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Finite Difference Analysis on Magnetohydrodynamic Fluid Flow Past an Infinite Continuous Moving Surface Embedded in Porous Medium.

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Abstract

An analysis has been carried out on the effects of variable viscosity on the problem of Magnetohydrodynamic fluid flow past an infinite continuous moving sheet embedded in porous media. The fluid viscosity is assumed to vary as an inverse linear function of temperature. The x – axis runs along the permeable continuous moving surface in the direction of the motion and the y – axis is perpendicular to it. The magnetic field is applied along the y axis. The governing equations have been reduced to the coupled non linear partial differential equations. The system of transformed non linear partial differential equations is solved numerically using the finite difference method. The effects of the various parameters on the flow are analyzed and discussed through graphs.

Keywords: Variable viscosity; MHD; porous surface; infinite plate.

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1. Introduction

Investigations of a steady, two dimensional laminar boundary flow of an electrically conducting fluid past a continuous moving surface embedded in porous media are important in many manufacturing processes, such as materials manufactured by polymer extrusion, artificial fibers, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning, cooling of metallic sheets. In addition, they also find very useful applications in the design of insulation systems employing porous media.

Most of the existing analytical studies are based on the constant physical properties of the fluid, [12]. However, it is known that these properties may change with temperature[4]. To accurately predict the flow it is necessary to take into account this variation of viscosity. [3] concluded that when the variable viscosity is included, the flow characteristics change substantially compared to the constant viscosity. [1] studied the effects of variable viscosity and nonlinear radiation on MHD flow with heat transfer over a surface stretching with a power-law velocity. [9] studied the influence of variable viscosity on laminar boundary layer flow and heat transfer due to a continuously moving plate.

Work regarding the boundary-layer behavior in moving surfaces in quiescent fluid was considered by [2]. [14] studied effects of variable viscosity on hydro magnetic boundary layer along a continuously moving vertical plate in the presence of radiation and chemical reaction. [5] studied the heat transfer in the steady two dimensional stagnation-point flow of an incompressible fluid over a stretching sheet considering the case of constant surface temperature taking into consideration the viscous dissipation of the fluid.

There is extensive literature on flow through porous media that is governed by generalized Darcy laws.[6]studied the flow of a non-Newtonian fluid over a wall with suction orblowing and [5] investigated the steady flow of a power-law fluid past an infinite porous flat plate subject to suction or blowing with heat transfer.[13]investigated boundary layer flow of Newtonian fluid through a highly porous medium. Later [11] used these equations to study the influence of free convective flow and mass transfer on flow through porous medium.[10] investigated oscillatory flow of a Newtonian fluid through a porous medium. [7] investigated MHD stokes fluid flow past a porous contracting surface with heat transfer. [8] studied MHD flow past an infinite continuous moving surface. In the present study we consider the laminar flow along permeable continuous infinite horizontal sheet taking into account the temperature dependent viscosity.

1.1 Formulation of the problem

We consider a steady two-dimensional flow of incompressible, viscous and electrically

Conducting fluid past a continuously moving plate of permeability k_p .The x axis runs along the continuous surface in the direction of the motion and y-axis is perpendicular to it. According to the assumption, the two-dimensional boundary layer equations for the flow of the fluid past a permeable continuous moving surface are as flows:

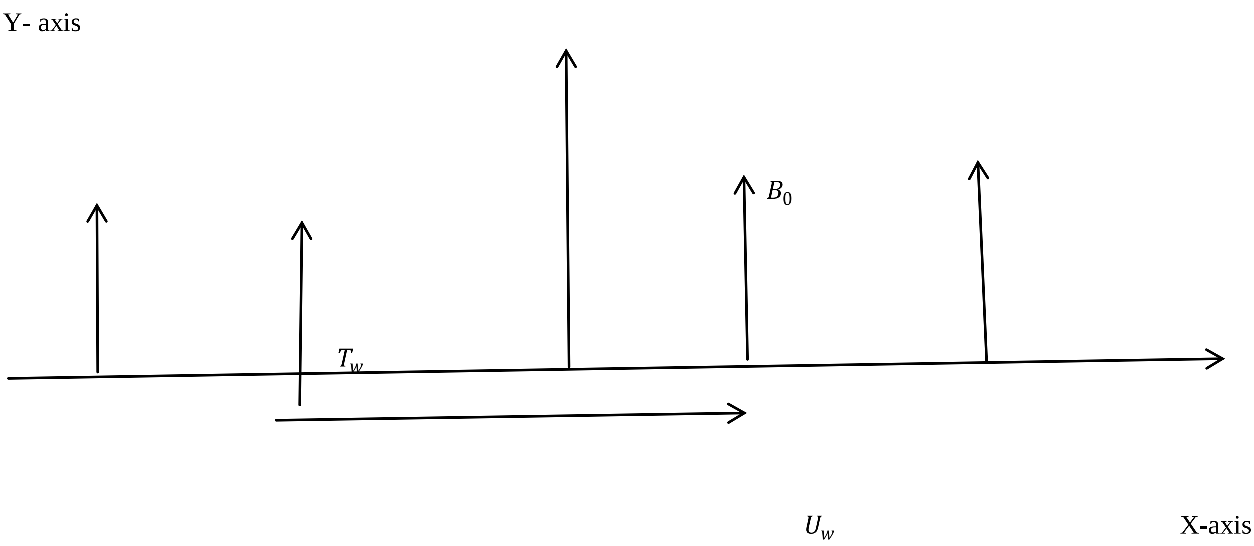


Figure 1: Flow configuration

$$\frac{du}{dx} + \frac{dv}{dy} = 0 \qquad (1)$$

$$v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma u {B_0}^2}{\rho} - u\frac{v}{k_p} \qquad (2)$$

$$\rho c_p \frac{DT}{Dt} = \frac{k\partial^2 T}{\partial y^2} + \left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

the fluid viscosity which is assumed to be inverse linear function of temperature, [7]

$$\mu = \frac{1}{\alpha(\mathrm{T}-\mathrm{T_r})} \qquad (4)$$

Both α and T_r, are constant and their values depend on the reference state and the thermal property of the fluid, i.e. a. In general $\alpha > 0$ for liquids and $\alpha < 0$ for gases.

$$\mathrm{v}\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\frac{1}{\alpha(T-T_r)}\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2 u}{\rho} \qquad (5)$$

Equation (3) and (5) reduces to

$$\frac{\partial u}{\partial y} + \mathrm{u}\frac{\partial \mathrm{u}}{\partial \mathrm{x}} = \frac{1}{\rho}\left(\frac{1}{\alpha(T-T_r)}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\alpha(T-T_r)^2}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}\right) - \frac{\alpha B_0^2 u}{\rho} - u\frac{v}{k_p} \qquad (6)$$

$$\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{1}{\alpha(T-T_r)\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (7)$$

Subject to the initial and boundary conditions of the problem are;

$y = 0,\ u = U_\infty, v = 0, T = T_w$

$$y \to \infty\ ,\ u = 0\ ,\ T = T_\infty \tag{8}$$

Where the quantities u and v are the velocity components in the x and y directions respectively, B_o is a constant magnetic field and all the other quantities have their usual meanings.

Introducing the following non-dimensional quantities;

$$\mathrm{x} = \mathrm{x}^*\mathrm{H},\quad \mathrm{y} = \mathrm{y}^*\mathrm{H},\quad \mathrm{u} = \mathrm{u}^*\mathrm{U}_\infty,\quad \mathrm{v} = \mathrm{v}^*\mathrm{U}_\infty T^* = \frac{T - T_\infty}{T_w - T_\infty} \tag{9}$$

where H,$\mathbf{U}_\infty$ and T are the characteristics length, velocity and temperature

Equation (6) and Equations (7) reduce to the following non-dimensional form:

$$\mathrm{v}^*\frac{\partial u^*}{\partial y^*} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\theta_r}\frac{1}{Re}\frac{\partial T^*\partial u^*}{\partial y^*\partial y^*} - Mu^* - Xiu^* \tag{10}$$

$$\mathrm{v}^*\frac{\partial \mathrm{T}^*}{\partial \mathrm{y}^*} = \frac{1}{\mathrm{Pe}}\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right) + \frac{\mathrm{Ec}}{\mathrm{Re}}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{11}$$

Where $\mathrm{R}e = \frac{\rho\, U_\infty H}{\mu} = \frac{1}{\rho H\, U_\infty\, \alpha\, (T - T_r)}, \theta_r = \frac{T - T_r}{T_w - T_\infty} = \frac{T - T_r}{\Delta T}$ and, θ_r is the viscosity parameter. Where Peclet number $Pe = RePr = \frac{\rho U_\infty H C_p}{k} = \frac{U_\infty H}{\alpha}$,

$Prandtl\ number\ \ Pr = \frac{C_p\mu}{k}, Eckert\ number, Ec = \frac{(U_\infty)^2}{C_p\Delta T}, Xi = \frac{vH}{U_\infty k_p}$ is the permeability parameter.

The corresponding initial and boundary conditions are;

y*=0, u*=1, v*=0, and T*=1

$$\text{y*} \to \infty,\ \text{u*=0, v*=0 and } \mathrm{T}^* = 0 \tag{12}$$

1.2 Method of Solution

Equating the equation (10) and (11) to a pseudo time derivative $\frac{\partial u}{\partial t}$ and $\frac{\partial T}{\partial t}$,

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{1}{\theta_r}\frac{1}{Re}\frac{\partial T^*\partial u^*}{\partial y^*\partial y^*} - \mathrm{v}^*\frac{\partial u^*}{\partial y^*} - Mu^* - Xiu^* \tag{13}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\mathrm{Pe}}\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right) + \frac{\mathrm{Ec}}{\mathrm{Re}}\left(\frac{\partial u^*}{\partial y^*}\right)^2 - \mathrm{v}^*\frac{\partial \mathrm{T}^*}{\partial \mathrm{y}^*} \tag{14}$$

The numerical solutions of the problem are obtained by solving the non-linear differential equations (13) and (14) subject to (12) using finite difference method together with cranks Nicolson.

Equations (13) and (14) in finite difference form and making $U_{i,j}^{k+1}$ and $T_{i,j}^{k+1}$ subject

$$U_{i,j}^{k+1} = U_{i,j}^{k} + \frac{\Delta t}{Re}\left(\frac{U_{i,j+1}^{k+1} + U_{i,j-1}^{k+1} + U_{i,j+1}^{k} - 2U_{i,j}^{k} + U_{i,j-1}^{k}}{2(\Delta y)^2}\right) - \frac{\Delta t}{Re\theta_r}\frac{T_p}{2\Delta y}\left(\frac{U_{i,j+1}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y}\right)$$

$$- \Delta t V_{i,j}^{k}\left(\frac{U_{i,j+1}^{k+1} + U_{i,j+1}^{k} - U_{i,j}^{k}}{2\Delta y}\right) - MU_{i,j}^{k} - XiU_{i,j}^{k}$$

$$/1+\frac{2\Delta t}{2Re(\Delta y)^2} - \frac{\Delta t}{\theta_r}\frac{1}{Re}\frac{T_p}{4(\Delta y)^2} - \frac{\Delta t}{2\Delta y}V_{i,j}^{k} \tag{15}$$

Where $T_p = T_{i,j+1}^{k+1} - T_{i,j}^{k+1} + T_{i,j+1}^{k} - T_{i,j}^{k}$

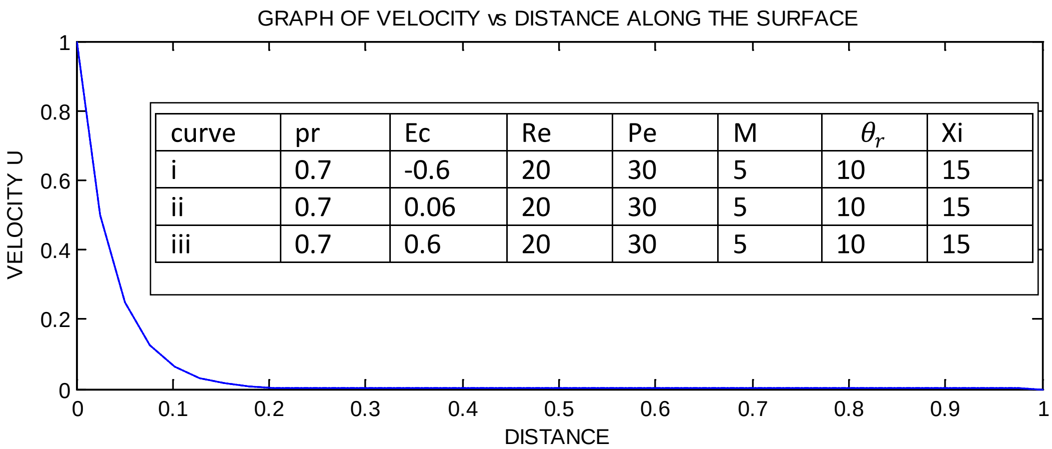
$$T_{i,j}^{k+1} = T_{i,j}^{k} + \frac{\Delta t}{Pe}\left(\frac{T_{i,j+1}^{k+1} + T_{i,j-1}^{k+1} + T_{i,j+1}^{k} - 2T_{i,j}^{k} + T_{i,j-1}^{k}}{2(\Delta y)^2}\right)$$

$$+ \Delta t\frac{Ec}{Re}\left(\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k+1} + U_{i,j+1}^{k} + U_{i,j}^{k}}{2\Delta y}\right)^2 - \Delta t V_{i,j}^{k}\left(\frac{T_{i,j+1}^{k+1} + T_{i,j+1}^{k} - T_{i,j}^{k}}{2\Delta y}\right)$$

$$/1 + \frac{\Delta t}{Pe}\frac{1}{(\Delta y)^2} - \frac{\Delta t}{2\Delta y} \tag{16}$$

2. Results and Discussion

To ensure stability and convergence of the finite difference scheme, the program is run using smaller values of Δt and Δy.It is observed that there were no significant changes in the results, which ensure that the finite difference method used in the problem converge and is stable.



curve	pr	Ec	Re	Pe	M	θ_r	Xi
i	0.7	-0.6	20	30	5	10	15
ii	0.7	0.06	20	30	5	10	15
iii	0.7	0.6	20	30	5	10	15

Figure 1.1: Velocity profiles for different values of Eckert number Ec.

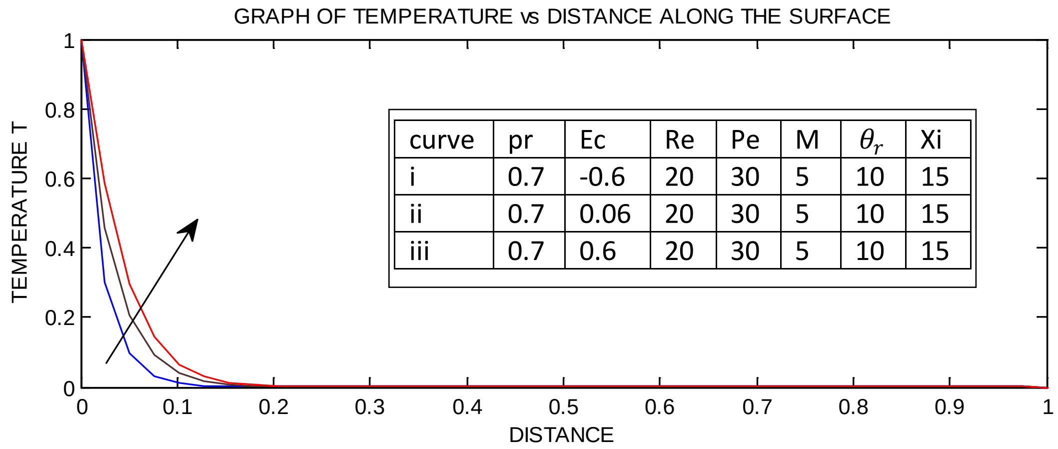


Figure 1.2: Temperature profiles for different values of Eckert number Ec

Figure 1.1 shows no change in velocity profile as Eckert number increases. Figure 1.2 shows that decrease in Eckert number Ec causes a decrease in temperature profiles. The Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against viscous fluid stress. A positive Eckert number implies cooling the sheet, implying heating the fluid. This causes a rise in temperature.

From figures 1.3 we noted that increase in Reynolds' number Re causes an increase in the velocity profiles. From figure 1.4 we noted that increase in Reynolds's number causes decrease in temperature profiles, but there is a cross over where temperature profile increases. The Reynolds's number represents the ratio of the inertial to viscosity forces. Increase in Re results to a large inertial force that in turn translates to a higher velocity.

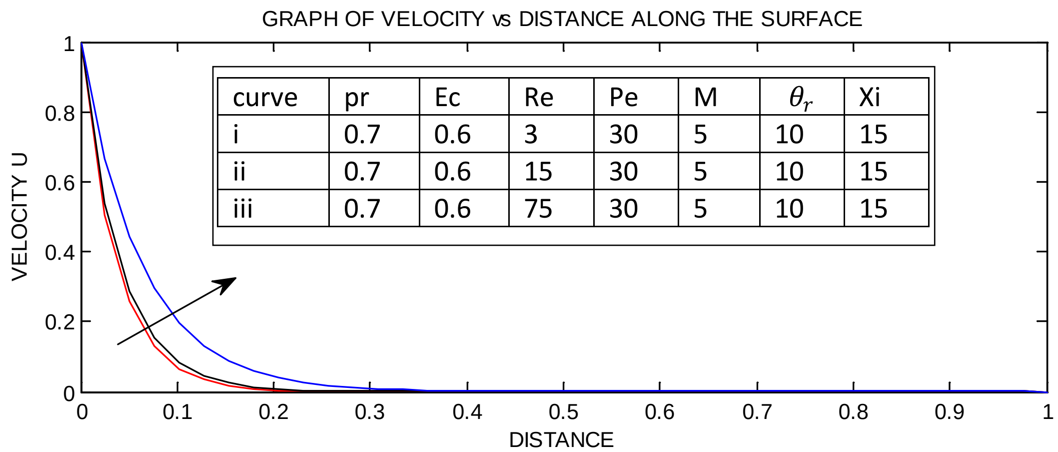


Figure 1.3: Velocity profiles for different values of Reynolds number Re

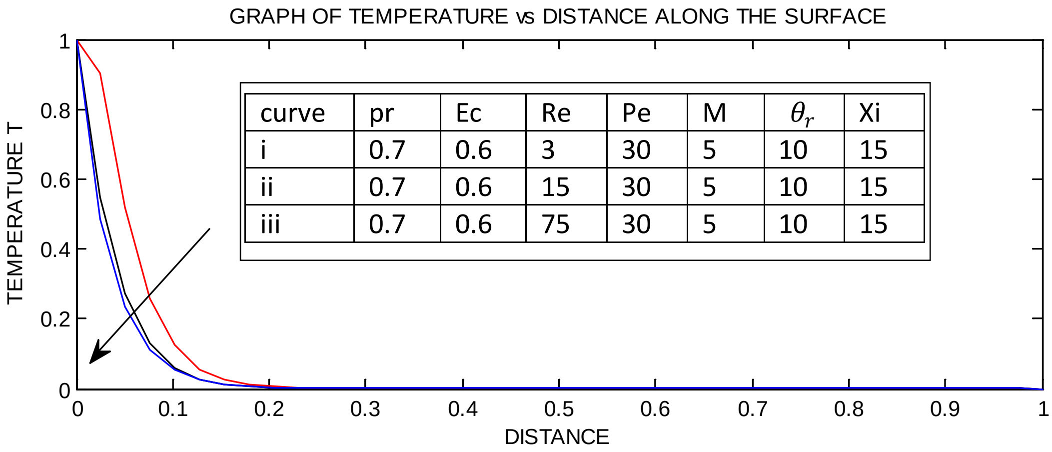


Figure 1.4: Temperature profiles for different values of Reynolds number Re

Figure 1.5 shows that increase in peclet number there is no increase in velocity profile. Figure 1.6 shows that increase in Peclet number increases temperature profiles. Peclet number is equivalent to the product of the Reynolds number and the Prandtl number. Thus, increasing thermal boundary layer.

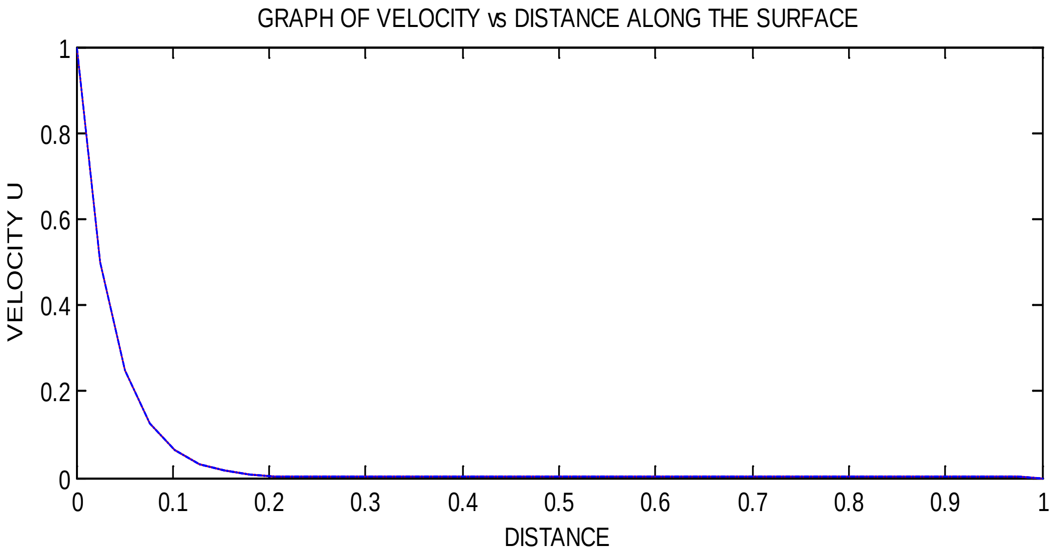


Figure 1.5: Velocity profiles for different values of Peclet number Pe

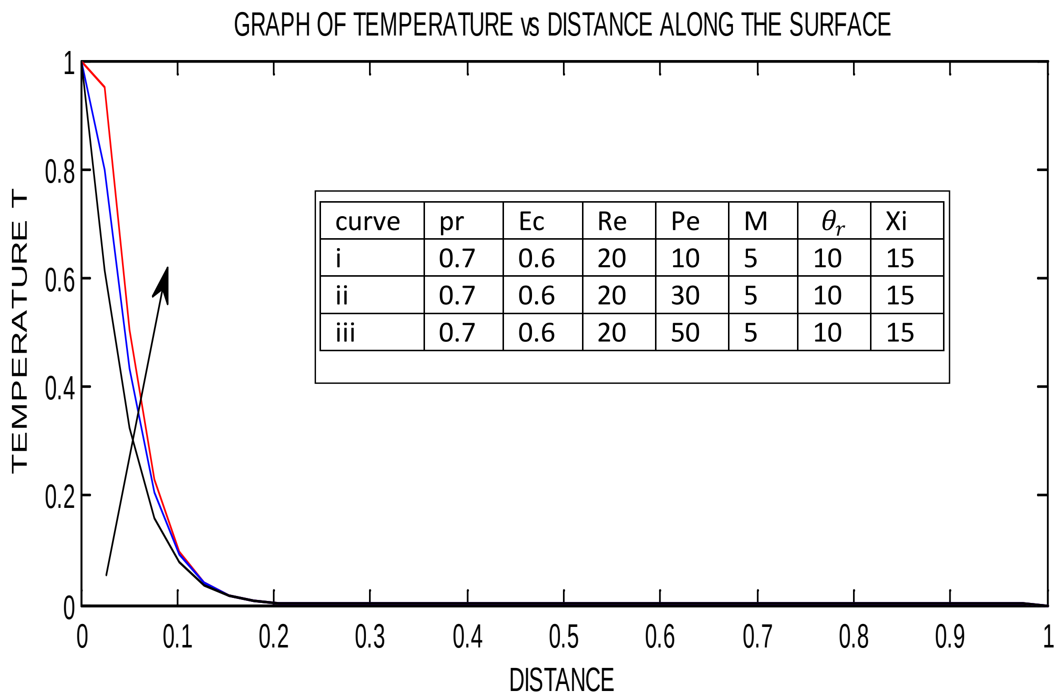


Figure 1.6: Temperature profiles for different values of peclet number Pe

From figure 1.7 and 1.8 we noted that increase in magnetic parameter M causes a decrease in the magnitude of both velocity and temperature profiles respectively. Application of a transverse magnetic field to an electrically conducting fluid give rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer, [8]. The reduced velocity by the frictional drag due to the Lorentz force is responsible for reducing thermal viscous dissipation in the fluid leading to a thinner thermal boundary layer. Magnetic field can therefore be employed to control the velocity and temperature boundary characteristics of a fluid.

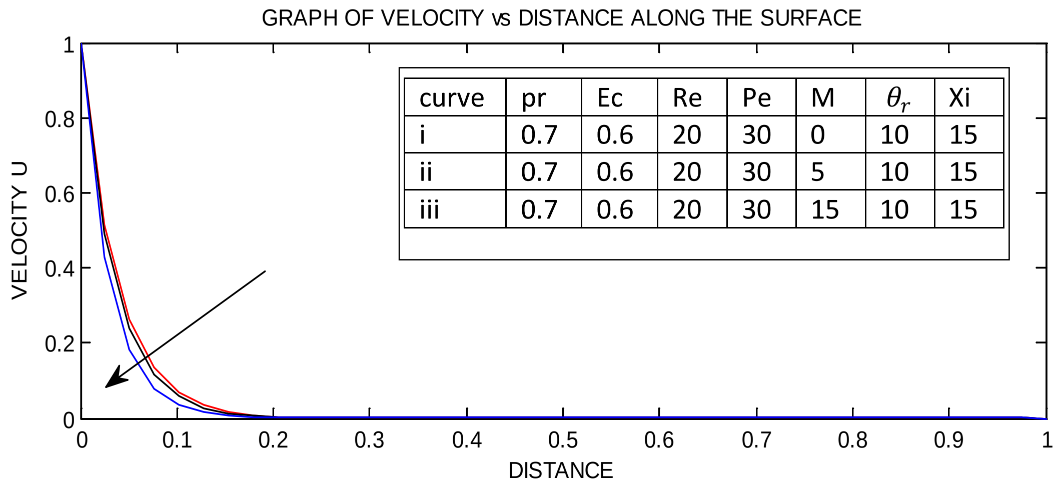


Figure 1.7: Velocity profiles for different values of magnetic parameter M

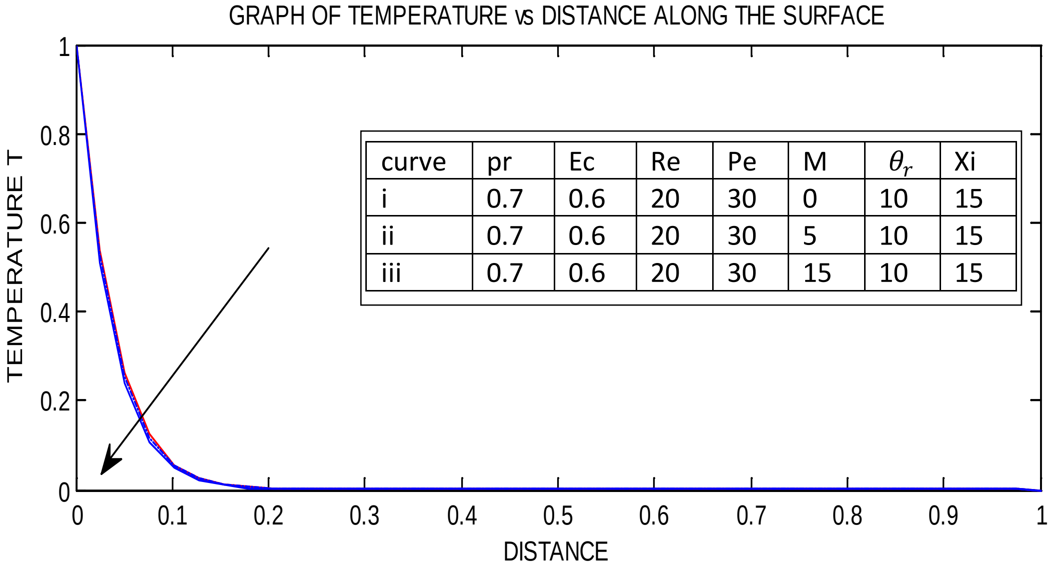


Figure 1.8: Temperature profiles for different values of magnetic parameter M

In figure 1.8 and figure 1.9 increase in permeability parameter lead to decrease in both velocity and temperature profiles. The permeability parameter Xi is inversely proportional to the actual permeability k_p of the porous medium. Increase in Xi leads to deceleration of the flow hence the velocity decreases.

Increasing Xi increases the resistance of the porous medium (as the permeability physically becomes less with increasing Xi). This decelerates the flow and reduces the magnitudes of both the velocity and temperature profiles respectively. Thus velocity and temperature profiles can be controlled by varying the permeability of the porous medium.

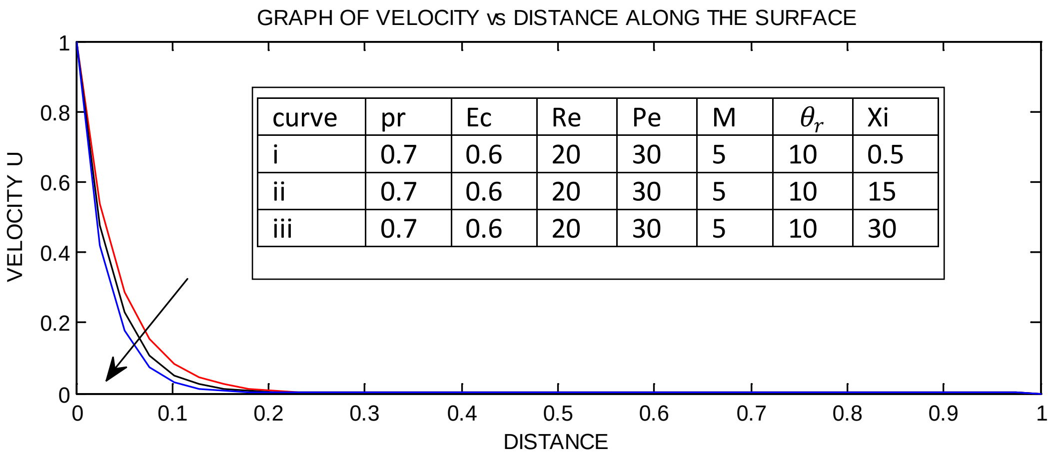
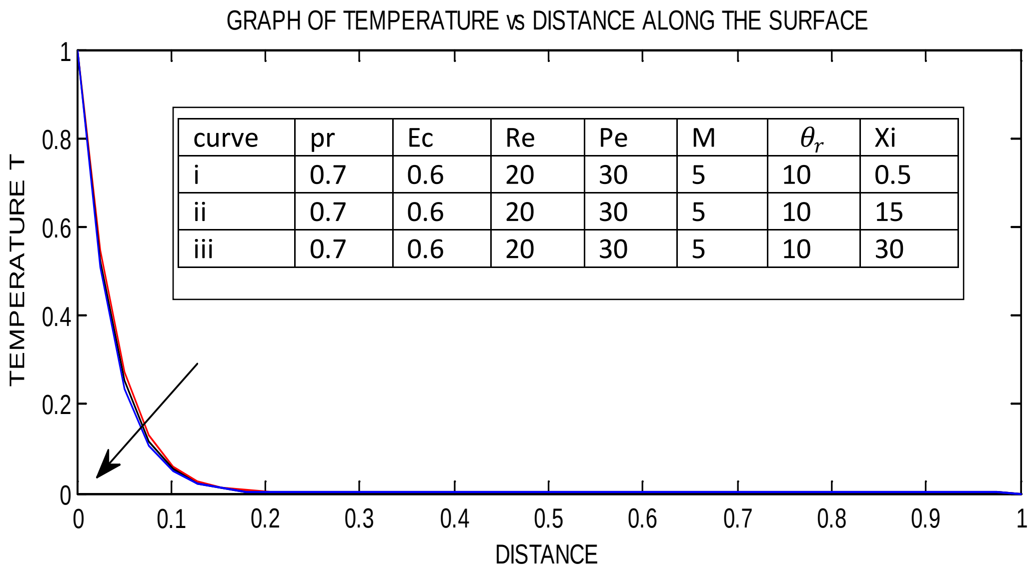


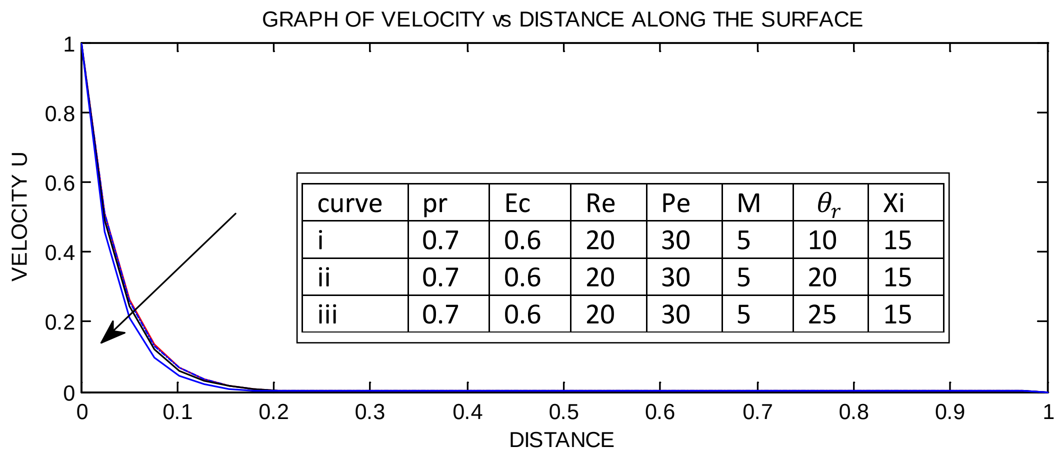
Figure 1.8: velocity profiles for different values of permeability parameter Xi



curve	pr	Ec	Re	Pe	M	θ_r	Xi
i	0.7	0.6	20	30	5	10	0.5
ii	0.7	0.6	20	30	5	10	15
iii	0.7	0.6	20	30	5	10	30

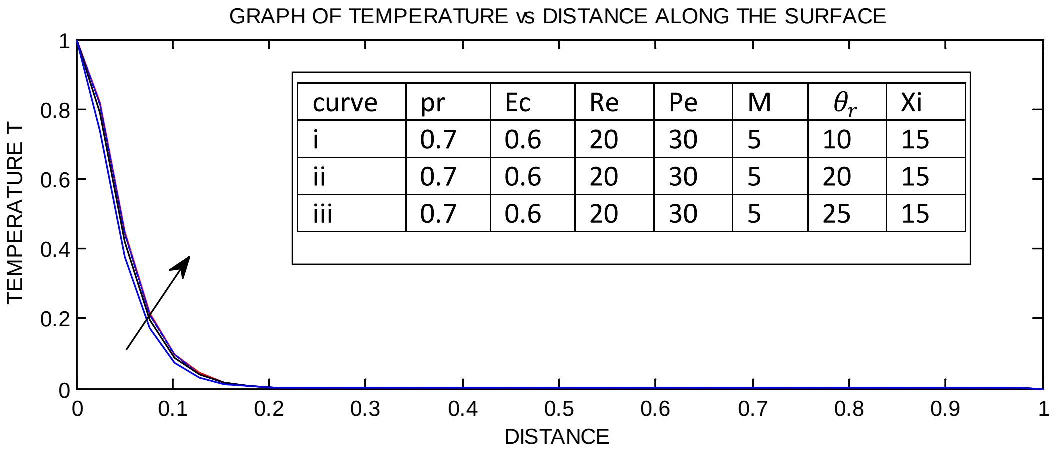
Figure 1.9: Temperature profiles for different values of permeability parameter Xi.

From figure 2.0 and 2.1 we noted that increase in viscosity parameter θ_r, causes a decrease in the magnitude of velocity profiles and an increase in temperature profiles.



curve	pr	Ec	Re	Pe	M	θ_r	Xi
i	0.7	0.6	20	30	5	10	15
ii	0.7	0.6	20	30	5	20	15
iii	0.7	0.6	20	30	5	25	15

Figure 2.0: velocity profiles for different values of viscosity parameter θ_r



curve	pr	Ec	Re	Pe	M	θ_r	Xi
i	0.7	0.6	20	30	5	10	15
ii	0.7	0.6	20	30	5	20	15
iii	0.7	0.6	20	30	5	25	15

Figure 2.1: Temperature profiles for different values of viscosity parameter θ_r

2.1 Nusselt Number And Shear Stress

The quantities of main engineering interest in the problem at hand are the local Nusselt number and the shearing stress on the continuous moving sheet. The Nusselt number physically indicates the rate of heat transfer. The shearing stress on the surface of the continuous surface is defined as.

$$\tau_y = \mu \frac{\partial u}{\partial y} \tag{17}$$

The local skin friction coefficients are defined as

$$\tau^*_y = \frac{1}{Re} \frac{\partial u}{\partial y} \tag{18}$$

Equation represents the respective local skin coefficient due to velocity profiles.

The local nusselt number Nu is expressed as

$$\text{Nu} = \frac{-1}{T_w - T_\infty} \frac{\partial T}{\partial y} \tag{19}$$

2.2 Variation Of Nusselt Number With Various Parameters

From table 1 and table 2 we noted the following:

- Increase in Eckert number leads to decrease in Nusselt number and an increase in shearing stress.
- Increase in Magnetic parameter number leads to increase in Nusselt number and shearing stress.

Table 1: Variation of Nusselt number with Ec, Re, Pe and M

NU	Ec	Re	Pe	M
5.608011	2	20	30	5
5.613877	2.000000e-002	20	30	5
3.866185	6.000000e-001	50	30	5
5.607203	6.000000e-001	20	70	5
6.181751	6.000000e-001	20	30	15
7.995947	6.000000e-001	20	30	30

Table 2: Variation of shearing stress with Ec, Pe and M.

Tau_x	**Ec**	Re	Pe	M
1.648487	1	20	50	5
1.649386	10	20	50	5
2.375860	1	20	50	10

3. Conclusion

The analysis of various parameters on steady laminar boundary layer flow of an incompressible, electrically conducting, and viscous Newtonian fluid past continuous electrically non conducting porous sheet has been carried out. The numerical solutions of velocity and temperature fields are obtained by finite difference method. [8] Analyzed MHD flow past an infinite continuous moving surface. The problem at hand has a porous plate. In order to validate the present results, the boundary condition for the sheet is changed so as there is no effects of porous sheet. In the absence of porous plate the results are compared with that of [8]; and agree. The main conclusions are as follow:

- Velocity profiles and temperature profiles decreases with increase in magnetic parameter M.
- Increase in Reynolds number Re leads to increase in velocity profiles and decrease in temperature profiles but there is a cross over.
- Increase in Eckert number Ec leads to no change in velocity profiles but to an increase in temperature profiles.
- Increase in peclet number Pe leads to increase in temperature profiles and no effects on velocity profiles.
- Increase in viscosity parameter θ_r, causes a decrease in the magnitude of velocity profiles and an increase in temperature profiles.

- Increase in Eckert number leads to decrease in Nusselt number and an increase in shearing stress.
- Increase in Magnetic parameter number leads to increase in Nusselt number and shearing stress.

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