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Global Chaos Synchronization of Two different Chaotic Systems Using Nonlinear Control

Israr Ahmad[a*], Azizan Bin Saaban[b], Adyda Binti Ibrahim[c], Mohammad Shehzad[d]

[a]School of Quantitative Sciences, College of Arts & Sciences, UUM, Malaysia, College of Applied Sciences Nizwa, Ministry of Higher Education, Sultanate of Oman.
[b, c]School of Quantitative Sciences, College of Arts & Sciences, UUM, Malaysia
[d]College of Applied Sciences Nizwa, Ministry of Higher Education, Sultanate of Oman.

[a] iak_2000plus@yahoo.com
[b] azizan@uum.edu.my
[c] adyda@uum.edu.my
[d] shehzad.niz@cas.edu.om,

Abstract

Synchronization between two chaotic systems occurs when the trajectory of one of the system asymptotically follows the trajectory of another system due to coupling or due to forcing. In this research paper, the synchronization problem between two identical Li and identical Lorenz systems and nonidentical Li and Lorenz Chaotic Systems have been addressed. In this study, the synchronization is performed through a nonlinear controller based on Lyapuonov Stability Theory to stabilize the error dynamics. It has been shown that the proposed strategies have excellent transient performances using less control effort with fast transient speed and has shown analytically as well as graphically that synchronization is asymptotically globally stable. Numerical simulations are carried out to verify and support the analytical results of the proposed methodology by using *mathematica* 9.

Keywords: Synchronization, Nonlinear Active Control, Lyapunov Stability Theory, Lorenz & Li Chaotic Systems.

1. Introduction

The Synchronization of Chaos is one way of explaining sensitive dependence on initial conditions which is known as the Butterfly Effect [1].

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*Corresponding author.
E-mail address: iak_2000plus@yahoo.com

Basically the issue of chaos synchronization is in designing a coupling between the two systems in such a way that the chaotic time evaluation becomes absolute. The output of the response (slave) system asymptotically follows the output of the drive (master) system [1]. After the pioneering work of Pecora and Carroll, increasing interest has been generated in chaotic synchronization and has attracted the attention of many researchers due to its powerful applications in physics and engineering sciences [2, 3]. In this line, many effective control techniques have been successfully applied to realize chaos synchronization [4]. Notable among these techniques, chaos synchronization using Nonlinear Active Control Techniques have recently been widely accepted as one of the powerful techniques in synchronizing two identical as well as nonidentical chaotic systems [5-7]. As most of the real-world control problems are nonlinear, Nonlinear Control Techniques take the advantage of the given nonlinear system dynamics to generate high-performance designs and no gain matrix or Lyapunov exponents are required for its execution. These characteristics free the designer to focus on the synchronization problem, leaving monotonous model manipulations [2].

The main focus of this study is to present new analysis to explain the stability of the error dynamics for the global chaos synchronization of two identical Lorenz [8] and two identical Li [9] systems and nonidentical Li and Lorenz chaotic systems by achieving two main objectives. The first objective is to utilize fewer control effort to synchronize two identical as well as nonidentical chaotic systems and the second objective is the fast synchronization of the error signals. This study can be considered as an improvement to the existing results in [10] and [11].

Based on the Lyapunov Stability Theory and using the approach in [7], a class of nonlinear control schemes will be designed to achieve the synchronization between two identical Lorenz and two identical Li systems and nonidentical Li and Lorenz Chaotic Systems. We will establish our results using Lyapunov Stability Theory [12] and will achieve asymptotically globally synchronization. Numerical simulations will be furnished to show the effectiveness of the proposed study and then to compare the performance of our proposed approach to those in [10] and [11].

The rest of the paper is organized as follows: Section 2, the problem statement and methodology for nonlinear control is introduced. Section 3, discusses the chaos synchronization of identical Lorenz and identical Li chaotic systems and nonidentical Li and Lorenz chaotic system. In section 4, numerical simulations are presented to verify the effectiveness of our proposed approaches and finally the concluding remarks are then given in section 5.

2. Designing of Nonlinear Active Controller

Most of the synchronization techniques belong to the master-slave system arrangement. By master-slave system arrangement means that two oscillators are coupled in a way in which one system is forced to track the other system and the states of two systems show similar behavior.

Thus let us consider a drive-response (master-slave) systems configuration for a chaotic system as,

$$\dot{x} = P_1 x + Q_1 F(x) \qquad \text{(Master system)}$$

and (1)

$$\dot{y} = P_1 y + Q_1 G(\mathrm{y}) + \phi(t) \qquad \text{(Slave system)}$$

Where, $x = [x_1,\ x_1, ..., x_n]^T$, $y = [y_1,\ y_1, ..., y_n]^T \in R^{n\times 1}$ are the corresponding state vectors, $P_1, P_2 \in R^{n\times n}$ are the matrices and $Q_1, Q_2 \in R^n$ are the vectors, $F, G: R^n \to R^n$ are the nonlinear continuous functions of the drive and response systems respectively and $\phi(t) = [\phi_1(t), \phi_2(t), ..., \phi_n(t)]^T \in R^{n\times 1}$ is an injected additive nonlinear controller to the controlled system.

If $F(\cdot) = G(\cdot)\ and\ /\ or\ P_1 = P_2, Q_1 = Q_2$, then x and y are the states of two identical (nearly identical) chaotic systems and if $F(\cdot) \neq G(\cdot)\ and\ /\ or\ P_1 \neq P_2, Q_1 \neq Q_2$, then x and y are the states of two nonidentical chaotic systems.

The error dynamics for the synchronization of (2.1) can be described as,

$$\dot{e}(t) = H(x, y, e) + \phi(t) \tag{2}$$

Where $H(x, y, e)$ that contains linear terms and nonlinear terms of the master and slave systems, and $e_i(t) = y_i(t) - x_i(t)$.

For the two (identical or non-identical) chaotic systems in the absence of a proper controller, ($\phi_i = 0$), if the initial conditions $\left(x_{1m}(0), x_{2m}(0), ..., x_{nm}(0) \neq y_{1s}(0), y_{2s}(0), ..., y_{ns}(0)\right)$, then the trajectories of the two the chaotic systems will quickly diverge from each other in all future states and will become uncorrelated. Hence the role of a proper feedback controller for the synchronization problem is, to force the error dynamics converges to zero for all initial conditions [10],

i.e., $\lim\limits_{t\to\infty} \|e_i(t)\| = \lim\limits_{t\to\infty} \|y_i(t) - x_i(t)\| = 0$, for all $e_i(0) \in R^n$,

then the two systems (2.1) are said to be synchronized.

Theorem 1 [7]. The trajectories of the two (identical or nonidentical) chaotic oscillators (1) for all initial conditions $\left(x_{1d}(0), x_{2d}(0), ..., x_{nd}(0) \neq y_{1r}(0), y_{2r}(0), ..., y_{nr}(0)\right)$ will be synchronized asymptotically globally with appropriate nonlinear regular stabilizing feedback controller,

$\phi(t) = [\phi_1(t), \phi_2(t), ..., \phi_n(t)]^T \in R^{n\times 1}$.

Proof: Let us construct a candidate Lyapunov Error Function as,

$$V(e) = e^T Me \tag{3}$$

where the matrix M is a positive definite matrix [7]. Further it is assumed that all the variables and parameters of the drive and response systems are available and measureable.

It may be noticed that, $V: R^n \to R^n$ is a positive definite function by construction. It may achieve the synchronization by selecting suitable non-linear controller '$\phi(t)$' to make $\dot{V}(e) = -e^T Ne$ be a positive definite function (i.e., the matrix N is also a positive definite matrix), then by the Lyapunov Stability Theory [12], the states of both drive and response systems (1) will be asymptotically globally synchronized.

3.1 Chaos Synchronization of Two Identical Lorenz Chaotic Systems

The main goal of this section is to focuses on applying Non-linear controller technique to study and investigate new results for the global chaos synchronization for two identical Lorenz systems [8]. For this purpose, let us consider a master for Lorenz chaotic system is given as follow;

$$\left.\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= cx_1 - y_1 - x_1 z_1 \\ \dot{z}_1 &= x_1 y_1 - bz_1 \end{aligned}\right\} \quad \text{(Master system)} \qquad (4)$$

and the slave system is described as

$$\left.\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + \phi_1 \\ \dot{y}_2 &= cx_2 - y_2 - x_2 z_2 + \phi_2 \\ \dot{z}_2 &= x_2 y_2 - bz_2 + \phi_2 \end{aligned}\right\} \quad \text{(Slave system)} \qquad (5)$$

Where $x_i, y_i, z_i \in R^n,\ i = 1,2$ are the corresponding state vectors of master and slave systems respectively, a, b and c are the system parameters, $\phi(t) = [\phi_1(t), \phi_2(t), \phi_3(t)]^T$ are the Nonlinear Controller that is yet to be designed. The Lorenz system exhibit a chaotic attractor with parameter values, $a = 10, b = 28\ and\ c = \frac{8}{3}$. The error dynamics for the two chaotic systems (4) and (5) is described as,

$$\left.\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + \phi_1 \\ \dot{e}_2 &= ce_1 - e_2 + x_1 z_1 - x_2 z_2 + \phi_2 \\ \dot{e}_3 &= -be_3 + x_2 y_2 - x_1 y_1 + \phi_3 \end{aligned}\right\}$$

where $\phi_1, \phi_2\ and\ \phi_3$ are the Nonlinear Active Controllers and, $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1$.

The main focus of this section is to investigate and analyze the synchronization of two identical chaotic systems (4) and (5) by designing such a Nonlinear controller that when synchronizing the two chaotic systems (4) and (5), the effect of nonlinearity of the chaotic systems is not neglected and the error signal of the two identical chaotic systems converge to the equilibrium (origin) asymptotically globally with less control effort and sufficient transient speed. To clinch this goal, let us assume the following theorem.

Theorem 2. The trajectories of the two Chaotic Systems (4) and (5) will achieve synchronization asymptotically and globally for any initial conditions $\left(x_1(0), y_1(0), z_1(0) \neq x_2(0), y_2(0), z_2(0)\right)$ with following Nonlinear Control law:

$$\phi_1(t) = 0\ ,\quad \phi_2(t) = -2ce_2 - x_1 z_1 + x_2 z_2\ ,\quad \phi_3(t) = x_1 y_1 - x_2 y_2$$

Proof: Let us construct a Lyapunov Error Function Candidate as;

$$V(e) = e^T Me \tag{6}$$

where $M = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is a positive definite function.

Now the time derivative of the Lyapunov function is,

$$\dot{V}(e) = -40e_1^{\,2} - 4e_2^{\,2} - 56e_3^{\,2} = -e^T \begin{pmatrix} 40 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 56 \end{pmatrix} e \prec 0$$

Therefore, $\dot{V}(e) = -e^T Ne$ and $N = \begin{pmatrix} 40 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 56 \end{pmatrix}$ which is also a positive definite matrix.

Hence based on Lyapunov stability theory [12], the error dynamics converges to the origin asymptotically. Thus the master and slave Lorenz chaotic systems are asymptotically globally synchronized.

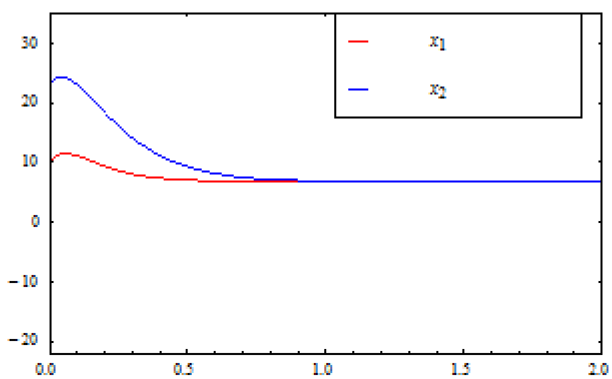

Fig. 1. Time series of x_1 & x_2 (identical Lorenz systems)

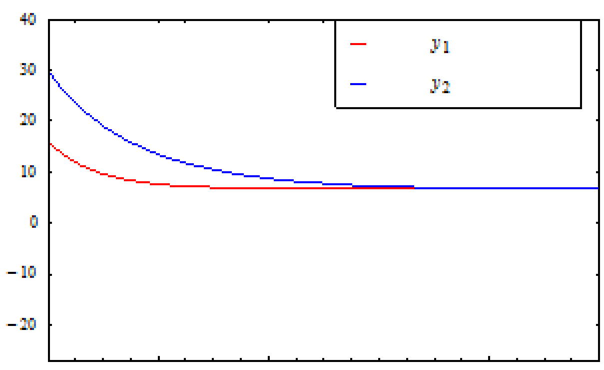

Fig. 2. Time series of y_1 & y_2 (identical Lorenz systems)

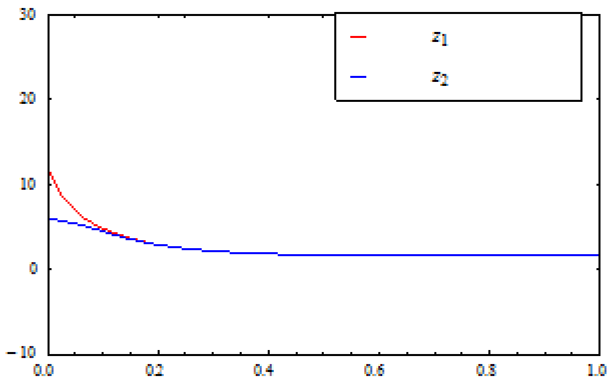

Fig. 3. Time series of z_1 & z_2 (identical Lorenz systems)

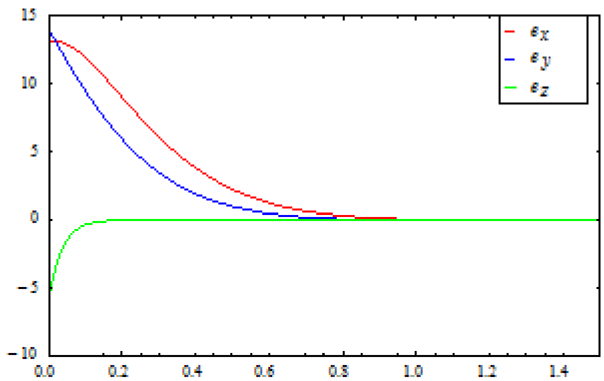

Fig. 4. Time series of errors, e_1, e_2 & e_3 (identical Lorenz systems)

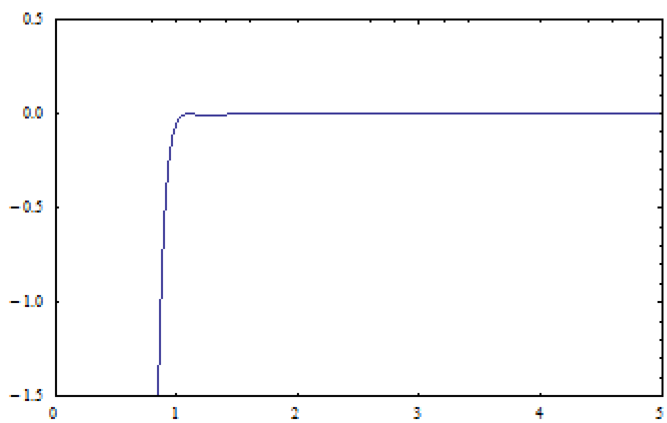

Fig. 5. Derivative of Lyapunov Errors Function (identical Lorenz systems)

3.2 Chaos Synchronization of Two Identical Li Systems

The main objective of this section is to study and investigate new results for the global chaos synchronization for two identical Li system [9]. For this purpose, let us consider the master-slave systems configuration for Li chaotic system is given as follow;

$$\left.\begin{aligned}\dot{x}_1 &= \alpha(y_1 - x_1)\\ \dot{y}_1 &= -y_1 + x_1 z_1\\ \dot{z}_1 &= \beta - x_1 y_1 - \gamma z_1\end{aligned}\right\} \quad \text{(Master system)} \qquad (7)$$

and

$$\left.\begin{aligned}\dot{x}_2 &= \alpha(y_2 - x_2) + \phi_1\\ \dot{y}_2 &= -y_2 + x_2 z_2 + \phi_2\\ \dot{z}_2 &= \beta - x_2 y_2 - \gamma z_2 + \phi_3\end{aligned}\right\} \quad \text{(Slave system)} \qquad (8)$$

where $x_1, y_1, z_1 \in R^n$ are the state variables of the master system with $\alpha, \beta \text{ and } \gamma$ are the system parameters and $x_2, y_2, z_2 \in R^n$ are the state variables of the corresponding slave system and $\phi(t) = [\phi_1(t), \phi_2(t), \phi_3(t)]^T \in R^{n\times 1}$ are the Active Feedback Controller. The Li system exhibits a chaotic attractor when the parameter values are taken as, $\alpha = 5$, $\beta = 16$ and $\gamma = 1$.

For chaotic synchronization of the above master-slave systems (7) and (8), the error dynamics is described as,

$$\left.\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1) + \phi_1\\ \dot{e}_2 &= -e_2 + x_2 z_2 - x_1 z_1 + \phi_2\\ \dot{e}_3 &= -\gamma e_3 - x_2 y_2 + x_1 y_1 + \phi_3\end{aligned}\right\} \qquad (9)$$

where $\phi_1, \phi_2 \text{ and } \phi_3$ are the Nonlinear Active Controllers and, $e_1 = x_2 - x_1, e_2 = y_2 - y_1, e_3 = z_2 - z_1$.

For the two identical chaotic systems (7) and (8), in the absence of a suitable controller, the trajectories of the two identical systems will diverge exponentially for all future states with a course of time and will become unsynchronized for all initial conditions. Thus the aim of the synchronization problem is to design a feedback controller $\phi(t)$ such that the error dynamics (9) converge to zero,

i.e., $$\lim_{t\to\infty} \|e(t)\| = 0, \quad \text{for all } e_i(0) \in R^n.$$

The aim of this section is, to focus on synchronizing two identical chaotic systems (7) and (8) by designing a nonlinear controller such that when synchronizing the two chaotic systems, the effect of nonlinearity of chaotic systems should not be neglected and the error dynamics convergence to the origin asymptotical globally with less control effort and enough transient speed. Therefore, using the approach in [7], selecting such a suitable Lyapunov error function candidate that will ensure asymptotically globally stability. For this purpose, let us assume the following theorem.

Theorem 3. The two Chaotic Systems (7) and (8) will approach global and asymptotical synchronization for any initial conditions $(x_1(0), y_1(0), z_1(0) \neq x_2(0), y_2(0), z_2(0))$ with following Control law;

$$\phi_1(t) = 0, \quad \phi_2(t) = -2e_1 - (x_2 z_2 - x_1 z_1), \quad \phi_3(t) = x_1 y_1 - x_2 y_2$$

Proof. Let us assume that the parameters of the master and slave systems are known and the states of both chaotic systems are measurable. Substituting the proposed controllers in (9), we have,

$$\left.\begin{aligned}\dot{e}_1 &= \alpha(e_2 - e_1)\\ \dot{e}_2 &= -2e_1 - e_2\\ \dot{e}_3 &= -\gamma e_3\end{aligned}\right\} \tag{10}$$

Let us construct the same Lyapunov Error Function Candidate as in (6) and constructing the matrix M as;

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}$$

Now the time derivative of the Lyapunov Error Function is,

$$\dot{V}(t) = -10e_1^2 - 5e_2^2 - e_3^2 = -e^T \begin{pmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} e \prec 0$$

i.e., $-\dot{V}(t) = e^T N e$ and N = $\begin{pmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ which is also a positive definite matrix.

We can see that $V(e)$ and $-\dot{V}(e)$ are positive definite functions. Hence the error states,

$$\lim_{t \to \infty} \| e_i(t) \| = 0$$

Thus by the Lypunov Stability Theory [12], the two identical Li chaotic systems are asymptotically globally synchronized.

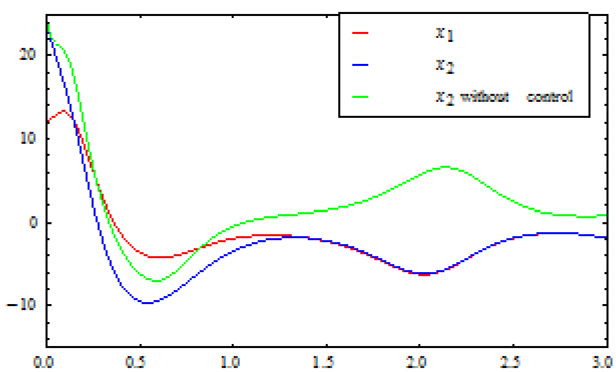

Fig. 6. Time series of x_1 and x_2 (Identical Li systems)

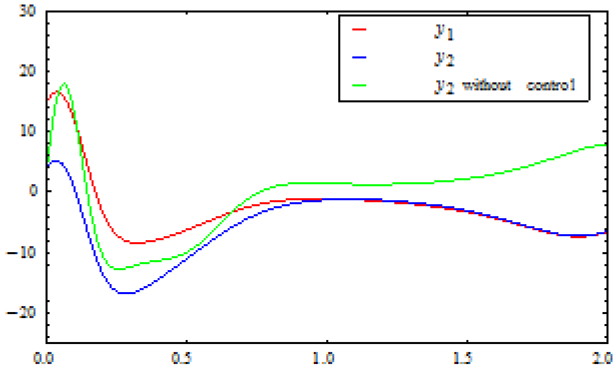

Fig. 7. Time series of y_1 and y_2 (Identical Li systems)

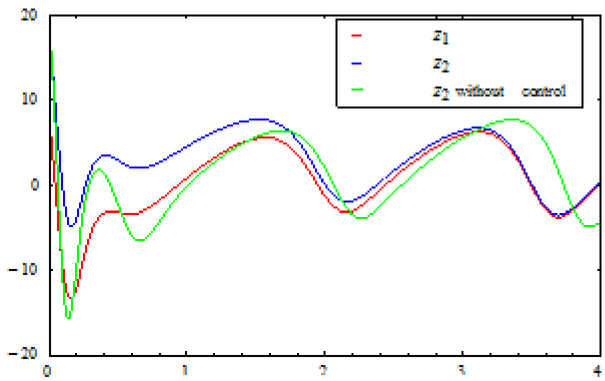

Fig. 8. Time series of z_1 and z_2 (Identical Li systems)

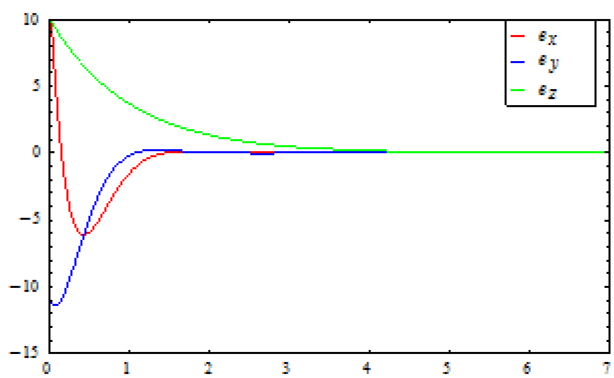

Fig. 9. Time series of errors e_1, e_2 & e_3 (Identical Li systems)

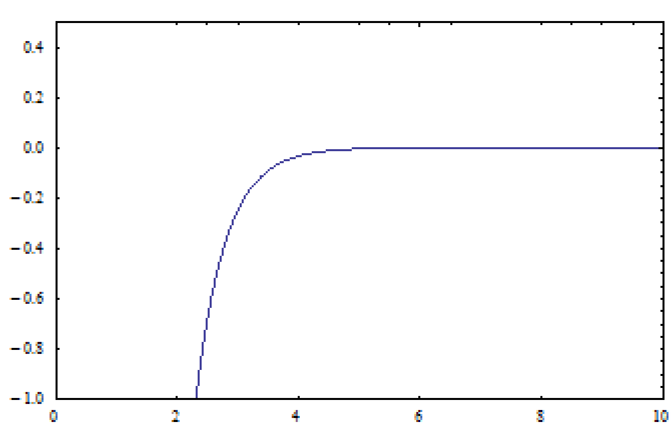

Fig. 10. Derivative of Lyapunov Errors Function (Identical Li systems)

3.3 Synchronization between Two Different Li and Lorenz Chaotic Systems

The main focus of this section is to analysis the switching synchronization between two nonidentical Li and Lorenz chaotic systems [9, 8] using the Nonlinear Control. To achieve this goal, it is assumed that the Li Chaotic System [9] drives the Lorenz Chaotic System [8]. Therefore, the master and slave systems arrangement is given as;

$$\left.\begin{aligned} \dot{x}_1 &= \alpha(y_1 - x_1) \\ \dot{y}_1 &= -y_1 + x_1 z_1 \\ \dot{z}_1 &= \beta - x_1 y_1 - \gamma z_1 \end{aligned}\right\} \quad \text{(Master system)} \qquad (11)$$

and

$$\left.\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + \phi_1 \\ \dot{y}_2 &= cx_2 - y_2 - x_2 z_2 + \phi_2 \\ \dot{z}_2 &= x_2 y_2 - bz_2 + \phi_3 \end{aligned}\right\} \quad \text{(Slave system)} \qquad (12)$$

Where $x_i, y_i, z_i \in R^n,\ i = 1, 2$ are the corresponding state vectors, α, β *and* γ and a, b and c are system parameters of the master and slave systems respectively, $\phi(t) = [\phi_1(t), \phi_2(t), \phi_3(t)]^T$ are the Nonlinear Controller that yet to be designed.

For chaotic synchronization of the above master-slave systems, the error dynamics is described as,

$$\left.\begin{aligned} \dot{e}_1 &= -ae_1 + \alpha e_2 + (\alpha - a)x_1 + (a - \alpha)y_2 + \phi_1 \\ \dot{e}_2 &= -e_2 + +ce_1 + cx_1 - (x_1 z_1 + x_2 z_2) + \phi_2 \\ \dot{e}_3 &= -be_3 + (1 - b)z_1 - \beta + x_1 y_1 + x_2 y_2 + \phi_3 \end{aligned}\right\} \qquad (13)$$

To achieve asymptotically globally stability of the error dynamics (13) using the Nonlinear Active Control, defining the controller $\phi(t) = [\phi_1(t), \phi_2(t), \phi_3(t)]^T \in R^{n\times 1}$ as,

$$\left.\begin{aligned} \phi_1(t) &= (a - \alpha)x_1 + (\alpha - a)y_2 \\ \phi_2(t) &= cx_1 - 2cx_2 + (x_1 z_1 + x_2 z_2) \\ \phi_3(t) &= (b - 1)z_1 + \beta - (x_1 y_1 + x_2 y_2) \end{aligned}\right\} \qquad (14)$$

Now let us take the same Lyapunov Error function as in (6) and selecting the matrix M as,

$$M = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{15}{32} & 0 \\ 0 & 0 & \frac{1}{7} \end{pmatrix}$$

The time derivative of the Lyapunov Error Function is,

$$\dot{V}(e) = -5{e_1}^2 - \frac{15}{16}{e_2}^2 - 8{e_3}^2 = -e^T \begin{pmatrix} 5 & 0 & 0 \\ 0 & \frac{15}{16} & 0 \\ 0 & 0 & 8 \end{pmatrix} e \prec 0$$

Therefore, $\dot{V}(e) = -e^T Be$ and $N = \begin{pmatrix} 5 & 0 & 0 \\ 0 & \frac{15}{16} & 0 \\ 0 & 0 & 8 \end{pmatrix}$ is also a positive definite matrix.

Hence based on Lyapunov Stability Theory [12], the origin of the error dynamics of two nonidentical chaotic systems (11) and (12) converge to the origin asymptotically. Thus the two chaotic systems (11) and (12) are asymptotically globally synchronized.

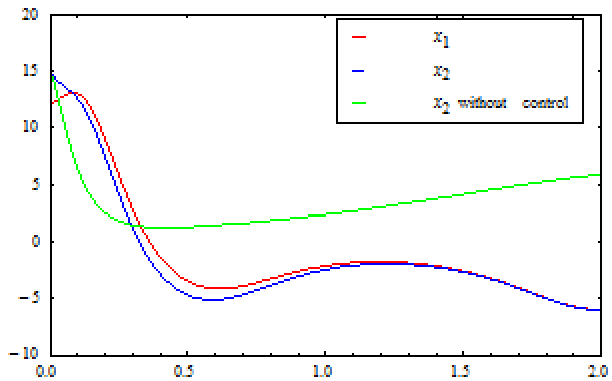

Fig. 11. Time series of x_1 and x_2 (Nonidentical Li & Lorenz systems)

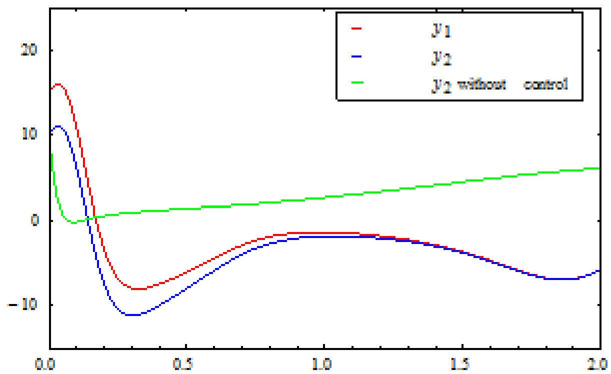

Fig. 12. Time series of y_1 and y_2 (Nonidentical Li & Lorenz systems)

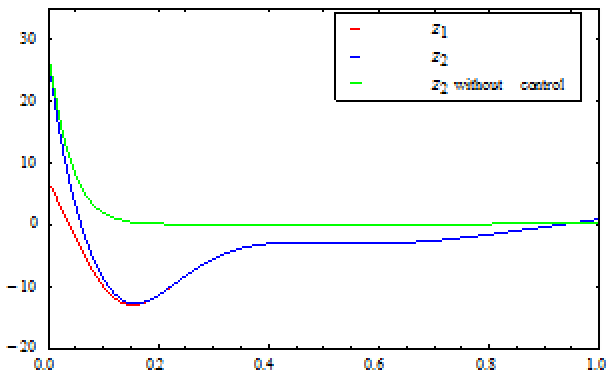

Fig. 13. Time series of z_1 and z_2 (Nonidentical Li & Lorenz systems)

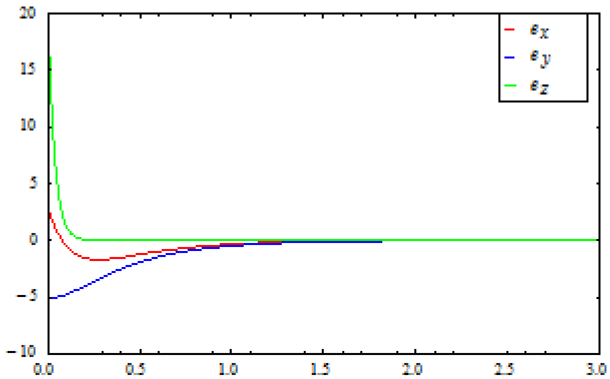

Fig. 14. Time series of errors e_1, e_2 & e_3 (Nonidentical Li & Lorenz systems)

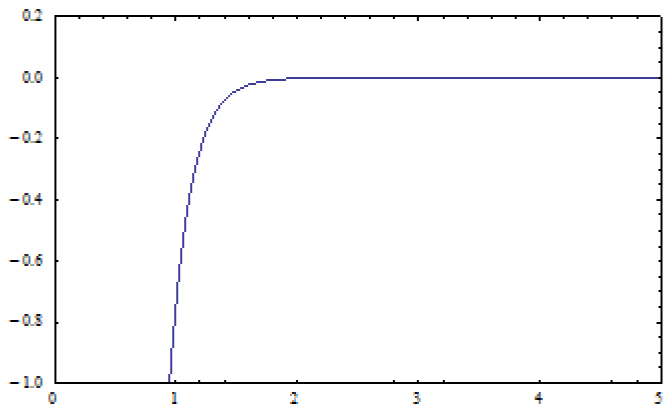

Fig. 15. Derivative of Lyapunov Errors Function (Nonidentical Li & Lorenz systems)

4. Numerical Simulations

Numerical simulations are furnished to verify the effectiveness of the proposed approach. For the Li systems the parameter values are taken as; $\alpha = 5$, $\beta = 16$ and $\gamma = 1$, with initial conditions, x1(0) = 12, y1(0) = 15, z1(0) = 7, x2(0) = -5, y2(0) = 0, z2(0) = -10.

For the Lorenz system, the parameters are taken as, a = 10, b = 28 and c = 8/3, with initial values, x1(0) = 10, y1(0) = 20, z1(0) = 30 and x2(0) = 2, y2(0) = 2, z2(0) = 2.

For the above chosen values, we have plotted the time series of states variables for identical systems (figures 1- 10) and for nonidentical systems (figures 11-15) whereas figure-4 and figure-9 depicts the time series of the errors dynamics for the two identical Lorenz and two identical Li chaotic systems and figure-14 for nonidentical systems and lastly, in order to discuss the stability, the time series of the derivatives of the Lyapunov errors functions have been plotted in figures-5, 10 and 15.

Fig. 4 shows the synchronization error for identical Lorenz Chaotic System when the controls are switched on at t = 0 s. It has been shown that the synchronization error has already achieved at t = 1s, while the synchronization error was achieved at t = 5s for [8] and thus the time delay is almost 4s. Moreover only two nonlinear controllers were applied to synchronize two identical Lorenz Chaotic Systems.

Fig. 9 shows the synchronization error for identical Li chaotic system when the controls were switched on at t = 0s. It has been shown that the synchronization error has already achieved at t = 4s, while the synchronization error was achieved at t = 8s for [9] and thus the time delay is almost 4s. On the other hand, only two controllers were applied to synchronize two identical Li Chaotic Systems.

Fig. 14 shows the synchronization error of two nonidentical Li and Lorenz Chaotic Systems. For the two different chaotic systems that contain parameters mismatch and different structures, the controller was used for synchronizing the states of master and slave systems asymptotically globally.

5. Summary and Conclusion

In the present study, simple but efficient Nonlinear Control Techniques were applied to achieve synchronization asymptotically globally between two identical Li and two identical Lorenz Chaotic Systems and two completely different chaotic Systems (Li and Lorenz). All results are furnished in graphical forms with time history in support of our analytical studies (Figures 1- 15).

In this study, it was found that only two nonlinear feedback controllers were utilized to achieve the synchronization asymptotically globally between the two identical Li and identical Lorenz Chaotic systems. This research study focused on selecting such a suitable Lyapunov error function candidate that ensured the global stability of the error dynamics of two coupled chaotic systems. It has been shown that the error signals converge to the origin very smoothly with minimum rate of decay, which shows that the investigated controller is more robust to accidental mismatch in the transmitter and receiver. In addition, the synchronization with negative derivative of the Lyapunov Errors Functions allows large synchronizable interval which is especially significant for engineering applications.

Since the synchronization of two identical as well as nonidentical chaotic systems assumes potential applications in the field of nonlinear dynamics, the result of this research work should be beneficial and could be employed in the field of secrete communication.

A comparative study can further be done to observe the robustness and stability of the proposed approach under the bounded noise.

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