

Diffractive elements designed to suppress unwanted zeroth order due to surface depth error

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Abstract. We consider a design approach to reduce unwanted zero-order intensity due to profile depth error in diffractive elements. Our method is based on addition of local bias phase to a binary element phase, leading to the introduction of a third phase level. We show theoretically and experimentally that gratings obtained with such modifications are more tolerant to profile depth error than conventionally designed binary or multilevel elements, thus reducing the appearance of unwanted zero order.

1. Introduction

In the past few years diffractive optics has increasingly found its way into practical applications in a wide range of fields [1, 2]. Following this development, practical issues such as, for example, element alignment, stray light and tolerances to typical fabrication errors with the most common fabrication methods have received increasing attention. In the field of beam splitter and beam shaper design several authors have recently investigated methods to develop algorithms that can be used to realize designs with relaxed fabrication and alignment tolerances [3, 4]. The common feature between all these approaches, which are closely related to methods proposed for design of multiple-colour diffractive optic elements (DOEs) [5–8], is that they utilize diffractive element potential for multi-functionality, i.e. the fact the diffractive optical elements can be designed to realize several optical functions at once. The trade-off with such multi-functional design approaches is the need for more design freedoms during the design procedure compared to traditional approaches. Consequently, the proposed approaches work best when the number of possible phase levels is relatively high; none of the authors demonstrate their approach with elements with only a few phase levels.

With most modern techniques used in fabrication of diffractive elements the surface profile can be typically realized with high lateral precision. However, vertical profile errors are more difficult to control. Moreover, fabrication errors in replication processes such as injection moulding used to realize large series of

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diffractive elements typically also lead to variations in the profile depth. In general, and especially in the case of binary diffractive elements, surface depth errors result in an increase of the zero-order efficiency [9, 10]. If the signal cannot be moved off-axis, as is often the case, such an increase is highly undesirable in applications, for example, in material processing or optical interconnections, where precise control of the efficiency of individual diffraction orders is required.

In this paper we consider a design approach that can be used to relax the fabrication tolerances of diffractive elements based on a binary phase profile, focusing especially on suppressing unwanted light in the zeroth order due to etch depth errors. It should be immediately noted that even though the considered elements are initially binary phased, the approach we propose to relax the fabrication tolerances leads, as a trade-off, to the introduction of a third phase level. Thus it should be stressed that we are not claiming to design binary elements with relaxed fabrication tolerances. However, since our approach uses a binary solution as a starting point, the elements proposed have some of the key beneficial properties of binary diffractive elements along with the added tolerance to profile depth error at the cost of a third phase level.

The paper is organized as follows. The basic concept of the proposed approach is presented in sections 2 and 3. In section 4 some example designs are considered as we present a numerical verification of the usefulness of the proposed method, followed by experimental verification in section 5. Finally, possible validity of the approach in light of rigorous diffraction theory as well as possible extensions are discussed in section 6.

2. Basic approach

It is generally known that phase functions which are identical in terms of the thin element approximation when no surface depth errors are present can have quite different behaviour when errors are taken into account. Ehbets *et al.* used this to minimize the uniformity error sensitivity of continuous-relief fan-out elements by introducing a constant bias to the element phase and then re-wrapping it to the interval $[0 \dots 2\pi]$, i.e. by effectively shifting the positions of $0-2\pi$ transitions within the grating period [11]. In the case of binary gratings, such an approach does not work, as the introduction of a constant bias phase cannot shift the surface transitions or change the relative phase difference between the levels even when depth errors are taken into account. It is, however, possible to introduce a local bias phase in a way that does change the element performance in connection to surface depth errors.

Let us consider a binary grating with phase levels 0 and π . According to thin element approximation, parts of the grating can be lowered (or raised) by 2π to introduce a third phase level, as shown in figure 1, without changing the performance of the perfectly fabricated grating. The same is also true for the case where the local bias phase of 2π is added to the grating in such a way that two new phase levels are introduced. However, as we will later show, the latter case has no beneficial effect with regard to surface depth errors. In figure 2 the complex-amplitudes connected to the phase levels of the element are presented in the complex plane. For clarity it should be noted that for three-level elements the point at the intersections of the negative part of the real axis and the unit circle actually represents two separate complex amplitudes with different phase values, indicated

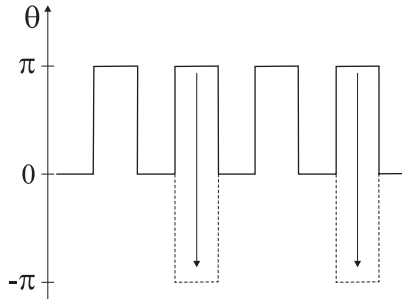


Figure 1. Schematic figure of the basic concept. Parts of the grating are lowered by 2π to introduce a third phase level.

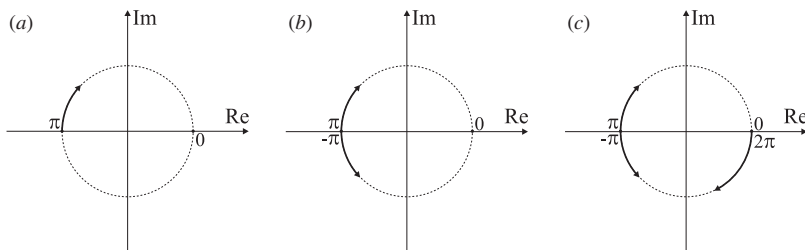


Figure 2. Effect of linear depth error in the complex plane with (a) two, (b) three and (c) four phase levels.

next to the axis, while for four level elements both intersections have two phase values, again indicated next to the axis. When a linear depth error in the form $h_{\text{actual}} = (1 + c)h_{\text{ideal}}$ is introduced, the complex amplitudes move on the unit circle as indicated by the arrows. If we consider the three different cases, i.e. binary, three and four level gratings, we see from figure 2 that the movement of the complex amplitudes due to depth error has inherent symmetry in the case of a three-level element, while with both binary and four elements no such symmetry can be seen. Intuitively this suggests that the zero-order efficiency, which is effectively the weighted average (with weights given by relative areas of each complex amplitude in the grating period) of the available complex amplitudes should be less sensitive to depth errors in the case of a three-level element than with binary or four level cases if the symmetry is properly utilized. We will now show that this is indeed true.

3. Mathematical formulation

For simplicity we consider a grating which is designed to have non-zero intensity only in the odd diffraction orders, i.e. so-called even orders missing (EOM) grating, with the understanding that the following can be easily generalized for an arbitrary binary grating with phase values 0 and π . An EOM grating exhibits symmetry which ensures that the total area of grating regions with a phase value of π is equal to the area with value 0, and consequently the zero-order efficiency of a perfectly fabricated element is zero [12]. Assuming illumination with a unit-amplitude plane wave and taking linear depth error into account, the efficiency

for the zeroth order of such an element is then, according to thin element approximation, given by

$$\eta_0 = |T_0|^2 = \left| \frac{1}{2} + \frac{1}{2} \exp [i(1+c)\pi] \right|^2 = \sin^2 (c\pi/2), \quad (1)$$

where c is the linear depth error factor in $h_{\text{actual}} = (1+c)h_{\text{ideal}}$. Note that this result is independent of the actual grating profile. We now proceed to introduce a third grating level by lowering some percentage of the grating areas with phase value π by 2π . Equation (1) then becomes

$$\begin{aligned} \eta_0 &= \left| \frac{1}{2} + \frac{1-M}{2} \exp [i(1+c)\pi] + \frac{M}{2} \exp [-i(1+c)\pi] \right|^2 \\ &= \sin^4 (c\pi/2) + \left(\frac{2M-1}{2} \right)^2 \sin^2 (c\pi), \end{aligned} \quad (3)$$

where M is a constant indicating what percentage of the grating regions with phase value π was lowered. From equation (2) we immediately see that by selecting $M = 1/2$, i.e. by lowering exactly half of the grating regions in question, we can reduce the unwanted zero order by a power of two compared to the binary case.

We next consider the case where some percentage of the whole grating is lowered by 2π to introduce 2 new phase levels. In this case equation (1) becomes

$$\begin{aligned} \eta_0 &= \left| \frac{1-M}{2} \{1 + \exp [i(1+c)\pi]\} + \frac{M}{2} \{1 + \exp [i(1+c)\pi]\} \exp [-i(1+c)2\pi] \right|^2 \\ &= \sin^2 (c\pi/2) [1 - 4(1-M)M \sin^2 (c\pi)], \end{aligned} \quad (3)$$

where M is now a constant indicating how large a portion of the entire grating was lowered. Again we see that the minimum is obtained by selection of $M = 1/2$, in which case the zero order is reduced by multiplication with factor $\cos^2 (c\pi)$ compared to the original binary case. With small depth errors this reduction is insignificant, as $\cos^2 (c\pi) \approx 1$. Thus we have confirmed through equations (1)–(3) that by introducing a third phase level and properly utilizing the symmetry seen in complex-amplitude change due to linear depth error, it is possible to significantly reduce the unwanted light in the zeroth order compared to both the binary and four level cases.

In the previous we considered only the zeroth order in connection with profile depth error. For binary elements this is sufficient, as depth error does not change the relative efficiency of the other diffraction orders. However, when additional phase level is introduced by lowering parts of the grating, this is no longer true and the change in relative efficiency of the other diffraction orders must be determined by taking into account both the amount of depth error and the grating profile considered. Therefore, in terms of uniformity, the optimal way to introduce the third phase level varies from grating to grating. It is, nevertheless, possible to outline some general rules. The three-level grating can be seen as a superposition of two binary gratings, one with depth corresponding to a phase of π and the other with depth matching a phase of 2π . The latter only appears when depth errors are present and deflects light from the zeroth order to higher orders. Thus the second grating, i.e. the parts of the original grating modified which are lowered, should be chosen so that the the light from the zeroth order is not deflected to any of the

signal orders. The easiest way to ensure this is to introduce local modifications with higher frequency than the main grating. Additionally, if the original grating is modulated only in one dimension, the second grating can be added in a perpendicular direction to further separate the signal and noise orders. Finally, it should be noted that since the local modifications are added to the original grating in a separate straightforward step, different strategies can be easily evaluated to find the optimum without a costly re-design step.

4. Theoretical results

We will now test the approach presented in the previous sections by considering some simple beam splitting designs. In the selected examples the incidence beam is split into 16 equal-intensity beams that are arranged in one ($1 \mapsto 16$) or two ($1 \mapsto 4 \times 4$) dimensional equally spaced array by means of a binary or a 16-level diffractive element. In all cases the designs were made using an iterative Fourier transform algorithm (IFTA) [13], and optimized in terms of both efficiency and uniformity error. Symmetries required to suppress all even orders were enforced during the design procedure, and consequently all designs have ideal zeroth order efficiency of 0%. The design values with binary elements for efficiency and uniformity error in the case of a one-dimensional array are $\eta = 80.8\%$ and $\Delta U = 0.3\%$, respectively, while for a two-dimensional case $\eta = 77.6\%$ and $\Delta U = 0.04\%$. For 16-level designs we have $\eta = 92.8\%$, $\Delta U = 0.05\%$ and $\eta = 91.4\%$, $\Delta U = 0.03\%$ for the 1D and 2D cases, respectively.

Figure 3 shows the grating profiles obtained from the binary designs by introduction of local bias phase, i.e. after the third phase level was introduced. In both cases the additional phase level was used to form a one-dimensional grating which has a frequency several times higher than the main grating. Additionally, in the case of the one-dimensional array, the main grating and the second grating formed by the addition of the third phase level were oriented perpendicularly to each other.

Performance of the three-level gratings was first evaluated by calculating the zeroth order efficiency, uniformity error and the diffraction efficiency as a function of the depth error coefficient c and comparing them to values obtained with the corresponding binary solution. Figures 4 and 5 show the results for the $1 \mapsto 16$ and the $1 \mapsto 4 \times 4$ designs, respectively. We see that in terms of zeroth order intensity, the three-level gratings created by introducing a local bias phase of 2π are superior compared to the corresponding binary elements. The trade-off of this performance improvement is the reduced performance in terms of diffraction efficiency in both cases and in terms of uniformity in the case for the two-dimensional array. However, for most applications utilizing beam splitters in the fields of material processing, optical interconnections or spatial filtering, the reductions in optical performance with the three-level element would be within acceptable specifications, whereas the rapid increase in the zeroth order efficiency seen in the case of the binary design would constitute a serious problem. Furthermore, it should be noted that the reduction in optical performance, especially in terms of the loss of uniformity, can be minimized by properly selecting the way the third phase level is introduced. Thus it can be concluded that optical performance of the three-level designs is more tolerant to surface profile depth error than the conventional binary solution.

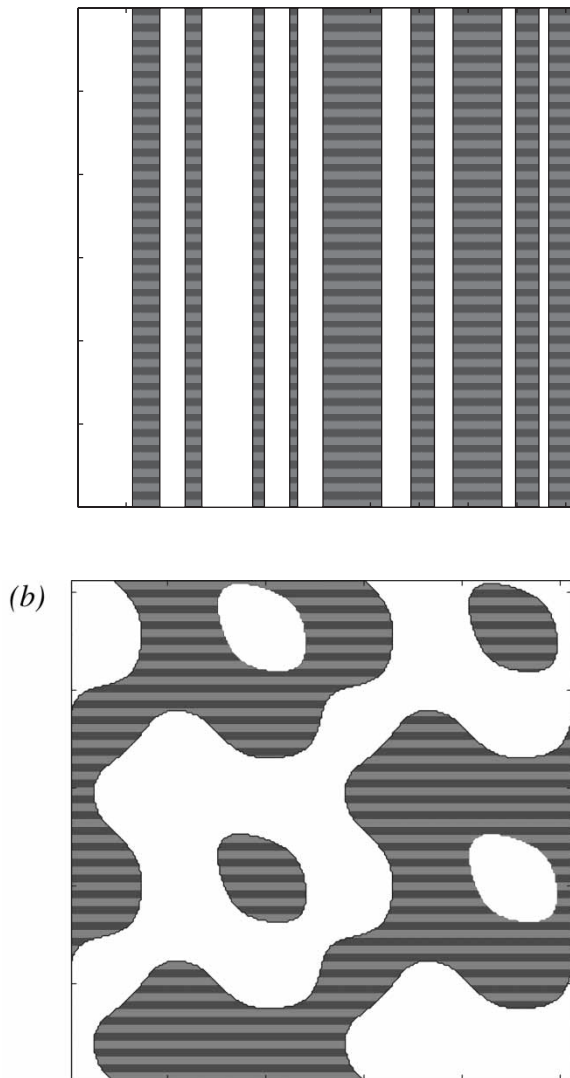


Figure 3. Considered three-level grating profile in the case of (a) $1 \mapsto 16$ and (b) $1 \mapsto 4 \times 4$ beam splitter. Phase levels: 0 (white), π (dark grey) and $-\pi$ (light grey).

Since fabrication of three-level gratings requires techniques that can be used with only little additional complexity to fabrication of elements with a higher number of phase levels, it is of interest to also compare these two approaches. Again the evaluation was done by calculating the zeroth order efficiency, uniformity error and the diffraction efficiency as a function of the depth error coefficient c . It should be noted that optimized global bias phases were added to the 16-level designs obtained with IFTA prior to the evaluation in order to increase the depth error tolerance of the designs in a manner suggested in [11]. The curves in the case of the 16-level designs can also be seen in figures 4 and 5 for the $1 \mapsto 16$ and the $1 \mapsto 4 \times 4$ designs, respectively. We again see that in terms of zeroth order intensity, the three-level gratings created by introducing a local bias phase of 2π are superior. The figures also show that, contrary to the comparison

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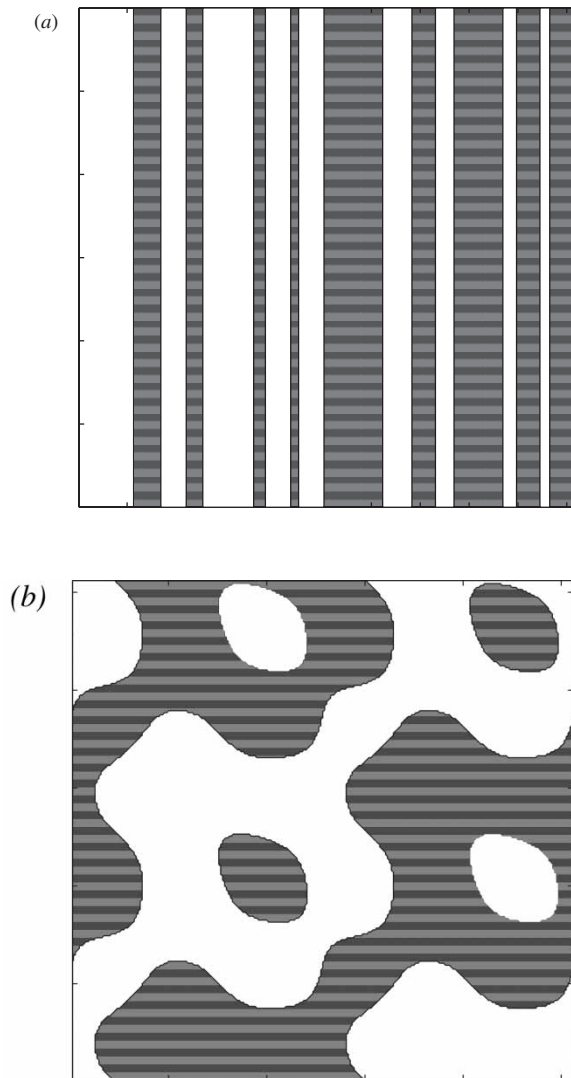


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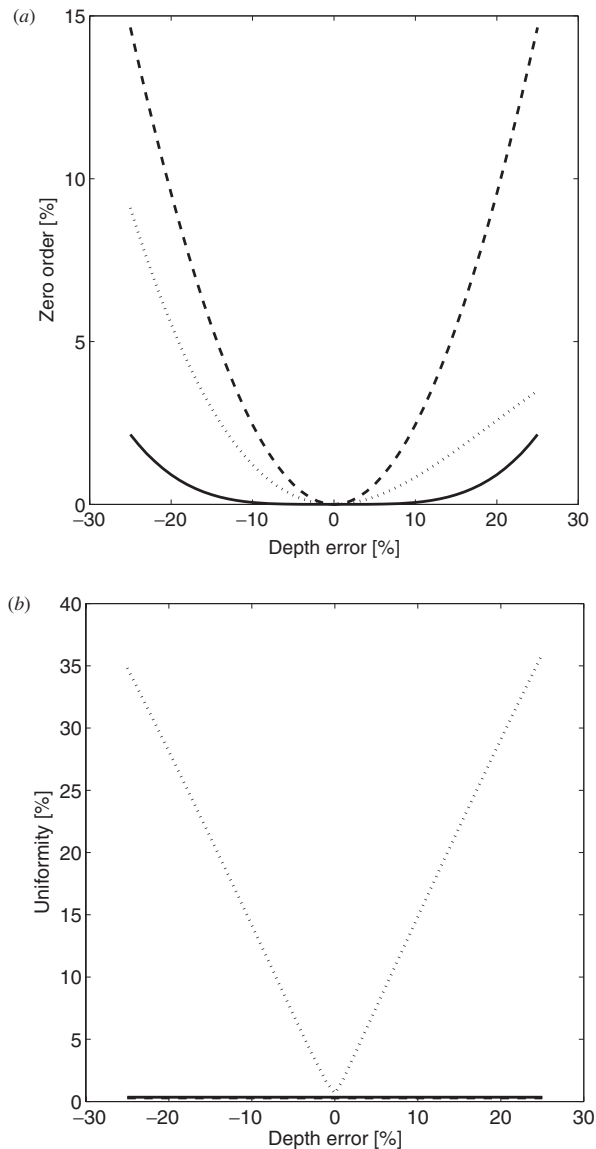


Figure 4. (a) Zeroth order efficiency, (b) uniformity error and (c) diffraction efficiency of the design as a function of depth error c for a three-level $1 \mapsto 16$ beam splitter. Corresponding binary and 16-level designs are shown with dashed and dotted lines, respectively, for comparison.

with binary gratings, the uniformity error of the three-level elements remains significantly smaller than that of the 16-level designs when profile depth errors are present in the element. This indicates that the important property of the depth errors affecting only the zero order, with other orders remaining unchanged in terms of the relative efficiency associated with binary gratings, is at least partially preserved when the additional phase level is introduced, making the elements more tolerant to surface profile depth errors. As expected, in terms of efficiency, the 16-level designs are greatly superior. Thus it can be concluded that for applications

sidewalls than in the case where the profile is realized using a single resist mask and proportional etching into the substrate. The size of the elements was $2.5 \mu\text{m} \times 2.5 \mu\text{m}$ and the period was $640 \mu\text{m} \times 640 \mu\text{m}$ with a pixel size of $5 \mu\text{m}$, hence, the elements operated well in the paraxial domain.

Each of the elements was of different depth with the depth errors selected to be within the range of $\pm 20\%$. To characterize the effect of the depth error on the signal, the optical function of the elements was determined by illuminating them with an expanded beam of a HeNe laser ($\lambda = 633 \text{ nm}$). The intensities of the generated diffraction orders were measured with an optical power meter in the far field and the results are shown in figure 6.

The experiments show excellent agreement with the theory for the zeroth order both in the case of the binary and the three-level elements, confirming that the zero order can indeed be reduced by introducing a third level into a binary grating in the manner described earlier in the paper. For the diffraction efficiency the match is less perfect, especially in the case of the binary elements, but the measured results still generally support the conclusion that suppression of the zeroth order is obtained at the cost of slightly lowered diffraction efficiency. In the case of the uniformity, the theoretical and measured results show weakest agreement which each other. This can be, however, attributed to the presence of other fabrication errors such as rounding of the surface profile, slight slanting of the vertical sidewalls due to anisotropic etching and small random variations in the filling factor due to positioning errors in the transition points defining the element. For example, the contribution of the latter to the uniformity error can be estimated to be nearly one percentage point even though the error itself

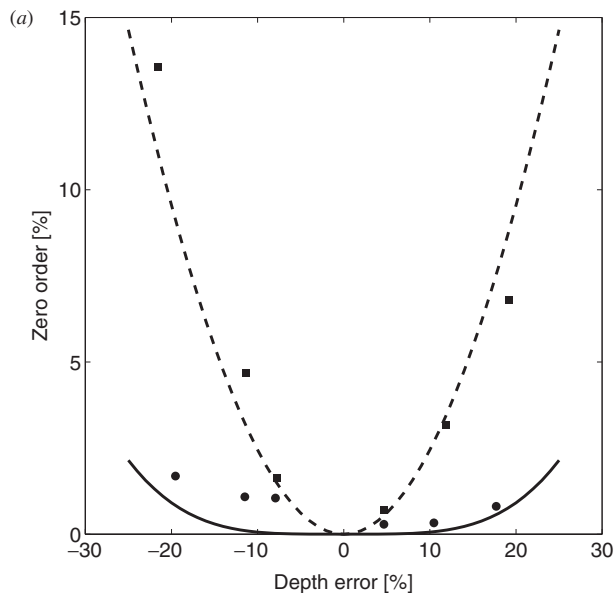


Figure 6. Measured (a) zeroth order efficiency, (b) uniformity error and (c) diffraction efficiency of the binary (boxes) and three-level (bullets) design as a function of depth error c for the $1 \rightarrow 4 \times 4$ beam splitter. Theoretical curves for binary and three-level designs are shown with dashed and solid lines, respectively.

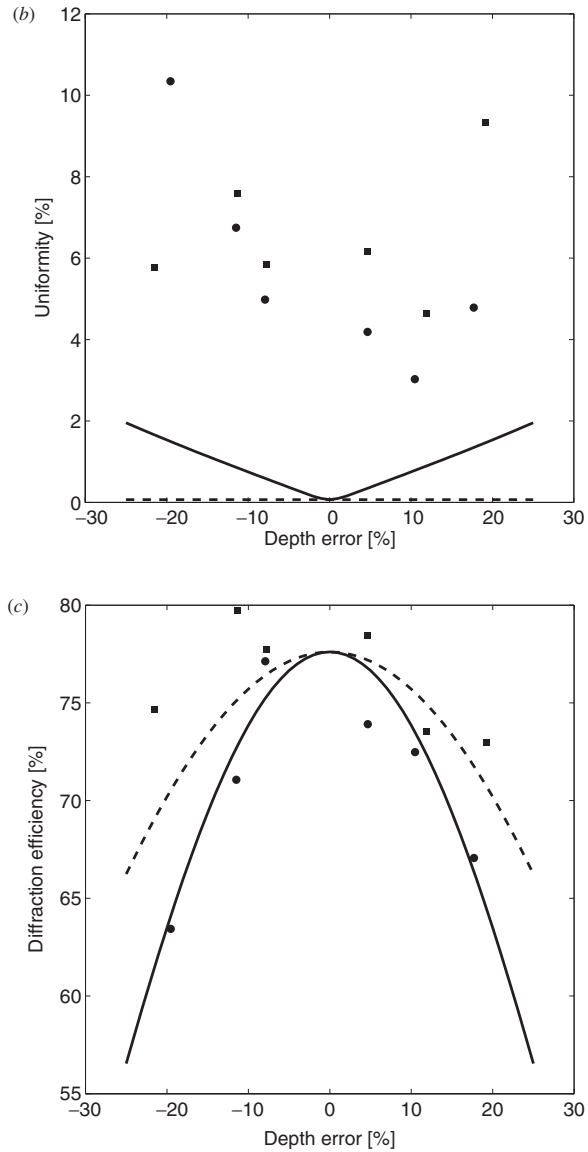


Figure 6. Continued.

is on average below 100 nm [9]. In the case of the three-level elements a small misalignment between the two masks and error in the relative height of the three-levels also contribute.

6. Discussion

It is generally accepted that with the modern techniques used in the fabrication of diffractive elements, the surface profile can be typically realized with high lateral precision, but such high precision cannot be achieved in the vertical direction. Therefore any method that significantly relaxes the requirements on the precision

needed to control the profile depth is, in general, valuable. This is especially true in the case of elements designed for the deep UV region, where an absolute error of a few nanometres is already proportionally significant with respect to the optical wavelength. We can also envision that a method such as the one proposed in this paper could be of interest for master elements used in mass production of diffractive elements. This is due to the fact that in mass production approaches, such as injection moulding, the tolerances of the replication process typically lead to profile depth variations from element to element, and a master with relaxed profile depth precision requirement could therefore directly influence the number of elements that either pass or fail the specified acceptance criteria. In other words, relaxing the fabrication tolerances of the master could lead to a direct increase in the yield of the process.

In terms of validity the proposed approach as discussed here is clearly dependent on the accuracy of the thin element approximation, i.e. it is only valid when the thin element approximation can be used for the modelling of the surface response. Since perturbation effects in the regions near the vertical surface transitions, shown to have a significant contribution in the break-down of thin element approximation in the non-paraxial domain [15], are ignored, the proposed approach also begins to fail when moving into the non-paraxial domain. Nevertheless, some zeroth order suppression can be obtained even when the minimum feature size of the element is only a few wavelengths, i.e. the design is deep inside the resonance domain. Furthermore, it should be possible to expand the proposed approach at least case by case to non-paraxial designs.

If one considers the trade-offs in terms of fabrication of proposed three-level elements, it is clear that the addition of the third level does increase the difficulty in fabrication. However, in the case of fabrication methods where the desired surface profile is produced in a single lithography step, e.g. electron-beam or laser beam writing or grey scale lithography, the increase does not present a significant problem. On the other hand, fabrication using optical lithography becomes a two mask process and other errors such as mask misalignment appear. The benefits gained using this approach in terms of zeroth order suppression must then be weighted against losses in diffraction efficiency and uniformity error due to other fabrication errors. A more detailed analysis of the fabrication trade-offs remains a subject for further study.

Previously discussion has been limited originally to gratings with a binary surface profile. We do not envision that the proposed approach could be straightforwardly expanded to multilevel or continuous gratings without increasing the phase range beyond 2π , i.e. without increasing the grating depth. Nevertheless expansion of the proposed method to multilevel or continuous profiles might be a useful alternative to the existing methods for design of fabrication error resistant diffractive elements.

7. Conclusions

We have shown theoretically and experimentally that the introduction of a third phase level to the original binary grating profile through a locally added bias phase of 2π can significantly reduce the design sensitivity to profile depth error and successfully suppress unwanted zeroth order light. Designs made using

the proposed method were found to be significantly improved compared to their binary counterparts.

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