International Journal of Sciences: Basic and Applied Research (IJSBAR)

ISSN 2307-4531
(Print & Online)

http://gssrr.org/index.php?journal=JournalOfBasicAndApplied

Differential Transform Method for Solving Mathematical Model of *SEIR* and *SEI* Spread of Malaria

Abioye Adesoye Idowu[a]*, Ibrahim Mohammed Olanrewaju[b], Peter Olumuyiwa James[c], Amadiegwu Sylvanus[d], Oguntolu Festus Abiodun[e]

[a,d]*Department of Mathematics, School of General Studies, Maritime Academy of Nigeria, Oron, Akwa Ibom State, Nigeria*

[b,c]*Department of Mathematics, University of Ilorin, Ilorin, Kwara State, Nigeria*

[e]*Department of Mathematics, Federal University of Technology, Minna, Nigeria*

[a]*Email: abioyeadesoye@gmail.com*, [b]*Email: moibraheem@yahoo.com*, [c]*Email: peterjames4real@gmail.com*, [d]*Email: amadiegwusylvanus@yahoo.com*, [e]*Email: festus.tolu@futminna.edu.ng*

Abstract

In this paper, we use Differential Transformation Method (DTM) to solve two dimensional mathematical model of malaria human variable and the other variable for mosquito. Next generation matrix method was used to solve for the basic reproduction number R_0 and we use it to test for the stability that whenever $R_0 < 1$ the disease-free equilibrium is globally asymptotically stable otherwise unstable. We also compare the DTM solution of the model with Fourth order Runge-Kutta method (R-K 4) which is embedded in maple 18 to see the e pattern which was found to be efficient and accurate.

Keywords: Malaria; *SEIR;SEI*; Differential Transformation Method; Runge-Kutta method; Reproduction number.

---

* Corresponding author.

E-mail address: *abioyeadesoye@gmail.com*.

1. Introduction

Differential Transformation Method is one of the methods used to solve differential equations and it was derived from Taylor series expansion. It is also a semi-analytical method of solving both linear and nonlinear system of ordinary differential equations (ODE) to obtain approximate series of solutions. [17] was the first scientist to introduce this method and he used it to solve both linear and nonlinear initial value problems in electrical circuit analysis. This method has been used to solve problems in Mathematics and Physics [6], Fractional Differential-Algebraic Equations[16], Fourth-order Parabolic Partial Differential Equations[12], Fractional-order integro-differential equations[10], Differential Equation[11] and problems in epidemic models [1,3,14,20].

SIR epidemic model was first introduced by [8, 9], extended this work by introducing a two dimensional model with one variable human and the other variable for mosquito. [2] introduced exposed class in human that is *SEIR* (Susceptible-Exposed-Infectious-Recovered or Removed) model and this caused reduction to the long term prevalence on both infected humans and mosquitoes. [5] considered stability in *SIR* and *SI* model of malaria with relapse . [7] considered SI_sI_rR and *SI* model where I_s and I_r are drug sensitive malaria strains and drug resistant malaria strains respectively in a human population. [13] considered $SEII_dR$ and *SEI* model and *SI* model where I and I_d are human with malaria symptoms and human with drug resistance symptoms. [16] studied simple *SEIR* model of malaria and [15] considered *SEIR* and *SEI* with non-linear forces of infection.

2. Model Formulation

Let S_h, E_h, I_h and R_h represent Susceptible, Exposed, Infected and Recovered in human (host) respectively, S_m, E_m and I_m represent Susceptible, Exposed, Infected and Recovered in mosquito (vector) respectively. All the variables are functions of t. The parameter Λ is the recruitment rate of humans into the population, ξ is the biting rate of mosquitoes, b_h is the Probability of transmission of infection from an infectious humans to a susceptible mosquitoes, τ is the exposed rate to become infected in human, μ is the rate of natural death in human, ε is the rate of recovery from infectious in humans, ρ_1 is the rate at which humans that loss their immunity moves from recovery state to susceptible, δ is the diseases induced death rate, ρ_2 is the rate at which the recovered human moved back to infectious class (that is Relapse), Γ is the recruitment rate of mosquitoes into the population, b_m is the Probability of transmission of infection from an infectious mosquito to a susceptible humans, η is the rate of natural death in mosquito, α is the exposed rate to become infected in mosquito, q is the number of mosquitoes per individual.

2.1 Basic Assumptions

The following assumptions are considered

- The population has a constant size.

- All parameters and variables are assumed to be positive.
- E_h and E_m are infected but not infectious.
- We assume that malaria is contacted only from infected mosquitoes.
- We assume that there is no permanent recovery after treatment.
- Entry into the population is through birth and exit is through natural death or malaria induced death.

The *SEIR* and *SEI* Relapse model with disease-induced death.

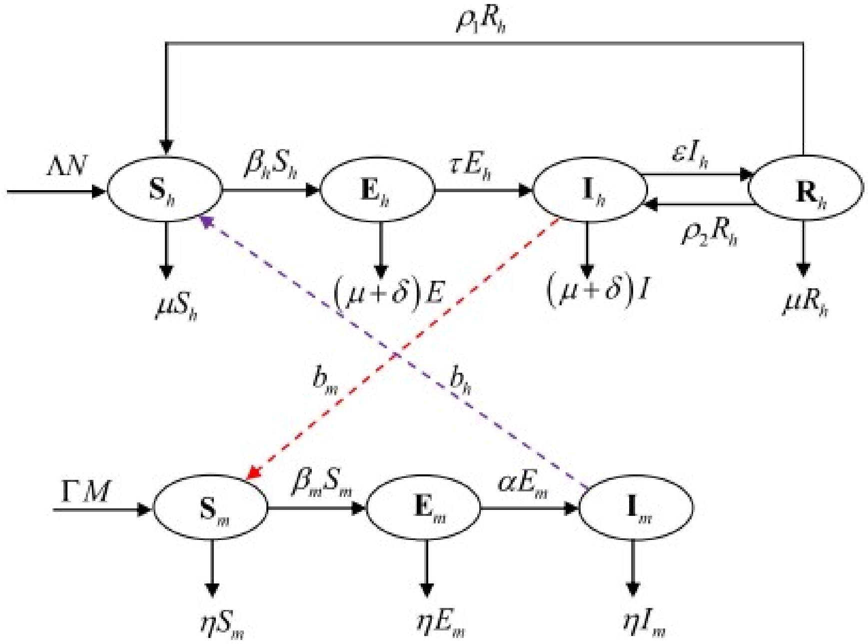

Figure 1: Flow chart of the model

2.2 Differential Equations of the Model

The diagram in figure 1 above is represented by the following differential equations.

$$\frac{dS_h(t)}{dt} = \Lambda N - \frac{\kappa_h S_h I_m}{N} - \mu S_h + \rho_1 R_h$$

$$\frac{dE_h(t)}{dt} = \frac{\kappa_h S_h I_m}{N} - (\tau + \mu + \delta) E_h$$

$$\frac{dI_h(t)}{dt} = \tau E_h + \rho_2 R_h - (\varepsilon + \mu + \delta) I_h$$

$$\frac{dR_h(t)}{dt} = \varepsilon I_h - (\rho_1 + \rho_2 + \mu) R_h \tag{1}$$

$$\frac{dS_m(t)}{dt} = \Gamma M - \frac{\kappa_m S_m I_h}{N} - \mu S_m$$

$$\frac{dE_m(t)}{dt} = \frac{\kappa_m S_m I_h}{N} - (\alpha + \eta) E_m$$

$$\frac{dI_m(t)}{dt} = \alpha E_m - \eta I_m$$

Where $\kappa_h = \xi b_h$ and $\kappa_m = \xi b_m$

Let N is the total population size for the humans and M is the total population size for the mosquitoes. Where M and N are constant.

If the total population of human and the total population of mosquito are denoted by N and M respectively, then the total population of both human and mosquito at time t, can be described as

$$\begin{aligned} N &= S_h(t) + E_h(t) + I_h(t) + R_h(t) \\ M &= S_m(t) + E_m(t) + I_m(t) \end{aligned} \tag{2}$$

Where N and M are constants.

Introducing new variables to normalize the system (1) as follows:

$$s_h = \frac{S_h(t)}{N},\ x_1 = \frac{E_h(t)}{N},\ x_2 = \frac{I_h(t)}{N},\ x_3 = \frac{R_h(t)}{N},\ s_m = \frac{S_m(t)}{M},\ y_1 = \frac{E_m(t)}{M},\ y_2 = \frac{I_m(t)}{M}$$

Where s_h, x_1, x_2, x_3, s_m, y_1, and y_2 are functions of t. System (1) becomes

$$\frac{ds_h(t)}{dt} = \Lambda - \xi b_h s_h y_2 - \mu s_h + \rho_1 x_3$$

$$\frac{dx_1(t)}{dt} = \xi b_h s_h y_2 - (\tau + \mu + \delta) x_1$$

$$\frac{dx_2(t)}{dt} = \tau x_1 + \rho_1 x_3 - (\varepsilon + \mu + \delta) x_2$$

$$\frac{dx_3(t)}{dt} = \varepsilon x_2 - (\rho_1 + \rho_2 + \mu) x_3 \tag{3}$$

$$\frac{ds_m(t)}{dt} = \Gamma - \xi b_m s_m x_2 - \mu s_m$$

$$\frac{dy_1(t)}{dt} = \xi b_m s_m x_2 - (\alpha + \eta) y_1$$

$$\frac{dy_2(t)}{dt} = \alpha y_1 - \eta y_2$$

And equation (2) respectively becomes

$$\begin{aligned} s_h &= 1 - x_1 - x_2 - x_3 \\ s_m &= 1 - y_1 - y_2 \end{aligned} \tag{4}$$

Equation (3) can be reduced by substituting equation (4) into it, to get

$$\frac{dx_1(t)}{dt} = \kappa q (1 - x_1 - x_2 - x_3) y_2 - (\tau + \mu + \delta) x_1$$

$$\frac{dx_2(t)}{dt} = \tau x_1 + \rho_2 x_3 - (\varepsilon + \mu + \delta) x_2$$

$$\frac{dx_3(t)}{dt} = \varepsilon x_2 - (\rho_1 + \rho_2 + \mu) x_3 \tag{5}$$

$$\frac{dy_1(t)}{dt} = \kappa (1 - y_1 - y_2) x_2 - (\alpha + \eta) y_1$$

$$\frac{dy_2(t)}{dt} = \alpha y_1 - \eta y_2$$

This is 5 – dimensional system of non-linear differential equations.

2.3 Basic Properties of the Model

2.3.1 Positivity of Solution

The region $\Omega \subset \mathbf{R}_+^5$ is positively invariant for the basic model (5) with non negative initial conditions in $\mathbf{R}_+^5$.

We considered the biological feasible solutions to show that

$$\Omega = \left\{\left(x_1(t), x_2(t), x_3(t), y_1(t), y_2(t)\right) \in \mathbf{R}_+^5 : x_1 + x_2 + x_3 \leq 1, y_1 + y_2 \leq 1\right\}$$

is positively invariant of $\mathbf{R}^5$ which is meaningful mathematically, biologically and epidemiologically in the domain Ω.

The solutions of equation (5) with the initial conditions are all positive $\forall\ t > 0$ and it is important in order to show that all variables have nonnegative.

Lemma

If $x_1(0) > 0,\ x_2(0) > 0,\ x_3(0) > 0,\ y_1(0) > 0$ and $y_2(0) > 0$ the solutions $x_1(t),\ x_2(t),\ x_3(t),$ $y_1(t)$ and $y_2(t)$ of equation (5) are positive $\forall\ t \geq 0$.

Proof

From the initial conditions given, we can show that the solutions of the equation (5) are positive; if not, we assume a contradiction in a way there exists a first time t_1 such that

$$\begin{aligned}
&x_1(t_1) = 0 \quad x_1'(t_1) \leq 0, \quad x_2(t_1) \geq 0, \quad x_3(t_1) \geq 0, \quad y_1(t_1) \geq 0,\\
&y_2(t_1) \geq 0 \quad x_2(t_1) + x_3(t_1) + y_1(t_1) + y_2(t_1) > 0 \quad x_2(t_1) > 0, \qquad (6)\\
&x_3(t_1) > 0, \quad y_1(t_1) > 0, \quad y_2(t_1) > 0 \quad t \in (0, t_1)
\end{aligned}$$

There exist a t_2 such that

$$\begin{aligned}
&x_2(t_2) = 0 \quad x_2'(t_2) \leq 0, \quad x_1(t_2) \geq 0, \quad x_3(t_2) \geq 0, \quad y_1(t_2) \geq 0,\\
&y_2(t_2) \geq 0 \quad x_1(t_2) + x_3(t_2) + y_1(t_2) + y_2(t_2) > 0 \quad x_1(t_2) > 0, \qquad (7)
\end{aligned}$$

$$x_3(t_2) > 0, \quad y_1(t_2) > 0, \quad y_2(t_2) > 0 \qquad t \in (0, t_2)$$

There exist a t_3 such that

$$\begin{aligned} &x_3(t_3) = 0 \quad x_3'(t_3) \le 0, \quad x_1(t_3) \ge 0, \quad x_2(t_3) \ge 0, \quad y_1(t_3) \ge 0, \\ &y_2(t_3) \ge 0 \quad x_1(t_3) + x_2(t_3) + y_1(t_3) + y_2(t_3) > 0 \qquad x_1(t_3) > 0, \\ &x_2(t_3) > 0, \quad y_1(t_3) > 0, \quad y_2(t_3) > 0 \qquad t \in (0, t_3) \end{aligned} \tag{8}$$

There exist a t_4 such that

$$\begin{aligned} &y_1(t_4) = 0 \quad y_1'(t_4) \le 0, \quad x_1(t_4) \ge 0, \quad x_2(t_4) \ge 0, \quad x_3(t_4) \ge 0, \\ &y_2(t_4) \ge 0 \quad x_1(t_4) + x_2(t_4) + x_3(t_4) + y_2(t_4) > 0 \qquad x_1(t_4) > 0, \\ &x_2(t_4) > 0, \quad x_3(t_4) > 0, \quad y_2(t_4) > 0 \qquad t \in (0, t_4) \end{aligned} \tag{9}$$

There exist a t_5 such that

$$\begin{aligned} &y_2(t_5) = 0 \quad y_2'(t_5) \le 0, \quad x_1(t_5) \ge 0, \quad x_2(t_5) \ge 0, \quad x_3(t_5) \ge 0, \\ &y_1(t_5) \ge 0 \quad x_1(t_5) + x_2(t_5) + x_3(t_5) + y_1(t_5) > 0 \qquad x_1(t_5) > 0, \\ &x_2(t_5) > 0, \quad x_3(t_5) > 0, \quad y_2(t_5) > 0 \qquad t \in (0, t_5) \end{aligned} \tag{10}$$

From the first case we can say

$$x_1'(t) = \kappa q(1 - x_2 - x_3) y_2 > 0 \tag{11}$$

This is a contradiction meaning that $x_1(t) > 0,\ t \ge 0$.

From the first case we can say

$$x_2'(t) = \tau x_1 + \rho_2 x_3 > 0 \tag{12}$$

This is a contradiction meaning that $x_2(t) > 0,\ t \geq 0$.

From the first case we can say

$$x_3'(t) = \varepsilon x_2 > 0 \tag{13}$$

This is a contradiction meaning that $x_3(t) > 0,\ t \geq 0$.

From the first case we can say

$$y_1'(t) = \kappa(1 - y_2)x_2 > 0 \tag{14}$$

This is a contradiction meaning that $y_1(t) > 0,\ t \geq 0$.

we say,

$$y_2'(t) = \alpha y_1 > 0 \tag{15}$$

This is a contradiction, meaning that $y_2(t) > 0,\ t \geq 0$. Therefore, the solutions $x_1(t),\ x_2(t),\ x_3(t),\ y_1(t)$ and $y_2(t)$ of equation (5) is still positive $\forall\ t \geq 0$.

2.3.2 Disease Free Equilibrium (DFE)

For equilibrium point, let the right hand side of equation (5) be equal to zero. That is

$$\begin{aligned}
0 &= \kappa q(1 - x_1 - x_2 - x_3)y_2 - (\tau + \mu + \delta)x_1 \\
0 &= \tau x_1 + \rho_2 x_3 - (\varepsilon + \mu + \delta)x_2 \\
0 &= \varepsilon x_2 - (\rho_1 + \rho_2 + \mu)x_3 \\
0 &= \kappa(1 - y_1 - y_2)x_2 - (\alpha + \eta)y_1 \\
0 &= \alpha y_1 - \eta y_2
\end{aligned} \tag{16}$$

By substituting $x_1 = x_1^*, x_2 = x_2^*, x_3 = x_3^*, y_1 = y_1^*$ and $y_2 = y_2^*$ into the equation (16), we obtain the

Endemic equilibrium points as

$$x_1^* = \frac{\kappa q\left(1 - x_2^* - x_3^*\right) y_2^*}{\left(\tau + \mu + \delta\right) + \kappa q y_2^*}$$

$$x_2^* = \frac{\tau x_1^* + \rho_2 x_3^*}{\left(\varepsilon + \mu + \delta\right)}$$

$$x_3^* = \frac{\varepsilon x_2^*}{\left(\rho_1 + \rho_2 + \mu\right)} \qquad (17)$$

$$y_1^* = \frac{\kappa\left(1 - y_2^*\right) x_2^*}{\left(\alpha + \eta\right) + \kappa x_2^*}$$

$$y_2^* = \frac{\alpha}{\eta} y_1^*$$

Therefore, equation (5) has shown that we can have two equilibriums in the non negative of $\mathbf{R}^5$ that is, the Disease Free Equilibrium (DFE) point $E_0 = \left(0, 0, 0, 0, 0\right)$ and the Endemic Equilibrium point, $E_1 = \left(x_1^*, x_2^*, x_3^*, y_1^*, y_2^*\right)$ which has been described in equation (5). The basic reproduction number of the DFE that is E_0 can be derived by using next generation matrix method and it is shown in equation (18) below

$$R_0 = \sqrt{\frac{\alpha \tau \kappa^2 q\left(\rho_1 + \rho_2 + \mu\right)}{\eta\left(\alpha + \eta\right)\left(\tau + \mu + \delta\right)\left[\left(\varepsilon + \mu + \delta\right)\left(\rho_1 + \rho_2 + \mu\right) - \varepsilon \rho_2\right]}} \qquad (18)$$

2.4 Local Asymptotic Stability of Disease Free Equilibrium (DFE)

Theorem 1

For Basic reproduction number (R_0) less than one $(R_0 < 1)$, the disease free equilibrium E_0 is locally asymptotically stable. Otherwise, it is unstable.

Proof

If $e_h = \frac{dx_1(t)}{dt}, i_h = \frac{dx_2(t)}{dt}, r_h = \frac{dx_3(t)}{dt}, e_m = \frac{dy_1(t)}{dt}$ and $i_m = \frac{dy_2(t)}{dt}$ then the system in equation (5)

linearised for the disease free equilibrium (DFE) is given by

$$e_h = \kappa q y_2 - (\tau + \mu + \delta) x_1$$

$$i_h = \tau x_1 + \rho_2 x_3 - (\varepsilon + \mu + \delta) x_2$$

$$r_h = \varepsilon x_2 - (\rho_1 + \rho_2 + \mu) x_3 \tag{19}$$

$$e_m = \kappa x_2 - (\alpha + \eta) y_1$$

$$i_m = \alpha y_1 - \eta y_2$$

The Characteristic equation of the polynomial of the above equation, that is $\left| \mathbf{J}(E_0) - \lambda I \right| = 0$ where λ is the eigenvalue and I is the Identity matrix is given by

$$\lambda^5 + A_4 \lambda^4 + A_3 \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0 \left(1 - R_0^2\right) = 0 \tag{20}$$

Where

$$A_4 = a_{11} + a_{22} + a_{33} + a_{44} + a_{55}$$

$$A_3 = a_{11} a_{22} + a_{33} (a_{11} + a_{22}) + (a_{11} + a_{22} + a_{33})(a_{44} + a_{55}) + (a_{44} a_{55} - \varepsilon \rho_2)$$

$$A_2 = a_{11} a_{22} (a_{33} + a_{44} + a_{55}) + a_{33} a_{44} (a_{11} + a_{22} + a_{55}) + a_{55} (a_{11} + a_{22})(a_{33} + a_{44}) - \varepsilon \rho_2 (a_{11} + a_{44} + a_{55})$$

$$A_1 = a_{44} a_{55} \left[a_{11} a_{22} + a_{33} (a_{11} + a_{22}) \right] + a_{11} a_{22} a_{33} (a_{44} + a_{55}) - \varepsilon \rho_2 \left[a_{11} (a_{44} + a_{55}) + a_{44} a_{55} + \alpha \tau \kappa^2 q \right]$$

$$A_0 = a_{11} a_{44} a_{55} (a_{22} a_{33} - \varepsilon \rho_2)$$

Using Routh-Hurwitz criterion, we discover that all roots of characteristics equation (20) have negative real part and show that $A_0 > 0, A_2 > 0, A_3 > 0$ and $A_4 > 0$. Therefore, the disease free equilibrium E_0 is locally asymptotically stable.

Table 1: Description of Parameters of the model

S/N	Parameter	Description	Value	Reference
1	ΛN	Recruitment rate of humans	1.2	[18]
2	β_h	Infection rate of humans	0.00638	[15]
3	τ	Developing rate of exposed in humans to infected	0.05	[15]
4	ε	Recovery rate	0.003704	[5]
5	ρ_1	Loss of immunity rate	0.0146	[5]
6	ρ_2	Relapse rate	0.004	[5]
7	μ	Natural death rate of human	0.03	[18]
8	δ	Disease-induced death	0.089	[18]
9	β_m	Infection rate of mosquitoes	0.0696	[16]
10	ΓM	Recruitment rate of mosquitoes	0.7	[16]
11	η	Natural death rate of mosquitoes	0.1429	[5]
12	α	Developing rate of exposed in mosquitoes to infected	0.083	[16]
13	ξ	Biting rate of Mosquitoes	0.29	[16]
14	b_h	Probability of transmission of infection from an infectious mosquitoes to a susceptible humans	0.022	[16]
15	b_m	Probability of transmission of infection from an infectious humans to a susceptible mosquitoes	0.24	[16]
16	N	Total population size for the human	1.0	[5]
17	M	Total population size for the mosquitoes	1.5	[5]
18	q	Number of mosquitoes per individual	1.5	[5]

3. Differential Transform Method (DTM)

The DTM can be derived from Taylor series expansion and it is a semi-analytical method of solving both linear and nonlinear system of ordinary differential equations (ODE) to obtain approximate series of solutions. The differential transform of the kth derivative of a function *f(t)* is given by;

$$F(k) = \frac{1}{k!}\left(\frac{d^k f(t)}{dt^k}\right)_{t=t_0} \tag{21}$$

Where$f(t)$ is the original function and F(k) the transformed function.

The inverse transformation is defined as follows;

$$f(t) = \sum_{k=0}^{\infty} t^k F[k] \tag{22}$$

Substituting Equation (21) into equation (22), we obtain

$$f(t) = \sum_{k=0}^{\infty} \left[\frac{t^k}{k!} \left(\frac{d^k f(t)}{dt^k} \right) \right]_{t=t_0} \tag{23}$$

It is shown from equation (5) that the DTM is derived from Taylor series expansion and it does not find numerical value for derivatives.

Basic rules of Differential Transform Method (DTM)

Given the arbitrary functions $f(t)$, $g(t)$ and $h(t)$ then the basic rules for manipulating differential transform method are as follows:

Rule 1: $f(t) = g(t) \pm h(t)$ then, $F[k] = G[k] \pm H[k]$

Rule 2: $f(t) = \alpha g(t)$ then, $F[k] = \alpha G[k]$ $\qquad$ α is a constant

Rule 3: $f(t) = \dfrac{d}{dx} g(t)$ then, $F[k] = (k+1)G[k+1]$

Rule 4: $f(t) = \dfrac{d^2}{dx^2} g(t)$ then, $F[k] = (k+1)(k+2)G[k+2]$

Rule 5: $f(t) = \dfrac{d^n}{dt^n} g(t)$ then, $F[k] = \dfrac{k!}{(k+n)!} G[k+n]$

Rule 6: $f(t) = g(t)h(t)$ then, $F[k] = \sum_{i=0}^{k} G[i]H[k-i]$

Rule 7: $f(t) = \dfrac{g(t)}{h(t)}$ then, $F[k] = \sum_{i=0}^{k} \dfrac{G[i]}{H[k-i]}$

Rule 8: $f(t) = t^{a}$ then $F[k]=\delta(k\text{-}a)$ where $\delta(k\text{-}a) = \begin{cases} 1, & k = a \\ 0, & k \neq a \end{cases}$

Rule 9: $f(t) = e^{at}$, then $F[k]=\dfrac{a^{k}}{k!}$

Rule 10: $f(t) = (1+t)^{a}$, then $F[k] = \dfrac{a(a-1)\ldots(a-k+1)}{k!}$

Rule 11: $f(t) = \sin(\alpha t + \beta)$, then $F[k] = \dfrac{\alpha^{k}}{k!}\sin\left(\dfrac{\pi}{2}k + \beta\right)$

Rule 12: $f(t) = \cos(\alpha t + \beta)$, then $F[k] = \dfrac{\alpha^{k}}{k!}\cos\left(\dfrac{\pi}{2}k + \beta\right)$

Rule 13: $f(x) = \sinh(ax)$, then $F[k] = \dfrac{1}{2k!}\left(a^{k} - (-a)^{k}\right)$

Rule 14: $f(x) = \cosh(ax)$, then $F[k] = \dfrac{1}{2k!}\left(a^{k} + (-a)^{k}\right)$

4. Numerical Solution and Results

Solution of the Model using Differential Transformation Method (DTM)

Let $S_h(k), X_1(k), X_2(k), X_3(k), S_m(k), Y_1(k)$ and $Y_2(k)$ represent the differential transform of the equation (2) and by applying the basic rules of DTM, each of the equation (2) has recurrence relation as follows:

$$S_h(k+1) = \frac{1}{k+1}\left[\Lambda\delta(k) - q\kappa_h\left\{\sum_{i=0}^{k} S_h(k-i)Y_2(i)\right\} - \mu S_h(k) + \rho_1 X_3(k)\right]$$

$$X_1(k+1) = \frac{1}{k+1}\left[q\kappa_h\left\{\sum_{i=0}^{k} S_h(k-i)Y_2(i)\right\} - (\tau+\mu+\delta)X_1(k)\right]$$

$$X_2(k+1) = \frac{1}{k+1}\left[\tau X_1(k) + \rho_2 X_3(k) - (\varepsilon+\mu+\delta)X_2(k)\right]$$

$$X_3(k+1) = \frac{1}{k+1}\left[\varepsilon X_2(k) - (\rho_1+\rho_2+\mu)X_3(k)\right]$$

$$S_m(k+1) = \frac{1}{k+1}\left[\Gamma\delta(k) - \kappa_m\left\{\sum_{i=0}^{k} S_m(k-i)X_2(i)\right\} - \eta S_m(k)\right]$$

$$Y_1(k+1) = \frac{1}{k+1}\left[\kappa_m\left\{\sum_{i=0}^{k} S_m(k-i)X_2(i)\right\} - (\alpha+\eta)Y_1(k)\right]$$

$$Y_2(k+1) = \frac{1}{k+1}\left[\alpha Y_1(k) - \eta Y_2(k)\right]$$

with the initial condition, where $S_h(0) = 0.83$, $X_1(0) = 0.08$, $X_2(0) = 0.07$, $X_3(0) = 0.02$, $S_m(0) = 0.7$, $Y_1(0) = 0.2$ and $Y_2(0) = 0.2$ according to [18]. Using the initial conditions above and the value of parameters in the Table 2, the results of iterations of differential transform of the equation (1) are obtained through Maple 2016 version. That is, when k = 0, the above equations become

$$S_h(1) = \Lambda - q\kappa_h S_h(0)Y_2(0) - \mu S_h(0) + \rho_1 X_3(0)$$

$$X_1(1) = q\kappa_h S_h(0)Y_2(0) - (\tau+\mu+\delta)X_1(0)$$

$$X_2(1) = \tau X_1(0) - (\varepsilon+\mu+\delta)X_2(0) + \rho_2 X_3(0)$$

$$X_3(1) = \varepsilon X_2(0) - (\rho_1+\rho_2+\mu)X_3(0)$$

$$S_m(1) = \Gamma - \kappa_m S_m(0)X_2(0) - \eta S_m(0)$$

$$Y_1(1) = \kappa_m S_m(0)X_2(0) - (\alpha+\eta)Y_1(0)$$

$$Y_2(1) = \alpha Y_1(0) - \eta Y_2(0)$$

When k = 1, we have

$$S_h(2) = -\frac{1}{2}\kappa_h q\left\{S_h(0)\left(\alpha Y_1(0) - \eta Y_2(0)\right) + \left(\Lambda - \kappa_h q S_h(0)Y_2(0) - \mu S_h(0) + \rho_1 X_3(0)\right)Y_2(0)\right\} -$$

$$\frac{1}{2}\mu\left\{\Lambda - \kappa_h q S_h(0)Y_2(0) - \mu S_h(0) + \rho_1 X_3(0)\right\} + \frac{1}{2}\rho_1\left\{\varepsilon X_2(0) - (\rho_1+\rho_2+\mu)X_3(0)\right\}$$

$$X_1(2)=\frac{1}{2}\kappa_h q\left\{S_h(0)\left(\alpha Y_1(0)-\eta Y_2(0)\right)+\left(\Lambda-\kappa_h q S_h(0)Y_2(0)-\mu S_h(0)+\rho_1 X_3(0)\right)Y_2(0)\right\}-$$

$$\frac{1}{2}(\tau+\mu+\delta)\left\{q\kappa_h S_h(0)Y_2(0)-(\tau+\mu+\delta)X_1(0)\right\}$$

$$X_2(2)=\frac{1}{2}\tau\left\{\kappa_h q S_h(0)Y_2(0)-(\tau+\mu+\delta)X_1(0)\right\}+\frac{1}{2}\rho_2\left\{\varepsilon X_2(0)-(\rho_1+\rho_2+\mu)X_3(0)\right\}-$$

$$\frac{1}{2}(\varepsilon+\mu+\delta)\left\{\tau X_1(0)+\rho_2 X_3(0)-(\varepsilon+\mu+\delta)X_2(0)\right\}$$

$$X_3(2)=\frac{1}{2}\varepsilon\left\{\tau X_1(0)+\rho_2 X_3(0)-(\varepsilon+\mu+\delta)X_2(0)\right\}-\frac{1}{2}(\rho_1+\rho_2+\mu)\left\{\varepsilon X_2(0)-\right.$$

$$\left.(\rho_1+\rho_2+\mu)X_3(0)\right\}$$

$$S_m(2)=-\frac{1}{2}\kappa_m\left\{S_m(0)\left(\tau X_1(0)+\rho_2 X_3(0)-(\varepsilon+\mu+\delta)X_2(0)\right)+\left(\Gamma-\kappa_m S_m(0)X_2(0)-\right.\right.$$

$$\left.\left.\eta S_m(0)\right)X_2(0)\right\}-\frac{1}{2}\mu\left(\Gamma-\kappa_m S_m(0)X_2(0)-\eta S_m(0)\right)$$

$$Y_1(2)=\frac{1}{2}\kappa_m\left\{S_m(0)\left(\tau X_1(0)+\rho_2 X_3(0)-(\varepsilon+\mu+\delta)X_2(0)\right)+\left(\Gamma-\kappa_m S_m(0)X_2(0)-\right.\right.$$

$$\left.\left.\eta S_m(0)\right)X_2(0)\right\}-\frac{1}{2}(\alpha+\eta)\left\{\kappa_m S_m(0)X_2(0)-(\alpha+\eta)Y_1(0)\right\}$$

$$Y_2(2)=\frac{1}{2}\alpha\left\{\kappa_m S_m(0)X_2(0)-(\alpha+\eta)Y_1(0)\right\}-\frac{1}{2}\eta\left\{\alpha Y_1(0)-\eta Y_2(0)\right\}$$

And so on for k = 2, 3, 4 …

We can now obtain the differential transform solution series as follows:

$$S_h(t)=S_h(0)+\left[\Lambda-q\kappa_h S_h(0)Y_2(0)-\mu S_h(0)+\rho_1 X_3(0)\right]t+\left[-\frac{1}{2}\kappa_h q\left\{S_h(0)\left(\alpha Y_1(0)-\right.\right.\right.$$

$$\eta Y_2(0)\big)+\big(\Lambda-\kappa_h q S_h(0)Y_2(0)-\mu S_h(0)+\rho_1 X_3(0)\big)Y_2(0)\Big\}-\frac{1}{2}\mu\Big\{\Lambda-\kappa_h q S_h(0)Y_2(0)-$$

$$\mu S_h(0)+\rho_1 X_3(0)\Big\}+\frac{1}{2}\rho_1\Big\{\varepsilon X_2(0)-\left(\rho_1+\rho_2+\mu\right)X_3(0)\Big\}\Bigg]t^2+\cdots$$

$$X_1(t)=X_1(0)+\left[q\kappa_h S_h(0)Y_2(0)-(\tau+\mu+\delta)X_1(0)\right]t+\Bigg[\frac{1}{2}\kappa_h q\Big\{S_h(0)\big(\alpha Y_1(0)-\eta Y_2(0)\big)+$$

$$\big(\Lambda-\kappa_h q S_h(0)Y_2(0)-\mu S_h(0)+\rho_1 X_3(0)\big)Y_2(0)\Big\}-\frac{1}{2}(\tau+\mu+\delta)\Big\{q\kappa_h S_h(0)Y_2(0)$$

$$-(\tau+\mu+\delta)X_1(0)\Big\}\Bigg]t^2+\cdots$$

$$X_2(t)=X_2(0)+\left[\tau X_1(0)-(\varepsilon+\mu+\delta)X_2(0)+\rho_2 X_3(0)\right]t+\Bigg[\frac{1}{2}\tau\Big\{\kappa_h q S_h(0)Y_2(0)-$$

$$(\tau+\mu+\delta)X_1(0)\Big\}+\frac{1}{2}\rho_2\Big\{\varepsilon X_2(0)-(\rho_1+\rho_2+\mu)X_3(0)\Big\}-\frac{1}{2}(\varepsilon+\mu+\delta)$$

$$\Big\{\tau X_1(0)+\rho_2 X_3(0)-(\varepsilon+\mu+\delta)X_2(0)\Big\}\Bigg]t^2+\cdots$$

$$X_3(t)=X_3(0)+\left[\varepsilon X_2(0)-(\rho_1+\rho_2+\mu)X_3(0)\right]t+\Bigg[\frac{1}{2}\varepsilon\Big\{\tau X_1(0)+\rho_2 X_3(0)-$$

$$(\varepsilon+\mu+\delta)X_2(0)\Big\}-\frac{1}{2}(\rho_1+\rho_2+\mu)\Big\{\varepsilon X_2(0)-(\rho_1+\rho_2+\mu)X_3(0)\Big\}\Bigg]t^2+\cdots$$

$$S_m(t)=S_m(0)+\left[\Gamma-\kappa_m S_m(0)X_2(0)-\eta S_m(0)\right]t+\Bigg[-\frac{1}{2}\kappa_m\Big\{S_m(0)\big(\tau X_1(0)+\rho_2 X_3(0)-$$

$$(\varepsilon+\mu+\delta)X_2(0)\big)+\big(\Gamma-\kappa_m S_m(0)X_2(0)-\eta S_m(0)\big)X_2(0)\Big\}-\frac{1}{2}\mu\big(\Gamma-$$

$$\kappa_m S_m(0)X_2(0)-\eta S_m(0)\big)\Bigg]t^2+\cdots$$

$$Y_1(t)=Y_1(0)+\left[\kappa_m S_m(0)X_2(0)-(\alpha+\eta)Y_1(0)\right]t+\Bigg[\frac{1}{2}\kappa_m\Big\{S_m(0)\big(\tau X_1(0)+\rho_2 X_3(0)$$

$$-(\varepsilon+\mu+\delta)X_2(0))+(\Gamma-\kappa_m S_m(0)X_2(0)-\eta S_m(0))X_2(0)\}-\frac{1}{2}(\alpha+\eta)$$

$$\{\kappa_m S_m(0)X_2(0)-(\alpha+\eta)Y_1(0)\}\big]t^2+\cdots$$

$$Y_2(t)=Y_2(0)+\left[\alpha Y_1(0)-\eta Y_2(0)\right]t+\left[\frac{1}{2}\alpha\{\kappa_m S_m(0)X_2(0)-(\alpha+\eta)Y_1(0)\}\right.$$

$$\left.-\frac{1}{2}\eta\{\alpha Y_1(0)-\eta Y_2(0)\}\right]t^2+\cdots$$

Using the following values $\Lambda=1.2,\ \kappa_h=0.00638,\ \Gamma=0.7,\ \mu=0.03,\ q=1.5,\ \rho_1=0.0146,$ $\rho_2=0.004,\ \tau=0.05,\ \delta=0.089,\ \varepsilon=0.003704,\ \kappa_m=0.0696,\ \alpha=0.083$ and $\eta=0.1429$. We obtain the differential transform solution series as follows

$$S_h(t)=0.83+1.174597690t-0.01819538748t^2+0.0001841729807t^3+$$

$$0.000003497073788t^4-0.0000006346815766t^5+0.00000004987487762t^6-$$

$$0.000000002784442912t^7+0.0000000001154782360t^8-3.337449766\times10^{-12}t^9+$$

$$4.151959351\times10^{-14}t^{10}+2.087513226\times10^{-15}t^{11}+\cdots$$

$$X_1(t)=0.08-0.012725690t+0.001646540080t^2-0.00009493055327t^3-$$

$$0.000000868278935t^4+0.0000006431329762t^5-0.00000006482083322t^6+$$

$$0.000000004135739122t^7-0.0000000001924002480t^8+6.564943952\times10^{-12}t^9-$$

$$1.424321951\times10^{-13}t^{10}-1.3326384\times10^{-17}t^{11}+\cdots$$

$$X_2(t)=0.07-0.00450928t-0.0000429143434t^2+0.00002920954574t^3-$$

$$0.000002082862206t^4+0.00000004245588104t^5+4.489970860\times10^{-9}t^6-$$

$$5.416878483\times10^{-10}t^7+3.415782323\times10^{-11}t^8-1.534707551\times10^{-12}t^9+$$

$$5.166238760\times10^{-14}t^{10}-1.223942376\times10^{-15}t^{11}+\cdots$$

$$X_3(t)=0.02-0.00071272t+0.000008967909440t^2-1.982650422\times10^{-7}t^3+$$

$$2.945695961\times10^{-8}t^4-1.829305969\times10^{-9}t^5+4.102680892\times10^{-11}t^6+$$

$$2.090992737\times10^{-12}t^7-2.635042547\times10^{-13}t^8+1.548076489\times10^{-14}t^9-$$

$$6.436921943\times10^{-16}t^{10}+2.024008403\times10^{-17}t^{11}+\cdots$$

$$S_m(t)=0.7+0.5965596t-0.04396755654t^2+0.002228830777t^3-0.00008569977010t^4+$$

$$0.000002424178504t^5-3.411109724\times10^{-8}t^6-1.048723261\times10^{-9}t^7+$$

$$7.734082631\times10^{-11}t^8-1.36765112\times10^{-13}t^9-2.982275032\times10^{-13}t^{10}+$$

$$2.825688864\times10^{-14}t^{11}+\cdots$$

$$Y_1(t)=0.2-0.0417696t+0.006061249445t^2-0.0005909215832t^3+0.00003944708700t^4-$$

$$0.000001757098465t^5+4.253000307\times10^{-8}t^6+3.72572990\times10^{-10}t^7-$$

$$6.912853686\times10^{-11}t^8+6.43890935\times10^{-13}t^9+2.856363805\times10^{-13}t^{10}-$$

$$3.024857484\times10^{-14}t^{11}+\cdots$$

$$Y_2(t)=0.1+0.00231t-0.001898487900t^2+0.0002581258750t^3-0.00002148316974t^4+$$

$$0.000001268810635t^5-5.452536872\times10^{-8}t^6+1.617380778\times10^{-9}t^7-$$

$$2.502501938\times10^{-11}t^8-2.401770322\times10^{-13}t^9+8.776424550\times10^{-15}t^{10}+$$

$$2.041242592\times10^{-15}t^{11}+\cdots$$

Table 2: For Human Population Model
Comparison Between Fourth Order Runge-kutta and Differential Transformation Methods

TIME day(s)	R-K 4 S[h](t)	DTM S[h](t)	R-K 4 E[h](t)	DTM E[h](t)	R-K 4 I[h](t)	DTM I[h](t)	R-K 4 R[h](t)	DTM R[h](t)
0	0.83000000	0.83000000	0.08000000	0.08000000	0.07000000	0.07000000	0.02000000	0.02000000
1	1.98658939	1.98658939	0.06882563	0.06882563	0.06547498	0.06547498	0.01929608	0.01929608
2	3.10792572	3.10792572	0.06037835	0.06037836	0.06101172	0.06101172	0.01860926	0.01860926
3	4.19516726	4.19516725	0.05412525	0.05412527	0.05671848	0.05671848	0.01793917	0.01793917
4	5.24946239	5.24946238	0.04959382	0.04959383	0.05266733	0.05266732	0.01728577	0.01728577
5	6.27192851	6.27192854	0.04638242	0.04638240	0.04890145	0.04890145	0.01664924	0.01664924
6	7.26364128	7.26364157	0.04416012	0.04415986	0.04544188	0.04544184	0.01602983	0.01602983
7	8.22563024	8.22563207	0.04266084	0.04265922	0.04229322	0.04229301	0.01542787	0.01542788
8	9.15887825	9.15888703	0.04167510	0.04166734	0.03944841	0.03944738	0.01484364	0.01484367
9	10.06432297	10.06435781	0.04104103	0.04101055	0.03689256	0.03688831	0.01427737	0.01427748
10	10.94285942	10.94297848	0.04063590	0.04053289	0.03460582	0.03459067	0.01372922	0.01372962

Table 3: For Mosquito Population Model
Comparison Between Fourth Order Runge-kutta and Differential Transformation Methods

TIME day(s)	R-K 4 S[m](t)	DTM S[m](t)	R-K 4 E[m](t)	DTM E[m](t)	R-K 4 I[m](t)	DTM I[m](t)
0	0.700000000	0.700000000	0.200000000	0.200000000	0.100000000	0.100000000
1	1.254737592	1.254737563	0.163738439	0.163738461	0.100649387	0.100649371
2	1.733783749	1.733783701	0.136556069	0.136556104	0.098784662	0.098784638
3	2.147769990	2.147769946	0.116087161	0.116087191	0.095344834	0.095344812
4	2.505793834	2.505793821	0.100557912	0.100557913	0.090985315	0.090985298
5	2.815639333	2.815639629	0.088651091	0.088650740	0.086155906	0.086155929
6	3.083969370	3.083972205	0.079397010	0.079393798	0.081159016	0.081159354
7	3.316491478	3.316509067	0.072087382	0.072067609	0.076192626	0.076194764
8	3.518099781	3.518184125	0.066208340	0.066113811	0.071381646	0.071391813
9	3.692995818	3.693330192	0.061389124	0.061015014	0.066800565	0.066840394
10	3.844791057	3.845933674	0.057363291	0.056086661	0.062489624	0.062624100

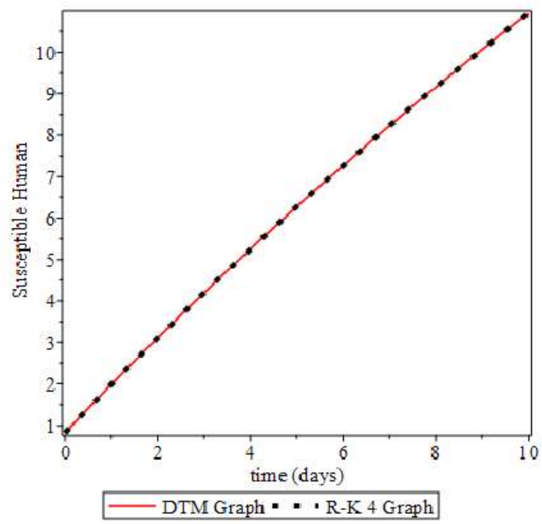

Figure 2: The graph of Susceptible Human against Time for both DTM and R-K 4

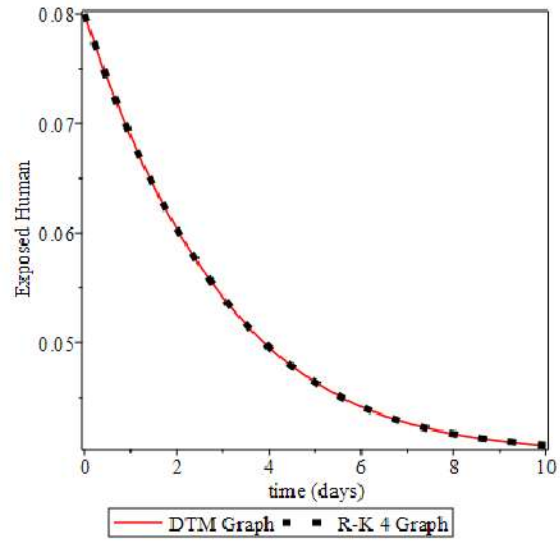

Figure 3: The graph of Exposed Human against Time for both DTM and R-K 4

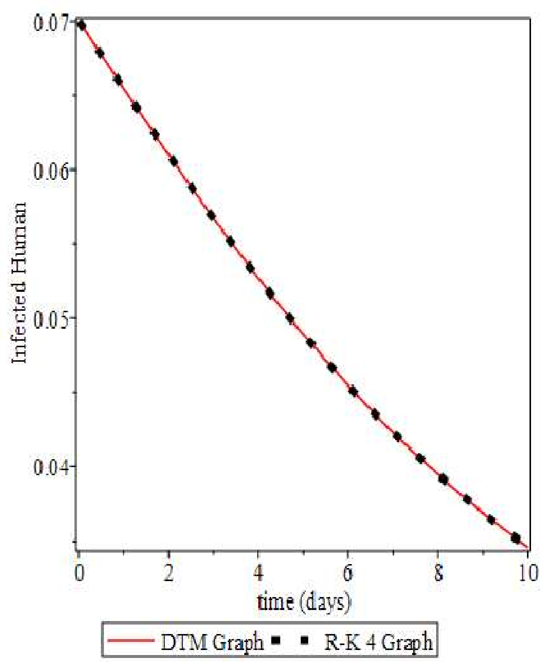

Figure 4: The graph of Infected Human Time for both DTM and R-K 4

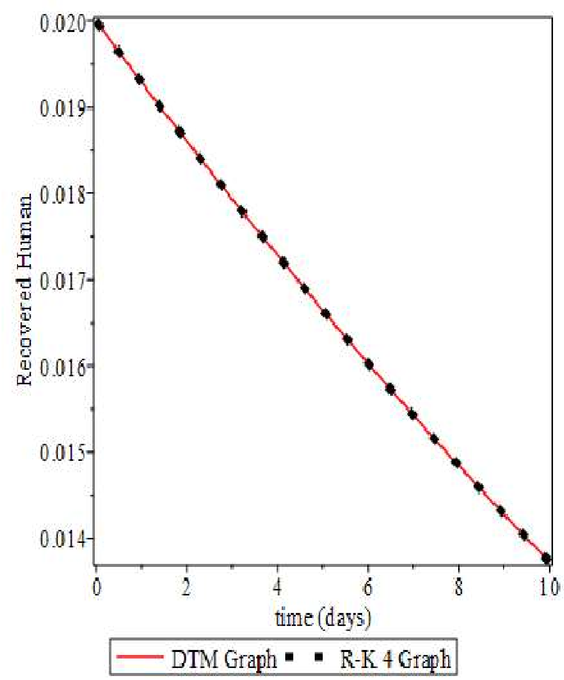

Figure 5: The graph of Recovered Humanagainst against Time for both DTM and R-K 4

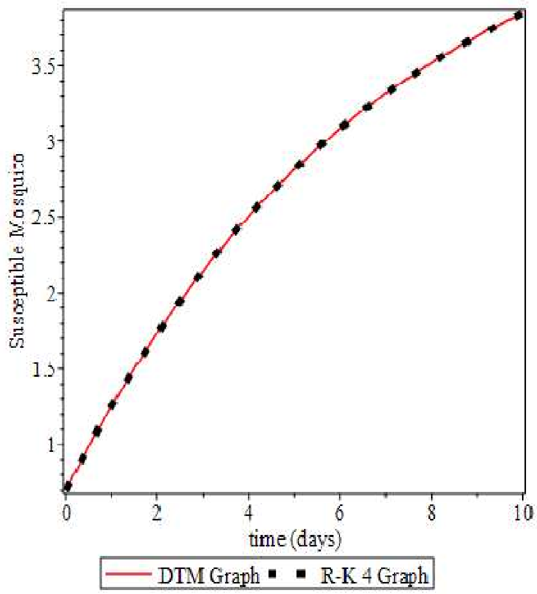

Figure 6: The graph of Susceptible Mosquito against Time for both DTM and R-K 4

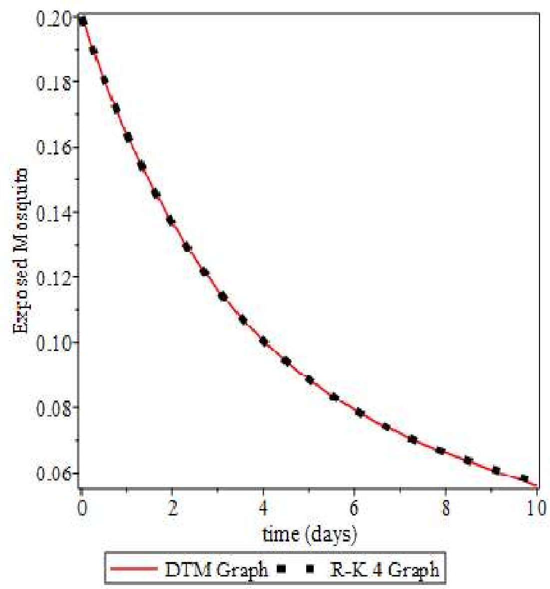

Figure 7: The graph of Exposed Mosquito against Time for both DTM and R-K 4

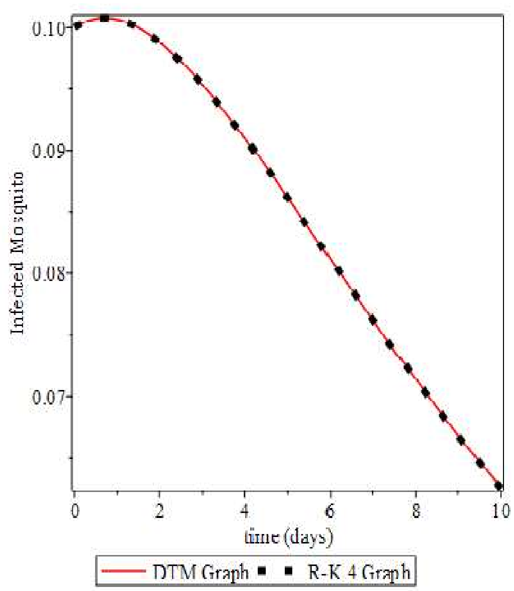

Figure 8: The graph of Infected Mosquito against Time for both DTM and R-K 4

5. Conclusion

Mathematical model of malaria is built on system of ordinary differential equation (ODE). Next generation matrix method was used to solve for the basic reproduction number R_0 and we use it to test for the stability that whenever $R_0 < 1$ the disease-free equilibrium is globally asymptotically stable otherwise unstable. These models were solved using DTM. The Solution of DTM compared favorably with the solution obtained by using classical fouth-other Runge-Kuta method. The solution of the two methods follows the same pattern and behavior.

References

[1] M. Z. Ahmad., D. Alsarayreh, A. Alsarayreh & I. Qaralleh. (2017, February). "Differential Transformation Method (DTM) for solving SIS and SI epidemic models", SainsMalaysiana, 46(10), pp. 2007-2017, http://dx.doi.org/10.17576/jsm-2017-4610-40

[2] R. M. Anderson & R. M. May (1991)."Infectious diseases of Humans: Dynamics and Control", New York, NY, Oxford University Press, December, 1992.ISBN 0-19-854599-1.

[3] F. S. Akinboro, S. Alao & F. O. Akinpelu (2014, March). "Numerical Solution of SIR Model using Differential Transformation Method and Variational Iteration Method" ,Gen. Math. Notes, 22(2), 82-92.

[5] H.-F. Huo & Q.-M.Qui (2014, February)."Stability of a Mathematical Model of Malaria Trans- mission with Relapse",Hindawi Publishing Corporation Abstract and Applied analysis, 2014(1) Article ID 289349, 9 pages, http://dx.doi.org/10.1155/2014/289349.

[6] B. Ibis, M. Bayram & G. Agargun. (2011). "Applications of Fractional Differential Transform Method to Fractional Differential-Algebraic Equations", European Journal of Pure and Applied Mathematics, 4(2), 129-141, ISSN 1307-5543.

[7] T. Julius, D.-M. Senelani & N. Farai. (2014, April)."A mathematical model for the transmission and spread of drug sensitive and resistant malaria strains within a human population", Hindawi Publishing Corporation, ISRN Biomathematics, Article ID 636973, pp. 1-12, http://dx.doi.org/10.1155/2014/636973.

[8] W. O. Kermack, W. O & A. G. McKendrick. (1927, August). "A contribution to the mathematical theory of epidemics", Proceeding of the Royal Society of London. Series A, Containing papers of a Mathematical and Physical Character, 115 (772): 700-721.

[9] G. Macdonald..The Epidemiology and control of malaria, Oxford University press, London, 1957.

[10] G. Methi (2016, June). "Solution of Differential Equation Using Differential Transform Method", Asian Journal of Mathematics & Statistics, 9(13), pp 1-5

[11] F. Mirzaee (2011, April). "Differential Transform Method for Solving Linear and Nonlinear Systems of Ordinary Differential Equations", Applied Mathematical Sciences, 5(70), 3465 - 3472.

[12] D. Nazari & S. Shahmorad. (2010, January)."Application of the fractional differential transform method to fractional-order integro-differential equations with nonlocal boundary conditions", Journal of Computational and Applied Mathematics, 234, 883-891.

[13] K. O. Okosun & O. D. Makinde (2011, September).“Modelling the impact of drug resistance in malaria”, Available online at http://www.academicjournals.org/IJPS, DOI: 10.5897/IJPS 10.542, ISSN 1992 - 1950.

[14] O. J. Peter & M. O. Ibrahim. (2017). Application of Differential Transform Method in Solving a Typhoid Fever Model. International Journal of Mathematical analysis and Optimization.1(1),250-260.

[15] S. Olaniyi & O. S. Olabiyi (2013, August).“Mathematical Model for Malaria Transmission Dynamics in Human and Mosquito Populations with Nonlinear Forces of Infection”, International Journal of Pure and Applied Mathematics 88(1), 125-156, doi: http://dx.doi.org/10.12732/ijpam.v88i1.10.

[16] M. A. E. Osman, K. K. Adu & C. Yang (2017, November).“A Simple SEIR Mathematical Model of Malaria Transmission”, Asian Research Journal of Mathematics 7(3): 1-22, Article no.ARJOM.37471, ISSN: 2456-477X.

[17] B. Soltanalizadeh (2012). “Application of Differential Transformation Method for Solving a Fourth-order Parabolic Partial Differential Equations”, International Journal of Pure and Applied Mathematics, Vol. 78(3), 299-308, ISSN: 1311-8080 (printed version).

[18] J. K. Zhou Differential Transformation and its Applications for Electrical Circuits.Huazhong University Press, Wuhan, China. (1986).

[19] M. Altaf Khan., A. Wahid., S. Islam., I. Khan., S.Shafie, & T. Gul. (2015). Stability analysis of an SEIR epidemic model with non-linear saturated incidence and temporary immunity. Int. J. Adv. Appl. Math. and Mech. 2(3), 1 – 14.

[20] O. J. Peter., M. O. Ibrahim ., F. A. Oguntolu.,O. B. Akinduko.,& S. T. Akinyemi. (2018). Direct and Indirect Transmission Dynamics of Typhoid Fever Model by Differential Transform Method. ATBU, Journal of Science, Technology & Education (JOSTE); 6 (1), 167-177.