Property of $cn - \bar{n}\bar{c}$ System in $\tilde{U}(12)$ -Scheme and X(3872)/Y(3940)

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abstract

The properties of four quark $cn \cdot \bar{n}c$ states are investigated as cn di-quark and $\bar{n}c$ di-antiquark system in $\tilde{U}(12)$ -classification scheme of hadrons. We consider the negative-parity di-quark and di-antiquark in ground states, and the properties of X(3872) and Y(3940) are consistent, respectively, with those of the $J^{PC} = 1^{++}$ and 2^{++} states from these negative-parity di-quark and di-antiquark. Their narrow-widths are explained from an orthogonality of spinor wave functions. The properties of ground-state $cs \cdot \bar{s}c$ system are also predicted in this scheme.

1 Introduction

The discovery of X(3872)[1] and Y(3940)[2] by Belle presented new problems on hadron spectroscopy. Because of their decay-properties, they are considered to be non- $c\bar{c}$ states. We consider the four-quark explanation[3] of X(3872)/Y(3940), where the relevant particles are considered as the cndi-quark and $\bar{n}\bar{c}$ di-antiquark system. This explanation is criticized from the observed decay widths. The $cn-\bar{n}\bar{c}$ system decays to $D\bar{D}^{(*)}$ and $J/\psi\rho$ (or $J/\psi\omega$). They are so-called OZI super-allowed processes, and proceed without no $q\bar{q}$ -pair production or annihilation. Their widths are expected to be so large as a few hundred MeV or more. On the other hand the observed widths of X(3872) and Y(3940) are very small, $\Gamma_X < 2.3 \text{MeV}[1]$ and $\Gamma_Y = 87 \pm 22 \pm 26 \text{MeV}[2]$, respectively. A few years ago[4], we have proposed a covariant level-classification scheme of hadrons based on $\tilde{U}(12)$ spin-flavor group. It is a relativistic generalization of the $SU(6)_{SF}$ -scheme in non-relativistic quark model(NRQM), and the squared-mass spectra of hadrons including light constituent quarks are classified as the representations of $\tilde{U}(12)$ [5]. Each spinor index corresponding to light constituent quark in composite hadron WF is expanded by free Dirac spinors $u_{r,s}(v_{\mu})$ called urciton spinors. At the rest frame $u_{+,s}(\mathbf{0})$ has the upper two-component which corresponds to the Pauli spinor appearing in NRQM, while the $u_{-,s}(\mathbf{0})$ has the lower two-component representing the relativistic effect. The r index of $u_{r,s}$ represents the eigen-value of ρ_3 , and the corresponding freedom is called ρ -spin, while the ordinary Pauli-spin freedom is called σ -spin. In the $\tilde{U}(12)$ -scheme, the $u_{-,s}$ is supposed to appear as new degrees of freedom for light constituents, being independent of $u_{+,s}$, and we predict the existence of a number of extra states out of the $SU(6)_{SF}$ -framework for meson and baryon systems, called chiral states.

The heavy-light $Q\bar{q}$ mesons in ground states are classified as 12^* , which includes the chiral $J^P = 0^+/1^+$ states with positive-parity as well as the negative-parity $0^-/1^-$ states[5]. The properties of $D_s(2317)/D_s(2460)$ are naturally explained as the chiral states in $c\bar{s}$ system. Applying the $\tilde{U}(12)$ scheme, the heavy-light Qq di-quark forms 12 in ground states, which includes chiral $J^P = 0^-/1^-$ states with negative parity as well as the positive parity $0^+/1^+$ states.

There is a kind of phenomenological conservation rule applicable to chiral states, called ρ_3 -line rule[6, 7]. In transitions of chiral states, the ρ_3 along each spectator-quark line is approximately conserved.

There is an interesting possibility that the observed X(3872) and Y(3940)are chiral states, which are made from negative-parity cn di-quark and $\bar{n}\bar{c}$ di-antiquark, and that their narrow-width properties are explained by the ρ_3 -line rule. In this talk we investigate this possibility, whether the X(3872)and Y(3940) are explained as $cn-\bar{n}\bar{c}$ states.

2 Di-quark and Di-antiquark Spinor Wave Function

The negative-parity cn di-quark $(\bar{n}\bar{c}$ di-antiquark), called chiral state, is denoted as $\Gamma^{\chi}_{cn}(\Gamma^{\chi}_{\bar{n}\bar{c}})$, which has negative $\rho^n_3(\rho^{\bar{n}}_3)$, while the ordinary positive-

parity di-quark(di-antiquark), called Pauli state, is denoted as $\Gamma_{cn}^{P}(\Gamma_{\bar{n}\bar{c}}^{P})$, which is composed of constituents both with positive ρ_{3} . Their explicit forms are given in ref.[8].

The Γ_{cn} is related with the WF of $c\bar{n}$ -meson $H_{c\bar{n}}$ in heavy quark effective theory, as $\Gamma_{cn} = H_{c\bar{n}}C^{\dagger}$. The WF of positive(negative)-parity di-quark corresponds to the WF of negative(positive)-parity meson. The positive-parity mesons, which are chiral states, become heavier[5] than the negative-parity Pauli mesons through spontaneous chiral symmetry breaking(χ SB), which is described by the Yukawa coupling to the scalar σ -meson nonet $s = s^i \lambda^i / \sqrt{2}$ in the framework of SU(3) linear σ model[9]. In Qq di-quark systems, because of the C^{\dagger} factor, the chiral states are expected to have smaller masses than the Pauli states. The mass shift $\delta M^{\chi,P}$ in χ SB is given by

$$\delta M^{\chi,P} = g_{\sigma} s_0 \left\langle \Gamma_{cn}^{\chi,P} \overline{\Gamma_{cn}^{\chi,P}} \right\rangle = g_{\sigma} s_0 \left\langle H_{c\bar{n}}^{\chi,P} C^{\dagger} C^{\dagger} \overline{H_{c\bar{n}}^{\chi,P}} \right\rangle = -g_{\sigma} s_0 \left\langle H_{c\bar{n}}^{\chi,P} \overline{H_{c\bar{n}}^{\chi,P}} \right\rangle, \quad (1)$$

where $\overline{\Gamma_{cn}^{\chi,P}} \equiv \gamma_4(\Gamma_{cn}^{\chi,P})^{\dagger}\gamma_4$, and the change of sign comes from $(C^{\dagger})^2 = -1$. The s_0 is vacuum expectation value of s and g_{σ} is the Yukawa coupling constant. The chiral mass splitting $\Delta M_{\chi} (\equiv \delta M^P - \delta M^{\chi}) = 2g_{\sigma}s_0$ is predicted with 242MeV[5].

Concerning $cn-\bar{n}\bar{c}$ system, we expect three combinations of $\Gamma_{cn}-\Gamma_{\bar{n}\bar{c}}$: The $\Gamma_{cn}^{P}-\Gamma_{\bar{n}\bar{c}}^{P}$ denoted as Pauli-Pauli states, the $\Gamma_{cn}^{P}-\Gamma_{\bar{n}\bar{c}}^{\chi}$ (or $\Gamma_{cn}^{\chi}-\Gamma_{\bar{n}\bar{c}}^{P}$) denoted as Pauli-chiral(or chiral-Pauli) states, and the $\Gamma_{cn}^{\chi}-\Gamma_{\bar{n}\bar{c}}^{\chi}$ denoted as chiral-chiral states. Among them the chiral-chiral states have the smallest masses, the chiral-Pauli states are heavier by ΔM_{χ} , and the Pauli-Pauli states are further heavier by ΔM_{χ} . The Pauli-Pauli(Pauli-chiral) states decay to Pauli-chiral(chiral-chiral) states through pion emission in S-wave, and the widths are expected to be so large as a GeV. Accordingly, the Pauli-Pauli states and Pauli-chiral states are not observed as resonances, but as non-resonant backgrounds. We focus our interests on chiral-chiral states in the following.

3 Oscillator WF and Ground-State Mass Spectra

We consider spin-spin interaction to predict the mass spectra of ground states. We use the space WF in joint spring quark model(JSQM)[10], where the cn di-quark ($\bar{n}\bar{c}$ di-antiquark) is in color $\mathbf{3}^*(\mathbf{3})$, and they are connected by spring potential. The space-coordinates of c, n, \bar{n} and \bar{c} are denoted,

type	J^{PC}	ΔM_{SS}	Mass	decay channels		
$\Gamma^{0^-}_{cn}$ - $\Gamma^{0^-}_{\bar{n}\bar{c}}$	0^{++}	-65	3824	$(J/\psi\rho),$	$\eta_c \pi$,	$Dar{D}$
$\frac{1}{\sqrt{2}} \left(\Gamma_{cn}^{0-} - \Gamma_{\bar{n}\bar{c}}^{1-} + \Gamma_{cn}^{1-} - \Gamma_{\bar{n}\bar{c}}^{0-} \right)$	1^{+-}	-27	3862	$J/\psi\pi,$	$\eta_c \rho$	
$\frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} \left(\Gamma_{cn}^{0-} - \Gamma_{\bar{n}\bar{c}}^{1-} - \Gamma_{cn}^{1-} - \Gamma_{\bar{n}\bar{c}}^{0-} \right)$	1^{++}	-17	<u>3872</u>	$J/\psi ho$		$D^0 \bar{D}^{0*}$
Γ^{1-}_{cn} - $\Gamma^{1-}_{ar{n}ar{c}}$	0^{++}	-22	3867	$(J/\psi\rho),$	$\eta_c \pi$,	$Dar{D}$
Γ^{1-}_{cn} - $\Gamma^{1-}_{ar{n}ar{c}}$	1^{+-}	0.	3889	$J/\psi\pi,$	$\eta_c \rho$,	$D\bar{D}^*$
Γ^{1-}_{cn} - $\Gamma^{1-}_{ar{n}ar{c}}$	2^{++}	+43	3932	$J/\psi\rho,$		$D\bar{D}^{(*)}(\text{D-wave})$

Table 1: Masses of ground-state $cn \cdot \bar{n}\bar{c}$ system (in MeV). Decay channels of I=1 states are shown. For I=0, $\rho(\pi)$ is replaced with $\omega(\eta)$.

respectively, as \mathbf{x}_1 , \mathbf{x}_2 , \mathbf{x}_3 and \mathbf{x}_4 , and the eigen-modes are numerically given by [8]

$$\mathbf{s} = (\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{x}_3 + \mathbf{x}_4)/\sqrt{2}, \quad \mathbf{R} = 0.785(\mathbf{x}_1 - \mathbf{x}_4) + 0.083(\mathbf{x}_2 - \mathbf{x}_3), \\ \mathbf{U} = 0.365(\mathbf{x}_1 - \mathbf{x}_4) - 0.862(\mathbf{x}_2 - \mathbf{x}_3).$$
(2)

We may regard **s** as relative coordinate in di-(anti)quark, $\mathbf{R}(\mathbf{U})$ approximately as relative coordinate between c and $\bar{c}(n$ and $\bar{n})$. The internal space WF is given by

$$f_s(\mathbf{s})f_R(\mathbf{R})f_U(\mathbf{U}) = \left(\frac{\beta_s\beta_R\beta_U}{\pi^3}\right)^{\frac{3}{4}}e^{-\frac{\beta_s}{2}\mathbf{s}^2}e^{-\frac{\beta_R}{2}\mathbf{R}^2}e^{-\frac{\beta_U}{2}\mathbf{U}^2},\qquad(3)$$

where $\beta_s = 0.129 \text{GeV}^2$, $\beta_R = 0.254 \text{GeV}^2$ and $\beta_U = 0.115 \text{GeV}^2$.

By using the WF (3), we can estimate the spin-spin interaction, $H_{SS} \propto -\frac{1}{m_i m_j} \delta^{(3)}(\mathbf{x}_i - \mathbf{x}_j) \boldsymbol{\sigma}^{(i)} \cdot \boldsymbol{\sigma}^{(j)}$. The proprional constant is fixed by using $m_{D^*} - m_D = 142 \text{MeV}$ as input, and the mass-shifts ΔM_{SS} in ground-states of $cn \cdot \bar{n}\bar{c}$ system are given in Table 1. The mass of X(3872), which plausibly $J^{PC} = 1^{++}[11]$, is used as input to fix overall mass-shift. The 2^{++} state is predicted with mass 3932 MeV, which is consistent with the experimental value of Y(3940), $M = 3943 \pm 11 \pm 13 \text{MeV}[2]$. At the same time, the existence of the other 4 states with $J^{PC} = 0^{++}$ and 1^{+-} is predicted in the same mass region.

4 Strong Decay Widths

(Overlapping of spinor WF) The decay mechanism of the relevant quarkrearrangement processes is unknown, however, it may be reasonable to assume that the decay matrix elements are proportional to the overlappings of the initial and final WFs. The overlapping of spinor WFs, denoted as A_s , are given for the decays to $c\bar{n} + n\bar{c}$ and $c\bar{c} + n\bar{n}$, respectively, by

$$\begin{aligned}
A_{s}(c\bar{n}+n\bar{c}) &= \langle \Gamma^{\chi}_{\bar{n}\bar{c}}\overline{H}_{\bar{D}}(v_{\bar{D}})(-iv_{\bar{D}}\cdot\gamma) {}^{t}\Gamma^{\chi}_{cn} {}^{t}[iv_{D}\cdot\gamma\overline{H}_{D}(v_{D})] \rangle \\
&= \langle \Gamma^{\chi}_{\bar{n}\bar{c}} B(v_{\bar{D}})\overline{H}_{\bar{D}}\bar{B}(v_{\bar{D}}) {}^{t}\Gamma^{\chi}_{cn} {}^{t}[B(v_{D})\overline{H}_{D}\bar{B}(v_{D})] \rangle \\
A_{s}(c\bar{c}+n\bar{n}) &= \langle \Gamma^{\chi}_{\bar{n}\bar{c}}\overline{H}_{\psi}(v_{\psi})\Gamma^{\chi}_{cn} {}^{t}[iv_{\rho}\cdot\gamma\overline{H}_{\rho}(v_{\rho})(-iv_{\rho}\cdot\gamma)] \rangle \\
&= \langle \Gamma^{\chi}_{\bar{n}\bar{c}} B(v_{\psi})\overline{H}_{\psi}\bar{B}(v_{\psi}) \Gamma^{\chi}_{cn} {}^{t}[B(v_{\rho})\overline{H}_{\rho}\bar{B}(v_{\rho})] \rangle, \quad (4)
\end{aligned}$$

where $D\bar{D}$ and $J/\psi\rho$ are taken as examples of $c\bar{n} + n\bar{c}$ and $c\bar{c} + n\bar{n}$. The $\overline{H}(v)$ is the spinor WF of the final meson with velocity $v_{\mu} (\equiv P_{\mu}/M)$, and \overline{H} is the one at the rest frame with velocity $v_{0\mu} = (0, 0, 0; i)$. They are related by Lorentz booster $B(v), \bar{B}(v) = ch\theta \pm \rho_1 \mathbf{n} \cdot \boldsymbol{\sigma} sh\theta$ as $\overline{H}(v) = B(v)\overline{H}\overline{B}(v)$, where the $ch\theta, sh\theta = \sqrt{\frac{E\pm M}{2M}}$. In Eq. (4), the change of ρ_3 on each quark-line in the decay occurs only through ρ_1 term in B(v) and $\overline{B}(v)$, which is proportional to $sh\theta$. It is related with the momentum \mathbf{P} of the final meson as $ch\theta sh\theta = \frac{\mathbf{P}}{2M}$, which is much smaller than unity for the relevant decays because of their small phase space. Thus, the ρ_3 quantum number along every the spectator-quark line is approximately conserved[6].

In the chiral-chiral states, the *n* in *cn* di-quark and the \bar{n} in $\bar{n}\bar{c}$ diantiquark are in negative ρ_3 state. Thus, their decay amplitudes to the Pauli mesons, of which ρ_3 of constituents are all positive, are doubly suppressed with the factor $(\frac{\mathbf{P}}{2M})^2$. In $\tilde{U}(12)$ -scheme, the WF of vector mesons, J/ψ , ρ , ω , D^* and \bar{D}^* , are commonly given by $\overline{H} = (1 + iv_0 \cdot \gamma)\frac{i\epsilon\cdot\gamma}{2\sqrt{2}}$, where the ρ_3 of constituents are all positive, $(\rho_3^q, \rho_3^{\bar{q}}) = (+, +)$. On the other hand, the WF of pseudoscalar mesons, π and η , which are Nambu-Goldstone bosons, are $\overline{H} = \frac{i\gamma_5}{2}$, which has doubly-negative ρ_3 component as $(\rho_3^q, \rho_3^{\bar{q}}) = \frac{1}{\sqrt{2}}((+, +) + (-, -))($, while the WF of *D* and η_c is $\overline{H} = (1 + iv_0 \cdot \gamma)\frac{\gamma_5}{2\sqrt{2}}$, which has $(\rho_3^c, \rho_3^{\bar{c}}) = (+, +))$. Thus, in the relevant decays of the chiral-chiral states, the transition amplitudes to $J/\psi\rho(\omega)$, $\eta_c\rho(\omega)$ and $D\bar{D}^{(*)}$ are strongly suppressed from ρ_3 -line rule, while the ones to $J/\psi\pi(\eta)$ and $\eta_c\pi(\eta)$ are not suppressed. The decay widths of the latter channels are expected to be a few hundred MeV or a GeV.

R	$J/\psi\omega$	$D\bar{D}$	$D\bar{D}^*$		$J/\psi ho$	$J/\psi\omega$
$\Gamma_Y(R)$	$60.3 \cos^2 \theta_Y$	20.1	6.6	$\Gamma_X(R)$	$(1.2 \sin^2 \theta_X)$	$(0.2 \cos^2 \theta_X)$

Table 2: Decay widths of Y(3940), Γ_Y , and X(3872), Γ_X in MeV. For the $\Gamma_Y(D\bar{D}^*)$, $\bar{D}D^*$ is also included. The $\cos\theta_{X,Y}(\sin\theta_{X,Y})$ are I=0(1) component of the flavor WFs of X, Y. See ref.[8] for details.

Thus, when the initial states have no $\pi(\eta)$ decay modes, they have narrow widths, and are expected to be observed as resonant particles. The possible decay channels of the ground six states are shown in Table 1. Among them, only two states with $J^{PC} = 1^{++}$ and 2^{++} have no $\pi(\eta)$ decay modes and are considered to be detected as resonances. This theoretical expectation is consistent with the present experimetnal situation: Only two states, X(3872)and Y(3940), are observed as the exotic candidates in the relevant mass region.

(*Decay widths*) We can also calculate the overlappings of space WFs, denoted as \tilde{F}_x . Their numerical values are $\tilde{F}_x(c\bar{n} + n\bar{c}) = 0.96 \text{GeV}^{-\frac{3}{2}}$ and $\tilde{F}_x(c\bar{c} + n\bar{n}) = 1.05 \text{GeV}^{-\frac{3}{2}}$, respectively. We neglect the difference between them, and use a common value $\tilde{F}_x \simeq 1 \text{GeV}^{-\frac{3}{2}}$.

The width Γ of the decay $X(\text{or } Y) \rightarrow A + B$ is given by

$$\Gamma = a |\mathbf{P}| (E_A E_B M \tilde{F}_x^2) |A_s|^2 \tag{5}$$

with one dimension-less parameter a, where M is the mass of initial 4 quark state, and $E_A(E_B)$ is the energy of the final meson A(B). The $|\mathbf{P}|$ is the momentum of final meson. The widths for the relevant decay modes of X(3872) and Y(3940) are given in Table 2. The parameter a is fixed with a = 18.8, which is determined by the the total width of Y(3940), $\Gamma_Y =$ 87MeV, as input. The partial decay rates of Y(3940) are fairly consistent with the present experiments. By taking the I=0 component $\cos\theta_Y \simeq 1$, the main decay mode of Y(3940) is considered to be $J/\psi\omega$. Experimentally the Y(3940) is observed in $J/\psi\omega$, not in $D\bar{D}$ and $D\bar{D}^*$.

5 Concluding Remarks

The X(3872) and Y(3940) have the properties of the four quark $cn-\bar{n}\bar{c}$ states with $J^{PC} = 1^{++}$ and 2^{++} , respectively, which come from $0^{-}-1^{-}$ and $1^{-}-1^{-}$ combinations of cn chiral di-quark and $\bar{n}\bar{c}$ chiral di-antiquark in $\tilde{U}(12)$ -

	$\Gamma_Y(D_s\bar{D}_s)$	$\Gamma_Y(D_s\bar{D}_s^*)$	$\Gamma_Y(J/\psi\phi)$		Γ_X^{tot}
$Y_{cs-\bar{s}\bar{c}}^{2^{++}}(4082)$	$23 \mathrm{MeV}$	8MeV	$11 \mathrm{MeV}$	$X_{cs-\bar{s}\bar{c}}^{1^{++}}(4152)$	$< 1 {\rm MeV}$

Table 3: The masses and widths for the ground $cs-\bar{s}\bar{c}$ states with $J^{PC} = 1^{++}$, $X^{1^{++}}_{cs-\bar{s}\bar{c}}$, and 2^{++} , $Y^{2^{++}}_{cs-\bar{s}\bar{c}}$. The total width of $Y^{2^{++}}_{cs-\bar{s}\bar{c}}$ is estimated by the sum as $\Gamma^{\text{tot}}_{Y^{2^{++}}_{cs-\bar{s}\bar{c}}} = 23 + 8 + 11 = 42 \text{MeV}$.

scheme. Their mass difference is consistently explained by the spin-spin interaction between constituents in JSQM. Their narrow widths are explained from a phenomenological selection rule, coming from an orthogonality of spinor WFs, called ρ_3 -line rule. Only these two states are expected to be observed as resonant particles in ground states, and the other states have very large widths and are not observed as resonances but as non-resonant backgrounds.

In order to check this interpretation, we present the predictions for the properties of ground $cs-\bar{s}\bar{c}$ states in Table 3. Their masses are estimated simply by adding $2(M_{D_s^*} - M_{D^{*0}}) \simeq 210 \text{MeV}$ to the corresponding $cn-\bar{n}\bar{c}$ states. Their decay widths are estimated by using Eq. (5) with the common parameter a. The mass of $X_{cs-\bar{s}\bar{c}}^{1++}$ is expected to be quite close to the $D_s\bar{D}_s^*$ threshold and its decay is strongly suppressed. The $J/\psi\phi$ is un-open channel. Thus, the main decay mode is considered to be electromagnetic ones of order keV. The main decay mode of $Y_{cs\bar{s}\bar{c}}^{2++}$ is $D_s\bar{D}_s$. In order to check the four quark nature of this state, it is necessary to observe $J/\psi\phi$ decay.

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