Risk Optimization of the CNSS' Portfolio Using a Return-Constrained Markowitz Model

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Abstract

In Morocco, the private pension fund is managed mainly by the National Social Security Fund (CNSS). CNSS is one of the most active institutional investors in the local market of Collective Investment in Transferable Securities Directives (UCITS) as it has a very important investment portfolio of cash surplus, in shares or units of UCITS. This work is a first step in the study of how modern techniques of portfolio management can be adopted to allow the establishment of a process that helps to monitor and optimize the investment decisions. In fact the results show a decrease in the variance and an improvement in the rate of return.

Keywords: Risk Optimization; Asset Allocation; Modern Portfolio Theory; Markowitz Model.

1. Introduction

Social security primarily refers to a social insurance program providing social protection, or protection against socially recognized conditions, including poverty, aging, disability, unemployment and others. Social security may also refer to social insurance, where people receive benefits or services in recognition of contributions to an insurance scheme. These services typically include provision for retirement pensions, disability insurance, survivor benefits and unemployment insurance.

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In Morocco, the National Social Security Fund (CNSS) is one of the most active institutional investors in the local market of Collective Investment in Transferable Securities Directives (UCITS/ French: OPCVM). The private pension system in Morocco is a market that is yet to be explored. However, there are numerous private life insurance products offered by private insurance companies. These private schemes do not go by the name of pension schemes but are operated as life insurance policies and provident funds.

The pension fund is managed by the National Social Security Fund (CNSS), whose primary role is to promote the economic security of the Moroccan workforce. It is the entity that takes decisions on the way the contributions of the workforce should be invested. The objective of this research is to investigate the method used by CNSS in managing its funds and make recommendations to better-off the results of its investments.

The products in which CNSS investments are permitted are the ones that are not very volatile. The allowed investment products are bonds, treasury bills (TB), debt Securities (DS), certificates of deposit (CDs) issued by banks and commercial paper. Despite this diversity of choice between these products, CNSS generally invests in mutual funds, money market or bond portfolios managed by professionals. It has a very important investment of cash surplus, in shares or units of UCITS, which prompted the idea on how modern techniques of portfolio management can be adopted to allow the establishment of a process that helps to monitor and optimize the portfolio.

2. Presentation of the CNSS' portfolio

CNSS' primary goal is to ensure an adequate and sustainable pension for the affiliated workforce. This explains the logical choice of investing in several mutual funds to synchronize financial flows and thus to face the challenges of social protection for the short and the long term.

Fund management of the CNSS is entrusted to several management companies. The benefits of this diversification have been defended by several theorists. In particular, Sharpe considered two determinant factors in investing in multiple management companies: specialization and diversification [8,9]. Thus, it is reasonable to seek the services of management companies that are able to make timely market decisions; and as a protection from eventual loss, to use several management companies in a good diversification plan.

CNSS invests in several funds to which it delegates the management of its cash surpluses. Thus, it behaves as a fund of funds. The portfolio of dedicated funds is composed of eight funds managed by different management companies. Table 1 presents the eight funds with their respective management companies.

Barry and Starks showed that the principal-agent relationship, namely considerations regarding the sharing of risks, may influence the decision to use multiple management companies [1]. They found that risk sharing is sufficient as a ground for making a decision to mandate multiple agents even in the absence of specialized managers. Moreover, the double diversification over time and through several management companies ensures proper management of resources while providing circulation of cash flows that are used to cover future liabilities.

Table 1: The composition of the CNSS' portfolio

Fund	Depositary	Management Company
AL IJTIMAII SECURITE	BMCE	BMCE CAPITAL
AL IJTIMAII INMAA	AWB	ATTIJARI Management
AL IJTIMAII AVENIR	SGMB	GESTAR
AL IJTIMAII ADDAMANE	CDG	Upline
AL IJTIMAII SOLIDARITE	CDG	CFG
AL IJTIMAII PATRIMOINE	CDG	VALORIS Management
AL IJTIMAII PROTECTION	CDG	CDG CAPITAL Gestion
CDG SECUR	CDG	CDG CAPITAL Gestion

3. Current management approach

The current management approach of the CNSS relies on measuring and monitoring performance as a criterion to arbitrate between the different funds. The inputs of the process are the net asset values and the benchmark chosen for each fund. The net asset values are reported every Friday and form the raw material of the decision-making process. They are used for calculating performance and risk indicators. The choice of the benchmark results from the composition of the funds. An investment committee formed by both CNSS and management companies agreed upon the benchmark to adopt. Thus, according to the committee, the appropriate and representative benchmark for is the Moroccan Bond Index (MBI).

The outputs of the process are performance indicators and different ratios. The objective is to find the composition that maximizes or minimizes a given ratio to optimize the portfolios. For instance, the aim will be to minimize the tracking error and maximize the information ratio. That been said, the next section is reserved to the implementation of this management process.

4. Methodology

The methodology is based on Markowitz's model [4], which corresponds to the following quadratic optimization problem:

$$\underset{w}{Min} \quad (w-b)^T \, \Omega (w-b) \tag{1}$$

Such that $(w-b)^T \, R = E(R_B)$ and $w \in D$ (2)

Where $D = \left\{ w = (w_1,...., w_N)^T \in R^N : \sum_{i=1}^{N} w_i = 1 \quad \text{and} \quad w \geq -b \right\}$ (3)

In an N asset portfolio, the asset allocation is characterized by the vector of weights w, with *b* being a vector of weights to limit short selling. The matrix Ώ is the variance-covariance matrix of the rates of returns of the assets. The right-hand side of equation (2) is the expected return of the Benchmark.

Since CNSS can earn a fixed annual rate of 4.56% from la "Caisse de Depot et Gestion (CDG)", our optimization model is:

$$\underset{w}{Min} \quad (w)^T \Omega (w)$$ (4)

Such that $(w)^T R > \dfrac{0.0456}{52}$ and $w \in D$ (5)

Where $D = \left\{ w = (w_1,...., w_N)^T \in R^N : \sum_{i=1}^{N} w_i = 1 \quad \text{and} \quad w \geq 0 \right\}$ (6)

b = 0 because the funds cannot be shorted.

In the calculations of the expected returns on the funds, R, we use the 9-point exponential moving average (EMA) as it gives the smallest MAPE, compared to other EMAs, using an out-of-sample of 52 historical data points.

The computational part of the problem is as follows: In each experience, we pick k successive historical rates of return of the assets from week j to week j+k-1. We use all these historical rates of return to calculate the (8x8) covariance matrix (we have 8 funds). We solve the optimization problem and take the solution w as the vector weights of the funds for week j+k then compare the rate of return of the portfolio for week j+k calculated with these weights and the actual rates of return of week j+k against the actual rate of return of the portfolio that was observed in week j+k. We try different values of k to see whether it is better to go far in history or not. k is taken equal to 52, 65, 78, 91, 104, 117, 130, and 143 since we have only 156 points (weeks) of historical data and as k increases the number of experiments decreases. For example in the case of k=143 we only have 156-143=13 experiments which really cannot confirm which method is better.

It is important to mention that even if excluding short selling is one of the constraints to the minimization equation used, we still got some very small, almost insignificant, negative weights. To remedy to this situation, the negligible negative weights were assumed to be equal to 0 and the positive weights were readjusted accordingly.

5. Results

To explain the results we define:

- Variance of the actual portfolio (VAP): The variance of the portfolio using the actual weights.
- Rate of return of the actual portfolio (RAP): The rate of return of the portfolio using the actual weights.
- Variance of the optimized portfolio (VOP): The variance of the portfolio using the weights we obtain from our optimization method.
- Rate of return of the optimized portfolio (ROP): The rate of return of the portfolio using the weights we obtain from our optimization method.
- Simple rate of return in week j (SRj): The weekly rate of return if we long the portfolio for one week (buy on the Friday of week j-1 then sell it on the Friday of week j).
- Compounded rate of return in week j (CRkj): The rate of return if we buy the portfolio on the Friday of week k and hold it for j weeks (buy on the Friday of week k then sell it on the Friday of week k+j).

The method was able to reduce the variance of the portfolio by an overall average of 26.48%. Figure 1 shows the percentage change in the variance of the portfolio (PCV) when using k = 52.

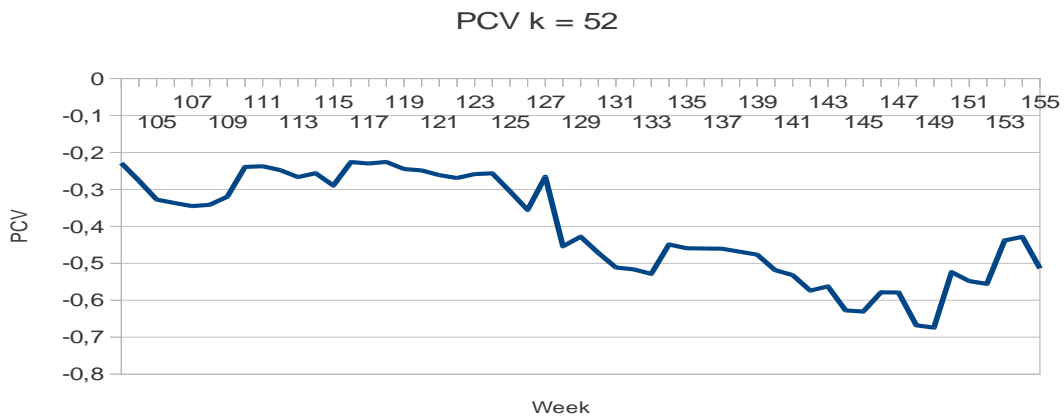$$PCV = \frac{VOP}{VAP} - 1 \qquad\qquad\qquad\qquad (7)$$

Figure 1: The percentage change in the variance when k = 52

For the different values of k, we obtain curves of PCV that are almost similar to the k = 65 curve in figure 1. This says that the optimization in fact reduced the variance of the portfolio and thus reduced the risk associated with the volatilities of the rates of return of the eight funds. On the other hand, we should make sure that this reduction in the variance does not bring a large drop in the rate of return of the portfolio. For this, we compare how the CRkj s and the SRj s of the RQP and ROP are. Tables 2 and 3 compare these rates.

Table 2: Comparison between the simple RAP and ROP

k	Number of observations	SRj of ROP > SRj of RAP	Frequency
52	104	51	49,0%
65	91	47	51,6%
78	78	42	53,8%
91	65	31	47,7%
104	52	25	48,1%
117	39	19	48,7%
130	26	12	46,2%
143	13	7	53,8%

Table 3: Comparison between the compounded RAP and ROP

k	Number of observations	CRkj of ROP > CRkj of RAP	Frequency
52	104	9	8,7%
65	91	78	85,7%
78	78	76	97,4%
91	65	42	64,6%
104	52	28	53,8%
117	39	6	15,4%
130	26	6	23,1%
143	13	12	92,3%

Table 2 (simple rates of return) indicates that our method does not really beat the actual portfolio. However, it is the compounded rate of return (Table 3) that really shows that if an investor decides to keep the portfolio for a long period, which is the case for the CNSS, it can largely beat (with a frequency exceeding 85%) the actual one for k = 65, 78, and 143. So our portfolio is overall doing much better in terms of return.

5. Conclusion and limitations

The simple optimization method that we use is able to not only reduce the variance of the portfolio but also increase its rate of return for the long run. This was achieved given that we base our forecasts on a basic tool which is the exponential moving average. Since this is just a first step in the portfolio's optimization, some of the main limitations of this work can be summarized as follows: (1) the exponential moving average model using for forecasting is among the most rudimentary options available, and it is certainly not sophisticated enough to allow for accurate prediction of trend, seasonality, and financial time series specific phenomena such

volatility clustering and changes; (2) Our study excluded fundamental analysis of the funds, as well as experts' opinions about the forecasts; and (3) the optimization problem at the heart of our Mean-Variance portfolio optimization model is elementary and does not include side constraints usually considered in practice by portfolio managers.

These limitations open the door to many future extensions concerning both the forecasting of the rates of return and the performance indicators and, depending on the CNSS' investment strategies, the formulation of the optimization model.

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