoly model with imperfect product information and a price inelastic monopoly segment, the firm with more (less) informed consumers has higher (lower) price, demand and profit, i.e.,*

$$p_i^A > (\leq)\, p_j^A,\ D_i^A > (\leq)\, D_j^A \text{ and } V_i^A > (\leq)\, V_j^A \quad \text{if } a_i > (\leq)\, a_j.$$

Thus, for the firm with more informed consumers price, demand and gross profit are larger. At a first glance, this might seem surprising, since prices are decreasing in the degree of consumer information. However, as the firm with more informed consumers has a larger monopoly segment, where demand is price inelastic, it can sustain a higher (relative) price.

Concerning demand, there are offsetting effects. The firm with more informed consumers enjoys a larger monopoly segment, but a lower market share in the competitive segment owing to the higher price. However, the increase in the monopoly segment always dominates the loss of competitive market share, implying that the firm with more informed consumers has a higher demand. Finally, since both price and demand is larger for the firm with more informed consumers, it also enjoys a greater gross profit.

Price elastic monopoly demand

Maximising (6), assuming that $v - t < p_i < v$, and solving the corresponding set of first-order conditions, yields the following price equilibrium:[9]

$$p_i^B = \frac{v\left(8 - 4a_i - 6a_j + 2a_i a_j\right) + t a_j \left(4 - a_i\right)}{16 - 8a_i - 8a_j + 3a_i a_j}, \quad i, j = 1, 2;\ i \neq j. \tag{14}$$

Inserting (14) into (2)-(6), we get the following equilibrium outcomes

$$\widehat{x}^B = \frac{1}{2} + \frac{(2t - v)\left(a_1 - a_2\right)}{t\left(16 - 8a_1 - 8a_2 + 3a_1 a_2\right)}, \tag{15}$$

$$\widetilde{x}_i^B = \left| \frac{v\left(2 - a_i\right)\left(4 - a_j\right) - t a_j \left(4 - a_i\right)}{t\left(16 - 8a_i - 8a_j + 3a_i a_j\right)} - z_i \right|, \tag{16}$$

$$D_i^B = \frac{a_i\left(2 - a_j\right)}{2t} \cdot \left(p_i^B\right), \tag{17}$$

$$V_i^B = \frac{a_i\left(2 - a_j\right)}{2t} \cdot \left(p_i^B\right)^2. \tag{18}$$

[9] The second order conditions are always fulfilled. Furthermore, the Jacobian is strictly positive, i.e., $J = \frac{a_i a_j (16 - 8a_i - 8a_j + 3a_i a_j)}{4t^2} > 0$, so we have a unique and stable equilibrium.

The price equilibrium, defined by (14), requires the following assumptions to be satisfied:[10]

$$\widetilde{x}_i^B < 1 \quad \Leftrightarrow \quad v < 2t, \tag{19}$$

$$\widehat{x}^B \in (0,1) \quad \Leftrightarrow \ t > \left| \frac{2\,(2t - v)\,(a_i - a_j)}{16 - 8a_i - 8a_j + 3a_i a_j} \right|, \tag{20}$$

$$u\left(\widehat{x}^B\right) > 0 \quad \Leftrightarrow \quad v > \frac{t}{2}\frac{(4 - a_i)\,(4 - a_j)}{8 - 3a_i - 3a_j + a_i a_j} \in \left(t, \frac{3t}{2}\right). \tag{21}$$

The impact of more consumers informed of own product (a_i) and rival product (a_j) on the equilibrium outcomes is found by taking the partial derivatives of (14)-(18), yielding the following result:

Proposition 3 *In a Hotelling duopoly model with imperfect product information and a price elastic monopoly segment, more information about own product (a_i) and rival product (a_j) have the following effects:*

$$\begin{aligned} \frac{\partial p_i^B}{\partial a_i} &> 0, \frac{\partial p_i^B}{\partial a_j} > 0, \frac{\partial \widehat{x}^B}{\partial a_1} > 0, \frac{\partial \widehat{x}^B}{\partial a_2} < 0, \ \frac{\partial \widetilde{x}_i^B}{\partial a_i} < 0, \\ \frac{\partial \widetilde{x}_i^B}{\partial a_i} &< 0, \frac{\partial D_i^B}{\partial a_i} > 0, \frac{\partial D_i^B}{\partial a_j} < 0, \frac{\partial V_i^B}{\partial a_i} > 0, \frac{\partial V_i^B}{\partial a_j} < 0. \end{aligned}$$

A proof is provided in the Appendix.

In contrast to the previous case (Proposition 1), we now find that equilibrium prices *increase* in the number of consumers being informed about the own or rival product. This result is *not* consistent with the pro-competitive finding by GS. The reason is that the marginal consumer informed about only one product is not just price responsive, but, in fact, *more* price responsive than the marginal consumer informed about both products. A marginal increase in price reduces demand in the monopoly segment with $-1/t$, while

[10] In this case, we do not need further conditions for existence. Consider a downward deviation from the equilibrium price, where firm 1 sets a low price p_1^D such that $\widehat{x} = 1$, i.e. it claims the full competitive segment when $p_2 = p_2^B$. Here, $p_1^D \leq p_2^B - t \leq v - t$ must hold, the first inequality being implied by $\widehat{x} = 1$, the second by (21). But then, $\widetilde{x}_1^D = 1$, so that $V_1\left(p_1^D, p_2^B\right) = p_1^D a_1$. Noting that $p_1^D = p_2^B - t$ is the deviation price attaining the highest level of profit, it is readily verified now that $p_1^D \leq 0 \Leftrightarrow v \leq 2t$, which is true from (20). But then, $V_1\left(p_1^D, p_2^B\right) \leq 0 < V_1\left(p_1^B, p_2^B\right)$, implying a deviation is never profitable.

in the competitive segment the effect is $-1/2t$.[11] The intuition is that consumers informed about only one product face on average higher transport costs (lower utility) than consumers informed about both products.

A greater number of consumers informed about the own product increases the size of both the competitive and monopoly segment. The indirect effects via prices are offsetting: The firm gains market share in the competitive segment, as the rival raises price by more than the firm itself. But the price increase leads to a loss of consumers in the monopoly segment. The net effect on demand, however, of own information is always positive. In contrast, demand falls as more consumers become informed about the rival product. The reason is that as a_j increases, consumers are shifted from firm i's monopoly segment to the competitive segment. The indirect effects are also negative: (i) demand in the monopoly segment drops due to higher prices; and (ii) the competitive segment market share is reduced due to changes in relative prices.

Finally, a greater number of consumers informed about the own product has a positive effect on own profits since both price and demand increase. The effect of more consumers being informed about the rival product is negative: although the own price can be increased, this is more than offset by the loss in demand.

Comparing the equilibrium outcomes, we obtain

$$p_i^B - p_j^B = \frac{2(a_j - a_i)(2t - v)}{16 - 8a_i - 8a_j + 3a_i a_j},$$

$$D_i^B - D_j^B = \frac{(a_i - a_j)(8v - 4va_i - 4va_j + ta_i a_j + va_i a_j)}{t(16 - 8a_i - 8a_j + 3a_i a_j)},$$

$$V_i^B - V_i^B = \frac{(a_i - a_j)(2v^2(2 - a_i - a_j) + a_i a_j t(2v - t))}{t(16 - 8a_i - 8a_j + 3a_i a_j)}.$$

We can now report the following results:

Proposition 4 *In a Hotelling duopoly model with imperfect information and*

[11] Alternatively, we can calculate the elasticities in the two segments. It is readily verified that $|\varepsilon_{\tilde{x}_i}| > |\varepsilon_{\hat{x}_i}| \Leftrightarrow v - t < p_j$, which is always true for the case of price elastic monopoly demand.

a price elastic monopoly segment, the firm with more (less) informed consumers has lower (higher) prices, but higher (lower) demand and profit, i.e.,

$$p_i^B < (\geq) \, p_j^B, \;\; D_i^B > (\leq) \, D_i^B \;\; \textit{and} \; V_i^B > (\leq) \, V_j^B \quad \textit{if} \;\; a_i > (\leq) \, a_j.$$

Proof. The price ranking follows from the equilibrium condition (19), i.e., $2t > v$, whereas the profit ranking follows from the equilibrium condition (21), i.e., $v > \frac{t}{2} \frac{(4-a_i)(4-a_j)}{8-3a_i-3a_j+a_ia_j} \in \left(t, \frac{3t}{2}\right)$. ■

In contrast to the previous case (Proposition 2), we now find that the firm with more informed consumers has a *lower* price. The reason is that demand is more price elastic in the monopoly than in the competitive segment, as explained above. As the firm with a larger fraction of informed consumers has a relatively larger monopoly segment, it charges a lower price.

As in the previous case, demand is higher for the firm with more informed consumers. If $a_i > a_j$, firm i has a larger monopoly segment than firm j. Furthermore, as firm i charges a relatively lower price, it attracts more consumers both within the monopoly and the competitive segment. All of these effects contribute unambiguously towards a higher demand. Finally, gross profit is also higher for the firm with more informed consumers. Although the firm charges a lower price, the increase in demand is always dominating.

2.2 Advertising and price competition

Let us now endogenise the degree of product information by allowing the firms to advertise. Employing the standard informative advertising model, as introduced by Butters (1977) and GS, we denote by $C(a_i, k)$ the cost of reaching with ads a fraction a_i of consumers. Advertising cost is assumed to be increasing and strictly convex in a_i.[12] To facilitate explicit solutions, we follow Tirole (1988) by assuming a quadratic function, i.e., $C(a_i, k) = ka_i^2/2$, where $k > 0$ is an advertising cost parameter. Firm i's profit function can now be written as:

$$\pi_i = p_i \cdot D_i - \frac{k}{2} a_i^2. \tag{22}$$

[12] For details about the advertising technology, see Grossman and Shapiro (1984).

As in GS and Tirole (1988), the firms set prices and advertising simultaneously and independently in order to maximise profits.

Price inelastic monopoly demand

Maximising (22) with respect to price and advertising, assuming that $v - t \geq p_i$, and solving the corresponding set of first-order conditions yields the following symmetric equilibrium:

$$p^C = t\left(\frac{2 - a^C}{a^C}\right) = \sqrt{2kt}, \tag{23}$$

$$a^C = \frac{2p^C}{2k + p^C} = \frac{2}{1 + \sqrt{2k/t}}, \tag{24}$$

which is identical to the one reported in Tirole (1988: 292-4). The following two assumptions ensure that (23) and (24) constitute an equilibrium:

$$\widetilde{x}^C = 1 \quad \Leftrightarrow \quad v \geq t + \sqrt{2kt}, \tag{25}$$

$$a^C \leq 1 \Leftrightarrow k \geq t/2. \tag{26}$$

Existence of the price-advertising equilibrium requires that (13) is satisfied for the equilibrium values $a_i = a_j = a^C$. Inserting (24) into (13), we get the following condition

$$v \leq t + \frac{2k}{\sqrt{2k/t} - 1}. \tag{27}$$

It can easily be shown that the RHS of (25) always is lower than the RHS of (27) for all $k \geq t/2$.

Assuming (25)-(27) to hold, we can analyse the equilibrium characteristics. First, we observe that $\partial p^C/\partial a^C < 0$, whereas $\partial a^C/\partial p^C > 0$. Hence, greater levels of advertising stimulate price competition (i.e. lower prices) and higher prices stimulate advertising competition (i.e. higher levels of advertising). We also see that price and advertising levels are increasing in product differentiation (t), whereas a more costly advertising technology (k) induces less advertising but higher prices. In the limit case, where $k = t/2$, so that $a^c = 1$, we get the full information outcome, with $p^C = t$.

Inserting (23) and (24) into (22), we obtain the firms' equilibrium profit:

$$\pi^C = \frac{2k}{\left(1+\sqrt{2k/t}\right)^2}. \tag{28}$$

As expected, profit increases in the degree of product differentiation, reflecting higher prices and a greater level of demand due to additional advertising. More surprisingly, however, profit also increases in the costliness of advertising, as measured by k. As firms engage in less advertising, the corresponding decrease in price competition overcompensates the direct tendency towards higher advertising costs. This is precisely the result found by GS and Tirole (1988). We can summarise in the following proposition:

Proposition 5 *The following holds for a Hotelling duopoly model with informative advertising and a price inelastic monopoly segment:*

(i) a higher advertising cost (k)*, lowers advertising, increases prices, and increases profits,*

(ii) more product differentiation (t)*, increases prices, advertising and profits.*

Price elastic monopoly segment

Maximising (22) with respect to price and advertising, assuming $v - t < p_i$, the symmetric price-advertising equilibrium is implicitly defined by the following two equations:[13,14]

$$Z^p \;:\; = 2(1-a)\,v + at - (4-3a)\,p = 0, \tag{29}$$

$$Z^a \;:\; = (2-a)\,p^2 - 2akt = 0. \tag{30}$$

From this we can express equilibrium price and advertising as

[13] We obtain the expression in (30) when substituting $2(1-a)\,v + at = (4-3a)\,p$ into the first-order condition with respect to a: $p\,[2(1-a)(v-p)+at] - 2akt = 0$, and rearranging.

[14] We show in the Proof of Proposition 6 that the Jacobian satisfies $J^D > 0$, which implies a unique and stable equilbrium.

$$p^D = \frac{2v\left(1 - a^D\right) + ta^D}{4 - 3a^D}, \tag{31}$$

$$a^D = \frac{2\left(p^D\right)^2}{2kt + \left(p^D\right)^2}. \tag{32}$$

For (29)-(30) to define an equilibrium, we need to assume[15]

$$\widetilde{x}_i \in \left(\frac{1}{2}, 1\right) \Leftrightarrow v \in \left[\underline{v}^D, 2t\right], \text{ with } \underline{v}^D = \frac{t\left(4 - a^D\right)}{2\left(2 - a^D\right)} \in \left(t, \frac{3}{2}t\right), \tag{33}$$

$$a^D \leq 1 \Leftrightarrow \left(p^D\right)^2 \leq 2kt. \tag{34}$$

Assuming the conditions (33)-(34) to hold, we can investigate the equilibrium characteristics. First, we observe that

$$\frac{\partial p^D}{\partial a^D} = \frac{2\left(2t - v\right)}{\left(4 - 3a^C\right)^2} > 0 \quad \text{and} \quad \frac{\partial a^D}{\partial p^D} = \frac{8p^D kt}{\left(2kt + \left(p^D\right)^2\right)^2} > 0.$$

Thus, in contrast to the previous case, advertising *relaxes* price competition, whereas higher prices continue to promote advertising competition. We also see that if $a^D \to 1$, then $p^D \to t$.

Inserting (31) and (32) into (22), we obtain the following equilibrium profit:

$$\pi^D = \frac{a^D\left(2 - a^D\right)}{4t} \cdot \left(p^D\right)^2. \tag{35}$$

We can now analyse the properties of the price-advertising equilibrium under price elastic demand in the monopoly segment. By applying Cramer's rule to the system (29) and (30), and differentiating (35), we obtain the following result:

Proposition 6 *In a Hotelling model with informative advertising and a price elastic monopoly demand segment,*

(i) a higher advertising cost (k) *lowers advertising, prices, and profits;*

[15] Note that $\widetilde{x}_i > \frac{1}{2}$ is equivalent to $u\left(\frac{1}{2}\right) > 0$. Note also that the condition in (34) is only implicit. As is readily verified this is satisfied if k is sufficiently large relative to v and t.

(ii) more product differentiation (t) *increases prices, and affects advertising and profits in the same yet ambiguous direction;*

(iii) a greater gross willingness to pay (v)*, increases advertising, prices, and profits.*

A proof is provided in the Appendix.

Similar to the previous case (Proposition 5), a higher advertising cost (k) has a negative impact on advertising. However, a higher advertising cost now leads to lower prices. A change in k has only an indirect effect on prices via advertising. As shown above, advertising and prices are now positively correlated. The reason is that at lower advertising levels, the monopoly segment is relatively larger than the competitive segment. Since price elasticity is higher in the monopoly segment, prices are increasing with advertising. The impact on profits of a higher advertising cost is then also negative due to lower prices and less advertising (demand).

As expected, product differentiation (t) has a positive impact on prices. The effect on advertising, however, is ambiguous. On the one hand, a higher t increases price and therefore renders advertising more attractive. On the other hand, (for a given price) a higher t depresses demand in the monopoly segment $\widetilde{x} = (v - p)/t$ and, thereby, renders advertising less attractive. The same two offsetting forces - higher price but lower demand in the monopoly segment - apply to the impact of t on profit. As it turns out product differentiation increases profit if and only if it also boosts advertising. We show in the proof that this is the case if and only if $a < 2v/(2v + t)$, i.e. if and only if advertising levels are sufficiently low. In this case the monopoly segment is small relative to the competitive segment, which clearly implies that the reduction of demand within the monopoly segment is dominated by the boost of prices. However, if the monopolistic segment is sufficiently large, product differentiation tends to stifle advertising and profits. This counter-intuitive finding stands again in contrast to the case of a perfectly price-inelastic monopolistic segment.

Finally, the impact of a higher gross willingness-to-pay is straightforward. A higher gross willingness to pay for the product, v, allows the firms to charge a higher price and to engage in more advertising. Both activities contribute towards a higher profit.

3 A generalised model

In this section we seek to establish a more general condition for informative advertising to be pro-competitive (or otherwise). Let firm i's demand in the monopoly and competitive segments be given by the continuous and twice differentiable functions $x_i(p_i)$ and $y_i(p_i, p_j)$ with the following properties:

Assumption 1: $\frac{\partial x_i}{\partial p_i} < 0, \frac{\partial y_i}{\partial p_i} < 0, \frac{\partial^2 x_i}{\partial p_i^2} \leq 0, \frac{\partial^2 y_i}{\partial p_i^2} \leq 0.$

Assumption 2: $\left|\frac{\partial y_i}{\partial p_i}\right| > \left|\frac{\partial y_i}{\partial p_j}\right|, \left|\frac{\partial^2 y_i}{\partial p_i^2}\right| > \left|\frac{\partial^2 y_i}{\partial p_j \partial p_i}\right|.$

Assumption 1 ensures profit maximum with respect to prices and Assumption 2 stability and uniqueness for the price equilibrium.

Firm i's demand function is given by:

$$D_i = a_i (1 - a_j) x_i (p_i) + a_i a_j y_i (p_i, p_j), \tag{36}$$

and has the following properties:

$$\frac{\partial D_i}{\partial p_i} = a_i (1 - a_j) \frac{\partial x_i}{\partial p_i} + a_i a_j \frac{\partial y_i}{\partial p_i} < 0, \; \frac{\partial D_i}{\partial p_j} = a_i a_j \frac{\partial y_i}{\partial p_j},$$

$$\frac{\partial D_i}{\partial a_i} = (1 - a_j) x_i (p_i) + a_j y_i (p_i, p_j) > 0,$$

$$\frac{\partial D_i}{\partial a_j} = -a_i [x_i (p_i) - y_i (p_i, p_j)] < 0.$$

If own price p_i is raised this reduces own demand in both the monopoly and the competitive segment. If products are substitutes then $\frac{\partial y_i}{\partial p_j} > 0$, i.e. a higher price of the rival product p_j increases own market share in the competitive segment. More consumers informed being informed about the own product (a_i) increase both the monopoly and the competitive segments. Finally, more consumers informed about the rival product (a_j) lower own demand because demand tends to shift from the monopoly segment to the competitive segment, where firm i faces a lower demand. Given that the underlying market demand functions are identical for the monopoly and competitive segments, it must necessarily be true that the firm captures at least as many consumers in the monopoly segment, as in the competitive segment, i.e. $x_i(p_i) \geq y_i(p_i, p_j)$ must be true.

3.1 Price equilibrium with exogenous information

Let us now derive the equilibrium when firms set prices simultaneously and independently taking the degree of product information as exogenously given. In this case, each firm i chooses the price that maximises the gross profit function

$$V_i = (p_i - c) D_i, \tag{37}$$

where c is a constant marginal cost parameter assumed to be identical across firms. The profit-maximising price of firm i is defined by the following first-order condition[16]

$$\frac{\partial V_i}{\partial p_i} = (1 - a_j)\left(x_i + (p_i - c)\frac{\partial x_i}{\partial p_i}\right) + a_j\left(y_i + (p_i - c)\frac{\partial y_i}{\partial p_i}\right) = 0, \quad i, j = 1, 2; j \neq i. \tag{38}$$

The profit-maximising price is balancing the marginal profitability from the monopoly (first-term) and the competitive (second-term) segments.[17]

Equation (38) implicitly defines a best-response function $p_i(p_j)$. By dif-

[16]The second-order condition requires that

$$\frac{\partial^2 V_i}{\partial p_i^2} = (1 - a_j)\left(2\frac{\partial x_i}{\partial p_i} + (p_i - c)\frac{\partial^2 x_i}{\partial p_i^2}\right) + a_j\left(2\frac{\partial y_i}{\partial p_i} + (p_i - c)\frac{\partial^2 y_i}{\partial p_i^2}\right) < 0,$$

which is satisfied by Assumption 1.

[17]Obviously, firm i could increase its profit by charging different prices to consumers in the monopoly and the competitive segment. However, we do not allow for price discrimination. As in other models of non-targetted advertising, uniform pricing is justified when firms are unable to observe individual consumers' information.

ferentiation, using the implicit-function rule, we obtain:

$$\frac{dp_i}{dp_j} = -\frac{\frac{\partial^2 V_i}{\partial p_j \partial p_i}}{\frac{\partial^2 V_i}{\partial p_i^2}} = -\frac{a_j\left(\frac{\partial y_i}{\partial p_j} + (p_i - c)\frac{\partial^2 y_i}{\partial p_j \partial p_i}\right)}{\frac{\partial^2 V_i}{\partial p_i^2}}. \tag{39}$$

Since the denominator is negative, the sign depends on the numerator. If the numerator is negative, then $\frac{dp_i}{dp_j} > 0$ and prices are *strategic complements.* Observe from (39) that all strategic interaction is going through the competitive segment. If $a_j = 0$, there is no strategic relationship in prices, and the firms set price as local monopolists.

The set of first-order conditions given by (38) implicitly defines the Nash-equilibrium in prices; $p_1^*(a_1, a_2)$ and $p_2^*(a_1, a_2)$. Using the (own-price) elasticities

$$\varepsilon_{x_i} := \frac{\partial x_i}{\partial p_i}\frac{p_i}{x_i} \quad \text{and} \quad \varepsilon_{y_i} := \frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i},$$

we can write the price equilibrium condition as:

$$(1 - a_j)\frac{\partial x_i}{\partial p_i}\left[\frac{1}{\varepsilon_{x_i}} + \frac{p_i^* - c}{p_i^*}\right] + a_j\frac{\partial y_i}{\partial p_i}\left[\frac{1}{\varepsilon_{y_i}} + \frac{p_i^* - c}{p_i^*}\right] = 0, \tag{40}$$

The price equilibrium is unique and stable if the Jacobian is strictly positive, i.e., if

$$J = \frac{\partial^2 V_i}{\partial p_i^2}\frac{\partial^2 V_j}{\partial p_j^2} - \frac{\partial^2 V_i}{\partial p_j \partial p_i}\frac{\partial^2 V_j}{\partial p_i \partial p_j} > 0$$

It is readily verified that Assumption 3 is sufficient to ensure that $J > 0$.[18]

From (40) we see that the equilibrium prices are determined by the relative sizes $(a_j, 1 - a_j)$ and price elasticities $(\varepsilon_{x_i}, \varepsilon_{y_i})$ of the competitive and the monopolistic segment. Define p_i^m as the price that would maximise profits in

[18]As in the Hotelling model, firms might have incentives to deviate, so that a pure strategy equilibrium might fail to exist. It can be shown that deviation does not arise if the difference between the elasticities ε_{x_i} and ε_{y_i} is not too large. Further details are available from the authors on request.

the monopoly segment[19]

$$x_i\left(p_i^m\right)+\left(p_i^m-c\right)\frac{\partial x_i\left(p_i^m\right)}{\partial p_i}=0\Leftrightarrow-\frac{p_i^m-c}{p_i^m}=\frac{1}{\varepsilon_{x_i}},\tag{41}$$

and p_i^c as the equilibrium price for the competitive segment

$$y_i\left(p_i^c,p_j^c\right)+\left(p_i^c-c\right)\frac{\partial y_i\left(p_i^c,p_j^c\right)}{\partial p_i}=0\Leftrightarrow-\frac{p_i^c-c}{p_i^c}=\frac{1}{\varepsilon_{y_i}}.\tag{42}$$

Obviously, if $a_j\to 0$, then $p_i^*\to p_i^m$, and if $a_j\to 1$, then $p_i^*\to p_i^c$. Using the definitions in (41)-(42), we can establish the following result:

Proposition 7 *The price equilibrium defined by (40) implies either of three possibilities: (i) If $\varepsilon_{x_i}=\varepsilon_{y_i}=const$, then*

$$-\frac{p_i^*-c}{p_i^*}=\frac{1}{\varepsilon_{x_i}}=\frac{1}{\varepsilon_{y_i}}\quad \text{and}\quad p_i^*=p_i^m=p_i^c.$$

(ii) If demand in the monopoly segment is less price elastic than in the competitive segment, i.e., if $0>\varepsilon_{x_i}|p_i>\varepsilon_{y_i}|p_i$, then

$$\frac{1}{\varepsilon_{x_i}}<-\frac{p_i^*-c}{p_i^*}<\frac{1}{\varepsilon_{y_i}}\quad \text{and}\quad p_i^c<p_i^*<p_i^m.$$

(iii) If demand in the monopoly segment is more price elastic than in the competitive segment, i.e., if $0>\varepsilon_{y_i}|p_i>\varepsilon_{x_i}|p_i$, then

$$\frac{1}{\varepsilon_{y_i}}<-\frac{p_i^*-c}{p_i^*}<\frac{1}{\varepsilon_{x_i}}\quad \text{and}\quad p_i^m<p_i^*<p_i^c.$$

Proof. Recalling that $\frac{\partial x_i}{\partial p_i}\leq 0$ and $\frac{\partial y_i}{\partial p_i}<0$, it follows that (40) implies either of three cases: (i) $\frac{1}{\varepsilon_{x_i}}=-\frac{p_i^*-c}{p_i^*}=\frac{1}{\varepsilon_{y_i}}$, (ii) $\frac{1}{\varepsilon_{x_i}}<-\frac{p_i^*-c}{p_i^*}<\frac{1}{\varepsilon_{y_i}}$, or (iii) $\frac{1}{\varepsilon_{y_i}}<-\frac{p_i^*-c}{p_i^*}<\frac{1}{\varepsilon_{x_i}}$. The outer (in-)equalities in these three expressions imply and are implied by the three cases given in the Proposition. ■

[19] Note that p_i^m is *not* equivalent to the monopoly price. Monopoly pricing is defined by the set of prices $\left(p_i^M,p_j^M\right):=\arg\max\left\{V_i\left(p_i,p_j\right)+V_j\left(p_i,p_j\right)\right\}$. It is easily verified that monopoly prices always exceed the equilibrium prices defined by (40).

Thus, the price effect of more information (or informative advertising) is not clear-cut as in GS (1984).[20] In fact, if demand in the competitive segment is less price elastic than demand in the monopoly segment, then a larger fraction of fully informed consumers will result in *higher* prices. We showed in section 2.1 that this was indeed the case in the Hotelling duopoly model with imperfect information and price elastic demand in the monopoly segment.

The impact of information / advertising

Let us analyse the impact of more consumer information on price competition. Performing comparative statics on the set of first-order conditions in (38) we obtain:[21]

$$\frac{dp_i^*}{da_i} = \frac{1}{J}\frac{\partial^2 V_j}{\partial a_i \partial p_j}\frac{\partial^2 V_i}{\partial p_j \partial p_i}, \tag{43}$$

$$\frac{dp_i^*}{da_j} = -\frac{1}{J}\frac{\partial^2 V_i}{\partial a_j \partial p_i}\frac{\partial^2 V_j}{\partial p_j^2}, \tag{44}$$

where $\frac{\partial^2 V_i}{\partial p_j \partial p_i} > 0$ if and only if prices are strategic complements, and where

$$\frac{\partial^2 V_i}{\partial a_j \partial p_i} = -\left(x_i + (p_i - c)\frac{\partial x_i}{\partial p_i}\right) + \left(y_i + (p_i - c)\frac{\partial y_i}{\partial p_i}\right).$$

Using (38), we get

$$\frac{\partial^2 V_i}{\partial a_j \partial p_i} = -\frac{1}{a_j}\left(x_i + (p_i - c)\frac{\partial x_i}{\partial p_i}\right)$$

[20] Note that the model by GS and, similarly, our model with a price inlastic monopoly segment is, indeed, a special case, for which condition (40) reads $(1 - a_j)\frac{x_i}{p_i^*} + a_j\frac{\partial y_i}{\partial p_i}\left[\frac{1}{\varepsilon_{y_i}} + \frac{p_i^* - c}{p_i^*}\right] = 0$. This obviously implies $-\frac{p_i^* - c}{p_i^*} < \frac{1}{\varepsilon_{y_i}}$ and, thus, $p_i^* > p_i^c$.

[21] Generally, we have that:

$$\frac{dp_i^*}{da_i} = -\frac{1}{J}\begin{vmatrix} \frac{\partial^2 V_i}{\partial a_i \partial p_i} & \frac{\partial^2 V_i}{\partial p_j \partial p_i} \\ \frac{\partial^2 V_j}{\partial a_i \partial p_j} & \frac{\partial^2 V_j}{\partial p_j^2} \end{vmatrix} \quad \text{and} \quad \frac{dp_i^*}{da_j} = -\frac{1}{J}\begin{vmatrix} \frac{\partial^2 V_i}{\partial a_j \partial p_i} & \frac{\partial^2 V_i}{\partial p_j \partial p_i} \\ \frac{\partial^2 V_j}{\partial a_j \partial p_j} & \frac{\partial^2 V_j}{\partial p_j^2} \end{vmatrix}$$

However, since $\partial^2 V_i / \partial a_i \partial p_i = \partial^2 V_j / \partial a_j \partial p_j = 0$, the comparative statics simplify to those reported in (43)-(44).

Using the elasticity, it follows that

$$\frac{\partial^2 V_i}{\partial a_j \partial p_i} > (<)\, 0 \Leftrightarrow \frac{1}{\varepsilon_{x_i}} > (<) - \frac{p_i - c}{p_i}.$$

We can thus conclude the following.

Proposition 8 *(i) Information about the own product reduces (increases) own price (through a strategic effect) if prices are strategic complements and the monopolistic segment is less (more) price elastic than the competitive segment. The opposite applies if prices are strategic substitutes.*

(ii) Information about the rival product reduces (increases) own price (through a direct effect) if and only if the monopolistic segment is less (more) price elastic than the competitive segment.

3.2 Price and advertising equilibrium

Let us now endogenise the degree of information in the market by allowing firms to advertise their products. We continue to denote by $C(a;k)$ the cost of reaching a fraction a of the population, where $C_a > 0$ and $C_{aa} > 0$.[22] We also assume that k is a shift parameter that increases advertising costs and marginal advertising costs, i.e., $C_k > 0, C_{ak} > 0$. The net profit of firm i is then given by

$$\pi_i = V_i - C(a_i;k)\,. \tag{45}$$

Following GS, we assume that the firms choose prices and advertising simultaneously and independently. Thus, each firm i chooses a_i and p_i in order to maximise profits taking the rival's decision as given. Since $\partial\pi_i/\partial p_i = \partial V_i/\partial p_i$, profit-maximising prices are defined by the first-order conditions in (38), whereas profit-maximising advertising levels are given by the set of first-order condition[23]

[22] For details about the underlying advertising technology, see Grossman and Shapiro (1984). Here, we simply adopt their cost function.

[23] The second-order conditions require that $\frac{\partial^2\pi_i}{\partial p_i^2} < 0$, $\frac{\partial^2\pi_i}{\partial a_i^2} = -C_{aa} < 0$, and $\frac{\partial^2\pi_i}{\partial p_i^2}\frac{\partial^2\pi_i}{\partial a_i^2} - \frac{\partial^2\pi_i}{\partial a_i\partial p_i}\frac{\partial^2\pi_i}{\partial p_i\partial a_i} > 0$. Since $\frac{\partial^2\pi_i}{\partial a_i\partial p_i} = 0$, the third condition is fulfilled by the previous two.

$$\frac{\partial \pi_i}{\partial a_i} = (p_i - c)\left[(1-a_j)\cdot x_i + a_j \cdot y_i\right] - C_a(a_i,k) = 0, \quad i,j = 1,2; j \neq i. \tag{46}$$

The symmetric price-advertising equilibrium $\{p^*(k), a^*(k)\}$ is then implicitly defined by the two following equations:

$$Z^p := (1-a)\left[x + (p^* - c)\frac{\partial x}{\partial p}\right] + a\left[y + (p^* - c)\frac{\partial y}{\partial p_i}\right] = 0, \tag{47}$$

$$Z^a := (p-c)\left[(1-a^*)\cdot x + a^* \cdot y\right] - C_a(a,k) = 0. \tag{48}$$

Using the implicit-function rule, we can analyse the relationship between equilibrium price and advertising

$$\begin{aligned}
\frac{\partial a}{\partial p} &= -\frac{Z_p^a}{Z_a^a} = \frac{-a^*(p^*-c)\frac{\partial y}{\partial p_j}}{(p^*-c)(y-x) - C_{aa}} > 0 \Leftrightarrow \frac{\partial y}{\partial p_j} > 0 \\
\frac{\partial p}{\partial a} &= -\frac{Z_a^p}{Z_p^p} = \frac{\left(x + (p^*-c)\frac{\partial x}{\partial p}\right) - \left(y + (p^*-c)\frac{\partial y}{\partial p_i}\right)}{\left\{\begin{array}{c} (1-a^*)\left[2\frac{\partial x}{\partial p} + (p^*-c)\frac{\partial^2 x}{\partial p^2}\right] \\ +a^*\left[2\frac{\partial y}{\partial p_i} + \frac{\partial y}{\partial p_j} + (p^*-c)\frac{d}{dp}\left(\frac{\partial y}{\partial p_i}\right)\right] \end{array}\right\}} \gtrless 0 \Leftrightarrow Z_a^p \gtrless 0
\end{aligned}$$

If (and only if) products are substitutes, then higher prices unambiguously fuel advertising competition. The impact of advertising on price competition depends again on the constellation of elasticities in the two segments. Recalling that we can rewrite $Z_a^p = \frac{1}{a_j}\left(x_i + (p_i - c)\frac{\partial x_i}{\partial p_i}\right)$, it follows from our previous argument that $Z_a^p > (<)\, 0$ if (and only if) the monopolistic segment is more (less) price elastic than the competitive segment. Hence, for a symmetric equilibrium it is then true that advertising stifles price competition if and only if the monopolistic market segment exhibits a higher price elasticity of demand. Interestingly, this is true irrespective of whether prices are strategic complements or substitutes.

The impact of advertising cost

Denoting the Jacobian for the system (47) and (48) by $J = Z_p^p Z_a^a -$

$Z_a^p Z_p^a > 0$, we can now examine the impact of the costliness of advertising on the equilibrium[24]

$$\begin{aligned} \frac{da}{dk} &= \frac{-Z_p^P Z_k^a}{J} < 0, \\ \frac{dp}{dk} &= \frac{Z_a^p Z_k^a}{J} = \frac{\partial p}{\partial a}\frac{da}{dk} \gtrless 0 \Leftrightarrow Z_a^p \gtrless 0. \end{aligned}$$

While a higher advertising cost always stifles advertising competition, the effect on price competition is now ambiguous. In contrast to GS, more costly advertising may well lead to a stifling of price competition if the monopolistic segment is *more* price elastic than the competitive segment. Note that this result is in line with our previous findings. If the monopolistic segment is relatively price elastic, then higher levels of both *own and rival's* informative advertising tend to boost prices. In this case, increases in advertising cost lower both equilibrium advertising and equilibrium price.

Consider now the effect of k on firm i's profit. Totally differentiating π_i and observing the envelope theorem ($\frac{d\pi_i}{da_i} = \frac{d\pi_i}{dp_i} = 0$) as well as symmetry ($\frac{\partial a_i^*}{\partial k} = \frac{\partial a_j^*}{\partial k} = \frac{da}{dk}$ and $\frac{\partial p_i^*}{\partial k} = \frac{\partial p_j^*}{\partial k} = \frac{dp}{dk}$) we obtain

$$\begin{aligned} \frac{d\pi_i}{dk} &= -C_k(a^*, k) + (p^* - c)\left[\frac{\overset{+}{\partial D_i}}{\partial p_j}\frac{dp}{dk} + \frac{\overset{-}{\partial D_i}}{\partial a_j}\overset{-}{\frac{da}{dk}}\right] \\ &= -C_k(a^*, k) + (p^* - c)\left[\frac{\partial p}{\partial a}\frac{\overset{+}{\partial D_i}}{\partial p_j} + \frac{\overset{-}{\partial D_i}}{\partial a_j}\right]\overset{-}{\frac{da}{dk}} \end{aligned}$$

As in GS, a higher advertising cost has, a direct and a strategic effect. A higher k (i) raises cost directly (provided $C_k(a, k) > 0$, which is true in general); (ii) increases own demand, as the rival engages in less advertising[25];

[24] It is easy to find conditions such that $J > 0$. For instance, $J > 0$ if price elasticities between the monopolistic and competitive segment do not vary much such that Z_a^p is small. Another case is a situation, where the advertising intensity a is low, implying that Z_p^a is small. In section 2.2 we have considered one case, where $J > 0$ is satisfied.

[25] Recall that for a given price $x \geq y$ is always true.

and (iii) increases own demand if and only if it raises the rival's price. The latter effect is true in GS, but not necessarily in our model. In GS, $\frac{dp}{dk} > 0$ due to the pro-competitive effect of advertising. More generally, however, the sign of $\frac{dp}{dk}$ depends on the relative price elasticities in the competitive and monopoly segments. If $\frac{dp}{dk} < 0$, then the effect of advertising cost on profit is ambiguous. For instance, if the level of demand in the competitive segment is close to the level of demand in the monopolistic segment, i.e. if $y \to x$, then $\overset{-}{\frac{\partial D_i}{\partial a_j}} = y - x \to 0$. In this case, the effect through price dominates, leading to an unambiguous reduction in gross (operating) profit and, for the corresponding increase in advertising cost, to a reduction in overall profit. Indeed, this was the case for the Hotelling-model with price elastic demand, as illustrated in section 2.2.[26]

We can summarise our general results as follows.

Proposition 9 *Consider a symmetric price-advertising equilibrium.*

(i) A higher advertising cost (k) always induces lower levels of advertising and induces a higher (lower) price if and only if the monopolistic segment is less (more) price elastic than the competitive segment.

(ii) A higher advertising cost (k) leads to a higher operating profit if the monopolistic segment is less price elastic than the competitive segment. Otherwise the effect on operating profit is indeterminate, as is the effect on overall profit.

4 Concluding Remarks

In this paper we have revisited the seminal work by Grossman and Shapiro (1984) in order to check the robustness of the pro-competitive effect of informative advertising. By using a Hotelling duopoly version, as presented in Tirole (1988), we have shown that the results are reversed once we allow for a price elastic monopoly demand segment, i.e., partially informed consumers

[26] In the duopoly version of GS, we have that $C_k(a,k) = (p-c)\overset{+}{\frac{\partial D_i}{\partial p_j}}\overset{+}{\frac{dp}{dk}}$ implying $\frac{d\pi_i}{dk} = (p-c)\overset{-}{\frac{\partial D_i}{\partial a_j}}\overset{-}{\frac{da}{dk}} > 0$. Obviously, this is not neccessarily true in a more general model.

trade-off prices against travelling costs. In this case, informative advertising in fact induces higher rather than lower prices. As demand also increases, firms' benefit from informative advertising, and advertising restrictions will be harmful for firms. The intuition behind this result is that partially informed consumers have on average a lower willingness to pay due to higher travelling (or mismatch) costs than fully informed consumers, and hence are more responsive to prices. We then generalize the model by allowing for general demand and cost functions, and derive exact conditions for informative advertising to be pro-competitive, as claimed in previous studies. A key result here is that if prices are strategic complements, then informative advertising will reduce (increase) prices if demand in the monopoly segment is less (more) price elastic than demand in the competitive segment.

By way of conclusion we would like to point out some issues for further research. First, there should be scope for further generalization of the model. For instance, one could allow for non-uniform distributions of consumers and/or targeted (non-random) advertising. Our focus was on the price inelasticity assumption in the previous models, so we have generalized the model along that direction. Second, there should be great scope for empirical testing, as the predictions in terms of prices and profits go in opposite directions. We leave these issues for future research.

5 Appendix

Proof of Proposition 1: Taking the partial derivatives of (7)-(10), we obtain

$$\frac{\partial p_i^A}{\partial a_i} = -\frac{2t}{3a_i^2} < 0,\ \frac{\partial p_i^A}{\partial a_j} = -\frac{4t}{3a_j^2} < 0,\ \frac{\partial \widehat{x}^A}{\partial a_1} = -\frac{1}{3a_1^2} < 0,\ \frac{\partial \widehat{x}^A}{\partial a_2} = \frac{1}{3a_2^2} > 0,$$

$$\frac{\partial D_i^A}{\partial a_i} = \frac{4-3a_j}{6} > 0,\ \frac{\partial D_i^A}{\partial a_j} = \frac{2-3a_i}{6},$$

$$\frac{\partial V_i^A}{\partial a_i} = t\frac{(4a_i - 2a_j - 3a_i a_j)(4a_i + 2a_j - 3a_i a_j)}{18a_i^2 a_j},$$

$$\frac{\partial V_i^A}{\partial a_j} = t\frac{\left(-4a_i + 2a_j - 3a_i a_j\right)\left(4a_i + 2a_j - 3a_i a_j\right)}{18 a_i a_j^2}.$$

All signs follow by inspection, except for $\partial V_i^A/\partial a_j$. Applying equilibrium condition (12), it is readily verified that $\left(-4a_i + 2a_j - 3a_i a_j\right) < 0$, which implies that $\partial V_i^A/\partial a_j < 0$ is true.

Proof of Proposition 3: Taking the partial derivatives of (14)-(18), we obtain:

$$\frac{\partial p_i^B}{\partial a_i} = \frac{2a_j\left(4 - a_j\right)\left(2t - v\right)}{\left(16 - 8a_i - 8a_j + 3a_i a_j\right)^2}, \quad \frac{\partial p_i^B}{\partial a_j} = \frac{4\left(4 - a_i\right)\left(2 - a_i\right)\left(2t - v\right)}{\left(16 - 8a_i - 8a_j + 3a_i a_j\right)^2},$$

$$\frac{\partial \widehat{x}^B}{\partial a_1} = \frac{\left(2t - v\right)\left(4 - 3a_2\right)\left(4 - a_2\right)}{t\left(16 - 8a_1 - 8a_2 + 3a_1 a_2\right)^2}, \quad \frac{\partial \widehat{x}^B}{\partial a_2} = -\frac{\left(2t - v\right)\left(4 - 3a_1\right)\left(4 - a_1\right)}{t\left(16 - 8a_1 - 8a_2 + 3a_1 a_2\right)^2},$$

$$\frac{\partial \widetilde{x}_i^B}{\partial a_i} = -t^{-1}\frac{\partial p_i^B}{\partial a_i}, \frac{\partial \widetilde{x}_i^B}{\partial a_j} = -t^{-1}\frac{\partial p_i^B}{\partial a_j},$$

$$\frac{\partial D_i^B}{\partial a_i} = \left(\frac{2 - a_j}{2t}\right)\left(p_i^B + a_i\frac{\partial p_i^B}{\partial a_i}\right), \quad \frac{\partial D_i^B}{\partial a_j} = -\frac{a_i}{2t}\left(p_i^B - \left(2 - a_j\right)\frac{\partial p_i^B}{\partial a_j}\right),$$

$$\frac{\partial V_i^B}{\partial a_i} = p_i^B\left(\frac{2 - a_j}{2t}\right)\left(p_i^B + 2a_i\frac{\partial p_i^B}{\partial a_i}\right), \quad \frac{\partial V_i^B}{\partial a_j} = p_i^B\frac{a_i}{2t}\left(2\left(2 - a_j\right)\frac{\partial p_i^B}{\partial a_j} - p_i^B\right).$$

Observing the equilibrium condition (19), i.e., $v < 2t$, all signs follow by inspection, except for $\partial V_i^B/\partial a_j$ and $\partial D_i^B/\partial a_j$. Noting that $sgn\partial V_i^B/\partial a_j = sgn\Omega$ with $\Omega := 2\left(2 - a_j\right)\partial V_i^B/\partial a_j - p_i^B$ we can verify $\Omega < 0$ by writing expressly

$$\begin{aligned} \Omega &= \left\{8\left(2 - a_j\right)\left(2 - a_i\right)\left(4 - a_i\right)\left(2t - v\right) - \Phi\left[2v\left(4 - 2a_i - 3a_j + a_i a_j\right) + t a_j\left(4 - a_i\right)\right]\right\}\Phi^{-2} \\ &= \left\{-2v\left[\left(8 - 3a_i - 3a_j + a_i a_j\right)\Phi + \left(4 - a_i\right)a_i a_j\right] + t\left(4 - a_i\right)\left[\left(4 - a_j\right)\Phi + a_i a_j\right]\right\}\Phi^{-2}, \end{aligned}$$

with $\Phi := 16 - 8a_i - 8a_j + 3a_i a_j$. From condition (21) it follows that $2v\left(8 - 3a_i - 3a_j + a_i a_j\right) \geq t\left(4 - a_i\right)\left(4 - a_j\right)$ implying that $\Omega \leq -\left(4 - a_i\right)a_i a_j\left(2v - t\right)\Phi^{-2} < 0$, where the second inequality is verified again from (21). But then $\partial V_i^B/\partial a_j < 0$.

If we write demand for product 1 as

$$D_1^B = a_1\left(1 - a_2\right)\widetilde{x}_1^B + a_1 a_2\widehat{x}^B,$$

and differentiate with respect to a_2, we get

$$\frac{\partial D_1^B}{\partial a_2} = -a_1\left(\widetilde{x}_1^B - \widehat{x}^B\right) + a_1\left(1-a_2\right)\frac{\partial \widetilde{x}_1^B}{\partial a_2} + a_1 a_2 \frac{\partial \widehat{x}^B}{\partial a_2}.$$

Since existence of competitive segment implies that $\widetilde{x}_1^B > \widehat{x}^B$ and since $\partial\widetilde{x}_1^B/\partial a_2 = -t^{-1}\partial p_1^B/\partial a_2 < 0$ and $\partial\widehat{x}_1^B/\partial a_2 < 0$ it follows that $\partial D_1^B/\partial a_2 < 0$. In the same way, it can be proven that $\partial D_2^B/\partial a_1 < 0$.

Proof of Proposition 6: By performing comparative statics on the system (29) and (30), using the Cramer's rule and the Jacobian $J^D = Z_p^p Z_a^a - Z_a^p Z_p^a = \frac{2p\left[2v\left(4-5a+2a^2\right)-ta(4-a)\right]}{a(4-3a)} > 0$, we obtain the following results:[27]

$$\begin{aligned}
\frac{dp^D}{dv} &= -\frac{Z_v^p Z_a^a - Z_a^p Z_v^a}{J^D} = \frac{4p^2\left(1-a\right)}{aJ^D} > 0, \\
\frac{dp^D}{dt} &= -\frac{Z_t^p Z_a^a - Z_a^p Z_t^a}{J^D} = \frac{2p^2\left[\left(2-a\right)v - at\right]}{t\left(4-3a\right)J^D} > 0, \\
\frac{dp^D}{dk} &= -\frac{Z_k^p Z_a^a - Z_a^p Z_k^a}{J^D} = -\frac{4ta\left(2t-v\right)}{\left(4-3a\right)J^D} < 0, \\
\frac{da^D}{dv} &= -\frac{Z_p^p Z_v^a - Z_v^p Z_p^a}{J^D} = \frac{4p\left(2-a\right)\left(1-a\right)}{J^D} > 0, \\
\frac{da^D}{dt} &= -\frac{Z_p^p Z_t^a - Z_t^p Z_p^a}{J^D} = \frac{p\left(2-a\right)\left[2\left(1-a\right)v - at\right]}{tJ^D} \gtrless 0, \\
\frac{da^D}{dk} &= -\frac{Z_p^p Z_k^a - Z_k^p Z_p^a}{J^D} = -\frac{2ta\left(4-3a\right)}{J^D} < 0.
\end{aligned}$$

Applying the equilibrium condition in (33), i.e., $v \in \left(\frac{t(4-a)}{2(2-a)}, 2t\right)$, all signs follow by inspection. Total differentiation of (35), yields the following results:

$$\begin{aligned}
\frac{d\pi^D}{dv} &= \frac{p^D}{2t}\left[\left(1-a^D\right)p^D\frac{da^D}{dv} + a^D\left(2-a^D\right)\frac{dp^D}{dv}\right] > 0, \\
\frac{d\pi^D}{dt} &= \frac{p^D}{2t}\left[\frac{-a^D\left(2-a^D\right)p^D}{2t} + \left(1-a^D\right)p^D\frac{da^D}{dt} + a^D\left(2-a^D\right)\frac{dp^D}{dt}\right] < 0 \Leftrightarrow \frac{da^D}{dt} < 0, \\
\frac{d\pi^D}{dk} &= \frac{p^D}{2t}\left[\left(1-a^D\right)p^D\frac{da^D}{dk} + a^D\left(2-a^D\right)\frac{dp^D}{dk}\right] < 0.
\end{aligned}$$

[27] It is readily verified that the equilibrium condition (33), i.e. $v \geq \frac{t(4-a)}{2(2-a)}$ implies $J^D > 0$.

Using the comparative statics results above, the sign of the first and third derivatives are immediate. Noting that

$$\begin{aligned}
& \frac{-a^D\left(2-a^D\right)p^D}{2t}+a^D\left(2-a^D\right)\frac{dp^D}{dt} \\
= \ & \frac{a^D\left(2-a^D\right)}{2t}\left(2t\frac{dp^D}{dt}-p^D\right)=\frac{-\left(2-a^D\right)\left(p^D\right)^2\left[2\left(1-a\right)v-at\right]}{tJ^D}=p^D\frac{da^D}{dt}
\end{aligned}$$

it follows that $\frac{d\pi^D}{dt}=\frac{\left(2-a^D\right)\left(p^D\right)^2}{2t}\frac{da^D}{dt}$, implying that $sgn\frac{d\pi^D}{dt}=\frac{da^D}{dt}$.

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