Higgs Mechanism via Bose-Einstein Condensation

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Recently the Bose-Einstein phenomenon has been proposed as possible physical mechanism underlying the spontaneous symmetry breaking in cold gauge theories. The mechanism is natural and we use it to drive the electroweak symmetry breaking. The mechanism can be implemented in different ways while here we review two simple models in which the Bose-Einstein sector is felt directly or indirectly by all of the standard model fields. The structure of the corrections due to the new mechanism is general and independent on the model used leading to experimental signatures which can be disentangled from other known extensions of the standard model.

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I. INTRODUCTION

The Standard Model of particle interactions has passed numerous experimental tests [1] but despite the experimental successes it is commonly believed that it is still incomplete. The spontaneous breaking of the electroweak symmetry, for example, has been the subject of intensive studies and different models have been proposed to try to accommodate it in a more general and satisfactory framework. Technicolor theories [2] and supersymmetric extensions [3] of the Standard Model are two relevant examples.

In [4, 5] we explored the possibility that the relativistic Bose-Einstein phenomenon [6, 7] is the cause of electroweak symmetry breaking. In [4] we introduced a new global symmetry of the Higgs field and associated a chemical potential μ with the generators of such a new symmetry. A relevant property of the theory was that the chemical potential induced a, non renormalizable and hence protected against quadratic divergences, direct negative mass squared for the Higgs field at the tree level destabilizing the symmetric vacuum and triggering symmetry breaking. The local gauge symmetries were broken spontaneously and the associated gauge bosons acquired a standard mass term. We also showed that while the properties of the massive gauge bosons at the tree level are identical to the ones induced by the conventional Higgs mechanism, the Higgs field itself has specific Lorentz non covariant dispersion relations. The Bose-Einstein mechanism occurs, in fact, in a specific frame the effects of which are felt by the other particles in the theory via electroweak radiative corrections. Some of these corrections have been explicitly computed in [4] and the model presented in [4] is very predictive.

In [5] we presented a new model in which a hidden Bose-Einstein sector drives the electroweak symmetry breaking while yielding small corrections to the standard Higgs dispersion relations. In this way the presence of a frame and hence Lorentz breaking corrections are suppressed with respect to the ones shown in [4]. In this class of models the Bose-Einstein mechanism operates on a complex scalar field neutral under all of the Standard Model interactions. On general grounds this field interacts with the ordinary Higgs and we used these interactions to trigger the ordinary electroweak symmetry breaking. Due to the Bose-Einstein nature of the new mechanism the form of the corrections is insensitive to its specific realization, hence the mechanism is distinguishable from other sources of beyond standard model physics. The two classes of models we presented in [4, 5] are schematically summarized in figure 1. In the first one the Higgs field feels directly the presence of a net background charge and communicates it to the gauge bosons and fermions via higher order corrections (left panel). In the second (the hidden case) a singlet field with respect to the standard model quantum numbers directly feels the effects of a net background charge. The ordinary Higgs field weakly feels the effects of a frame via the interactions with the singlet field. Finally the rest of the standard model particles will be affected via higher order corrections (right panel). Since the chemical potential

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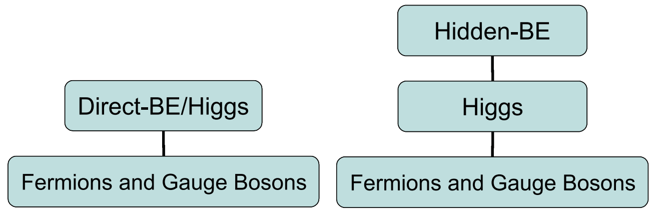


FIG. 1: Schematic representation of the direct and indirect Bose-Einstein mechanism for triggering electroweak symmetry breaking.

differentiates between time and space, and we have a scalar vacuum, all of the dispersion relations are isotropic in the Bose-Einstein frame. Theories with condensates of vectorial type have been studied in different realms of theoretical physics [8, 9, 10, 11, 12, 13, 14]. It is also worth mentioning that, although in a different framework, effects of a large lepton number on the spontaneous gauge symmetry breaking for the electroweak theory at high temperature relevant for the early universe have been studied [8, 10, 11, 15, 16]. In [17] the concept of Bose-Einstein condensation in gravitational systems was considered.

II. MODEL I: DIRECT BOSE-EINSTEIN MECHANISM

The Higgs sector of the Standard Model possesses, when the gauge couplings are switched off, an $SU_L(2)\times SU_R(2)$ symmetry. The full symmetry group is mostly easily recognized when the Higgs doublet field

$$H=\frac{1}{\sqrt{2}}\begin{pmatrix}\pi_2+i\,\pi_1\\ \sigma-i\,\pi_3\end{pmatrix} \tag{1}$$

is represented as a two by two matrix in the following way:

$$M\equiv\frac{1}{\sqrt{2}}\left(\sigma+i\,\vec{\tau}\cdot\vec{\pi}\right)=\frac{1}{\sqrt{2}}\begin{pmatrix}\sigma+i\,\pi_3 & \pi_2+i\,\pi_1\\ -\pi_2+i\,\pi_1 & \sigma-i\,\pi_3\end{pmatrix}\equiv[i\,\tau_2 H^*\ ,\ H]\ , \tag{2}$$

where the two columns of the two by two matrix are identified with $i\tau_2 H^*$ and H. The $SU_L(2)\times SU_R(2)$ group acts linearly on M according to:

$$M\to g_L M g_R^{\dagger}\qquad \text{and}\qquad g_{L/R}\in SU_{L/R}(2)\ . \tag{3}$$

It is easy to verify that:

$$M\frac{(1-\tau^3)}{2}=[0\ ,\ H]\ ,\qquad M\frac{(1+\tau^3)}{2}=[i\,\tau_2 H^*\ ,\ 0]\ , \tag{4}$$

where the notation follows from (2) and the zeros represent an entire column of the two by two matrix. The $SU_L(2)$ symmetry is gauged by introducing the weak gauge bosons W^a with $a=1,2,3$. The hypercharge generator is taken to be the third generator of $SU_R(2)$. The ordinary covariant derivative acting on the Higgs, in the present notation, is:

$$D_\mu M=\partial_\mu M-i\,g\,W_\mu M+i\,g' M\,B_\mu\ ,\qquad \text{with}\qquad W_\mu=W_\mu^a\frac{\tau^a}{2}\ ,\quad B_\mu=B_\mu\frac{\tau^3}{2}\ . \tag{5}$$

At this point one simply *assumes* that the mass squared of the Higgs field is negative and this leads to the electroweak symmetry breaking and more generally to the successful Standard Model as we know. However, theoretically a more satisfactory explanation of the origin of the Higgs mechanism is needed. In the literature many models have been proposed in order to explain the emergence of such a negative mass squared. Technicolor theories, for example, assume a dynamical mechanism identical to spontaneous chiral symmetry breaking in quantum chromodynamics [2]. Supersymmetric extensions of the Standard Model [3] explain the negative mass squared as due to the running of the masses from high scales down to the electroweak one.

We now review the first model in which the Higgs mechanism is driven by the Bose-Einstein phenomenon [4]. To illustrate the idea we consider a Higgs sector with the symmetry group $SU_L(2)\times SU_R(2)\times U_A(1)$ where the $SU_L(2)$ is later on gauged and the $U_Y(1)$ is associated to the $T^3=\frac{\tau^3}{2}$ generator of $SU_R(2)$ while $U_A(1)$ remains a global symmetry. Now we introduce a chemical potential μ_A associated to $U_A(1)$. When the chemical potential is sufficiently large $SU_L(2) \times SU_R(2) \times U_A(1)$ breaks spontaneously to $SU_V(2)$ and we have four goldstones. It is advantageous to use nonlinear realizations for the Higgs field with:

$$M = \frac{\sigma}{\sqrt{2}} U_\eta U \qquad \text{with} \qquad U_\eta = e^{i\frac{\eta}{v}} \qquad U = e^{i\frac{\pi}{v}} \ , \qquad \text{and} \qquad \pi = \tau^a \pi^a \ . \tag{6}$$

In the above equation v is the vacuum expectation value of σ. In the linearly realized case we should also include the heavy $U_A(1)$ partners of the π field which we have taken to be more massive than the neutral Higgs particle and hence we have decoupled them. The η field is the the goldstone boson associated to the spontaneous breaking of the global $U_A(1)$ symmetry.

The gauge interactions as well as the chemical potential are introduced via the following covariant derivative:

$$\mathcal{D}_\mu M = \partial_\mu M - i\, g\, W_\mu\, M + i\, g'\, M\, B_\mu - i\mathcal{A}_\mu M \equiv D_\mu M - i\mathcal{A}_\mu M \ , \quad \text{with} \quad \mathcal{A}_\mu = \mu_A(1, \vec{0}) \ . \tag{7}$$

Electroweak breaking is now forced by the introduction of the chemical potential for the extra global symmetry. For $\mu_A^2 > m^2$ we have Bose-Einstein condensation together with the ordinary spontaneous breaking of the internal symmetry $SU_L(2) \times SU_R(2) \times U_A(1) \to SU_V(2)$ with 4 null curvatures corresponding to the four broken generators. In the unitary gauge the three fields π^a are absorbed into the longitudinal components of the three massive gauge boson fields while the field η remains massless. In the unitary gauge the quadratic terms are:

$$\begin{aligned} \mathcal{L}_{\text{quadratic}} &= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \eta \partial^\mu \eta - \mu_A \left(h\partial_0 \eta - \eta \partial_0 h\right) \\ &+ \frac{v^2}{8}\left[g^2\left(W^1_\mu W^{\mu,1} + W^2_\mu W^{\mu,2}\right) + \left(g\, W^3_\mu - g'\, B_\mu\right)^2\right] - (\mu_A^2 - m^2)h^2 \end{aligned} \tag{8}$$

with

$$\langle\sigma\rangle^2 = v^2 = \frac{\mu_A^2 - m^2}{\lambda} \ , \qquad \text{and} \qquad \sigma = v + h \ , \tag{9}$$

where h is the Higgs field. The Z_μ and the photon A_μ gauge bosons are:

$$Z_\mu = \cos\theta_W\, W^3_\mu - \sin\theta_W B_\mu \ , \qquad A_\mu = \cos\theta_W\, B_\mu + \sin\theta_W W^3_\mu \ , \tag{10}$$

with $\tan\theta_W = g'/g$ while the charged massive vector bosons are $W^{\pm}_\mu = (W^1 \pm i\, W^2_\mu)/\sqrt{2}$. The bosons masses, due to the custodial symmetry, satisfy the tree level relation $M_Z^2 = M_W^2/\cos^2\theta_W$ with $M_W^2 = g^2\, v^2/4$.

Except for the presence of an extra degree of freedom and the $h-\eta$ mixing term one recovers the correct electroweak symmetry breaking pattern. The third term in the Lagrangian signals an explicit breaking of the Lorentz symmetry in the Higgs sector. Such a breaking is due to the introduction of the chemical potential and hence it happens in a very specific and predictive way so that all of the features can be studied. Note that in our theory Lorentz breaking, at the tree level, is confined only to the Higgs sector of the theory which is also the least known experimentally. The rest of the theory is affected via weak radiative corrections. As previously emphasized, in general, the introduction of the chemical potential at zero temperature does not introduce new ultraviolet divergences and hence does not spoil renormalizability. Besides, diagonalizing the quadratic terms the spectrum and the propagators for h and η yields:

$$E_h^2 = \Delta^2 + \left(1 + 4\frac{\mu_A}{\Delta^2}\right)p^2 - \frac{\mu_A^4}{\Delta^6}p^4 + \cdots \ , \tag{11}$$

$$E_\eta^2 = \left(1 - 4\frac{\mu_A^2}{\Delta^2}\right)p^2 + \cdots \ . \tag{12}$$

with

$$\Delta^2 = 2(3\mu_A^2 - m^2) = 4\mu_A^2 + 2\left(\mu_A^2 - m^2\right) \ . \tag{13}$$

The second term in the expression for Δ^2 is the potential curvature evaluated on the ground state which in the absence of the chemical potential is the mass of h. Note that the energy gap (energy at zero momentum) of the Higgs Δ is larger than the one predicted by just assuming a change in the sign of the mass squared coefficient.

Some important features of the present Bose-Einstein condensation mechanism are: a mass Δ for h larger than the one in vacuum i.e. $M_h^2 = 2\left(\mu_A^2 - m^2\right)$, more specifically $\Delta \geq \sqrt{3}M_h$, and a massless goldstone state η. For example if the Higgs mass, in vacuum, is about 90 GeV the Bose-Einstein mechanism leads to a mass larger than 155 GeV. The spontaneous breaking of the $U_A(1)$ symmetry requires the η field to be massless. Phenomenologically one may need to add a mass, small with respect to the chemical potential, for the η field [4].

III. THE NON HIGGS SECTOR

We have shown that, at the tree level, the gauge bosons acquire the ordinary electroweak masses and dispersion relations while we argued that deviations with respect to the ordinary Higgs mechanism, in our theory, arise when considering higher order corrections. In this section we analyze some of the effects of spontaneous symmetry breaking via a nonzero $U_A(1)$ charge density on the non Higgs sector of the electroweak theory due to such higher order effects. We first investigate the gauge boson sector and then the fermion one. On general grounds we expect the presence of the chemical potential to induce different time and spatial corrections while keeping rotational invariance intact. In Fig. 2 we schematically show how the corrections due to the Bose-Einstein induced Higgs mechanism propagate to the rest of the standard model fields.

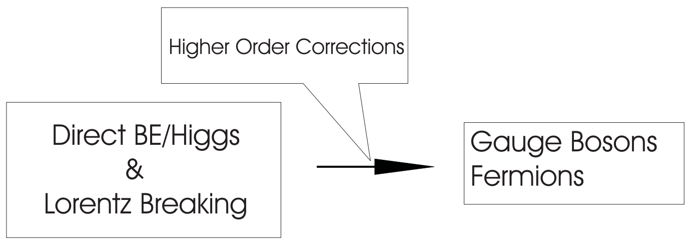


FIG. 2: Schematic representation of the way the corrections due to the direct Bose-Einstein condensation are felt by the non Higgs sector of the standard model.

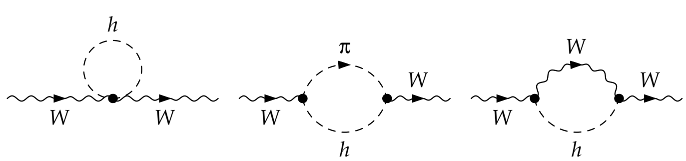
A. The Gauge Bosons

The Higgs propagator is modified in the presence of the chemical potential and assumes the form:

$$- - - - - - \;=\; i\,\frac{p^2}{p^4-2(\mu_A^2-m^2)p^2-4\mu_A^2 p_0^2}\;. \tag{14}$$

All of the loops containing this propagator are affected by the presence of the chemical potential. The Landau gauge [18] is chosen to evaluate the relevant contributions although our results are gauge independent.

We are interested in computing the new physics corrections for the W vacuum polarization due to a different Higgs sector with respect to the conventional Standard Model one. The diagrams needed are [19, 20]:



(15)

The major difference with respect to known possible extensions is the appearance of a new type of dispersion relations for the Higgs. The η particle does not appear in the previous diagrams since we used a polar decomposition for the M field.

In the leading logarithmic approximation, when the external momentum vanishes and assuming an expansion in the gauge bosons masses with respect to the Higgs mass and setting $m = 0$, the result is:

$$\begin{aligned}\Pi^{\mu\nu}_{WW}(0) \;&=\; \frac{3\,g^2}{4\,(4\pi)^2}\,\log\left(\frac{\mu_A^2(2+\sqrt{3})}{\Lambda^2}\right)\left[g_{\mu\nu}\,\left(M_W^2+\frac{\mu_A^2}{9}\right)-\frac{4\,\mu_A^2}{9}\,V_\mu V_\nu\right] \\ &+\; \mathcal{O}\left(\frac{M_W^2}{\mu_A^2}\right)\end{aligned} \tag{16}$$

where Λ is the renormalization scale and $V_\mu = (1,\mathbf{0})$. The photon vacuum polarization is not affected at zero momentum due to the ordinary Ward identities [20]. The first diagram on the right hand side of eq. (15) does not contribute to the Lorentz non covariance of the vacuum polarization which is entirely due to the second and third diagrams. Note that the specific combination $\mu_A^2\left(2+\sqrt{3}\right)$ appearing in any logarithmic corrections is consequence of the fact that in the presence of the chemical potential the particle gaps are not the curvatures evaluated on the minimum.

The contribution to the Z vacuum polarization is obtained by replacing in the previous expressions g^2 with $g^2/\cos^2\theta_W$ and M_W^2 with M_Z^2. The onset of Lorentz breaking in this sector is small especially if one chooses Λ of the order of μ_A. In general it is possible to define a new set of oblique parameters capable to capture the relevant corrections due to this type of spontaneous symmetry breaking. Here, for illustration, we consider the following straightforward extension of the parameter which measures deviations with respect to the breaking of the custodial symmetry i.e. the parameter T [19]:

$$\alpha\, T^{\mu\nu} = \frac{e^2}{\sin^2\theta_W\, \cos^2_W\, M_Z^2}\left(\Pi_{11}^{\mu\nu,\ new}(0) - \Pi_{33}^{\mu\nu,\ new}(0)\right) \ , \tag{17}$$

with $\alpha = e^2/4\pi$ the fine structure constant. We also used $g^2\,\Pi_{11}^{\mu\nu}(0) = \Pi_{WW}^{\mu\nu}(0)$ and $g^2\Pi_{33}^{\mu\nu}(0) = \cos^2\theta_W \Pi_{ZZ}^{\mu\nu}(0)$ since the photon vacuum polarization at zero momentum vanishes.

This parameter is equal to $\alpha\, T\, g^{\mu\nu}$ for any Lorentz preserving extension of the Higgs sector. The newly defined parameter is not directly a measure of the amount of Lorentz breaking but rather it estimates the amount of custodial symmetry breaking for the different spacetime components of the vacuum polarizations. Here we still have $T^{\mu\nu} = T\, g^{\mu\nu}$ but due to the fact that we have different dispersion relations and a different gap structure with the respect to the Standard Model masses the value of T, although very small, is not zero:

$$T \approx -\frac{3}{16\pi}\frac{1}{\cos^2\theta_W}\log\left(1 + \frac{\sqrt{3}}{2}\right) \approx -0.048\ . \tag{18}$$

Here we assumed the Standard Model Higgs mass to be given by the expression $M_H^2 = 2\mu_A^2$ which is the curvature evaluated on the minimum.

B. The Fermions

The fermions constitute a very interesting sector to be explored since it can be used experimentally to test the idea presented in this paper. In order to understand the type of corrections we start with recalling that the chemical potential explicitly breaks $SL(2,C)$ to $SO(3)$. So the corrections must differentiate time from space in the fermion kinetic term according to:

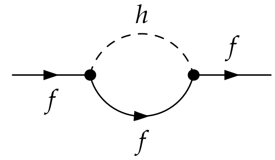
$$(1 - a_0)\,\bar{f}\gamma^0\partial_0\, f + (1 - a)\,\bar{f}\gamma^i\partial_i\, f \to \bar{f}\gamma^0\partial_0\, f + v_f\,\bar{f}\gamma^i\partial_i\, f\ , \qquad v_f \simeq 1 - (a - a_0) \tag{19}$$

where a and a_0 are the corrections induced by loop contributions and f represents a generic Standard Model fermion. In the last expression we have rescaled the fermion wave function and used the fact that the a's are small calculable corrections. In the difference $a - a_0$ all of the Lorentz covariant corrections disappear while only the Lorentz breaking terms survive.

In order for the fermions to receive one loop corrections sensitive to the chemical potential they need to couple at the tree level with the Higgs. This is achieved via the Yukawa interactions (for a more detailed discussion see [4]):

$$\widetilde{Y}_f\, h\bar{f}f\ , \quad \text{with} \quad \widetilde{Y}_f \simeq \frac{m_f}{v}\ . \tag{20}$$

To determine v_f at the one loop we need to compute the contributions to the fermion self energy which break Lorentz invariance. In the present case the one loop diagram is



, (21)

where the solid line represent the fermion and the dashed one the Higgs field. The diagram is evaluated in detail in [4] and yields the following contribution to the a and a_0 coefficients:

$$a \ = \ \frac{\widetilde{Y}_f^2}{2(4\pi)^2}\left[\log\left(\frac{\mu_A^2(2+\sqrt{3})}{\Lambda^2}\right) - \frac{1}{6}\right]\ , \tag{22}$$

$$a_0 \ = \ \frac{\widetilde{Y}_f^2}{2(4\pi)^2}\left[\log\left(\frac{\mu_A^2(2+\sqrt{3})}{\Lambda^2}\right) + 4\sqrt{3} - \frac{15}{2}\right]\ . \tag{23}$$

If only the leading logarithmic corrections are kept we have that the fermions still obey standard Lorentz covariant dispersion relations. The dispersion relations are modified when considering the finite contributions. To estimate the size of possible departures from the standard dispersion relations we keep the constant terms and determine

$$v_e \simeq 1 - \widetilde{Y}_e^2 \frac{0.4}{2(4\pi)^2} \simeq 1 - 5 \times 10^{-15} \ , \tag{24}$$

where the numerical evaluation has been performed for the electron. The present formula is valid practically for all of the fermions. For the muon the corrections are of the order of 8×10^{-10}. The induced corrections for the photon due to fermion loops are further suppressed by powers of the fine structure constant $\alpha = e^2/4\pi$.

IV. MODIFIED FERMI THEORY

Another class of indirect corrections to the fermion sector are the ones induced by modified gauge vector propagators discussed in the previous section. To illustrate these effects we use the low energy electroweak effective theory for the charged currents. The neutral currents will be affected in a similar way. The chemical potential leaves intact the rotational subgroups of the Lorentz transformations, so the effective Lagrangian modifies as follows:

$$\mathcal{L}_{\rm Eff}^{\rm CC} = -2\sqrt{2}\, G J_\mu^+ J^{\mu -} \Rightarrow -2\sqrt{2}\, G \left[{J_\mu}^+ J^{\mu -} + \delta\, J_i^+ J^{i\,-} \right] \ , \tag{25}$$

where δ is a coefficient effectively measuring the corrections due to modified dispersion relations for the gauge vectors. Using the previous Lagrangian the decay rate for the process $\mu \to e \bar{\nu}_e \nu_\mu$ is:

$$\Gamma[\mu \to e \bar{\nu}_e \nu_\mu] = \frac{G^2 M_\mu^5}{192 \pi^3} \left(1 + \frac{3}{2} \delta \right) \ , \tag{26}$$

where M_μ is the muon mass and we neglected the electron mass. However the effects of a nonzero electron mass are as in the Standard Model case [1]. The parameter δ can be estimated using the vacuum polarizations presented in the previous section yielding:

$$\delta \approx \frac{g^2}{3(4\pi)^2} \frac{\mu_A^2}{M_W^2} \log \left(\frac{(2+\sqrt{3})\mu_A^2}{\Lambda^2} \right) . \tag{27}$$

By choosing the renormalization scale Λ to be of the order of the electroweak scale $\sim M_Z$ and $\mu_A \simeq 150$ GeV we determine $\delta \simeq 0.007$. Precise and independent measurements of M_W and the muon decay rate may observe deviations with respect to the Standard Model. Finally we expect sizable corrections to the fermion dispersion relations in eq. (19) induced by the gauge boson exchanges. These arise in the fermion vacuum polarization at the two loop level and are expected to be of the order of $g^2 \delta/(4\pi)^2$.

V. MODEL II: HIDDEN BOSE-EINSTEIN MECHANISM

Another model was proposed in [5]. Now the Standard Model is extended by adding a new complex bosonic field ϕ which is a singlet under all of the standard model charges. In this way we also introduce an extra $U(1)$ symmetry. The most general renormalizable potential involving the standard model Higgs and the field ϕ respecting all of the symmetries at hand is:

$$\begin{aligned} V\left[\phi, M\right] \ &= \ \frac{1}{2}\left(M_H^2 - 8\hat{g}\ |\phi|^2 \right) {\rm Tr}\left[M^\dagger M \right] + m^2 |\phi|^2 \\ &+ \ \hat{\lambda}\, |\phi|^4 + \lambda {\rm Tr}\left[M^\dagger M \right]^2 \ , \end{aligned} \tag{28}$$

where $M \equiv \frac{1}{\sqrt{2}} \left(\sigma + i\, \vec{\tau} \cdot \vec{\pi} \right)$. We have assumed the potential to be minimized for a zero vacuum expectation values of the fields and the couplings to be all positive. With this choice it is clear that if ϕ acquires a non zero vacuum expectation value the ordinary Higgs also acquires a negative mass square contribution.

We introduce a non zero background charge for the field ϕ, which modifies the ϕ kinetic term as follows [4, 6, 7]:

$$\mathcal{L}_{\rm charge} = \mathcal{D}_\mu \phi^* \mathcal{D}^\mu \phi \ , \tag{29}$$

with

$$\mathcal{D}_\nu \phi = \partial_\nu \phi - i \mathcal{A}_\nu \phi \ , \qquad \mathcal{A}_\nu = \mu \left(1, \vec{0}\right) \ , \tag{30}$$

and μ is the associated chemical potential. Substituting (30) in (29) we have:

$$\mathcal{L}_{\rm charge} = \partial_\mu \phi^* \partial^\mu \phi + i\,\mu \left(\phi^* \partial_0 \phi - \partial_0 \phi^* \phi\right) + \mu^2 \left|\phi\right|^2 \ . \tag{31}$$

The introduction of the chemical potential has broken Lorentz invariance $SO(1,3)$ to $SO(3)$ while providing a negative mass squared contribution to the ϕ boson. When $\mu > m$ the spontaneous breaking of the $U(1)$ invariance is a necessity. Once the bosonic field has acquired a vacuum expectation value and for $8\hat{g}\,\langle|\phi|\rangle^2 > M_H^2$ the Higgs field condenses as well.

In this model the Bose-Einstein mechanism, although indirectly, still triggers electroweak symmetry breaking while the effects of Lorentz breaking induced by the presence of the chemical potential are attenuated. The latter are controlled by the new coupling constant $\hat{g}$ as well as the ordinary higher order electroweak corrections. Our model potential is similar in spirit to the one used in hybrid models of inflation [21].

For the parameter values $M_H^2 = m^2 = 0$ it was shown in [5] that the ground state of the theory has

$$\langle|\phi|\rangle^2 = \frac{\mu^2}{2\hat{\lambda}} + \mathcal{O}(\epsilon^4) \ , \qquad \langle\sigma\rangle^2 = \frac{\mu^2}{2\hat{\lambda}}\,\epsilon^2 + \mathcal{O}(\epsilon^6) \ ,$$

where $\epsilon^2 = 2\hat{g}/\lambda \ll 1$. Assuming the new physics scale associated to $\langle|\phi|\rangle$ to be within reach of LHC, i.e., of the order of $1-10$ TeV while the Higgs scale is $\langle\sigma\rangle \simeq 250$ GeV we determine:

$$\epsilon \simeq 0.25 - 0.025 \ . \tag{32}$$

All the corrections will be proportional to the fourth power of ϵ. We adopt the unitary gauge for the Higgs field and use the polar decomposition for the field ϕ:

$$\sigma = \langle\sigma\rangle + h \ , \quad \phi = \frac{\left(\sqrt{2}\langle\phi\rangle + \psi\right)}{\sqrt{2}}\, e^{i\frac{\eta}{\sqrt{2}\langle\phi\rangle}} \ . \tag{33}$$

In the broken phase h and ψ are not mass eigenstates. These are related to h and ψ:

$$h = \cos\theta\,\widetilde{h} - \sin\theta\,\widetilde{\psi} \ , \qquad \psi = \cos\theta\,\widetilde{\psi} + \sin\theta\,\widetilde{h} \ . \tag{34}$$

Working out the details of the diagonalization [5], one finds that the new neutral Higgs field $\widetilde{h}$ due to the mixing with ψ feels feebly but directly the presence of the net background charge. The propagator for $\widetilde{h}$ is:

$$\frac{i}{p^2 - m^2_{\widetilde{h}} - 4p_0^2\mu^2\,\sin^2\theta\,\mathcal{F}\left[p,p_0\right]} \ , \tag{35}$$

with

$$\mathcal{F}\left[p,p_0\right] = \frac{m^2_{\widetilde{\psi}} - p^2}{\left(m^2_{\widetilde{\psi}} - p^2\right)\,p^2 + 4p_0^2\,\mu^2\,\cos^2\theta} \ . \tag{36}$$

We take ψ to be heavy so that the only corrections will be the ones induced by the small mixing between h and ψ. The function (36) for large μ is $\mathcal{F} \approx 1/(3p_0^2 - \mathbf{p}^2)$ and resembles the pole associated to the gapless state η. Again, if phenomenologically needed we can give a small mass (with respect to the scale μ) to η [4]. The rest of the standard model particles are affected by the presence of a frame via weak radiative corrections and these corrections can be computed as in [4]. The Higgs propagator here is more involved than the one in [4]. However the relevant point is that the effects of the non standard dispersion relations in the model II are damped with respect to the ones determined in model I [4] due to the presence of the suppression factor $\sin^2\theta \approx \epsilon^6$ in the $\widetilde{h}$ propagator. Indeed since $m^2_{\widetilde{h}} \approx \epsilon^2\mu^2$ the term in the propagator bearing information of the explicit breaking of Lorentz invariance due to the Bose-Einstein mechanism is down by ϵ^4 with respect to the mass term. So we have a large suppression of Lorentz breaking effects induced by the presence of a frame needed for the Bose-Einstein mechanism to take place. Clearly also the dispersion relations of the ordinary Higgs are modified only slightly with respect to the standard scenario. The corrections due to the mixing with the ψ field should also be taken into account although in general they will be further suppressed due to the assumed hierarchy $\mu \gg m_h$.

Let us roughly estimate, using the results of [4], the size of the corrections to some observables. We first consider the modification of the low energy Fermi theory due to the new sector. Recall that the parameter δ controlling the corrections in the low energy theory (25) was found to be $\delta \approx 0.007$ in the direct Bose-Einstein model. For the hidden Bose-Einstein sector this result, on general grounds, is further suppressed by a multiplicative factor of the order ϵ^4 yielding a new δ of the order of $2.7 \times 10^{-5} - 2.7 \times 10^{-9}$ for $\langle|\phi|\rangle \approx 1 - 10$ TeV.

The fermion sector also bears information of the direct or indirect nature of the underlying Bose-Einstein phenomenon. We demonstrated for the direct case in [4] that the one loop corrections to the fermion velocities due to the exchange of the Higgs are tiny ($\simeq 10^{-15}$ for the electron). This is due to the smallness of the associated Yukawa's couplings. However we have also argued that the higher order corrections due to the modified gauge boson dispersion relations may be relevant and estimated in this case a correction of the order of $\delta\, g^2/4\pi$ with g the weak coupling constant. In the hidden Bose-Einstein case also the corrections to this sector are further suppressed by a factor of the order of ϵ^4 with respect to the findings in [4].

VI. CONCLUSIONS

The Bose-Einstein condensation phenomenon has been proposed as possible physical mechanism underlying the spontaneous symmetry breaking of cold gauge theories. We have explicitly considered two possible realizations of this picture. In model I [4] the Higgs field was assumed to carry global and local symmetries and was identified with the Bose-Einstein field. The effects of a non zero background charge were, in this way, maximally felt in the Higgs sector and then communicated via weak interactions to the other standard model particles. In model II [5] the Higgs mechanism was triggered by a hidden Bose-Einstein sector. The relevant feature of this model is that the effects of modified dispersion relations for the standard model fields are strongly suppressed with respect to the ones in model I. We have predicted the general form of the corrections and estimated their size in both cases. The fermion sector of the Standard Model, for example, is affected via higher order corrections leading to the appearance of modified dispersion relations of the type $E^2 = v_f^2\, p^2 + m_f^2$. The deviation with respect to the speed of light for the fermions is small when considering the direct Bose-Einstein mechanism [4] while it is further suppressed by a factor ϵ^4 in the model presented here.

We emphasize that the form of the corrections, induced by the modified propagators in Eqs. (35) and (36), are general and dictated solely by the Bose-Einstein nature of our mechanism. The strength of the coupling between the standard model fields and the Bose-Einstein field is measured by the parameter ϵ which enters in some of the physical observables. Experiments can be used to constrain the possible values of ϵ.

An interesting consequence of the direct (i.e. model I) Bose-Einstein condensation mechanism is that the mass Δ for the neutral Higgs h is predicted to be larger than the one in vacuum i.e. $M_h^2 = 2\left(\mu_A^2 - m^2\right)$. More specifically $\Delta \geq \sqrt{3} M_h$. If, for example, the in vacuum Higgs mass is chosen to be about 90 GeV the Bose Einstein mechanism yields a mass larger than 155 GeV.

Spontaneous breaking of a gauge theory via Bose-Einstein condensation necessarily introduces Lorentz breaking since a frame must be specified differentiating time from space. We recall that the issue of Lorentz breaking has recently attracted much theoretical [22, 23, 24] and experimental attention [25]. In the Bose-Einstein case the underlying gravitational theory is not the cause of Lorentz breaking which is instead due to having immersed the theory in a background charge.

Modified dispersion relations for the standard model particles have also cosmological consequences since now one cannot simply assume the velocity of light as the common velocity for all of the massless elementary particles. In fact different particles will have, in general, different dispersion relations and hence speed. This is relevant, for example, when observing neutrinos from distant sources. Using the present model as guide for the structure of the corrections experiments can test the Bose-Einstein mechanism as possible source of electroweak symmetry breaking.

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