# $B$ Physics (Theory)' 

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#### Abstract

Some theoretical aspects of $B$ physics are reviewed. These include a brief recapitulation of information on weak quark transitions as described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix, descriptions of CP asymmetries in $B$ decays to CP eigenstates and to self-tagging modes, a discussion of final-state phases in $B$ and charm decays, some topics on $B_{s}$ properties and decays, prospects for unusual excited $B$ states opened by discovery of some narrow $c \bar{s}$ resonances, and the search for heavier $Q=1 / 3$ quarks predicted in some extended grand unified theories.


## 1. INTRODUCTION

The physics of $B$ mesons (those containing the $b$ [bottom or beauty] quark) has greatly illuminated the study of the electroweak and strong interactions. This brief review is devoted to some theoretical aspects of $B$ physics, with emphasis on current questions for $e^{+} e^{-}$and hadron collider experiments. Section 2 reviews weak quark transitions. We note in Section 3 progress and puzzles in the study of $B^{0}$ decays to CP eigenstates, turning in Section 4 to direct CP asymmetries which require strong final-state phases for their observation. Some aspects of these phases are described in in Section 5. We devote Section 6 to the strange $B$ mesons, with Section 7 treating the possibility of narrow $b \bar{s}$ states suggested by the recent observation of narrow $c \bar{s}$ mesons. Section 8 discusses the prospects for seeing heavier $Q=1 / 3$ quarks. We summarize in Section 9. This review updates and supplements Refs. [1, 2].

## 2. WEAK QUARK TRANSITIONS

The relative strengths of charge-changing weak quark transitions are shown in Fig. It is crucial to describe this pattern precisely in order to distinguish among theories which might predict it, and to see whether it can reproduce all weak phenomena including CP violation or whether some new ingredient is needed.

[^0]

FIGURE 1. Charge-changing weak transitions among quarks. Solid lines: relative strength 1; dashed lines: relative strength 0.22 ; dot-dashed lines: relative strength 0.04 ; dotted lines: relative strength $\leq 0.01$.

### 2.1. The CKM matrix

The interactions in Fig. 1 may be parametrized by a unitary Cabibbo-KobayashiMaskawa (CKM) matrix which can be written approximately [3, 4] in terms of a small expansion parameter $\lambda$ as

$$
V_{\mathrm{CKM}}=\left[\begin{array}{ccc}
1-\frac{\lambda^{2}}{2} & \lambda & A \lambda^{3}(\rho-i \eta)  \tag{1}\\
-\lambda & 1-\frac{\lambda^{2}}{2} & A \lambda^{2} \\
A \lambda^{3}(1-\bar{\rho}-i \bar{\eta}) & -A \lambda^{2} & 1
\end{array}\right],
$$

where $\bar{\rho} \equiv \rho\left(1-\frac{\lambda^{2}}{2}\right)$ and $\bar{\eta} \equiv \eta\left(1-\frac{\lambda^{2}}{2}\right)$. The columns refer to $d, s, b$ and the rows to $u, c, t$. The parameter $\lambda=0.224$ [4] is $\sin \theta_{c}$, where $\theta_{c}$ is the Cabibbo angle. The value $\left|V_{c b}\right| \simeq 0.041$, obtained from $b \rightarrow c$ decays, indicates $A \simeq 0.82$, while $\left|V_{u b} / V_{c b}\right| \simeq 0.1$, obtained from $b \rightarrow u$ decays, implies $\left(\rho^{2}+\eta^{2}\right)^{1 / 2} \simeq 0.45$. We shall generally use the CKM parameters quoted in Ref. [5].

### 2.2. The unitarity triangle

The unitarity of the CKM matrix can be expressed in terms of a triangle in the complex $\bar{\rho}+i \bar{\eta}$ plane, with vertices at $(0,0)$ (angle $\left.\phi_{3}=\gamma\right),(1,0)$ (angle $\phi_{1}=\beta$ ), and $(\bar{\rho}, \bar{\eta})$ (angle $\left.\phi_{2}=\alpha\right)$. The triangle has unit base and its other two sides are $\bar{\rho}+i \bar{\eta}=-\left(V_{u b}^{*} V_{u d} / V_{c b}^{*} V_{c d}\right)$ $\left.\phi_{1}=\beta\right)$ and $1-\bar{\rho}-i \bar{\eta}=-\left(V_{t b}^{*} V_{t d} / V_{c b}^{*} V_{c d}\right)$. The result is shown in Fig. 2.


FIGURE 2. The unitarity triangle [1]. Ranges of angles allowed at $95 \%$ c.l. [5] are $78^{\circ}<\alpha<122^{\circ}$, $20^{\circ}<\beta<27^{\circ}$, and $38^{\circ}<\gamma<80^{\circ}$.

Flavor-changing loop diagrams provide further constraints. CP-violating $K^{0}-\bar{K}^{0}$ mixing is dominated by $\bar{s} d \rightarrow \bar{d} s$ with virtual $t \bar{t}$ and $W^{+} W^{-}$intermediate states. It constrains $\operatorname{Im}\left(V_{t d}^{2}\right) \sim \bar{\eta}(1-\bar{\rho})$, giving a hyperbolic band in the $(\bar{\rho}, \bar{\eta})$ plane. $B^{0}-\bar{B}^{0}$ mixing is dominated by $t \bar{t}$ and $W^{+} W^{-}$in the loop diagram for $\bar{b} d \rightarrow \bar{d} b$, and thus constrains $\left|V_{t d}\right|$ and hence $|1-\bar{\rho}-i \bar{\eta}|$. By comparing $B_{s} \bar{B}_{s}$ and $B^{0}-\bar{B}^{0}$ mixing, one reduces dependence on matrix elements and learns $\left|V_{t s} / V_{t d}\right|>4.4$ or $|1-\bar{\rho}-i \bar{\eta}|<1$. The resulting constraints are shown in Fig. [3] [5].

## 3. $B$ DECAYS TO CP EIGENSTATES

The decays of neutral $B$ mesons to CP eigenstates $f$, where $C P|f\rangle=\xi_{f}|f\rangle, \xi_{f}= \pm 1$, provide direct information on CKM phases without the need to understand complications of strong interactions. As a result of $B^{0}-\bar{B}^{0}$ mixing, a state $B^{0}$ at proper time $t=0$ evolves into a mixture of $B^{0}$ and $\bar{B}^{0}$ denoted $B^{0}(t)$. Thus there will be one pathway to the final state $f$ from $B^{0}$ through the amplitude $A$ and another from $\bar{B}^{0}$ through the amplitude $\bar{A}$, which acquires an additional phase $2 \phi_{1}=2 \beta$ through the mixing. The interference of these two amplitudes can differ in the decays $B^{0}(t) \rightarrow f$ and $\bar{B}^{0}(t) \rightarrow f$, leading to a time-integrated rate asymmetry

$$
\begin{equation*}
A_{C P} \equiv \frac{\Gamma\left(\bar{B}^{0} \rightarrow f\right)-\Gamma\left(B^{0} \rightarrow f\right)}{\Gamma\left(\bar{B}^{0} \rightarrow f\right)+\Gamma\left(B^{0} \rightarrow f\right)} \tag{2}
\end{equation*}
$$

as well as to time-dependent rates

$$
\left\{\begin{array}{l}
\Gamma\left[B^{0}(t) \rightarrow f\right]  \tag{3}\\
\Gamma\left[\bar{B}^{0}(t) \rightarrow f\right]
\end{array}\right\} \sim e^{-\Gamma t}\left[1 \mp A_{f} \cos \Delta m t \mp S_{f} \sin \Delta m t\right],
$$

where

$$
\begin{equation*}
A_{f} \equiv \frac{|\lambda|^{2}-1}{|\lambda|^{2}+1}, \quad S_{f} \equiv \frac{2 \operatorname{Im} \lambda}{|\lambda|^{2}+1}, \quad \lambda \equiv e^{-2 i \beta} \frac{\bar{A}}{A} \tag{4}
\end{equation*}
$$

where $S_{f}^{2}+A_{f}^{2} \leq 1[6,7]$.


FIGURE 3. Constraints in the $(\bar{\rho}, \bar{\eta})$ plane as of July 2002 (from the web page of Ref. [5]).

## 3.1. $B^{0} \rightarrow J / \psi K_{S}$ and $\phi_{1}=\beta$

For this decay one has $\bar{A} / A \simeq \xi_{J / \psi K_{S}}=-1$. The time-integrated asymmetry $A_{C P}$ is proportional to $\sin \left(2 \phi_{1}\right)=\sin (2 \beta)$. Using this and related decays involving the same quark subprocess, $\mathrm{BaBar}[8]$ finds $\sin (2 \beta)=0.741 \pm 0.067 \pm 0.033$ while Belle [9] finds $0.719 \pm 0.074 \pm 0.035$. The world average [10] is $\sin (2 \beta)=0.734 \pm 0.054$, consistent with other determinations [5, 11, 12].

$$
\text { 3.2. } B^{0} \rightarrow \pi^{+} \pi^{-} \text {and } \phi_{2}=\alpha
$$

Two amplitudes contribute to the decay: a "tree" $T$ and a "penguin" $P$ :

$$
\begin{equation*}
A=-\left(|T| e^{i \gamma}+|P| e^{i \delta}\right), \quad \bar{A}=-\left(|T| e^{-i \gamma}+|P| e^{i \delta}\right) \tag{5}
\end{equation*}
$$

where $\delta$ is the relative $P / T$ strong phase. The asymmetry $A_{C P}$ would be proportional to $\sin (2 \alpha)$ if the penguin amplitude could be neglected. One way to account for its contribution is via an isospin analysis [13] of $B$ decays to $\pi^{+} \pi^{-}, \pi^{ \pm} \pi^{0}$, and $\pi^{0} \pi^{0}$, separating the amplitudes for decays involving $I=0$ and $I=2$ final states. Information can then be obtained on both strong and weak phases. Since the branching ratio of $B^{0}$ to $\pi^{0} \pi^{0}$ may be very small, of order $10^{-6}$, alternative methods [14, 15] may be useful in which flavor $\operatorname{SU}(3)$ symmetry is used to estimate the penguin contribution [16, 17, 18].

The tree amplitude for $B^{0}(=\bar{b} d) \rightarrow \pi^{+} \pi^{-}$involves $\bar{b} \rightarrow \pi^{+} \bar{u}$, with the spectator $d$ quark combining with $\bar{u}$ to form a $\pi^{-}$. Its magnitude is $|T|$; its weak phase is $\operatorname{Arg}\left(V_{u b}^{*}\right)=\gamma$; by convention its strong phase is 0 . The penguin amplitude involves the flavor structure $\bar{b} \rightarrow \bar{d}$, with the final $\bar{d} d$ pair fragmenting into $\pi^{+} \pi^{-}$. Its magnitude is $|P|$. The dominant $t$ contribution in the loop diagram for $\bar{b} \rightarrow \bar{d}$ can be integrated out and the unitarity relation $V_{t d} V_{t b}^{*}=-V_{c d} V_{c b}^{*}-V_{u d} V_{u b}^{*}$ used. The $V_{u d} V_{u b}^{*}$ contribution can be absorbed into a redefinition of the tree amplitude, after which the weak phase of the penguin amplitude is $0(\bmod \pi)$. By definition, its strong phase is $\delta$.

The time-dependent asymmetries $S_{\pi \pi}$ and $A_{\pi \pi}$ specify both $\gamma($ or $\alpha=\pi-\beta-\gamma)$ and $\delta$, if one has an independent estimate of $|P / T|$. One may obtain $|P|$ from $B^{+} \rightarrow K^{0} \pi^{+}$ using flavor $\operatorname{SU}(3)$ [16, 17, 19] and $|T|$ from $B \rightarrow \pi l v$ using factorization [20]. (An alternative method discussed in Refs. [15, 18] uses the measured ratio of the $B^{+} \rightarrow K^{0} \pi^{+}$ and $B^{0} \rightarrow \pi^{+} \pi^{-}$branching ratios to constrain $|P / T|$.)

In addition to $S_{\pi \pi}$ and $A_{\pi \pi}$, a useful quantity is the ratio of the $B^{0} \rightarrow \pi^{+} \pi^{-}$branching ratio $\mathscr{B}\left(\pi^{+} \pi^{-}\right)$(unless otherwise specified, branching ratios refer to CP averages) to that due to the tree amplitude alone:

$$
\begin{equation*}
R_{\pi \pi} \equiv \frac{\mathscr{B}\left(\pi^{+} \pi^{-}\right)}{\left.\mathscr{B}\left(\pi^{+} \pi^{-}\right)\right|_{\text {tree }}}=1+2\left|\frac{P}{T}\right| \cos \delta \cos \gamma+\left|\frac{P}{T}\right|^{2} \tag{6}
\end{equation*}
$$

One also has

$$
\begin{gather*}
R_{\pi \pi} S_{\pi \pi}=\sin 2 \alpha+2\left|\frac{P}{T}\right| \cos \delta \sin (\beta-\alpha)-\left|\frac{P}{T}\right|^{2} \sin 2 \beta  \tag{7}\\
R_{\pi \pi} A_{\pi \pi}=-2|P / T| \sin \delta \sin \gamma \tag{8}
\end{gather*}
$$

We take $\beta=23.6^{\circ}$. The value of $|P / T|$ (updating [14, 15]) is $0.28 \pm 0.06$. Taking the central value, we plot in Fig. 4 trajectories in the ( $S_{\pi \pi}, A_{\pi \pi}$ ) plane for $-\pi \leq \delta \leq \pi$.

As shown in Table 1 BaBar [21] and Belle [22] obtain different asymmetries, especially $S_{\pi \pi}$. Even once this conflict is resolved, there are discrete ambiguities, since curves for different $\alpha$ intersect one another. These can be resolved with the help of $R_{\pi \pi}=0.62 \pm 0.28$, as shown in Fig. 5] The present value favors large $|\delta|$ and


FIGURE 4. Curves depicting dependence of $S_{\pi \pi}$ and $A_{\pi \pi}$ on $\delta$ [1]. From right to left the curves correspond to $\phi_{2}=\left(120^{\circ}, 105^{\circ}, 90^{\circ}, 75^{\circ}, 60^{\circ}\right)$. Plotted point: average of BaBar and Belle values (see text). As $|\delta|$ increases from 0 to $\pi$, the values of $S_{\pi \pi}$ become more positive, while the magnitudes $\left|A_{\pi \pi}\right|$ increase from zero and then return to zero. Positive values of $A_{\pi \pi}$ correspond to negative values of $\delta$.

TABLE 1. Values of $S_{\pi \pi}$ and $A_{\pi \pi}$ quoted by BaBar and Belle and their averages. Here we have applied scale factors of $\sqrt{\chi^{2}}=(2.31,1.24)$ to the errors for $S_{\pi \pi}$ and $A_{\pi \pi}$, respectively.

| Quantity | BaBar [21] | Belle [22] | Average |
| :---: | :---: | :---: | ---: |
| $S_{\pi \pi}$ | $0.02 \pm 0.34 \pm 0.05$ | $-1.23 \pm 0.41_{-0.07}^{+0.08}$ | $-0.49 \pm 0.61$ |
| $A_{\pi \pi}$ | $0.30 \pm 0.25 \pm 0.04$ | $0.77 \pm 0.27 \pm 0.08$ | $0.51 \pm 0.23$ |

$\phi_{2}=\alpha>90^{\circ}$, but with large uncertainty. It is not yet settled whether $A_{\pi \pi} \neq 0$, corresponding to "direct" CP violation.

Does the tree $(T)$ amplitude alone account for the $B^{0} \rightarrow \pi^{+} \pi^{-}$rate (corresponding to $R_{\pi \pi}=1$ ) or is there destructive interference with the penguin terms (corresponding to $R_{\pi \pi}<1$ )? Recently Zumin Luo and I [23] have combined the $B \rightarrow \pi l v$ spectrum reported by the CLEO Collaboration [24] with information on the $B^{+} \rightarrow \pi^{+} \pi^{0}$ rate, estimates of the ratio of color-suppressed to color-favored amplitude in this process, other determinations of $\left|V_{u b}\right|$, and lattice gauge theory predictions of the $B \rightarrow \pi l v$ form


FIGURE 5. Curves depicting dependence of $R_{\pi \pi}$ on $S_{\pi \pi}$ for various values of $\delta[1]$. The plotted point is the average of BaBar and Belle values for $S_{\pi \pi}$ (see text).
factor at high momentum transfer, to find that $R_{\pi \pi}=0.87_{-0.28}^{+0.11}$. The corresponding fit to the $B \rightarrow \pi l v$ spectrum is shown in Fig. 6, while the fit to the lattice predictions is shown in Fig. 7 For massless leptons (a good approximation), the differential decay rate is governed by a single form factor $F_{+}\left(q^{2}\right)$ :

$$
\begin{equation*}
\frac{d \Gamma}{d q^{2}}\left(B^{0} \rightarrow \pi^{-} \ell^{+} v_{\ell}\right)=\frac{G_{F}^{2}\left|V_{u b}\right|^{2}}{24 \pi^{3}}\left|\vec{p}_{\pi}\right|^{3}\left|F_{+}\left(q^{2}\right)\right|^{2}, \tag{9}
\end{equation*}
$$

where we take the simple form $F_{+}\left(q^{2}\right)=[F(0)]\left(1+a q^{2} / m_{B^{*}}^{2}\right) /\left(1-q^{2} / m_{B^{*}}^{2}\right)$. We find $a=1.14_{-0.42}^{+0.72}, F_{+}(0)=0.23 \pm 0.04$. The evidence for destructive tree-penguin interference in $B^{0} \rightarrow \pi^{+} \pi^{-}$is not overwhelming. A more definite conclusion will be possible when improved $B \rightarrow \pi l v$ spectra become available.
3.3. $B^{0} \rightarrow \phi K_{S}$ vs. $B^{0} \rightarrow J / \psi K_{S}$

In $B^{0} \rightarrow \phi K_{S}$, governed by the $\bar{b} \rightarrow \bar{s}$ penguin amplitude, the standard model predicts the same CP asymmetries as in those processes (like $B^{0} \rightarrow J / \psi K_{S}$ ) governed by $\bar{b} \rightarrow \bar{s} c \bar{c}$. In both cases the weak phase is expected to be $0(\bmod \pi)$, so the indirect CP asymmetry should be governed by $B^{0}-\bar{B}^{0}$ mixing and thus should be proportional to $\sin 2 \beta$. There should be no direct CP asymmetries (i.e., $A \simeq 0$ ) in either case. This is true for $B \rightarrow$ $J / \psi K ; A$ is consistent with zero in the neutral mode, while the direct CP asymmetry is consistent with zero in the charged mode [8]. However, a different result for $B^{0} \rightarrow \phi K_{S}$ could point to new physics in the $\bar{b} \rightarrow \bar{s}$ penguin amplitude [25].


FIGURE 6. Fit to $\int d q^{2} \frac{d \mathscr{B}}{d q^{2}}\left(B^{0} \rightarrow \pi^{-} l^{+} v_{l}\right)$ values [23] obtained for three $q^{2}$ bins in Ref. [24]. Points with errors correspond to data; the histogram represents the fit.

TABLE 2. Values of $S_{\phi K_{S}}$ and $A_{\phi K_{S}}$ quoted by BaBar and Belle and their averages. We have applied a scale factor of $\sqrt{\chi^{2}}=2.29$ to the error on $A_{\phi K_{S}}$.

| Quantity | BaBar [26] | Belle [27] | Average |
| :---: | :---: | :---: | :---: |
| $S_{\phi K_{S}}$ | $-0.18 \pm 0.51 \pm 0.07$ | $-0.73 \pm 0.64 \pm 0.22$ | $-0.38 \pm 0.41$ |
| $A_{\phi K_{S}}$ | $0.80 \pm 0.38 \pm 0.12$ | $-0.56 \pm 0.41 \pm 0.16$ | $0.19 \pm 0.68$ |

The experimental asymmetries in $B^{0} \rightarrow \phi K_{S}$ [26, 27] are shown in Table 2 For $A_{\phi K_{S}}$ there is a substantial discrepancy between BaBar and Belle. The value of $S_{\phi K_{S}}$, which should equal $\sin 2 \beta=0.734 \pm 0.054$ in the standard model, is about $2.7 \sigma$ away from it. If the amplitudes for $B^{0} \rightarrow \phi K^{0}$ and $B^{+} \rightarrow \phi K^{+}$are equal (true in many approaches), the time-integrated CP asymmetry $A_{C P}$ in the charged mode should equal $A_{\phi K_{S}}$. The BaBar Collaboration [28] has recently reported $A_{C P}\left(\phi K^{+}\right)=0.039 \pm 0.086 \pm 0.011$.

Many proposals for new physics can account for the departure of $S_{\phi K_{S}}$ from its expected value of $\sin 2 \beta$ [29]. A method for extracting a new physics amplitude has been developed [30], using the measured values of $S_{\phi K_{S}}$ and $A_{\phi K_{S}}$ and the ratio

$$
\begin{equation*}
R_{\phi K_{S}} \equiv \frac{\mathscr{B}\left(B^{0} \rightarrow \phi K_{S}\right)}{\left.\mathscr{B}\left(B^{0} \rightarrow \phi K_{S}\right)\right|_{\mathrm{std}}}=1+2 r \cos \phi \cos \delta+r^{2} \tag{10}
\end{equation*}
$$

where $r$ is the ratio of the magnitude of the new amplitude to the one in the standard


FIGURE 7. Comparison of lattice data points with best-fit form factor $F_{+}\left(q^{2}\right)$ [23]. Lattice data are from UKQCD (squares), APE (stars), Fermilab (circles) and JLQCD (diamonds) (see [23]).
model, and $\phi$ and $\delta$ are their relative weak and strong phases. For any values of $R_{\phi K_{S}}$, $\phi$, and $\delta$, Eq. 10) can be solved for the amplitude ratio $r$ and one then calculates the asymmetry parameters as functions of $\phi$ and $\delta$. The $\phi K_{S}$ branching ratio in the standard model is calculated using the penguin amplitude from $B^{+} \rightarrow K^{* 0} \pi^{+}$and an estimate of electroweak penguin corrections. Various regions of $(\phi, \delta)$ can reproduce the observed values of $S_{\phi K_{S}}$ and $A_{\phi K_{S}}$. Typical values of $r$ are of order 1; one generally needs to invoke new-physics amplitudes comparable to those in the standard model.

The above scenario envisions new physics entirely in $B^{0} \rightarrow \phi K^{0}$ and not in $B^{+} \rightarrow$ $K^{* 0} \pi^{+}$. An alternative is that new physics contributes to the $\bar{b} \rightarrow \bar{s}$ penguin amplitude and thus appears in both decays. Again, $S_{\phi K_{S}}$ suggests an amplitude associated with new physics [30], but one must wait until the discrepancy with the standard model becomes more significant. At present both the decays $B^{0} \rightarrow K_{S}\left(K^{+} K^{-}\right)_{C P=+}$ and $B^{0} \rightarrow \eta^{\prime} K_{S}$ display CP asymmetries consistent with standard expectations.

TABLE 3. Values of $S_{\eta^{\prime} K_{S}}$ and $A_{\eta^{\prime} K_{S}}$ quoted by BaBar and Belle and their averages. We have applied scale factors $\sqrt{\chi^{2}}=(1.48,1.15)$ to the errors for $S_{\eta^{\prime} K_{S}}$ and $A_{\eta^{\prime} K_{S}}$, respectively.

| Quantity | BaBar [26] | Belle [27] | Average |
| :---: | :---: | :---: | :---: |
| $S_{\eta^{\prime} K_{S}}$ | $0.02 \pm 0.34 \pm 0.03$ | $0.76 \pm 0.36_{-0.06}^{+0.05}$ | $0.37 \pm 0.37$ |
| $A_{\eta^{\prime} K_{S}}$ | $-0.10 \pm 0.22 \pm 0.03$ | $0.26 \pm 0.22 \pm 0.03$ | $0.08 \pm 0.18$ |

$$
\text { 3.4. } B^{0} \rightarrow K_{S}\left(K^{+} K^{-}\right)_{C P=+}
$$

The Belle Collaboration [27] finds that for $K^{+} K^{-}$not in the $\phi$ peak, most of the decay $B^{0} \rightarrow K_{S} K^{+} K^{-}$involves even CP for the $K^{+} K^{-}$system $\left(\xi_{K^{+} K^{-}}=+1\right)$. It is found that

$$
\begin{align*}
-\xi_{K^{+} K^{-}} S_{K^{+} K^{-}} & =0.49 \pm 0.43 \pm 0.11_{-0.00}^{+0.33}  \tag{11}\\
A_{K^{+} K^{-}} & =-0.40 \pm 0.33 \pm 0.10_{-0.26}^{+0.00} \tag{12}
\end{align*}
$$

where the third set of errors arise from uncertainty in the fraction of the CP-odd component. Independent estimates of this fraction have been performed in Refs. [31] and [32]. The quantity $-\xi_{K^{+} K^{-}} S_{K^{+} K^{-}}$should equal $\sin 2 \beta$ in the standard model, but additional non-penguin contributions can lead this quantity to range between 0.2 and 1.0 [32].

## 3.5. $B \rightarrow \eta^{\prime} K$ (charged and neutral modes)

At present neither the rate nor the CP asymmetry in $B \rightarrow \eta^{\prime} K$ present a significant challenge to the standard model. The rate can be reproduced with the help of a modest contribution from a "flavor-singlet penguin" amplitude [33, 34, 35, 36, 37]. (An alternative treatment [38] finds an enhanced standard-penguin contribution to $B \rightarrow \eta^{\prime} K$.) The CP asymmetry is not a problem; the ordinary and singlet penguin amplitudes have the same weak phase $\operatorname{Arg}\left(V_{t s}^{*} V_{t b}\right) \simeq \pi$ and hence one expects $S_{\eta^{\prime} K_{S}} \simeq \sin 2 \beta, A_{\eta^{\prime} K_{S}} \simeq 0$. The experimental situation is shown in Table 3. The value of $S_{\eta^{\prime} K_{S}}$ is consistent with the standard model expectation at the $1 \sigma$ level, while $A_{\eta^{\prime} K_{S}}$ is consistent with zero.

The singlet penguin amplitude may contribute elsewhere in $B$ decays. It is a possible source of a low-effective-mass $\bar{p} p$ enhancement [39] in $B^{+} \rightarrow \bar{p} p K^{+}$[40].

## 4. DIRECT CP ASYMMETRIES

Decays such as $B \rightarrow K \pi$ (with the exception of $B^{0} \rightarrow K^{0} \pi^{0}$ ) are self-tagging: Their final states indicate the flavor of the decaying state. For example, the $K^{+} \pi^{-}$final state is expected to originate purely from a $B^{0}$ and not from a $\bar{B}^{0}$. Such self-tagging decays involve both weak and strong phases. Several methods permit one to separate these from one another.

$$
\text { 4.1. } B^{0} \rightarrow K^{+} \pi^{-} \text {vs. } B^{+} \rightarrow K^{0} \pi^{+}
$$

The decay $B^{+} \rightarrow K^{0} \pi^{+}$is a pure penguin $(P)$ process, while the amplitude for $B^{0} \rightarrow$ $K^{+} \pi^{-}$is proportional to $P+T$, where $T$ is a (strangeness-changing) tree amplitude. The ratio $T / P$ has magnitude $r$, weak phase $\gamma \pm \pi$, and strong phase $\delta$. The ratio $R_{0}$ of these two rates (averaged over a process and its CP conjugate) is

$$
\begin{equation*}
R_{0} \equiv \frac{\bar{\Gamma}\left(B^{0} \rightarrow K^{+} \pi^{-}\right)}{\bar{\Gamma}\left(B^{+} \rightarrow K^{0} \pi^{+}\right)}=1-2 r \cos \gamma \cos \delta+r^{2} \geq \sin ^{2} \gamma \tag{13}
\end{equation*}
$$

where the inequality holds for any $r$ and $\delta$. For $R_{0}<1$ this inequality implies a constraint on $\gamma$ [41]. Using branching ratios [42, 43, 44] averaged in Ref. [45] and the $B^{+} / B^{0}$ lifetime ratio from Ref. [46], one finds $R_{0}=0.948 \pm 0.074$, which is consistent with 1 and does not permit application of the bound. However, using additional information on $r$ and the CP asymmetry in $B^{0} \rightarrow K^{+} \pi^{-}$, one can obtain a constraint on $\gamma$ [14, 47].

In Refs. [14, 47] we defined a "pseudo-asymmetry" normalized by the rate for $B^{0} \rightarrow K^{0} \pi^{+}$, a process which should not have a CP asymmetry since only the penguin amplitude contributes to it:

$$
\begin{equation*}
A_{0} \equiv \frac{\Gamma\left(\bar{B}^{0} \rightarrow K^{-} \pi^{+}\right)-\Gamma\left(B^{0} \rightarrow K^{+} \pi^{-}\right)}{2 \bar{\Gamma}\left(B^{+} \rightarrow K^{0} \pi^{+}\right)}=R_{0} A_{C P}\left(K^{+} \pi^{-}\right)=-2 r \sin \gamma \sin \delta \tag{14}
\end{equation*}
$$

One can eliminate $\delta$ between this equation and Eq. (13) and plot $R_{0}$ as a function of $\gamma$ for the allowed range of $\left|A_{0}\right|$. For a recent analysis based on this method see [1]. Instead we shall directly use $A_{C P}\left(K^{+} \pi^{-}\right)$, as in Refs. [2] and [48].

The value of $r$, based on present branching ratios and arguments given in Refs. [1, 14, 47]) is $r=0.17 \pm 0.04$. BaBar and Belle data imply $A_{C P}\left(K^{+} \pi^{-}\right)=-0.09 \pm$ 0.04 , leading us to take its magnitude as less than 0.13 at the $1 \sigma$ level. Curves for $A_{C P}\left(K^{+} \pi^{-}\right)=0$ and $\left|A_{C P}\left(K^{+} \pi^{-}\right)\right|=0.13$ are shown in Fig. 8 [45]. The lower limit $r=$ 0.13 is used to generate these curves since the limit on $\gamma$ will be the most conservative.

Using the $1 \sigma$ constraints on $R_{0}$ and $\left|A_{C P}\left(K^{+} \pi^{-}\right)\right|$one finds $\gamma \gtrsim 50^{\circ}$. No bound can be obtained at the $95 \%$ confidence level, however. Further data are needed in order for a useful constraint to be obtained.

$$
\text { 4.2. } B^{+} \rightarrow K^{+} \pi^{0} \text { vs. } B^{+} \rightarrow K^{0} \pi^{+}
$$

The comparison of rates for $B^{+} \rightarrow K^{+} \pi^{0}$ and $B^{+} \rightarrow K^{0} \pi^{+}$also gives information on $\gamma$. The amplitude for $B^{+} \rightarrow K^{+} \pi^{0}$ is proportional to $P+T+C$, where $C$ is a colorsuppressed amplitude. It was suggested in [49] that this amplitude be compared with $P$ from $B^{+} \rightarrow K^{0} \pi^{+}$and $T+C$ taken from $B^{+} \rightarrow \pi^{+} \pi^{0}$ using flavor $\mathrm{SU}(3)$ and a triangle construction to determine $\gamma$. Electroweak penguin amplitudes contributing in the $T+C$ term [50] may be taken into account [51] by noting that since $T+C$ corresponds to isospin $I(K \pi)=3 / 2$ for the final state, the strong-interaction phase of its EWP contribution is the same as that of the rest of the $T+C$ amplitude.


FIGURE 8. Behavior of $R_{0}$ for $r=0.134$ and $A_{0}=0$ (dashed curves) or $\left|A_{0}\right|=0.13$ (solid curve) as a function of the weak phase $\gamma$ [45]. Horizontal dashed lines denote $\pm 1 \sigma$ experimental limits on $R_{0}$, while dot-dashed lines denote $95 \%$ c.l. $( \pm 1.96 \sigma)$ limits.

New data on branching ratios and CP asymmetries permit an update of previous analyses [14, 51]. One makes use of the quantities (see [37] and [45])

$$
\begin{gather*}
R_{c} \equiv \frac{2 \bar{\Gamma}\left(B^{+} \rightarrow K^{+} \pi^{0}\right)}{\bar{\Gamma}\left(B^{+} \rightarrow K^{0} \pi^{+}\right)}=1.24 \pm 0.13  \tag{15}\\
\mathscr{A}_{C P}\left(K^{+} \pi^{0}\right)=-\frac{2 r_{c} \sin \delta_{c} \sin \gamma}{R_{c}}=0.035 \pm 0.071 \tag{16}
\end{gather*}
$$

where $r_{c} \equiv|(T+C) / P|=0.20 \pm 0.02$, and a strong phase $\delta_{c}$ is eliminated by combining (15) and (16). One must also use an estimate [51] of the electroweak penguin parameter $\delta_{\mathrm{EW}}=0.65 \pm 0.15$. One obtains the most conservative (i.e., weakest) bound on $\gamma$ for the maximum values of $r_{c}$ and $\delta_{\text {EW }}$ [14]. The resulting plot is shown in Fig. 9 [1, 45].) One obtains a bound at the $1 \sigma$ level very similar to that in the previous case: $\gamma \gtrsim 52^{\circ}$. The bound is actually set by the curve for zero CP asymmetry, as emphasized in Ref. [51].

### 4.3. Asymmetries in $B^{+} \rightarrow\left(\pi^{0}, \eta, \eta^{\prime}\right) K^{+}$

The amplitudes for the decays $B^{+} \rightarrow M^{0} K^{+}\left[\left(M^{0}=\left(\pi^{0}, \eta, \eta^{\prime}\right)\right]\right.$ all are dominated by penguin amplitudes and can be expressed as

$$
\begin{equation*}
A\left(B^{+} \rightarrow M^{0} K^{+}\right)=a\left(e^{i \gamma}-\delta_{E W}\right) e^{i \delta_{T}}-b \tag{17}
\end{equation*}
$$



FIGURE 9. Behavior of $R_{c}$ for $r_{c}=0.22$ ( $1 \sigma$ upper limit) and $\mathscr{A}_{C P}\left(K^{+} \pi^{0}\right)=0$ (dashed curves) or $\left|\mathscr{A}_{C P}\left(K^{+} \pi^{0}\right)\right|=0.11$ (solid curve) as a function of the weak phase $\gamma$ [45]. Horizontal dashed lines denote $\pm 1 \sigma$ experimental limits on $R_{c}$, while dotdashed lines denote $95 \%$ c.l. $( \pm 1.96 \sigma)$ limits. We have taken $\delta_{E W}=0.80$ (its $1 \sigma$ upper limit), which leads to the most conservative bound on $\gamma$.
where $a$ and $b$ may be calculated using flavor $\mathrm{SU}(3)$ from other processes [37], and $\delta_{T}$ is a strong phase. The allowed ranges of the resulting CP asymmetries are shown in Fig. 10 [37]. The asymmetries are sensitive to $\delta_{T}$ but vary less significantly with $\gamma$ over the $95 \%$ c.l. allowed range [5] $38^{\circ}<\gamma<80^{\circ}$. For illustration we have chosen $\gamma=60^{\circ}$.

The constraints on $\delta_{T}$ from $A_{C P}\left(\pi^{0} K^{+}\right)$are $-34^{\circ} \leq \delta_{T} \leq 19^{\circ}$ and a region of comparable size around $\delta_{T}=\pi$. The allowed range of $A_{C P}\left(\eta K^{+}\right)$restricts these regions further, leading to net allowed regions $-7^{\circ} \leq \delta_{T} \leq 19^{\circ}$ or a comparable region around $\delta_{T}=\pi$. These regions do not change much if we vary $\gamma$ over its allowed range. The scheme of Ref. [38] predicts an opposite sign of $A_{C P}\left(\eta K^{+}\right)$to ours for a given sign of $\delta_{T}$ and hence the constraints will differ.

$$
\text { 4.4. } B^{+} \rightarrow \pi^{+} \eta
$$

The possibility that several different amplitudes could contribute to $B^{+} \rightarrow \pi^{+} \eta$, thereby leading to the possibility of a large direct CP asymmetry, has been recognized for some time [19, 33, 34, 52, 53]. Contributions can arise from a tree amplitude (colorfavored plus color-suppressed) $T+C$, whose magnitude is estimated from that occurring in $B^{+} \rightarrow \pi^{+} \pi^{0}$, a penguin amplitude $P$, obtained via flavor $\mathrm{SU}(3)$ from $B^{+} \rightarrow K^{0} \pi^{+}$, and a singlet penguin amplitude $S$, obtained from $B \rightarrow \eta^{\prime} K$.

In Table 4 we summarize branching ratios and CP asymmetries obtained for the decay


FIGURE 10. Predicted $C P$ rate asymmetries when $\gamma=60^{\circ}$ for $B^{+} \rightarrow \pi^{0} K^{+}$(top), $B^{+} \rightarrow \eta K^{+}$(middle), and $B^{+} \rightarrow \eta^{\prime} K^{+}$(bottom) [37]. Horizontal dashed lines denote $95 \%$ c.l. ( $\pm 1.96 \sigma$ ) upper and lower experimental bounds, leading to corresponding bounds on $\delta_{T}$ denoted by vertical dashed lines. Arrows point toward allowed regions.

TABLE 4. Branching ratios and CP asymmetries for $B^{+} \rightarrow \pi^{+} \eta$.

|  | $\mathscr{B}\left(10^{-6}\right)$ | $A_{C P}$ |
| :--- | :---: | :---: |
| CLEO [54] | $1.2_{-1.2}^{+2.8}(<5.7)$ | - |
| BaBar [55] | $4.2_{-0.0 \pm}^{+1.0} \pm 0.3$ | $-0.51_{-0.18}^{+0.20}$ |
| Belle [43] | $5.2_{-1.7}^{+2.0} \pm 0.6$ | -.0 |
| Average | $4.1 \pm 0.9$ | $-0.51_{-0.18}^{+0.20}$ |
| $\|T+C\|^{2}$ alone | 3.5 | 0 |
| $\|P+S\|^{2}$ alone | 1.9 | 0 |

$B^{+} \rightarrow \pi^{+} \eta$ by CLEO [54], BaBar [55], and Belle [43]. We assume that the $S$ and $P$ amplitudes have the same weak and strong phases. The equality of their weak phases is quite likely, while tests exist for the latter assumption [37].

If an amplitude $A$ for a process receives two contributions with differing strong and weak phases, one can write

$$
\begin{equation*}
A=a_{1}+a_{2} e^{i \phi} e^{i \delta}, \bar{A}=a_{1}+a_{2} e^{-i \phi} e^{i \delta} . \tag{18}
\end{equation*}
$$

The CP-averaged decay rate is proportional to $a_{1}^{2}+a_{2}^{2}+2 a_{1} a_{2} \cos \phi \cos \delta$, while the CP asymmetry is

$$
\begin{equation*}
A_{C P}=-\frac{2 a_{1} a_{2} \sin \phi \sin \delta}{a_{1}^{2}+a_{2}^{2}+2 a_{1} a_{2} \cos \phi \cos \delta} . \tag{19}
\end{equation*}
$$

In the case of $B^{+} \rightarrow \pi^{+} \eta$ the rates and CP asymmetry suggest that $|\sin \phi \sin \delta|>$ $|\cos \phi \cos \delta|$.

By combining the branching ratio and $C P$ rate asymmetry information of the $\pi^{ \pm} \eta$ modes, one can extract the values of the relative strong phase $\delta$ and the weak phase $\alpha$, assuming maximal constructive interference between ordinary and singlet penguin amplitudes. On the basis of the range of amplitudes extracted from other processes, we find that the rates and CP asymmetries for $B^{+} \rightarrow \pi^{ \pm} \eta$ and $B^{+} \rightarrow \pi^{ \pm} \eta^{\prime}$ are correlated with one another [37]:

$$
\begin{gather*}
A_{C P}\left(\pi^{+} \eta\right)=-(0.91 \sin \delta \sin \alpha) /(1-0.91 \cos \delta \cos \alpha)  \tag{20}\\
A_{C P}\left(\pi^{+} \eta^{\prime}\right)=-(\sin \delta \sin \alpha) /(1-\cos \delta \cos \alpha)  \tag{21}\\
\mathscr{B}\left(\pi^{+} \eta\right)=5 \times 10^{-6}(1-0.91 \cos \delta \cos \alpha)  \tag{22}\\
\mathscr{B}\left(\pi^{+} \eta^{\prime}\right)=3.4 \times 10^{-6}(1-\cos \delta \cos \alpha) \tag{23}
\end{gather*}
$$

where $\mathscr{B}$ refers to a CP-averaged branching ratio. One finds that $\mathscr{B}\left(B^{+} \rightarrow \pi^{+} \eta^{\prime}\right)=$ $(2.7 \pm 0.7) \times 10^{-6}$ (below current upper bounds) and that $A_{C P}\left(\pi^{+} \eta^{\prime}\right)=-0.57 \pm 0.23$.

The amplitudes for $B^{ \pm} \rightarrow \pi^{ \pm} \eta$ may be written in the form

$$
\begin{equation*}
A\left(\pi^{ \pm} \eta\right) \sim e^{ \pm i \gamma}\left[1-r_{\eta} e^{i( \pm \alpha+\delta)}\right] \tag{24}
\end{equation*}
$$

where $r_{\eta}$ (estimated in Ref. [37] to be $0.65 \pm 0.06$ ) is the ratio of penguin to tree contributions to the $B^{ \pm} \rightarrow \pi^{ \pm} \eta$ decay amplitudes. We define $R_{\eta}$ as the ratio of the observed $C P$-averaged $B^{ \pm} \rightarrow \pi^{ \pm} \eta$ decay rate to that which would be expected in the limit of no penguin contributions and find

$$
\begin{equation*}
R_{\eta}=1+r_{\eta}^{2}-2 r_{\eta} \cos \alpha \cos \delta=1.18 \pm 0.30 \tag{25}
\end{equation*}
$$

One can then use the information on the observed $C P$ asymmetry in this mode to eliminate $\delta$ and constrain $\alpha$. (For a related treatment with a different convention for penguin amplitudes see Ref. [48].) The asymmetry is

$$
\begin{equation*}
A_{\eta}=-2 r_{\eta} \sin \alpha \sin \delta / R_{\eta}=-0.51 \pm 0.19 \tag{26}
\end{equation*}
$$

so one can either use the result

$$
\begin{equation*}
R_{\eta}=1+r_{\eta}^{2} \pm \sqrt{4 r_{\eta}^{2} \cos ^{2} \alpha-\left(A_{\eta} R_{\eta}\right)^{2} \cot ^{2} \alpha} \tag{27}
\end{equation*}
$$

with experimental ranges of $R_{\eta}$ and $A_{\eta}$ or solve (27) for $R_{\eta}$ in terms of $\alpha$ and $A_{\eta}$. The result of this latter method is illustrated in Fig. [11] [37].


FIGURE 11. Predicted value of $R_{\eta}$ (ratio of observed $C P$-averaged $B^{ \pm} \rightarrow \pi^{ \pm} \eta$ decay rate to that predicted for tree amplitude alone) as a function of $\alpha$ for various values of $C P$ asymmetry $\left|A_{\eta}\right|$ [37]. (The values 0.70 and 0.32 correspond to $\pm 1 \sigma$ errors on this asymmetry.)

The range of $\alpha$ allowed at $95 \%$ c.l. in standard-model fits to CKM parameters is $78^{\circ} \leq \alpha \leq 122^{\circ}$ [5]. For comparison, Fig. 11]permits values of $\alpha$ in the three ranges

$$
\begin{equation*}
14^{\circ} \leq \alpha \leq 53^{\circ}, \quad 60^{\circ} \leq \alpha \leq 120^{\circ}, \quad 127^{\circ} \leq \alpha \leq 166^{\circ} \tag{28}
\end{equation*}
$$

if $R_{\eta}$ and $\left|A_{\eta}\right|$ are constrained to lie within their $1 \sigma$ limits. The middle range overlaps the standard-model parameters, restricting them slightly. Better constraints on $\alpha$ in this region would require reduction of errors on $R_{\eta}$.

## 5. FINAL-STATE PHASES

## 5.1. $B$ decays

We have seen that final-state phases are needed in order to observe direct CP asymmetries. It is interesting to obtain information on such phases in those $D$ decays in which weak phases are expected to play little role, so that magnitures of amplitudes directly reflect relative strong phases. As one example we illustrate such phases in the decays of $B \rightarrow D \pi$ and related processes in Fig. 12 [56]. The color-suppressed amplitude $C$ is found to have a non-trivial strong phase with respect to the color-favored tree amplitude $T$, with a small exchange amplitude $E$ (governing $B^{0} \rightarrow D_{s}^{-} K^{+}$) at an even larger phase with respect to $T$. Such large phases can signal strong rescattering effects.


FIGURE 12. Amplitude triangle for $\bar{B} \rightarrow D \pi$ and related decays [56]. The amplitude $E$ points from either $O$ or $O^{\prime}$ to the center of the small circle. The amplitudes $T$ and $C$ are shown only for the first of these two solutions. Here $A\left(B^{0} \rightarrow D^{-} \pi^{+}\right)=T+E, A\left(B^{+} \rightarrow \bar{D}^{0} \pi^{+}\right)=T+C, \sqrt{2} A\left(B^{0} \rightarrow \bar{D}^{0} \pi^{0}\right)=C-E$, $\sqrt{3} A\left(B^{0} \rightarrow \bar{D}^{0} \eta\right)=-(C+E)$, and $A\left(B^{0} \rightarrow D_{s}^{-} K^{+}\right)=E$.

### 5.2. Charm decays

In one method for measuring the weak phase $\gamma$ in $B^{ \pm} \rightarrow K^{ \pm}\left(K K^{*}\right)_{D}$ decays, the relative strong phase $\delta_{D}$ in $D^{0} \rightarrow K^{*+} K^{-}$and $D^{0} \rightarrow K^{*-} K^{+}$decays (equivalently, in $D^{0} \rightarrow K^{*+} K^{-}$and $\bar{D}^{0} \rightarrow K^{*+} K^{-}$) plays a role [57]. A study of the Dalitz plot in $D^{0} \rightarrow K^{+} K^{-} \pi^{0}$ can yield information on this phase [58]. By comparing such Dalitz plots for constructive and destructive interference between the two $K^{*}$ bands one finds that a clear-cut distinction is possible between $\delta_{D}=0$ and $\delta_{D}= \pm \pi$ with a couple of thousand decays.

## 6. $B_{S}$ MIXING AND DECAYS

## 6.1. $B_{s}-\bar{B}_{s}$ mixing

The ratio of the $B_{s}-\bar{B}_{s}$ mixing amplitude $\Delta m_{s}$ to the $B^{0}-\bar{B}^{0}$ mixing amplitude $\Delta m_{d}$ ( $B_{d} \equiv B^{0}$ ) is given by

$$
\begin{equation*}
\frac{\Delta m_{s}}{\Delta m_{d}}=\frac{f_{B_{s}}^{2} B_{B_{s}}}{f_{B_{d}}^{2} B_{B_{d}}} \frac{m_{B_{s}}}{m_{B_{d}}}\left|\frac{V_{t s}}{V_{t d}}\right|^{2} \simeq 48 \times 2^{ \pm 1} . \tag{29}
\end{equation*}
$$

Here $f_{B_{d, s}}$ are meson decay constants, while $B_{B_{d, s}}$ express the degree to which the mixing amplitude is due to vacuum intermediate states. A lattice estimate of the ratio $\xi \equiv$ $\left(f_{B_{s}} / f_{B_{d}}\right) \sqrt{B_{B_{s}} / B_{B_{d}}}$ is $1.21 \pm 0.04 \pm 0.05$ [59]. We have taken $\left|V_{t d}\right|=A \lambda^{3}|1-\bar{\rho}-i \bar{\eta}|=$ $(0.8 \pm 0.2) A \lambda^{3}$ with $\left|V_{t s}\right|=A \lambda^{2}$ and $\lambda=0.22$. With [46] $\Delta m_{d}=0.502 \pm 0.006 \mathrm{ps}^{-1}$ one then predicts $\Delta m_{s}=24 \mathrm{ps}^{-1} \times 2^{ \pm 1}$. The lower portion of this range is already excluded by the bound [46] $\Delta m_{s}>14.4 \mathrm{ps}^{-1}$ ( $95 \%$ c.l.). When $\Delta m_{s}$ is measured it will constrain $\bar{\rho}$ significantly.

### 6.2. Decays to CP eigenstates

$$
\text { 6.2.1. } B_{s} \rightarrow J / \psi \phi, J / \psi \eta, \ldots
$$

Since the weak phase in $\bar{b} \rightarrow \bar{c} c \bar{s}$ is expected to be zero while that of $B_{s}-\bar{B}_{s}$ mixing is expected to be very small, one expects $C P$ asymmetries to be only a few percent in the standard model for those $B_{s}$ decays dominated by this quark subprocess. The $B_{s} \rightarrow J / \psi \phi$ final state is not a CP eigenstate but the even and odd CP components can be separated using angular analyses. The final states of $B_{s} \rightarrow J / \psi \eta$ and $B_{s} \rightarrow J / \psi \eta^{\prime}$ are CP-even so no such analysis is needed.

$$
\text { 6.2.2. } B_{s} \rightarrow K^{+} K^{-} \text {vs. } B^{0} \rightarrow \pi^{+} \pi^{-}
$$

A comparison of time-dependent asymmetries in $B_{s} \rightarrow K^{+} K^{-}$and $B^{0} \rightarrow \pi^{+} \pi^{-}$[60] allows one to separate out strong and weak phases and relative tree and penguin contributions. In $B_{s} \rightarrow K^{+} K^{-}$the $\bar{b} \rightarrow \bar{s}$ penguin amplitude is dominant, while the strangenesschanging tree amplitude $\bar{b} \rightarrow \bar{u} u \bar{s}$ is smaller. In $B^{0} \rightarrow \pi^{+} \pi^{-}$it is the other way around: The $\bar{b} \rightarrow \bar{u} u \bar{d}$ tree amplitude dominates, while the $\bar{b} \rightarrow \bar{d}$ penguin is Cabibbo-suppressed. The U-spin subgroup of $\mathrm{SU}(3)$, which interchanges $s$ and $d$ quarks, relates each amplitude in one process to that in the other apart from the CKM factors.

$$
\text { 6.2.3. } \bar{B}_{s}, B^{0} \rightarrow K^{+} \pi^{-} .
$$

In comparing $B_{s} \rightarrow K^{+} K^{-}$with $B^{0} \rightarrow \pi^{+} \pi^{-}$, the mass peaks will overlap with one another if analyzed in terms of the same final state (e.g., $\pi^{+} \pi^{-}$) [61]. Thus, in the absence of good particle identification, a variant on this scheme employing the decays $B^{0} \rightarrow K^{+} \pi^{-}$and $B_{s} \rightarrow K^{-} \pi^{+}$(also related to one another by U-spin) may be useful [62]. For these final states, kinematic separation may be easier. One can also study the timedependence of $B_{s} \rightarrow K^{+} K^{-}$while normalizing the penguin amplitude using $B_{s} \rightarrow K^{0} \bar{K}^{0}$ [63].

### 6.3. Other $\mathrm{SU}(3)$ relations

The U-spin subgroup of $\mathrm{SU}(3)$ allows one to relate many other $B_{s}$ decays besides those mentioned above to corresponding $B_{d}$ decays [64]. Particularly useful are relations between CP-violating rate differences. One thus will have the opportunity to perform many tests of flavor $\operatorname{SU}(3)$ and to learn a great deal more about final-state phase patterns when a variety of $B_{s}$ decays can be studied.

## 7. EXCITED STATES

### 7.1. Flavor tagging for neutral $B$ mesons

A promising method for tagging the flavor of a neutral $B$ meson is to study the charge of the leading light hadron accompanying the fragmentation of the heavy quark [65, 66, 67]. For example, an initial $b$ will fragment into a $\bar{B}^{0}$ by "dressing" itself with a $\bar{d}$. The accompanying $d$, if incorporated into a charged pion, will end up in a $\pi^{-}$. Thus a $\pi^{-}$is more likely to be "near" a $\bar{B}^{0}$ than to a $B^{0}$ in phase space. This correlation between $\pi^{-}$and $\bar{B}^{0}$ (and the corresponding correlation between $\pi^{+}$and $B^{0}$ ) is also what one would expect on the basis of non-exotic resonance formation. Thus the study of the resonance spectrum of the excited $B$ mesons which can decay to $B+\pi$ or $B^{*}+\pi$ is of special interest [68]. The lowest such mesons are the P-wave levels of a $\bar{b}$ antiquark and a light ( $u$ or $d$ ) quark.

### 7.2. Excited $D_{s}$ states below $D^{(*)} K$ threshold

In April of this year the BaBar Collaboration [69] reported a narrow resonance at 2317 MeV decaying to $D_{s} \pi^{0}$. This state was quickly confirmed by CLEO [70], who also presented evidence for a narrow state at 2463 MeV decaying to $D_{s}^{*} \pi^{0}$. Both states have been confirmed by Belle [71]. We mention briefly why these states came as surprises.

The previously known P-wave levels of a charmed quark $c$ and an antistrange $\bar{s}$ were candidates for $J=1$ and $J=2$ states at 2535 and 2572 MeV [72]. These levels have narrow widths behave as expected if the spin of the $\bar{s}$ and the orbital angular momentum were coupled up to $j=3 / 2$. (One expects $j-j$ rather than $L-S$ coupling in a light-heavy system [73, 74, 75].) If the $j=1 / 2$ states were fairly close to these in mass one would then expect another $J=1$ state and a $J=0$ state somewhere above 2500 MeV . Instead, the candidate for the $J=0 c \bar{s}$ state is the one at 2317 MeV , with the state at 2463 MeV the candidate for the second $J=1$ level. Belle's observation of the decay $D_{s J}(2463) \rightarrow D_{s} \gamma$ reinforces this interpretation [71]. Both states are narrow since they are too light to decay respectively to $D K$ or $D^{*} K$. They decay instead via isospin-violating transitions. They are either candidates for $D^{(*)} K$ molecules [76], or indications of a broken chiral symmetry which places them as positive-parity partners of the $D_{s}$ and $D_{s}^{*}$ negative-parity $c \bar{s}$ ground states [77]. Indeed, the mass splittings between the parity partners appear to be exactly as predicted ten years ago [78]. Potential-based quarkonium models have a hard time accommodating such low masses [79, 80, 81],

There should exist non-strange $j=1 / 20^{+}$and $1^{+}$states, lower in mass than the $j=3 / 2$ states at 2422 and 2459 MeV [72] but quite broad since their respective $\bar{D} \pi$ and $\bar{D}^{*} \pi$ channels will be open. The study of such states will be of great interest since the properties of the corresponding $B$-flavored states will be useful in tagging the flavor of neutral $B$ mesons.

### 7.3. Narrow positive-parity states below $\bar{B}{ }^{(*)} K$ threshold?

If a strange antiquark can bind to a charmed quark in both negative- and positive-parity states, the same must be true for a strange antiquark and a $b$ quark. One should then expect to see narrow $J^{P}=0^{+}$and $1^{+}$states with the quantum numbers of $\bar{B} K$ and $\bar{B}^{*} K$ but below those respective thresholds. They should decay to $\bar{B}_{s} \pi^{0}$ and $\bar{B}_{s}^{*} \pi^{0}$, respectively. To see such decays one will need a multi-purpose detector with good charged particle and $\pi^{0}$ identification!

## 8. EXOTIC $Q=-1 / 3$ QUARKS

Might there be heavier quarks visible at hadron colliders? At present we have evidence for three families of quarks and leptons belonging to 16 -dimensional multiplets of the grand unified group $\mathrm{SO}(10)$ (counting right-handed neutrinos as a reasonable explanation of the observed oscillations between different flavors of neutrinos). Just as $\mathrm{SO}(10)$ was pieced together from multiplets of $\mathrm{SU}(5)$ with dimensions 1,5 , and 10 , the smallest representation of a still larger grand unified group could contain the 16-dimensional $\mathrm{SO}(10)$ spinor. Such a group is $\mathrm{E}_{6}$ [82]. Its smallest representation, of dimension 27, contains a 16 -dimensional spinor, a 10 -dimensional vector, and a singlet of $\mathrm{SO}(10)$. The 10-dimensional vector contains vector-like isosinglet quarks " $h$ " and antiquarks $\bar{h}$ of charge $Q= \pm 1 / 3$ and isodoublet leptons. The $\mathrm{SO}(10)$ singlets are candidates for sterile neutrinos, one for each family.

The new exotic $h$ quarks can mix with the $b$ quark and push its mass down with respect to the top quark [83]. Troy Andre and I are looking at signatures of $h \bar{h}$ production in hadron colliders, to either set lower mass limits or see such quarks through their decays to $Z+b, W+t$, and possibly Higgs $+b$. The $Z$, for example, would be identified by its decays to $v \bar{v}, \ell^{+} \ell^{-}$, or jet + jet, while the Higgs boson would show up through its $b \bar{b}$ decay if it were far enough below $W^{+} W^{-}$threshold.

## 9. SUMMARY

The process $B^{0} \rightarrow J / \psi K_{S}$ has provided spectacular confirmation of the KobayashiMaskawa theory of CP violation, measuring $\beta$ to a few degrees. Now one is entering the territory of more difficult measurements.

The decay $B^{0} \rightarrow \pi^{+} \pi^{-}$can give useful information on $\alpha$. One needs either a measurement of $\mathscr{B}\left(B^{0} \rightarrow \pi^{0} \pi^{0}\right)$ [13], probably at the $10^{-6}$ level (present limits [42, 43, 44] are several times that), or a better estimate of the tree amplitude from $B \rightarrow \pi l v$ [20]. Indeed, such an estimate has been presented recently [23]. The BaBar and Belle experimental CP asymmetries [21, 22] will eventually converge to one another, as did the initial measurements of $\sin 2 \beta$ using $B^{0} \rightarrow J / \psi K_{S}$.

The $B \rightarrow \phi K_{S}$ decay can display new physics via special $\bar{b} \rightarrow \bar{s} s \bar{s}$ operators or effects on the $\bar{b} \rightarrow \bar{s}$ penguin. Some features of any new amplitude can be extracted from the data in a model-independent way if one uses both rate and asymmetry information [30].

While the effective value of $\sin 2 \beta$ in $B^{0} \rightarrow \phi K_{S}$ seems to differ from its expected value by more than $2 \sigma$, CP asymmetries in $B \rightarrow K_{S}\left(K^{+} K^{-}\right)_{C P=+}$ do not seem anomalous.

The rate for $B \rightarrow \eta^{\prime} K_{S}$ is not a problem for the standard model if one allows for a modest flavor-singlet penguin contribution in addition to the standard penguin amplitude. The CP asymmetries for this process are in accord with the expectations of the standard model at the $1 \sigma$ level or better. Effects of the singlet penguin amplitude may also be visible elsewhere, for example in $B^{+} \rightarrow p \bar{p} K^{+}$.

Various ratios of $B \rightarrow K \pi$ rates, when combined with information on CP asymmetries, show promise for constraining phases in the CKM matrix. These tests have steadily improved in accuracy in the past couple of years. One expects further progress as $e^{+} e^{-}$ luminosities increase, and as hadron colliders begin to provide important contributions. The decays $B^{+} \rightarrow \pi^{+} \eta$ and $B^{+} \rightarrow \pi^{+} \eta^{\prime}$ show promise for displaying large CP asymmetries [37] since they involve contributions of different amplitudes with comparable magnitudes. Strong final-state phases, important for the observation of direct CP violation, are beginning to be mapped out in $B$ decays.

In the near term the prospects for learning about the $B_{s} \bar{B}_{s}$ mixing amplitude are good. The potentialities of hadron colliders for the study study of CP violation and branching ratios in $B_{s}$ decays will be limited only by the versatility of detectors. Surprises in spectroscopy, as illustrated by the low-lying positive-parity $c \bar{s}$ candidiates, still can occur, and one is sure to find more of them. Finally, one can search for objects related to the properties of $b$ quarks, such as the exotic isosinglet quarks $h$, with improved sensitivity in Run II of the Tevatron and with greatly expanded reach at the LHC.

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