Light-light and heavy-light mesons in the model of QCD string with quarks at the ends

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Abstract. The variational einbein field method is applied to the model of the QCD string with quarks at the ends for the case of light–light and heavy–light mesons. Special attention is payed to the proper string dynamics. The correct string slope of the Regge trajectories is reproduced for light–light states which comes out from the picture of rotating string. Masses of several low-lying orbitally and radially excited states in the D, D_s, B, and B_s meson spectra are calculated and a good agreement with the experimental data as well as with recent lattice calculations is found. The role of the string correction to the interquark interaction is discussed at the example of the identification of $D^{*\prime}(2637)$ state recently claimed by DELPHI Collaboration. For the heavy–light mesons the standard constants used in Heavy Quark Effective Theory are extracted and compared to the results of other approaches.

INTRODUCTION

One of the most beautiful phenomena observed in QCD — namely, the formation of an extended string between the colour sources, implies that the string degrees of freedom in hadrons should be taken into account in the proper way. In the present contribution the model of the QCD string with quarks at the ends is used to investigate the spectra of the light–light and heavy–light mesons. The variational einbein field method (see [1] and references therein) is used in numerical calculations of the spectra.

LIGHT-LIGHT MESONS

Starting from the gauge invariant Green's function of a $q\bar{q}$ meson, performing integration in the path integral over the fermionic and gluonic fields, and using the minimal area law assumption in the latter case, one can extract the Lagrangian of the spinless quark-antiquark system in the form (see *e.g.* [2])

$$L(t) = -m_1\sqrt{\dot{x}_1^2} - m_2\sqrt{\dot{x}_2^2} - \sigma\int_0^1 d\beta\sqrt{(\dot{w}w')^2 - \dot{w}^2 w'^2}, \tag{1}$$

where we synchronize the quark times, $x_{10} = x_{20} = t$, and choose the minimal-string profile function in the straight-line form, $w_\mu(t,\beta) = \beta x_{1\mu}(t) + (1-\beta)x_{2\mu}(t)$.

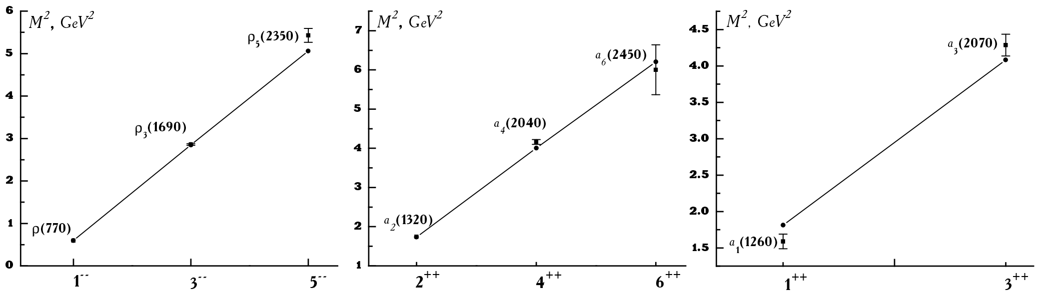

FIGURE 1. The lowest Regge trajectories for the light–light mesons (experimental data are given by boxes with error bars).

If the einbeins $\mu_{1,2}$ and $\nu(\beta)$ are now introduced in (1) to get rid of the square roots, then the centre-of-mass Hamiltonian for the case of massless quarks reads $(\mu_1 = \mu_2 = \mu)$

$$H = \frac{p_r^2}{\mu} + \mu + U(\mu, r), \qquad U(\mu, r) = \frac{\sigma r}{y} \arcsin\, y + \mu y^2, \tag{2}$$

where the extremum in ν is already taken at the level of the Hamiltonian yielding $\nu_{\rm ext}(\beta) = \sigma r(1 - 4y^2(\beta - \frac{1}{2})^2)^{-1/2}$, so that y is the solution to the transcendental equation

$$\frac{L}{\sigma r^2} = \frac{1}{4y^2}(\arcsin\, y - y\sqrt{1-y^2}) + \frac{\mu y}{\sigma r}. \tag{3}$$

The spectrum of the Hamiltonian (2) is found using the quasiclassical method with the consequent minimization of each eigenvalue with respect to the einbein μ, which is thus treated as a variational parameter, playing the role of the effective constituent mass of the quark. It appears due to the interaction and takes the value of $200 - 300 MeV$ even if one starts with zero current quark mass. The results of numerical calculations with $\sigma = 0.17 GeV^2$ are given in Fig.1. The interested reader can find details in papers [1]. Note, that the Regge trajectories for the light–light mesons remain nearly straight-line up to very low momenta and the only fitting parameter is the overall negative shift $\Delta M^2 \sim -1 GeV$, one and the same for all three trajectories. Another important comment is that the correct string slope of the trajectories, $2\pi\sigma$, appears quite naturally in the given approach as a consequence of the rotating string inertia properly taken into account in the Hamiltonian (2), so that the form of the effective potential $U(\mu, r)$ does not amount to the naive sum of the centrifugal barrier for the quarks and the linearly rising potential σr [1].

HEAVY-LIGHT MESONS

In the calculations of the light-light meson spectra we totally ignored the quark spin, concentrating on the proper account of the string dynamics. Now, to have reliable

TABLE 1. **Splittings for the D, D_s, B, and B_s mesons in MeV.**

Splitting	D_s–D	D_s^*–D^*	D^*–D	D_s^*–D_s	B_s–B	B_s^*–B^*	B^*–B	B_s^*–B_s
Experiment	99	102	141	144	90	91	46	47
Theory	114	115	146	147	100	102	63	65

predictions for the heavy–light meson masses, we supply the Hamiltonian for spinless quarks connected by the string,

$$H_0=\sum_{i=1}^{2}\left(\frac{\vec{p}^2+m_i^2}{2\mu_i}+\frac{\mu_i}{2}\right)+\sigma r-\frac{\sigma(\mu_1^2+\mu_2^2-\mu_1\mu_2)}{6\mu_1^2\mu_2^2}\frac{\vec{L}^2}{r}, \tag{4}$$

by spin-dependent corrections due to confining and the OGE interaction ($\kappa=-\frac{4}{3}\alpha_s$),

$$V_{sd}=\frac{8\pi\kappa}{3\mu_1\mu_2}(\vec{S}_1\vec{S}_2)\,|\psi(0)|^2+\frac{\kappa}{\mu_1\mu_2r^3}\Big(3(\vec{S}_1\vec{n})(\vec{S}_2\vec{n})-(\vec{S}_1\vec{S}_2)\Big)-\frac{\sigma}{2r}\left(\frac{\vec{S}_1\vec{L}}{\mu_1^2}+\frac{\vec{S}_2\vec{L}}{\mu_2^2}\right) \tag{5}$$
$$+\frac{\kappa}{r^3}\left(\frac{1}{2\mu_1}+\frac{1}{\mu_2}\right)\frac{\vec{S}_1\vec{L}}{\mu_1}+\frac{\kappa}{r^3}\left(\frac{1}{2\mu_2}+\frac{1}{\mu_1}\right)\frac{\vec{S}_2\vec{L}}{\mu_2}+\frac{\kappa^2}{2\pi\mu^2r^3}\Big(\vec{S}\vec{L}\Big)\,(1.43-\ln(\mu r)),$$

as well as by the Coulomb interaction $-\frac{4}{3}\frac{\alpha_s}{r}$ and the overall negative constant shift $-C_0$. The latter remains the only fitting parameter, whereas we use the standard values for others: $\sigma=0.17GeV^2$, $m_u=5MeV$, $m_d=9MeV$, $\alpha_s=0.4$ for D mesons and $\alpha_s=0.39$ for B's. The last term on the r.h.s. of equation (4) is the string correction which is always negative and accounts for the proper string dynamics. The results of numerical calculations for the spectra of orbitally and radially excited D, D_s, B, and B_s mesons, as well as comparison with the lattice data and results of other approaches, can be found in [1, 3]. In Table 1 we give the splittings for the above-mentioned mesons compared to the experimental values. Let us also quote the masses of the radially $(n=1)$ and orbitally $(l=2)$ excited D mesons: $M(0^-)=2664MeV$, $M(2^-)=2663MeV$, $M(3^-)=2654MeV$. Thus the 3^- state is the lightest one and it is the most probably candidate for the resonance $D^{*\prime}(2637)$ recently claimed by the DELPHI Collaboration [4]. Such an identification resolves the problem usually encountered in the framework of the quark models: being too narrow this meson can not be associated with the first radial excitation, whereas model predictions for orbitally excited states lie about $50-60MeV$ higher than needed. In our approach the proper string dynamics, in the form of the correction to the Hamiltonian, lowers the energy of the orbitally excited state 3^-. It is also instructive to note, that the fitted value of the parameter C_0 is insensitive to the heavy quark and depends only on the light-quark content of the meson, that supports the idea that it is due to the self-energy of the latter.

BRIDGE TO HEAVY QUARK EFFECTIVE THEORY

The suggested approach allows us to find analytic formulae for the constants used in the Heavy Quark Effective Theory, as well as to evaluate them numerically. In the standard

TABLE 2. **Standard parameters used in HQET.**

	$m_1 \to \infty$ $m_2 \to 0$	**Fit (7)**	**Sum rules [5]**	***B* mesons decays [6]**	**DS equation [7]**
$\bar{\Lambda}$, *GeV*	0.471	0.485	$0.4 \div 0.5$	0.39 ± 0.11	0.493/0.288
λ_1, GeV^2	-0.506	-0.379	-0.52 ± 0.12	-0.19 ± 0.10	-
λ_2, GeV^2	0.21	0.17	0.12	0.12	-

parameterization the mass of a heavy-light meson is

$$M_{hl} = m_Q + \bar{\Lambda} - (\lambda_1 + d_H\lambda_2)/2m_Q + O\left(1/m_Q^2\right) \tag{6}$$

with d_H being +3 for 0^- states or -1 for 1^- ones. For the idealized case ($m_1 \to \infty$, $m_2 = 0$) our formulae simplify considerably, giving analytical expressions for all three constants [1]. A more reliable way to estimate two of them is to find the best fit of the form

$$M_{fit} = m_Q + \bar{\Lambda} + C_0 - \lambda_1/2m_Q \tag{7}$$

with C_0 fixed by fitting the experimental spectrum, as discussed above, and varying m_Q around the bottom quark mass. The results are listed in Table 2, where they are compared with those of other approaches, demonstrating good agreement with the latter.

CONCLUSIONS

In conclusion let us emphasize that the proper dynamics of the gluonic degrees of freedom in hadrons, taken into account in the form of an effective QCD string between quarks, are of paramount importance in establishing the hadron spectra of mass and their properties. Account for the inertia of the rotating string inside mesons allowed us to reproduce the correct string slope of the Regge trajectories, as well as to fit the spectrum of *D* and *B* mesons, and to resolve the problem of identification of the state recently claimed by the DELPHI Collaboration. The variational einbein field method used in the calculations is proved to be efficient and accurate, that allows one to use it in investigations of various relativistic systems.

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