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Application of Generalized Estimating Equations to Non-Life Insurance Claims Reserve in Ghana -A Case Study

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Abstract

Claim reserves often take greater percentage of the liabilities of non-life insurance. The chain ladder method is the most widely used method for estimating these reserves though this method omits the possible existence of correlation within accident years. In Ghana, it is difficult to determine which methods are used in estimating claims reserve as almost all insurance companies are adamant to give any information on claims reserve. In this paper the Generalised Estimating Equation (GEE) framework is used to estimate claims reserve using data from SIC insurance company (Bolgatanga Branch) in Ghana. GEE allows for the incorporation of dependencies within accident years. The Quasi-Likelihood Information Criterion (QIC) and Correlation Information Criterion (CIC) were used as the criteria for model comparison and selection. The results show that the canonical Chain Ladder method and the GEE techniques can be used in Ghana to estimate insurance claims reserve. However, the GEE technique provides better estimate than the canonical Chain Ladder method.

Keywords: Reserve; Accident Years; Chain Ladder Method; Generalised Estimating Equation.

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1. Introduction

In return for the payment of premium, an insurance company accept the liability of making payments to the insured on the occurrence of one or more specified event(s) over a given time frame. Generally, payment of claims is delayed due to reasons such as reporting delays and settlement delays [1].

Payment of insurance claims in Ghana faces a lot of challenges. There is a common perception that it is difficult to make claims in non-life insurance [2]. Furthermore, [2], outlined some of the challenges that contribute largely to delays in claim payments. These challenges include the ignorance of procedures for making claims, inflation of prices of damaged items, inclusion of damaged part which is not due to the accidents in the accident repair works and repair works which were not inspected by mutual assessors before commencement. One factor which has not been considered is how insurance companies in Ghana compute reserves for the eventual payment of claims which could also form part of the problems that makes payment of claims unduly delay in Ghana.

Setting suitable reserves to meet prospective claim payment is one of the major undertakings of non-life insurance actuaries [3]. In fact, claim reserves take a greater percentage most of the time in the liabilities of non-life insurance [4]. Indeed, there are several methods for setting appropriate loss reserve in non-life insurance. The Chain Ladder method is the most widely used method for estimating claim reserve [5]. Despite its popularity, there are some limitations such as independence assumption within accident years which needs to be catered for. [6], indicated that accident years can be correlated because of factors such as inflation and court jurisprudence. Estimating claim reserves with the Chain Ladder method under correlated accident years can lead to overestimation or underestimation of the reserve. According to [7], consistent overestimation of claim reserves leads to inefficient capital allocation and overall cost of capital while consistent underestimation also leads to downgraded credit and in worse scenarios insurance company insolvency. Therefore, estimating sufficient outstanding claims is important for every non-life insurance company.

Majority of reserving models such as the chain ladder method requires the losses to be independent [8]. In reality this assumption often does not apply and therefore it is imperative to find a method which helps to manage such situations. The Generalised Estimating Equation (GEE) which is an extension of the Generalised Linear Model (GLM) helps to model data which are likely to be correlated.

Thus, in this paper, the Generalized Estimating Equation technique is applied to a non-life insurance data from Ghana to determine the usefulness of this technique in claims reserve in Ghana and the results compared to the results of the canonical chain ladder method. This paper is restricted to only a single portfolio, that is, motor insurance data.

2. Resources and Methods Used

2.1 Source of Data

Data which spans from 2012 to 2016 was collected from the SIC insurance company (Bolgatanga Branch) in Ghana. The data was divided into 6 accident years and 6 development lags.

2.2 Notation for Claims Reserving

The canonical claims reserving notation and terminology is introduced. The outstanding claims are structured in a triangular form known as the run-off triangle. Table 1 shows a run-off triangle with 6 accident years and development lags. In Table 1, $X_{i,j}$ represent the claims amount in accident year i and development year *j*. The upper part of Table 1 shows the observed claims while the lower part shows the outstanding claims to be estimated. From Table 1, it is observed that there is no outstanding claim in accident year 1.

Table 1: Run-off Triangle for Incremental Claims with 6 Accident Years

	Development lag j					
Accident Year i	1	2	3	4	5	6
1	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$X_{1,6}$
2	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	
3	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$		
4	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$			
5	$X_{5,1}$	$X_{5,2}$				
6	$X_{6,1}$					

2.3 The Univariate Chain Ladder Method

The univariate chain ladder model as presented by [9] has the following assumptions;

- For each $k \in \{1, \dots, n-1\}$ and $i \in \{1, \dots, n\}$ unknown constants $F_1, \dots, F_{n-1}$ with

$$\left(C_{i,k} \middle| \varsigma_{k-1}\right) = C_{i,k-1} F_k \tag{1}$$

- Different accident years $\{i \neq j\}$ are independent. That is

$$Cov\left(C_{i,k}, C_{j,k} \middle| \varsigma_{k-1}\right) = 0 \tag{2}$$

- There exist Unknown constants $\alpha_1, \dots, \alpha_{n-1}$ with

$$Var\left(C_{i,k} \middle| \varsigma_{k-1}\right) = C_{i,k} \alpha_k^2 \tag{3}$$

Again, for $i \in \{0,1, \dots, n\}$ and $k \in \{1, \dots, n\}$ where n is the number of development years such that $i + k \geq n + 1$ the univariate estimator (development factor) is expressed as

$$\hat{F}_k = \frac{\sum_{i=0}^{n-k} C_{i,k}}{\sum_{i=0}^{n-k} C_{i,k-1}} \tag{4}$$

Thus, the chain-ladder estimator is given by;

$$\hat{C}_{i,k}^{CL} = C_{i,n-i} \prod_{l=n-i+1}^{k} \hat{F}_l \tag{5}$$

Where $\hat{C}_{i,k}^{CL}$ is the estimated reserve at the accident year i and development lag k, $\hat{F}$ is the development factor and $C_{i,n-i}$ is the cumulative loss at accident year i and development lag $n-i$.

2.4 Generalised Estimating Equation (GEE)

GEE is used for estimating the parameters of a GLM with a possible unknown correlation between outcomes. Thus, GEE is an extension of the generalised linear models. Let N be the number of cases. For each case, there are $k = 1, \dots, n$ observed values. In the GEE framework Y can either be continuous or categorical. For example, $Y = Y_{ik}$ response for case i measured at different occasions. The expected value $\mu_{ij} = E(Y_{ij})$ is a function of the linear predictor η_{ij} given by

$$\mu_{ij} = g^{-1}(\eta_{ij}) = g^{-1}(X^T\beta) \tag{6}$$

The variance structure of Y_{ij} is given by

$$Var(Y_{ij}) = \phi u(\mu_{ij}) \tag{7}$$

The function u is the variance function and the parameter ϕ is the dispersion parameter.

2.5 The Covariance Structure

The variance function $V(\mu_i)$ is a diagonal matrix and according to [10], can be factorised into Equation (7)

$$V_i = \phi \nabla_i^{1/2} R_i(\alpha) \nabla_i^{1/2} \tag{8}$$

where ∇_i is a diagonal matrix with the element $u(\mu_{ij})$ and $R_i(\alpha)$ is the working covariance structure. The parameter α is assumed to be same for all cases.

2.6 GEE Parameter Estimation

The parameters to be estimated in the GEE framework are α (which determines the strength of the dependency between two subject), ϕ(the dispersion parameter) and the regression parameter β.The dispersion parameter ϕ is estimated by

$$\hat{\phi} = \frac{1}{N-p} \sum_{i=1}^{k} \sum_{j=1}^{n} e_{ij}^2 \tag{9}$$

where $N = \sum_{i=1}^{k} n_i$ is the aggregated number of subjects, p equals the number of regression parameters and e_{ij} is the Pearson residual.

The parameter α is also estimated using the Pearson residual.

The generalised estimating equation for estimating a vector of regression parameter β given the estimate $\hat{\alpha}$ and $\hat{\phi}$ is defined by

$$S(\beta) = \sum_{i=1}^{N} D_i^T V_i^{-1}(X_i - \mu_i) \quad (10)$$

where $D_i = \frac{\partial \mu}{\partial \beta}$. According to [9] Liang and Zeger (1986), the variance estimates for $\hat{\beta}$ can be obtained via the sandwich estimator

$$cov(\hat{\beta}) = A^{-1} M A^{-1} \quad (11)$$

where $A = \sum_{i=1}^{N} D_i^T V_i^{-1} D_i$, $M = \sum_{i=1}^{N} D_i^T V_i^{-1}(X_i - \mu_i)(X_i - \mu_i)^T V_i^{-1} D_i$

The mean structure for the logarithmic link function, that is, $\log(\mu_{ij}) = \gamma + \lambda_i + \beta_j$ and three working covariance structures namely the independent (Ind), exchangeable (Exch) and AR(1) covariance structures were used under Poisson (Pois) and Gamma (Gam) distributions for all the GEE models.

Here, γ represent the influence of accident year 1 and development lag 1 known as the intercept, λ_i is the effect of accident year $i\,(2 \leq i \leq n)$ and β_j represents the effect of development lag j $(2 \leq j \leq n)$ on the estimated reserve. These parameters are the regression parameters.

3. Results and Discussions

The data obtained was divided into two, the upper part and the lower part.

The upper part was used to represent the observed part of the data and the lower part taken off to be estimated by the models and to check precision of the estimate to the actual reserves.

3.1 Parameters Estimation

Table 2, shows the estimates for the parameter γ which represent the influence of accident year 1 and development lag 1 on the reserve estimate.

The differences in values under the different covariance structures are as a result of the possible unobserved correlation within accident years.

Table 2: Estimate of γ

Ind		**Exch**		**AR(1)**	
Pois	Gamm	Pois	Gam	Pois	Gam
0.541	0.485	0.634	0.485	0.531	0.418

Table 3 shows the estimates for the parameter λ_i which represent the influence of accident years on the estimated reserve.

Table 3: Estimate of λ_i

Accident Years	Ind		Exch		AR(1)	
	Pois	Gam	Pois	Gam	Pois	Gam
2	0.64	0.65	0.59	0.65	0.67	0.75
3	0.92	0.55	0.71	0.55	0.94	0.77
4	0.89	1.03	0.84	1.03	0.88	1.07
5	0.84	0.70	0.73	0.70	0.84	0.78
6	-0.87	-0.81	-0.96	-0.81	-0.86	-0.74

Table 4 shows the estimates for the parameter β_j. It depicts the effect of the development lags on the estimated reserve.

Table 4: Estimate of β_j

Development lag	Ind		Exch		AR(1)	
	Pois	Gam	Pois	Gam	Pois	Gam
2	0.48	0.72	0.48	0.72	0.48	0.69
3	0.56	0.64	0.56	0.64	0.55	0.54
4	-0.87	-0.70	-0.88	-0.70	-0.83	-0.68
5	-0.77	-0.41	-0.73	-0.41	-0.62	-0.24
6	1.51	1.57	1.47	1.57	1.61	1.83

3.2 Estimating Reserves

The Gamma and Poisson distribution were used because they have quadratic and linear variance functions respectively.

Thus, in addition to the Chain ladder method, 6 different models with the same mean structure were fitted to the data and competing models compared using the Quasi-likelihood Information Criterion (QIC) and Correlation Information Criterion (CIC). The reserves estimated by the six models considered under the GEE and the Chain Ladder method are presented in Table 5.

It can be observed that GEE model with independence covariance structure under Poisson produced the same results as the Chain Ladder method. Moreover, the estimate by exchangeable under Gamma mirrors the estimate by independent under Gamma. However, there is some variation in their QIC and CIC values presented in Table 6. Again, the closest estimated total reserves to the total actual reserves was the estimated value by the exchangeable covariance structure under Poisson.

Table 5: Estimated Reserves

Accident year	Actual Reserve	Chain Ladder	GEE					
			Ind		Exch		AR(1)	
			Pois	Gam	Pois	Gam	Pois	Gam
2	25263	51363	51363	51519	51071	51519	57615	69493
3	73576	74502	74502	53348	64244	53348	83523	79903
4	40828	78197	78197	93462	79014	93462	85354	115575
5	69518	98517	98517	88576	94702	88576	104850	105604
6	24734	22020	22020	24762	21539	24762	23364	28106
Total	233919	324599	324599	311667	310570	311667	354706	398681

3.3 Model Diagnostics

The residuals as shown in Fig 1 are normally distributed as there are no specific trend in the residual plots for all the models. This means that all the models are quite a good fit.

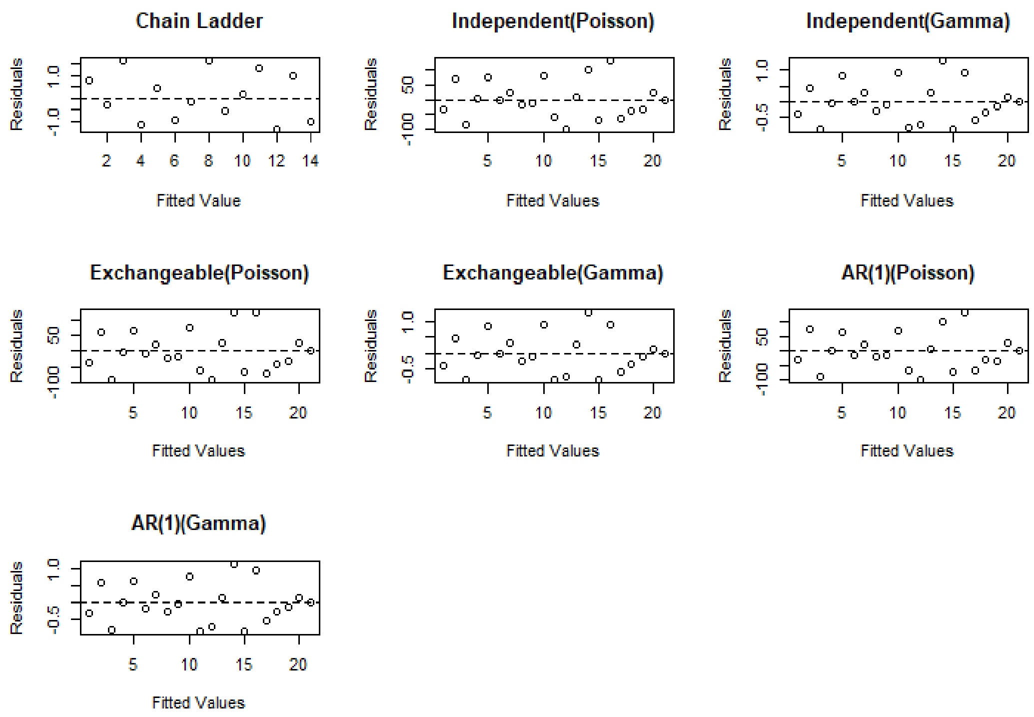

Figure 1: Residual Plots

QIC and CIC criteria for model comparison and selection are given in Table 6. The QIC values for independent covariance structure under Poisson and that of AR(1) covariance structure under Poisson are the same. They have the smallest QIC values making them preferable for prediction. However, the difference between their values and that of the exchangeable covariance structure under Poisson is small and therefore, makes the exchangeable covariance structure under Poisson also reasonable for prediction. In addition, the CIC values for exchangeable covariance structure under Poisson has the smallest value among all the values and therefore its predictions will be preferable.

Table 6: Criteria for Model Comparison and Selection

Selection Criteria	**Chain Ladder**	**GEE**					
		Ind		Exch		AR(1)	
		Pois	Gam	Pois	Gam	Pois	Gam
QIC	-5069000	-5069000	55.052	-5068000	52.05	-5069000	52.09
CIC	5.96	5.961	6.503	4.581	5.002	5.316	5.46

4. Conclusions

The GEE modelling technique and the canonical Chain Ladder method are applicable for claims reserving in Ghana. The canonical Chain Ladder method assumes independence within accident years. In situations in which this assumption does not hold, misleading inferences can be made. However, the GEE techniques help to model hidden dependencies between claims of the development lags within each accident year. These dependencies are modelled through working covariance structures. Misspecification of the working covariance structures has very little effect on the estimated claims. Again, Table 5 and Table 6 show that the estimated reserve for exchangeable covariance structure under Poisson model gives the better estimate since it has the closest estimated value to the actual reserve and have the smallest CIC value and comparable QIC value to the other models. Thus for better estimation claims reserve, insurance companies in Ghana can apply the GEE techniques.

5. Recommendations

Insurance companies in Ghana can adapt the GEE approach to estimate their non-life insurance claim reserve. This paper was restricted to only a single portfolio (motor insurance). The reserve estimates of several portfolios can be considered for future work. Again, other methods such as the collective risk model can be used to estimate the insurance claim reserve and the results compared to the canonical Chain Ladder and the GEE approach.

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