International Journal of Sciences: Basic and Applied Research (IJSBAR)

ISSN 2307-4531

(Print & Online)

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

Properties of the International Standard Serial Number

David Gikunju Muriuki[a*], Peter Kamaku Waweru[b], Augustus Wali Nzomo[c]

[a,b]*Department of Pure and Applied Mathematics, Jomo Kenyatta University of Agriculture and Technology, P.O Box 62000, Juja-00200, Nairobi, Kenya*

[c]*Department of Mathematics and Actuarial sciences, South Eastern Kenya University, P.O Box 90200, Kitui-170, Nairobi, Kenya*

[a]*Email: gdavvie@gmail.com*

[b]*Email: wkamaku@jkuat.com*

[c]*Email: nzomoa@gmail.com*

Abstract

This paper, analyses the properties of international standard serial number in terms of error detection, correction and the size of the code dictionary. Through weight check equation the ability of ISSN code to detect and correct errors is determined. The total number of code words is established by how many digits permute. By using permutation methods, the length of the ISSN dictionary is determined. ISSN detects errors but only corrects single and transposition errors.

Keywords: ISSN; error detection; error correction; single error; transposition errors.

1. Introduction

A digit is an element of a code word and a code word is an element of a code.Total number of digits in a code word are referred as the length of a code. Error correcting coding is an effective technique of detecting and correcting errors which may occur due to environmental interference or physical defects such as human errors in the communication channels.

----

* Corresponding author.

Error correcting coding makes a communication channel reliable by ensuring that the information/message the receiver gets is the correct information/message the sender/source intended the receiver to get. The International Standard Serial Number (ISSN) code is internationally used for identifying the title of serial publications. A good communication channel should have a good error coding scheme which is excellent in error detection and correction capabilities.

1.1 Types of error in a code

1. Single error: A single error occurs when one digit is incorrectly typed in a code word.
2. Double error: A double error occurs when two digits in a code word are incorrectly typed. In case of a double error, the parity check equation does not hold.
3. Silent error: A silent error occurs when the parity check equation holds despite of the code word having error(s).
4. Transpose error: A transpose error may occur due to interchanging of digits in a code word [3].
5. Jump transpose error: A jump transpose error may occur when three digits are reversely interchanged.
6. Jump twin error: A jump twin error may occur when two equal digits are interchanged for another.
7. Phonetic error: A phonetic error may occur when a digit is replaced with another that sounds almost the same.
8. Twin error: A twin error may occur when a pair of similar digits are replaced with another pair.
9. Omission or insertion of a digit(s) error: These are errors that occur when a digit(s) are omitted orextra digit(s) are added.

2. Assumptions

1. Errors in different positions in a code word are independent; the existence of an error in one position in the code word does not affect the probability of an error in another position.
2. Each element $f \in (F, q)^n$ has the same probability r of being inaccurately transmitted. The assumption is made that the probability of error is small, r $\ll \frac{1}{2}$.
3. If an element $f \in (F, q)^n$ is transmitted, then all $q-1$ remaining elements are equally likely to be received. Such channels are known as $q-\ ary$ symmetric channels.

2.1 Number of code words in ISSN Code

Total number of code words that can be generated by a certain code depends on the following factors:

1. The length *n* of the code words / the number of elements in a code word.
2. The order of the field q.
3. The number of digits in a code word which can permute.
4. Whether repetition of digits is allowed.

Theorem 2.1. *ISSN code has a dictionary of upper bound of* 10^7

Proof. ISSN code has a length of 8 and it is a finite field of order 11 but the elements of ISSN can be chosen from {0, 1, 2,..., 9} except for the check digit number where $A = 10$ is used. Check digit number does not permute for it is not chosen but calculated on the basis of weight check equation. Repetition of the permuting digits is allowed therefore there are 10^7 permutations yielding the dictionary of ISSN be $10^7 = 10{,}000{,}000$. □

2.2 Calculation of the check digit in ISSN

Let the code word for ISSN be $X = x_1, x_2, \ldots x_7$ without the check digit. To compute the check digit x_8, calculate

$$\sum_{i=1\, j=9-i}^{7} jx_i \text{ . Let } \xi = \sum_{i=1\, j=9-i}^{7} jx_i \quad \text{then } x_8 + \xi \equiv 0(mod\ 11).$$

Alternatively, the check digit x_8 can be computed by calculating $\sum_{i=1\, j=8-(i-1)}^{7} (11-j)x_i$ or $\sum_{i=1}^{7}(2+\mathrm{i})x_i$. Let

$$\xi = \sum_{i=1\, j=8-(i-1)}^{7} (11-j)x_i \quad \text{or } \xi = \sum_{i=1}^{7}(2+\mathrm{i})x_i \quad \text{then}$$

$x_8 + \xi \equiv 0(mod\ 11)$. Now since 0 is the additive identity of $(\mathbb{Z}, +)$ therefore x_8 is the additive inverse of $\xi(mod\ 11$ therefore

$x_8 \equiv -\xi\ (mod\ 11)$. If compute the table of the additive inverse of the *modulo* 11

Table 2.1: Additive inverse *modulo* 11

ξ	1	2	3	4	5	6	7	8	9	10
x_8	10=A	9	8	7	6	5	4	3	2	1

When the check digit is 10 , replace it with an upper case A.

Table 2.2: Multiplicative inverse *modulo* 11

x	1	2	3	4	5	6	7	8	9	10
x^{-1}	1	6	4	3	9	2	8	7	5	10

2.3 Error detection in ISSN

Let the code word for ISSN be $X = x_1, x_2, \ldots x_8$, then the weight check is computed as $\sum_{i=1}^{n} \sum_{j=n-(i-1)} jx_i \equiv 0 \pmod{11}$. Since $j = n - (i-1)$ this equation can be further simplified to $\sum_{i=1}^{8} (10 - \mathrm{n} + \mathrm{i}) x_i \equiv 0 \pmod{11}$ but in this case $n = 8$ therefore the weight check is computed as $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \equiv 0 \pmod{11}$. If $\sum_{i=1}^{8} \sum_{j=9-i} jx_i \neq 0 \pmod{11}$ then the code has detected error(s). An ISSN code has a length of only eight digits if the code has a length of more or less than eight then there is an error. Alternatively, let the code word for ISSN be $X = x_1, x_2, \ldots, x_8$, then the weight check is computed as $\sum_{i=1}^{8} \sum_{j=9-i} (11 - j) x_i \equiv 0 \pmod{11}$ or $\sum_{i=1}^{8} (2+i) x_i \equiv 0 \pmod{11}$. If $\sum_{i=1}^{8} (2+i) x_i \neq 0 \pmod{11}$ then the code has detected error(s).

2.3.1 Single error detection in ISSN

Proposition 2.3.1 *ISSN code detects any single error in the code*

Proof. Suppose $X = x_1, x_2, \ldots, x_8$ is the ISSN code and $Y = x_1, x_2, \ldots, x_{\tau-1}, y_\tau, x_{\tau+1}, \ldots, x_8$ with $y_\tau = x_\tau + \alpha$, $\alpha \neq 0$ be the ISSN code with a single error that has occurred in digit x_τ. Then

$$\Rightarrow \left(\sum_{i=1}^{8} (2+i) y_i\right)$$

$$\Rightarrow \left(\sum_{i=1}^{8} (2+i) y_i\right) + (2+\tau)\alpha \neq 0 \pmod{11}$$

$$\Rightarrow (2+\tau)\alpha \neq 0 \pmod{11}$$

Hence the single error is detected.

Alternately, Suppose $X = x_1, x_2, \ldots, x_8$ is the ISSN code and $Y = x_1, x_2, \ldots, x_{\tau-1}, y_\tau, x_{\tau+1}, \ldots, x_8$ with $y_\tau = x_\tau + \alpha, \alpha \neq 0$ Be the ISSN code with a single error that has occurred in digit x_τ then

$$\Rightarrow \left(\sum_{i=1}^{8} \sum_{j=9-i} jy_i\right)$$

$$. \Rightarrow \left(\sum_{i=1}^{8} \sum_{j=9-i} jy_i\right) + (9-\tau)\alpha \neq 0 \pmod{11}$$

$$\Rightarrow (9-\tau)\alpha \neq 0 \pmod{11}$$

Therefore the single error is detected.

Corollary 2.3.1 *ISSN code cannot detect single silent error*

Proof. Suppose a single error has occurred in ISSN code but $\sum_{i=1\, j=9-i}^{8} jy_i \equiv 0 \pmod{11}$. Then if $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \alpha \neq 0$ is the ISSN code with a single error that has occurred in digit x_τ then

$$\Rightarrow (\sum_{i=1\, j=9-i}^{8} jy_i)$$
$$.\Rightarrow (\sum_{i=1\, j=9-i}^{8} jy_i) + (9-\tau)\alpha \equiv 0 \pmod{11}$$
$$\Rightarrow (9-\tau)\alpha \equiv 0 \pmod{11}$$

This implies that either $9-\tau$ or α is a multiple of 11 or 0. Since $9-\tau$ cannot be a multiple of 11 or 0 hence and no error. This is a contradiction. Conversely if $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0 \pmod{11}$ this implies there is no any error in the code but it is a contradiction. □

Remark 2.3.1. Corollary 2.3.1 can also be proved as following: Suppose a single error has occurred in ISSN code but $\sum_{i=1\, j=9-i}^{8} (11-j)x_i \equiv 0 \pmod{11}\, or \sum_{i=1}^{8} (2+\mathrm{i})x_i \equiv 0 \pmod{11}$ then

$$\Rightarrow (\sum_{i=1}^{8} (2+i)y_i)$$
$$\Rightarrow (\sum_{i=1}^{8} (2+i)y_i) + (2+\tau)\alpha \equiv 0 \pmod{11}$$
$$\Rightarrow (2+\tau)\alpha \equiv 0 \pmod{11}$$

This implies that either $2+\tau$ or α is a multiple of 11 or 0. Since $2+\tau$ cannot be a multiple of 11 or 0 hence and no error. This is a contradiction. Conversely if

$\sum_{i=1\, j=9-i}^{8} (11-j)x_i \equiv 0 \pmod{11}\, or \sum_{i=1}^{8} (2+\mathrm{i})x_i \equiv 0 \pmod{11}$ this implies there is no any error in the code but it is a contradiction.

2.3.2 Correction of single error in ISSN

Proposition 2.3.2. *ISSN code can correct a single error only if the position of the error is known*

proof. Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code and $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \alpha \neq 0$ be the ISSN code with a single error that has occurred in digit x_τ. If the error has been detected and the position of the error is known . Then the error can be corrected by

$$\Rightarrow \sum_{i=1 i\neq\tau\ j=9-\tau}^{8} jx_i + (9-\tau)\,\mathrm{y}_\tau \equiv 0(\mathrm{mod}11)$$

$$\Rightarrow (9-\tau)\,\mathrm{y}_\tau \equiv -\left[\left(\sum_{i=1 i\neq\tau\ j=9-\tau}^{8} jx_i \right)(\mathrm{mod}11) \right]$$

$$\Rightarrow y_\tau = (9-\tau)^{-1} \times -\left[\left(\sum_{i=1 i\neq\tau\ j=9-\tau}^{8} jx_i \right)(\mathrm{mod}11) \right]$$

Since $y_\tau + \alpha, \alpha = 0$ hence $y_\tau = x_\tau$ yielding the original ISSN code.

Example 2.3.2.1 Let the ISSN code 2 0 4 9 3 6 3 0. Assume that there is an error in x_5. If the code word with the error is 2 0 4 9 7 6 3 0 . By

$$\Rightarrow \sum_{i=1\ j=9-i}^{8} jx_i \neq 0(\mathrm{mod}11)$$

$$\Rightarrow 137 \neq 0\,(\mathrm{mod}11) \qquad \text{Therefore}$$

$$\Rightarrow 5 \neq 0\,(\mathrm{mod}11)$$

$$\Rightarrow \sum_{i=1 i\neq\tau\ j=9-\tau}^{8} jx_i + (9-\tau)y_\tau \equiv 0\,(\mathrm{mod}11)$$

$$\Rightarrow 4y_5 \equiv -109(\mathrm{mod}11)$$

$$\Rightarrow 4y_5 \equiv -10(\mathrm{mod}11)$$

$$\Rightarrow y_5 = 4^{-1} \times -10(\mathrm{mod}11)$$

$$\Rightarrow y_5 = 4^{-1} \times 1(\mathrm{mod}11)$$

$$\Rightarrow y_5 = 3 \times 1(\mathrm{mod}11)$$

$$\Rightarrow y_5 = 3$$

Hence the error is corrected. Alternatively, Proposition 2.3.2 can be proved by

Proof: Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code and $Y = x_1, x_2, ..., x_{\tau-1}, y_\tau, x_{\tau+1}, ..., x_8$ with $y_\tau = x_\tau + \alpha, \alpha \neq 0$ be the ISSN code with a single error that has occurred in digit x_τ. If the error has been detected and the position of the error is known . Then the error can be corrected by

$$\Rightarrow \sum_{i=1 i\neq\tau\ j=9-\tau}^{8} (11-j)x_i + (2+\tau)\,\mathrm{y}_\tau \equiv 0(\mathrm{mod}11)$$

$$\text{or } \sum_{i=1 i\neq\tau}^{8} (2+\mathrm{i})x_i + (2+\tau)\,\mathrm{y}_\tau \equiv 0(\mathrm{mod}11)$$

$$\Rightarrow (2+\tau)\,\mathrm{y}_\tau \equiv -\left[\left(\sum_{i=1 i\neq\tau}^{8} (2+i)x_i \right)(\mathrm{mod}11) \right]$$

$$\Rightarrow y_\tau = (2+\tau)^{-1} \times -\left[\left(\sum_{i=1 i\neq\tau}^{8} (2+i)x_i \right)(\mathrm{mod}11) \right]$$

Example 2.3.2.2 ISSN code 2 0 4 9 3 6 3 0. Assume that there is an *error* in x_5. If the code word with the error is

2 0 4 9 7 6 3 0 . By

$$\Rightarrow \sum_{i=1}^{8} (2+i)x_i \neq 0(\mathrm{mod}11)$$
$$\Rightarrow 137 \neq 0\,(\mathrm{mod}11)$$
$$\Rightarrow 5 \neq 0\,(\mathrm{mod}11)$$

Therefore

$$\Rightarrow \sum_{i=1}^{8} (2+i)x_i + (2+\tau)y_\tau \equiv 0\,(\mathrm{mod}11)$$
$$\Rightarrow 7y_5 \equiv -555(\mathrm{mod}11)\,\text{since } \tau = 5$$
$$\Rightarrow 7y_5 \equiv -1(\mathrm{mod}11)$$
$$\Rightarrow y_5 = 7^{-1} \times -1(\mathrm{mod}11)$$
$$\Rightarrow y_5 = 7^{-1} \times 10(\mathrm{mod}11)$$
$$\Rightarrow y_5 = 8 \times 10(\mathrm{mod}11)$$
$$\Rightarrow y_5 = 80(\mathrm{mod}11)$$
$$\Rightarrow y_5 = 3$$

Hence the error is corrected.

2.3.3 Transposition error detection in ISSN

Proposition 2.3.3. *ISSN code detects any transposition error in the code.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code and $Y = x_1, x_2, ..., x_\beta, x_\tau ..., x_8$ with x_τ *and* x_β be the exchanged digits of the ISSN code. Then

$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i$$
$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i + (\tau-\beta)x_\tau + (\beta-\tau)x_\beta \neq 0 \pmod{11}$$
$$\Rightarrow (\tau-\beta)(x_\tau - x_\beta) \neq 0 \pmod{11}$$

Provided $\tau \neq \beta$ and $x_\tau \neq x_\beta$. Therefore transposition error of x_τ and x_β detected.

Example 2.3.3.1 Suppose that 03178471 is the correct ISSN code. Now if x_4 and x_5 adjacently transposed

then the ISSN code is 03187471, yielding $x_4 = 8$ and $x_5 = 7$. The weight sum equation $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 1 \pmod{11}$. Therefore $\sum_{i=1\, j=9-i}^{8} jx_i \neq 0 \pmod{11}$. Now, because of transposition error β comes first before τ hence $\beta = 4$ *and* $\tau = 5$ then

$$\Rightarrow (\tau-\beta)(x_\tau - x_\beta) \neq 0 \pmod{11}$$
$$\Rightarrow (5-4)(7-8) = (1)(-1) = -1$$
$$\Rightarrow -1 \neq 0 \pmod{11}$$

Therefore adjacent transposition error detected.

Alternatively, proposition 2.3.3. can be proved by

$$\Rightarrow (\sum_{i=1\, j=9-i}^{8} (11-j)y_i) \text{ or } (\sum_{i=1}^{8} (2+\mathrm{i})y_i$$
$$\Rightarrow (\sum_{i=1\, j=9-i}^{8} (11-j)\,\mathrm{x}_i) + (\beta-\tau)x_\tau + (\tau-\beta)x_\beta \neq 0 \pmod{11}$$
$$\Rightarrow (\beta-\tau)(x_\tau - x_\beta) \neq 0 \pmod{11}$$

Provided $\tau \neq \beta$ and $x_\tau \neq x_\beta$. Therefore transposition error of x_τ and x_β detected.

Example 2.3.3.2. Suppose that 03178471 is the correct ISSN code. Now if x_4 and x_5 adjacently transposed

Then the ISSN code is 03187471, yielding $x_4 = 8$ and $x_5 = 7$. the weight sum equation $\sum_{i=1\, j=9-i}^{8} (11-j)x_i \neq 0 \pmod{11}$.

Now, because of transposition error β comes first before τ hence $\beta = 4$ *and* $\tau = 5$

then

$$\Rightarrow (\beta-\tau)(x_\tau - x_\beta) \neq 0 \,(\text{mod}\,11)$$
$$\Rightarrow (4-5)(7-8) = (-1)(-1) = 1$$
$$\Rightarrow 1 \neq 0 \,(\text{mod}\,11)$$

Therefore adjacent transposition error detected.

Remark 2.3.3. There is no way can $\tau = \beta$ for □ and □ are from *i* in the equation $\sum_{i=1\, j=9-i}^{8} jx_i \neq 0 \,(\text{mod}\,11)$ therefore

$1 \le \tau \le 7, 1 \le \beta \le 7, 0 \le x_\tau \le 9, 0 \le x_\beta \le 9$. Furthermore, $\tau \neq 8 \, and \, \beta \neq 8$ for x_8 is the check digit therefore it does not permute rather computed.

Corollary 2.3.3. *If transposition error occurs in the ISSN code but* $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0 \,(\text{mod}\,11)$ *then the transposed digits are equal.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code and $Y = x_1, x_2, ..., x_\beta, x_\tau ..., x_8$ with $x_\tau \, and \, x_\beta$ be the exchanged digits of the ISSN code. Then

$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i$$
$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i + (\tau-\beta)x_\tau + (\beta-\tau)x_\beta \equiv 0 \,(\text{mod}\,11)$$
$$\Rightarrow (\tau-\beta)(x_\tau - x_\beta) \equiv 0 \,(\text{mod}\,11)$$
$$\Rightarrow (\tau-\beta)(x_\tau - x_\beta) = 0 \text{ since } 0 \le x_\tau \le 9 \, and \; 0 \le x_\beta \le 9$$
$$\Rightarrow (x_\tau - x_\beta) = 0 \text{ since } (\tau-\beta) \neq 0$$
$$\Rightarrow x_\tau = x_\beta$$

Hence transposition error has occurred but since $x_\tau = x_\beta$ the weight check equation $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0 \,(\text{mod}\,11)$ holds. Alternatively, Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code and $Y = x_1, x_2, ..., x_\beta, x_\tau ..., x_8$ with $x_\tau \, and \, x_\beta$ be the exchanged digits of the ISSN code. Then

$$\Rightarrow (\sum_{i=1}^{8}\sum_{j=9-i}(11-j)y_i) \text{ or } (\sum_{i=1}^{8}(2+i)y_i$$

$$\Rightarrow (\sum_{i=1}^{8}(2+i)x_i) + (\beta-\tau)x_\tau + (\tau-\beta)x_\beta \equiv 0(\text{mod}11)$$

$$\Rightarrow (\beta-\tau)(x_\tau - x_\beta) \equiv 0(\text{mod}11)$$

$$\Rightarrow (\beta-\tau)(x_\tau - x_\beta) = 0 \text{ since } 0 \le x_\tau \le 9 \, and \; 0 \le x_\beta \le 9$$

$$\Rightarrow (x_\tau - x_\beta) = 0 \text{ since } (\tau-\beta) \ne 0$$

$$\Rightarrow x_\tau = x_\beta$$

Hence transposition error has occurred but since $x_\tau = x_\beta$ the weight check equation $\sum_{i=1}^{8}(2+i)x_i \equiv 0(\text{mod}11)$ holds.

2.3.4 Correction of Transposition error in ISSN

Proposition 2.3.4. *ISSN code corrects transposition error if only the error has been detected in the code.*

Proof. Suppose $X = x_1, x_2, ..., x_8$ is the ISSN code and $Y = x_1, x_2, ..., x_\beta, x_\tau ..., x_8$ with $x_\tau \; and \; x_\beta$ be the exchanged digits of the ISSN code. If the transposition error has been detected, then

$$\Rightarrow \sum_{i=1}^{8}\sum_{j=9-i} jy_i$$

$$\Rightarrow \sum_{i=1}^{8}\sum_{j=9-i} jy_i + (\tau-\beta)x_\tau + (\beta-\tau)x_\beta \ne 0(\text{mod}11)$$

$$\Rightarrow (\tau-\beta)(x_\tau - x_\beta) \ne 0(\text{mod}11)$$

$$\Rightarrow \sum_{i=1}^{8}\sum_{j=9-i} jy_i + (\tau-\beta)(x_\tau - x_\beta) \ne 0(\text{mod}11)$$

Let $x_\tau - x_\beta = H, \text{since } \tau-\beta = 1$ hence

$$\Rightarrow \sum_{i=1}^{8}\sum_{j=9-i} jy_i \ne 0(\text{mod}11)$$

$$\Rightarrow \sum_{i=1}^{8}\sum_{j=9-i} jy_i = \sum_{i=1}^{8}\sum_{j=9-i} jx_i + H$$

Where $0 \le H \le \pm 9$ and $\sum_{i=1}^{8}\sum_{j=9-i} jx_i \equiv 0(\text{mod}11)$.

If $H > 0$ then this then this implies that $x_\beta > x_\tau$ hence $x_\beta - x_\tau = H$. To find the position of x_β *and* x_τ , consider the ISSN code word $Y = x_1, x_2, ..., x_\beta, x_\tau ..., x_8$ then find the position where there is $x_i - x_{i+1} = H$. Finally after finding the position of the transposed digits, the transposition error is corrected by just simply exchanging the transposed digits, then the weight check equation

$\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0 (\mathrm{mod}11)$ is checked. If $H < 0$ then this implies that $x_\beta < x_\tau$ hence $x_\tau - x_\beta = H$. To find the position of x_β *and* x_τ , consider the ISSN code word $Y = x_1, x_2, ..., x_\beta, x_\tau ..., x_8$ then find the position where there is $x_i - x_{i-1} = H$. Finally after finding the position of the transposed digits, the transposition error is corrected by just simply exchanging the transposed digits, then the weight check equation is $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0 (\mathrm{mod}11)$ checked. *Remark 2.3.4.* If after the transposition error is corrected when $H > 0$ but $\sum_{i=1\, j=9-i}^{8} jx_i \neq 0 (\mathrm{mod}11)$ then consider $H < 0$.

Example 2.3.4.1 Suppose $X = 03178471$ is ISSN code. Now if x_4 *and* x_5 are adjacently transposed then the ISSN code is

$Y = 03187471$, yielding $x_4 = 8$ *and* $x_5 = 7$. To correct this transpose error

$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i \neq 0 (\mathrm{mod}11)$$
$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i = 1 (\mathrm{mod}11)$$
$$\Rightarrow \mathrm{H} = 1$$

Since $H > 0$ this implies $x_\beta > x_\tau$ therefore consider $x_i - x_{i+1} = 1$ in the code word $Y = 03187471$. Here there is only one case where $x_i - x_{i+1} = 1$ that is $8 - 7$ in the position x_4 *and* x_5 . To correct the error exchange 8 and 7 yielding 03178471 then weight check equation $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0 (\mathrm{mod}11)$ holds, therefore transposition error corrected.

Example 2.3.4.2 Suppose $X = 03178471$ is ISSN code. Now if x_2 *and* x_3 are adjacently transposed then the ISSN code is

$Y = 01378471$, yielding $x_2 = 1$ *and* $x_3 = 3$. To correct this transpose error:

$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i \neq 0 \,(\text{mod}11)$$
$$\Rightarrow \sum_{i=1\, j=9-i}^{8} jy_i = 9\,(\text{mod}11)\,\text{or} - 2\,(\text{mod}11)$$
$$\Rightarrow \text{H} = 9\, or\ -2$$

For $H = 9$ does not work since $Y = 01378471$ for there is no digits in Y such that $x_i - x_{i+1} = 9$. Consider $H = -2$ it implies that $x_\beta < x_\tau$ therefore $x_i - x_{i-1} = 2$. Here there is only one case where $x_i - x_{i-1} = 2$ that is $3 - 1$ in the position x_2 *and* x_3 . To correct the error exchange 3 and 1 yielding 01378471 then weight check equation $\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0\,(\text{mod}11)$ holds, therefore transposition error corrected.

Proposition 2.3.5. *ISSN code detects jump transposition error*

Proof. (**By contradiction**)

Proof. Suppose that $X = x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$ is ISSN code and $Y = x_1, x_2, x_5, x_4, x_3, x_6, x_7, x_8$ be ISSN code with jump transposition error. The check sum of X and Y are $8x_1 + 7x_2 + 6x_3 + 5x_4 + 4x_5 + 3x_6 + 2x_7 + x_8$ and

$8x_1 + 7x_2 + 6x_5 + 5x_4 + 4x_3 + 3x_6 + 2x_7 + x_8$ Respectively. Assume that X and Y are multiple of 11 that is both are

$\sum_{i=1\, j=9-i}^{8} jx_i \equiv 0\,(\text{mod}11)$. Consider $-Y$. After cancellation

$$X - Y = (6x_3 + 4x_5) - (6x_5 + 4x_3)$$
$$\Rightarrow 2x_3 - 2x_5$$
$$\Rightarrow 2(x_3 - x_5)$$

Since $x_3 - x_5$ is a one digit number, then $-9 \le x_3 - x_5 \le 9$. The only multiple of 11 between -9 and 9 is *zero*. Therefore $x_3 - x_5 = 0$, then $x_3 = x_5$ meaning no error has occurred. This is a contradiction.

3. Conclusion

i. ISSN can detect any error in the code if and only if the parity equation does not hold.

ii. ISSN can detect and correct all single error in the code if and if the position of the error is known.

ii. ISSN can detect and correct all transposition errors in the code even if the position of errors is unknown.

iii. ISSN can detect and correct all double errors in the code if and only if the position of errors in known.

iv. ISSN cannot detect and correct silent error(s).

v. ISSN code cannot correction jump transpose error, double error, jump twin error, phonetic error, twin error, and omission and insertion error.

References

[1] Kamaku, W. (2012). Error Detection and Correction on the International Standard

[2] Huffman, W. C., & Pless, V. (2010). *Fundamentals of Error-Correcting Codes*. Cambridge university press.

[3] Raymond, H. (1986). *A first Course in Coding Theory*. CLARENDON Press, U.S.A.

[4] Skrzypczak, J. (2014). International Standard Serial Number (ISSN).

[5] Stein, W. (2008). *Elementary Number Theory: Primes, Congruences, and Secrets: a Computational Ap- proach*. Springer Science & Business Media.