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Dynamic soil-pile-interaction effects on eigenfrequency and damping of slender structures

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ABSTRACT: Single pile foundations have been widely used as a support solution for offshore wind turbines (OWT), where the design has been driven towards large diameter monopiles in order to satisfy the deformation limitations in the superstructure. The aim of the current study is to illustrate the effect of the dynamic soil-pile-interaction on the natural vibration characteristics of the flexibly supported structure. For this purpose a two-step iterative procedure has been developed based on two analytical solutions. The frequency dependent dynamic stiffness and damping coefficients are taken into consideration after a rigorous solution of horizontal soil – pile vibration, while the modified SSI eigenperiod and damping are calculated accounting for the cross coupling stiffness and damping terms of the soil – pile system. Disregarding the off diagonal terms is considered inappropriate since it results to non-conservative overestimation of the eigenfrequency and underestimation of damping especially for small slenderness ratios and high flexibility factor of the soil – pile system (short, rigid piles). The observed trends become even more prominent as the height of the slender structure increases. The effect of the monopile foundation properties on the natural vibration characteristics was also examined. The influence of the frequency dependent impedances was proven significant, since the modified SSI eigenfrequency decreased substantially, when the structural eigenfrequency was set between the first and the second eigenfrequency of the soil layer.

KEY WORDS: SSI eigenfrequency; SSI damping; soil-pile interaction, Offshore wind turbines.

1 INTRODUCTION

Dynamic soil-structure-interaction is a phenomenon affecting the response of structures subjected to dynamic loading, the latter arising from earthquakes, impacts, explosions, or environmental conditions like wind. The effect of the soil properties and the soil-foundation-structure interaction, in the estimation of the eigenfrequency and damping (natural vibration characteristics) is of primary interest. Slender structures like offshore wind turbines (OWTs) are subjected to dynamic loading, while the site and soil specific conditions should be accounted for, as delineated in certification and design guidelines [1].

Moreover the accurate estimation of the natural vibration characteristics of OWTs becomes critical given the narrow window of target design eigenfrequencies, which is left between the rotor and the blade passing frequency. Therefore recent research has been directed towards the development of analytical and experimental methods allowing for the calculation of the eigenfrequency of the wind turbine, while incorporating also the influence of the foundation stiffness [2,3,4]. With the majority of the installed offshore wind turbines being supported by monopiles, the soil-pile-interaction has been incorporated in the adopted modelling, while the effect of different approaches has been also examined [5,6]. Three major modelling approaches have been identified in the aforementioned studies: (a) the apparent fixity model, where the soil – pile system is replaced by an equivalent cantilever beam, (b) the coupled or uncoupled (translational and rotational) springs model (elsewhere referred as stiffness matrix model), and (c) the distributed springs model. The first two modelling approaches rely on the former analysis of the soil – pile interaction and the estimation of the corresponding stiffness coefficients at the mudline. The resulting impedances provide the basis for the calibration of the corresponding model parameters, leading thus to a two-step procedure.

The motivation of the current study emerges from the design of OWTs, which resembles the case of slender structures supported by single, large diameter pile foundations. A rigorous analytical formulation is presented, which comprises of two coupled analytical solutions, the first one providing the dynamic impedances of the soil-pile system and the second one the modified SSI eigenperiod and damping. Thereafter an iterative procedure considering the variation of the dynamic stiffness and damping coefficients with frequency is followed. The effect of the pile diameter, slenderness ratio, and relative flexibility is further discussed giving special consideration to the height of the structure.

2 ANALYTICAL FORMULATION

The substructuring method has been employed in the current study to analyze the dynamic soil-structure-interaction (SSI) of slender structures, an approach which has prevailed the past decades in the earthquake response analysis [7]. The analytical formulation of the problem comprises of two steps: (a) dynamic response analysis of the soil-pile system and (b) dynamic equilibrium of the flexibly supported structure. The first step provides the frequency dependent dynamic impedances at the pile head, which represent the required stiffness and damping properties at the foundation of the structure, while the SSI eigenfrequency and the damping are

calculated in the second step. A schematic illustration of the analytical formulation of the problem is shown in Figure 1. The coupling of the two analytical approaches is justified by the linear elastic response of the soil and all the structural components of the problem. Furthermore the estimation of the natural vibration characteristics (eigenfrequency and damping) allows for the consideration of harmonic loading.

The dynamic soil-pile-interaction has been investigated by several researchers while the employed models differentiate in the implementation and definition of the soil resistance to dynamic loading. The most popular approach is the beam on Winkler foundation, wherein the pile is considered as a beam embedded in soil and the soil resistance as independent elastic springs distributed along the pile. By including the inertial terms in the differential equation of the static lateral pile response [8], analytical solutions for the dynamic response have been derived [9,10]. The major shortcomings of this method in the case of dynamic response are: (a) the model formulation is equivalent to plane strain response, hence the wave fronts are prescribed as parallel planes, and (b) the soil reaction of each finite layer is independent, hence shear stresses are not transmitted between them. On the other hand this mathematical formulation has provided the framework for the development and implementation of nonlinear near field soil-pile-interaction models, which have been combined with far field elastic models [11,12]. The continuum approach in either an analytical [13,14] or a numerical formulation [15,16] has been shown to overcome the abovementioned limitations.

The analytical assessment of the dynamic SSI effects on the eigenfrequency and damping of structures has been substantiated by a single degree of freedom (SDOF) system supported by translational and rotational springs and dashpots [17,18]. The concept of a replacement oscillator with equivalent damping and stiffness has been employed and analytical expressions for the modified SSI eigenperiod and damping have been proposed [19]. Recently the abovementioned expressions have been improved to account for the double damping terms [20], and furthermore the cross-coupling (roto-translational) stiffness and damping component [21].

The current analytical formulation comprises of the calculation of the dynamic impedances based on the continuum approach proposed by Novak and Nogami [14], and the analytical SSI eigenfrequency and damping proposed by Zania [21]. Hereafter only an outline of the two methods is presented.

2.1 Dynamic impedances (Novak and Nogami[14])

The dynamic soil – pile interaction is performed in a 'decoupled' manner, since the soil response to the dynamic load is firstly established [13] and then the dynamic pile response to the exerted pressure is calculated, assuming a full compliance of the pile with the surrounding soil [14]. The soil resistance (Equation 1) is estimated as a sum of the contributions of individual wave modes, after the solution of the wave propagation equations for appropriate boundary conditions.

$$p(z)=\sum_{n=1}^{\infty}\pi r_0 G\left[(1+\mathrm{i}D_s)h_n^2-\left(\frac{\omega}{V_s}\right)^2\right]T_n U_n \sinh_n z \qquad (1)$$

where U_n is the modal amplitude independent of the depth z, r_0 is the pile radius, G is the shear modulus of the soil, D_s is the hysteretic damping ratio associated with shear strains (tangent of loss angle), $h_n = \pi(2n–1)/2H$, H is the depth of the soil layer, V_S is the shear wave velocity and T_n is given by:

$$T_n=\frac{4K_1(q_n r_0)K_1(s_n r_0)+s_n r_0 K_1(q_n r_0)K_0(s_n r_0)+q_n r_0 K_0(q_n r_0)K_1(s_n r_0)}{q_n K_0(q_n r_0)K_1(s_n r_0)+s_n K_1(q_n r_0)K_0(s_n r_0)q_n s_n r_0 K_0(q_n r_0)K_0(s_n r_0)} \qquad (2)$$

where K_m is the modified Bessel function of second kind and order m. The variables q_n and s_n are functions of the dimensionless frequency $\alpha_0=\omega H/V_S$, the wave number of the eigenmodes h_n, and the damping ratio. It is noteworthy that even though the harmonic wave propagation equations of the soil layer are formulated considering the vertical displacements associated with horizontal pile vibration as negligible, the approximation is considered rational when the pile deforms in bending without substantial shear deformations.

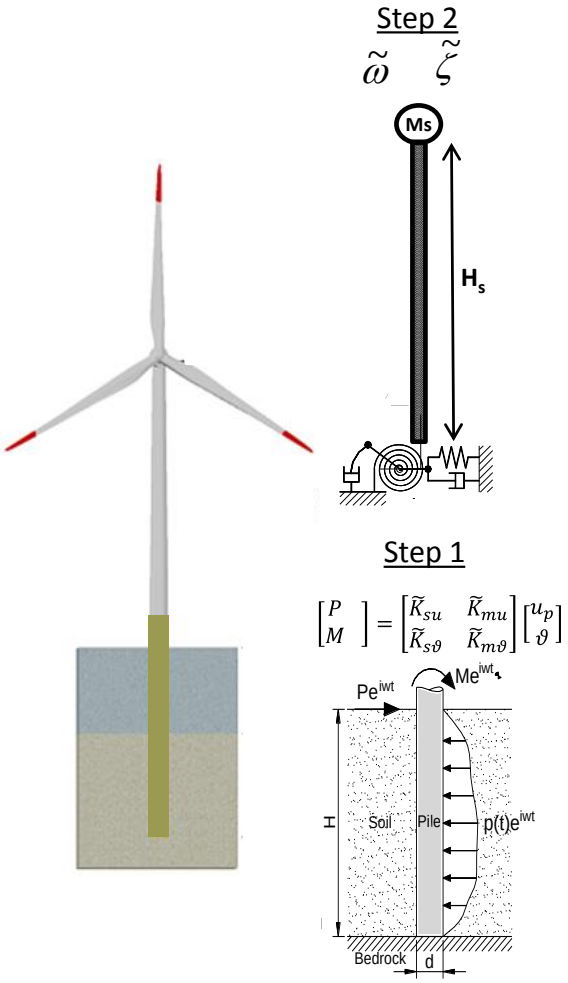

Figure 1. Basis for the analytical formulation of the dynamic soil-structure-interaction for slender structures like OWTs.

Thereafter the governing equation of the dynamic response of the pile subjected to a pressure distribution equal to the soil resistance can be formulated:

$$E\,I\,\frac{\partial^4}{\partial z^4}\left(u\mathrm{e}^{\mathrm{i}\omega t}\right)+m\,\frac{\partial^2}{\partial t^2}\left(u\mathrm{e}^{\mathrm{i}\omega t}\right)=-p\,(z)\mathrm{e}^{\mathrm{i}\omega t} \qquad (3)$$

The solution of the partial differential equation is a summation of the complete solution of the homogeneous equation and the particular solution of the non–homogeneous equation. Assuming the motion of the soil identical to the motion of the pile at the contact interface, and using Fourier expansions for the trigonometric functions an analytical expression of the displacement is derived. The classical beam theory is thereafter employed in order to define the angle of rotation, the bending moment and the shear force distributions along the pile. The boundary conditions at the pile tip and the pile head provide the necessary equations for calculating the integration constants. Finally the impedance functions are estimated as the forces/moments at the pile head for unit displacement/rotation. Since the displacement is expressed in the frequency domain via a modal summation, the derived impedance functions are also frequency dependent, while the real part resembles the stiffness and the imaginary part the damping (both material and radiation components).

2.2 SSI eigenfrequency and damping (Zania[21])

In a recent publication by the author a new analytical solution for the SSI eigenfrequency and damping has been proposed [21]. The dynamic equilibrium of a SDOF system has been solved accounting for the full impedance matrix at the foundation level (pile head). Hence improved analytical expressions for the modified SSI eigenfrequency and damping have been suggested which account for the roto-translational impedances. The flexibly supported SDOF attains three dynamic degrees of freedom. After the formulation of the three dynamic equilibrium equations, the displacement and the rotation at the pile head are related to the structural distortion, while the latter is related to the ground motion. Using the notion of the equivalent oscillator [19] the modified SSI eigenfrequency and damping are obtained. The presentation of the analytical expressions is omitted for brevity; however the simplified expression of SSI eigenfrequency by neglecting the double damping terms as insignificant, is:

$$\tilde{\omega}^2 = \frac{\omega_s^2}{1 + k_s \left[\dfrac{K_{m\vartheta} - 2H_s K_{s\vartheta} + H^2{}_s K_{su}}{K_{su} K_{m\vartheta} - K^2{}_{s\vartheta}} \right]} \tag{4}$$

where k_s is the structural stiffness, H_s the height of the structure, K_{su} the translational stiffness, $K_{s\theta}$ the roto-translational stiffness, and $K_{m\theta}$ the rotational stiffness.

The dynamic impedances (both real and imaginary part) are frequency dependent, which implies that an iterative procedure should be adopted. The process of the SSI eigenfrequency and damping calculation is repetitive, by tuning the impedances to the obtained value of the SSI eigenfrequency until the difference between two successive steps is small enough.

3 EFFECT OF SOIL-PILE-INTERACTION ON NATURAL VIBRATION OF OWT

In order to assess the effects of dynamic soil-pile interaction on the natural vibration characteristics of OWT, a parametric study was performed by implementing the abovementioned analytical method. The investigated parameters include the diameter of the pile (d), the soil/pile height (H), the shear wave velocity (V_S), and the height of the OWT. The selected properties are listed in Table 1, while the wall thickness is equal to d/100, and five intervals were considered within the reported range of shear wave velocity.

Table 1. Soil Properties and dimensionless parameters.

Diameter (m)	V_S (m/s)	H/d	K_r [23]
d_1=4	44-294	7	0.037-0.0008
6	60-400	6	0.037-0.0008
d_2=6	44-294	7	0.037-0.0008

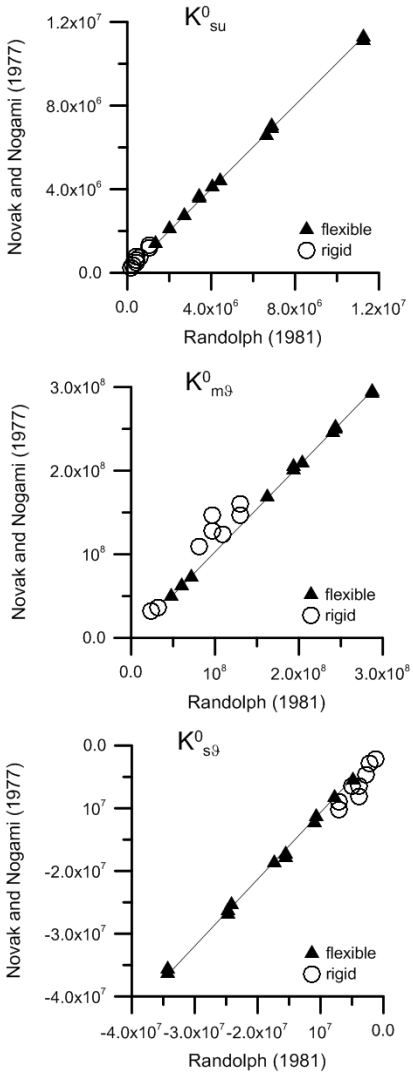

Figure 2. Comparison of stiffness coefficients obtained after the analytical solution [14] and the fitted expressions to numerical results [22].

Note that the pile flexibility factor K_r [23] is identical for the selected pile configurations and varies only with V_S. The categorization scheme proposed by Poulos and Davis [23] has been adopted in the current study. Thus the terms 'rigid' piles refers to K_r>0.01, referring to the lowest range of V_S. Furthermore the frequency range of the calculated impedances varied as the dimensionless soil frequency is $0.1<\alpha_0<8.0$. The

three OWTs which were selected for the purposes of this study, cover a range of 0.66-2MW power supply: (a) OWT 1: Hs=50m, D=40m, (b) OWT 2: Hs=70m, D=70m, and (c) OWT 3: Hs=100m, D=100m. The total mass was considered lumped at the top node and comprised of the sum of the mass of the rotor (hub and blades), the nacelle and 22.7% of the tower mass. The bending stiffness of the tower varied to obtain structural fixed- base eigenfrequencies ranging between 0.2Hz and 1Hz.

For small values of frequency the dynamic impedances approximate the static stiffness coefficients, thereafter the obtained values were compared with the reported expressions in the literature, which had been proposed after finite element analysis [22]. An excellent agreement of the current results was found with the corresponding static stiffness coefficients after Randolph [22] in the case of flexible piles (Figure 2), while the deviation increased at 25% for the translational and the rocking coefficients and at 45% for the cross coupling coefficient in the case of rigid piles. The discrepancy between the two methods is attributed to the increase of the static stiffness coefficients as the slenderness ratio decreases (increase of the exponent on the relative flexibility term of stiffness expressions). The derived expressions [22] do not account for the influence of the slenderness ratio since they had been proposed for flexible piles.

Even though the dynamic stiffness coefficients of the first and last pile configurations (Table 1) were found identical [21], the static stiffness coefficients are proportional to the diameter (translational), it's second (roto-translational) and third power (rotational). This implies that the first pile configuration attains lower impedances than the other two configurations, leading thus to smaller SSI eigenfrequencies. Figures 3 and 4 illustrate the normalized modified SSI eigenfrequency with respect to the normalized frequency, separately for rigid (K_r>0.01) and flexible piles. The modified SSI eigenfrequency of a coupled system founded on flexible piles appears to be decreased only marginally by 5%-10%, for large diameter piles. On the other hand the decrease may become 20% of the fixed base eigenfrequency for smaller diameter piles over the same normalized frequency range. In the case of rigid piles the same trends are preserved, while the decrease of the eigenfrequency appears more prominent. This is attributed to the different frequency range applicable in each case. The structural eigenfrequency lies below the first eigenfrequency of the soil layer for the examined cases of flexible piles (Figure 3b), while in rigid piles it lies around and above the first eigenfrequency and below the second eigenfrequency of the soil layer (Figure 3a). For both rigid and flexible piles the modified SSI eigenfrequency decreases for higher slenderness ratios even though the relative flexibility is identical. On the other hand the decrease of the relative flexibility (increase of V_S) leads to increase of the SSI eigenfrequency. Comparing Figures 3 and 4 it becomes apparent that the effects of the diameter, the slenderness ratio H/d and the relative flexibility are independent of the height of the structure. The normalized eigenfrequency is larger for the taller OWT, implying that the dynamic SSI effects are stronger.

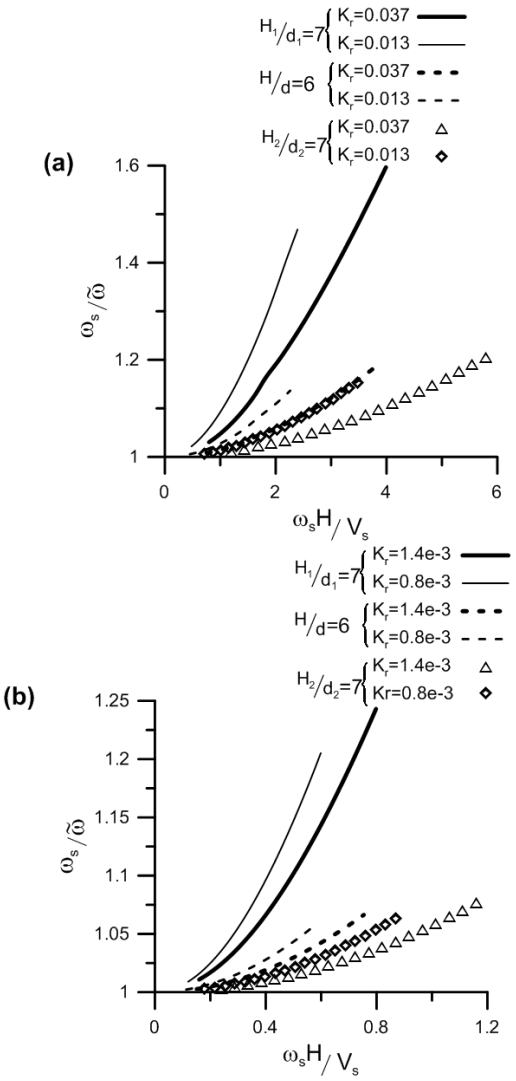

Figure 3. Ratio of fixed base eigenfrequency to the modified SSI eigenfrequency with respect to the normalized eigenfrequency. Results are presented for OWT of 70m height founded on (a) rigid and (b) flexible piles.

Moreover, the increase of the structural damping expressed as the ratio of the modified SSI relative damping ratio to the corresponding of the fixed base SDOF, is presented in Figure 5. The results correspond to all three OWTs supported by rigid piles. It is evident that the modified SSI damping increases with respect to the structural eigenfrequency for frequencies higher than the first natural eigenfrequency of the soil layer (which is almost equal to $\omega_s H/V_s=\pi/2$). As presented in the methodology the dynamic impedances are tuned at the modified SSI eigenfrequency, this implies that the first eigenfrequency of the soil layer is met when $\tilde{\omega} = V_s*\pi/(2H)$ (resonance of structure and soil layer). For the smallest OWT the SSI damping increases as the slenderness ratio and the relative flexibility factor increase, when referring to the same relative flexibility factor and slenderness ratio respectively. Additionally the decrease of the diameter, for constant slenderness ratio and relative flexibility factor, raises the SSI damping to even triple the structural fixed base damping.

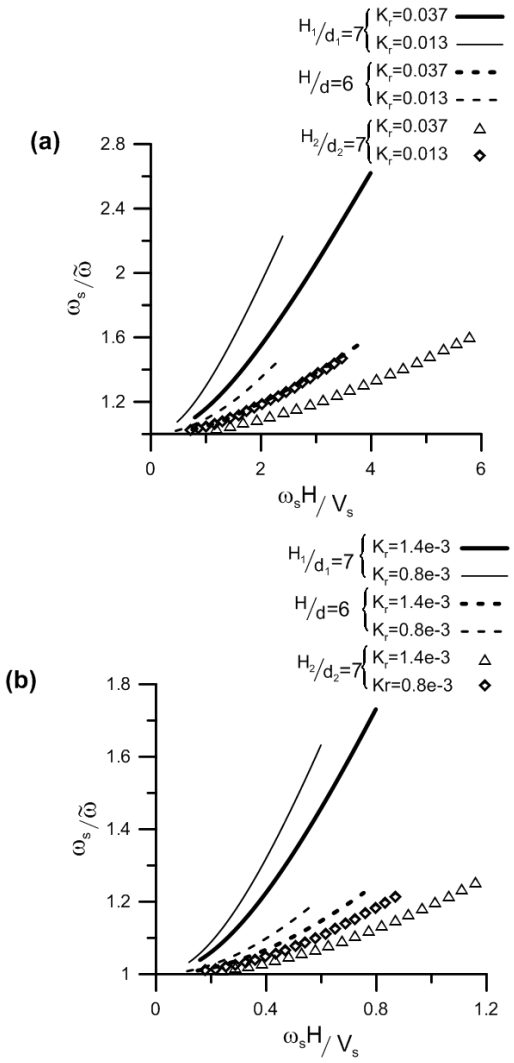

Figure 4. Ratio of fixed base eigenfrequency to the modified SSI eigenfrequency with respect to the normalized eigenfrequency. Results are presented for OWT of 100m height founded on (a) rigid and (b) flexible piles.

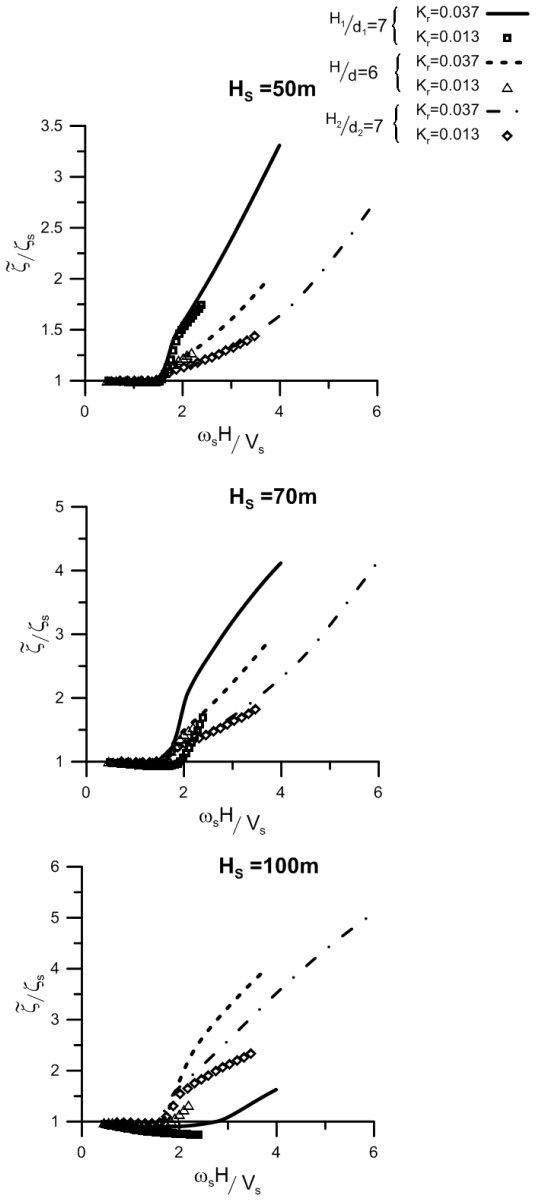

Figure 5. Ratio of modified SSI damping to structural damping ratio with respect to the normalized eigenfrequency.

Observing Figure 5 it becomes apparent that, the increase of the height of the SDOF system increases the modified SSI damping while the abovementioned trends (in regards with the slenderness ratio and the flexibility factor) are preserved for the large diameter pile. However, as the diameter of the pile decreases, the increase of the height of the OWT results to reduction of the SSI damping. This effect is more prominent for the tallest OWT (H_S=100m). The modified SSI damping is proportional to the second power of the SSI eigenfrequency to the fixed based eigenfrequency. For the presented rigid pile configurations the decrease of the SSI eigenfrequency is substantial (note also the larger normalised frequency at which the damping increases), minimizing thus the contribution of the radiation damping to the total SSI damping.

Figure 6 illustrates only results obtained for rigid piles of two different diameters for all the examined OWTs and the two softer soil profiles. The normalised eigenfrequency variation for the two pile configurations shows clearly the effect of the height of the structure. This trend is preserved even when the soil stiffness increases (V_{S2}>V_{S1}). This comparison illustrates the effect of the cross coupling and translational stiffness and damping terms, as their contribution to the modified SSI eigenfrequency is enhanced with the increase of the height of the structure (see equation 4).

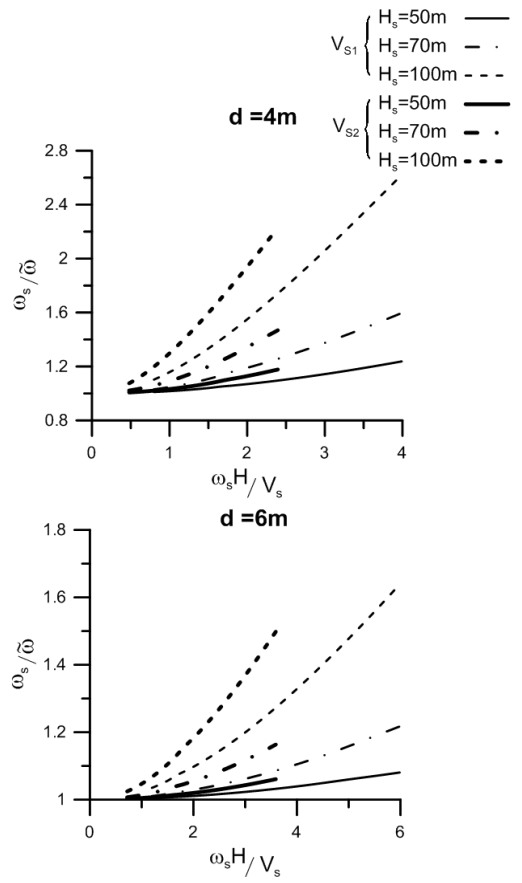

Figure 6. Effect of diameter on the modified SSI eigenfrequency of OWT founded on rigid piles.

4 COMPARISON WITH APPROXIMATE SOLUTION

The derived equations of the modified SSI eigenfrequency and damping incorporate the off diagonal stiffness and damping coefficients; hence it is worthwhile to illustrate the divergence of the results when these terms are disregarded as in previous solutions [19]. Figure 7 presents the ratio of the rigorous SSI eigenfrequency to the approximate value (no cross coupling terms) with respect to the dimensionless eigenfrequency of the structure. Note that the cases illustrated here refer to rigid piles, two different diameters, and the same slenderness ratio. The approximate solution is shown to overestimate the eigenfrequency in all cases, and it may be even 40% larger than the corresponding rigorous. Moreover the discrepancy of the two methods becomes more intense as the pile diameter decreases and as the flexibility ratio increases ($V_{s1}< V_{s2}$). The increase of the height of the structure enhances the divergence between the results obtained after the rigorous and the approximate method. This implies a stronger effect of the off diagonal impedance terms as the height of the structure increases. Figure 8 illustrates the ratio of the rigorous to the approximate modified SSI damping. It is shown that disregarding the off diagonal stiffness and damping coefficients would imply underestimation of the damping for frequencies higher than the first natural eigenfrequency of the soil layer. This trend is mostly relevant for large pile diameter and high flexibility factor. On the other hand for smaller pile diameters, embedded in very soft soil layers, the approximate solution may overestimate the damping especially for tall structures with fixed base eigenfrequency higher than 0.5Hz.

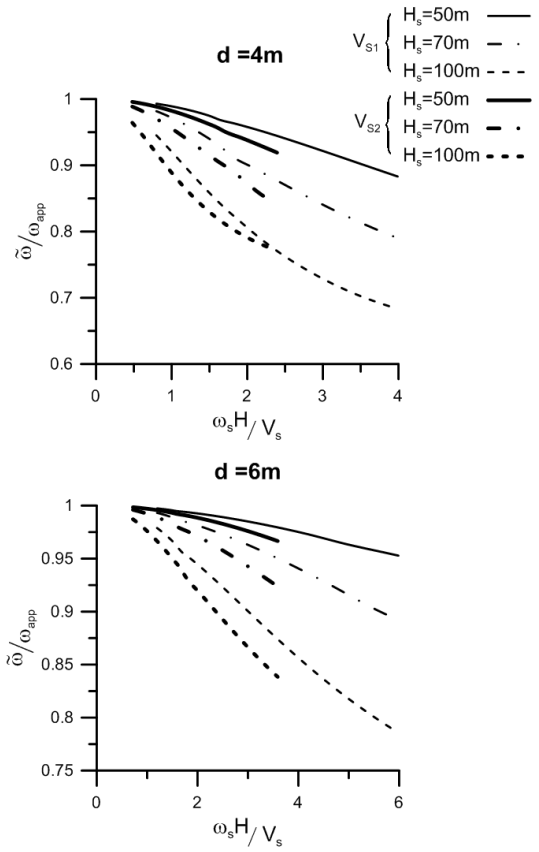

Figure 7. Effect of diameter on the ratio of the rigorous to the approximate modified SSI eigenfrequency of OWT founded on rigid piles.

5 CONCLUSIONS

The natural vibration characteristics of slender structures founded on large diameter piles has been investigated. For this purpose a two-step iterative procedure has been employed based on the analytical methods suggested in the literature for the dynamic pile impedances and for the calculation of the modified SSI eigenfrequency and damping of the structure. The new analytical formulation allows for the consideration of the off diagonal terms of the dynamic impedances, and provides a rigorous solution of the natural vibration characteristics of the replacement oscillator. The inaccuracy introduced on the estimated SSI eigenfrequency and damping, when disregarding the off diagonal impedance terms, was examined for large diameter piles. The results of the current study have demonstrated that not accounting for the cross coupling terms, would imply overestimation of the eigenfrequency of the structure and underestimation of the damping, especially for large diameter piles, small slenderness ratios and high flexibility factor of the soil – pile system (short, rigid piles) and for increasing height of the structure.

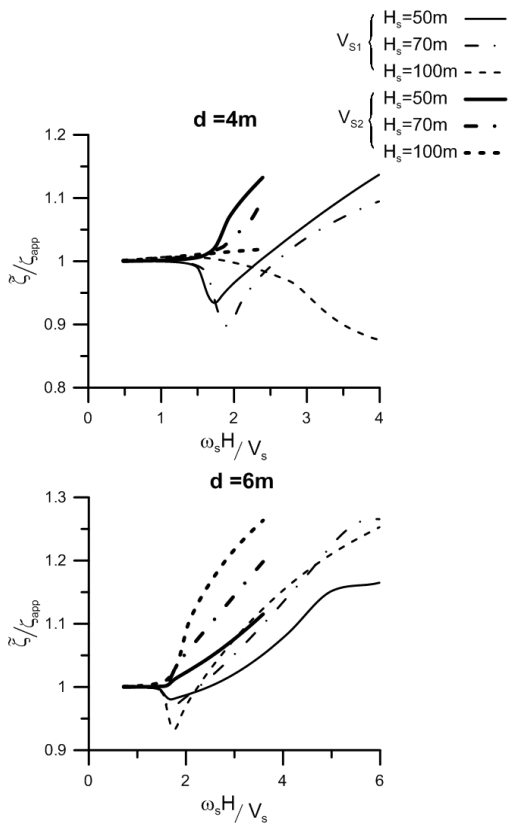

Figure 8. Effect of diameter on the ratio of the rigorous to the approximate modified SSI damping of OWT founded on rigid piles.

The effect of the monopile foundation properties on the natural vibration characteristics was also examined. The influence of the frequency dependent impedances was proven significant, since the modified SSI eigenfrequency decreased substantially, when the structural eigenfrequency was set between the first and the second eigenfrequency of the soil layer. At the same frequency range the modified SSI damping increased, especially as the pile diameter decreased, and as the slenderness ratio of the pile and the height of the structure increased.

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