

# Microscopic origin of the magnetic field in compact stars

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**Abstract.** A magnetic aspect of quark matter is studied by the Fermi liquid theory. The magnetic susceptibility is derived with the one-gluon-exchange interaction, and the critical Fermi momentum for spontaneous spin polarization is found to be  $1.4\text{fm}^{-1}$ . A scenario about the origin of magnetic field in compact stars is presented by using this result.

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## INTRODUCTION

Nowadays the phase diagram of QCD in the density-temperature plane has been explored by many people: the high-temperature region is relevant for relativistic heavy-ion collisions or cosmological phase transition in early universe, while the high-density region for compact stars. In the high-density and low-temperature region many exciting phenomena have been expected due to the large and sharp Fermi surface, such as color superconductivity, chiral density waves or ferromagnetism. Here we are concentrated in the magnetic aspect of quark matter: we are interested in the spin degree of freedom [1]. If magnetism is realized in quark matter, it should be interesting theoretically, but have important implications for the magnetic properties of compact stars.

Recent discoveries of magnetars, compact stars with huge magnetic field of  $O(10^{15}\text{G})$ , seem to enforce us to reconsider the origin of the magnetic field in compact stars. They have firstly observed by the  $P - \dot{P}$  curve, and some cyclotron absorption lines have been recently observed [2]. A naive working hypothesis of conservation of magnetic flux during their evolution from the main-sequence progenitors cannot be applied to magnetars, because the resultant radius is too small for  $O(10^{15}\text{G})$ . The dynamo mechanism may work in compact stars, but it might look to be unnatural to produce such a huge magnetic field. The typical energy scale is given by the interaction energy with this magnetic field, which amounts to  $O(\text{MeV})$  for electrons, and  $O(\text{keV})$  to  $O(\text{MeV})$  for nucleons or quarks. On the other hand, considering the atomic energy scale or the strong-interaction energy scale, we can say that  $O(10^{15}\text{G})$  is very large for electrons, but not so large for nucleons or quarks. Hence it would be interesting to consider a microscopic origin of magnetic field as an alternative: if ferromagnetism or spontaneous spin polarization occurs inside compact stars, it may be a possible candidate <sup>1</sup>. For

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<sup>1</sup> In a recent paper Makishima also suggested a hadronic origin from the observation of X-ray binaries [3].

nuclear matter, there have been done many calculations with different nuclear forces and different methods since the first discoveries of pulsars in early seventies, but all of them have given negative results so far [4]. In the following we discuss the magnetic aspect of quark matter and consider a possibility of spontaneous spin polarization.

## RELATIVISTIC FERROMAGNETISM

We have considered the possibility of ferromagnetism in quark matter interacting with the one-gluon-exchange (OGE) interaction [5] or with an effective interaction [6], and suggested that quark matter has a potentiality to be spontaneously polarized. To understand the magnetic properties of quark matter more realistically, especially near the critical point, some non-perturbative consideration about the instability of the Fermi surface is indispensable. Recently there are some studies about the effective interaction near the Fermi surface [7, 8], using the idea of the renormalization group [9]. Here we apply the Fermi liquid theory to derive the magnetic susceptibility and discuss the spontaneous spin polarization, considering quarks as quasiparticles [10, 11, 12].

### Fermi liquid theory

In the Fermi liquid theory the total energy is given as a functional of the distribution function. As is already shown in ref. [13], the spin degree of freedom is specified by the three vector  $\zeta$  in the rest frame. Then the quasi-particle energy and the effective interaction near the Fermi surface can be written as,

$$\varepsilon(\mathbf{k}\zeta ci) = \frac{\delta E}{\delta n(\mathbf{k}\zeta ci)}, \quad f_{\mathbf{k}\zeta ci, \mathbf{q}\zeta' dj} = \frac{\delta \varepsilon(\mathbf{k}\zeta ci)}{\delta n(\mathbf{q}\zeta' dj)}, \quad (1)$$

where the subscripts  $c(d)$  and  $i(j)$  denotes the color and flavor degrees of freedom. The Landau Fermi liquid interaction  $f_{\mathbf{k}\zeta ci, \mathbf{q}\zeta' dj}$  is related to the forward scattering amplitude for two quarks on the Fermi surface.<sup>2</sup> In QCD the interaction is flavor independent,  $f_{\mathbf{k}\zeta ci, \mathbf{q}\zeta' dj} = \delta_{ij} f_{\mathbf{k}\zeta c, \mathbf{q}\zeta' d}$ . Since there is no direct interaction due to color neutrality, the Fock exchange interaction gives the leading contribution in the weak coupling limit, i.e. the color symmetric OGE interaction can be written as

$$f_{\mathbf{k}\zeta, \mathbf{q}\zeta'}^S \equiv \frac{1}{N_c^2} \sum_{c,d} f_{\mathbf{k}\zeta c, \mathbf{q}\zeta' d} = \frac{m}{E_k} \frac{m}{E_q} M_{\mathbf{k}\zeta, \mathbf{q}\zeta'}, \quad (2)$$

with the Lorentz invariant matrix element,

$$M_{\mathbf{k}\zeta, \mathbf{q}\zeta'} = g^2 \frac{N_c^2 - 1}{4N_c^2 m^2} [2m^2 - k \cdot q - m^2 a \cdot b] \frac{1}{(k - q)^2}, \quad (3)$$

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<sup>2</sup> Note that it is also only the non-relevant interaction at the Fermi surface in the context of the renormalization group approach [9].

where we used the Feynman gauge for the gluon propagator. The term including the inner product  $a \cdot b$  represents the spin dependence. The spin vector  $a^\mu$  is explicitly given as a function of  $\zeta$  and momentum [13]. There are many possible forms about  $a^\mu$ , but we here use the simplest one,

$$a^0 = \frac{\mathbf{k} \cdot \boldsymbol{\zeta}}{m}, \mathbf{a} = \boldsymbol{\zeta} + \frac{\mathbf{k}(\boldsymbol{\zeta} \cdot \mathbf{k})}{m(E_k + m)}. \quad (4)$$

From the invariance of the properties of quark matter under the Lorentz transformation, the Fermi velocity can be written [10] as

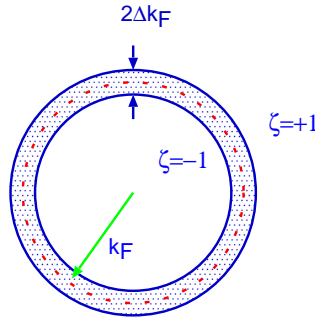
$$v_F^{-1} = \left( \frac{\partial k}{\partial \varepsilon(\mathbf{k}\boldsymbol{\zeta})} \right)_{k_F} = \frac{\mu}{k_F} \left( 1 + \frac{1}{3} F_1^S \right) \quad (5)$$

with the spin-symmetric Landau parameter  $F_1^S$  defined by

$$F_1^S = N(0) f_1^S, f_1^S = -\frac{3}{4} \frac{g^2 (N_c^2 - 1) m^2}{4 N_c^2 \mu^2 k_F^2} \int_{-1}^1 du u \frac{1}{1-u}, \quad (6)$$

for OGE. Here  $N(0)$  is the density of states at the Fermi surface,  $N(0) = 2N_c k_F^2 / 2\pi^2 (\partial k / \partial \varepsilon(\mathbf{k}\boldsymbol{\zeta}))_{k_F}$ . Note that  $f_1^S$  clearly shows log divergence reflecting the gauge interaction. When we take into account the higher-order corrections for the gluon propagator, the electric propagator is screened by the Debye mass, while the magnetic one receives only the Landau damping. This fact exhibits the non Fermi liquid nature of quark matter. However, we shall see that the magnetic susceptibility becomes finite even in this case.

Applying the weak magnetic field to quark matter, we consider the energy change (see Fig. 1). Using the Gordon identity, the QED interaction Lagrangian can be recast as



**FIGURE 1.** Modification of the Fermi surface in the presence of the weak magnetic field.

$$\int d^4x \mathcal{L}_{\text{ext}}^{\text{QED}} = \sum_f \mu_q^f \int d^4x \bar{\psi}_f [-i\mathbf{r} \times \nabla + \boldsymbol{\sigma}] \times \mathbf{B} \psi_f \quad (7)$$

with the magnetic moment,  $\mu_q^f = e_q^f / 2m$ . Since the orbital angular momentum gives null contribution on average, we hereafter only consider the spin contribution. In the

following we consider only one flavor without loss of generality. For the energy to be minimum (chemical equilibrium),

$$\varepsilon(k_F + \Delta k_F, \zeta = +1) = \varepsilon(k_F - \Delta k_F, \zeta = -1), \quad (8)$$

i.e.,

$$\begin{aligned} & - \frac{g_D \mu_q B}{2} + \left( \frac{d\varepsilon}{dk} \right)_{k_F} \Delta k_F + (\bar{f}_{++} - \bar{f}_{+-}) \Delta N \\ & = \frac{g_D \mu_q B}{2} - \left( \frac{d\varepsilon}{dk} \right)_{k_F} \Delta k_F + (\bar{f}_{-+} - \bar{f}_{--}) \Delta N, \end{aligned} \quad (9)$$

where  $\Delta N = N_c k_F^2 \Delta k_F / 2\pi^2$ , and  $g_D$  is the gyromagnetic ratio [13],

$$g_D = 2 \int \frac{d\Omega_k}{4\pi} \left( a_z - \frac{k_F}{\mu} \cos \theta a_0 \right), \quad (10)$$

which is reduced to be 2 in the non-relativistic limit,  $m \gg k_F$ . The angle-averaged Fermi liquid interactions  $\bar{f}_{\zeta\zeta'}$  are given as

$$\begin{aligned} \bar{f}_{++} = \bar{f}_{--} &= \frac{(N_c^2 - 1)g^2}{4N_c^2\mu^2} \left[ \frac{1}{2} - \frac{m^2}{k_F^2} - \frac{1}{3} \frac{m(\mu - m)}{k_F^2} \right] + \frac{f_1^S}{3} \\ \bar{f}_{+-} = \bar{f}_{-+} &= \frac{(N_c^2 - 1)g^2}{4N_c^2\mu^2} \left[ \frac{1}{2} + \frac{1}{3} \frac{m(\mu - m)}{k_F^2} \right], \end{aligned} \quad (11)$$

by the use of the standard spin configuration (4). The latter is reduced to null in the non-relativistic limit, which implies there is no interaction between quarks with different spins [11]. From Eqs. (9) and (11) we find the spin susceptibility,

$$\begin{aligned} \chi_{\text{spin}} &\equiv g_D \mu_q \Delta N / VB \\ &= \chi_{\text{free}} \left[ 1 - \frac{4\alpha_c}{3\mu\pi} \frac{m(2\mu + m)}{3k_F} \right]^{-1} \end{aligned} \quad (12)$$

for  $N_c = 3$ , with the corresponding one without any interaction,  $\chi_{\text{free}} = g_D^2 \mu_q^2 \mu k_F / 4\pi^2$ . Note that log divergence is included in the Fermi liquid interaction (11), but it is canceled by the one coming from the Fermi velocity (5). Fig. 2 shows the ratio of spin susceptibility as a function of the Fermi momentum. It diverges around  $k_F = 1.4 \text{fm}^{-1}$ , which is a signal of the spontaneous magnetization.

It would be interesting to compare the above result with the previous one given by the perturbative calculation with OGE [5], where we can see the *weakly first-order* phase transition at a certain low density, and that the critical density is similar to that given by the Fermi liquid theory. Thus two calculations are consistent with each other in the weak coupling limit.

Here we have discussed only the lowest order contribution, but a recent paper has also suggested the phase transition at low densities by including the higher-order effects [14].

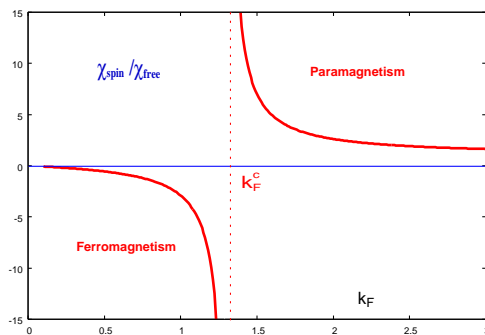


FIGURE 2. Spin susceptibility as a function of the Fermi momentum.

## ASTROPHYSICAL IMPLICATIONS

There are two possibilities about the existence of quark matter in compact stars: one is in the quark stars, which are composed of low density strange matter, and the other is in the core region of neutron stars, which are called hybrid stars. Consider magnetars as quark stars or hybrid stars. Then we can easily estimate their magnetic field near the surface, if ferromagnetism is realized in quark matter. The maximum strength of the magnetic field on the surface  $r = R$  is estimated by

$$B_{\max} = \frac{8\pi}{3} \left( \frac{r_Q}{R} \right)^3 \mu_Q n_Q, \quad (13)$$

where  $r_Q$  is the radius of quark lump with density  $n_Q$  and  $\mu_Q$  the single quark magnetic moment; it amounts to  $O(10^{15-17})\text{G}$  for  $n_Q = 0.1\text{fm}^{-3}$ , which might be enough for magnetars. It should be interesting to observe the braking indexes about magnetars; if their values are near three, the dipole radiation is a good picture and we may say that the above scenario looks more realistic.

There may be left another interesting problem about hierarchy of magnetic field in compact stars (Table 1). Unfortunately, the idea of ferromagnetism may not be sufficient for explaining it, and we need to consider the global magnetic structure and some dynamical mechanisms, e.g. formation of magnetic domain or existence of metamagnetism, besides it.

TABLE 1. Hierarchy of magnetic field in compact stars.

	millisecond pulsars	usual radio pulsars	magnetars
Magnetic field [G]	$10^9$	$10^{12}$	$10^{15}$
Period [sec]	$10^{-2}$	$10^0$	$10^1$
Age [year]	$10^9$	$10^6$	$10^3$

We might also consider a scenario about the cosmological magnetic field in the galaxies and extra galaxies. It is well known that magnetic fields are present in all galaxies and galaxy clusters, which are characterized by the strength,  $10^{-7} - 10^{-5}\text{G}$ , with the spatial scale,  $\leq 1\text{Mpc}$  [15]. The origin of such magnetic fields is still unknown,

but the first magnetic fields may have been created in the early universe. If magnetized quark lumps are generated during the QCD phase transition, they can give the seed fields.

## CONCLUDING REMARKS

Magnetic properties of quark matter have been discussed by applying the Fermi liquid theory, which gives one of the non-perturbative tool to analyze them. Spin couples with motion in the relativistic theories, and we must extend the Fermi liquid interaction accordingly. We have seen that the magnetic susceptibility can be given by applying a weak magnetic field and considering the energy change in the tiny region near the Fermi surface. The Landau parameters may be log divergent for the gauge interaction, but the magnetic susceptibility is given to be finite by the cancellation. The critical point is found to be the same order with the one given by the perturbative evaluation, which may support the relevance of the Fermi liquid theory in this problem. We may extend the present analysis by including higher order diagrams within the Fermi liquid theory or using the renormalization group.

If ferromagnetism is realized in quark matter, we may consider various scenarios: it might give a microscopic origin of the magnetic field in compact stars, and a seed of the primordial magnetic field during the cosmological QCD phase transition. It may also predict the production of small magnets composed of strange matter during the relativistic heavy-ion collisions.

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