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Dependence Analysis in BRICS Stock Markets: A Vine Copula Approach

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Declaration

I certify that the minor dissertation submitted by me for the degree Master's of Commerce (Financial Economics) at the University of Johannesburg is my independent work and has not been submitted by me for a degree at another university.

LIANG TANG

Abstract

This study makes use of three types of vine copulas, c-vine, d-vine and r-vine copulas, to investigate the dependence structure in the BRICS stock markets using daily stock market price data spanning from 28-12-2000 to 10-08-2018. To account for the dynamic effects in dependence measures, the study divides the sample period into three sub-samples: the pre-crisis period (from 28-12-2000 to 31-01-2007), the crisis period (from 01-02-2007 to 29-12-2011), and the post-crisis period (from 04-01-2012 to 10-08-2018). The price data is first converted to return series and filtered using different ARIMA-GARCH models in order to remove the autocorrelation and heteroscedasticity effects. During this process, it was found that most of the return series exhibited leverage effects, an indication that bad news in the stock markets leads to larger spikes in volatility than good news does. To understand the implication of this effect on the dependence structure of stock markets in the BRICS countries, the c-vine, d-vine and r-vine copulas are used. The use of vine copulas has some significant advantages over traditional copulas as they model the dependence in the BRICS using pairwise copula constructions. The results show that the three types of vine copula models suggest that Student's t and the SBB7 copulas best describe the dependence structure in the BRICS markets. Unlike other studies, our findings show the existence of a very strong dependence between South Africa and Russia, South Africa and India, and South Africa and Brazil during the pre-crisis, the crisis and the post-crisis periods, suggesting a financial integration between these three countries. Furthermore, we find strong dependence between China and the rest of BRICS markets only during a financial crisis. The study identifies two types of dependence in the BRICS stock markets: the first is among small economies (South Africa, Brazil and Russia) and the second one among large economies (China and India). Small economies tend to co-move during bull and bear markets while large economies co-move with the rest only during bear market periods.

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CHAPTER I: INTRODUCTION

One of the growing fields of research in modern financial economics and risk management is that of financial market dependence. Knowing the dependence structure between different stock markets has important implications for researchers, individual investors, risk managers and policy makers. Understanding the underlying linkages and dependence structures between different markets can help individual investors, risk managers and policy makers to diversify their investment portfolios, minimize their investment risks and implement adequate economic policies respectively.

Correlation measures have been used in past research to study dependence structure, co-movement and linkages between stock markets. However, Lahrech and Sylwester (2011) and Syllignakis and Kouretas (2011) have shown that Pearson's constant correlation is not an appropriate measure of stock market dependence structure as it is often unable to capture a nonlinear relationship amongst stock markets. The limitation of the Pearson's constant correlation led to the use of rank correlation measures of dependence such as Kendall's tau and Spearman's rho as well as the dynamic constant correlation (DCC) regression model to capture nonlinearity in the relationships between stock markets.

However, the DCC model assumes that the asset returns follow a symmetric multivariate distribution such as Student's t-distribution or normal distribution. Furthermore, the empirical distribution of asset returns is often characterized by excess skewness, high kurtosis and heavy tails, which suggest that the dependence structure between asset returns is usually asymmetric and nonlinear (Embrechts, Mcneil and Straumann, 2002).

Against this background, this thesis attempts to model dependence in the BRICS stock markets by making use of vine copulas.

According to Sklar's theorem (1959), a multivariate joint distribution can be decomposed into a copula function and a set of univariate marginal distributions. The marginal distribution of each asset return can follow a different theoretical distribution and be linked by the copula function. The copula coefficient represents the measure of the dependence structure between the marginal distributions of asset returns.

There are two types of copula families, namely the elliptical and Archimedean. The elliptical family includes the Gaussian copula and Student's t copula while the Archimedean family includes, for instance, the Frank, Clayton and Gumbel copulas.

Although copula method has many advantages over the use of, for example, constant correlation in modelling dependence structure, it is difficult to use the copula method when the dimensions of the data become larger. In addition, the use of copulas under Sklar's theorem (1959) assumes that there is a unique copula function that models the dependence structure between all marginal distributions. This assumption is empirically too strong and unrealistic as two or more marginal distributions can exhibit a dependence structure that is totally different from the rest of the set of marginal distributions.

To overcome these two limitations, i.e. dimensionality and uniqueness of the dependence measure, Joe (1996) proposes the use of the bivariate copula construction. This process involves the construction of bivariate copulas to model each pair of marginal distributions. This technique is known as the vine copula. The method uses graphical representation of the decomposition of multivariate copulas into bivariate copulas known simply as pair copulas. The pair copula construction method allows each bivariate copula the flexibility to choose independently of each other a different copula type, even for modeling asymmetry and tail dependence.

There exist three types of vine copulas, namely r-vine (regular vine), c-vine (canonical vine) and d-vine (drawable vine) copulas (Brechmann and Schepsmeier, 2013). More detail about each type of vine copula is provided in the methodology section of this thesis (Chapter III).

As yet, no study has used all three vine copulas simultaneously to model dependence structure in the BRICS stock markets, especially from a South African perspective. This thesis attempts to fill this existing gap by analyzing and assessing the dependence structures between the BRICS countries by using the three types of vine copula methods.

To do so, different autoregressive integrated moving average – generalised autoregressive conditional heteroscedasticity (ARIMA-GARCH) models are first used to remove the autocorrelation and heteroscedasticity effects from the BRICS stock market data that spans from 28-12-2000 to 10-08-2018. For the purpose of robustness, the entire sample period is divided into three sub-samples, which represents the pre-crisis period (from 28-12-2000 to 31-01-2007), the crisis period (from 01-02-2007 to 29-12-2011), and the post-crisis period (from 04-01-2012 to 10-08-2018). Ideally, we want to investigate whether the dependence structure in the BRICS stock markets changes over time, which might provide implications for portfolio diversification and economic policy implementation.

The results suggest that the Student's t and the SBB7 copulas best describe the dependence structure in the BRICS stock markets. In contrast to previous studies, our findings show the existence of a very strong dependence structure between South Africa and Russia, South Africa and India, and South Africa and Brazil during the pre-crisis, the crisis and the post-crisis periods, suggesting a financial integration between these three countries. Furthermore, we find strong dependence between China and the rest of BRICS markets only during financial crisis. This study identifies two types of dependence structures in the BRICS stock markets: the first is between small economies (South Africa, Brazil and Russia), and the second is between large economies (China and India). Small economies tend to co-move during bull and bear markets while large economies co-move with the rest only during bear market periods.

The rest of the thesis is organized as follows: Chapter II presents the literature review, Chapter III presents the methodology used in the study, Chapter IV provides the empirical analysis, while Chapter V concludes the thesis.

CHAPTER II: LITERATURE REVIEW

This chapter presents the literature review on the dependence structure between stock markets. The review deals with three main concepts, namely dependence structure using correlation, dependence structure using copulas, and dependence structure using vine copula.

2.1 Dependence Analysis Using Correlation

In this section, a number of previous studies that have used correlation as a measure of dependence structure are reviewed. The first is Agmon (1972), who made use of constant correlation to investigate co-movements between United States and three other stock markets: the United Kingdom, Japan and Germany. The study uses a regression model and monthly return data spanning from 1961 to 1966. The results show that German stock market had the strongest co-movement with the United States stock market. The co-movement between the United States and United Kingdom markets and between the United States and Japanese markets were similar. The study also indicated that the price change of the United Kingdom and German markets followed the price change of the United States stock market within one period.

Arshanapalli and Doukas (1993) examined the stock market interdependence between the US and German, UK, Japan and France. The study used daily closing data of stock indices from 1980 to 1990, and the study period was divided into two as pre-crash and post-crash. Empirical results from co-integration test revealed that the link between the US and other stock markets was weak during the pre-crash period. However, in the post-crash period, stock markets of German, UK and France co-integrated with the US markets, except Japan.

Jain (2014) investigated the interdependence among BRICS stock markets. Daily closing stock price indices from 2003 to 2014 was used and the study period was divided into two as pre-crisis and post-crisis. Empirical results from correlation analysis indicated during post-crisis period, correlation increased significantly between each country compared to pre-crisis period. Co-integration analysis revealed that Brazil, India and China co-integrated with each other only for pre-crisis period. Russia and South Africa didn’t co-integrated with each other in pre-crisis period but became more integrated in post-crisis period. There was no significant increase for the linkage level between the BRICS stock markets for post-crisis period, except for South Africa and China and South Africa and Russia.

Syllignakis and Kouretas (2011) used the DCC GARCH model to investigate the conditional correlation between the market return of the United States, German and Russian stocks and the stock returns from Central and Eastern European (CEE) markets using weekly data of stock price indices from 1997 to 2009. They found that conditional correlation increased for all examined pairs except the Russia-Czech Republic pair. The study indicated that condition correlations between the CEE stock markets and the United States reached their peak, which was matched with the 2008 stock market crash.

In a similar study, Lahrech and Sylwester (2011) used weekly data from 1988 to 2004 to investigate the correlation between four Latin American and the United States stock market returns using the DCC GARCH model. The study showed that United States stock markets returns had the highest conditional correlation with Mexico, followed by Brazil, Argentina and Chile. The study also applied the smooth transition model for the return series and found out that the integration between Latin American stock markets and the Unites States market were increasing.

Baumohl and Lyocsa (2014) also used DCC models to examine the relationship between conditional volatility and correlation between 32 frontier and emerging stock markets and the world stock index of MSCI. Weekly stock market returns were used from 2000 to 2012. The study indicated that the correlations between the developed markets and the emerging or frontier markets increased if the volatility increased.

Zhang, Li and Yu (2013) also applied DCC models, but this time on returns of the stock markets for the BRICS countries as well as developed markets. For developed markets, S&P 500 and MSCI Europe were used as proxies of the region. Daily data was used from 2000 to 2012. The study demonstrated that the correlation between the developed stock markets and BRICS countries increased over the 12 years, and 2008 financial crisis affected the correlation between the BRICS and the developed stock market returns.

Kenourgios, Samitas and Paltalidis (2010) applied the regime switching copula model together with the AG-DCC model to test the dependence structure between the BRIC stock markets and the stock markets of United States and United Kingdom. Weekly data from 1995 to 2006 was used, and the dataset was divided into five crisis periods. The study aimed at comparing the correlation between crisis and non-crisis periods. Empirical results indicated that correlation increased from non-crisis to crisis periods. Dependence changes among the BRIC markets were larger than the dependence changes between them and the United States and United Kingdom stock markets. The AG-DCC model indicated that the dependence between the stock markets was high in crisis periods. This paper revealed that the dependence level obtained from the regime switching copula model was higher than the one from the AG-DCC model.

2.2 Dependence Analysis Using Copulas

In this section, a number of previous studies that make use of a copula as a measure of the dependence structure are reviewed. Firstly, Mensah and

Alagidede (2016) used the daily stock markets returns from 2000 to 2014 to examine the dependence level between four African countries (South Africa, Kenya, Nigeria and Egypt) and the United States and United Kingdom stock markets. The bivariate copula method was used for estimating the dependence. The Gaussian and Student’s t copulas of the elliptical class were used as time-invariant copulas. The Archimedean Gumbel and rotated Gumbel copula were then used to check tail dependence. Time-varying copulas were obtained by using the generalized autoregressive score model. The empirical results revealed that dependence structures between these four African countries and stock markets of the United States and United Kingdom was generally weak. Weak and asymmetric tail dependence was found for all markets.

Yang and Hamori (2013) used daily stock return data from 2002 to 2013 to examine the dependence structure among developed countries (Japan, the European Union bloc, the United Kingdom and the United States), the emerging markets (the BRIC bloc), and the interdependence between them. The study used normal and Student’s t copulas to capture the dependence. Gumbel and Clayton copulas were used to test asymmetric tail dependence. Empirical results showed that dependence among the developed countries was high compared to the emerging markets. All pairs together with the Russian market were not statistically significant. The authors asserted that culture and geographical distance were important for determining dependence.

Dharmawan, Harini and Sumarjaya (2015) examined stock market pair dependence among five stock markets, using the JKST (Jakarta Stock Exchange), Hang Seng Index, KOSPI (Korea Composite Stock Price Index), Nikkei 225, and STI (Straits Times Index). They used the Gaussian copula, rotated Gumbel copula and symmetrized Joe Clayton (SJC) copula. According to the Akaike information criterion (AIC), Bayesian information criterion (BIC) and log-likelihood (LL) for JKSE-STI pair, the SJC copula fit the best. The rests of the pairs were better fit by the Gaussian copula.

Reboredo, Tiwari and Albulescu (2015) used daily data from 2000 to 2013 for investigating the dependence structure of four stock markets: Hungary, Czech Republic, Romania and Poland. The study used both the time-variant and time-invariant copula methods. The empirical findings showed that the dependence between these four markets were positive and significant except for the Romanian market. Moreover, the Czech Republic and Romanian markets showed symmetric tail dependence. The time-varying copula method showed that the dependence between the countries was significantly reinforced since the onset of 2008 financial crisis.

Wang, Chen and Huang (2011) examined the dependence structure between the Chinese stock market and six other indices: MSCI AcWorld, MSCI European, MSCI Pacific, MSCI Unites States, MSCI Japan and MSCI World. The study used daily data from 2000 to 2009. Unconditional copula models indicated that the Chinese market had the highest dependence level with the Pacific market, followed by with the Japanese market. This dependence was caused by regional economic developments and geographical proximity. The conditional copula method showed that the Chinese stock markets had the highest dependence level and greatest dependence variability level with the Japanese and the Pacific markets.

Hussain and Li (2018) used daily stock return data from 2005 to 2015 to examine the dependence structures between China and Australia, Japan, Germany, Canada, the United Kingdom and the United States. The study used both the constant copula method and the time-varying copula method. The results from the constant copula showed that China and Australia pair had the strongest overall dependence and tail dependence. The weakest overall dependence or tail dependence was found for the China and United States pair. The results from time-varying copula method indicated that the China and Australia pair had the strongest dependence, which could be due to the strong trade and economic relations between the two countries.

Aloui, Aissa and Nguyen (2011) examined tail dependence between the stock markets returns of the BRIC countries and the United States. The study used daily data from year 2004 to 2009. Gumbel and Galambos copulas were used to test the upper tail dependence and lower tail dependence. Empirical results indicated that there was extreme co-movement for all stock market pairs, both in the upper and the lower tail. Furthermore, the dependence of the United States and Brazil and United States and Russia were higher than the pairs of United States and China and United States and India.

2.3 Dependence Analysis Using Vine Copulas

This section reviews studies that made use of vine copulas as measures of the dependence structure. These studies include Brechmann and Schepsmeier (2013), who used stock markets daily returns from the United States, Japan, China, Germany, France and the United Kingdom from 2009 to 2010 to test the dependence structure among them, using the c-vine and d-vine copulas. Empirical results indicated that dependence levels were high among European stock markets, and France's stock market index was treated as the central market for interpreting overall dependence. Asymmetric tail dependence was found and, using the Vuong test, the study could not distinguish between these vine copulas.

Allen, Ashraf, McAleer, Powell and Singh (2013) applied the r-vine method to investigate the interdependence between 30 stocks selected from the Dow Jones. The data ranged from 2005 to 2011, and was divided into the same three sub-sample periods as this current study, called by Allen et al. the pre- global financial crisis (GFC), GFC, and post-GFC. The first step in all three was to use the Student's t copula for fat tail distribution. Times reduced post-GFC than in the GFC period when using the Student's t copula. Empirical results indicated that in pre-GFC period, different types of dependence were being used. During GFC period, student's t copulas were most used while the usages of Gaussian copulas were low. The reliance of student's t copula decreased in post-GFC.

Maya, Gomez-Gonzalez and Velandia (2015) used exchange rate data from six Latin American economics (Brazil, Colombia, Mexico, Peru, Argentina and Chlie) from 2005 to 2012 to examine the tail dependence between them. Dependence parameters from bivariate copulas were obtained using the r-vine copula. After estimating the r-vine copula, a simulation procedure was implemented to calculate tail dependence. The empirical results showed that lower tail dependences were significant between all Latin American countries expect the pair of Peru and Argentina, which means that when the exchange rate experienced large appreciation, there is contagion effect among Latin American countries. The insignificant lower tail dependence between Peru and Argentina could be explained by the dollarized economy of Peru and the debt restructuring program in Argentina.

Dibmann, Brechmann, Czado and Kurowicka (2013) applied the r-vine copula on 16 international indices, using daily data from five equity indices, nine bond indexes and two commodity indices from 2001 to 2009. The paper introduced the r-vine selection approach, which involved sequentially finding a maximum spanning tree using the graph theoretic algorithm. Five r-vine classes were chosen, which were the mixed r-vine, mixed c-vine, all t r-vine, mixed d-vine and multivariate Guass. Vuong tests indicated that mixed r-vine model was preferred to the multivariate Guass and mixed d-vine models. When used the Schwarz correction, the mixed r-vine model was superior to the mixed c-vine and all t r-vine models. The results confirmed the ability of the r-vine copulas in modelling dependence structure.

Aas, Czado, Frigessi and Bakken (2009) tested tail dependence between four indices: the Norwegian stock index, the Norwegian bond index, the MSCI World Index, and SSBWG hedge index, using daily data from 1999 to 2003. All pairs of d-vine decomposition used Student's t copula and the result were compared with those from the multivariate Student's t copula with four dimensions. Empirical results from d-vine copula with all student's t pair copula decomposition model indicated that the dependence level was strongest between SSBWG hedge index and MSCI world index, Norwegian stock index and MSCI world index, and Norwegian stock index and Norwegian bond index. The study also compared the tail dependence result between d-vine model and student copula. Empirical evidence revealed that the d-vine copula method is preferred to the multivariate Student's t copula method when testing the tail dependence.

Feng and Hayes (2016) used the r-vine copula to test the dependence between annual land returns of 24 states in the United States from 1967 to 2014. The results from the r-vine copula was compared to those from both the Gaussian copula method and Student's t copula method. AIC criteria suggested that the r-vine copula model is superior to the other two, and the r-vine result was used for portfolio construction.

Czado, Schepsmeier and Min (2012) used four models to test for dependence between the exchange rates between the United States and eight other currencies using daily data from 2005 to 2009. Mixed c-vine models were used, meaning that the pair copulas were allowed to choose individually. The study first used both sequential estimation and maximum likelihood estimation. For model one, the mixed c-vine, the dependence was low. Model two used the same mixed c-vine as model one but included the independence test to see whether the independence copula needed to replace some of the insignificant pair copulas. Model three used the t copula for all pair copulas. Model four changed all pair copulas to Gaussian copulas. The dependence level from model one was often very low for each pair. According to the AIC and BIC, model two is the best-fitting model, and this result was also confirmed by Vuong and Clarke tests.

Brechmann, Czado and Aas (2012) applied the r-vine copulas on 19 indices of international and Norwegian financial variables, using daily data from 2003 to 2008. The paper investigated the most appropriate truncation level or simplification level for the r-vine copula. R-vine truncated at tree level K means that all pair copulas in which conditioning is set equal or larger than tree level K are replaced by independence copulas. R-vine simplified at tree level K means that all pair copulas in which conditioning is set equal or larger than tree level K are replaced by Gaussian copulas. Empirical analysis from this study indicated that the most important dependence could be captured by tree four to tree six, which meant that the r-vine could be truncated at either level four or level six. For simplification, the r-vine could be simplified at level two The paper also compared Student's t copula to the truncated r-vine model and the simplified r-vine model, and the result revealed that statistically the Student's t copula is equivalent or inferior to other two models.

Vesper (2012) provided a time-varying vine copula method to investigate the dependence structure of 16 firms selected from S&P 100. Monthly equity returns were chosen from 1990 to 2010. The Markov Chain Monte Carlo was used to draw inference using a Bayesian approach. Empirical results showed high level of tail dependence for pair copula, and the correlation between equities have grown in the past twenty years. The mean square error for out of sample data showed that the dynamic d-vine copula outperformed the static-vine copula method.

Sithole (2014) applied c-vine and d-vine copulas on daily data from six industrial indices from the JSE from 1998 to 2004 for the purposes of portfolio optimization. For the c-vine, the financial sector was chosen as the root node of the first tree. Comparing the sharp ratio of c-vine, d-vine and mean-variance models, the empirical analysis indicated that d-vine had the highest sharp ratio, followed by the c-vine model. The efficient frontier in the mean variance model had the highest variance with the lowest return.

Geidosch and Fischer (2016) used d-vine and r-vine copulas for testing the dependence structure of credit portfolios. A total of 40 companies were drawn from Euro Stoxx 50 and 75 companies were drawn from S&P 500 to form a loan portfolio. Month-end equity log returns from 1999 to 2011 were chosen for the study. According to the AIC, for the Euro Stoxx 50 portfolio, for the traditional copula model Student’s t copula outperformed both the Clayton and the Gaussian copulas. The d-vine copula fit better than the traditional copula only in the Clayton case. Flexible r-vine outperformed the flexile d-vine in both portfolios. Empirical evidence indicated that the economic capital was underestimated by the Gaussian copula. However, economic capital increased when using the r-vine copula. Overall, the flexible r-vine was the best fit for estimating economic capital. The study revealed that the framework of vine copulas was stable even when extending the time series period.

CHAPTER III: METHODOLOGY

This chapter presents different methodologies used in this study to measure dependence structure among the stock markets. Firstly, the traditional method of measuring dependence structures using correlation measure such as the Pearson's correlation and the rank correlation are discussed. This section is followed by a discussion on the use of copulas in modelling dependence structure amongst stock markets. Lastly, a discussion on the use of vine copulas, including the r-vine, c-vine and d-vine copulas, is provided.

3.1 Pearson Correlation and Rank Correlation

The methodology presented in this section follows Embrechts et al. (2002), Malevergne and Sornette (2006) and Mwamba (2012). Statistically speaking, random variable X and random variable Y are defined as independent if:

$$P(X \le x \text{ and } Y \le y) = P(X \le x)P(Y \le y) \qquad (1)$$

Consequently, two random variables will be referred to as dependent if they are not independent. A number of methods can be used for measuring the dependence structure. The Pearson correlation expresses the linear correlation between two random variables as follows:

$$\rho(X, Y) = \frac{Cov(X, Y)}{\sqrt{\sigma^2(X)\sigma^2(Y)}} \qquad (2)$$

where $\sigma^2(X)$ is the variance of X and $\sigma^2(Y)$ is the variance of Y. $Cov(X,Y)$ represents the covariance between X and Y. However, Muteba Mwamba (2012) points in his thesis that the Pearson correlation as a measure of dependence structure has a number of serious shortcomings, including the fact that the correlation is not invariant under non-linear strictly increasing transformation. For instance, zero correlation between two random variables does not necessary mean independence of those random variables.

Rank correlation is an alternative method provided in the literature to overcome the shortcomings of the Pearson correlation. Rank correlation is used in dependence analysis for measuring the concordance (when two random variables move in the same direction) and discordance of the random variables. Two often-used rank correlation measures are Kendall's tau and Spearman's rho. They are equal to zero if the two random variables X and Y are independent. Muteba Mwamba (2012) defined Kendall's tau and Spearman's rho as follows:

Given independent pairs (X_i, Y_i) of the two random variables (X, Y), if the random variables are continuous, the Kendall's tau coefficient is given by:

$$\tau = 2P[(X_1 - X_2)(Y_1 - Y_2) > 0] - 1 \qquad (3)$$

If the marginal distribution is under monotonic transformation, the Kendall's tau is invariant. It varies between -1 to 1. The Spearman's rho $\rho_S(X, Y)$ is given by:

$$\rho_S = 3P[(X_1 - X_2)(Y_1 - Y_3) > 0] - P[(X_1 - X_2)(Y_1 - Y_3) < 0] \qquad (4)$$

As discussed above, traditionally dependence structure has been studied by making use of Pearson's linear correlation. The use of linear correlation to measure the dependence structure does, however, have its problems. To overcome the disadvantage of linear correlation in modeling the dependence between the BRICS countries, this thesis discusses an alternative method of measuring dependence known as vine copulas. Before addressing the mathematics behind vine copulas, we first present a short discussion of the copula function.

3.2 Copulas

The word copula was firstly coined by SKlar (1959) who proved that a collection of marginal distribution can be coupled together via a copula to form a multivariate distribution. Unlike the Pearson correlation, copula methods are indifferent to continuous increasing monotonic transformation, which gives them the ability to precisely describe the dependence structure in both bull and bear markets. The methodology presented in this part mainly follows Embrechts et al. (2002), Malevergne and Sornette (2006) and Mwamba (2012).

According to Malevergne and Sornette (2006), $C:[0,1]^n \rightarrow [0,1]$ is an n-dimensional copula function if it satisfies the following properties:

$$\circ\ \forall \mathrm{u} \in [0,1],\ \mathrm{C}(1,....,1,\mathrm{u},1,....,1)=u, \tag{5}$$

$\circ\ \forall \mathrm{u_i} \in [0,1],\ \mathrm{C}(\mathrm{u}_1,u_2,u_3,u_4.....,u_n)=0$, if at least one of the u_i's equal to zero.

There are two families of copulas: the Elliptical and Archimedean. Two most important examples of the former are the Gaussian, also known as normal, and Student's t copulas. The Archimedean family of copula includes, among others, the Clayton copula, Gumbel copula and Frank copula. The Frank copula can be used to model symmetric dependence. The Clayton copula can model the lower tail and the Gumbel the upper tail.

3.2.1 Sklar's Theorem (1959)

Sklar (1959) showed that if F is an n-dimensional joint distribution function with continuous marginal of $F_1, F_2, ..., \mathrm{F_n}$, then there exists a unique copula defined as $C:[0,1]^n \rightarrow [0,1]$ such that:

$$F(x_1, x_2,...., x_n) = C(F_1(x_1), \mathrm{F_2}(\mathrm{x_2}), ..., \mathrm{F_n}(x_n)) \tag{6}$$

Conversely, given a multivariate distribution function F with marginals

$F_1, F_2, ..., F_n$, for any $(u_1, u_2, ..., u_n)$ in $[0,1]^n$

$$C(u_1, u_2, ..., u_n) = C\left(F_1^{-1}(u_1), F_2^{-1}(u_2), ..., F_n^{-1}(u_n)\right) \quad (7)$$

C represents the continuous copula function that links these marginals. The copula C can be either of the two families of copulas, i.e. elliptical copulas or Archimedean copulas. Elliptical copulas model the dependence structure of a distribution that is spread symmetrically to the center. Figure 3.1 below exhibits both a symmetrical dependence structure (the panel in the middle uses Franck copula) and asymmetric dependence structures (panel in the far left uses the Clayton copula and panel in the far right uses the Gumbel copula). This figure indicates that Archimedean copulas are able to model random variable dependence structure that is concentrated in both tails (such as the Frank copula) or only concentrated in one tail (such as the Clayton copula for the upper tail and the Gumbel for the lower tail).

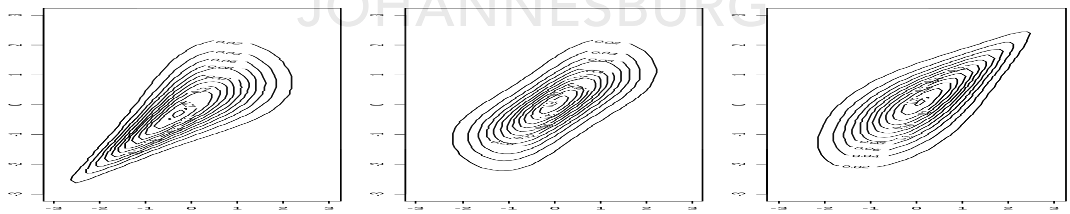

Figure 3.1: Contour plots of the Clayton, Frank and Gumbel copulas respectively.

3.2.2 Dependence Structure Analysis Using Copulas

The dependence structure analysis using copulas is made possible by expressing the rank correlation coefficients (Kendall's tau and Spearman's rho) in terms of copula C as in equation (6). For example:

$$\tau(C_t) = 4 \iint_{[0,1]^2} C(u,v)\,dC(u,v) - 1$$: the Kendal's τ with t-copula (8)

$$\rho_S(C_t) = 12 \iint_{[0,1]^2} C(u,v)\,du\,dv - 3$$:Spearman's ρ_S with t-copula (9)

$$\tau(C_F) = 1 - \frac{4}{\theta}\left[1 - D_1(\theta)\right]$$: the Kendal's τ for t-copula for the Frank copula (10)

$$\rho_S(C_F) = 1 - \frac{12}{\theta}\left[D_1(\theta) - D_2(\theta)\right]$$: the Spearman's ρ_S for Frank copula (11)

where $D_k(x)$ denotes the "Debye" function: $D_k(x) = \frac{k}{x}\int_0^x \frac{t^k}{(e^t - 1)}dt$ (Genest and MacKay, 1986).

3.2.3 Estimation of Copula Parameters

This thesis makes use of the maximum likelihood method to estimate the parameters of the copula in equation (6) above. The maximum likelihood method proceeds as follows. Let F be a multivariate distribution function with continuous marginal F_i and copula C. Take the first derivative of equation (6) to obtain the joint distribution function f:

$$f(x_1, \ldots, x_n) = \frac{\partial^n F(x_1, \ldots, x_n)}{\partial x_1, \ldots, \partial x_n} = \left[\prod_{k=1}^{n} f_k(x_k)\right] \times C(F_1(x_1), \ldots, F_n(x_n)) \quad (12)$$

The first derivative of the cumulative distribution function of equation (6) above is $\frac{\partial^n F(x_1, \ldots, x_n)}{\partial x_1, \ldots, \partial x_n}$. The probability density of $F_1(x_1)$ is $f_k(x_k)$. C is either an elliptical or Archimedean copula. Density of the copula is c, which is given by :

$$c(u_1, \ldots, u_n) = \frac{\partial C(u_1, \ldots, u_n)}{\partial u_1 \ldots \partial u_n} \quad (13)$$

All the parameters that need to be estimated are given by the vector

$\delta = (\beta_1,....., \beta_2, \alpha)$. The parameters for marginal distribution F_i is given by the vector β_i, and the copula parameters is given by the vector α. The log-likelihood of the equation (12) can be written as:

$$l(\delta) = \sum_{t=1}^{T} \ln c\left(F_1\left(x_1^t;\beta_1\right),...., F_n\left(x_n^t;\beta_n\right); \alpha\right) + \sum_{t=1}^{T}\sum_{i=1}^{n} \ln\left(f_i\left(x_i^t;\beta_i\right)\right) \quad (14)$$

3.3 Dependence Structure Using Vine Copulas

In equation (6), C is assumed to be unique for all marginals. However, this become inflexible in high dimensional data since some pair variables might exhibit different dependence structures. For example, if one uses Sklar's theorem in equation (6) to model the dependence structure in the BRICS stock markets, it is assumed that only one copula type measures the dependence among all five countries. Practically, this is unlikely since it is possible to find pair-wise markets exhibiting different copula types. To overcome this issue, this study will focus on building pair-wise copulas that can exhibit different dependence structures between pair variables. The method for building pair-wise copulas is known as the vine copula method, which was pioneered by Joe (1996).

In order to understand the logic behind the use of vine copula construction, we use equation (12) in a two dimensional framework:

$$f(x_1, x_2) = C_{12}\{F_1(x_1).F_2(x_2)\}.f_1(x_1)f_2(x_2) \quad (16)$$

It the two random variables are dependent, then:

$$f(x_2|x_1) = \frac{f(x_1,x_2)}{f_1(x_1)} \quad (17)$$

The conditional probability distribution is of $f(x_2|x_1)$. $f(x_1,x_2)$ is the joint distribution of x_1 and x_2. The vine copula corresponding to Equation (17) is given by:

$$f(x_2|x_1) = \frac{C_{12}\{F_1(x_1).F_2(x_2)\}.f_1(x_1)f_2(x_2)}{f_1(x_1)} \qquad (18)$$

or

$$f(x_2|x_1) = C_{12}\{F_1(x_1).F_2(x_2)\}.f_2(x_2) \qquad (19)$$

In three-dimension framework, the vine copula in equation (19) can be written as:

$$f(x_1, x_2, x_3) =$$

$$f_1(x_1).f_2(x_2).f_3(x_3).C_{1,2}(F_1(x_1).F_2(x_2).C_{2,3|1}F_1(x_2|x_1).F_2(x_3|x_1).C_{13}\{F_1(x_1).F_3(x_3)\}$$

(20)

Equation (20) has some significant advantages as it helps to build the bivariate pair-copulas $C_{1,2}, C_{2,3|1}$, and C_{13} which will capture dependence structures that may exist between pair variables. The generalization of the vine copulas in the n-dimension is provided by Joe (1996) and is expressed as:

$$F(x|v) = \frac{\partial C_{x,v_j|v_{-j}}(F(x|v_{-j}),F(v_j|v_{-j}))}{\partial F(v_j|v_{-j})} \qquad (21)$$

$C_{ij|k}$ represents a bivariate copula. v_j is an arbitrarily chosen component from vector v, and v_{-j} excludes components v_j will left.

Equation (20) can be equivalently generalized to:

$$f(x|v) = C_x, v_j|v_{-j}\{F(x|v_{-j}), F(v_j|v_{-j}).f(x|v_{-j})\} \qquad (22)$$

It is worth knowing that the construction of the pair-wise/vine copula is not unique. It depends on the conditional distribution and prior information. For example, in equation (18), x_1 is the prior information. However, if x_2 becomes the prior information, equation (18) will be changed. In this context, many vine copulas can be constructed. In this thesis, three types of vine copulas are discussed, namely the r-vine, c-vine and d-vine.

3.3.1 R-Vine Copula

According to Bedford and Cooke (2001), Kurowicka and Cooke (2002), Bedford and Cooke (2006) and Aas (2016), an r-vine (or regular vine) copula of a $n-dimension$ variable is a pair-wise constructed copula made of trees $T_1, \dots, T_{n-1}$. Let E_i and N_i be the sets of edges and nodes for tree T_{i}. The following conditions will be satisfied:

i. T_1 has nodes $N_i = \{1, \dots, n\}$ and a set of edges denoted by E_i;
ii. For $i = 2, \dots, n-1$, T_i has nodes $N_i = E_{i-1}$ and edge set E_i; and
iii. In tree T_i if there are two edges are to be joined in tree $T_{i+1,}$ as nodes, in tree $T_{\ i}$ they need to share a common node (proximity condition).

The edges of an r-vine tree can be uniquely identified by the conditioned and conditioning nodes. R-vine copula density is expressed as:

$$f(x_1, \dots, x_n) = \prod_{k=1}^{n} f_{(k)}(x_k) \times \prod_{i=1}^{n-1} \prod_{e \in E_i} c_{j(e),k(e)} | D_{(e)} (F(x_{j(e)} | x_{D(e)}), F(x_{k(e)} | x_{D(e)})) \tag{23}$$

A hypothetic graphical representation is shown below of an r-vine with five random variables:

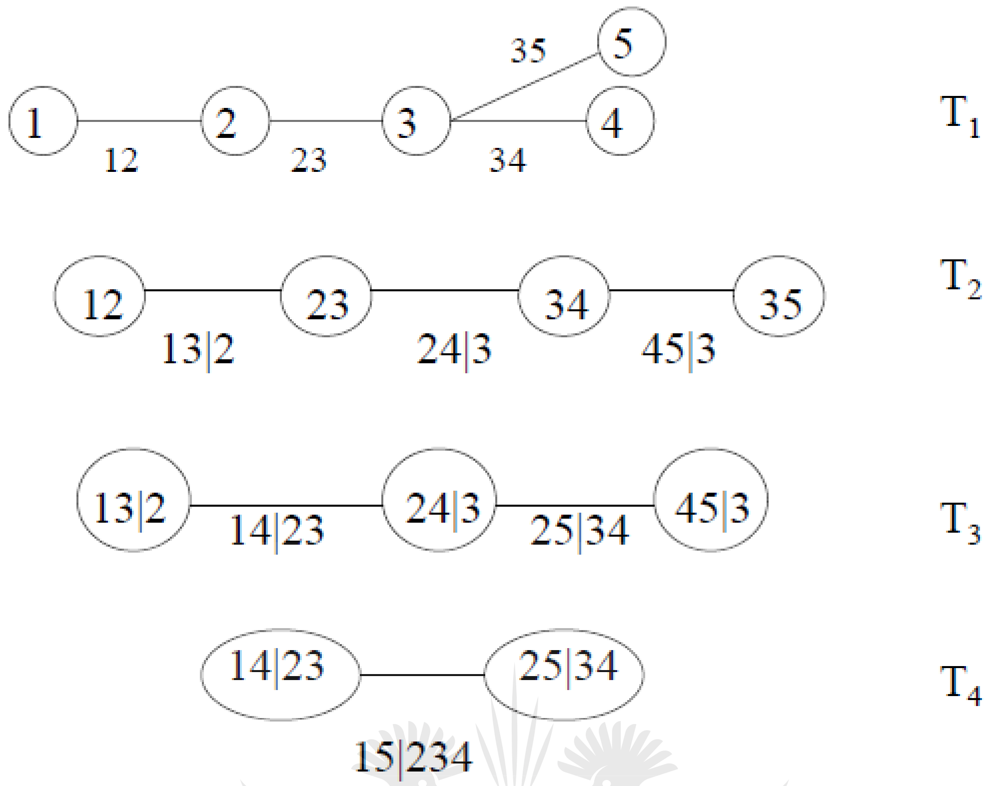

Figure 3.2: A hypothetical R-vine copula graphical representation

Source: Aas, Czado, Frigessi and Bakken (2009).

3.3.2 C-Vine Copula

According to Aas (2016), the n dimensional density for a canonical vine is expressed as:

$$\prod_{k=1}^{n} f(x_k) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{j,j+i|1,\ldots,j-1} \left\{F(x_j|x_1,\ldots,x_{j-1}), F(x_{j+i}|x_1,\ldots,x_{j-1})\right\}$$

(24)

The corresponding graphical representation of a c-vine with five random variables:

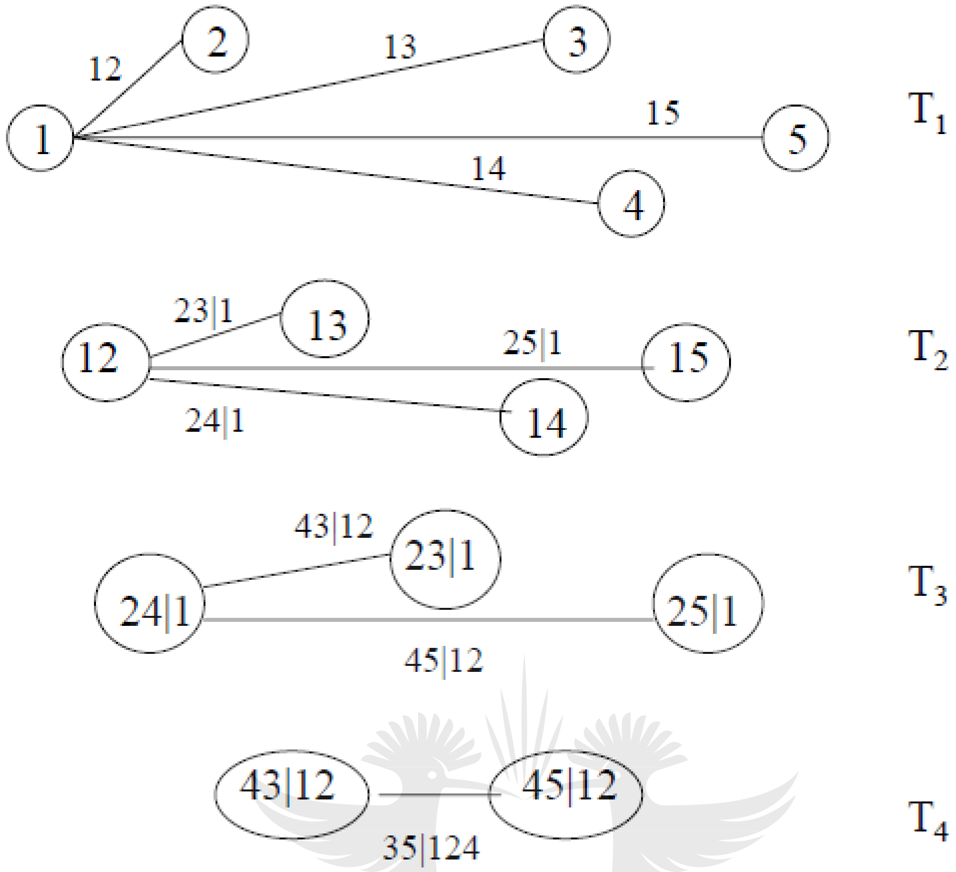

Figure 3.3: C-vine graphical representation

3.3.3 D-Vine Copula

According to Aas (2016), the n dimensional density for a d-vine is expressed as:

$$\prod_{k=1}^{n} f(x_k) \prod_{i=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|i+1,\ldots,i+j-1} \left\{ F(x_i|x_{i+1},\ldots,x_{i+j-1}), F(x_{i+j}|x_{i+1},\ldots,x_{i+j-1}) \right\} \quad (25)$$

The corresponding graphical representation of a d-vine with five random variables:

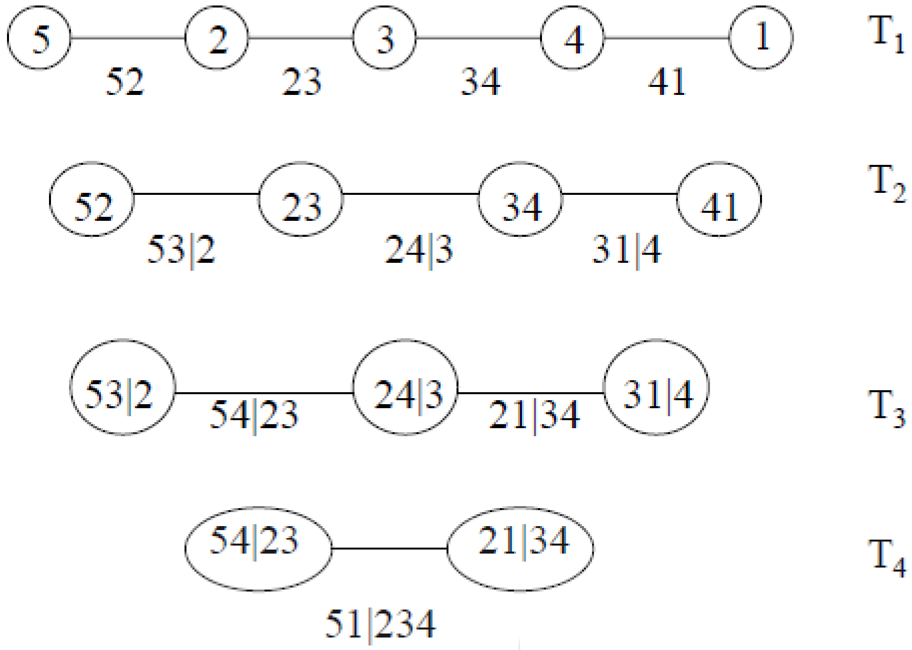

Figure 3.4: A D-vine graphical representation

CHAPTER IV: EMPIRICAL ANALYSIS

4.1 Preliminary Analysis

Daily stock price data was collected from the investing.com website for the five BRICS countries' stock markets. The data spanned from 28-12-2000 to 10-08-2018 and included the following indices: Bovespa (Brazil), Moex (Russia), Nifty 50 (India), Shanghai (China), and All Share Index (ALSI) (South Africa). Stock return was calculated as $R_t = \ln(\frac{P_n}{P_{n-1}}) \times 100$. The whole sample period was divided into three sub-samples to represent the pre-crisis (from 28-12-2000 to 31-01-2007), the crisis (from 01-02-2007 to 29-12-2011), and the post-crisis period (from 04-01-2012 to 10-08-2018). It is worth noting that the main objective of this study was not to determine the exact dates that correspond to each sub-sample periods. Instead the study attempted to investigate the changing dynamics of BRICS stock markets' dependence structure during these sub-sample periods. The study used the software R for implementation and deployment of the vine-copula methodologies. The descriptive statistics for each sub-sample period are reported in Table 4.1, Table 4.2 and Table 4.3 below.

Table 4.1: Descriptive statistics: Pre-crisis period

	Brazil	**Russia**	**India**	**China**	**South Africa**
Mean	0.0856	0.1951	0.0954	0.0241	0.0902
Std Dev.	1.9796	2.1984	1.6136	1.5361	1.2868
Kurtosis	2.3025	4.2271	7.4107	5.4557	2.6014
Skewness	-0.0695	-0.2099	-0.5704	0.7930	0.0386
Minimum	-9.6286	-10.481	-13.054	-6.8814	-6.7003
Maximum	10.6213	14.6083	10.2473	9.5746	5.8895

During this period, Russia exhibited the largest mean return followed by India. China had the lowest return during this sub-sample period. Using the standard deviation as a measure of risk, it is clear that Russia had the highest risk followed by Brazil. South Africa had the lowest risk in this period. The empirical distribution of Chinese stock market was most positively skewed, followed by that of South Africa. The rest of the markets were negative skewed, with India having the largest negative skewness. Russia, India and China all had kurtosis greater than 3, which suggests a significant deviation from the normal probability distribution.

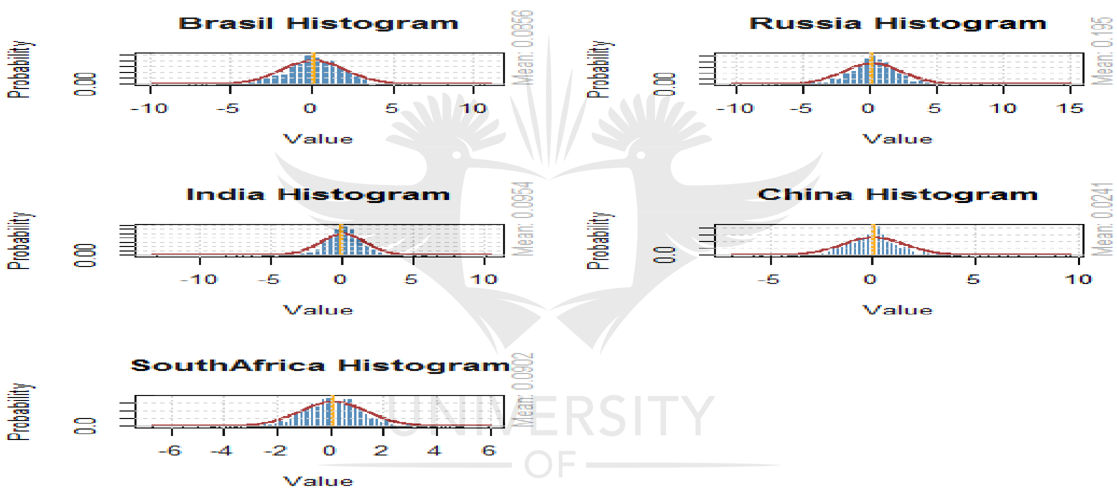

Figure 4.1: Histograms during the pre-crisis period

Figure 4.1 shows that the empirical distributions of all five markets tended to follow a symmetric distribution with skewness close to zero. However, it is clear that kurtosis level was high for Russia, India and China.

Table 4.2: Descriptive statistics: Crisis period

	Brazil	Russia	India	China	South Africa
Mean	0.024	-0.017	0.013	-0.024	0.024
Std Dev.	2.271	3.161	2.066	2.190	1.698
Kurtosis	8.765	27.888	6.787	1.902	7.832
Skewness	-0.515	-1.408	-0.060	-0.299	-0.619
Minimum	-18.749	-36.109	-13.014	-9.256	-15.307
Maximum	13.678	25.226	16.334	9.034	6.834

During this crisis period, the mean return of all five markets decreased compared to in the previous period. Russia and China had negative mean returns. In terms of risk as represented by the standard deviation, the risk level for all five BRICS countries' stock markets increased compared to in the pre-crisis period. South Africa had the lowest risk while Russia had the highest level of risk with a minimum return of -36.12%. All BRICS countries exhibited negative skewness indicating that the likelihood of losses was high during this sub-sample period. Brazil, Russia and India had kurtosis greater than 3, suggesting significant deviation from the normal probability distribution.

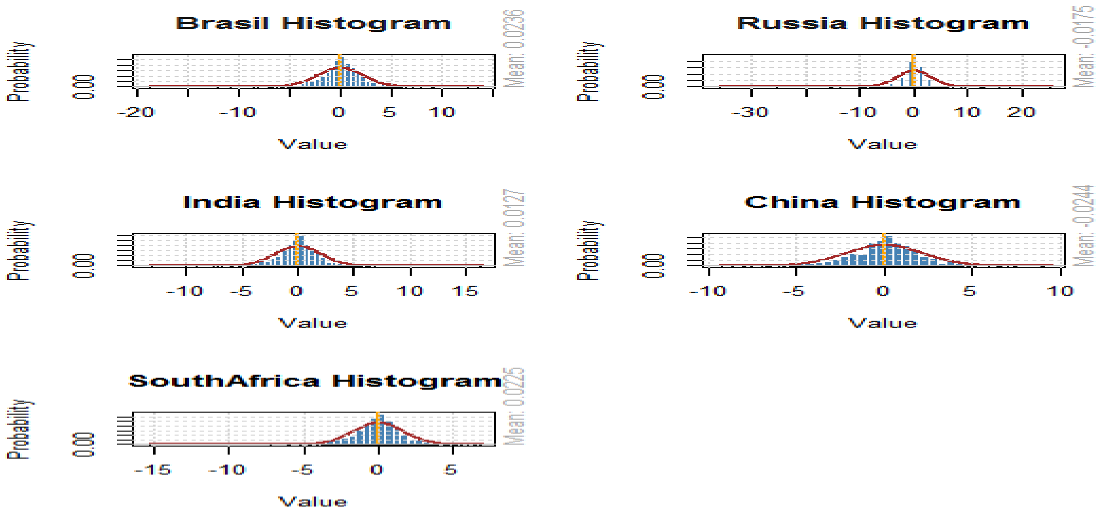

Figure 4.2: Histograms during the crisis period

Figure 4.2 above matches with the findings from Table 4.2. Significant skewness and high kurtosis indicate high probability of losses during the crisis period.

Table 4.3: Descriptive statistics: Post-crisis period

	Brasil	Russia	India	China	SouthAfrica
Mean	0.021	0.035	0.065	0.018	0.042
Std Dev.	1.547	1.241	0.971	1.545	0.965
Kurtosis	2.196	3.854	2.863	8.316	2.164
Skewness	0.043	-0.206	-0.265	-0.820	-0.208
Minimum	-9.211	-8.025	-6.097	-10.83	-4.872
Maximum	8.601	7.654	5.185	10.045	5.132

In the post-crisis period, there was a significant recovery for all the BRICS stock markets. All mean returns increased from the crisis period. Standard deviations as a measure of risk decreased than in the crisis period, with India having the lowest standard deviation of 0.971%, down from 2.066%. Russia, India and South Africa showed slightly negative skewness. The kurtosis of Russia and China was greater than 3, which indicates significant deviation of the normal probability distribution.

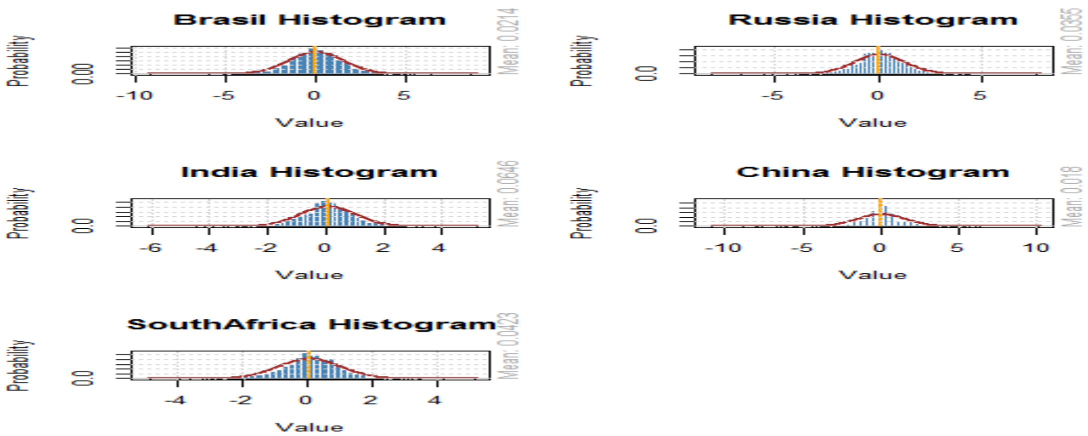

Figure 4.3: Histograms during the post-crisis period

Figure 4.3 shows that the empirical distributions during the post-crisis period exhibited a relatively symmetric distribution with a low level of skewness. However, it is clear that the kurtosis level was high for Russia and China.

Risk and Return Analysis

Next, we look at each sub-sample period in terms of risk and return. We plot the risk-reward plots in Figure 4.4, Figure 4.5 and Figure 4.6 below.

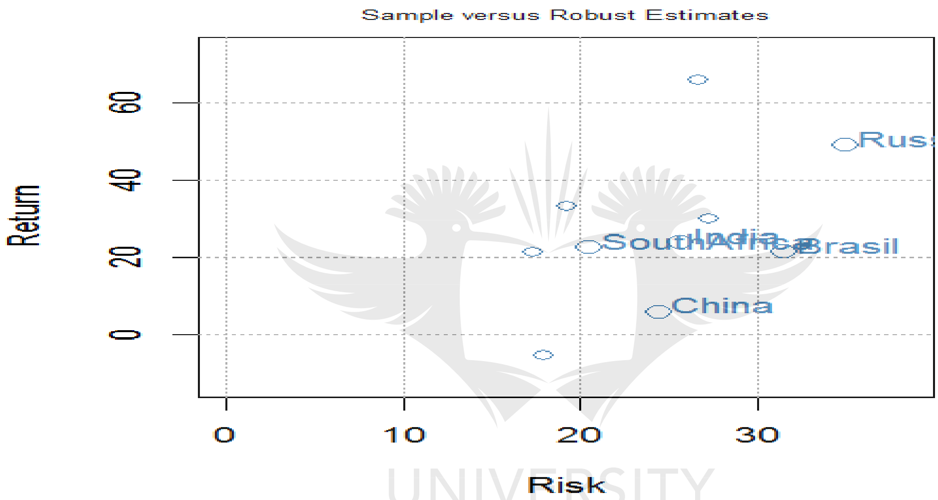

Figure 4.4: Risk-reward plot: Pre-crisis period

Figure 4.4 shows that during the pre-crisis period, Russia had the highest level of return with the highest level of risk. China had the lowest return with a relatively low level of risk. South Africa, India and Brazil had relatively similar levels of return with different levels of risk. South Africa had the lowest risk, followed by India and then Brazil.

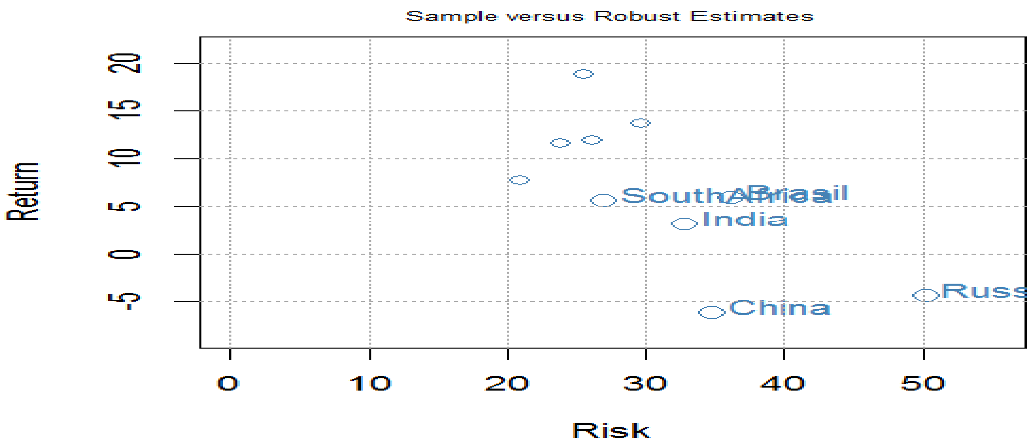

Figure 4.5: Risk-reward plot: Crisis period

Figure 4.5 shows that during the crisis period, Russia had the lowest level of return but the highest level of risk. China's stock market return in the crisis period was low and similar to that of Russia, but had a relatively low level of risk compared to Russia. South Africa and Brazil had the highest rates of return. South Africa exhibited the lowest level of risk among all BRICS stock markets.

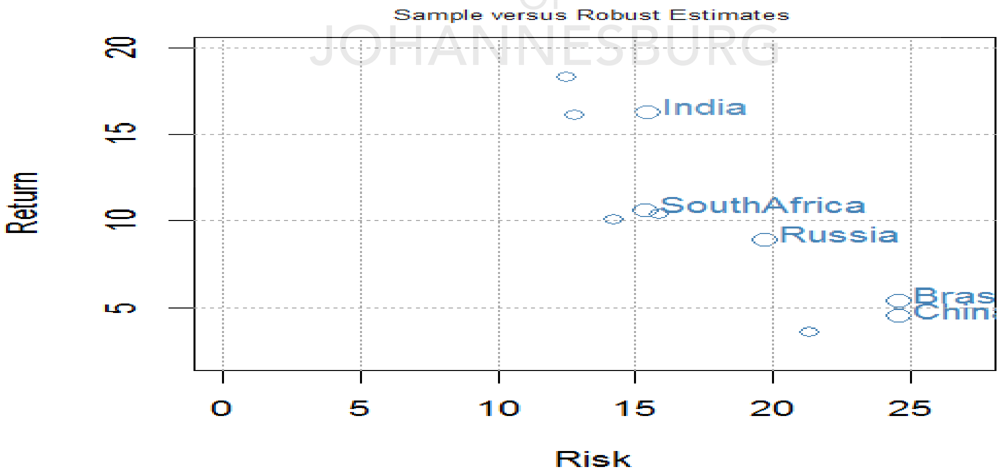

Figure 4.6: Risk-reward plot: Post-crisis period

Figure 4.6 shows that during the post-crisis period, India had the higher return among the BRICS countries but with the same risk level as South Africa. Brazil and China had the same level of risk. The risk level in Brazil was slightly higher than that in China.

In the following figures, we plot the pie charts of each stock market. The pie chart show four moments of each stock market. The first moment is the mean return of each stock market. The second moment is the standard deviation representing the risk level. The third moment is the skewness that measures the shape of the empirical distribution. The fourth moment is the kurtosis, which provides the likelihood of large swing in the tails of the empirical distribution.

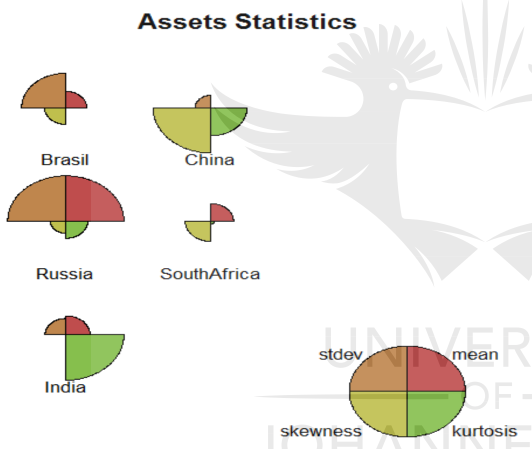

Figure 4.7: Asset statistics plot: Pre-crisis period

Figure 4.7 shows that during the pre-crisis period, South Africa was the only counties among the bloc that had insignificant risk.

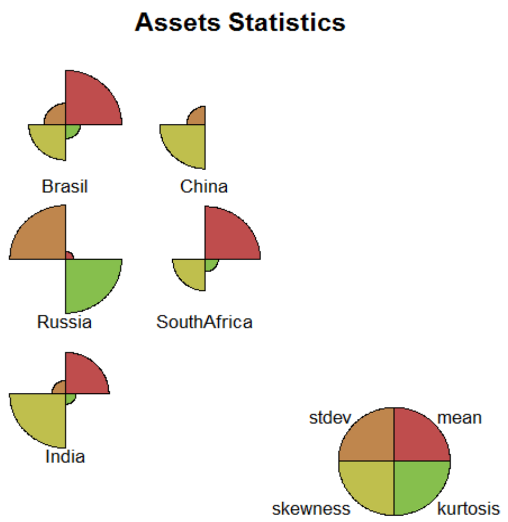

Figure 4.8: Asset statistics plot: Crisis period

Figure 4.8 shows that during crisis period, Russia exhibited the greatest kurtosis. Mean return and kurtosis of China were also both insignificant. The risk level was not significant for South Africa.

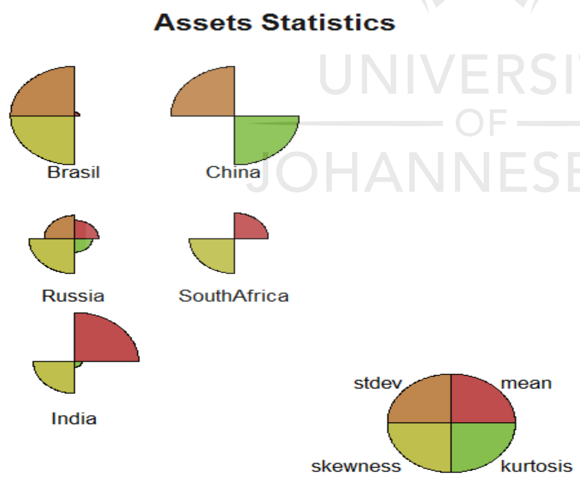

Figure 4.9: Asset statistics plot: Post-crisis period

Figure 4.9 shows that during post-crisis period, the mean return of China and Brazil were insignificant. India exhibited the highest mean return among the BRICS.

Based on the above preliminary analysis, the study investigated the dependence structure among the BRICS stock markets during three different sub-sample periods. We first look at the Pearson correlation measure of the sample periods before we begin the dependence structure analysis of the BRICS.

Dependence Structure Analysis with Pearson Correlation

Figure 4.10, Figure 4.11 and Figure 4.12 report the correlation between the BRICS stock markets for different sub-sample periods.

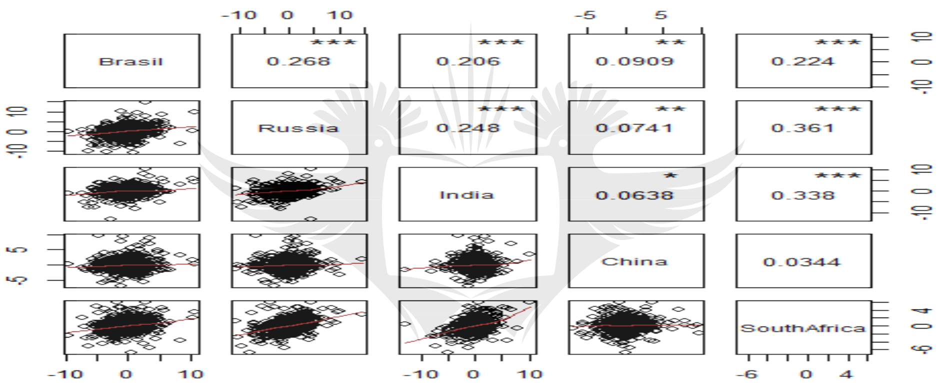

Figure 4.10: Pearson correlation plot: Pre-crisis period

As shown in this Figure 4.10, during the pre-crisis period, the correlations between each country were small, indicating that each country might have following its own domestic policy for reaching economic growth. For instance, the correlation between China and other BRICS stock markets was almost below 10%.

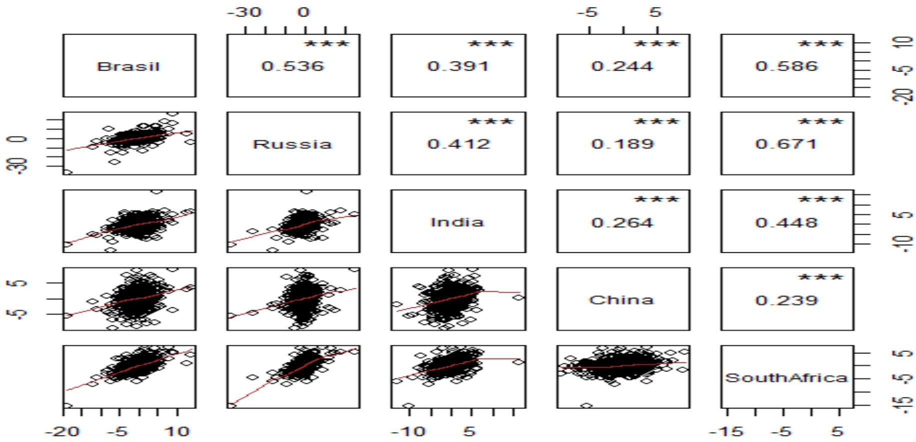

Figure 4.11: Pearson correlation plot: Crisis period

Figure 4.11 shows that during crisis period, the correlation between each BRICS stock market increased dramatically compare to pre-crisis levels. For instance, the Pearson correlation between Russia and South Africa increased from 36% during the pre-crisis period to 67% during the crisis period.

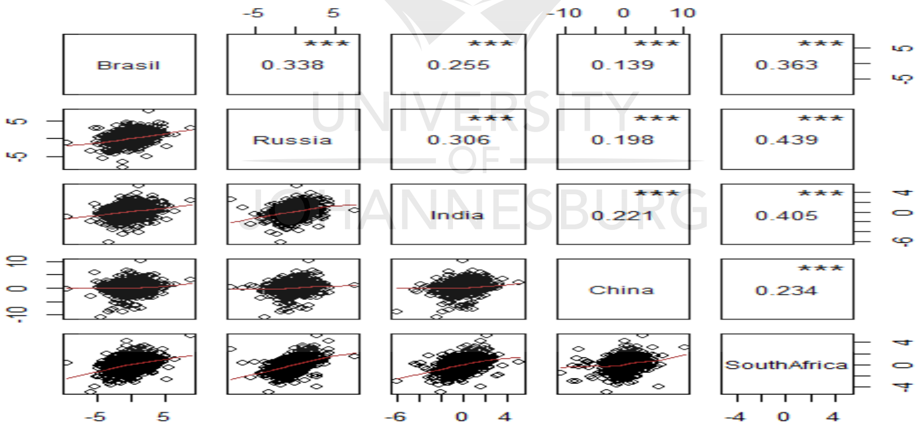

Figure 4.12: Pearson correlation plot: Post-crisis period

Figure 4.12 shows that the correlation between each stock markets decreased from the crisis period level and returned to pre-crisis levels.

The dependence structure analysis using the Pearson correlation measure shows that during financial crisis period, all stock markets of the BRICS countries tended to co-move, while decoupling during normal market period. This phenomenon might suggest that during normal market periods, each country follows its own domestic policy in order to reach its own economic targets.

Figures 4.10, 4.11 and 4.12 suggest that each pair of countries is likely to exhibit a particular dependence structure that is specific to their bilateral relationship. For instance, India and China showed particular relationship either during normal market periods or during the crisis period. This phenomenon leads us to analysis the dependence structure between the BRICS stock markets by making use of the pair-wise copula construction method, also known as vine copula method.

4.2 Dependence Structure Analysis with Vine Copulas

The use of vine copulas allows us to distinguish different types of pair-wise dependence that might exist among the BRICS stock markets. In order to estimate the vine copulas, we need to first filter the returns of each stock market in order to remove the effects of autocorrelation and heteroscedasticity in the return series. This is done by fitting an autoregressive (AR) moving average (MA) and generalized autoregressive conditional heteroscedasticity (GARCH) model to each return series.

There are two different groups of GARCH models. The first type includes the symmetric GARCH models as they assume that the conditional distribution of the error terms is symmetric (normal or t distribution). The second type includes asymmetric GARCH models, which assume that the conditional distributions of the error terms are not symmetric because bad news and good news each affect volatility differently.

Without getting into details of the volatility modeling using the GARCH models, is it sufficient to state that this study uses both symmetric and asymmetric GARCH models based on different error term distributions in order to filter each return series. The estimated coefficients of each fitted GARCH mode is reported in the Appendix D in Table 4.4 to Table 4.18. The resulting residuals of each model are standardized and used for the estimation of the vine copulas.

Estimation of Vine Copulas

In this section, we use c-vine, d-vine, and r-vine copula methods discussed in Chapter III to build the pair-wise copulas that independently capture the dependence structure between each pair of markets. The estimation of the vine copula is done in two stages. The first stage consists of selecting the pair of vines (c-, d-, and r-vines) that minimize some information criteria such as AIC. The second stage consists in estimating the parameters of the selected vines using the maximum likelihood method. The estimated results for c-, d- and r-vine copulas are presented below.

4.2.1 C-Vine Copulas

The estimation of the c-vine copula is done in two stages. The first stage consists of selecting the pair of vine copulas that minimize some information criteria such as the BIC and AIC. The second stage consists of estimating the parameters of the selected vines using the maximum likelihood method. All stages are presented in the tables below.

Each selection and estimation stage results in a number of trees, labeled as tree 1, tree 2, tree 3, etc. Each tree is made of a number of nodes and edges. Each node represents a country or a group of countries from the BRICS markets. "**1**" represents Brazil, "**2**" represents Russia, "**3**" India, "**4**" China and "**5**" South Africa. The dependence between two nodes is referred to as the "*edge*". For instance, edge 5,3 represents the dependence between South

Africa and India, while edge 2,3;5 represents the dependence between Russia and India conditioning on South Africa. Tree 1 has 4 nodes, tree 2 has 3 nodes, tree 3 has two nodes, and tree 4 has only one node.

Each copula type has a specific family. For the full list of families of the copulas used in this thesis, the reader is referred to the VineCopula package[1]. “**Par**”, “**par2**” are the first and second parameters of the selected copula function. It is worth noting that some copula families do not have two parameters. “**Tau**” is the Kendal Tau corresponding to the selected copula family. “**Utd**” and “**ltd**” represent the upper tail dependence and the lower tail dependence respectively. Elliptical copulas have zero dependence, while Archimedean copulas have either upper or lower tail dependence or both.

Step 1: Selection Stage for C-Vine Copula

Table 4.19: C-vine selection: Pre-crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**	**Utd**	**Ltd**
1	5,3	t	0.3	4.46	0.19	0.14	0.14
	5,1	t	0.26	5.02	0.17	0.11	0.11
	5,2	t	0.34	4.65	0.22	0.15	0.15
	5,4	t	0.03	14.25	0.02	0	0
2	2,3;5	t	0.13	8.75	0.08	0.02	0.02
	2,1;5	BB1	0.12	1.06	0.11	0.08	0
	4,2;5	C	0.1	0	0.05	-	0
3	1,3;2,5	SJ	1.08	0	0.04	-	0.1
	4,1;2,5	F	0.45	0	0.05	-	-
4	4,3;1,2,5	I	-	-	0	-	-

The log-likelihood value of the selection is 308.93 with the following criteria: AIC: -587.86; BIC: -510.79.

[1] https://github.com/tnagler/VineCopula.

Table 4.19 shows that for the first tree Student’s t copula is selected since it minimizes the AIC, the BIC, and/or maximizes the log-likelihood value. The rest of the trees use different families of copulas. Tree 4 suggests that the dependence structure for the whole sample of BRICS markets is independent.

Table 4.20: C-vine selection: Crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	5,2	t	0.66	3.29	0.46	0.4	0.4
	5,1	SBB7	1.59	0.48	0.36	0.24	0.46
	5,3	t	0.45	3.18	0.3	0.28	0.28
	5,4	t	0.24	6.99	0.16	0.06	0.06
2	3,2;5	t	0.18	5.48	0.12	0.08	0.08
	3,1;5	t	0.16	4.52	0.1	0.1	0.1
	4,3;5	t	0.21	8.27	0.13	0.04	0.04
3	1,2;3,5	t	0.21	6.18	0.13	0.07	0.07
	4,1;3,5	BB7	1.08	0.05	0.07	0.1	0
4	4,2;1,3,5	J90	-1.04	0	-0.02	-	-

The log-likelihood value of the selection is 851.02 with the following information criteria: AIC: -1664.05; BIC: -1507.48.

Table 4.20 shows that trees 1 and 2 suggest that the dependence structure is best modelled using Student’s T copula expect for edge (5,1) which suggests a family of copulas.

Table 4.21: C-vine selection: Post-crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	5,1	SBB7	1.28	0.26	0.23	0.07	0.28
	5,3	t	0.41	8.55	0.27	0.07	0.07
	5,2	t	0.46	8.62	0.31	0.09	0.09
	5,4	t	0.25	6.25	0.16	0.07	0.07
2	2,1;5	t	0.23	11.39	0.14	0.02	0.02
	2,3;5	t	0.15	7.9	0.09	0.03	0.03
	4,2;5	t	0.1	7.84	0.06	0.03	0.03
3	3,1;2,5	N	0.1	0	0.06	-	-
	4,3;2,5	t	0.11	15.91	0.07	0	0
4	4,1;3,2,5	Tawn180	5.88	0	0	-	0

The log-likelihood value of the selection is 577.67 with the following criteria: AIC: -1117.33; BIC: -1017.77. As in the previous table, it is clear that Student's t copula is found to be best to model the dependence structure for most of pairs in the BRICS stock markets.

Step 2: Estimation Results of C-Vine Copulas

Table 4.22: C-vine estimation: Pre-crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	5,3	t	0.29 (0.03)	4.44 (0.71)	0.19	0.14	0.14
	5,1	t	0.26 (0.03)	5.02 (0.86)	0.17	0.11	0.11
	5,2	t	0.33 (0.03)	4.63 (0.75)	0.22	0.15	0.15
	5,4	t	0.03 (0.03)	10.00 (2.60)	0.02	0.01	0.01
2	2,3;5	t	0.13 (0.03)	8.72 (2.60)	0.08	0.02	0.02
	2,1;5	BB1	0.12 (0.05)	1.06 (0.02)	0.11	0.08	0
	4,2;5	C	0.08 (0.04)	-	0.04	-	0
3	1,3;2,5	SJ	1.06 (0.03)	-	0.03	-	0.07
	4,1;2,5	F	0.44 (0.17)	-	0.05	-	-
4	4,3;1,2,5	I	-	-	0	-	-

The log-likelihood value of the selection is 307.74 with the following criteria: AIC: -585.48; BIC: -508.4. Table 4.22 reports the estimated c-vine copula parameters for the four trees. It can be seen that the Student's t copula dominates tree 1. The estimated dependence structures are shown in the column labelled tau. The estimated standard errors are shown in the parentheses. The first parameter for the Student's t copula is statistically significant everywhere. This parameter represents the correlation for the Student's t copula. The second parameter represents the degree of freedom for the Student's t copula. Table 4.22 shows that the strongest dependence was between Russia and South Africa (edge: 5,2), with 0.22. The rest of the pairs exhibited weak dependence structure. The estimated C-vine copula during the pre-crisis period can be represented graphically as shown in Appendix A.

Table 4.23: C-vine estimation: Crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**	**Utd**	**Ltd**
1	5,2	t	0.66 (0.02)	3.26 (0.42)	0.46	0.4	0.4
	5,1	SBB7	1.59 (0.06)	0.48 (0.06)	0.36	0.24	0.46
	5,3	t	0.45 (0.03)	3.17 (0.42)	0.3	0.28	0.28
	5,4	t	0.24 (0.03)	6.99 (1.81)	0.16	0.06	0.06
2	3,2;5	t	0.18 (0.04)	5.47 (1.16)	0.12	0.08	0.08
	3,1;5	t	0.15 (0.04)	4.51 (0.77)	0.1	0.1	0.1
	4,3;5	t	0.21 (0.03)	8.27 (2.34)	0.13	0.04	0.04
3	1,2;3,5	t	0.21 (0.03)	6.19 (1.34)	0.14	0.07	0.07
	4,1;3,5	BB7	1.08 (0.03)	0.05 (0.03)	0.07	0.1	0
4	4,2;1,3,5	J90	-1.00 (0.03)	-	0	-	-

The log-likelihood value of the selection is 849.56 with the following criteria: AIC: -1661.12; BIC: -1567.55. As in the previous table, it is clear that Student's t copula is found to be best to be model the dependence structure for most of pairs in the BRICS stock markets during the crisis period.

Table 4.23 shows that the strongest dependence was between South Africa and Russia of 0.46. There was a weak dependence structure between South Africa and China. The corresponding graphical representation of this C-vine copula during the crisis period is provided in Appendix A.

Table 4.24: C-vine estimation: Post-crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	5,1	SBB7	1.28 (0.04)	0.26 (0.05)	0.23	0.07	0.28
	5,3	t	0.41 (0.02)	8.51 (2.17)	0.27	0.07	0.07
	5,2	t	0.46 (0.02)	8.53 (2.10)	0.3	0.09	0.09
	5,4	t	0.24 (0.03)	6.23 (1.29)	0.16	0.07	0.07
2	2,1;5	t	0.23 (0.03)	10.00 (2.81)	0.15	0.02	0.02
	2,3;5	t	0.14 (0.03)	7.85 (1.94)	0.09	0.03	0.03
	4,2;5	t	0.10 (0.03)	7.82 (2.00)	0.07	0.03	0.03
3	3,1;2,5	N	0.10 (0.03)	-	0.06	-	-
	4,3;2,5	t	0.11 (0.03)	10.00 (2.39)	0.07	0.01	0.01
4	4,1;3,2,5	Tawn180	2.91 (NA)	0.00 (0.00)	NA	-	0

The log-likelihood value of the selection is 575.45 with the following criteria: AIC: -1112.91; BIC: -1013.35. As in the previous table, it is clear that Student's t copula was the best in modelling the dependence structure for most of pairs in the BRICS stock markets, even during the post-crisis period.

Table 4.24 shows that the strongest dependence was between South Africa and Russia of 0.3. The corresponding graphical representation of this C-vine copula estimation during the post-crisis period is shown in Appendix A.

4.2.2 D-Vine Copulas

Using the same selection and estimation steps, the d-vine copulas were selected and estimated as reported below.

Step 1: Selection of d-vine copulas

Table 4.25: D-vine selection: Pre-crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**
1	4,5	BB7	1.12786807	0.2616807	0.02
	3,4	t	0.20612203	4.3794402	0.02
	2,3	t	0.03676189	10.2765316	0.13
	1,2	t	0.02826363	14.2527808	0.17
2	3,5;4	Rotated Joe	1.11042967	0	0.19
	2,4;3	Clayton	0.1108798	0	0.05
	1,3;2	t	0.29247555	4.5686597	0.06
3	2,5;3,4	Frank	0.46895716	0	0.18
	1,4;2,3	t	0.28102183	7.3553437	0.05
4	1,5;2,3,4	t	0.1611543	9.45156	0.1

Table 4.25 shows that for the pre-crisis period, the d-vine copula uses a mixture of elliptical and Archimedean copulas to model the pair-wise dependence structure among the BRICS stock markets.

Table 4.26: D-vine selection: Crisis period

Tree	Edge	Cop	Par	Par2	Tau
1	4,5	t	0.47871872	2.9368	0.16
	3,4	t	0.43563889	3.606792	0.18
	2,3	t	0.28277126	5.576078	0.29
	1,2	t	0.24398118	6.994293	0.32
2	3,5;4	t	0.18282604	4.770521	0.27
	2,4;3	t	0.07365544	9.909738	0.05
	1,3;2	t	0.41758635	3.61814	0.12
3	2,5;3,4	t	0.09630345	12.425629	0.4
	1,4;2,3	t	0.58485363	5.424054	0.06
4	1,5;2,3,4	t	0.25487683	5.906942	0.16

Table 4.26 shows that the Student's t copula is best in modelling the dependence structure for most of pairs in the BRICS stock markets during this crisis period.

Table 4.27: D-vine selection: Post-crisis period

Tree	Edge	Cop	Par	Par2	Tau
1	4,5	t	0.3439789	5.918663	0.16
	3,4	t	0.3142296	6.680936	0.14
	2,3	t	0.2177598	8.724694	0.2
	1,2	t	0.2477202	6.249978	0.22
2	3,5;4	t	0.1606922	19.733812	0.24
	2,4;3	t	0.1509238	9.568435	0.1
	1,3;2	t	0.3729364	9.679816	0.1
3	2,5;3,4	Frank	0.2306871	0	0.24
	1,4;2,3	t	0.3740071	13.934593	0.03
4	1,5;2,3,4	t	0.1805529	10.072126	0.12

Table 4.27 shows that the Student's t copula is the best in modelling the

dependence structure for most of pairs in the BRICS stock markets, expect for tree 3.

Estimation results of d-vine copulas

Table 4.28: D-vine estimation: Pre-crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**
1	4,5	BB7	1.12786807 (0.03327391)	0.2616807 (0.04375102)	0.02
	3,4	t	0.20612203 (0.03102889)	4.3794402 (0.69738144)	0.02
	2,3	t	0.03676189 (0.03056953)	10.2765316 (3.48639983)	0.13
	1,2	t	0.02826363 (0.02999567)	14.2527808 (6.41570439)	0.17
2	3,5;4	Rotated Joe	1.11042967 (0.02768119)	0.0000000 (0.0000000)	0.19
	2,4;3	Clayton	0.11087980 (0.03383648)	0.0000000 (0.0000000)	0.05
	1,3;2	t	0.29247555 (0.02903188)	4.5686597 (0.76877745)	0.06
3	2,5;3,4	Frank	0.46895716 (0.17030715)	0.0000000 (0.0000000)	0.18
	1,4;2,3	t	0.28102183 (0.02790675)	7.3553437 (1.79467669)	0.05
4	1,5;2,3,4	t	0.1611543 (0.02952757)	9.45156 (2.77029866)	0.1

Table 4.28 shows that the estimated Kendal tau is very small indicating that weak dependence structure existed among the BRICS stock markets during

the pre-crisis period. A mixture of copula families is also suggested to be best to model the dependence structure during this sub-sample period. A graphical representation of the d-vine during this sub-sample period is shown in Appendix B.

Table 4.29: D-vine estimation: Crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**
1	4,5	t	0.47871872 (0.02904936)	2.9368 (0.3991512)	0.16
	3,4	t	0.43563889 (0.02874479)	3.606792 (0.5280007)	0.18
	2,3	t	0.28277126 (0.03183859)	5.576078 (1.1804083)	0.29
	1,2	t	0.24398118 (0.03211330)	6.994293 (1.8148388)	0.32
2	3,5;4	t	0.18282604 (0.03434067)	4.770521 (0.8620367)	0.27
	2,4;3	t	0.07365544 (0.03364259)	9.909738 (3.4006689)	0.05
	1,3;2	t	0.41758635 (0.02933008)	3.61814 (0.5505812)	0.12
3	2,5;3,4	t	0.09630345 (0.03331507)	12.425629 (5.4415226)	0.4
	1,4;2,3	t	0.58485363 (0.02130110)	5.424054 (1.0580346)	0.06
4	1,5;2,3,4	t	0.25487683 (0.03270670)	5.906942 (1.4724412)	0.16

Table 4.29 shows the Student's t copula is the best copula for modelling the

dependence structure for most of pairs in the BRICS stock markets for the crisis period. Strong dependence structure was found between Russia, South Africa, India and China (edge: 2,5;3,4), followed by Brazil and Russia. India, China and South Africa also showed strong dependence structure (edge: 3,5;4). The corresponding graphical representation of the d-vine copula during this sub-sample period is provided in Appendix B.

Table 4.30: D-vine estimation: Post-crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**
1	4,5	t	0.3439789 (0.026)	5.918663 (1.187171)	0.16
	3,4	t	0.3142296 (0.026)	6.680936 (1.494270)	0.14
	2,3	t	0.2177598 (0.02745289)	8.724694 (2.552578)	0.2
	1,2	t	0.2477202 (0.02771633)	6.249978 (1.305692)	0.22
2	3,5;4	t	0.1606922 (0.02709088)	19.733812 (12.301148)	0.24
	2,4;3	t	0.1509238 (0.0282479)	9.568435 (2.937187)	0.1
	1,3;2	t	0.3729364 (0.02386836)	9.679816 (2.733854)	0.1
3	2,5;3,4	Frank	0.2306871 (0.16226891)	0.000000 (0.000000)	0.24
	1,4;2,3	t	0.3740071 (0.0231799)	13.934593 (5.368231)	0.03
4	1,5;2,3,4	t	0.1805529 (0.02761281)	10.072126 (3.188567)	0.12

Table 4.30 shows the Student's t copula is the best copula for modelling the

dependence structure for most of pairs in the BRICS stock markets in the post-crisis period. Table 4.30 reports weak dependence structure almost for every pair except for India, China and South Africa (edge: 3,5;4) and Russia, India, China and South Africa (edge: 2,5;3,4). The corresponding graphical representation of the D-vine copula during the post-crisis period is reported in Appendix B.

4.2.3 R-Vine Copulas

Using the same selection and estimation procedures, the r-vine copulas were selected and estimated as shown below.

Step 1: Selection of R-vine Copulas

Table 4.31: R-vine selection: Pre-crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	1,4	t	0.09	8.85	0.06	0.02	0.02
	5,1	t	0.26	5.02	0.17	0.11	0.11
	5,2	t	0.34	4.65	0.22	0.15	0.15
	5,3	t	0.3	4.46	0.19	0.14	0.14
2	5,4;1	I	-	-	0	-	-
	2,1;5	BB1	0.12	1.06	0.11	0.08	0
	3,2;5	t	0.13	8.75	0.08	0.02	0.02
3	2,4;5,1	C	0.08	0	0.04	-	0
	3,1;2,5	SJ	1.08	0	0.04	-	0.1
4	3,4;2,5,1	I	-	-	0	-	-

The log-likelihood value of the selection is 312.47 with the following criteria: AIC: -596.94; BIC: -525. Table 4.31 shows that for pre-crisis period, Student's t copula was chosen for tree 1, and the rest of the trees suggest other families of copulas.

Table 4.32: R-vine selection: Crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	3,4	t	0.28	5.58	0.18	0.1	0.1
	5,1	SBB7	1.59	0.48	0.36	0.24	0.46
	5,2	t	0.66	3.29	0.46	0.4	0.4
	5,3	t	0.45	3.18	0.3	0.28	0.28
2	5,4;3	t	0.12	9.9	0.08	0.01	0.01
	2,1;5	t	0.24	4.72	0.15	0.11	0.11
	3,2;5	t	0.18	5.48	0.12	0.08	0.08
3	2,4;5,3	t	0	12.28	0	0	0
	3,1;2,5	t	0.12	6.55	0.08	0.04	0.04
4	1,4;2,5,3	t	0.07	11.33	0.05	0.01	0.01

The log-likelihood value of the selection is 847.93 with the following criteria: AIC: -1655.85; BIC: -1557.36. Table 4.32 shows that the Student's t copula is the best in modelling the dependence structure for most of pairs in the BRICS stock markets during the crisis period.

Table 4.33: R-vine selection: Post-crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	5,1	SBB7	1.28	0.26	0.23	0.07	0.28
	5,2	t	0.46	8.62	0.31	0.09	0.09
	5,3	t	0.41	8.55	0.27	0.07	0.07
	5,4	t	0.25	6.25	0.16	0.07	0.07
2	2,1;5	t	0.23	11.39	0.14	0.02	0.02
	3,2;5	t	0.15	7.9	0.09	0.03	0.03
	4,3;5	t	0.12	11.57	0.08	0.01	0.01
3	3,1;2,5	N	0.1	0	0.06	-	-
	4,2;3,5	t	0.08	9.33	0.05	0.01	0.01
4	4,1;3,2,5	Tawn180	5.9	0	0	-	0

The log-likelihood value of the selection is 847.52 with the following criteria: AIC:

-1655.03; BIC: -1556.54. As in most tables, Table 4.33 shows that the Student's t copula is the best in modelling the dependence structure for most of pairs in the BRICS stock markets during post-crisis period.

Step 2: Estimation Results of R-Vine Copulas

Table 4.34: R-vine estimation: Pre-crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	1,4	t	0.09 (0.03)	8.85 (2.65)	0.06	0.02	0.02
	5,1	t	0.26 (0.03)	5.02 (0.86)	0.17	0.11	0.11
	5,2	t	0.33 (0.03)	4.63 (0.75)	0.22	0.15	0.15
	5,3	t	0.29 (0.03)	4.44 (0.71)	0.19	0.14	0.14
2	5,4;1	I	-	-	0	-	-
	2,1;5	BB1	0.12 (0.05)	1.06 (0.02)	0.11	0.08	0
	3,2;5	t	0.13 (0.03)	8.72 (2.60)	0.08	0.02	0.02
3	2,4;5,1	C	0.06 (0.04)	-	0.03	-	0
	3,1;2,5	SJ	1.06 (0.03)	-	0.03	-	0.07
4	3,4;2,5,1	I	-	-	0	-	-

The log-likelihood value of the selection is 311.81 with the following criteria: AIC: -592.62; BIC: -523.69. Table 4.34 shows that only Russia and South Africa had strong dependence (edge: 5,2); the rest of the pairs exhibited weak dependence structure. The graphical representation of this dependence structure during the pre-crisis period is reported in Appendix C.

Table 4.35: R-vine estimation: Crisis period

Tree	Edge	Cop	Par	Par2	Tau	Utd	Ltd
1	3,4	t	0.28 (0.03)	5.58 (1.17)	0.18	0.1	0.1
	5,1	SBB7	1.59 (0.06)	0.48 (0.06)	0.36	0.24	0.46
	5,2	t	0.66 (0.02)	3.26 (0.42)	0.46	0.4	0.4
	5,3	t	0.45 (0.03)	3.17 (0.42)	0.3	0.28	0.28
2	5,4;3	t	0.12 (0.03)	9.89 (3.37)	0.08	0.01	0.01
	2,1;5	t	0.24 (0.03)	4.72 (0.81)	0.15	0.11	0.11
	3,2;5	t	0.18 (0.04)	5.47 (1.16)	0.12	0.08	0.08
3	2,4;5,3	t	0.01 (0.03)	10.00 (3.12)	0	0.01	0.01
	3,1;2,5	t	0.12 (0.04)	6.57 (1.49)	0.08	0.04	0.04
4	1,4;2,5,3	t	0.07 (0.03)	10.00 (3.04)	0.04	0.01	0.01

The log-likelihood value of the selection is 847.93 with the following criteria: AIC: -1655.85; BIC: -1557.36. Table 4.35 shows that during crisis period, the dependence structure between South Africa and Russia was strong – almost double its pre-crisis level. The second-highest dependence structure was between South Africa and Brazil. There was asymmetric dependence between South Africa and Brazil, with lower dependence of 0.46 and upper tail dependence of 0.24. The graphical representation of the R-vine copula during crisis period is shown in Appendix C.

Table 4.36: R-vine estimation: Post-crisis period

Tree	**Edge**	**Cop**	**Par**	**Par2**	**Tau**	**Utd**	**Ltd**
1	5,1	SBB7	1.28 (0.04)	0.26 (0.05)	0.23	0.07	0.28
	5,2	t	0.46 (0.02)	8.53 (2.10)	0.3	0.09	0.09
	5,3	t	0.41 (0.02)	8.51 (2.17)	0.27	0.07	0.07
	5,4	t	0.24 (0.03)	6.23 (1.29)	0.16	0.07	0.07
2	2,1;5	t	0.23 (0.03)	10.00 (2.81)	0.15	0.02	0.02
	3,2;5	t	0.14 (0.03)	7.85 (1.94)	0.09	0.03	0.03
	4,3;5	t	0.12 (0.03)	10.00 (2.81)	0.08	0.01	0.01
3	3,1;2,5	N	0.10 (0.03)	-	0.06	-	-
	4,2;3,5	t	0.08 (0.03)	9.35 (2.81)	0.05	0.01	0.01
4	4,1;3,2,5	Tawn180	20.00 (0.00)	0.01 (NaN)	0.01	-	0.01

The log-likelihood value of the selection is 110.43 with the following criteria: AIC: -182.86; BIC: -83.3. Table 4.36 shows that for post-crisis period, most of the dependence level decreased compare to their crisis period levels. Relatively strong dependence was found between South Africa and Russia followed by between South Africa and Brazil. The graphical representation of this R-vine copula during the post-crisis period is shown in Appendix C.

AIC for Estimating Vine Models

In this section, the AIC criterion was used to compare different vine copula models during different sub-sample periods. These AIC figures are reported in Table 4.37 below.

Table 4.37: Comparison of vine copulas

	Vines	AIC
Pre-Crisis	R	-596.9361
	C	-587.8597
	D	-574.9371
Crisis	R	-1655.853
	C	-1664.046
	D	-1654.828
Post-Crisis	R	-1116.92
	C	-1117.33
	D	-1107.132

It can be clearly seen in Table 4.37 that for during the pre-crisis period, the r-vine copula model best fit the dependence structure in the BRICS stock markets, while the c-vine copula model best fit the dependence structure in the BRICS market during both crisis and post-crisis periods.

CHAPTER V: CONCLUSION

This thesis intended to simultaneously use the c-vine, d-vine and r-vine copula models to investigate the dependence structure among the BRICS stock markets. Daily stock price data spanning from 28-12-2000 to 10-08-2018 was used. The entire sample data was then divided into three sub-samples in order to understand the dynamics of the dependence structure during different economic periods. The three sub-sample periods were the crisis period (from 01-02-2007 to 29-12-2011), the pre-crisis period (from 28-12-2000 to 31-01-2007) and the post-crisis period (from 04-01-2012 to 10-08-2018). The price data was firstly converted to return series and filtered using different ARIMA-GARCH models in order to remove the autocorrelation and heteroscedasticity effects. The filtered returns series were thereafter obtained and used in the modelling the dependence analysis using the three types of vine copulas named above.

Empirical results showed that during the pre-crisis period when c-vine copula model was used, the Student's t copula was found to best model the dependence structure in the first tree while the rest of the trees used different families of copulas. However, when the d-vine copula model was used during the same sub-sample period, all trees used a mixture of elliptical and Archimedean copulas to model the dependence structure. During the same sub-sample period, the use of the r-vine copula model suggests that the dependence structure in the first tree is best modelled by a Student's t copula, while the rest of the trees show a mixture of families of copulas.

The dependence structure during the pre-crisis period was found to be weak for all three types of vine copulas, except the dependence between South Africa and India and South Africa and Russia, which exhibited strong dependence structure during this sub-sample period.

During the crisis period when the c-vine copula model was used, results suggest that the dependence structures in tree 1 and tree 2 were best modelled using Student's t copula. However, when the d-vine copula model was employed, it was found that the dependence structure in all trees was best modelled using the Student's t copula. Furthermore, when the r-vine copula model was used in the same sub-sample period, it was found that the dependence structures in tree 1 and tree 2 were best modelled using the Student's t copula.

The overall dependence structure during the crisis period was found to be increasingly strong for all three types of vine copula models – an indication that during crisis period stock markets tend to co-move more than during bull market periods. Strong dependence structure was found between South Africa and Russia, South Africa and Brazil, Brazil and Russia, etc. For example, the dependence between South Africa and Russia increased from 0.22 to 0.46, and the dependence between South Africa and Brazil increased from 0.17 to 0.36 when the c-vine copula model was used. The dependence between Brazil and Russia increased from 0.17 to 0.32, and the dependence between Russia, India, China and South Africa increased from 0.18 to 0.4 when the d-vine copula model was used. An asymmetric dependence between South Africa and Brazil increased from 0.16 to 0.36 with a strong lower tail when the r-vine copula model was used.

However, during the post-crisis period the dependence structures in most of the trees were found to be best modelled using Student's t copula for all the three types of vine copula models.

The dependence structure during the post-crisis period was found to decrease from the crisis period for all the three types of vine copula models. For example, the dependence between South Africa and Russia decreased from 0.46 to 0.3

when the c-vine copula model was used. The dependence between South Africa, Russia, China and India decreased from 0.4 to 0.24 when the d-vine was used, and the dependence between South Africa and Brazil decreased from 0.36 to 0.23 when the r-vine copula model was used.

This thesis attempted to identify the vine copula model that can best fit the dependence structure in stock markets during a specific economic period i.e. bull, bear or stable period. For this purpose, the thesis compared the AIC generated by each type of vine copula model. It was found that the r-vine copula model best fit the dependence structure in stock markets during pre-crisis period, whereas the c-vine copula model best fit the dependence structure in stock markets during both the crisis and post-crisis periods. These findings are very important not only for portfolio diversification purposes but also for economic planning.

Overall, the findings of this thesis showed a very strong dependence structure between South Africa and Russia, South Africa and India, and South Africa and Brazil during the pre-crisis, the crisis and the post-crisis periods, suggesting a financial integration between these three countries. In addition, a strong dependence structure was found between China and the rest of BRICS markets only during financial crisis.

The thesis identified two types of dependence structure in the BRICS stock markets: the first was between small economies (South Africa, Brazil and Russia), and the second was between large economies (China and India). Small economies tend to co-move during bull and bear markets while large economies co-move with the rest only during bear market periods.

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APPENDIX A: C-VINE GRAPHICAL REPRESENTATION

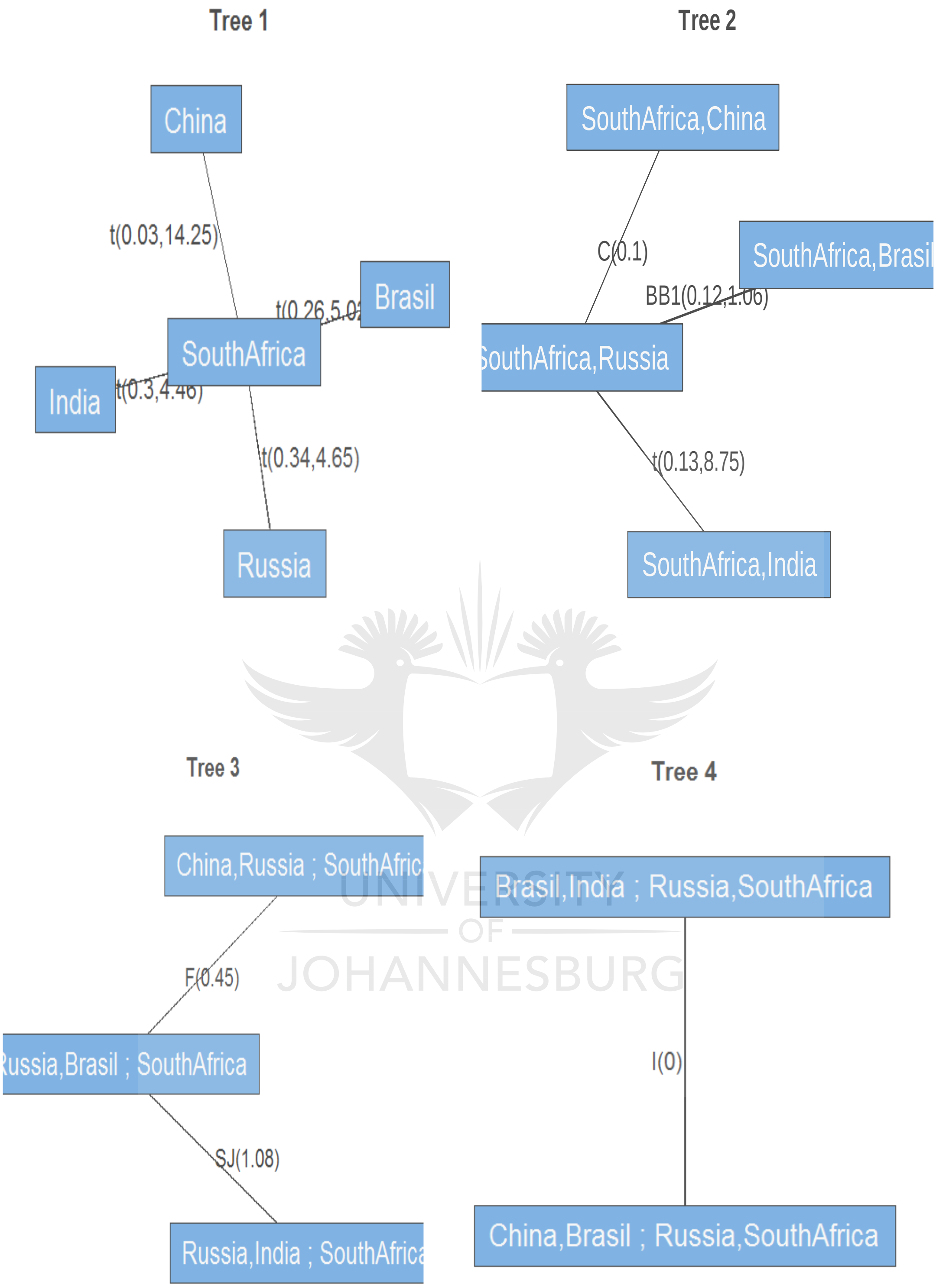

Figure 4.13: Tree plots of c-vine: Pre-crisis period

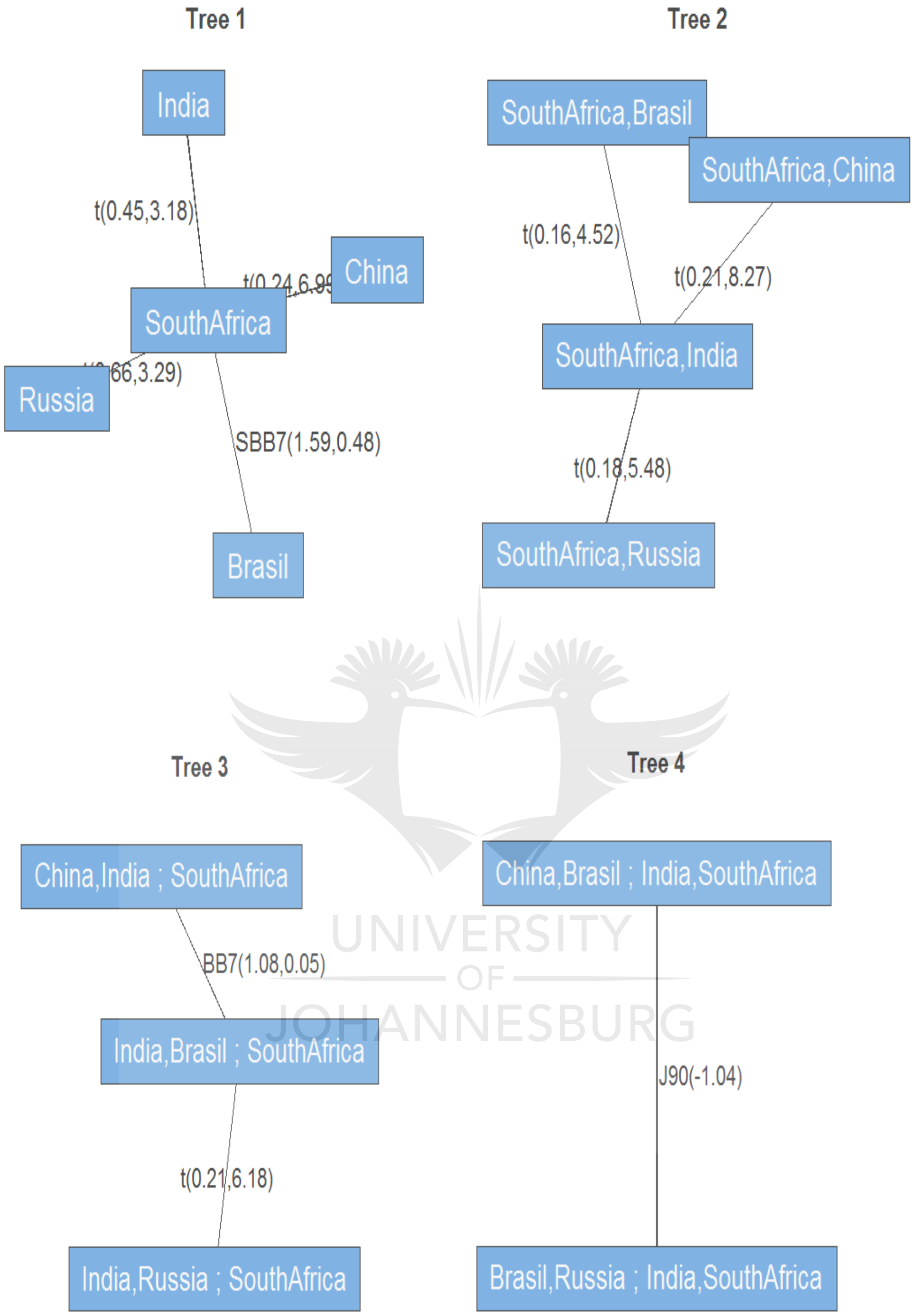

Figure 4.14: Tree plots of c-vine: Crisis period

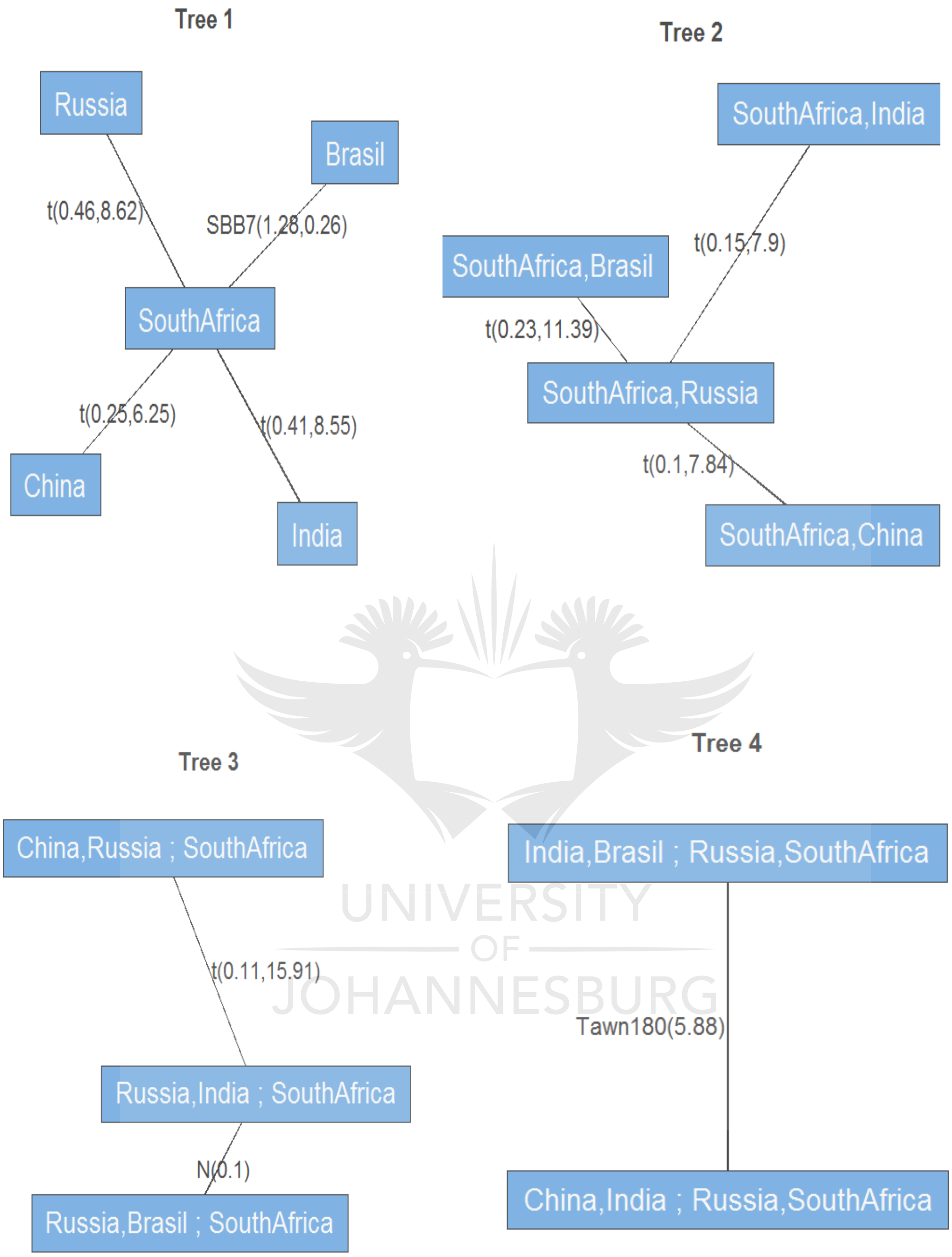

Figure 4.15: Tree plots of c-vine: Post-crisis period

APPENDIX B: D-VINE GRAPHICAL REPRESENTATION

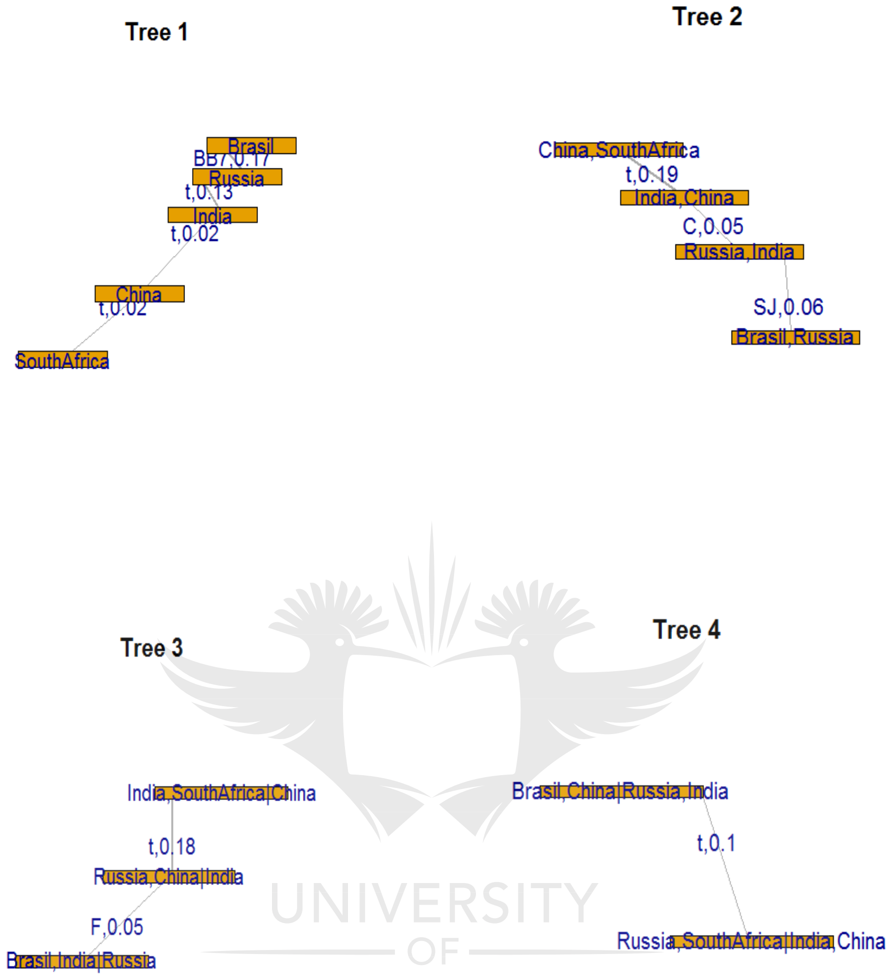

Figure 4.16: Tree plot of d-vine: Pre-crisis period

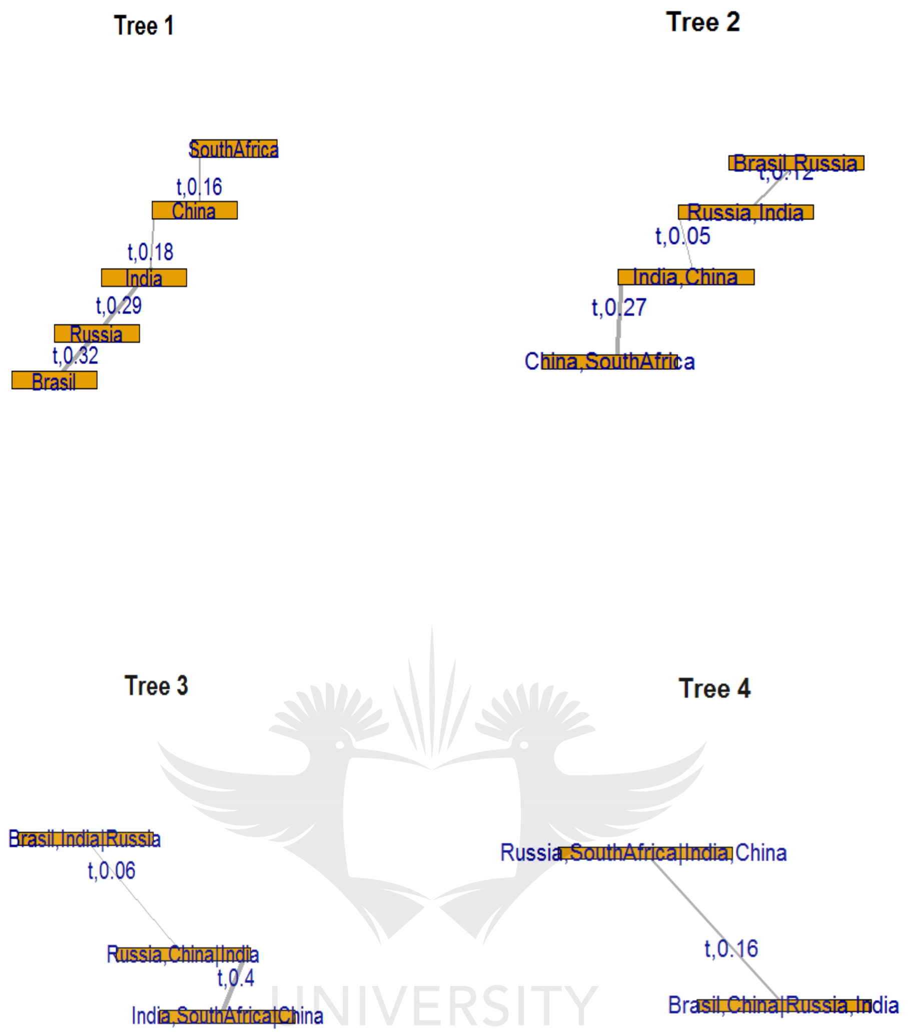

Figure 4.17: Tree plots of d-vines: Crisis period

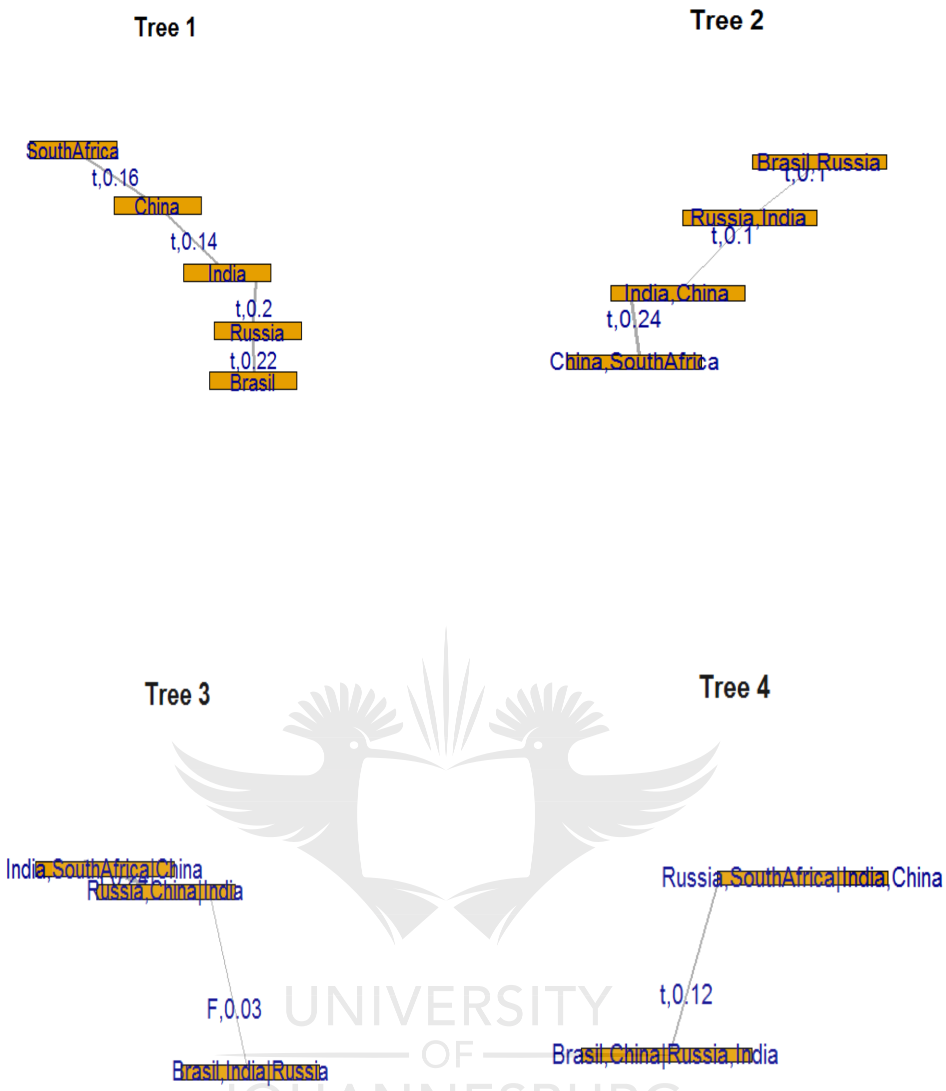

Figure 4.18: Tree plots of d-vine: Post-crisis period

APPENDIX C: R-VINE GRAPHICAL REPRESENTATION

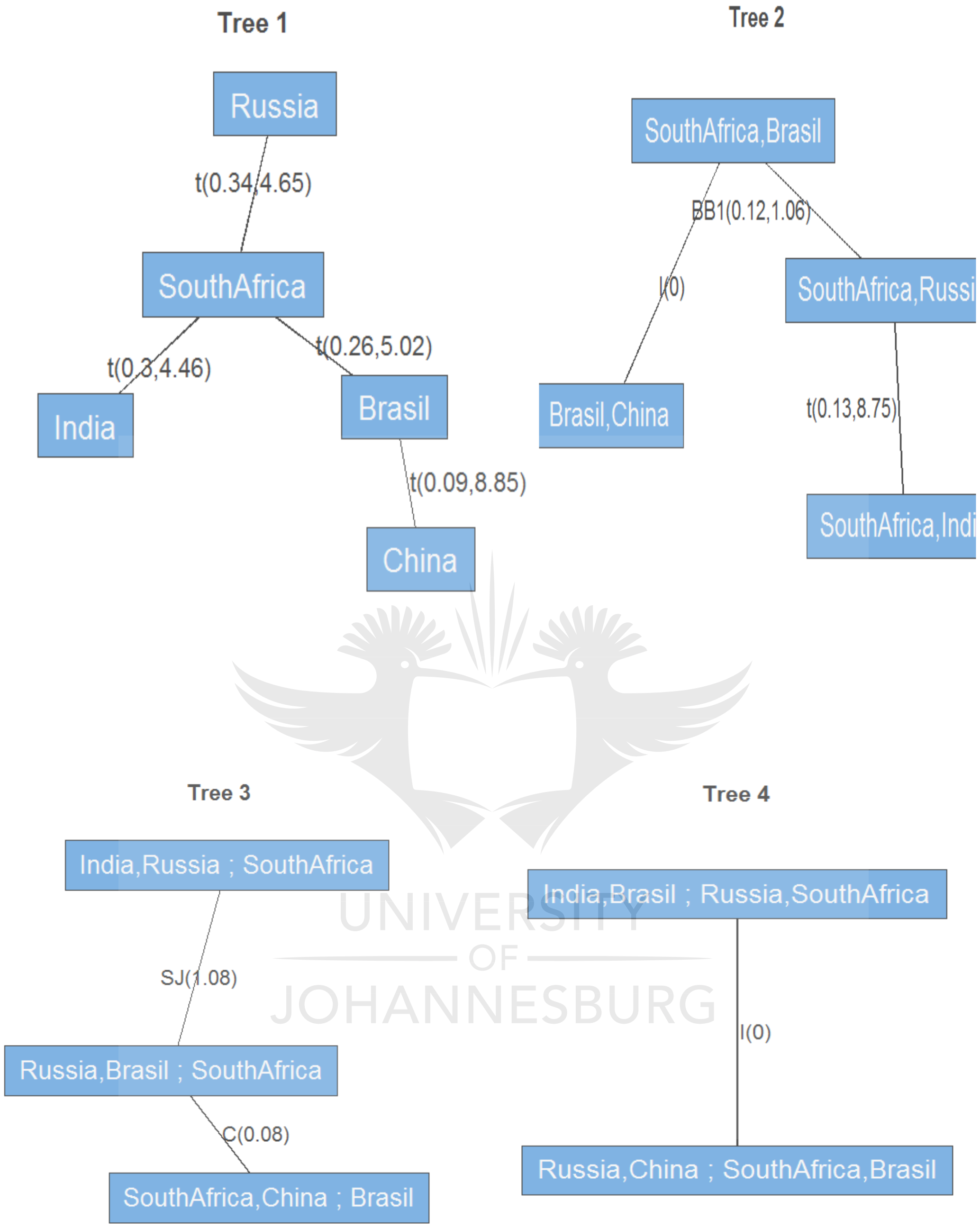

Figure 4.19: Tree plots of r-vine: Pre-crisis period

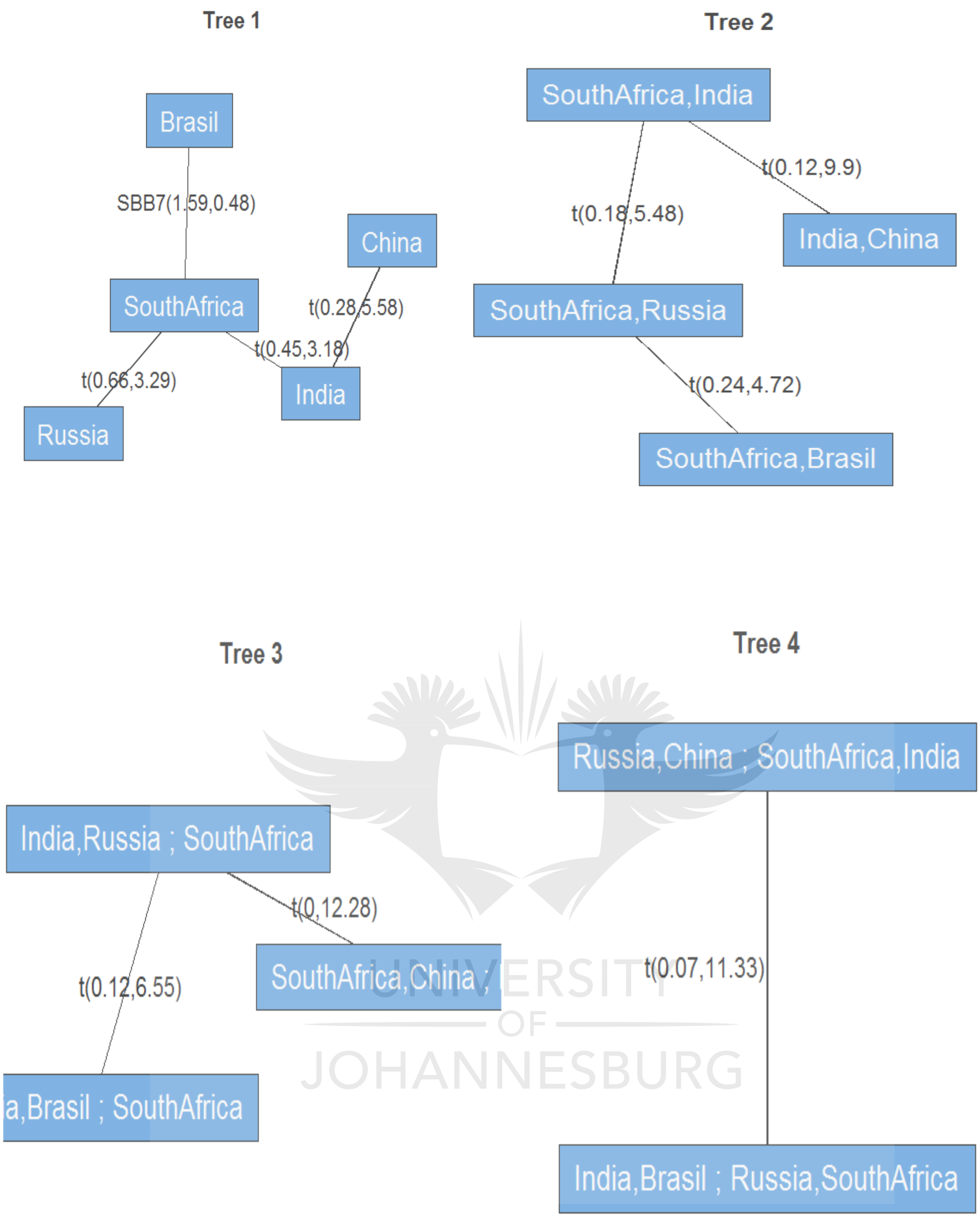

Figure 4.20: Tree plots of r-vine: Crisis period

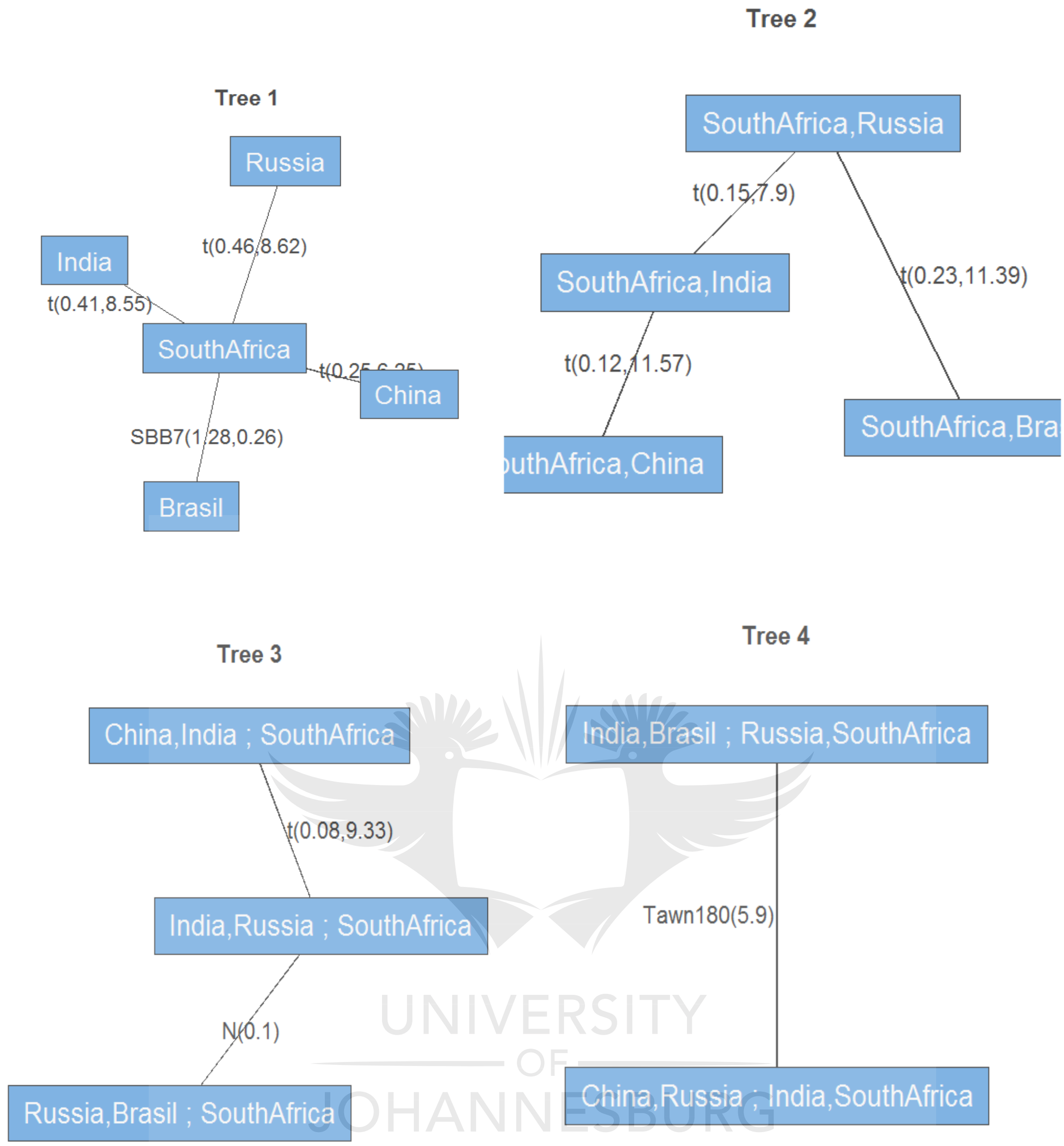

Figure 4.21: Tree plots of r-vine: Post-crisis period

APPENDIX D: ESTIMATED ARIMA-GARCH MODEL

Pre-crisis period

All GARCH models are GJR-GARCH with Student t-distributions.

Table 4.4: GARCH fit for Brazil: Pre-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.131358	0.04732	2.77598	0.005504
ar1	0.727001	0.127303	5.71077	0
ma1	-0.75228	0.123306	-6.10096	0
omega	0.101436	0.052404	1.93565	0.052911
alpha1	0.003553	0.007731	0.45961	0.645797
beta1	0.938031	0.020013	46.87014	0
gamma1	0.05986	0.02444	2.44926	0.014315
shape	6.371173	1.087523	5.85842	0

Table 4.5: GARCH fit for Russia: Pre-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.292065	0.047656	6.1286	0
ar1	-0.98775	0.002897	-340.9133	0
ma1	0.995135	0.000176	5648.8987	0
omega	0.347852	0.1376	2.528	0.011471
alpha1	0.066609	0.02734	2.4363	0.014839
beta1	0.798478	0.043641	18.2965	0
gamma1	0.133577	0.052619	2.5386	0.011131
shape	4.475133	0.559505	7.9984	0

Table 4.6: GARCH fit for India: Pre-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.137552	0.038972	3.5295	0.000416
ar1	-0.63762	0.118052	-5.4012	0
ma1	0.728504	0.104192	6.992	0
omega	0.238702	0.049986	4.7754	0.000002
alpha1	0.040894	0.023011	1.7772	0.075543
beta1	0.713797	0.03709	19.2452	0
gamma1	0.285955	0.066274	4.3147	0.000016
shape	6.133245	0.963582	6.365	0

Table 4.7: GARCH fit China: Pre-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	-0.03186	0.039239	-0.81192	0.416841
ar1	-0.97758	0.004618	-211.69218	0
ma1	0.986947	0.00019	5196.56248	0
omega	0.092324	0.038558	2.39442	0.016646
alpha1	0.064131	0.018599	3.44809	0.000565
beta1	0.874903	0.024839	35.22308	0
gamma1	0.070936	0.040806	1.73837	0.082146
shape	4.173259	0.485967	8.58754	0

Table 4.8: GARCH fit for South Africa: Pre-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.093742	0.035613	2.632273	0.008482
ar1	0.020528	0.30672	0.066928	0.946639

ma1	0.031774	0.305254	0.10409	0.917098
omega	0.078806	0.032282	2.441177	0.014639
alpha1	0.039006	0.029298	1.331338	0.183078
beta1	0.849506	0.036568	23.230994	0
gamma1	0.128712	0.036392	3.536835	0.000405
shape	7.649627	1.310941	5.83522	0

Crisis period

All models are GJR-GARCH with GED distribution.

Table 4.9: GARCH fit for Brazil: Crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.1027	0.051104	2.0097	0.044465
ar1	0.70749	0.08094	8.741	0
ma1	-0.74827	0.074012	-10.110	0
omega	0.10037	0.036458	2.753	0.005905
alpha1	0.10371	0.026797	3.8701	0.000109
beta1	0.87447	0.024418	35.8118	0
shape	1.37354	0.086472	15.8841	0

Table 4.10: GARCH fit for Russia: Crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.100226	0.056817	1.764	0.07773
ar1	-0.917	0.065439	-14.01	0
ma1	0.888852	0.074192	11.9805	0
omega	0.06373	0.026919	2.3675	0.017909
alpha1	0.096709	0.024238	3.9899	0.000066

beta1	0.895755	0.019543	45.8342	0
shape	1.376283	0.096383	14.2793	0

Table 4.11: GARCH fit for India: Crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.084585	0.048369	1.7487	0.080336
ar1	0.23983	0.017602	13.6254	0
ma1	-0.2213	0.016375	-13.52	0
omega	0.053713	0.024006	2.2375	0.025254
alpha1	0.093603	0.021566	4.3403	0.000014
beta1	0.896922	0.020592	43.557	0
shape	1.303019	0.108076	12.0565	0

Table 4.12: GARCH fit for China: Crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.067156	0.066914	1.0036	0.315561
ar1	-0.046	0.019663	-2.3212	0.020276
ma1	0.036085	0.015694	2.2993	0.021488
omega	0.019042	0.013533	1.4071	0.159397
alpha1	0.042998	0.008612	4.9926	0.000001
beta1	0.95293	0.008687	109.6997	0
shape	1.277575	0.067643	18.887	0

Table 4.13: GARCH fit for South Africa: Crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.088995	0.036898	2.4119	0.015868
ar1	-0.2574	0.018998	-13.55	0
ma1	0.268679	0.019214	13.9837	0
omega	0.044718	0.017669	2.5308	0.01138
alpha1	0.10996	0.022731	4.8375	0.000001
beta1	0.875815	0.02092	41.8647	0
shape	1.552062	0.122031	12.7186	0

Post-crisis period

Except Brazil, GARCH fit for Russia, India, China and South Africa all use sGARCH with std and no mean equation.

Table 4.14: GARCH fit for Brazil: Post-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.073335	0.020691	3.5443	0.000394
ar1	-0.978	0.005254	-186.1124	0
ma1	0.990773	0.000133	7469.4586	0
omega	0.025469	0.011154	2.2833	0.02241
alpha1	0.073679	0.015037	4.8999	0.000001
beta1	0.900688	0.021576	41.7446	0
shape	6.797314	1.204904	5.6414	0

Table 4.15: GARCH fit for Russia: Post-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.038734	0.027009	1.4341	0.151547
omega	0.03591	0.021799	1.6473	0.099489
alpha1	0.053397	0.017694	3.0178	0.002546
beta1	0.923432	0.028153	32.8004	0
shape	5.486152	0.805749	6.8088	0

Table 4.16: GARCH fit for India: Post-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.08654	0.022628	3.8245	0.000131
omega	0.012171	0.006037	2.016	0.043805
alpha1	0.039055	0.008507	4.591	0.000004
beta1	0.948145	0.009461	100.2119	0
shape	5.960128	0.959924	6.209	0

Table 4.17: GARCH fit for China: Post-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.045223	0.022904	1.9745	0.04833
omega	0.011965	0.006346	1.8853	0.059385
alpha1	0.062316	0.014429	4.3189	0.000016
beta1	0.936684	0.012073	77.5858	0
shape	4.015723	0.393194	10.2131	0

Table 4.18: GARCH fit for South Africa: Post-crisis period

Robust	Standard	Errors:		
	Estimate	Std. Error	t value	Pr(>|t|)
mu	0.07381	0.02072	3.5622	0.000368
omega	0.025188	0.016811	1.4983	0.134044
alpha1	0.071106	0.023862	2.9799	0.002883
beta1	0.903689	0.038432	23.5142	0
shape	6.562716	1.125472	5.8311	0