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# Theoretical Challenges for a Precision Measurement of the W Mass at Hadron Colliders\*

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We summarize the status of calculations of the electroweak radiative corrections to W and Z boson production via the Drell-Yan mechanism at hadron colliders. To fully exploit the precision physics potential of the high-luminosity environment of the Fermilab Tevatron  $p\bar{p}$  (Run II) and the CERN LHC  $pp$  colliders, it is crucial that the theoretical predictions are well under control. The envisioned precision physics program includes a precise measurement of the W boson mass and the (leptonic) weak mixing angle, as well as probing the Standard Model (SM) of electroweak interactions at the highest accessible center-of-mass energies. Some numerical results are presented.

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# 1 Introduction

The Standard Model of electroweak interactions (SM) so far withstood all experimental challenges and is tested as a quantum field theory at the 0.1% level [1]. However, the mechanism of mass generation in the SM predicts the existence of a Higgs boson which, so far, has eluded direct observation. Direct searches at LEP2 give a (preliminary) 95% confidence-level lower bound on the mass of the SM Higgs boson of  $M_H > 113.5$  GeV [2]. Indirect information on the mass of the Higgs boson can be extracted from the  $M_H$  dependence of radiative corrections to the W boson mass. With the present knowledge of the W boson and top quark masses, and the electromagnetic coupling constant,  $\alpha(M_Z^2)$ , the SM Higgs boson mass can be indirectly constrained to  $M_H = 77_{-39}^{+69}$  GeV [1] by a global fit to all electroweak precision data. Future more precise measurements of the W boson and top quark masses are expected to considerably improve the present indirect bound on  $M_H$ : with a precision of 30 MeV for the W boson mass,  $M_W$ , and 2 GeV for the top quark mass which are target values for Run II of the Tevatron [3],  $M_H$  can be predicted with an uncertainty of about 30%. In addition, the confrontation of a precisely measured W boson mass with the indirect SM prediction from a global fit to all electroweak precision data,  $M_W = 80.385 \pm 0.022$  GeV [1], will provide a stringent test of the SM. A detailed discussion of the prospects for the precision measurement of  $M_W$ , and of the (leptonic) effective weak mixing angle,  $\sin^2 \theta_{eff}^l$ , at Run II and the LHC is given in Refs. [3] and [4], respectively.

In order to measure  $M_W$  with high precision in a hadron collider environment it is necessary to fully control higher order QCD and electroweak radiative corrections to the W and Z production processes. The status of the QCD corrections to W and Z boson production at hadron colliders is reviewed in Refs. [5,6]. Here we discuss the electroweak  $\mathcal{O}(\alpha)$  corrections to  $p\bar{p}^{(-)} \rightarrow W^\pm \rightarrow l^\pm \nu_l$  and  $p\bar{p}^{(-)} \rightarrow \gamma^*, Z \rightarrow l^+ l^-$  ( $l = e, \mu$ ) as presented in detail in Refs. [7,8] and [9,10].

## 2 Electroweak $\mathcal{O}(\alpha)$ Corrections to $p\bar{p}^{(-)} \rightarrow W^\pm \rightarrow l^\pm \nu$

The full electroweak  $\mathcal{O}(\alpha)$  corrections to resonant W boson production in a general four-fermion process were calculated in Ref. [7] with special emphasis on obtaining a gauge invariant decomposition into a photonic and non-photonic part. It was shown that the cross section for resonant W boson production via the Drell-Yan mechanism at parton level,  $q_i \bar{q}_{i'} \rightarrow f \bar{f}'(\gamma)$ , can be written in the following form [8]:

$$\begin{aligned}
 d\hat{\sigma}^{(0+1)} &= d\hat{\sigma}^{(0)} [1 + 2\mathcal{R}e(\tilde{F}_{weak}^{initial}(\hat{s} = M_W^2) + \tilde{F}_{weak}^{final}(\hat{s} = M_W^2))] \\
 &+ \sum_{\substack{a=initial,final, \\ interf.}} [d\hat{\sigma}^{(0)} F_{QED}^a(\hat{s}, \hat{t}) + d\hat{\sigma}_{2 \rightarrow 3}^a], \quad (1)
 \end{aligned}$$

where the Born cross section,  $d\hat{\sigma}^{(0)}$ , is of Breit-Wigner form, and  $\hat{s}$  and  $\hat{t}$  are the usual Mandelstam variables in the parton center of mass frame. The (modified) weak corrections and the virtual and soft photon emission from the initial and final state fermions (as well as their interference) are described by the form factors  $\tilde{F}_{weak}^a$  and  $F_{QED}^a$ , respectively. The IR finite contribution  $d\hat{\sigma}_{2\rightarrow 3}^a$  describes real photon radiation away from soft singularities. Mass singularities of the form  $\ln(\hat{s}/m_f^2)$  arise when the photon is emitted collinear with a charged fermion and the resulting singularity is regularized by retaining a finite fermion mass ( $m_f$ ).  $F_{QED}^{initial}$  and  $d\hat{\sigma}_{2\rightarrow 3}^{initial}$  still include quark-mass singularities which need to be extracted and absorbed into the parton distribution functions (PDFs). The absorption of the quark-mass singularities into the PDFs can be done in complete analogy to gluon emission in QCD, thereby introducing a QED factorization scheme dependence. Explicit expressions for the W production cross section in the QED DIS and  $\overline{\text{MS}}$  scheme are provided in Ref. [8]. So far, in the extraction of the PDFs from data as well as in the PDF evolution, QED corrections are not taken into account. The latter result in a modified scale dependence of the PDFs, which is expected to have a negligible effect on the observable cross sections [4]. The numerical evaluation of the cross section is done with the parton level Monte Carlo program WGRAD [8]<sup>1</sup>, and results have been obtained for a variety of interesting W boson observables at the Tevatron [8] and the LHC [4].

In the past, fits to the distribution of the transverse mass of the final-state lepton neutrino system,  $M_T(l\nu)$ , have provided the most accurate measurements of  $M_W$  [11]. Photonic initial state and initial-final state interference corrections were found to have only a small effect on the  $M_T$  distribution, and weak corrections uniformly reduce the cross section by about 1%. However, final-state photon radiation significantly distorts the shape of the  $M_T$  distribution, and thus considerably affects the extracted value of  $M_W$ . In the electron case, when taking into account realistic lepton identification requirements to simulate the detector acceptance, the electroweak radiative corrections are strongly reduced because electron and photon momenta are combined for small opening angles between the two particles. This eliminates the mass singular terms associated with final state radiation. The ratio of the full  $\mathcal{O}(\alpha^3)$  and lowest order differential cross section as a function of  $M_T(l\nu)$  with and without lepton identification requirements taken into account is shown in Fig. 1.

A previous approximate calculation [12] took only the real photonic corrections properly into account while the effect of soft and virtual virtual photonic corrections were estimated from the inclusive  $W \rightarrow l\nu(\gamma)$  width. Weak corrections were ignored in Ref. [12]. Comparing the W mass shifts obtained using the calculations of Refs. [12] and [8], one finds that the proper treatment of virtual and soft corrections and the inclusion of weak corrections induces an additional shift of  $\mathcal{O}(10 \text{ MeV})$  in the extracted W boson mass.

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<sup>1</sup>WGRAD is available from the authors.

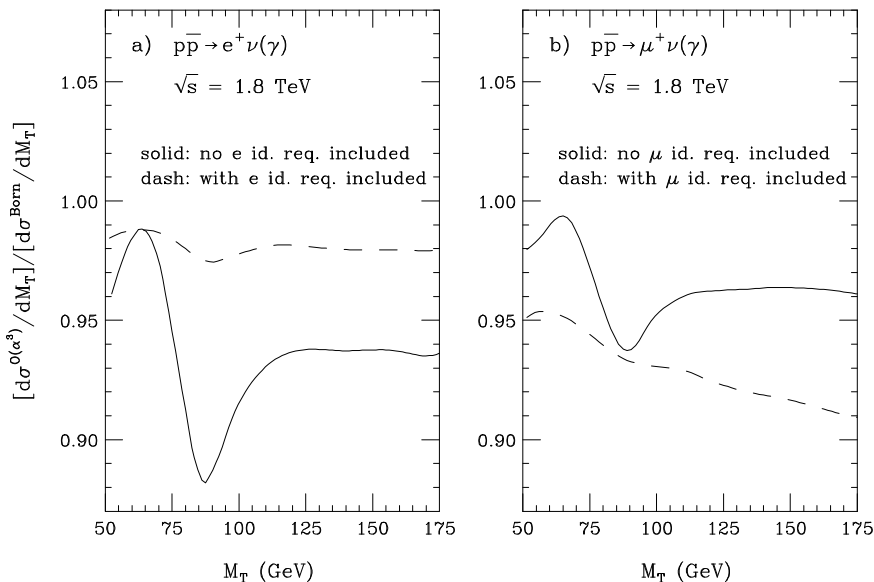


Figure 1: The relative corrections to the  $M_T(l\nu)$  distributions at the Tevatron when taking into account the full electroweak  $\mathcal{O}(\alpha)$  corrections (from Ref. [8]).

### 3 Electroweak $\mathcal{O}(\alpha)$ Corrections to $p\bar{p} \rightarrow \gamma^*, Z \rightarrow l^+l^-$

Neutral-current Drell-Yan production is interesting for several reasons:

1. Future precise measurements of the W boson mass at hadron colliders depend on a precise knowledge of the Z boson production process. When compared to the values measured at LEP, the measured Z boson mass and width help to determine the energy scale and resolution of the electromagnetic calorimeter.
2. Ratios of W and Z boson observables may yield a more precise measurement of  $M_W$  than the traditional technique of fitting the  $M_T$  distribution [3,13].
3. The forward-backward asymmetry in the vicinity of the Z resonance can be used to measure the (leptonic) effective weak mixing angle [4,9]. Studying the forward-backward asymmetry above the Z resonance probes the  $\gamma, Z$  interference at the highest available energies.
4. Finally, at large di-lepton invariant masses,  $m(l^+l^-)$ , deviations from the SM prediction could indicate the presence of new physics, such as new heavy gauge bosons  $Z'$  or extra spatial dimensions.

It is therefore important to determine the electroweak corrections for this process.

The electroweak  $\mathcal{O}(\alpha)$  corrections to neutral-current Drell-Yan processes naturally decompose into QED and weak contributions, i.e. they build gauge invariant subsets and thus can be discussed separately. The observable next-to-leading order (NLO) cross section is obtained by convoluting the parton cross section with the quark distribution functions  $q(x, Q^2)$  ( $\hat{s} = x_1 x_2 S$ ) [10]

$$d\sigma(S) = \int_0^1 dx_1 dx_2 q(x_1, Q^2) \bar{q}(x_2, Q^2) [d\hat{\sigma}^{(0+1)}(\hat{s}, \hat{t}) + d\hat{\sigma}_{\text{QED}}(\mu_{\text{QED}}^2, \hat{s}, \hat{t})], \quad (2)$$

where  $d\hat{\sigma}^{(0+1)}$  comprises the NLO cross section including weak corrections, and  $d\hat{\sigma}_{\text{QED}}$  describes the QED part, i.e. virtual corrections and real photon emission off the quarks and charged leptons. The PDFs depend on the QCD renormalization and factorization scales which we choose to be equal; the common scale is denoted by  $Q^2$ . The radiation of collinear photons off quarks requires the factorization of the arising mass singularities into the PDFs which introduces a dependence on the QED factorization scale,  $\mu_{\text{QED}}$ . The treatment of mass singularities is universal and thus the same as in the  $W$  case. The QED  $\mathcal{O}(\alpha)$  corrections to  $p\bar{p} \rightarrow \gamma^*, Z \rightarrow l^+l^-$  ( $l = e, \mu$ ) have been calculated and implemented in the parton level Monte Carlo program ZGRAD [9]<sup>2</sup> and their impact on the di-lepton invariant mass spectrum, the lepton transverse momentum distribution, and on the forward-backward asymmetry,  $A_{\text{FB}}$ , has been studied. In addition, the prospects for a precision measurement of  $\sin^2 \theta_{\text{eff}}^l$  extracted from  $A_{\text{FB}}$  at the Z resonance at the LHC were investigated.

In Fig. 2 we show the effect of the QED corrections on the invariant mass distribution of the final state lepton pair. Similar to the transverse mass distribution in the charged-current Drell-Yan process, final-state photon radiation strongly affects the shape of the  $m(l^+l^-)$  distribution. When lepton identification requirements are taken into account, the large contributions from mass singular logarithms largely cancel in the electron case. As in the charged-current Drell-Yan process, initial-final state interference is negligible, and the impact of initial-state radiation is small after factorizing the collinear singularities into the PDFs. The difference in the extracted Z boson mass when comparing the approximate calculation of Ref. [12] with the full calculation of the  $\mathcal{O}(\alpha)$  QED corrections is of  $\mathcal{O}(10 \text{ MeV})$ . Since the detector response is calibrated using Z boson observables, the shift in the Z boson mass is expected to slightly modify the W mass extracted from experiment.

For precision physics away from the Z resonance, the (non-universal) weak corrections must also be included. These corrections become important at large values of the di-lepton invariant mass due to the presence of large Sudakov-like electroweak logarithms of the form  $\ln(m(l^+l^-)/M_V)$ ,  $V = W, Z$ , which eventually may be resummed [15]. A calculation of the non-universal weak corrections in  $p\bar{p} \rightarrow \gamma^*, Z \rightarrow l^+l^-$  is currently in progress [10]. In the implementation of the weak corrections we

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<sup>2</sup>ZGRAD is available from the authors.

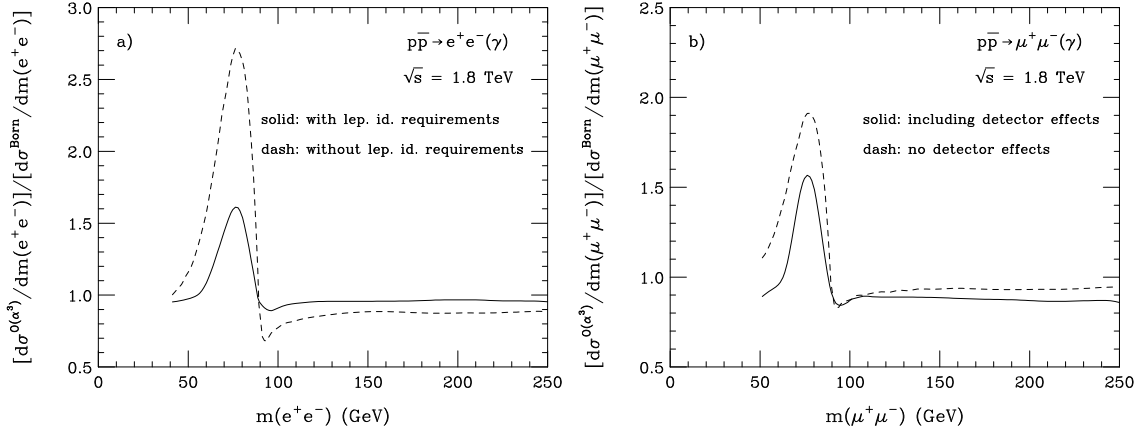


Figure 2: The relative corrections to the  $m(e^+e^-)$  and  $m(\mu^+\mu^-)$  distributions in Drell-Yan production at the Tevatron due to the  $\mathcal{O}(\alpha)$  QED corrections (from Ref. [9]).

closely follow Ref. [18], in particular for the treatment of higher-order corrections, which are important for a precise description of the Z resonance.

The NLO parton differential cross section, including weak  $\mathcal{O}(\alpha)$  and leading  $\mathcal{O}(\alpha^2)$  corrections, which enters eq. (2) is of the form [10]

$$d\hat{\sigma}^{(0+1)} = dP_{2f} \frac{1}{12} \sum |A_\gamma^{(0+1)} + A_Z^{(0+1)}|^2(\hat{s}, \hat{t}) + d\hat{\sigma}_{\text{box}}(\hat{s}, \hat{t}), \quad (3)$$

where the sum is taken over the spin and color degrees of freedom, and  $dP_{2f}$  denotes the two-particle phase space.  $d\hat{\sigma}_{\text{box}}$  describes the contribution of the box diagrams involving two massive gauge bosons. The matrix elements  $A_{\gamma,Z}^{(0+1)}$  comprise the Born matrix elements, the  $\gamma$ ,  $Z$ ,  $\gamma Z$  self energy insertions including a leading-log resummation of the terms involving the light fermions, and the one-loop vertex corrections. While  $A_{\gamma,Z}^{(0+1)}$  can be expressed in terms of effective vector and axial-vector couplings, the box contribution  $d\hat{\sigma}_{\text{box}}$  cannot be absorbed in effective couplings. However, in the Z resonance region the box diagrams can be neglected and the NLO cross section  $d\hat{\sigma}^{(0+1)}$  of eq. (3) has a Born-like structure. The leading universal electroweak corrections, i.e. the running of the electromagnetic charge and corrections connected to  $\Delta\rho$ , can be included in form of an effective Born approximation (EBA). Comparing results of the calculation which includes the full  $\mathcal{O}(\alpha)$  corrections with those obtained using the EBA together with the pure QED corrections reveals the effects of the genuine non-universal electroweak corrections such as box diagrams.

The weak corrections to neutral-current Drell-Yan processes as described above are currently being implemented in the parton level MC program ZGRAD2 [10]. A detailed numerical discussion of the effects of the electroweak  $\mathcal{O}(\alpha)$  corrections on distributions in  $p\bar{p} \rightarrow \gamma^*, Z \rightarrow l^+l^-(\gamma)$ ,  $l = e, \mu$  at the Tevatron and the LHC will be

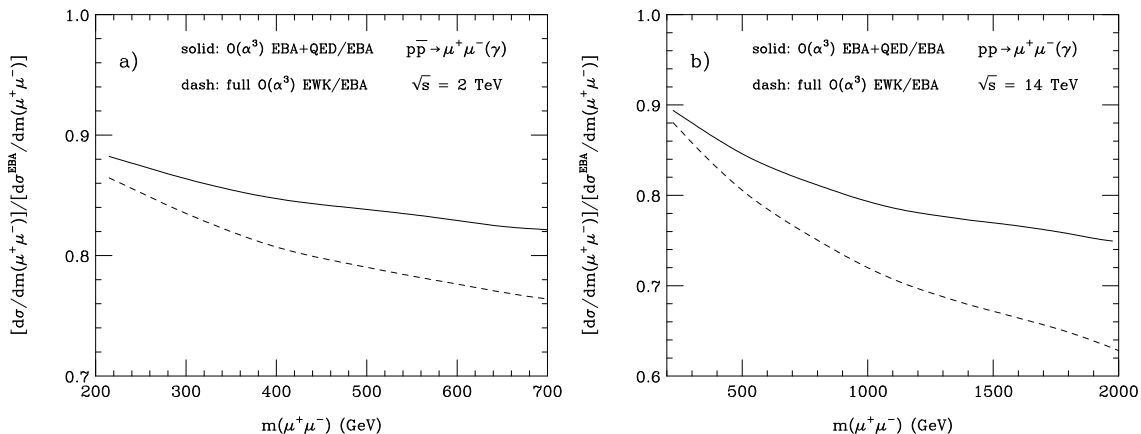


Figure 3: The relative corrections to the  $m(\mu^+\mu^-)$  distribution a) at the Tevatron and b) at the LHC when taking into account the universal corrections entering the EBA and QED corrections only (solid line), and when the full  $\mathcal{O}(\alpha)$  electroweak corrections are included in the calculation (dashed line).

given in Ref. [10]. Here we present some selected preliminary results for the di-lepton invariant mass distribution and the forward-backward asymmetry.

In Fig. 3 we show the  $\mu^+\mu^-$  invariant mass distribution including the full  $\mathcal{O}(\alpha)$  corrections normalized to the differential cross section in the EBA for large di-lepton invariant masses at the Tevatron and the LHC. Separation cuts and lepton identification requirements to simulate the detector acceptance as described in Ref. [9] (Tevatron) and Ref. [4] (LHC) are taken into account in Fig. 3. For comparison the relative corrections including the QED corrections only are also shown. As expected from the presence of large electroweak Sudakov-like logarithms, the weak corrections strongly increase in magnitude with increasing  $m(\mu^+\mu^-)$ , reaching about 10% at  $m(\mu^+\mu^-) = 1$  TeV. Both, the QED and the genuine weak corrections reduce the differential cross section. Qualitatively similar results are obtained in the  $e^+e^-$  case.

In Fig. 4, we show how the purely weak corrections affect the forward backward asymmetry at the LHC<sup>3</sup>. To illustrate the effect of the non-universal weak corrections, we plot the difference of the forward backward asymmetry including the full  $\mathcal{O}(\alpha)$  corrections, and the asymmetry which only takes into account QED corrections and the universal corrections which are included in the EBA. A genuine non-universal electroweak effect can be observed in the vicinity of  $m(l^+l^-) = M_W$  and  $2M_W$ , which is due to threshold effects in the box diagrams involving two W bosons. Results qualitatively similar to those shown in Fig. 4 are also obtained for the Tevatron.

<sup>3</sup>For a definition of  $A_{FB}$  at the LHC, see Ref. [9].

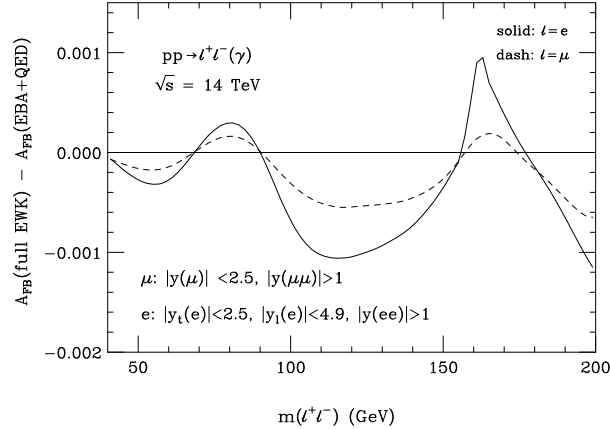


Figure 4: The forward-backward asymmetry including NLO electroweak corrections at the LHC, imposing the cuts and lepton identification requirements of Ref. [16]. The EBA and QED contribution have been subtracted (preliminary results).

The forward backward asymmetry at the LHC is very sensitive to the rapidity coverage of the leptons assumed. In Fig. 4, we have used the lepton rapidity coverages foreseen for the ATLAS detector [4,16]. For muon pairs, both muons are required to have rapidity  $|y(\mu)| < 2.5$ . For  $e^+e^-$  pairs, the leptons are required to have  $|y_l(e)| < 4.9$ , with one of them having to fulfill the more stringent requirement  $|y_t(e)| < 2.5$ . In addition, the lepton pair rapidity has to be  $|y(ll)| > 1$  for both electrons and muons in the final state. This cut substantially increases the magnitude of  $A_{\text{FB}}$  at the LHC [17].

It is interesting to check whether the threshold effect at  $m(l^+l^-) = 2M_W$  will be observable. In the electron case, the expected statistical uncertainty in  $A_{\text{FB}}$  for  $m(e^+e^-) = 2M_W \pm 5$  GeV and  $100 \text{ fb}^{-1}$  at the LHC is about  $(3 - 4) \times 10^{-3}$  per experiment. The size of the non-universal electroweak corrections in the region are of the order of  $10^{-3}$ . In a realistic calculation, contributions from  $W^+W^- \rightarrow l^+\nu_l l^-\bar{\nu}_l$ ,  $ZZ \rightarrow l^+l^-\bar{\nu}\nu$  and  $\bar{t}t$  production to the forward backward asymmetry need to be taken into account, which could well be of the same order as the genuine weak corrections. It will thus be difficult to observe a clear signal of the threshold effects originating from the box diagrams involving two W bosons in  $A_{\text{FB}}$  at the LHC. On the other hand, given the expected statistical precision, the genuine weak corrections cannot be neglected when comparing data with the SM prediction.



## 4 Conclusions

Our results show that, for the precision obtained in previous Tevatron runs, the existing calculations for W and Z boson production are sufficient. However, for future precision measurements the full electroweak  $\mathcal{O}(\alpha)$  corrections and probably also multiple photon radiation effects should be taken into account. The inclusion of the non-resonant contributions to W production in WGRAD is in progress [14] (see also Ref. [19]). As a first step towards a calculation of the  $\mathcal{O}(\alpha^2)$  QED corrections, the effects of two-photon radiation in W and Z boson production at hadron colliders have been computed in Ref. [20].

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