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1 Characterisation of crowd lateral dynamic forcing from full-scale 2 measurements on the Clifton Suspension Bridge

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5 **Abstract**

6 Lateral loading of bridges by a crowd of walking pedestrians is of serious concern as it can lead to a
7 sudden growth in the amplitude of structural oscillations, i.e. lateral dynamic instability. A vibration
8 amplitude threshold, marking a qualitative change in pedestrians behaviour, is then usually proposed
9 beyond which the likelihood of structural instability is said to increase. To verify this presumption,
10 measurements were taken during a crowd loading event on Clifton Suspension Bridge in Bristol, UK.
11 Two lateral modes of the bridge were studied, previously found susceptible to pedestrian-induced ex-
12 citation. A novel procedure is proposed based on time-frequency analysis enabling, for the first time,
13 the average equivalent added mass per pedestrian to be identified from measurements on a full-scale
14 structure. Previous measurements on Clifton Suspension Bridge during crowd loading leading to the
15 onset of large-amplitude vibrations revealed an increase in the natural frequency of one from the two
16 considered modes. The proposed time-frequency analysis procedure has successfully identified the
17 additional mass, due to the pedestrians, that is effectively negative. Cycle-by-cycle energy analysis
18 per mode confirms the presence of additional damping of the pedestrians at low vibration amplitudes,
19 that is also effectively negative. Although some of the results are uncertain quantitatively, there is no
20 evidence of the amplitude threshold at which the human-structure interaction phenomenon occurs.

21 **1 Introduction**

22 Human-Structure Interaction (HSI) is a phenomenon of increasing interest for researchers from civil,
23 structural and mechanical engineering disciplines. The literature has documented many cases of large
24 amplitude lateral bridge oscillations in the presence of pedestrians. The most comprehensive studies in-
25 clude the London Millennium Footbridge (LMF) [1], Toda Park Bridge (TPB) [2], Solferino Footbridge
26 (SF) [3], Pedro e Inês Footbridge [4], Clifton Suspension Bridge (CSB) [5] and the Singapore Airport
27 Changi Mezzanine Bridge (CMB) [6]. The cause of excessive response of these bridges is thought to be
28 negative damping provided by pedestrians. The framework of modelling humans as negative dampers
29 was first suggested by Arup [1] from analysis of the experimental data on the behaviour of LMF. This
30 framework was expanded to account for the component of pedestrian force in phase with structural ac-
31 celeration (or displacement) after some tests on an instrumented treadmill by Pizzimenti and Ricciardelli
32 [7]. After scaling, this force component can be expressed as equivalent added mass (or stiffness). While
33 added damping and stiffness are conventionally adopted in wind engineering when modelling aeroe-
34 lasticity, [8] [9], it is not entirely clear how these force components arise from the action of a crowd.
35 Nevertheless, in the human structure interaction literature [1, 5, 6, 7, 10, 11, 12, 13, 14, 15] it is con-
36 ventional to model the effects of pedestrians as equivalent added damping and mass or stiffness (which
37 can be positive or negative) to the structure. A plausible explanation of the added damping effect was
38 provided by Macdonald [10] who, expanding a simpler model by Barker [16], built a highly reduced or-
39 der pedestrian model inverted pendulum model (IPM). The IPM can capture pedestrian dynamics in the
40 frontal plane when walking on a rigid ground [17, 18]. When applied to a laterally oscillating structure,
41 IPM is capable of generating negative added damping and hence cause the onset of divergent amplitude
42 vibrations, even without pedestrians synchronising their stride frequency to that of the structure [10, 19].

43 **1.1 Negative damping model**

44 Controlled pedestrian loading tests were carried out on the LMF in which the density and number of
45 walkers were varied [1]. Detailed analysis of the collected data revealed that the lateral force amplitude
46 per pedestrian is approximately linearly correlated to the local lateral velocity amplitude of the deck. The
47 linear force-velocity relationship implied that lateral pedestrian loading could be treated as equivalent to
48 the action of negative dampers that tend to amplify the bridge responses. Thus, Arup proposed that it was
49 a result of pedestrians acting as ‘negative dampers and synchronising with bridge motion’ which caused

50 the growth of large amplitude dynamic vibrations, termed a lateral instability [1]. Using data from the
51 full-scale tests, Arup were able to estimate the average negative damping coefficient per pedestrian.
52 They named this the ‘lateral walking force coefficient’, k (see [1]). If each person introduces negative
53 damping, then when N_{crit} pedestrians are on a structure the sum of pedestrians and structural damping
54 equals zero, for a given mode. Thus, the formulation (eqn. (1)) estimates the number N_{crit} of pedestrians,
55 having uniform spatial density, that is necessary to generate a structural system with zero total modal
56 damping. Any pedestrian number greater than this N_{crit} would result in a growth of large amplitude
57 dynamic vibrations.

$$N_{q,crit} = \frac{4\pi f_{q,n} \zeta_{q,b} M_{q,b}}{k\psi}, \quad \psi = \int_0^{L_b} \frac{1}{L_b} \phi_q^2(x) dx \quad (1)$$

58 where $f_{q,n}$ is the modal natural frequency [Hz], $\zeta_{q,b}$ is the damping ratio, $M_{q,b}$ is the bridge modal mass,
59 k is the negative damping coefficient per person, ψ accounts for the distribution of pedestrians along the
60 bridge, L_b is the length of the bridge, $\phi_q(x)$ is the lateral mode shape and x is the coordinate along the
61 bridge length, and q is the mode number. For uniform mass per unit length, the bridge modal masses,
62 $M_{q,b}$, are defined as

$$M_{q,b} = \frac{M_b}{L_b} \int_0^{L_b} \phi_q^2(x) dx \quad (2)$$

63 where M_b is the overall bridge mass.

64 Using the value of $k = 300\text{Ns/m}$ derived from the LMF [1] it was possible to predict the number of
65 people for which the initiation of large amplitude vibrations occurred in the case of the Pedro an Inês
66 Footbridge in Portugal [4]. Measurements taken on the Changi Mezzanine Bridge [6] and Clifton Sus-
67 pension Bridge [5] agreed with the above negative damping model, although the derived values of the
68 damping coefficient differed. The initiation of divergent lateral vibrations on the Clifton Suspension
69 Bridge was found to occur for 150 and 240 pedestrians for the second and third lateral mode, respec-
70 tively. It was estimated that 70 pedestrians were required for the onset of large amplitude vibrations to
71 occur on the Changi Mezzanine Bridge [6].

72 **1.2 Aims**

73 This paper explores the amplitude dependency for the observed effects of HSI from measurement taken
74 on the Clifton Suspension Bridge. The frequency spectrum is investigated during the crowd loading
75 event to identify any subtle HSI effects, including shifts in natural frequencies. The aim is to determine
76 negative or positive equivalent added damping and mass due to pedestrians contributing to the lateral
77 bridge response. A novel application of the Hilbert transform is employed to estimate the added mass
78 and damping due to HSI. New results presented here, from a crowd loading event in 2017, are compared
79 with previous data collected during the International Balloon Fiesta in 2003 [5].

80 The aims of this paper are to explore the following research questions:

- 81 • Is human-structure interaction observed at low-amplitude lateral bridge vibration?
- 82 • Is Arup's negative damping model applicable for low-amplitude lateral bridge vibrations?
- 83 • Counter intuitively, can a structure's modal mass appear to decrease with an increase of pedestrian
84 numbers?

85 **2 Experimental method**

86 An experiment on the Clifton Suspension Bridge, Bristol, England, was carried out on Sunday 15th
87 October 2017. A structural health monitoring system (SHM) was deployed to investigate the bridge
88 response during the crowd loading event. Crowd monitoring by GoPro video cameras was carried out in
89 synchrony with the SHM. This experiment was reviewed by the faculty research ethics board. The size
90 and structure of the bridge also makes it convenient for study: small enough that the complete bridge can
91 be monitored and understood, but large enough and flexible enough to exhibit some interesting dynamic
92 behaviour [5, 9].

93 **2.1 Bridge description**

94 The Clifton Suspension Bridge spans the River Avon bridging from Clifton, Bristol, to Leigh Woods,
95 North Somerset. It is approximately 2km west of the centre of Bristol. The main span is 214.35m,

96 from centreline to centreline of the towers, with the suspended bridge length spanning 193.85m, L_b .
 97 The roadway is 6.1m wide between the two longitudinal stiffening girders. These are supported by
 98 vertical suspension rods spaced 2.44m apart from each other along the bridge. Total deck width is
 99 9.46m including 1.1m footways either side. The deck design comprises of timber with wrought iron
 100 lattice cross-girders. Lateral restraint is provided at either end by a tongue and groove design. No
 101 direct vertical or torsional restraint is provided by the abutment; the vertical loads are fully carried by
 102 the suspension rods allowing for relative motion. The chains account for approximately half the dead
 103 load of the main span. The overall bridge mass is approximately 1150 tonnes, M_b . A more complete
 104 description of the structure is given by Barlow [20]. The bridge layout and locations of the monitoring
 105 instruments are displayed in Figure 1.

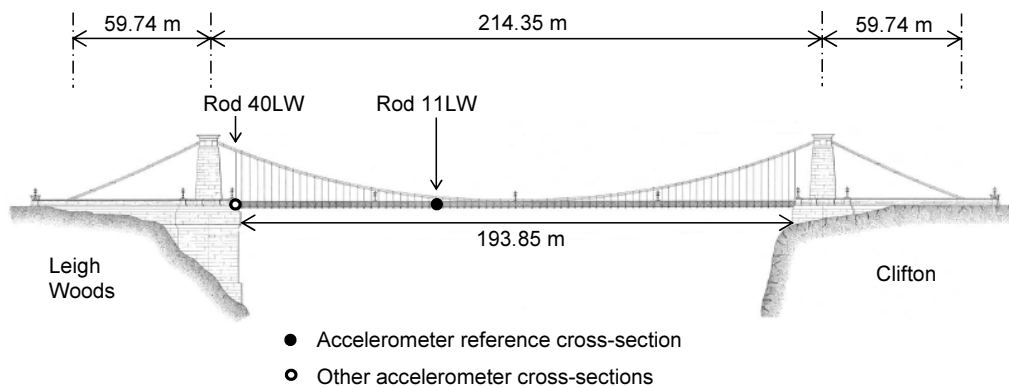


Figure 1: *Clifton Suspension Bridge diagram with associated rod(hanger) references, based on a figure from Barlow [20] (used with permission)*

106 2.2 Structural health monitoring (SHM) system and data acquisition

107 The SHM was setup at two (hanger) locations, illustrated in Figure 1, Rod 11LW and Rod 40LW. Rod
 108 11LW, located 26.7m from the bridge midpoint, was previously identified by Macdonald [5] as a suitable
 109 point for motion measurement in all lateral vibration modes below 3Hz. Rod40LW was selected as it
 110 was believed that this would show the greatest vertical transient response from pedestrians exciting
 111 the suspended deck span [21]. The CSB is well documented and previous work carried out by both
 112 Macdonald [5] and Gunner et al. [21] allowed for the efficient installation in one afternoon only requiring
 113 four people.

114 The sensors used as part of the SHM comprised of four accelerometers, including three uniaxial (Tokyo

115 measuring instruments lab) ARF-A low capacity acceleration transducers and a single triaxial accelerom-
 116 eter, (Lord Microstrain) G-LINK-200-8g, and two displacement transducers. At Rod 40LW the triaxial
 117 accelerometer was positioned above bridge deck level. Parallel to this, two displacement transducers
 118 and a single acceleration transducer were positioned below bridge deck level on the articulation span.
 119 These measured vertical displacements and accelerations. At Rod 11LW, three uniaxial accelerometers
 120 were positioned below bridge level. Two of these measured vertical motions on either side of the bridge
 121 deck, from which the pure vertical and torsional components of motion could be determined. The third
 122 accelerometer measured lateral accelerations. Sampling was configured to a rate of 64 Hz.

123 Four GoPro cameras were set up, two at either end of the bridge, mounted on the towers above pedestrian
 124 level. The GoPro data analysis was carried out to correlate the manual pedestrian counts and to ensure
 125 the validity of the timestamp data. An FE (Finite Element) model has been previously constructed by
 126 COWI [22] for the Clifton Suspension Bridge corresponding to the unloaded case. This model has been
 127 employed to estimate the mode shapes and modal masses for the lateral modes of interest. Modes shapes
 128 were scaled to a maximum amplitude of unity. Table 1 characterises the second and third lateral modes
 129 with the corresponding mode shapes shown in Figure 2. Lateral modes are labelled L1, L2, L3 etc, where
 130 L1 is the lowest frequency lateral mode. Modes L2 and L3 were found to have the lowest damping ratios
 131 which is significant as these modes experienced large-amplitude pedestrian-induced vibrations during
 132 the Balloon Fiesta in 2003 [5]. It is not surprising that these two modes were thus excited again during
 133 this crowd loading event.

Table 1: *Lateral bridge modes L2 and L3, using COWI's Finite Element model [22]*

Mode	Modal frequency, $f_{q,n}$ [Hz] (measured [5])	Modal damping ratio, $\zeta_{q,b}$ [%] (measured [5])	Modal mass, $M_{q,b}$ [tonnes] (FE [22])
L2	0.524	0.580	691.9
L3	0.746	0.680	698.7

134 **2.3 Crowd monitoring**

135 The crowd loading event took place for a duration of 19 minutes between 11:29am and 11:48am. The
 136 bridge was closed to all vehicles during the event, allowing pedestrians to walk along the roadway.
 137 Low volume footpath traffic was active with pedestrians not taking part in the prescribed event. GoPro
 138 footage showed that pedestrian flow along these pathways exhibited larger walking speeds than pedestri-
 139 ans travelling along the roadway. Pedestrians were limited to groups of 25, set off at 30s intervals, with

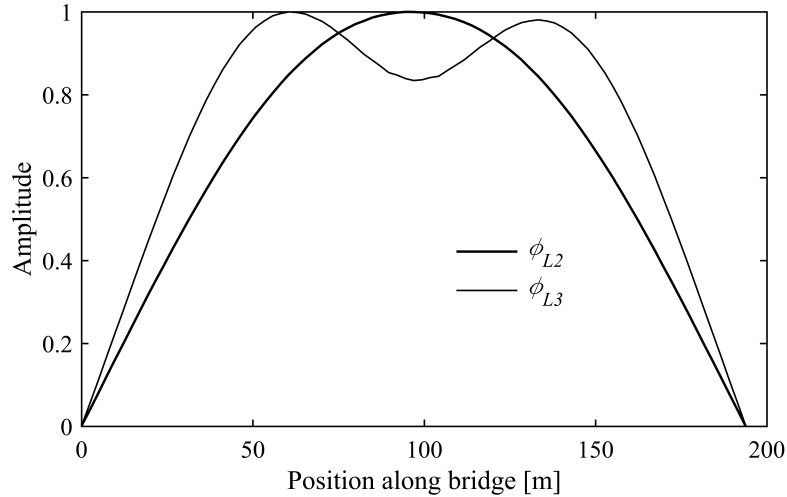


Figure 2: Clifton Suspension Bridge lateral mode shapes L2 and L3 identified using COWI's Finite Element model [22, 5]

140 the aim of limiting the total number of pedestrians on the bridge, given the large amplitude vibrations
 141 experienced during the previous crowd loading event [5]

142 To measure the number of pedestrians on the bridge as a function of time, four team members counted
 143 the flow of people using the smartphone timestamp application TimeStamp [23]. Two team members
 144 were situated at each end of the bridge, counting people stepping onto or off the suspended span, since
 145 the pedestrian traffic was bi-directional. The positive direction of progression was defined as Clifton to
 146 Leigh Woods. Pedestrians moving in the opposite direction (Leigh Woods to Clifton) were denoted as
 147 a counter-flow. This was found to be as low as 1-4 pedestrians on the bridge at any one time and could
 148 therefore be assumed to be negligible in the analysis. A total of 780 people were recorded crossing the
 149 bridge over the 19-minute period. This included pedestrians not participating in the official event. The
 150 number of people on the bridge as a function of time was evaluated using the summation of the people
 151 counted on (Clifton) and off (Leigh Woods). At any time during the event the maximum total number
 152 of people on the bridge was 151. This includes people on both the roadway and footways. This number
 153 was only sustained for short time periods with the number fluctuating greatly, as can be seen in Figure
 154 3.

155 2.4 Crowd dynamics and kinematics

156 The average velocity of each pedestrian was estimated using the raw timestamp data [23] assuming there
 157 was, on average, no overtaking on the bridge. The corresponding positions of each individual pedestrian

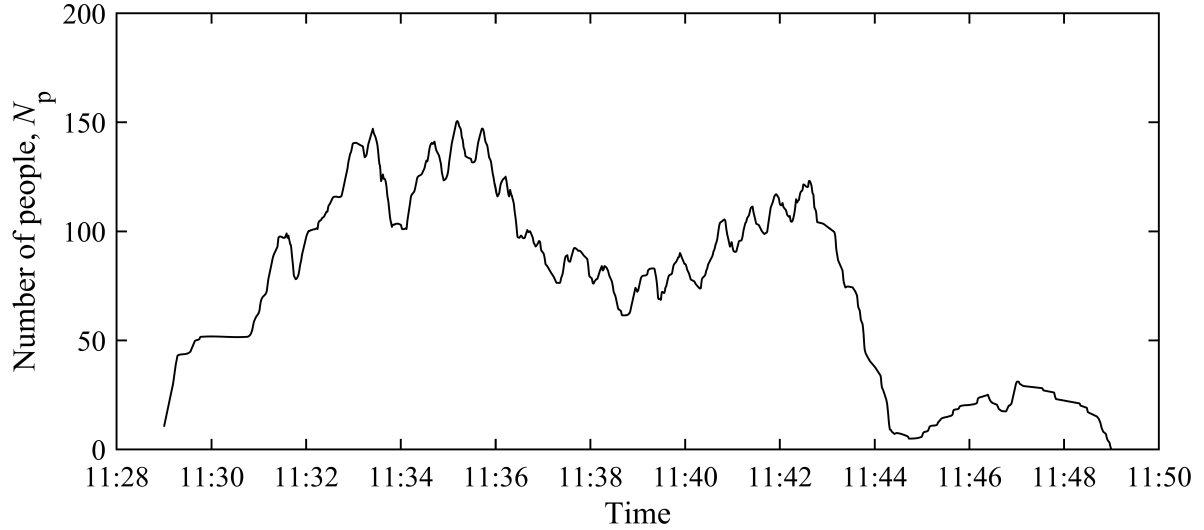


Figure 3: Number of people on the Clifton Suspension Bridge during the crowd loading event

158 along the suspended bridge length, as a function of time, during the crowd loading event were evaluated
 159 as,

$$x_j(t) = (t - t_{arr,j})v_j, \quad v_j = \frac{L_b}{t_{dep,j} - t_{arr,j}} \quad (3)$$

160 where $x_j(t)$ is the position of the j^{th} pedestrian, t is time, $t_{arr,j}$ and $t_{dep,j}$ are the arrival and departure
 161 times of the j^{th} pedestrian respectively. v_j is the estimated average velocity of the j^{th} pedestrian.

162 During the crowd loading event pedestrians were found to take, on average, 102.5s to cross the 193.85m
 163 suspended span. This equates to an average walking speed of 1.89m/s, which is larger than the typical
 164 preferred walking speed for humans, which is roughly 1.4m/s [24, 25]. However, humans are capable
 165 of walking at speeds upwards of 2.5m/s, and there were a few runners during the event which would
 166 increase the average speed [26]. Individuals find slower or faster speeds uncomfortable. This too agrees
 167 with data collected and analysed by Pachi and Ji [27]. Their results indicated pedestrian's walked over
 168 footbridges in the velocity range 0.93-1.8m/s.

169 2.5 Effective number of people (per mode) distributed on bridge

170 The effective number of people loading each mode accounts for the distribution of pedestrians on the
 171 bridge relative to the mode shape. Using mode shapes L2 and L3, shown in Figure 2 it was possible to
 172 estimate the effective number of people, $N_{q,eff}$, contributing to each mode of interest as follows

$$N_{q,\text{eff}} = \sum_{j=1}^{N_p} \phi_q^2(x_j(t)) \quad (4)$$

173 where $N_{q,\text{eff}}$ is the effective number of people loading the q^{th} mode and N_p is the number of people.
 174 For uniformly distributed pedestrians the effective number is found to be roughly half the total number,
 175 on the bridge at that time, based on the mode shapes normalised to a maximum magnitude of one. The
 176 maximum effective number of people during the crowd loading event at a given time was found to be 87
 177 and 94 people for modes L2 and L3 respectively. These correspond to 58% and 39% of the pedestrians
 178 required for the onset of large amplitude vibrations ($N_{\text{crit}} > N_p$) according to equation (1), based on
 179 $k = 300Ns/m$). This is represented in Figure 4 showing the effective number of people for both modes
 180 as a function of time during the crowd loading event.

181 The mass ratio, μ_q , (for the q^{th} mode) is the ratio of the pedestrian modal mass to the structural modal
 182 mass for a particular mode and is defined as,

$$\mu_q = \frac{M_{q,p}}{M_{q,b}} \quad (5)$$

183 where the pedestrian modal mass is $M_{q,p}$ is defined as,

$$M_{q,p} = \sum_{j=1}^{N_p} m_j \phi_q^2(x_j) \approx \bar{m}_p \sum_{j=1}^{N_p} \phi_q^2(x_j) = \bar{m}_p N_{q,\text{eff}} \quad (6)$$

184 where m_j is the mass of the j^{th} pedestrian and \bar{m}_p is the average pedestrian mass. It should be noted
 185 that each pedestrian is modeled as a lumped mass m_j , in equation 6, where m_j is the actual total mass
 186 of the j^{th} pedestrian. We make no attempt to model each pedestrian in a more complex biomechanical
 187 fashion as a multi-degree of freedom system.

188 National Health Service (UK) statistics report that the average mass of the general population, in the UK,
 189 is approximately 76kg [28]. Assuming this value for the average pedestrian mass, \bar{m}_p , the pedestrian
 190 modal mass, $M_{q,p}$, can be simply evaluated by multiplying it by the effective number of people, seen in
 191 Figure 4, as indicated in eqn. (5). The maximum modal pedestrian masses are then estimated to be 6.38
 192 tonnes for mode L2, and 6.31 tonnes for mode L3, occurring at approximately 11:33am and 11:35am
 193 respectively. These correspond to mass ratios of 0.0096 and 0.0102 respectively.

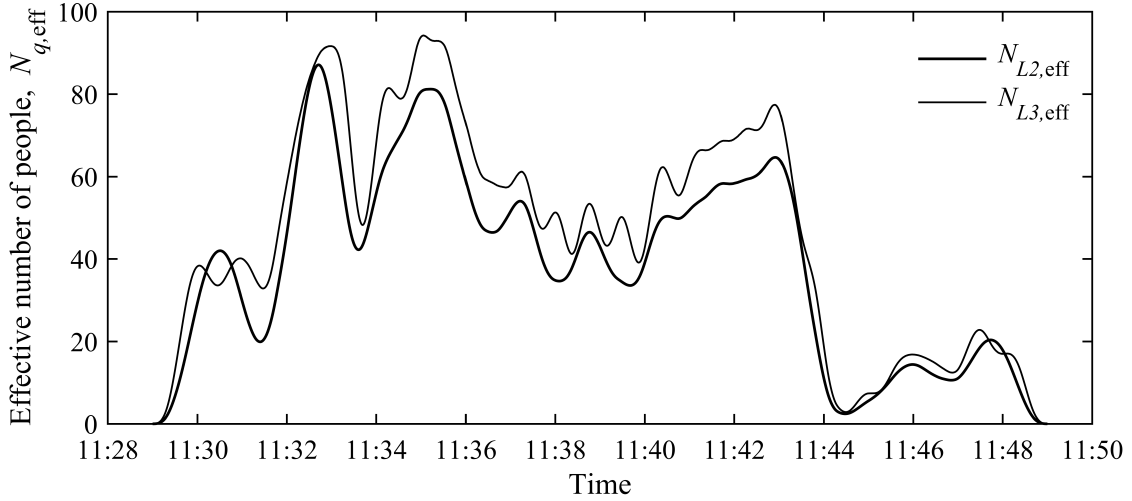


Figure 4: *Effective number of people loading modes L2 & L3 during the event*

194 **3 Experimental data analysis**

195 A 40-minute window of data collected by the structural health monitoring system was retrieved around
 196 the crowd loading event. Figure 5 shows the complete band-pass filtered lateral acceleration time-history,
 197 measured at Rod 11LW, illustrating five periods of differing loading. The band pass filter contained a
 198 low-cut filter at 0.2Hz (to remove quasi-static effects, [5]) and a high-cut filter at 5Hz (as only low-
 199 frequency modes are of interest).

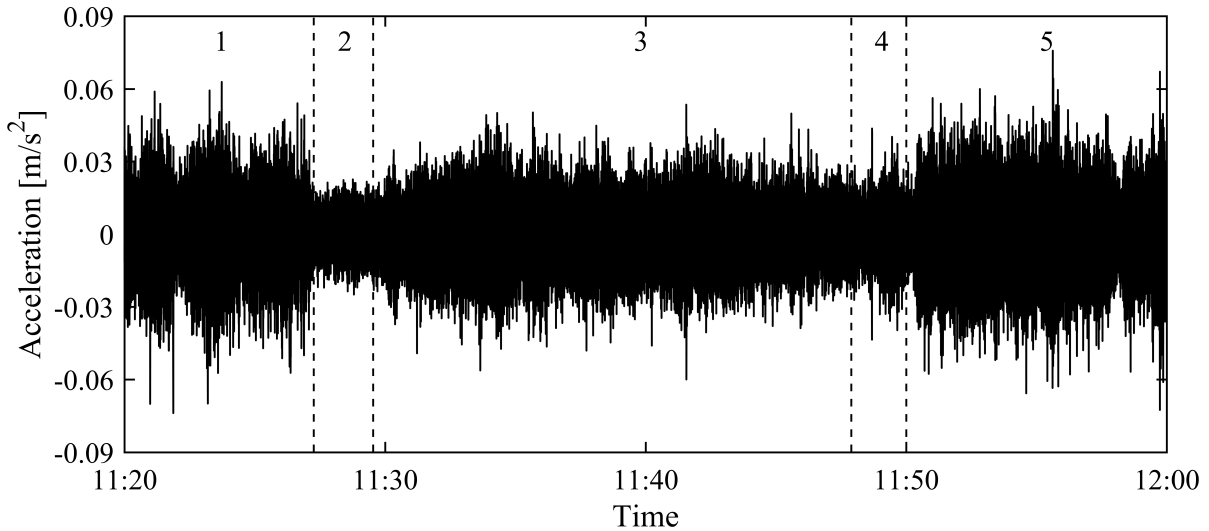


Figure 5: *Band-pass filtered lateral acceleration at Rod 11 of the Clifton Suspension Bridge over 40min period of monitoring*

200 Table 2 identifies the loading conditions for each of the five periods. This allowed for efficient post-
 201 processing of the data in comparing the conditions of the bridge before, during and after the crowd

202 loading event. Using these specified conditions, the data were split into three datasets corresponding
 203 to periods 1, 3 and 5 respectively. Note that the ambient ('unloaded') vibration cases 2 and 4 were not
 204 long enough in duration to extract useful resolution frequency information for the modes of interest, so
 205 are not considered here. Differences in the power spectral densities could then be identified between the
 206 vehicle and pedestrian loading conditions. The implications of these are discussed in Section 3.1. The
 207 maximum lateral acceleration observed during the crowd loading event was approximately 2.35 times
 208 the maximum response in ambient conditions.

Table 2: Sections of measurements corresponding to band-pass filtered lateral acceleration time-history in Figure 5

Period	Description	RMS Acceleration (m/s ²)	Key Times
1	Traffic loading	0.013	Bridge Closure: 11:27
2	Unloaded (nominal conditions)	0.007	Start of crowd event: 11:29
3	Crowd loading event	0.011	End of crowd event: 11:48
4	Intermediate loading	0.009	
5	Traffic loading	0.014	Bridge Reopened: 11:50

209 3.1 Data Processing

210 To investigate the crowd loading effects on specific critical modes, the response accelerations were
 211 band-pass filtered in the frequency ranges 0.45-0.65 Hz and 0.65-0.83 Hz to isolate the responses of
 212 modes L2 and L3 respectively. The filters used were zero-phase 6th order Butterworth filters. The
 213 amplitude envelopes of these band-filtered responses are displayed in Figure 6 for the crowd loading
 214 period (period 3 in Figure 5 and Table 2). The amplitude envelope in this figure was obtained using the
 215 Hilbert Transform [29, 30]. The short-time Fourier transform [31] is not used because it suffers from a
 216 loss of time-frequency resolution due to the Heisenberg's uncertainty principle. The Hilbert transform
 217 allows a much higher time-single frequency resolution. Section 3.2 discusses the implementation of the
 218 Hilbert transform used in this paper.

219 Table 3 summarises the maximum lateral dynamic responses measured at Rod 11LW for both crowd
 220 loading events. Displacements estimates were calculated by double integration of the measured accel-
 221 eration at Rod11LW. There is a significant difference in magnitude of the amplitudes observed during
 222 each crowd loading event. The 2003 event had an estimated maximum number of 488 pedestrians on the
 223 bridge, at any one time, equating to an average pedestrian density over the two footways of 1.1people/m²,
 224 the roadway was kept closed. In contrast, the 2017 event had a maximum number of 151 pedestrians

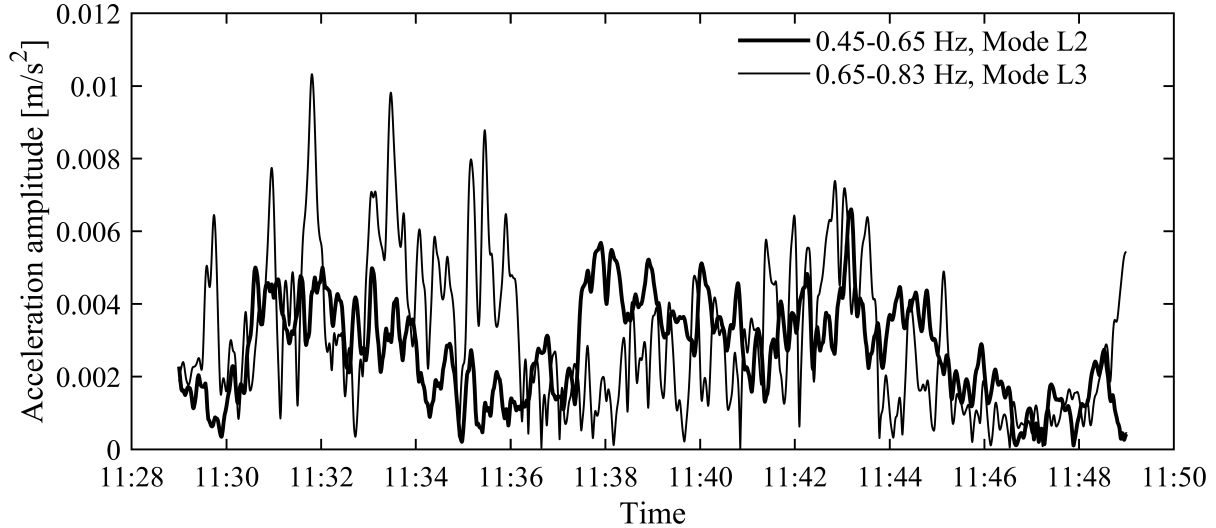


Figure 6: Acceleration amplitude envelopes for lateral modes L2 and L3 during crowd loading period

225 on the bridge, at any one time. The average pedestrian density over the roadway was found to be 0.13
 226 people/m². An advantage of the 2017 data is that pedestrians were counted on and off the suspended
 227 bridge span, so the pedestrian numbers are reliable, whereas for the 2003 data the pedestrian numbers
 228 were only estimated from CCTV footage of people approaching the bridge.

Table 3: Comparison of maximum lateral dynamic responses measured at Rod 11LW

	2003 event [5]		2017 event	
	Peak Disp.	Peak Acc.	Peak Disp.	Peak Acc.
	mm	m/s ²	mm	m/s ²
Total (0.2-5Hz)	11.7	0.190	1.5	0.054
Mode L2 (0.45-0.65 Hz)	10.2	0.110	0.59	0.007
Mode L3 (0.65-0.83 Hz)	4.7	0.110	0.43	0.010

229

230 Figure 7 displays the power spectral density (PSDs) of lateral accelerations (using Welch's algorithm
 231 [32]) for the time periods before (period 1), during (period 3) and after (period 5) the event. These
 232 power spectra indicate small increases in frequency of mode L2 and L3 modes during the event. This
 233 may be due to fact that the traffic loading (in periods 1 and 5) has a larger mass than the crowd (in period
 234 3). The resonance peaks of L2 and L3 modes appear to be slightly narrower for crowd loading than
 235 for traffic loading. This is suggestive of a reduction in damping, although the frequency resolution of
 236 the spectra is not sufficient for robust estimates of damping to be made from them. The spectral power
 237 observed around 0.3-0.4Hz is the lateral component of torsional mode 'T1' (0.356Hz), as identified by
 238 Macdonald [5].

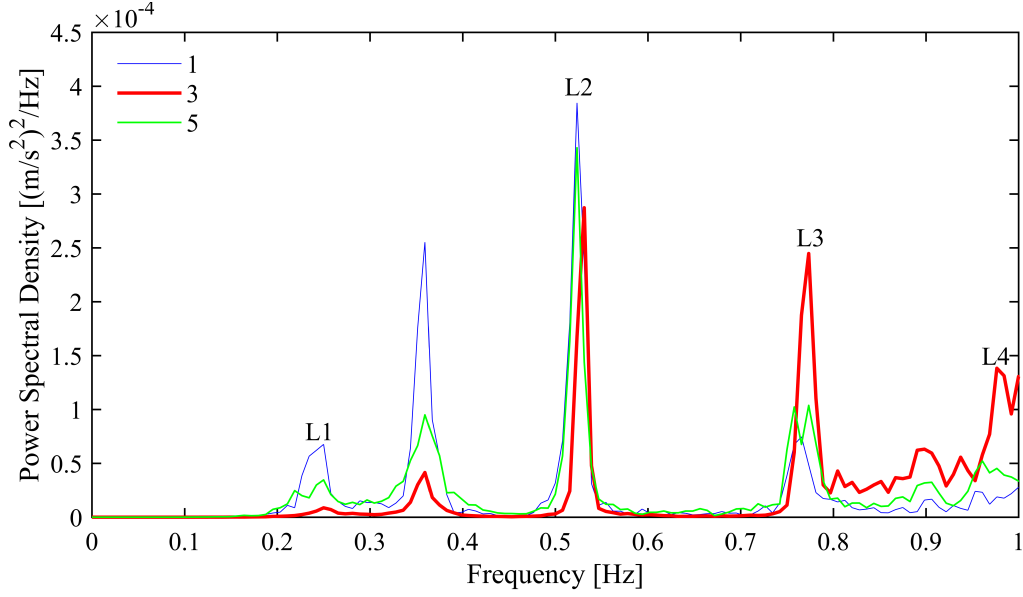


Figure 7: Power Spectral Densities of lateral acceleration with labelled modes of vibration

239 3.2 Time-Frequency Analysis: Hilbert Transform

240 By using the Hilbert transform the time-varying (instantaneous) phase, frequency, and amplitude (en-
 241 velope) of a real time-series can be calculated. The analysis of the instantaneous frequency allows the
 242 characterisation of any fluctuations observed in the bridge modal frequencies. These fluctuations could
 243 illustrate the presence of equivalent added mass (negative or positive) of the pedestrians, even at low-
 244 amplitudes. The modal acceleration signals of L2 and L3, during period 3 of Figure 5 and Table 3 (11:29
 245 - 11.49), were used for the time-frequency analysis, Hilbert transform. These transformed signals were
 246 used to characterise the mean equivalent added mass per pedestrian.

247 Formally, let $x(t)$ represent a timeseries. An analytical signal $s(t)$ of this timeseries $x(t)$ (computed
 248 directly using the *hilbert()* function in Matlab [29, 30]), is defined as

$$s(t) = x(t) + iy(t) \quad (7)$$

249 where $y(t)$ is the Hilbert transform of $x(t)$ and $i = \sqrt{-1}$. The instantaneous natural frequency is given
 250 by,

$$f(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}, \quad \theta(t) = \arctan \left(\frac{y(t)}{x(t)} \right) \quad (8)$$

251 where $f(t)$ and $\theta(t)$ are the instantaneous natural frequency and phase respectively. The instantaneous

252 amplitude envelope is given by

$$A(t) = |s(t)| = \sqrt{x(t)^2 + y(t)^2} \quad (9)$$

253 Eqn. (8) can be further expanded (using complex number algebra, see [33]) as

$$f(t) = \frac{1}{2\pi} \frac{x(t)\dot{y}(t) - y(t)\dot{x}(t)}{A(t)^2} \quad (10)$$

254 This form avoids the direct use of the $\arctan()$ function. It also indicates three key problems with this
255 instantaneous frequency estimate, that are

- 256 (i) The instantaneous frequency estimate is a single-valued function in time. Hence, it is only possible
257 to estimate one instantaneous frequency at a particular point in time t . In the case of multi-
258 frequency component signals $x(t)$ eqns (8) and (10) will produce some weighted average of all
259 components at t .
- 260 (ii) Computing $y(t)$ from $x(t)$ makes use of the Fast Fourier Transform (FFT) and this is subject to
261 its well documented spectral leakage [34]. For finite length signals spectral leakage can introduce
262 significant errors in the instantaneous frequency at the beginning and end of the signal.
- 263 (iii) When the amplitude of the signal $A(t)$ tends to zero it is likely that the frequency estimate will
264 tend to $\pm\infty$. This leads to spikes in instantaneous frequency estimates.

265 To alleviate these three problems the follow strategies have been adopted:

- 266 (i) The instantaneous frequencies for a single modal acceleration component \ddot{x}_q are calculated, that is
267 obtained by band-pass filtering previously discussed. Therefore, we limit the averaging of multi-
268 components signals
- 269 (ii) The filtered (mode q) acceleration signal $\ddot{x}_q(t)$ is multiplied by a Tukey windowing function $w(t)$
270 to attenuate (spectral leakage) at the beginning and end of the signal
- 271 (iii) A threshold is applied to the instantaneous frequency data f_q . This means the validity of the
272 instantaneous frequency estimate is only accepted if its corresponding instantaneous amplitude is
273 above a threshold level. A threshold value of 25% of signal maximum was found to be a reasonable

274 compromise between spike removal while keeping a large enough sample size. Equation (11)
 275 defines thresholding

$$f_q(t) = \begin{cases} f_q(t) & : A_q(t) > 0.25 \max(A_q(t)) \\ \emptyset & : \textit{otherwise} \end{cases} \quad (11)$$

276 where \emptyset signifies a null set, i.e. not a number (NaN) within MatLab.

277 The thresholded amplitudes and instantaneous natural frequencies of mode L2 and L3 are illustrated in
 278 Figure 8 and 9 as green lines overlaying the instantaneous amplitude and natural frequency (black lines
 279 in both Figure 8a, 8b, 9a and 9b). The thresholded quantities are taken as 25% of each maximum modal
 280 acceleration response corresponding to 0.007m/s^2 and 0.010m/s^2 , for modes L2 and L3 respectively.

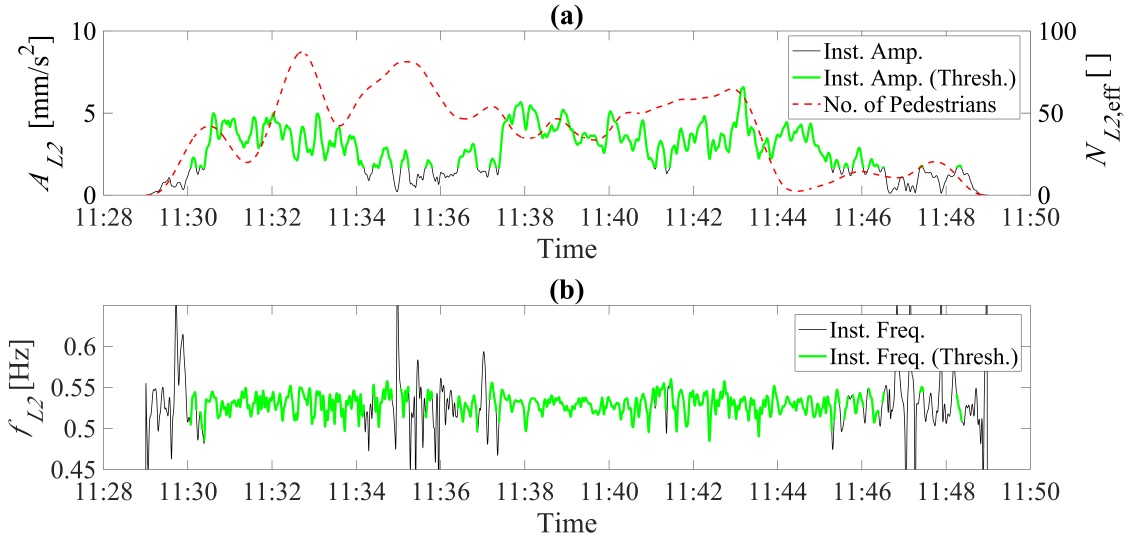


Figure 8: Comparison plots, Mode L2 (a) Instantaneous Amplitude, effective number of Pedestrians vs time (b) Instantaneous natural frequency vs time

281 3.3 Statistical analysis of Human-Structure Interactions

282 The bridge modal responses for the 2017 crowd loading event are low in amplitude and the number
 283 of pedestrians are well below the critical threshold ($N_p < N_{crit}$). Nevertheless, statistical analysis is
 284 performed to determine whether there is any evidence suggestive of a correlation between the effective
 285 number of people, the observed instantaneous amplitudes and natural frequencies, for modes L2 and L3.

286 Figures 8 and 9 show a direct comparison between the pedestrian loading numbers ($N_{q,eff}$) and estimated

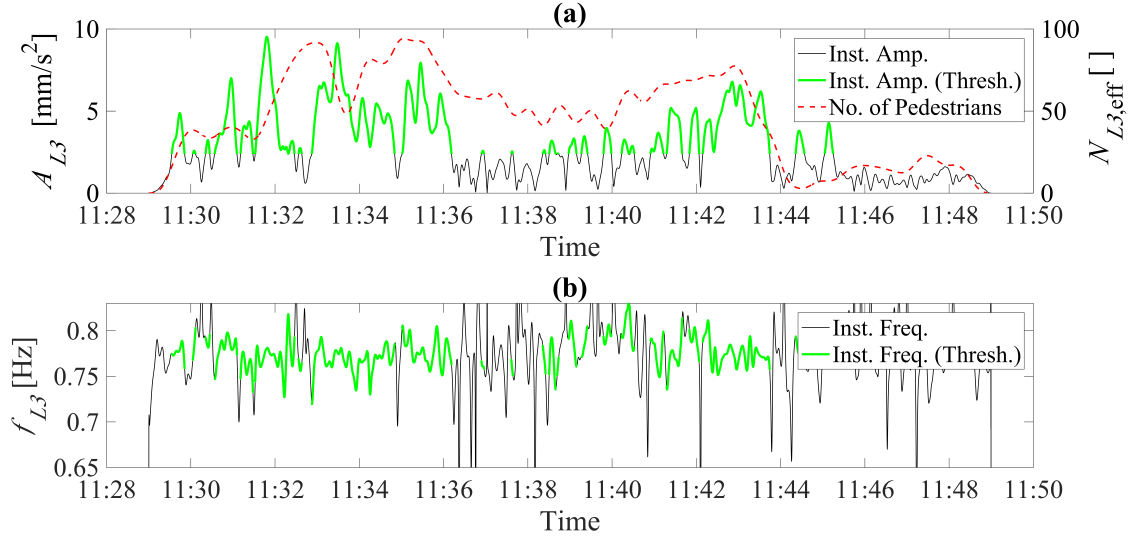


Figure 9: Comparison plots, Mode L3 (a) Instantaneous Amplitude, effective number of Pedestrians vs time (b) Instantaneous natural frequency vs time

287 bridge's instantaneous amplitude A_q and natural frequency f_q of mode q). The correlations between
 288 these quantities are determined by evaluating the Pearson correlation coefficient r [35]. Table 4 displays
 289 the respective correlation coefficients of the instantaneous amplitude and natural frequency with the
 290 effective number of people, for lateral modes L2 and L3. The fourth column identifies the **minimum**
 291 value required for the correlation, between the effective number of people and instantaneous frequency,
 292 to be considered statistically significant [36].

293 A correlation exists between the instantaneous response amplitude A_q and effective number of pedes-
 294 trians $N_{q,eff}$ for mode L3. There is a weaker correlation between instantaneous **natural** frequency and
 295 effective number of pedestrians, although still significant (statistically at a 99% confidence level, as
 296 shown in Table 4). However, in Figure 8, the increase in the effective number of people is subtly mir-
 297 rored in the instantaneous **natural** frequency and amplitude in the time 11:30-11:34am for mode L2. The
 298 interesting finding here is that the correlation between $N_{q,eff}$ and f_q for mode L2 is positive while it is
 299 negative for mode L3.

Table 4: Correlation coefficients for comparisons of instantaneous frequencies and amplitudes with the effective number of pedestrians

	Pearson correlation coefficients		Significant r at 99% confidence
	Inst Freq. $f_q(t)$	Inst. Amp. $A_q(t)$	
$N_{L2,eff}$	0.123	0.252	0.019
$N_{L3,eff}$	0.130	0.549	0.014

300 3.4 Estimating effective added mass per mode $M_{q,p}^*$

301 In this section we first estimate the change in modal frequencies due to the addition of the pedestrian
 302 masses from a theoretical point of view. We then seek to validate this expression (modal frequency
 303 change vs. the effective number of pedestrians) with experimental data. Results indicate that pedestrians
 304 appear to act as both positive or negative equivalent added mass.

305 The equivalent added mass is estimated through fluctuations in the bridge modes' instantaneous natural
 306 frequencies. The unloaded natural modal frequency of mode q is given as

$$f_q = \frac{1}{2\pi} \sqrt{\frac{K_{q,b}}{M_{q,b}}} \quad (12)$$

307 where $K_{q,b}$ is the bridge modal stiffness.

308 Treating the crowd simply as added mass, the loaded natural modal frequency of mode q is given by

$$2\pi f_q^* = \sqrt{\frac{K_{q,b}}{M_{q,b} + M_{q,p}^*}} = \sqrt{\frac{K_{q,b}}{M_{q,b}(1 + \alpha_q \mu_q)}} = \sqrt{\frac{K_{q,b}}{M_{q,b}}} (1 + \alpha_q \mu_q)^{-\frac{1}{2}} \quad (13)$$

309 Using the binomial expansion, for the case of small μ_q , we obtain the following estimated change in the
 310 natural frequency due to the crowd

$$f_q^* \simeq f_q \left(1 - \frac{1}{2} \alpha_q \mu_q + \mathcal{O}(\mu_q^2)\right) \quad (14)$$

311 where $M_{q,p}^*$ is the effective crowd modal mass which is equal to $\alpha_q M_{q,p}$, that is a certain proportion α_q
 312 of the crowd modal mass $M_{q,p}$ given in eqn (6). The modal mass ratio of crowd to bridge is μ_q and
 313 is defined in eqn (5). This mass ratio can also be expressed in terms of the effective number of people
 314 $N_{\text{eff},q}$, average person mass \bar{m}_p and the bridge modal mass $M_{q,b}$ hence

$$f_q^* \simeq f_q - \alpha_q \left(\frac{f_q \bar{m}_p}{2M_{q,b}}\right) N_{q,\text{eff}} = a + b N_{q,\text{eff}} \quad (15)$$

315 Equation (15) demonstrates a linear relationship between the effective number of pedestrians (for mode
 316 q), $N_{q,\text{eff}}$, and the instantaneous modal natural frequency f_q^* . The coefficients of the linear fit, a and

317 b, are the intercept and gradient respectively.

318 Figures 10 and 11 illustrate the relationships between the effective number people and instantaneous
319 frequency for modes L2 and L3 for both low and high amplitude response crowd loading events, in
320 2017 and 2003 respectively. Figures 10(a) and 11(a) both show a positive linear correlation however
321 the gradient values for mode L2, indicated in Table 5, are dissimilar. The 95% confidence limits of
322 Figure 10(a) are significantly larger than Figure 11(a) suggesting tentative evidence for equivalent added
323 mass by pedestrians during the 2017 crowd loading event. The linear trend in Figure 11(a) is very
324 clear. The scatter of data in these figures can be partly explained by the instantaneous fluctuations in
325 pedestrian stepping behaviour which can cause the equivalent added mass to vary from step to step.
326 Nevertheless, when averaged over a long time record, this would reveal the underlying relationship
327 between the instantaneous **natural** frequency, hence equivalent added mass, and the effective number of
328 people.

329 Evaluating the mean equivalent added mass per person, $\alpha_q \bar{m}_p$, using equation 15, we obtain values
330 of -164kg and for -71.5kg. These are considerably different. During the 2003 crowd loading event
331 a significant number of people were observed over a greater time span which would suggest a larger
332 pedestrian density. This correlates to slower pedestrian walking speeds, which according to the IPM,
333 changes the equivalent added mass (and damping) per person [19]. In comparison, the recent crowd
334 loading event observed less people interacting on the bridge over a shorter time span equating to a much
335 smaller pedestrian density promoting larger pedestrian walking speeds. According to the IPM, mode
336 L2's natural frequency, 0.524Hz, is within the bandwidth where the equivalent negative added mass is
337 negative [19].

338 The correlations of mode L3 in Figures 10(b) and 11(b) are inconclusive however there is some tentative
339 suggestion for the presence of equivalent added mass by pedestrians. A negative linear correlation is
340 observed for the 2017 event whilst a positive linear trend is illustrated for the 2003 event. Evaluating
341 the mean equivalent added mass per person, using eqn 15, we obtain values of -6.7kg and 174kg. The
342 gradient of mode L3 is steep and negative for 2017 data however for 2003 the slope is very small but still
343 positive. Comparing this to the IPM, mode L3's natural frequency, 0.746Hz, is within the bandwidth
344 close to 0kg equivalent added mass for a lateral pacing frequency of 0.6Hz [19]. The polarity of this
345 value is dependent on the walking frequency according to the IPM. Ref [37] suggests that an increase in
346 crowd density decreases the pedestrian walking velocity. The 2003 event observed large crowd densities

347 on average on the walkways, $1.1\text{people}/\text{m}^2$ at the estimated maximum number of people. This may
 348 have resulted in slow walking speeds potentially causing smaller pacing frequencies. However, the 2017
 349 event observed low pedestrian densities on average on the roadway in comparison, $0.13\text{people}/\text{m}^2$ at
 350 the estimated maximum number of people. This may have promoted faster walking speeds resulting
 351 in larger pacing frequencies (on average) suggesting equivalent negative added mass by pedestrians,
 352 according to the IPM [19]. This qualitatively agrees with the observed trend.

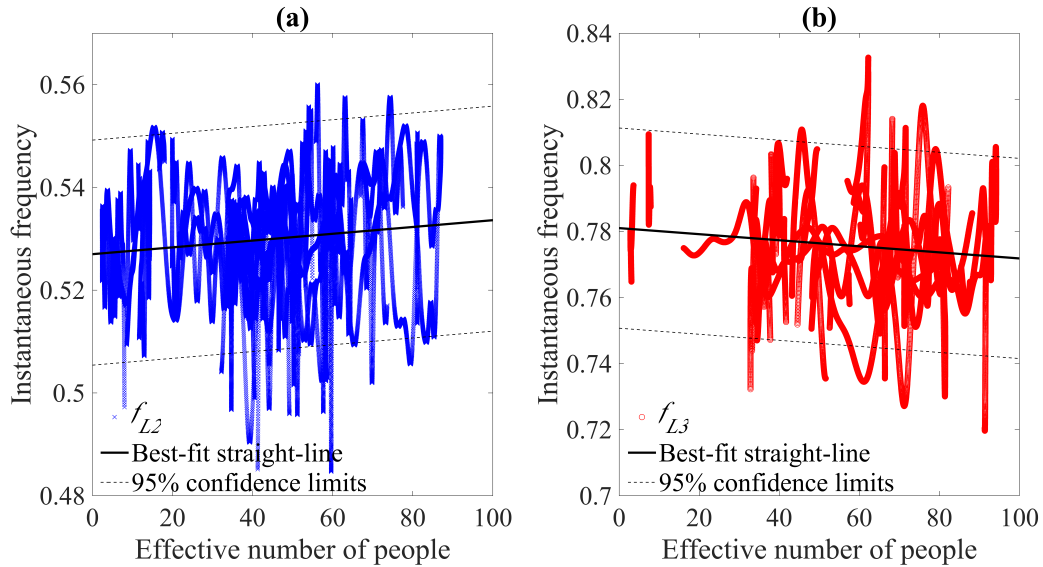


Figure 10: 2017 event - relationship between instantaneous frequency and effective number of pedestrians (a) Mode L2;(b) Mode L3

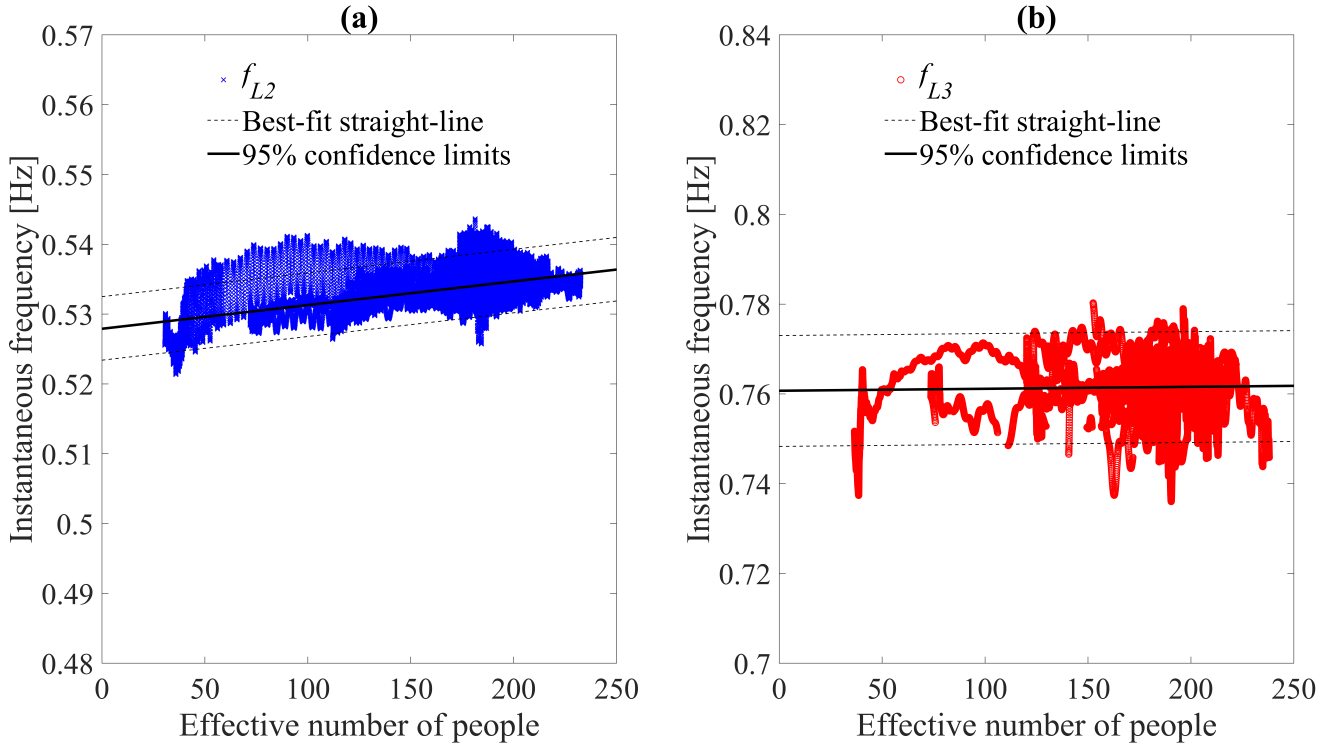


Figure 11: 2003 event - relationship between instantaneous frequency and effective number of pedestrians (a) Mode L2;(b) Mode L3

Table 5: Comparison of instantaneous frequency for 2003 and 2017 crowd loading events

Dataset	L2			L3		
	correl. coeff. r	gradient, b [Hz/pedestrian] (10^{-5})	intercept, a [Hz]	correl. coeff. r	gradient, b [Hz/pedestrian] (10^{-5})	intercept, a [Hz]
2003	0.501	1.617	0.528	0.027	0.214	0.7607
2017	0.123	6.610	0.527	-0.13	-9.180	0.781

353 3.5 Arup's Negative Damping Model

354 This method of analysis considers the principle of conservation of energy assuming the energy **input**
355 by pedestrians is **output** into the bridge directly corresponding to changes in the observed vibration
356 amplitude from cycle-to-cycle. An equivalent force is assumed to account for the pedestrians' lateral
357 forcing on the bridge deck. The equation used to identify the amplitude of the equivalent generalised
358 lateral excitation force per pedestrian in phase with the bridge velocity, for each vibration cycle, is given
359 by [1]:

$$F_{\text{ped,v}}(t) = \frac{M_{q,b}}{N_{q,\text{eff}}\sqrt{\psi}} \left(2\zeta_{q,b}A + \frac{\Delta A}{\pi} \right) \quad (16)$$

360 where A is the generalised acceleration vibration amplitude, ΔA is the increase in the amplitude from
 361 one cycle to the next.

362 The velocity amplitude used for this method of analysis was evaluated using the Hilbert Transform
 363 which is discussed in Section 3.2. Both the force and velocity amplitudes are scaled by $\sqrt{2}$ [1, 5] to be
 364 consistent with Arup's procedure of analysis. The full procedure can be found in reference [1]

365 The equation of motion of the bridge, for the q^{th} mode, is given by eqn (17). The forcing by the
 366 pedestrians, on the right-hand side of eqn (17), comprises three force component terms. A motion-
 367 independent static force, equivalent to that generated while walking on rigid ground, $F_{st}(t)$, and two
 368 motion-dependent force components of which one can be expressed as equivalent added mass, $F_{\text{ped,a}}(t)$,
 369 and the other as equivalent added damping, $F_{\text{ped,v}}(t)$. Arup's negative damping coefficient, k , corre-
 370 sponds to minus the average value of c_j per person ($\bar{c}_j \approx -k$). It is given by,

$$M_{q,b}\ddot{X}_q(t) + C_{q,b}\dot{X}_q(t) + K_{q,b}X_q(t) = F_{st}(t) - F_{\text{ped,a}}(t) - F_{\text{ped,v}}(t) \quad (17)$$

$$F_{\text{ped,a}}(t) = \ddot{X}_q(t)\phi_q^2(x_j) \sum_{j=1}^{N_p} \alpha_{q,j}m_j(\omega_b), \quad F_{\text{ped,v}}(t) = \dot{X}_q(t)\phi_q^2(x_j) \sum_{j=1}^{N_p} c_j(\omega_b) \quad (18)$$

371 where dots denote derivatives with respect to time, $C_{q,b}$ and is the bridge modal damping coefficient,
 372 $X_q(t)$ is the bridge modal displacement, $\alpha_{q,j}m_j$ is the equivalent added mass per pedestrian and ω_b is the
 373 circular vibration frequency of the bridge.

374 Arup's procedure (eqn (16)) is a method of approximating the third term on the right-hand side of the
 375 equation of motion (eqn (17)), $F_{\text{ped,v}}(t)$, directly from measured bridge deck acceleration. This is an es-
 376 timation which accepts that $F_{st}(t)$, cycle-to-cycle, contributes to the estimation of $F_{\text{ped}}(t)$. The external
 377 forcing $F_{st}(t)$ is random in nature, due to the variability in step-by-step locomotion, which, on average,
 378 is zero. This can be described as a random walk [38]. For low-amplitude bridge vibrations, this makes
 379 it difficult to distinguish $F_{\text{ped,v}}(t)$ from $F_{st}(t)$ for this method of analysis due to the significant scatter
 380 observed.

381 To evaluate the “lateral walking coefficient” (negative damping coefficient), $F_{ped,v}(t)$, (eqn (16) is plot-
 382 ted as a function of the locally scaled bridge deck velocity amplitude, as previously mentioned. The
 383 negative damping coefficient is directly evaluated from the gradient of this relationship. This gradient
 384 has been previously shown to be linear [1, 5, 6]. To reduce the effect of $F_{st}(t)$, in the identification of the
 385 negative damping coefficients of mode L2 and L3, $F_{ped}(t)$ and $\dot{X}_{q,amp}(t)$ measurements were allocated
 386 into bins of $1 \times 10^{-4} \text{ m/s}^2$ intervals and averaged, accordingly producing a single data point per bin.

387 The results from the procedure of Arup’s negative damping model are illustrated in Figure 12. Figure
 388 12(a) and Figure 12(b) display a linear trend for both modes L2 and L3 respectively, with 95% confi-
 389 dence limits. This qualitatively agrees with the literature [1, 4, 5, 6, 11, 12, 13, 14, 15]. The applicability
 390 of the negative damping model for low-amplitude bridge vibrations suggests that a threshold amplitude
 391 is not required for the HSI phenomenon observed during lateral bridge excitation. Outliers were still
 392 present using this method of analysis and were therefore omitted during the regression to obtain best ap-
 393 proximates for the gradient ‘ k ’ values, negative damping coefficients per pedestrian. These values were
 394 evaluated as -685Ns/m and -970Ns/m for modes L2 and L3 respectively. These are significantly larger
 395 than Macdonald’s findings, from the 2003 experimental data [5], of -160Ns/m and -210Ns/m, for modes
 396 L2 and L3 respectively, and Arup’s value of -300Ns/m from measurements on the LMF[1]. However,
 397 Macdonald suggested that at low amplitudes the gradient of mode L2 could possibly be significantly
 398 steeper [5]. From a series of measurements on a laterally oscillating treadmill, Ingólfsson et al. [11]
 399 considered the amplitude dependency of the added damping and mass coefficients, suggesting that they
 400 differ for low and high amplitude vibrations. Also, the level of noise observed (noise-signal ratio) in the
 401 measurements could cause inaccuracies which are amplified through the analysis, lading to uncertainty
 402 in the evaluation of the gradients in Figure 12.

403 Table 6 summarises the evaluated equivalent added mass and damping coefficients per pedestrian for
 404 both crowd loading events, 2003 and 2017 respectively.

Table 6: Comparison of equivalent added mass and damping coefficients for 2003 and 2017 crowd loading events

	2003		2017	
	k (Ns/m)	$\alpha_q \bar{m}_p$ (kg)	k (Ns/m)	$\alpha_q \bar{m}_p$ (kg)
L2	160	-71.5	685	-164
L3	210	6.70	970	174

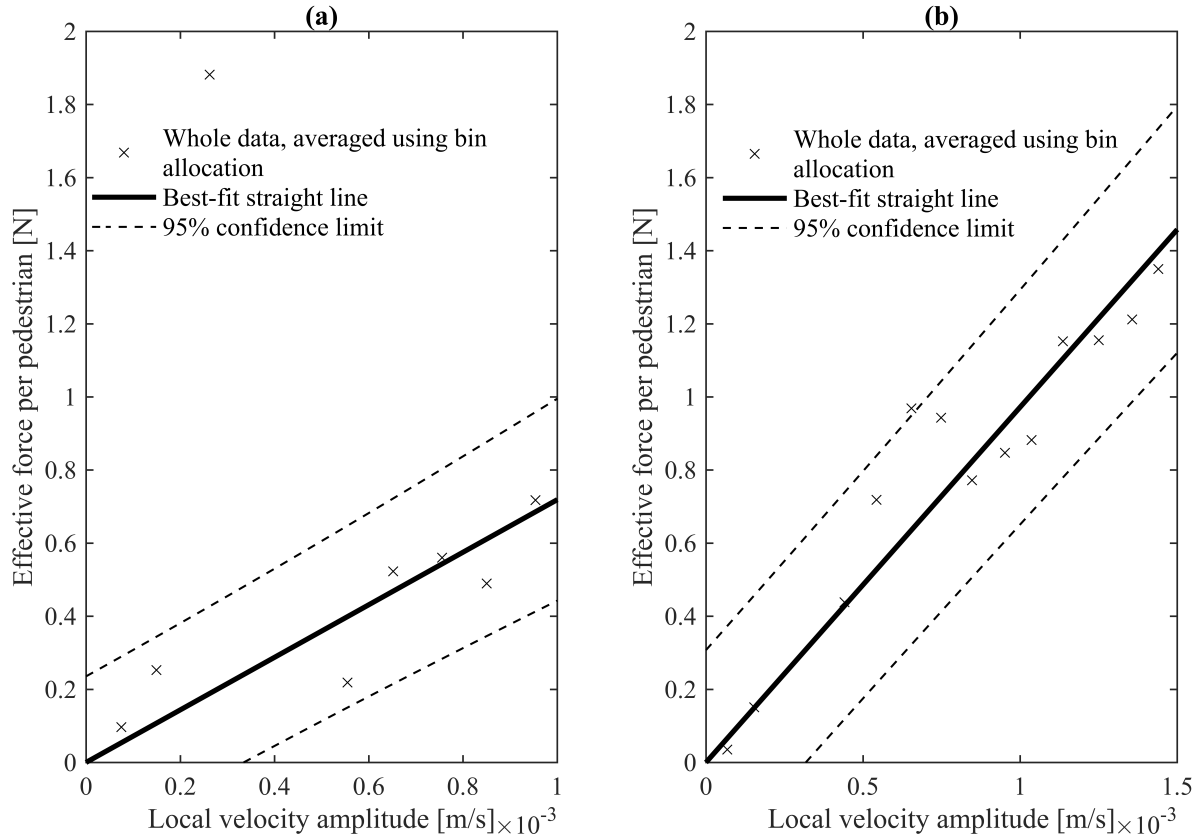


Figure 12: Relationship between bridge velocity amplitude and force amplitude (in phase with velocity) per pedestrian: (a) Mode L2; (b) Mode L3

405 4 Conclusions

406 A novel procedure has been presented to evaluate the equivalent added mass of pedestrians during crowd
 407 loading using a time-frequency analysis approach. This has enabled the mean equivalent added mass per
 408 person to be identified from full-scale data for the first time. Although some of the results are uncertain
 409 quantitatively, there is no evidence of a threshold amplitude at which the HSI phenomenon starts. Sta-
 410 tistical analysis suggests tentative evidence for human-structure interactions observed at low amplitudes
 411 during the 2017 crowd loading event. Qualitatively this agrees well with the literature however, the
 412 quantification of the equivalent added mass and damping per pedestrian gives significantly large values.
 413 They have been difficult to identify accurately due to the noise-to-signal ratio and large scatter of the
 414 results at low amplitude.

415 A structure's total modal mass has been shown to decrease for an increase in the number of pedestrians
 416 for mode L2 in the 2003 crowd loading event. This is observed through an increase in the resonant

417 frequency of this mode for an increase in the number of people. The mean equivalent added mass per
418 pedestrians has been approximated as 71.5kg for this mode.

419 Arup's negative damping model is inconclusive for the 2017 crowd loading event however equivalent
420 negative added damping by pedestrians is observed. The applicability of this model for low-amplitude
421 bridge responses is therefore uncertain. The estimation of the equivalent added damping includes the
422 variation in amplitude from cycle-to-cycle, which at low amplitudes ignores information in the bridge re-
423 sponse during each cycle. The bridge-independent forcing by pedestrians causes difficulty in extracting
424 the negative damping coefficient observed during crowd loading.

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