Localization and Coherent Structures in Wave Dynamics via Multiresolution

Antonina N. Fedorova, Michael G. Zeitlin

Mathematical Methods in Mechanics Group
Institute of Problems of Mechanical Engineering (IPME RAS)
Russian Academy of Sciences
Russia, 199178, St. Petersburg, V.O., Bolshoj pr., 61
zeitlin@math.ipme.ru, anton@math.ipme.ru
http://www.ipme.ru/zeitlin.html
http://www.ipme.nw.ru/zeitlin.html

Abstract

We apply variational-wavelet approach for constructing multiscale high-localized eigenmodes expansions in different models of nonlinear waves. We demonstrate appearance of coherent localized structures and stable pattern formation in different collective dynamics models.

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We apply variational-wavelet approach for constructing multiscale high-localized eigenmodes expansions in different models of nonlinear waves. We demonstrate appearance of coherent localized structures and stable pattern formation in different collective dynamics models.

We consider the applications of a new numerical-analytical technique based on the methods of local nonlinear harmonic analysis or wavelet analysis to nonlinear wave dynamics problems. Such approach may be useful in all models in which it is possible and reasonable to reduce all complicated problems related with statistical/stochastic distributions to the problems described by systems of nonlinear ordinary/partial differential equations with or without some (functional) constraints (e.g. Kuramoto-Sivashinsky (KS) equation as a model of weak turbulence). Wavelet analysis gives us the possibility to work with well-localized bases in functional spaces and gives for the general type of operators (differential, integral, pseudodifferential) in such bases the maximum sparse forms. For KS equation

$$\Psi_t = -\Psi_{xxx} - \Psi_{xx} - \Psi\Psi_x \tag{1}$$

and in related models we use the following variational approach. Let L be an arbitrary (non) linear (polynomial/rational) differential/integral operator with matrix dimension d, which acts on some set of functions $\Psi \equiv \Psi(t,x) = \Big(\Psi^1(t,x),\dots,\Psi^d(t,x)\Big)$, $t,x \in \Omega \subset \mathbf{R}^{n+1}$ from $L^2(\Omega)$:

$$L\Psi \equiv L(Q,t,x)\Psi(t,x) = 0, \qquad Q \equiv Q_{d_1,d_2}(t,x,\Psi,\partial/\partial t,\partial/\partial x) = \sum_{i_1,i_2}^{d_1,d_2} a_{i_1 i_2}(t,x.\Psi)\Big(\frac{\partial}{\partial t}\Big)^{i_1}\Big(\frac{\partial}{\partial x}\Big)^{i_2} \tag{2}$$

Let us consider now the N mode approximation for solution as the following ansatz (in the same way we may consider different ansatzes):

$$\Psi^N(t,x) = \sum_{r,k}^{N} a_{rk} A_r \otimes B_k(t,x), \qquad k \in Z^d \tag{3}$$

We shall determine the coefficients of expansion from the following conditions (different related variational approaches are considered in [1]-[4]):

$$\ell^N_{k\ell} \equiv \int (L\Psi^N) A_k(t) B_\ell(x) \mathrm{dtdx} = 0 \tag{4}$$

So, we have exactly dN^{n+1} algebraical equations for dN^{n+1} unknowns a_{rk}. Such variational approach reduces the initial problem to the problem of solution of functional equations at the first stage and some algebraical problems at the second stage. The solution is parametrized by solutions of two set of reduced algebraical problems, one is linear or nonlinear (depends on the structure of operator L) and others are some linear problems related to computation of coefficients of algebraic equations (4). These coefficients can be found by some wavelet methods by using compactly supported wavelet basis functions for expansions (3). The constructed solution has the following multiscale/multiresolution decomposition via nonlinear high-localized "eigenmodes"

$$\Psi(t,x) = \sum_{(i,j)\in Z^{n+1}} a_{ij} A^i(t) B^j(x), \tag{5}$$

$$A^i(t) = A^{i,slow}_N(t) + \sum_{r\geq N} A^i_r(\omega^1_r s),\ \omega^1_r \sim 2^r, \qquad B^j(x) = B^{j,slow}_M(x) + \sum_{l\geq M} B^j_l(k_l x),\ k_l \sim 2^l$$

which corresponds to the full multiresolution expansion in all underlying time/space scales. Formula (5) gives us expansion into the slow part $\Psi^{slow}_{N,M}$ and fast oscillating parts for arbitrary N, M. So, we may move from coarse scales of resolution to the finest one to obtain more detailed information about our dynamical process. The first terms in the RHS of formulae (5) correspond on the global level of function space decomposition to resolution space and the second ones to detail space. This representation (5) provides the solution as in linear as in nonlinear cases without any perturbation technique but on the level of expansions in (functional) space of solutions. The using of wavelet basis with high-localized properties provides good convergence properties of constructed solution (5). As a result of good (phase)space/time localization properties we can construct high-localized coherent structures in spatially--extended stochastic systems with collective behaviour. In all these models numerical modelling demonstrates the appearance of coherent high-localized structures (Fig.1) and stable patterns formation (Fig.2).

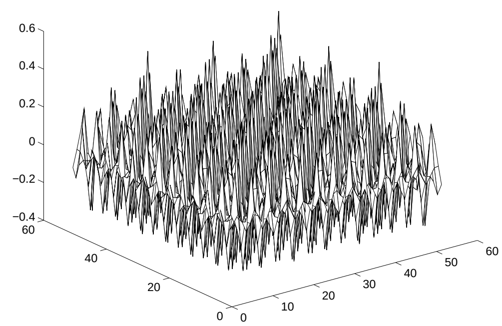

Figure 1: Appearance of coherent structure

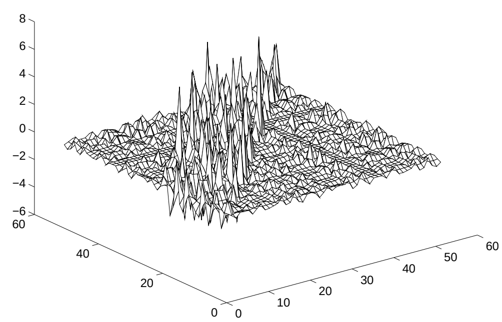

Figure 2: Stable pattern

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Antonina Fedorova, Russian Academy of Sciences, Institute of Problems of Mechanical Engineering,
V.O., Bolshoj pr., 61, 199178, St. Petersburg, Russia
email: anton@math.ipme.ru, http://www.ipme.ru/zeitlin.html, http://www.ipme.nw.ru/zeitlin.html