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Preference disaggregation for multiple criteria sorting with partial monotonicity constraints: Application to exposure management of nanomaterials

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We propose a novel approach to multiple criteria sorting incorporating a threshold-based value-driven procedure. The parameters deciding upon the shape of marginal value functions and separating class thresholds are inferred through preference disaggregation from the Decision Maker's incomplete assignment examples and partial requirements on the type of (non-)monotonicity for each marginal value function. These types include standard monotonic shapes, level-monotonic functions, A- and V-types combining increasing and decreasing value trends, and unknown monotonicity constraints. A representative instance of the sorting model compatible with the preference information is constructed by solving a dedicated Mixed-Integer Linear Programming problem. Its complexity is controlled by minimizing the number of changes in monotonicity between all subsequent sub-intervals of marginal value functions. The assignments derived using the constructed representative model are validated against the outcomes of robustness analysis. The proposed method is applied to a real-world problem of exposure management of engineered nanomaterials. We develop a model for predicting precaution level while handling nanomaterials in certain conditions using a respirator. The model captures interrelations between ten accounted evaluation criteria, including both monotonic and non-monotonic criteria, and the recommended class assignment. This makes it suitable for the management of exposure scenarios, which have not been directly judged by the experts.

1. Introduction

Multiple Criteria Decision Aiding (MCDA) is one of the fastest developing sub-fields of computer science and operational research [16]. Its importance derives from offering a diversity of approaches for structuring decision problems involving multiple criteria and carrying forward their solution. As the criteria used to represent pertinent viewpoints on the quality

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of considered alternatives usually do not align to indicate the most preferred alternative, arriving at the problem's solution requires involvement of Decision Maker (DM). (S)he is expected to exchange information with the method in a way ensuring that a recommendation constructed in the course of a decision aiding process is feasible and consistent with his/her value system [36].

The two major components of decision aiding approaches are responsible for querying the DM for suitable inputs and performing the analysis of his/her feedback to produce a recommendation in function of the specific problem to solve [23]. When it comes to the required inputs, their characteristics may be two-fold. On the one hand, they may be imposed by the context of a particular decision problem, hence referring to the characteristics of criteria, type of performances, or specificity of expected results. On the other hand, the inputs may represent the DM's subjective preferences indicating his/her priorities, requirements, and choices that should be respected when deriving the recommendation. Processing such diverse information consists in constructing a preference model of the DM in the context of the considered decision problem, and exploiting this model to produce numerical and other arguments supporting the recommendation.

Most traditional MCDA methods incorporate complete information about the problem and model parameters. Such information takes the form of precise performances of alternatives, well-defined preference directions for all criteria, exact requirements imposed on the provided outcomes, or exact values of preference model parameters [37]. The assumption on availability of such complete information may be questioned on many grounds. When it comes to the model parameters, it may not be possible to obtain their reliable exact estimates from the DM due to a misunderstanding of their meaning, a prohibitively high cognitive effort related to their elicitation, a lack of DM's confidence in providing precise inputs, or an application of some arbitrary transformation of the incomplete judgments to the complete ones (e.g., converting ordinal scales of criteria to cardinal weights). For this reason, the interest in recently developed MCDA approaches has been shifted to acquiring partial preference information at an affordable effort [5,37].

The terms of *incomplete* or *partial information* can be interpreted in two interrelated ways [13,28]. On the one hand, they indicate that the DM's preferences – usually modeled in form of some constraints – can be satisfied by more than one set of parameter values. This implies multiplicity of preference model instances compatible with the DM's statements [37]. On the other hand, incompleteness or partiality of preference information emphasizes that its use may not lead to a univocal recommendation [5]. However, the latter can be made robust by eliciting richer (i.e., more complete) information from the DM [3].

As far as MCDA methods incorporating partial preference information are concerned, the preference disaggregation approaches have been prevailing in the recent years [5,21]. They assume that the DM's preferences have the form of example holistic decisions concerning a subset of reference alternatives. Such judgments may come from historical data, from the DM's better knowledge of some alternatives, or can be implied by a relative easiness of performing a comprehensive evaluation of such alternatives [39].

In this paper, we consider multiple criteria sorting problems oriented toward an assignment of alternatives to pre-defined and preference ordered decision classes [47]. For this purpose, we use a threshold-based value-driven sorting procedure [14, 46]. It incorporates a preference model composed of an additive value function and thresholds separating the classes on a scale of a comprehensive value. The parameter values deciding upon the shape of marginal value functions and separating thresholds are inferred indirectly from the assignment examples, which are composed of reference alternatives and their desired class assignments [14]. The latter ones should be reproduced in the final recommendation, while additionally delimiting the space of admissible values of preference model parameters and influencing the sorting of non-reference alternatives.

The preference disaggregation paradigm has been so far mostly applied in the context of monotone learning data, i.e., criteria with well-defined preference directions [14,26]. These include gain and cost criteria, on which one prefers, respectively, greater or lesser performances. However, the recent trend in MCDA (see, e.g., [11,25,34]) – motivated by numerous real-world applications – consists in accounting for the non-monotonic criteria [1].

The framework proposed in this paper accounts for a wide spectrum of types of monotonic and non-monotonic marginal value functions within a preference disaggregation framework. These types admit specification of partial information concerning the DM's per-criterion preferences implied by the problem's peculiarity. In particular, we consider both gain- and cost-type criteria as well as preference-ordered attributes for which the direction of monotonicity cannot be specified a priori. Furthermore, we account for A- and V-type functions, which combine increasing and decreasing trends in disjoint sub-ranges of the performances scale. We also generalize the latter functions to level-monotonic characteristics, which correspond to the shapes assigning the same marginal value to all performances in a certain performance sub-region, but adhering to monotonicity constraints in the other region [34]. For example, the level-decrease function assigns the same maximal marginal value to a subset of the least performances, while systematically decreasing it from a certain point of the performance scale down to zero being associated with the greatest performance. Finally, we also account for the criteria with unknown monotonicity constraints [25], for which the respective marginal functions are allowed to take any shape.

Similarly to Kliegr [25], we aim at constructing a model whose complexity is controlled by the number of changes in monotonicity between all subsequent sub-intervals of marginal value functions. Minimizing this number, we implement the prudence principle in MCDA, while adjusting the model's complexity to the available incomplete preferences. Hence, the lack of complete information about the monotonicity of particular criteria offers different means for ensuring consistency between the DM's preference information and the model than in traditional MCDA approaches. Indeed, it opposes to both consistency restoration which eliminates the conflicting DM's statements [30,31] and consistency preservation enforcing compatibility of the new DM's judgments with the previously elicited statements [2,4]. To adjust the non-monotonic character of the marginal value functions to the available assignment examples, we use Mixed-Integer Linear Programming (MILP).

The proposed basic model constructs a single additive value function and a vector of precise class thresholds. However, when using indirect preference information, there may exist multiple instances of the sorting model that would be compatible with it, hence restoring the DM's assignment examples [14,17,22]. In our case, a set of compatible instances of the sorting model is delimited by the minimal number of changes in monotonicity for all marginal value functions. The application of such model instances on the set of non-reference alternatives may lead to different assignments [14,26]. From the viewpoint of robustness analysis, it is thus advisable to examine how the sorting recommendation changes when the complexity of compatible model instances varies within the plausible limits. The results of such an examination take the form of possible assignments, which indicate classes to which a given alternative is assigned by at least one instance of the compatible sorting model. Such assignments can be interpreted as robust conclusions which are supported by the DM's partial preference information.

The proposed method is applied to a real-world problem of exposure management of Engineered Nanomaterials (ENMs). Nowadays, such materials are commonly used in consumer products like cosmetics, clothes and food, which implies that the number of workers exposed to such materials is increasing each year [12,27]. The available approaches proposed for controlling exposure to nanomaterials include the use of personal protective equipment, administrative and work practices control and engineering controls [33]. We develop a model for assessing the suitability of a particular Risk Management Measure (RMM) for exposure management during the manufacturing of ENMs. Specifically, we focus on the use of a respirator while handling nanomaterials in certain conditions. The input preference information concerns a holistic assessment of a subset of exposure scenarios to nanomaterials conducted by a team of experts in view of the recommended level of the selected RMM [32]. In addition, ten descriptors are included in the model development. They include seven monotonic criteria of either gain- or cost-type, a single level-increase criterion, and two non-monotonic variables. The role of constructed model is to capture the interrelations between the evaluation criteria and the recommended level of use of the considered RMM. In this way, the model explains the expert judgments, but it can also be used to assess other exposure scenarios to ENMs. The obtained recommendation is validated against the outcomes of robustness analysis in view of the plurality of sorting model instances compatible with the assignment examples.

The remainder of this paper is organized as follows. In Section 2, we describe the mathematical models underlying the proposed method and review the existing preference disaggregation methods that are able to handle non-monotone data. Section 3 discusses the results of its application to exposure management of engineered nanomaterials. The last section concludes and outlines avenues for future research.

2. Construction of threshold-based value-driven sorting model with partially known monotonicity constraints based on the Decision Maker's assignment examples

Let us use the following notation [23]:

- $A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\}$ – a finite set of n alternatives;
- $A^R = \{a^*, b^*, \ldots\} \subseteq A$ – a finite set of reference alternatives, which the DM accepts to critically judge in a holistic way;
- $G = \{g_1, g_2, \ldots, g_j, \ldots, g_m\}$ – a finite set of m evaluation criteria, $g_j : A \to \mathbb{R}$ for all $j \in J = \{1, \ldots, m\}$;
- $X_j = \{x_j \in \mathbb{R} : g_j(a_i) = x_j, a_i \in A\}$ – a set of all different performances on g_j, $j \in J$;
- $x_j^1, x_j^2, \ldots, x_j^{n_j(A)}$ – increasingly ordered values of X_j, $x_j^k < x_j^{k+1}, k = 1, 2, \ldots, n_j(A) - 1$, where $n_j(A) = |X_j|$ and $n_j(A) \leq n$;
- $C_1, C_2, \ldots, C_p$ - p pre-defined, preference ordered classes, where C_{h+1} is preferred to C_h, $h = 1, \ldots, p - 1$ ($H = \{1, \ldots, p\}$).

2.1. Sorting model

To comprehensively assess the quality of alternatives, we use an additive value function defined as follows [24,39]:

$$U(a_i) = \sum_{j=1}^{m} u_j(g_j(a_i)) = \sum_{j=1}^{m} u_j(a_i) \in [0, 1], \tag{1}$$

where u_j is a marginal value associated with criterion g_j, $j = 1, \ldots, m$. It is used to evaluate alternatives $a_i \in A$ from a specific point of view. Observe that in Eq. (1) and in the following with the notation $u_j(a)$ we mean $u_j(g_j(a))$. For all criteria, we use general functions with all unique performances corresponding to the characteristic points [14]. Hence, the shape of $u_j(a_i)$ is determined by $u_j(x_j^k)$, $k = 1, 2, \ldots, n_j(A)$.

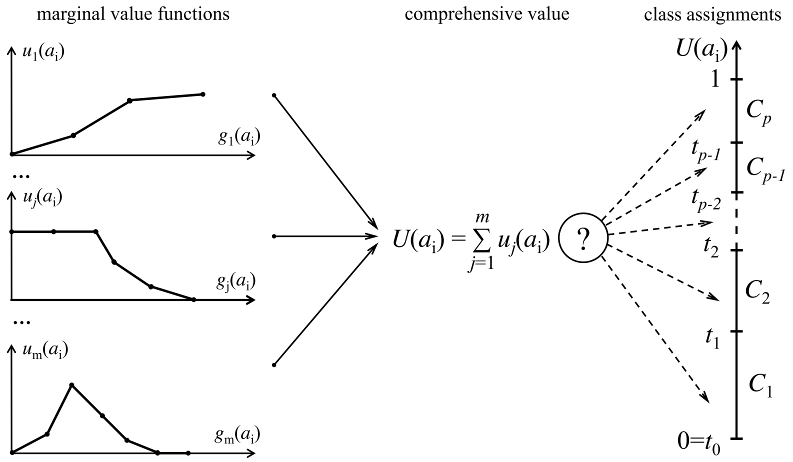


Fig. 1. Value-driven threshold-based sorting procedure.

To classify the alternatives, we use a value-driven threshold-based sorting procedure in which the boundaries between the classes are defined with a vector of thresholds $t_0, t_1, \ldots, t_h, \ldots, t_p$, such that t_{h-1} and t_h are, respectively, the lower and upper bounds on a scale of a comprehensive value for class C_h, $h = 1, \ldots, p$ [14,46]. Alternative $a_i \in A$ is assigned to class C_h in case $t_{h-1} \leq U(a_i) < t_h$. Such a procedure is presented graphically in Fig. 1. The set of constraints defining the basic assumptions of the underlying preference model is as follows:

$$\left.\begin{array}{l} U(a_i) = \sum_{j=1}^{m} u_j(a_i), \text{ for all } a_i \in A, \\ t_h - t_{h-1} \geq \varepsilon, \ h = 1, \ldots, p, \\ t_0 = 0, \ t_p \geq 1 + \varepsilon, \end{array}\right\} E^{MODEL} \quad (2)$$

where ε is an arbitrarily small positive value.

In the following subsections, we discuss constraints that reconstruct the DM's preference information and define a set of compatible value functions. We also present the mathematical models for both selection of a single representative sorting model as well as robustness analysis whose results are quantified by means of possible assignments.

2.2. Preference information

The parameters of an assumed sorting model are inferred indirectly from the DM's assignment examples specifying for each reference alternative $a_i^* \in A^R$ its desired class $C_{DM(a_i^*)}$ (e.g., alternative a_1^* should be assigned to class C_2, whereas alternative a_2^* should be sorted into class C_4) [14,26]. The assignment examples are translated to the following constraints:

$$\left.\begin{array}{l} \text{for all } a_i^* \in A^R: \\ U(a_i^*) \geq t_{DM(a_i^*)-1}, \\ U(a_i^*) + \varepsilon \leq t_{DM(a_i^*)}. \end{array}\right\} E^{ASS-EX} \quad (3)$$

Thus, a comprehensive value of a reference alternative assigned to C_{DM} should be within the bounds associated with this class.

2.3. Compatible sorting model instances

In the proposed approach, we consider a wide spectrum of types of monotonic and non-monotonic marginal value functions within a preference disaggregation framework. These types include standard monotonic shapes, level-monotonic functions, A- and V-types combining increasing and decreasing value trends, and unknown monotonicity constraints.

The existing preference disaggregation methods that are able to handle non-monotone data can be classified into different streams. Firstly, one has proposed to use some specific forms of non-monotonicity or pre-defined shapes of non-monotonic marginal value functions. In this regard, Despotis and Zopounidis [6] and Guo et al. [17] considered the criteria with some mid-point corresponding to the most preferred performance, whereas Rezaei [34] accounted for a rich spectrum of precisely specified shapes including, e.g., A- or V-type functions. Secondly, some more general algorithms have been devised to avoid dealing solely with some specific form of non-monotonicity. In particular, Doumpos [7] used a differential evolution algorithm and Ghaderi et al. [10] introduced a mathematical programming model for constructing non-monotonic functions, while not directly restraining the model's complexity. The last group of methods aimed at disaggregating holistic judgments while not making any assumptions on the shape of marginal value functions, but controlling their complexity. In this regard, Kliegr [25] penalized the changes of non-monotonicity in the shape of marginal functions using MILP models, whereas Ghaderi et al. [11] and Liu et al. [29] considered minimization of the variation in slope with, respectively, Linear Programming (LP) techniques or a quadratic optimization problem.

In what follows, we discuss constraints that define the shape of marginal value functions depending on the desired types of (non-)monotonicity, and normalize comprehensive values within the [0, 1] range.

Shape of marginal value functions. For each criterion g_j, $j = 1, \ldots, m$, the DM is expected to define the respective requirements on monotonicity of marginal values which are assigned to the respective performances $x_j^1, x_j^2, \ldots, x_j^{n_j(A)}$. These are implied by the type associated with a given criterion. We consider the following types: gain, cost, monotonic non-defined, A, V, increase-level, decrease-level, level-increase, level-decrease, and non-monotonic. In what follows, we explain their meaning and discuss the respective constraints. Whichever the criterion's type, we require all marginal values to be non-negative:

$$\left. u_j(x_j^k) \geq 0,\ j = 1, ..., m, k = 1, ..., n_j(A). \right\} E^{NON-NEG}$$

The set of constraints involving $E^{NON-NEG}$ as well as the constraints related to the type of (non-)monotonicity for all criteria will be denoted by E^{MON}.

- *Gain type* means that the greater $g_j(a_i)$, the more preferred alternative a_i on criterion g_j, thus implying the non-decreasing trend for the marginal values with the increase in $g_j(a_i)$ (see Fig. 2a):

$$\left. u_j(x_j^k) \geq u_j(x_j^{k-1}),\ k = 2, ..., n_j(A). \right\} E^{MON}_{GAIN}$$

- *Cost type* implies that the greater $g_j(a_i)$, the less preferred alternative a_i on criterion g_j, thus implying the non-increasing trend for the marginal values with the increase in $g_j(a_i)$ (see Fig. 2b):

$$\left. u_j(x_j^k) \leq u_j(x_j^{k-1}),\ k = 2, ..., n_j(A). \right\} E^{MON}_{COST}$$

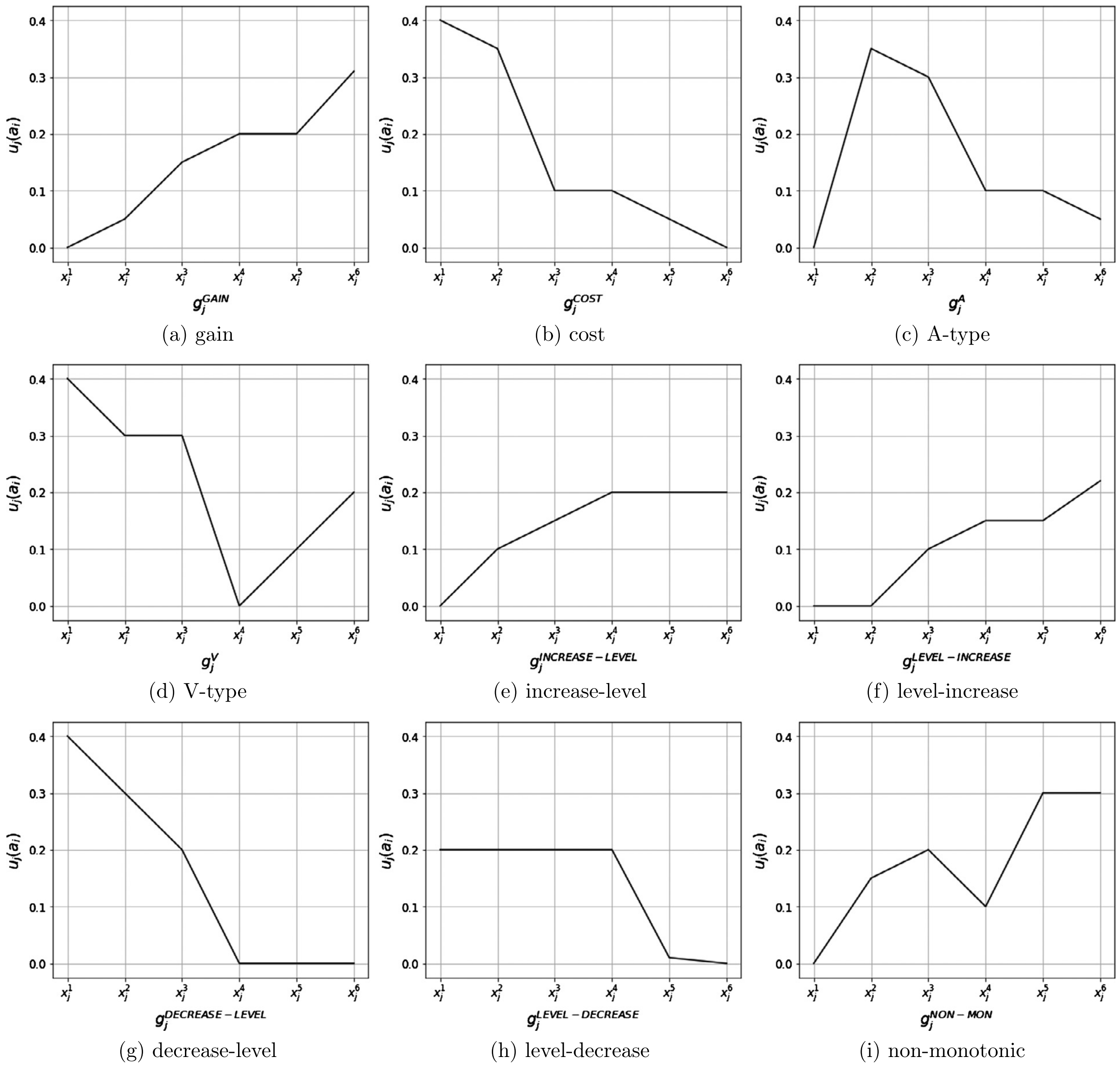
- *Monotonic non-defined type* implies that the preference on g_j adheres to the monotonicity constraints, but whether it is of gain or cost type cannot be specified a priori:

$$\left.\begin{array}{l} u_j(x_j^k) = u_j^{\uparrow}(x_j^k) + u_j^{\downarrow}(x_j^k),\ k = 1, ..., n_j(A), \\ u_j^{\uparrow}(x_j^k) \geq u_j^{\uparrow}(x_j^{k-1}),\ k = 2, ..., n_j(A), \\ u_j^{\downarrow}(x_j^k) \leq u_j^{\downarrow}(x_j^{k-1}),\ k = 2, ..., n_j(A), \\ u_j^{\uparrow}(x_j^k), u_j^{\downarrow}(x_j^k) \geq 0,\ k = 1, ..., n_j(A), \\ u_j^{\uparrow}(x_j^{n_j(A)}) \leq M \cdot (1 - v^{mon}_{j,cost}), \\ u_j^{\downarrow}(x_j^1) \leq M \cdot v^{mon}_{j,cost}, \\ v^{mon}_{j,cost} \in \{0, 1\}, \end{array}\right\} E^{MON}_{NON-DEF}$$

  where M is an arbitrarily large positive constant. The marginal value function u_j is modeled as a sum of values derived from the assumption that g_j is either of gain ($u_j^{\uparrow}$) or cost ($u_j^{\downarrow}$) type. However, only one of them can be activated with the binary variable $v^{mon}_{j,cost}$. Specifically, if $v^{mon}_{j,cost} = 1$, g_j is of cost type. Then, $u_j^{\uparrow}(x_j^{n_j(A)}) = 0$ and, thus, all marginal values $u_j^{\uparrow}(\cdot)$ are equal to 0. Otherwise, $u_j^{\downarrow}(x_j^1) = 0$ and, thus, all marginal values $u_j^{\downarrow}(\cdot)$ are equal to 0. This, in turn, implies that g_j is of gain type.

  When modelling marginal value functions for the criteria of gain, cost, or monotonic non-defined types, we required that monotonicity is non-strict. This admits marginal values assigned to a pair of performances x_j^{k-1} and x_j^k for $k = 2, \ldots, n_j(A)$, to be equal. In case the DM would expect the marginal function to be strictly monotonic, the respective weak inequalities should be replaced with their strict counterparts involving ε. For example, for gain-type criteria, constraint $u_j(x_j^k) \geq u_j(x_j^{k-1})$ contained in E^{MON}_{GAIN} should be replaced with $u_j(x_j^k) \geq u_j(x_j^{k-1}) + \varepsilon$.

- *A-type* means that the most preferred performance potentially does not align with any extreme performance, hence admitting at most one change of monotonicity from non-decreasing to non-increasing (see Fig. 2c):



(a) gain (b) cost (c) A-type

(d) V-type (e) increase-level (f) level-increase

(g) decrease-level (h) level-decrease (i) non-monotonic

Fig. 2. Example marginal value functions representing different types of requirements with respect to their monotonicity.

$$\left.\begin{array}{l} M \cdot \sum_{p=2}^{k} v_{j,p}^{opt} + u_j(x_j^k) \geq u_j(x_j^{k-1}),\ k = 2, ..., n_j(A), \\ u_j(x_j^k) \leq u_j(x_j^{k-1}) + M \cdot (1 - \sum_{p=2}^{k} v_{j,p}^{opt}),\ k = 2, ..., n_j(A). \\ \sum_{p=2}^{n_j(A)} v_{j,p}^{opt} \leq 1, \\ v_{j,p}^{opt} \in \{0, 1\},\ p = 2, ..., n_j(A). \end{array}\right\} E_A^{MON}$$

Note that $v_{j,p}^{opt}$ is allowed to be 1 for at most one $p \in \{2, \ldots, n_j(A)\}$. If $v_{j,p}^{opt} = 1$, then the following constraints hold:

$$\left.\begin{array}{l} u_j(x_j^k) \geq u_j(x_j^{k-1}),\ \text{if } p \geq 3,\ k = 2, ..., p-1, \\ u_j(x_j^k) \leq u_j(x_j^{k-1}),\ k = p, ..., n_j(A). \end{array}\right\}$$

Thus, if $v_{j,2}^{opt} = 1$, u_j is non-increasing (i.e., g_j is of cost type); if $v_{j,p}^{opt} = 1$, for $3 \leq p \leq n_j(A)$, then u_j is of pure A-type, whereas $v_{j,p}^{opt} = 0$ for $p \in \{2, \ldots, n_j(A)\}$ implies that u_j is non-decreasing (i.e., g_j is of gain type).

- *V-type* means that the least preferred performance potentially does not align with any of the extreme performances, hence admitting at most one change of monotonicity from non-increasing to non-decreasing (see Fig. 2d):

$$\left.\begin{array}{l} u_j(x_j^k) \leq u_j(x_j^{k-1}) + M \cdot \sum_{p=2}^{k} v_{j,p}^{opt},\ k = 2, \ldots, n_j(A). \\ M \cdot (1 - \sum_{p=2}^{k} v_{j,p}^{opt}) + u_j(x_j^k) \geq u_j(x_j^{k-1}),\ k = 2, \ldots, n_j(A), \\ \sum_{p=2}^{n_j(A)} v_{j,p}^{opt} \leq 1, \\ v_{j,p}^{opt} \in \{0, 1\}, p = 2, \ldots, n_j(A). \end{array}\right\} E_V^{MON}$$

The role of binary variable $v_{j,p}^{opt}$ is analogous to the case of A-type function.

- *Increase-level type* implies that u_j is non-decreasing up to a certain (though not indicated a priori) performance and then reaches saturation, hence remaining constant from this point up to the greatest performance (see Fig. 2e). This type of function can be enforced by putting together the requirements for A- and gain-type functions, i.e.:

$$E_A^{MON},\ E_{GAIN}^{MON}.\ \}\ E_{INC-LEV}^{MON}$$

- *Level-increase type* implies that u_j is constant up to a certain performance (thus, assigning zero to the respective marginal values), and non-decreasing in the range between this point and the greatest performance (see Fig. 2f). This type of function can be enforced by putting together the requirements for V- and gain-type functions, i.e.:

$$E_V^{MON},\ E_{GAIN}^{MON}.\ \}\ E_{LEV-INC}^{MON}$$

- *Decrease-level type* implies that u_j is non-increasing up to a certain performance and then assigns zero to marginal values corresponding to all remaining performances (see Fig. 2g), i.e.:

$$E_V^{MON},\ E_{COST}^{MON}.\ \}\ E_{DEC-LEV}^{MON}$$

- *Level-decrease type* implies that u_j is constant up to a certain performance (thus, assigning the maximal value to the respective marginal values), and non-increasing in the range between this point and the greatest performance (see Fig. 2h), i.e.:

$$E_A^{MON},\ E_{COST}^{MON}.\ \}\ E_{LEV-DEC}^{MON}$$

- *Non-monotonic type* means that there is no prior information on the monotonicity of criterion g_j (see Fig. 2i). In general, it would be possible to avoid defining any constraints for such functions, but since we aim at controlling the complexity of the inferred marginal value functions, we will include the following constraint set which captures the number of changes in monotonicity between the neighboring performance sub-intervals:

$$\left.\begin{array}{l} M \cdot (1 - v_{j,mon-dir}^{k,k-1}) + u_j(x_j^k) \geq u_j(x_j^{k-1}),\ k = 2, \ldots, n_j(A), \\ u_j(x_j^k) \leq u_j(x_j^{k-1}) + M \cdot v_{j,mon-dir}^{k,k-1}\ k = 2, \ldots, n_j(A), \\ v_{j,mon-dir}^{k,k-1} - v_{j,mon-dir}^{k-1,k-2} + M \cdot v_{j,change-mon}^{k,k-2} \geq 0,\ k = 3, \ldots, n_j(A), \\ v_{j,mon-dir}^{k,k-1} - v_{j,mon-dir}^{k-1,k-2} - M \cdot v_{j,change-mon}^{k,k-2} \leq 0,\ k = 3, \ldots, n_j(A), \\ v_{j,mon-dir}^{k,k-1} \in \{0, 1\}, k = 2, \ldots, n_j(A), \\ v_{j,change-mon}^{k,k-2} \in \{0, 1\}, k = 3, \ldots, n_j(A). \end{array}\right\} E_{NON-MON}^{MON}$$

If $u_j(x_j^k) \geq u_j(x_j^{k-1})$, then $v_{j,mon-dir}^{k,k-1} = 1$ and u_j is non-decreasing between characteristic points x_j^{k-1} and x_j^k. If $u_j(x_j^k) \leq u_j(x_j^{k-1})$, then $v_{j,mon-dir}^{k,k-1} = 0$ and u_j is non-increasing between characteristic points x_j^{k-1} and x_j^k. If there is a change in the monotonicity direction of u_j between three characteristic points x_j^{k-2}, x_j^{k-1}, and x_j^k (i.e., either $v_{j,mon-dir}^{k,k-1} = 1$ and $v_{j,mon-dir}^{k-1,k-2} = 0$, or $v_{j,mon-dir}^{k,k-1} = 0$ and $v_{j,mon-dir}^{k-1,k-2} = 1$), then $v_{j,change-mon}^{k,k-2} = 1$. Otherwise (i.e., either $v_{j,mon-dir}^{k,k-1} = 1$ and $v_{j,mon-dir}^{k-1,k-2} = 1$, or $v_{j,mon-dir}^{k,k-1} = 0$ and $v_{j,mon-dir}^{k-1,k-2} = 0$), $v_{j,change-mon}^{k,k-2} = 0$ and there is no change in the monotonicity direction of u_j between x_j^{k-2} and x_j^k. Thus, the sum of $v_{j,change-mon}^{k,k-2} \in \{0, 1\}$, for $k = 3, \ldots, n_j(A)$, represents the number of changes in the monotonicity of u_j.

Normalization. For the sake of interpretability, an additive value function is normalized to the [0, 1] interval. This is attained by means of two types of constraints. On the one hand, the marginal values of the least preferred performances on all criteria need to be zero. In this way, a comprehensive value of an anti-ideal alternative is also equal to zero. On the other hand, the marginal values assigned to the most preferred performances on all criteria need to sum up to one, i.e.:

$$\left.\sum_{j=1}^{m} u_j^{best} = 1, \right\} E^{NORM-1}$$

where u_j^{best} is the greatest marginal value for criterion g_j, $j = 1, ..., m$. The set of constraints involving E^{NORM-1} as well as dedicated normalization constraints related to the type of (non-)monotonicity for all criteria will be denoted by E^{NORM}. The respective constraints which allow to identify the least and the most preferred performances which are assigned, respectively, zero and a maximal marginal value are discussed individually for each criterion type:

- For gain, increase-level, and level-increase criteria, the least performance is assigned a marginal value of zero, i.e.:

  $$\left. u_j(x_j^1) = 0, \right\} E_{GAIN}^{NORM-0}$$

  whereas the greatest performance is the most preferred one, i.e.:

  $$\left. u_j(x_j^{n_j(A)}) = u_j^{best}. \right\} E_{GAIN}^{NORM-1}$$

- For cost, decrease-level, and level-decrease criteria, the greatest performance is assigned a marginal value of zero, i.e.:

  $$\left. u_j(x_j^{n_j(A)}) = 0, \right\} E_{COST}^{NORM-0}$$

  whereas the least performance is the most preferred one, i.e.:

  $$\left. u_j(x_j^1) = u_j^{best}. \right\} E_{COST}^{NORM-1}$$

- For monotonic criteria with non-defined type of monotonicity, the less preferred performance is either the least (if $v_{j,cost}^{mon} = 0$) or the greatest one (if $v_{j,cost}^{mon} = 1$), i.e.:

  $$\left.\begin{array}{l} u_j^{\downarrow}(x_j^{n_j(A)}) \leq M \cdot (1 - v_{j,cost}^{mon}), \\ u_j^{\uparrow}(x_j^1) \leq M \cdot v_{j,cost}^{mon}, \end{array}\right\} E_{NON-DEF}^{NORM-0}$$

  whereas the most preferred performance is either the greatest (i.e., $v_{j,cost}^{mon} = 0$) or the least one (if $v_{j,cost}^{mon} = 1$), i.e.:

  $$\left.\begin{array}{l} u_j^{best} - u_j^{\downarrow}(x_j^1) \geq -M \cdot (1 - v_{j,cost}^{mon}), \\ u_j^{best} - u_j^{\downarrow}(x_j^1) \leq M \cdot (1 - v_{j,cost}^{mon}), \\ u_j^{best} - u_j^{\uparrow}(x_j^{n_j(A)}) \geq -M \cdot v_{j,cost}^{mon}, \\ u_j^{best} - u_j^{\uparrow}(x_j^{n_j(A)}) \leq M \cdot v_{j,cost}^{mon}. \end{array}\right\} E_{NON-DEF}^{NORM-1}$$

- For A-type criteria, the least preferred performance is either x_j^1 (if $v_{j,norm-0} = 0$) or $x_j^{n_j(A)}$ (if $v_{j,norm-0} = 1$), i.e.:

  $$\left.\begin{array}{l} u_j(x_j^1) \leq u_j(x_j^{n_j(A)}) + M \cdot v_{j,norm-0}, \\ M \cdot (1 - v_{j,norm-0}) + u_j(x_j^1) + \varepsilon \geq u_j(x_j^{n_j(A)}), \\ u_j(x_j^1) \leq M \cdot v_{j,norm-0}, \\ u_j(x_j^{n_j(A)}) \leq M \cdot (1 - v_{j,norm-0}), \\ v_{j,norm-0} \in \{0, 1\}, \end{array}\right\} E_{A}^{NORM-0}$$

  whereas the most preferred performance is either x_j^{k-1} (if $v_{j,k}^{opt} = 1$ for some $k = 2, ..., n_j(A)$) or $x_j^{n_j(A)}$ (if $v_{j,k}^{opt} = 0$ for all $k = 2, ..., n_j(A)$), i.e.:

  $$\left.\begin{array}{l} u_j^{best} - u_j(x_j^{k-1}) \geq -M \cdot (1 - v_{j,k}^{opt}),\ k = 2, ..., n_j(A), \\ u_j^{best} - u_j(x_j^{k-1}) \leq M \cdot (1 - v_{j,k}^{opt}),\ k = 2, ..., n_j(A), \\ u_j^{best} - u_j(x_j^{n_j(A)}) \geq -\sum_{k=2}^{n_j(A)} v_{j,k}^{opt}, \\ u_j^{best} - u_j(x_j^{n_j(A)}) \leq \sum_{k=2}^{n_j(A)} v_{j,k}^{opt}. \end{array}\right\} E_{A}^{NORM-1}$$

- For V-type criteria, the less preferred performance is either x_j^{k-1} (if $v_{j,k}^{opt}=1$ for some $k=2,...,n_j(A)$) or $x_j^{n_j(A)}$ (if $v_{j,k}^{opt}=0$ for all $k=2,...,n_j(A)$):

$$\left.\begin{array}{l} u_j(x_j^{k-1}) \le 1 - v_{j,k}^{opt},\ k=2,...,n_j(A), \\ u_j(x_j^{n_j(A)}) \le \sum_{k=2}^{n_j(A)} v_{j,k}^{opt}, \end{array}\right\} E_V^{NORM-0}$$

whereas the most preferred performance is either x_j^1 (if $v_{j,norm-1}=1$) or $x_j^{n_j(A)}$ (if $v_{j,norm-0}=0$), i.e.:

$$\left.\begin{array}{l} u_j(x_j^1) \le u_j(x_j^{n_j(A)}) + M \cdot v_{j,norm-1}, \\ M \cdot (1 - v_{j,norm-1}) + u_j(x_j^1) \ge u_j(x_j^{n_j(A)}), \\ u_j^{best} - u_j(x_j^{n_j(A)}) \le M \cdot v_{j,norm-1}, \\ u_j^{best} - u_j(x_j^{n_j(A)}) \ge -M \cdot v_{j,norm-1}, \\ u_j^{best} - u_j(x_j^1) \le -M \cdot (1 - v_{j,norm-1}), \\ u_j^{best} - u_j(x_j^1) \ge M \cdot (1 - v_{j,norm-1}), \\ v_{j,norm-1} \in \{0,1\}. \end{array}\right\} E_V^{NORM-0}$$

- For non-monotonic criteria $u_j(x_j^k)$ needs to be equal to 0 for at least one characteristic point x_j^k, $k=1,...,n_j(A)$, such that $v_{j,norm-0}^k=1$:

$$\left.\begin{array}{l} u_j(x_j^k) - M \cdot (1 - v_{j,norm-0}^k) \le 0,\ k=1,...,n_j(A), \\ \sum_{k=1}^{n_j(A)} v_{j,norm-0}^k \ge 1,\ k=1,...,n_j(A), \\ v_{j,norm-0}^k \in \{0,1\},\ k=1,...,n_j(A). \end{array}\right\} E_{NON-MON}^{NORM-0}$$

Similarly, the maximal marginal value needs to be assigned to at least one characteristic point x_j^k, $k=1,...,n_j(A)$, such that $v_{j,norm-1}^k=1$:

$$\left.\begin{array}{l} \text{for } k=1,...,n_j(A): \\ \quad u_j(x_j^k) \ge u_j(x_j^i) - M \cdot (1 - v_{j,norm-1}^k),\ i=1,...,k-1,k+1,...,n_j(A), \\ \quad u_j^{best} - u_j(x_j^k) \le M \cdot v_{j,norm-1}^k, \\ \quad u_j^{best} - u_j(x_j^k) \ge -M \cdot v_{j,norm-1}^k, \\ \sum_{k=1}^{n_j(A)} v_{j,norm-1}^k \ge 1, \\ v_{j,norm-1}^k \in \{0,1\},\ k=1,...,n_j(A). \end{array}\right\} E_{NON-MON}^{NORM-1}$$

Overall, a set of sorting model instances (i.e., additive value functions and class thresholds) compatible with the DM's assignment examples and requirements on the (non-)monotonicity of particular criteria can be defined as follows:

$$E^{A^R} = E^{MODEL} \cup E^{ASS-EX} \cup E^{MON} \cup E^{NORM}.$$

2.4. Sorting recommendation

In this section, we discuss two complementary ways of exploiting a set of compatible sorting model instances. Arbitrary selection of a single representative instance leads to precise assignments for all alternatives, whereas robustness analysis reveals all possible sorting recommendations that follow the DM's preference information and the use of an assumed preference model.

2.4.1. Selection of a single representative sorting model

To select a representative sorting model, we minimize the number of changes in monotonicity for all marginal value functions u_j, $j=1,...,m$, by solving the following optimization problem:

$$Minimize: NM = \sum_{j \in G_A \cup G_V} \sum_{p=2}^{n_j(A)} v_{j,p}^{opt} + \sum_{j \in G_{NON-MON}} \sum_{k=3}^{n_j(A)} v_{j,change-mon}^{k,k-2},\ \text{s.t. } E^{A^R},$$

where G_A and G_V are subsets of, respectively, A- and V-type criteria admitting at most one change in monotonicity (their number is represented by $\sum_{p=2}^{n_j(A)} v_{j,p}^{opt}$), and $G_{NON-MON}$ is a subset of criteria for which no monotonicity requirements have been specified (in this case, the number of changes in monotonicity is captured by $\sum_{k=3}^{n_j(A)} v_{j,change-mon}^{k,k-2}$). Let us denote the minimal number of such changes by NM^*.

Note that the above objective function is applicable only when at least one criterion is of A-, V- or non-monotonic type. Otherwise, there are no changes in monotonicity for any marginal value function and hence NM is equal to zero. Then, a standard approach to derive a representative sorting model consists in treating ε contained in E^{A^R} as a variable and solving the following problem:

$$Minimize : \varepsilon, \text{ s.t. } E^{A^R}.$$

2.4.2. Robustness analysis

Solving the problems presented in Section 2.4.1 leads to a selection of some arbitrary marginal value functions and class thresholds compatible with the DM's partial preference information. Its analysis is beneficial in terms of providing precise recommendation along with information on the importance of particular criteria, trade-offs between criteria, or distribution of class thresholds [15]. However, in view of the incompleteness of DM's preferences, there exist multiple compatible instances of the sorting model whose recommendation for the non-reference alternatives may be different. To verify the stability of sorting recommendation, we refer to the concept of *possible assignment*, which indicates a set of classes to which a given alternative can be assigned by at least one compatible instance of the sorting model [14,22]. The validity of such an assignment for alternative $a \in A$ and class C_h, $h = 1, \dots, p$, can be verified by considering the following set of constraints, which exploits a set of models with the minimal number of changes in monotonicity for all marginal value functions:

$$\left.\begin{array}{l} E^{A^R}, \\ NM^* = \sum_{j \in G_A \cup G_V} \sum_{p=2}^{n_j(A)} v_{j,p}^{opt} + \sum_{j \in G_{NON-MON}} \sum_{k=3}^{n_j(A)} v_{j,change-mon}^{k,k-2}, \\ U(a) \geq t_{h-1},\ U(a) + \varepsilon \leq t_h. \end{array}\right\} E(a \rightarrow^P C_h)$$

If $E(a \rightarrow^P C_h)$ is feasible and $\varepsilon^* =$ max ε, s.t. $E(a \rightarrow^P C_h)$ is greater than 0, a can be possibly assigned to C_h. In case $E(a \rightarrow^P C_h)$ is infeasible or $\varepsilon^* \leq 0$, a cannot be assigned to C_h with any compatible instance of the sorting model. The set of all classes to which a can be possibly assigned is denoted by $C_P(a)$. In case $C_P(a)$ is a singleton, a is assigned to a class contained in $C_P(a)$ by all compatible instances of the sorting model. Such an assignment can be deemed as robust or necessary.

Note that the possible assignment $C_P(a)$ for each alternative $a \in A$ is a union of intervals, one for each possible type of function. However, since such a union cannot be ensured to be an interval on its own, we cannot guarantee "the no jump property" for the possible assignments [14]. Therefore, in what follows, all possible assignments are represented as sets of classes (e.g., $C_P(a_{35}) = \{C_3, C_4, C_5\}$) rather than intervals (e.g., $C_P(a_{35}) = [C_3, C_5]$). In what follows, we provide a detailed discussion on "the no jump property" in the context of the method introduced in this paper.

Let us denote by $\mathcal{U}$ a set of all possible value functions, by $\mathcal{T}$ – a set of all possible thresholds vectors and by $\mathcal{V}$ – a set of all possible binary vectors. Now, let us denote by $\mathcal{P} \subseteq \mathcal{U} \times \mathcal{T} \times \mathcal{V}$ a set of all triples $(U, \mathbf{b}, \mathbf{v})$ satisfying constraints in E^{A^R}, that is, all models (value functions, vectors of thresholds, binary vectors) compatible with the preference information provided by the DM.

Let us suppose $(U_1, \mathbf{t}_1, \mathbf{v}_1), (U_2, \mathbf{t}_2, \mathbf{v}_2) \in \mathcal{P}$ and that a is assigned to C_h w.r.t. $(U_1, \mathbf{t}_1, \mathbf{v}_1)$, while a is assigned to C_k w.r.t. $(U_2, \mathbf{t}_2, \mathbf{v_2})$, with $h, k \in [1, \dots, p]$ such that $h > k + 1$.

We have to distinguish two cases:

1) $\mathbf{v}_1 = \mathbf{v}_2$: in all criteria, the two functions U_1 and U_2 present the shape and the monotonicity changes exactly in the same characteristic points;
2) $\mathbf{v}_1 \neq \mathbf{v}_2$: in at least one criterion, the two functions U_1 and U_2 have a different shape or, they are of the same shape but the monotonicity changes in different characteristic points.

Let us prove that for all $l \in]h, k[$, there exists $(U, \mathbf{b}, \mathbf{v}) \in \mathcal{P}$ such that a is assigned to C_l w.r.t. $(U, \mathbf{b}, \mathbf{v})$.

Proposition 1. *Let $a \in A$, $(U_1, \mathbf{t}_1, \mathbf{v}_1), (U_2, \mathbf{t}_2, \mathbf{v}_2) \in \mathcal{P}$ and $h, k \in [1, \dots, p]$, such that:*

1) *a is assigned to C_h w.r.t. $(U_1, \mathbf{t}_1, \mathbf{v}_1)$,*
2) *a is assigned to C_k w.r.t. $(U_2, \mathbf{t}_2, \mathbf{v}_2)$,*
3) *$\mathbf{v}_1 = \mathbf{v}_2$,*
4) *$h > k + 1$,*

then for all $l \in]k, h[$ there exists $(U, \mathbf{b}, \mathbf{v_1}) \in \mathcal{P}$ such that a is assigned to C_l w.r.t. $(U, \mathbf{b}, \mathbf{v_1})$.

Proof. The first two hypotheses are equivalent to the following:

$$t_{1,h-1} \leqslant U_1(a) < t_{1,h} \qquad \text{and} \qquad t_{2,k-1} \leqslant U_2(a) < t_{2,k}.$$

Since $l \in]k, h[$ and because of the thresholds monotonicity, we have:

$$U_1(a) \geqslant t_{1,h-1} \geqslant t_{1,l} > t_{1,l-1}, \tag{4}$$

and

$$t_{2,l} > t_{2,l-1} \geqslant t_{2,k} > U_2(a). \tag{5}$$

Let $\alpha \in]0, 1[$ and let us define the corresponding convex combinations of $t_{1,l}$ and $t_{1,l-1}$ on one hand and of $t_{2,l}$ and $t_{2,l-1}$ on the other hand, that is,

$$t_{1\alpha l} = \alpha t_{1,l} + (1-\alpha)t_{1,l-1}$$

and

$$t_{2\alpha l} = \alpha t_{2,l} + (1-\alpha)t_{2,l-1}.$$

Let us consider the triple $(\lambda U_1 + (1-\lambda)\, U_2, \lambda \mathbf{t}_1 + (1-\lambda)\, \mathbf{t}_2, \mathbf{v_1})$ with $\lambda \in \mathbb{R}$ such that

$$\lambda U_1(a) + (1-\lambda)\, U_2(a) = \lambda t_{1\alpha l} + (1-\lambda)\, t_{2\alpha l}$$

from which

$$\lambda = \frac{t_{2\alpha l} - U_2(a)}{(U_1(a) - t_{1\alpha l}) + (t_{2\alpha l} - U_2(a))}.$$

Observing that $t_{1\alpha l} \in]t_{1,l-1}, t_{1,l}[$ and $t_{2\alpha l} \in]t_{2,l-1}, t_{2,l}[$, by Eqs. (4) and (5), we get

$$U_1(a) > t_{1\alpha l}$$

and

$$t_{2\alpha l} > U_2(a),$$

from which we get $\lambda \in]0, 1[$. Consequently, since any subset of $\mathcal{P}$ containing all the triples $(U, \mathbf{b}, \mathbf{v})$ with $\mathbf{v} = \overline{\mathbf{v}}$ for some $\overline{\mathbf{v}}$ is convex, then $(\lambda U_1 + (1-\lambda)\, U_2, \lambda \mathbf{t}_1 + (1-\lambda)\, \mathbf{t}_2, \mathbf{v_1}) \in \mathcal{P}$.

Observing that:

- the component $l-1$ of the vector $\lambda \mathbf{t}_1 + (1-\lambda)\, \mathbf{t}_2$ is $\lambda t_{1,l-1} + (1-\lambda)\, t_{2,l-1}$,
- the component l of the vector $\lambda \mathbf{t}_1 + (1-\lambda)\, \mathbf{t}_2$ is $\lambda t_{1,l} + (1-\lambda)\, t_{2,l}$,
- $\lambda U_1(a) + (1-\lambda)\, U_2(a) = \lambda t_{1\alpha l} + (1-\lambda)\, t_{2\alpha l} = \alpha \left(\lambda t_{1,l} + (1-\lambda) t_{2,l}\right) + (1-\alpha)(\lambda t_{1,l-1} + (1-\lambda) t_{2,l-1})$,
- $\alpha \in]0, 1[$,

then

$$\lambda t_{1,l-1} + (1-\lambda)\, t_{2,l-1} \leqslant \lambda U_1(a) + (1-\lambda)\, U_2(a) < \lambda t_{1,l} + (1-\lambda)\, t_{2,l}$$

implying that a is assigned to C_l w.r.t. $(\lambda U_1 + (1-\lambda)\, U_2, \lambda \mathbf{t}_1 + (1-\lambda)\, \mathbf{t}_2, \mathbf{v_1})$. □

Corollary 1. *If for all $(U, \mathbf{t}, \mathbf{v}) \in \mathcal{P}$, $\mathbf{v} = \overline{\mathbf{v}}$, then for all $a \in A$, $C_P(a)$ is an interval of classes without any jump, that is*

$$C_P(a) = \{C_{L(a)}, C_{L(a)+1}, \ldots, C_{R(a)}\}$$

where

$$L(a) = \min\{h : C_h \in C_P(a)\},$$
$$R(a) = \max\{h : C_h \in C_P(a)\}.$$

Let us denote by $C_P^{\mathcal{Q}}(a)$ the set of possible classes to which a can be assigned by at least one triple $(U, \mathbf{t}, \mathbf{v})$ in $\mathcal{Q} \subseteq \mathcal{P}$. In particular, $C_P(a) = C_P^{\mathcal{P}}(a)$. The following holds:

Corollary 2. *Let $\mathcal{P} = \mathcal{P}_1 \cup \mathcal{P}_2 \cup \ldots \cup \mathcal{P}_r$ where, for all $i = 1, \ldots, r$, for all $(U, \mathbf{b}, \mathbf{v}) \in \mathcal{P}_i$, $\mathbf{v} = \overline{\mathbf{v}}_i$; then:*

- *$C_P^{\mathcal{P}_1}(a), C_P^{\mathcal{P}_2}(a), \ldots, C_P^{\mathcal{P}_r}(a)$, are intervals without jumps and $C_P(a) = C_P^{\mathcal{P}_1}(a) \cup C_P^{\mathcal{P}_2}(a) \cup \ldots \cup C_P^{\mathcal{P}_r}(a)$,*
- *if $\overline{\mathbf{v}}_1 = \overline{\mathbf{v}}_2 = \ldots = \overline{\mathbf{v}}_r$, then $C_P(a)$ is an interval without jumps.*

Since, in general, a union of intervals is not necessarily an interval, if in the previous corollary, $\overline{\mathbf{v}}_i \neq \overline{\mathbf{v}}_j$ for some $i, j \in \{1, \ldots, r\}$, nothing can be said about the presence or absence of jumps in $C_P(a)$.

3. Application to exposure management of engineered nanomaterials

Engineered nanomaterials (ENMs) are materials with at least one dimension in the range of 1-100 nanometers, though larger ones are usually included in this definition. One distinctive feature of these materials is that their physicochemical properties are significantly different from materials of larger sizes. This makes them suitable for the development of products with enhanced performances in several areas including construction, electronics, environmental management and healthcare [35,44,45]. As a result, the number of workers exposed to such materials is rising [12]. Even though ENMs enable the development of high performance products, there is a lot discussion and concern about their potential impacts on human health and the environment [9]. This motivates the development of risk assessment and management strategies to handle the risks that ENMs can cause. Risk assessment is the tool that has been advanced to assess and manage risks of ENMs and it is composed of a hazard and an exposure assessment part. In this paper, we focus on the latter and contribute to the development of decision support systems to manage exposure to ENMs manufacturing by recommending RMMs [42].

3.1. Decision classes, criteria, and alternatives

We present the model for assessing the need of using a risk management measure (i.e., a respirator) by workers exposed to nanomaterials during their manufacturing. We incorporate real-world data elaborated by Naidu [32], who developed a set of exposure scenarios to ENMs and received expert recommendations on several risk management measures.

Classes. The considered problem is formulated in terms of multiple criteria sorting with five preference ordered classes referring to the requirement of precautions: C_1 (required; the least preferred class), C_2 (might be required), C_3 (optional), C_4 (might be optional), and C_5 (not required; the most preferred class).

Criteria. We consider a set of exposure scenarios to ENMs, which are characterized with ten descriptors. These criteria refer to the following characteristics of the materials and the exposure conditions:

- Particle size (g_1) evaluated on a 6-point ordinal scale. Since most studies suggest that toxicity is higher for smaller sizes [41], but does not differ for greater sizes, we assumed an increase-level marginal value function.
- Toxicity (g_2; cost type) defined on a 3-point scale (from low through medium to high) determines what type of effect the ENM has on human health [8].
- Airborne capacity (g_3; cost type) – expressed on 4-point scale from none (preferred) to high (not preferred) – characterizes the capacity of the ENMs to spread in the workplace through the air stream [18].
- Detection limit (g_4; gain type), defined on a qualitative 4-point scale (none (not preferred), low, moderate, and good (preferred)), relates to the capacity of the exposure assessment tools to identify ENMs.
- Exposure limit (g_5; cost type) indicates an assumed level of exposure among five ranges based on asbestos, which is a widely accepted reference [32].
- Quantity (g_6; cost type) refers to the quantity (in kg) of ENM handled in the scenario (lesser quantities imply smaller chance of exposure [19]).
- Number of employees (g_7) considers the number of people involved in handling of the ENMs. Due to a lack of clear indication how this number affects the exposure management, we consider it as potentially non-monotonic criterion.
- Engineering controls (g_8; non-monotonic) indicates the laboratory setting in which the manufacturing tasks are conducted among four possible combinations referring to positive (PP) or negative (NP) pressure as well as open (O) or closed (C) system.
- Duration of exposure (g_9; cost type) to the nanomaterials during the manufacturing tasks (the shorter the duration, the lesser the risk of exposure [19]).
- Multiple exposure (g_{10}; cost type) concerns the frequency of exposure [43] (unknown value is considered as the least preferred performance).

In Appendix A, we summarize the characteristics of all criteria as well as the encoding of respective performances. Overall, the considered descriptors involve both monotonic and non-monotonic criteria, which justifies an employment of the proposed methodological framework.

Alternatives. To demonstrate the framework's applicability, we consider a set of 51 exposure scenarios (for their performances, see Tables 1 and 2). In terms of MCDA, these are interpreted as decision alternatives $a_1 - a_{51}$.

Table 1
The performances of 30 reference exposure scenarios on ten criteria, their desired class assignments C_{DM}, and comprehensive values $U(a_i)$ according to a representative sorting model.

a_i	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	C_{DM}	$U(a_i)$	a_i	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	$C_{DM}(a_i)$	$U(a_i)$
a_1	5	2	0	0	2	4	1	2	3	1	2	0.3859	a_{16}	1	2	1	0	1	4	3	3	5	1	3	0.4007
a_2	5	2	3	2	4	1	2	5	5	0	1	0.3101	a_{17}	6	3	3	2	4	4	4	4	2	2	3	0.4181
a_3	4	2	2	3	5	3	4	5	5	0	3	0.4617	a_{18}	1	1	3	3	4	5	1	2	2	0	5	0.5523
a_4	1	3	3	0	3	1	2	2	4	3	1	0.1228	a_{19}	6	1	0	2	4	1	3	3	4	3	5	0.6550
a_5	3	1	1	0	4	5	1	4	1	3	3	0.4617	a_{20}	6	3	1	3	3	1	1	4	5	0	5	0.5531
a_6	1	2	1	3	3	5	3	1	3	2	3	0.4617	a_{21}	6	1	0	2	2	2	3	5	4	0	5	0.5523
a_7	4	2	0	1	5	5	1	1	5	1	3	0.4007	a_{22}	6	3	1	1	1	4	1	3	1	3	5	0.7404
a_8	1	3	0	2	4	3	4	5	3	3	3	0.4007	a_{23}	1	1	1	2	2	3	2	2	1	0	5	0.6411
a_9	2	2	3	0	1	3	2	3	5	2	1	0.2936	a_{24}	2	2	3	3	5	1	1	3	3	0	5	0.5523
a_{10}	4	1	3	3	5	4	4	1	3	3	3	0.4617	a_{25}	4	1	2	1	3	5	4	2	4	1	3	0.4617
a_{11}	3	1	1	1	4	4	3	2	5	0	5	0.5523	a_{26}	3	1	3	1	3	3	1	1	5	3	2	0.3249
a_{12}	3	1	2	1	3	1	1	5	1	1	5	0.5653	a_{27}	6	1	3	0	5	3	3	1	2	1	1	0.3101
a_{13}	3	2	1	3	4	4	2	5	4	1	5	0.5523	a_{28}	6	2	2	2	5	4	1	1	1	2	4	0.4765
a_{14}	2	1	0	3	3	2	4	4	5	1	5	0.6524	a_{29}	5	1	0	1	3	5	2	4	2	2	4	0.5375
a_{15}	1	2	3	1	3	2	4	5	1	2	3	0.4007	a_{30}	1	3	2	0	4	4	1	4	3	1	1	0.1394

Table 2
The performances of 21 non-reference exposure scenarios on ten criteria, their representative $C(a_i)$ and possible $C_P(a_i)$ assignments.

a_i	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	$C(a_i)$	$C_P(a_i)$	a_i	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	$C(a_i)$	$C_P(a_i)$
a_{31}	1	0	2	2	1	0	3	0	2	2	3	2, 3	a_{42}	4	2	2	1	1	3	3	4	4	3	2	1, 2
a_{32}	3	2	3	1	0	0	3	2	2	0	4	3, 4	a_{43}	2	1	2	1	4	0	1	1	3	2	2	1, 2
a_{33}	1	2	0	1	2	3	2	0	3	0	4	3, 4	a_{44}	2	2	0	0	1	4	3	0	1	0	3	2, 3
a_{34}	3	0	1	2	4	1	1	4	1	3	4	4, 5	a_{45}	4	2	2	3	0	1	2	0	1	3	5	4, 5
a_{35}	5	0	1	0	0	0	3	4	1	0	4	3, 4, 5	a_{46}	2	2	3	3	4	3	0	1	3	2	3	3, 4
a_{36}	5	0	2	2	1	2	0	3	3	1	3	3	a_{47}	2	2	2	1	4	1	0	3	2	0	2	1, 2
a_{37}	5	2	2	1	4	4	1	2	4	0	3	2, 3	a_{48}	2	2	3	0	1	4	3	0	3	1	1	1
a_{38}	0	0	2	3	1	0	2	2	0	0	5	5	a_{49}	0	2	0	1	1	0	0	0	1	2	3	3, 4
a_{39}	4	2	1	3	4	4	2	4	3	2	4	4, 5	a_{50}	0	1	0	0	3	1	3	2	3	1	3	2, 3
a_{40}	4	0	1	0	1	4	3	1	0	2	4	4, 5	a_{51}	5	2	3	1	3	4	2	4	2	1	1	1, 2
a_{41}	4	1	0	2	2	1	0	1	4	0	4	4, 5													

3.2. Preference information

For thirty scenarios ($a_1 - a_{30}$), we consider the expert input in form of class assignments. The respective classes capture the recommended precaution level for RMM (see Table 1). Overall, the distribution of classes in the reference judgments is as follows: C_1 – 5, C_2 – 2, C_3 – 10, C_4 – 2, and C_5 – 11. Hence, the use of a respirator has been judged as optional (C_3) or not required (C_5) in the context of the greatest number of exposure scenarios. In addition, for better discrimination between the classes the minimal difference between the neighboring thresholds has been assumed to be 0.07.

3.3. Results

3.3.1. Representative sorting model

The expert input has been used to develop a model to recommend a precaution level for workers exposed to nanomaterials. In this way, we account for the interrelations between the ten descriptors of the exposure scenarios and the recommended risk management measure. Fig. 3 exhibits the marginal value functions which can constitute a part of the representative sorting model. They indicate two changes in the monotonicity, from decreasing to increasing for g_7 (engineering controls) and from increasing to decreasing for g_8 (number of employees). For all remaining criteria, the shape of marginal function adheres to the pre-defined monotonicity constraints. Specifically, for g_1 (particle size) – it is increase-level, for g_4 (detection limit) – it is increasing, whereas for the remaining criteria – it is either strictly decreasing or non-increasing.

Although all criteria contribute to the comprehensive values, one can observe significant differences in the maximal shares of respective marginal value functions. Specifically, the greatest maximal shares correspond to detection limit (0.2682), duration of exposure (0.1776), and airborne capacity (0.1454). This confirms a substantial impact that these criteria have on the classification. On the contrary, the least maximal shares are noted for quantity (0.0209) and frequency of exposure (0.0296), thus indicating their marginal role in deciding upon the sorting of considered scenarios.

When it comes to the variation of marginal values, it also differs vastly from one criterion to another. The greatest difference of marginal values can be observed for:

- airborne capacity (g_3) when moving from moderate (2) to low (1) capacity of the nanomaterial to spread in the workplace;
- detection limit (g_4) when attaining poor (1) rather than none (0) or good (3) rather than moderate (2) capacity of the exposure assessment tool to identify the nanomaterial;
- number of employees (g_8) when moving to an intermediate level ($11 - 50$) from both lower and higher numbers;
- duration of exposure (g_9) when reducing the time from less than one hour (3) to less than 15 minutes (2) and further to incidental occurrence (1).

These differences indicate the transitions where a high gain in the reduction of precaution level can be attained. On the contrary, the least or null differences of marginal values can be observed for particles sizes (g_1) greater than 2 nm (2), at least moderate (2) toxicity (g_2), intermediate ($2 - 4$) exposure limits (g_5), quantities (g_6) less than 10 tons (4), number of employees (g_8) not less than 51 (4), and duration exposure (g_9) not less than one minute (3). Consequently, changes of performances within these ranges do not influence at all or much the comprehensive values and resulting assignments.

The comprehensive values computed according to a representative value function for the reference exposure scenarios ($a_1 - a_{30}$) are presented in Table 1. They need to be interpreted jointly with the following thresholds which set the boundaries for the ranges of comprehensive values judged as typical for particular classes:

$$t_0 = 0,\ t_1 = 0.3175,\ t_2 = 0.3933,\ t_3 = 0.4691,\ t_4 = 0.5449. \tag{6}$$

For example, all scenarios with comprehensive values not less than 0.3933 and less than 0.4691 are assigned to class C_3. Clearly, these thresholds were not pre-defined, but rather constructed by the method to reproduce – when coupled with an additive value function – all 30 assignment examples.

To support understanding of the employed threshold-based value-driven sorting procedure, Fig. 4 presents five example reference alternatives with different assignments along with their marginal values and thresholds separating the classes. Firstly, this figure demonstrates to which degree different criteria contribute to the comprehensive values of particular alternatives. Secondly, it clarifies that the assignment is derived from attaining a comprehensive value in a particular range. Thirdly, it exhibits the differences between the alternatives for which the requirement of precaution is, e.g., obligatory (a_9), optional (a_3), or not needed (a_{18}).

In this perspective, an assignment of alternative a_{18} to class C_5 ($U(a_{18}) = 0.5522 \geq t_4 = 0.5449$) was largely due to its highly preferred performances on g_2, g_4, g_8, and g_9. In particular, its best performance with respect to the detection limit (g_4) contributes already almost half of the comprehensive value needed for the assignment to the most preferred class. Furthermore, alternatives a_3 and a_{29} attained high marginal value on a subset of criteria (for a_3 – g_1, g_4, g_7, and g_{10}; for a_{29} – g_1, g_2, g_3, g_4 and g_9), but scored relatively worse on the remaining criteria (including four criteria with marginal values equal to zero), which justifies their assignment to the intermediate classes. When it comes to a_1, eight

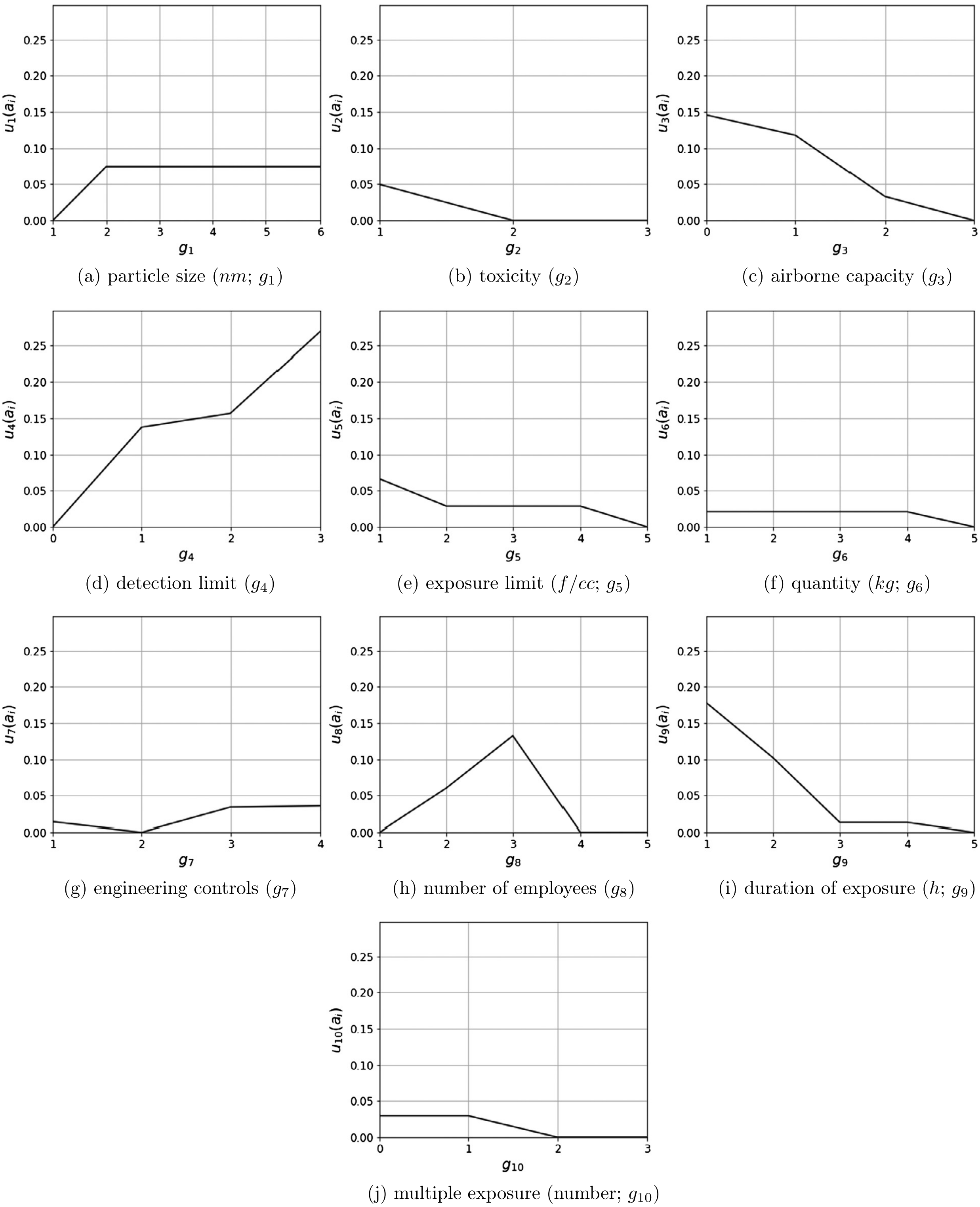


Fig. 3. Marginal value functions for the exposure management of nanomaterials in the context of using a respirator.

criteria contribute to its comprehensive quality with a marginal value greater than zero. However, only for three of them (g_1, g_3, g_8), these contributions can be viewed as relatively high. As a result, the comprehensive value of a_1 is rather low and sufficient only for granting a place in class C_2 ($t_1 = 0.3175 \leq U(a_1) = 0.3858 < t_2 = 0.3933$). Finally, alternative a_9

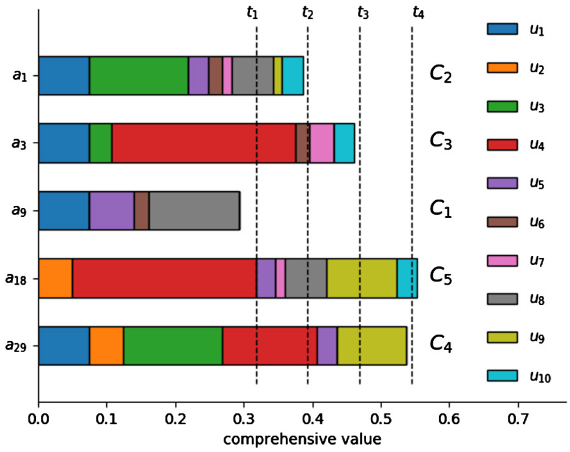
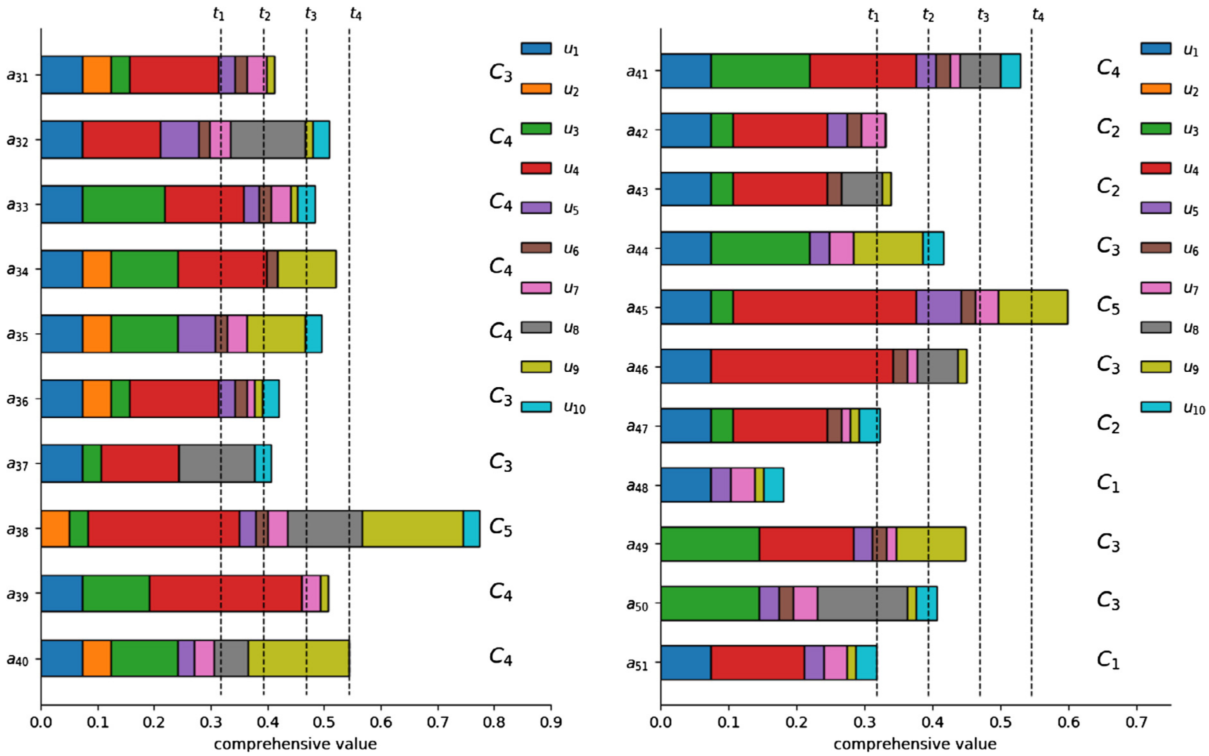


Fig. 4. Marginal and comprehensive values as well as class assignments for the five example reference exposure scenarios. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)



(a) $a_{31} - a_{41}$ (b) $a_{42} - a_{51}$

Fig. 5. Marginal and comprehensive values as well as class assignments for the 21 non-reference exposure scenarios.

attained a marginal value of zero on six criteria (g_2, g_3, g_4, g_7, g_9, and g_{10}), which implied its assignment to the least preferred class C_1.

As far as non-reference exposure scenarios (a_{31} - a_{51}) are concerned, their comprehensive values and class assignments are presented in Table 2. The explanation of these assignments is enhanced by Fig. 5, which collates the comprehensive values with the class thresholds, while additionally decomposing them into the marginal values. For the 21 non-reference alternatives, the distribution of class assignments is as follows: C_1 - 2, C_2 - 3, C_3 - 7, C_4 - 7, and C_5 - 2. Hence, the precaution level of the greatest number of exposure scenarios is judged as optional (C_3) or might be optional (C_4), whereas only a pair of alternatives is assigned to either of extreme classes.

Let us explain the classification for a few selected non-reference exposure scenarios by referring to the contribution of particular criteria to their comprehensive values. When it comes to a pair of alternatives (a_{48} and a_{51}) assigned to C_1, they perform relatively well only on the less important criteria, while not attaining any positive marginal values on g_2, g_3, g_4 (only for a_{48}), g_6, and g_8. This justifies their low comprehensive values and sorting into the least preferred class. When compared with a_{48}, a_{44} is characterized by more advantageous performances on g_3 and g_9, which is sufficient for attaining an intermediate class (C_3). As for the two alternatives (a_{38} and a_{45}) placed in C_5, 9 and 7 criteria, respectively, contribute to their comprehensive values. However, the main role in exceeding the lower threshold of the most preferred class can be attributed to: for a_{38} – g_2, g_4, g_8, and g_9, and for a_{45} – g_1, g_4, g_5, and g_9, on which they reached the maximal values. The alternatives assigned to the least preferred classes either lack any contribution from the significant share of criteria (see, e.g., a_{37} – C_2 or a_{39} – C_4), or have rather unbalanced performance profiles with significant contribution from some criteria and relative poor evaluations on the remaining ones (e.g., a_{50} – C_3 or a_{32} – C_4), or simply attain average performances on the vast majority of criteria, hence lacking substantial contribution from the most important descriptors (see, e.g., a_{36} – C_3 or a_{41} – C_4).

3.3.2. Robustness analysis

The recommendation obtained for the non-reference exposure scenarios with the representative instance of the sorting model is validated against the outcomes of robustness analysis. The possible assignments obtained through the analysis of all compatible instances of the sorting model admitting two changes in monotonicity for all marginal value functions are presented in Table 2. These possible assignments are precise only for 3 out of 21 alternatives. Hence, the assignment of a_{36} to C_3, a_{38} to C_5, and a_{48} to C_1 can be judged as robust in view of the incompleteness of the DM's assignment examples and multiplicity of compatible sorting models.

For the remaining non-reference exposure scenarios, the possible assignments are imprecise, thus indicating a hesitation with respect to the recommended class. However, for 17 alternatives just two classes are possible, and only for a_{35} – three classes can be recommended depending on the choice of a compatible sorting model. The additional classes contained in the possible assignment are more or less preferred than the classes suggested by the representative model for, respectively, 8 (e.g., a_{34} and a_{46}) and 11 (e.g., a_{31} and a_{45}) alternatives.

The analysis of such possible assignments allows to indicate the classes that cannot be viewed as an admissible result, because they are not confirmed by any compatible instance of the sorting model. For example, since a_{34} is possibly assigned to C_4 or C_5, the recommended precaution is surely not required (C_1), not might be required (C_2), nor optional (C_3). Similarly, since $C_P(a_{51}) = \{C_1, C_2\}$, the following requirements of precautions are excluded: optional (C_3), might be optional (C_4), and not required (C_5).

3.4. Discussion

The proposed model could be a first tiered solution to exposure management of nanomaterials, similarly to the step-wise strategies proposed for the exposure assessment phase [20,38]. It could be used to provide an initial indication of concern regarding specific tasks performed by the workers. In this way, when the proposed model indicates that the assigned class is at most C_3 (indicating required, potentially required, or optional precaution level), these tasks should be given priority and further investigated as they can be seen as "safety warning flags". Obviously, the less preferred the class, the greater attention should be paid to the analysis of a respective task. For such alternatives, the choices of the health managers could be directed towards working on the criteria of the model, i.e., characteristics of the materials and the exposure conditions, to see whether any of them can be modified to increase a comprehensive value and to trigger a more preferred class. The analysis of marginal value functions provides directions on which performance changes offer the greatest gains in this regard and which modifications do not lead to significant improvements.

Let us also emphasize that the primary aim of Section 3 was to illustrate the applicability of the proposed method in a standard MCDA setting. In this setting, the DM's preference information is used to construct a preference model compatible with the DM's value system. Such a model is subsequently employed to evaluate the non-reference alternatives that have not been directly judged by the DM, in a way that would be acceptable for him/her, being consistent with his/her preferences. Thus, in typical MCDA applications the objective truth to be discovered does not exist as the true classification for the non-reference alternatives is not known. However, it can be analyzed for the considered study, because the most appropriate assignments for the non-reference exposure scenarios have also been determined by the experts. In this regard, for 12 out of 21 non-reference scenarios (a_{36}, a_{37}, a_{38}, a_{42}, a_{44}, a_{45}, a_{46}, a_{47}, a_{48}, a_{49}, a_{50}, and a_{51}) the assignments obtained with a representative sorting model instance agree with the actual ones. Moreover, for the remaining 9 non-reference scenarios, the actual class is contained in the set of possible assignments, hence being confirmed by at least one compatible sorting model instance.

4. Conclusions

We proposed a novel approach for multiple criteria sorting incorporating a threshold-based value-driven procedure. The parameters of the constructed model deciding upon the shape of marginal value functions and separating class thresholds are inferred through disaggregation of the assignment examples provided by the DM. This is attained by solving dedicated MILP problems. Apart from accounting for the incomplete preference information, the method allows the DM to specify partial requirements on the assumed type of (non-)monotonicity for the individual criteria. Specifically, we considered gain and cost attributes, monotonic functions without a pre-defined preference direction, level-monotonic shapes, A- and V-types combining increasing and decreasing trends within a single function, and lack of monotonicity constraints. In this perspective, to control the complexity and interpretability of the inferred model, we minimized the number of changes in monotonicity for all marginal value functions.

The characteristic of the provided results is two-fold. On the one hand, we derive univocal assignments with a representative instance of the sorting model. The analysis of such a model allows capturing the trade-offs between different criteria, assessment of their relative importance, and indication of performance changes that can be viewed as the most advantageous in terms of improving a comprehensive quality. On the other hand, we perform robustness analysis and quantify its results by means of possible assignments. They confirm which classes are admissible for a given alternative for at least one compatible instance of the sorting model. Moreover, they allow to reject the hypotheses concerning the assignments which are not confirmed by any model.

The proposed approach can be seen as a sorting counterpart of the method proposed by Rezaei [34]. However, it does not require to pre-define the exact shapes of marginal value functions, tolerating instead partial information on the monotonicity constraints. Moreover, it extends the algorithm introduced by Kliegr [25] to a broad family of shapes of marginal functions as well as to a robustness analysis which accounts for all compatible instances of the sorting model with the minimal possible complexity.

Apart from the methodological advances, the paper contributes to the literature by exhibiting its applicability to analysis of a real-world sorting problem. Specifically, we considered the problem of exposure management for engineered nanomaterials, and used expert judgments to develop a model for predicting precaution level while handling nanomaterials in certain conditions using a respirator. The model was able to capture the interrelations between ten criteria – including monotonic descriptors, a single increase-level criterion, and a pair of non-monotonic attributes – and the recommended assignments.

The analysis of a representative instance of the sorting model allowed to identify the criteria that significantly affected the recommended sorting. These descriptors involved detection limits, airborne capacity, and duration of exposure. On the contrary, quantity of nanomaterial, frequency of exposure, and engineering controls had the least share in the comprehensive values of exposure scenarios. The classes were well-separated due to a significant difference between the inferred thresholds. In this way, each class accommodated multiple alternatives with diverse characteristics on the individual criteria that considered jointly could be holistically judged as required, optional, or absolutely redundant with respect to the precaution level. The case study also demonstrated how the representative and univocal results can be enriched with the outcomes of robustness analysis. Specifically, we showed the usefulness of possible assignments for capturing the uncertainty related to the recommended classification as a consequence of incompleteness of the DM's preference information.

The potential extensions of the proposed method and the considered case study are five-fold. Firstly, the practical applicability of our approach is limited due to a high number of binary variables. In fact, it depends on the numbers of criteria and unique performances of alternatives. Hence, a potential revision of the method needs to control the model's complexity and impose monotonicity constraints without incorporating binary variables. Secondly, we assumed that the model's complexity can be adjusted to reproduce all assignment examples by increasing the number of changes in monotonicity. In case this is not possible, one can apply the standard procedures for eliminating the minimal number of assignment examples underlying the inconsistency [30,31]. However, since they are based on MILP and associate a unique binary variable with each assignment example, their applicability is also limited to few hundred of holistic judgments. Dealing with larger sets of potentially inconsistent example assignments requires the development of some dedicated heuristic approaches.

Furthermore, the family of shapes of marginal value functions can be extended to account for polynomials and splines, whose interpretability is desirable in many real-world applications [40]. Moreover, the method can be easily adapted to multiple criteria ranking and choice. Instead of incorporating the DM's assignment examples, it should accept pairwise comparisons of reference alternatives.

Finally, motivated by the peculiarity of exposure management of nanomaterials, the method can be enriched to account for a few decision attributes simultaneously. In this specific application, they would represent different risk management measures [32]. For each classification problem, one should construct a dedicated sorting model reproducing the provided assignment examples. However, the individual models should be interrelated to reflect the dependencies between classes desired for the same alternative on various decision attributes.

Declaration of competing interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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Table A.3
Encoding of performances on ten criteria used for the management of exposure scenarios to nanomaterials.

g_j	Criterion	Preference	Performance	Code
g_1	Particle size (nm)	increase-level	< 2	1
			$2 - 10$	2
			$10 - 100$	3
			$100 - 500$	4
			$500 - 1000$	5
			> 1000	6
g_2	Toxicity	cost	Low	1
			Moderate	2
			High	3
g_3	Airborne capacity	cost	None	0
			Low	1
			Moderate	2
			High	3
g_4	Detection limit	gain	None	0
			Poor	1
			Moderate	2
			Good	3
g_5	Exposure limit (fiber/cc)	cost	< 0.1	1
			$0.1 - 0.2$	2
			$0.2 - 0.5$	3
			$0.5 - 1.0$	4
			> 1.0	5
g_6	Quantity (kg)	cost	< 1	1
			$1 - 100$	2
			$100 - 1000$	3
			$1000 - 10000$	4
			> 10000	5
g_7	Engineering controls	non-monotonic	Open-PP	1
			Open-NP	2
			Closed	3
			Closed-NP	4
g_8	Number of employees	non-monotonic	$1 - 3$	1
			$3 - 10$	2
			$11 - 50$	3
			$51 - 100$	4
			$101 - 500$	5
g_9	Duration of exposure (h)	cost	incidental	1
			< 0.25	2
			< 1	3
			$1 - 5$	4
			$5 - 8$	5
g_{10}	Multiple exposure (number)	cost	none	0
			$1 - 3$	1
			> 3	2
			unknown	3

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Appendix A. Encoding of performances for the case study

Table A.3 summarizes the characteristics of ten criteria used for the management of exposure scenarios to nanomaterials as well as the encoding of respective performances.

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