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Applying the Polynomial Model in Simplex –Centroid Design to Formulate the Optimum Dairy Feed

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Abstract

The component proportions of dairy feeds using simplex- centroid design approach was considered. Soya beans, maize jam, cotton seed and fish meal were blended at each design point of the Simplex-Centroid and the responses in terms of dairy animal productivity was considered. The main objective of the study was to test a simplex- centroid design and search for optimal mixture of feed ingredients to the outcome on milk productivity of dairy animals using a polynomial model. Research therefore seek to derive a polynomial model for the four components simplex- centroid design. Using the collected data, fit the derived model and determine the coefficient at each design point and test for their significance using R-software. The result proved that the feed supplement had a significance effect on the milk productivity. However ANOVA was run to improve the precision of the model by taking into account the constant term. This proved that also other feeding practices were significantly important in the productivity. The result showed that the blend of soya beans and Fish meal supplement has significance contribution at 0.05. The concentrate of soya beans, cotton seed and fish meal supplement shows significance contribution at 0.05. Again at the centroid point where all the four ingredient are blended was significance at 0.05.

Keywords: Simplex-centroid; Polynomial; Design.

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1. Introduction

This section gives a background of the study by describing the dairy meal concentrates, Mixture experiments and their applications on formulation of dairy meal concentrates. The statement of the problem is discussed. From the statement of the problem, the objectives of the study which guided the researcher throughout the study were formulated. The scope, assumptions and the significance of the study were discussed.

1.1 Background of the study

Dairy meals are mixtures used to complement natural animal feeding. The agricultural research has to consider the formulation of dairy meals, their process as well as their acceptability by farmers and agricultural experts. The multiple characteristics of meals and cost efficiency are major issues of agricultural research.

In dairy production, feeding program affects profitability more than any other single factor. Without good feeding programmes, the benefits of good breeding and management programme cannot be realized. Practical feeding of dairy cows has four main themes; assessment of nutritional value of feedstuff, description of the nutrient requirements of animals, ratio formulation and diagnosis, prognosis and prevention of disorders of nutrition and metabolism [1].The nutrients should be supplied in their required amount to meet specific performance targets.

Due to high costs and non-availability of those sources, readily available supplements are required to optimally feed dairy cows particularly during dry season [2].

Dairy animals require nutrients for maintenance, growth, reproduction and milk production. The nutrient requirement of the animal depends on its physiological state. Forages which form the basic diet of ruminants may not contain sufficient nutrients and minerals to meet the feed requirements for dairy animals. Concentrates, rich in the nutrients that are deficient in forages, balance the diet, they also improve forage intake especially that of low quality, as is the case under smallholder production systems of Eastern Africa. However, too high a proportion of concentrates in the diet interferes with rumen fermentation and decreases digestion efficiency.

Concentrates contain high-energy and protein rich feedstuffs that are added to a ration primarily to increase its energy and protein density. They are mostly cereals or cereal by-products, oil seed cakes, roots and tubers, molasses, fats and oils. However, these energy and protein sources also contain small quantities of minerals and vitamins. Although minerals make up a small portion of the diet, their functional contribution is significant and hence their supplementation is necessary. The mineral level in plant feed in turn depends on the mineral content of the soil. The main diet should meet basic individual mineral requirements which depend on the age of the dairy animal and the level of production.

A number of non-conventional feed resources are being used as protein concentrates for farmers home-mixing. However, there is a problem to formulate diets that are balanced with respect to protein, energy, vitamins and minerals and at the same time being low-cost.

Various evaluation techniques have been used in the formulation of dairy feeds. In [3], Wagner used Pearson square. However this technique can only balance one nutrient at a time. Least cost formulation based on linear programming has also been used in [4]. This technique requires commercial feed software which is costly for most extension organization in developing countries and return on investment when using them on a small scale does not justify its purchase.

Castro, and his colleagues in [5] stated that optimization can be defined as the choice of the best alternative, from some specified set of possibilities. In their research, optimization entails developing a formula, which through the optimization process, brings about the optimum levels of the key components (independent variables) and the desired product (response factor). This is expected to yield a more nutritious concentrate that gives maximum response in terms of dairy products.

Dean, and his colleagues in [6] used least- cost feed formulation for dairy cattle to the next logical step of profit maximization. Linear programming model presented, selects simultaneously the concentrate and roughage components of the ration, the roughage-concentrate ratio, level of feeding per cow and quantity of milk production maximizing income above feed costs. They presented the nutritional and economic aspects of the model in mathematical form. This was followed by illustrations on nutritional specifications which include body maintenance requirements and milk production response curve associated with alternative levels of energy and protein fed. They recommended an improvement of the model by research designed to define more precisely the milk production response relationships for cows of different ability, size, breed, stage of lactation and other determinants.

Chakeredza, and his colleagues in [7] used linear programming to formulate a least cost ration. The least cost formulation was described as a mathematical solution based on linear programming

Mahmut, and his colleagues in [8] used Mixture design to investigate the effects of four different gums (xanthan gum, guar gum, alginate and locust bean gum) and their combinations on the rheological properties of a prebiotic model instant hot chocolate beverage (including 3.5% inulin) and to determine their interactions in the model beverage. Simplex centroid mixture design was applied to predict the physicochemical (soluble solids, pH, colour properties) and rheological parameters (consistency index (K), flow behaviour index (n) and apparent viscosity (η_{50})) of the samples. In the model, the optimum gum combination was found by simplex centroid mixture design as 59% xanthan gum and 41% locust bean gum, and the highest K value was 33.56 Pa s^n. The increase of guar gum and alginate in the gum mixture caused a decrease in the K value of the sample.

Ashwini, and his colleagues in [8], worked on Optimization of Carboxymethyl-Xyloglucan-Based Tramadol Matrix Tablets Using Simplex Centroid Mixture Design.The aim was to determine the release-modifying effect of carboxymethyl xyloglucan for oral drug delivery. Sustained release matrix tablets of tramadol HCl were prepared by wet granulation method using carboxymethyl xyloglucan as matrix forming polymer. HPMC K100M was used in a small amount to control the burst effect which is most commonly seen with natural hydrophilic polymers. A simplex centroid design with three independent variables and two dependent variables was employed to systematically optimize drug release profile. Carboxymethyl xyloglucan, HPMC K100M, and dicalcium phosphate were taken as independent variables. The dependent variables selected were percent of drug release at 2nd hour and at 8th hour .They developed Response surface plots and optimum formulations were selected on the basis of desirability. The formulated tablets showed anomalous release mechanism and followed matrix drug release kinetics, resulting in regulated and complete release from the tablets within 8 to 10 hours. The polymer carboxymethyl xyloglucan and HPMC K100M had significant effect on drug release from the tablet. Polynomial mathematical models, generated for various response variables using multiple regression analysis, were found to be statistically significant. The statistical models developed for optimization were found to be valid.

Okpala and Okoli in [10] used three ingredients.Biscuits were produced by blends of pigeon pea, sorghum and cocayan flours. Ten formulations were obtained from this design.

In their paper “A V-optimal design for Scheffe’ polynomial model”, Shuangzhe and Heinz (1993), applied the weighted simplex centroid design to obtain V-optimal allocations of the observations and showed optimality over the entire simplex using the equivalence theorem.

Wang and Fang in [11] Studied component proportions of medicine SIBELIUM capsule and concluded that the quality of medicine depends on the proportions of the components. They analyzed the component of the medicine. The design points of symmetric-simplex design and design points generated by XVERT algorithm were both used. For the two quality characteristics being considered, it was found that the two optimization methods produced similar results.

Gaylor and Sweeny in [12], studied optimum allocation. They said that a region of interest does not necessarily correspond to the region available for experimentation$0 \leq x \leq 1$. They said that the allocation of experimental data points minimizes the average variance of the predicted values occurring according to the density function in the region of prediction is derived as$\hat{y} = \alpha + \beta z$. The errors of this relation were assumed to be uncorrelated and of a common varianceσ_y^2.

Castro and his colleagues [5] tested a complex constrained simplex using direct search sequential method for the optimization of a ternary mixture of protein ingredients used in a formulation for the preparation of a milk drink regularly consumed in institutional nutritional programmes. They mixed three protein and mixtures containing different proportions of the three ingredients submitted to sensory, nutritional and economic evaluations. From the result they suggested that the simplex method is efficient and flexible enough for multiresponse optimization.

In dairy farming, nutritional problems in terms of quantity and quality of feeds are the most critical constraints to milk productions. In Kenya for example, where most dairy farmers practice on small scale, the current low levels of milk production do not reflect the genetic potential of most dairy breeds of cattle reared. In dairy production, the feeding programme affects profitability more than any other single factor. The costs of feeding make up 60-80% of the variable costs of milk production [13].

This research adopts a mixture experiment design which is a special type of a response surface experiment in which the factors are the ingredients or components of the mixture; the response is a function of the proportions of each ingredient. The proportional amounts of each ingredient are typically measured by weight, volume, and ratio. They also defined Response surface methodology as a collection of statistical and mathematical techniques useful for developing, improving and optimizing process, and response is the performance measure of a given process [14].

Therefore, the research seeks to determine an optimal dairy feed concentrate using four components (ingredients) simplex-centroid design by applying the polynomial model. This will help in determining the best optimal mixture in terms of milk productivity of dairy cows.

1.2 Scope of the study

A four component mixture experiment is considered where the experimental domain τ is given by

$$\tau = \{t\epsilon[0,1]^m : 1'_m t = 1\} \tag{1.1}$$

A polynomial equation will be derived and data will later be fitted on the model where at the points of simplex, data on the response will be collected.

At this point the coefficients of the model were numerically determined using R software and their significant tested.

The data used was collected from the Best feed company based in Meru County. Best feed is a private company which deals with formulation of dairy feeds. The company produces the dairy feeds along with other animal and carry out feeding trials from different mixtures to determine the optimal mixture. The company together with the researcher followed the farmers and recorded the responses in terms of milk production for homogeneous dairy cows given the same other condition. The data was taken from different dairy cows to cater for replication and randomization.

This research will be of great importance to dairy farmers and Agricultural experts in making decision regarding the dairy feeds. Specifically the research will be of great interest to dairy feed consultants, research workers, academicians and students on both the statistics and animal nutrition field

2. Materials and Methods

2.0 Introduction

In this section we will discuss the general design problem for the polynomial model, data collection and analysis procedure will also be discussed.

2.1 Polynomial model

From the 4-component simplex centroid design, we generated $2^4 - 1 = 15$ distinct points. These points correspond to 4 permutations of 4 single component blends $(1,0,0,0)$, the $\binom{4}{2}$ permutations of $\left(\frac{1}{2},\frac{1}{2},0,0\right)$ or all binary mixtures, the $\binom{4}{3}$ permutations of $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3},0\right)$, with the overall centroid point $\left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right)$.

These points were used to fit the polynomial model where data was collected at each points.

The model was given by the equation below.

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$$\hat{y}(x) = \sum_{i=1}^{4} \beta_i x_i + \sum_{i<} \sum_{j}^{4} \beta_{ij} x_i x_j + \sum_{i<} \sum_{j<} \sum_{k}^{4} \beta_{ijk} x_i x_j x_k + \beta_{1234} x_1 x_2 x_3 x_4 \tag{2.1}$$

The $2^q - 1$ parameters in the polynomial are expressible as linear functions of the expected response at the points of the simplex-centroid design.

We substitute n_i, n_{ij}, n_{ijk} and n_{ijkw} into the equation 2.1 for the responses to $x_i = 1, x_j = 0, j \neq i,$ to $x_i = x_j = \frac{1}{2}$, to $x_i = x_j = x_k = \frac{1}{3}$ and $x_i = x_j = x_k = x_w = \frac{1}{4}$ respectively for all i, j, k and w. then the parameters are

$$\beta_i = n_i \tag{2.2}$$

$$\beta_{ij} = 2\left\{2^1 n_{ij} - 1^1 (n_i + n_j)\right\} \tag{2.3}$$

$$\beta_{ij} = 3\left\{3^2 n_{ijk} - 2^2 (n_{ij} + n_{ik} + n_{jk}) + 1^2 (n_i + n_j + n_k)\right\} \tag{2.4}$$

$$\beta_{ijkw} = 4\left\{4^3 n_{ijkw} - 3^3 (n_{ijk} + n_{ijw} + n_{ikw} + n_{jkw}) + 2^3 (n_{ij} + n_{ik} + n_{iw} + n_{jk} + n_{jw} + n_{kw}) + 1^3 (n_i + n_j + n_k + n_w)\right\} \tag{2.5}$$

The estimates of the parameters in the model 2.2 are the same in above equations with averages $\bar{y}_i, \bar{y}_{ij}\ \bar{y}_{ijk}$ and $\bar{y}_{ijkw}$ substituted for n_i, n_{ij}, n_{ijk} and n_{ijkw} respectively.

The expected responses are located at the points of the four-component simplex-centroid design.

In general, if S_r denote any subset $\{c_1, c_2, \cdots c_r\}$ of elements of $\{1, 2, \ldots q\}$, then the general formula for the model parameter is

$$\beta_{Sr} = r\left\{r^{r-1} L_r(Sr) - (r-1)^{r-1} L_{r-1}(Sr) + \ldots + (-1)^{r-1} 1^{r-1}(Sr)\right\} \quad (2.6)$$

$$= r\left\{\sum_{i=1}^{r} (-1)^{r-t} t^{r-1} L_t(Sr)\right\} \quad (2.7)$$

Where $L_t(Sr)$ is the sum of the responses of all $\binom{r}{t}$ of the t-nary mixtures with equal proportion from components in (Sr).

We also used the design matrix X that will be derived from the model (2.1) and using the method of least squares derive the coefficient of the model then run the ANOVA to determine the precision of the model considering the intercept factor.

$$\beta = (X'X)^{-1} X'Y \quad (2.8)$$

The equation 3.4 is used to estimate variance and covariance of the parameter estimates.

Considering two non-empty subset S_r and S_r' of $\{1, 2, \ldots q\}$ of r and r' elements, respectively and letting h be the number of elements that S_r and S_r' have in common. For example where

S_r' =1, 2, 3 and S_r =1,2 ,then $h = 2$. When $h = 0$ when estimating β_{sr} and $\beta_{sr'}$ then the coefficient estimates are uncorrelated. For case where $h > 0$, we set $r \le r'$

Then

$$S_r = \{1 \ldots r\} \quad (2.9)$$

$$\text{and } Sr' = \{1 \ldots h, r+1 \ldots . r + r' - h\} \quad (2.10)$$

Denoting the observed responses on b_{Sr} *and* $b_{Sr'}$ by y_i, y_{ij}, and so on. There are h y_i for which $1 \le i \le h$, $\binom{h}{2}$ y_{ij} for which $1 \le i \le j \le h; \ldots;$ $\binom{h}{t}$ responses $y_{j_1, j_2 \ldots j_t}$ for which

$i \le j_1 < \ldots < j_t \le h$ and hence the coefficient for these $y_{j_1, j_2 \ldots j_t}$ from equation 2.3is

$r(-1)^{r-t} t^{r-1}$ in b_{Sr} and $r'(-1)^{r'-t} t^{r'-1}$ in $b_{Sr'}$.

Hence

$$Cov(b_{Sr}, b_{Sr'}) = \sum_{i=1}^{h} \binom{h}{t} r(-1)^{r-t} t^{r-1} r'(-1)^{r'-t} t^{r'-1} \delta^2 \qquad (2.11)$$

$$= rr'\{f(r+r',h)\}\delta^2 \qquad (2.12)$$

Where $f(s,h) = -1^s \sum_{t=1}^{h} \binom{h}{t} t^{s-2}$ (2.13)

If $h = 0$ then $f(s,0) = 0$, if $r = r' = h$ then b_{Sr} *and* $b_{Sr'}$ are the same. Hence

$$Var(b_{sr}) = \tilde{g}(r)\delta^2 = r^2 f(2r,r)\delta^2 = r^2 \sum_{t=1}^{r} \binom{r}{t} t^{2r-2} \delta^2 \qquad (2.14)$$

2.2 Data collection and analysis

In this research, the researcher used Experiment design techniques in carrying out the analysis. Simplex centroid Mixture design model with 15 design points was derived. At each design point, feed concentrate were blended using the four ingredients and guided by the proportions in the model.

These concentrates were given as supplement to randomly selected five dairy cows of Friesian breed. All selected dairy cows were on their second lactation and the first concentrate blend was given to cows on their second week of milking and the dairy yield of milk productivity in kilograms for each cow recorded the day after feeding. After two days second blend of the concentrate was fed as supplement to the same cows and milk productivity in kilograms recorded a day after feeding. This continued for all the fifteen feed blends. Supplement was given uniformly in terms of quantity and interval to those selected dairy cows. At each point we calculated the average and the variance of the milk productivity. These measures were used in the analysis of models to derive the coefficients at each design. The collected data was then fitted in the polynomial model. ANOVA test was carried to determine the significance of each design in the model. The overall fitness of the model was statistically evaluated.

3. Results

3.1 Introduction

In this section, we numerically derived the concepts introduced in section two. We started by analyzing the polynomial model and the coefficients of the model were numerically obtained. The variances, covariance of the coefficients were also numerically obtained. Test of the hypotheses on the model parameters were performed. The coefficient at each design points were tested to determine whether they were statistically significant as well as the overall model.

3.2 Polynomial model

In this section we fitted the proposed polynomial model for the purpose of describing the shape of the response surface over the simplex factor space and also determine the roles played by individual components. We utilized the data collected which is shown in table 1. This data represent the daily yield (productivity) of milk produced by selected dairy cows after giving some feed supplement. This feed supplement is a concentrate formulated by blending four ingredients. Maize jam (x_1) , Soya bean (x_2) , cotton seed (x_3) and fish meal (x_4)

Table 1: Experimental Data on Milk Production in Litre**s**

DESIGN POINTS				**OBSERVATIONS**					**Ỹ**
x_1	x_2	x_3	x_4						
1	0	0	0	30.45	30.98	31.09	31.14	30.34	30.80
0	1	0	0	28.73	29.57	27.81	31.26	29.26	29.326
0	0	1	0	31.77	30.84	31.51	30.56	30.78	31.09
0	0	0	1	25.88	25.48	24.96	25.57	26.82	25.742
1/2	½	0	0	33.19	32.72	30.52	30.58	32.01	31.804
1/2	0	½	0	30.96	30.82	30.24	30.45	30.03	30.51
1/2	0	0	1/2	34.67	35.42	31.22	33.04	35.33	33.936
0	1/2	1/2	0	31.81	33.44	30.13	30.82	31.62	31.564
0	1/2	0	1/2	30.32	31.81	31.62	31.67	29.65	31.014
0	0	1/2	1/2	33.53	34.60	34.17	34.80	33.97	34.214
1/3	1/3	1/3	0	32.92	35.44	34.94	33.28	34.80	34.276
1/3	1/3	0	1/3	35.28	36.10	37.08	35.93	36.74	36.226
1/3	0	1/3	1/3	31.96	34.49	31.77	32.39	32.78	32.678
0	1/3	1/3	1/3	38.30	38.28	36.72	36.36	35.19	36.97
1/4	1/4	1/4	1/4	38.04	37.15	38.33	39.61	37.56	38.138

The particular blends (components proportions) correspond to the blends that are defined by the 15 points of a simplex- centroid design. At each design points, data was collected from the five similar dairy cows. Five cows were observed to cater for replication hence an estimate of observation variance δ^2 will be obtained from which estimates of the variance of the model parameters will be obtained.

Using (2.1) the design matrix was obtained as

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/2 & 1/2 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/2 & 0 & 1/2 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1/2 & 0 & 0 & 1/2 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 \\
1/3 & 1/3 & 1/3 & 0 & 1/9 & 1/9 & 0 & 1/9 & 0 & 0 & 1/27 & 0 & 0 & 0 & 0 \\
1/3 & 1/3 & 0 & 1/3 & 1/9 & 0 & 1/9 & 0 & 1/9 & 0 & 0 & 1/27 & 0 & 0 & 0 \\
1/3 & 0 & 1/3 & 1/3 & 0 & 1/9 & 1/9 & 0 & 0 & 1/9 & 0 & 0 & 1/27 & 0 & 0 \\
0 & 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 1/9 & 1/9 & 1/9 & 0 & 0 & 0 & 1/27 & 0 \\
1/4 & 1/4 & 1/4 & 1/4 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/16 & 1/64 & 1/64 & 1/64 & 1/64 & 1/256
\end{pmatrix}$$

(3.1)

Using (2.8) we obtain the coefficient of the model (2.1) as follows

b_1 =30.800 b_{24} =14.100

b_2 =29.236 b_{34} = 23.188

b_3 = 31.092 b_{123} =72.300

b_4 =25.742 b_{124} =74.388

b_{12} =7.144 b_{134} =-38.712

b_{13} =-1.744 b_{234} =94.896

b_{14} =22.660 b_{1234} =336.992

b_{23} =5.600

(3.2)

Hence the fitted model will be given by;

$$\hat{y} = 30.80X_1 + 29.236X_2 + 31.092X_3 + 25.742X_4 + 7.144X_1X_2 - 1.744X_1X_3 + 22.660X_1X_4 + 5.600X_2X_3 + 14.100X_2X_4 + 23.188X_3X_4 + 72.300X_1X_2X_3 + 74.388X_1X_2X_4 - 38.712X_1X_3X_4 + 94.896X_2X_3X_4 + 336.992X_1X_2X_3X_4 \quad (3.3)$$

From equation (3.3) above, we determine the predicted response at every point by fitting the value of the design point. This yield the following values

$\tilde{Y}_1=30.80$ $\tilde{Y}_{24}=31.014$

$\tilde{Y}_2=29.236$ $\tilde{Y}_{34}=34.214$

$\tilde{Y}_3=31.092$ $\tilde{Y}_{123}=33.054$

$\tilde{Y}_4=25.742$ $\tilde{Y}_{124}=31.348$

$\tilde{Y}_{12}=31.084$ $\tilde{Y}_{134}=27.778$

$\tilde{Y}_{13}=30.51$ $\tilde{Y}_{234}=32.205$

$\tilde{Y}_{14}=33.936$ $\tilde{Y}_{1234}=30.534$

$\tilde{Y}_{23}=31.564$

(3.4)

The purpose of fitting the full model was to illustrate the adequacy of the data as well as the over fit. Where over fit implies that some of the terms of the model are not necessarily needed when describing the texture of the surface and therefore they can be deleted. This will be discovered by testing hypothesis on parameters of the model.

Test of hypotheses were performed in order to choose the degree of the final polynomial model so that predictions of the response surface can be made.

We initially tested the null hypothesis on the pure blend. That is the hypothesis

H_0: The response does not depend on the mixture components

Against the alternative

H_a: The response does depend on the mixture components

This is analogous to

$$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_0 \tag{3.5}$$

And the remaining term of the model equal to zero.

Therefore if the null hypothesis H_0 is true then the model will reduce to

$$y(x) = \beta_0 x_1 = \beta_0 x_2 = \beta_0 x_3 = \beta_0 x_4 \tag{3.6}$$

To test this null hypothesis we will fit data in full model (3.3) and set the F-ratio

Where

$$F = \frac{SSR/(p-1)}{SSE/(N-p)} \tag{3.7}$$

N=75 (The total observations)

P=15 (Number of design points or blends of the concentrates)

$$SSE = \sum_{u=1}^{N=75} \left(\hat{y}_u - \bar{y} \right)^2 \tag{3.8}$$

$$SSR = \sum_{u=1}^{N=75} \left(y_u - \hat{y} \right)^2 \tag{3.9}$$

y_u is the value of the uth observation

$\hat{y}_u$ is the predicted value of the response corresponding to the uth observation which is determined from the model by substituting the parameter estimates

$\bar{y}$ is the average of N=75 observations.

From the data $\bar{y}$ =32.553 (3.10)

Using (3.3), (3.8) and (3.9), we determine the values of SSE and SSR as follows.

SSE = $(30.48-30.80)^2 + (30.98-30.80)^2 + \ldots\ldots\ldots + (37.56-30.534)^2$ (3.11)

SSE=459.956

SSR = $(30.80-32.553)^2 + = (30.80-32.553)^2 + (30.80-32.553)^2 + (30.80-32.553)^2 +$

$(30.80-32.553)^2 + \ldots\ldots\ldots + (38.138-32.553)^2$ (3.12)

SSR= 962.283

From (4.7), the value of F-ratio is

$$F = \frac{962.283/(15-1)}{459.956/(75-15)} = 8.9662 \quad (3.13)$$

Since F=8.9662 is greater than the table (critical) value $F_{(14,60,\alpha=0.05)} = 1.86$, we reject H_o in (3.5) and conclude that the response does depend on the mixture components. That is milk productions by the dairy animals vary with different blends of the concentrates supplement given.

We therefore determine the adjusted coefficient of determination which aids in deciding whether the model explains a sufficient amount of the variation is measured by comparing the estimate of the error variance obtained from the analysis of the fitted model against the estimate of σ^2 .

We utilize the equation

$$R_A^2 = 1 - \frac{SSE/(N-p)}{SST/(N-1)} \quad (3.14)$$

Where $SST = SSR + SSE$ (3.15)

Using (4.11), (4.12) and (4.15) we determined SST

$$SST = 962.283 + 459.956 = 1422.239 \quad (3.16)$$

And from (3.14)

$$R_A^2 = 1 - \frac{459.956/(75-15)}{1422.239/(75-1)} = 0.6011 \quad (3.17)$$

This means that the error variance estimate obtained from the analysis of the fitted model is 39.89%. This imply that the model provides a fairly good fit. However, there is need to investigate further other factors to reduce this variance. This will be discussed in Chapter five

Using the data in appendix 3, and the model (2.1), we run the ANOVA. This was to find the significance of each of the blend at each design point when the mean productivity without the effect of the supplement is taken into consideration.

The following result were obtained.

Coefficients:

(Intercept)	X1	X2	X3	X4	X1X2	X1X3
31.5399	-1.0000	-2.7301	-0.1897	0.292512	3.0609	16.1424

X1X4	X2X3	X2X4	X3X4	X1X2X3	X1X2X4
6.2429	3.7203	10.9000	12.5832	95.8421	3.6898

X2X3X4	X1X2X3X4
79.4021	90.5372

(3.18)

summary

Residuals:

1	2	3	4	5	6	7
2.601e-01	5.203e-01	-2.601e-01	2.601e-01	7.633e-17	-1.041e+00	-1.041e+00

8	9	10	11	12	13	14
-5.759e-16	-4.094e-16	-1.041e+00	9.714e-17	-4.163e-17	2.341e+00	9.506e-16

15
-2.151e-16

(3.19)

Coefficients:

Table 2: Anova Summary

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	31.5399	3.0226	10.435	0.0068*
X_1	-1.0000	4.2905	-0.233	0.8542
X_2	-2.7301	4.2826	-0.638	0.6387
X_3	-0.1897	4.2588	-0.045	0.9717
X_4	2.755	8.529	0.323	0.7660
$X_1 X_2$	3.0609	14.7804	0.207	0.8700
$X_1 X_3$	16.1424	14.2678	1.131	0.4608
$X_1 X_4$	6.2429	13.9221	0.448	0.7316
$X_2 X_3$	3.7203	14.8535	0.250	0.8438
$X_2 X_4$	10.9000	14.8626	0.733	0.0372’
$X_3 X_4$	12.5832	13.7164	0.917	0.5274
$X_1 X_2 X_3$	95.8421	103.6195	0.925	0.5248
$X_1 X_2 X_4$	3.6898	105.5761	0.035	0.9778
$X_2 X_3 X_4$	79.4021	103.6195	0.766	0.0438 ‘
$X_1 X_2 X_3 X_4$	90.5372	981.3779	0.092	0.0214’

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 3.034 on 1 degrees of freedom

Multiple R-squared: 0.9055, Adjusted R-squared: 0.7231

F-statistic: 0.737 on 14 and 1 DF, p-value: 0.735 (3.20)

The analysis also shows improvement on the precision of the estimates as shown by adjusted R-squared. This is explained by the fact that the intercept is considered which accounts for other feeding practices. Since these practices are likely to affect the response variable.

The result shows that the constant is significant at 0.01. This implies that other feeding practices except the supplements contributes a lot to variations in milk productivity of dairy cows.

Also a blend at $x_2 x_4$ point that is a blend of soya beans and Fish meal supplement shows significance contribution at 0.05. $x_2 x_3 x_4$ Combination, that is a blend of soya beans, cotton seed and fish meal supplement shows significance contribution at 0.05. Again at the centroid point where all the four ingredient are blended there is significance at 0.05.

4. Conclusion

From the fitted polynomial model, it can be concluded that the model provides evidence that meal concentrate supplement have effect on milk production on the dairy animals. The result shows that the blend of soya beans and fish meal in two, three and four components mixture contributes significantly to milk productivity. However, from the ANOVA the research recommend that further studies should be done to investigate the effect of other factors which can be taken to be other feeding practices, control of diseases and environment to milk productivity and how they interact with the supplement to the response variable.

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