Epoch Dependent Dark Energy

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Abstract. We present a model in which the parameter w approaches -1 near a particular value of z, and has significant negative values in a restricted range of z. For example, one can have $w \approx -1$ near $z = 1$, and $w > -0.2$ from $z = 0$ to $z = 0.3$, and for $z > 9$. The ingredients of the model are neutral fermions (which may be neutrinos, neutralinos, etc) which are very weakly coupled to a light scalar field.

This model emphasises the importance of the proposed studies of the properties of dark energy into the region $z > 1$.

INTRODUCTION

About twenty five years ago the possibility of neutrinos interacting by exchange of very light neutral scalars to produce interesting cosmological and astrophysical effects was considered by Kawasaki, Murayama and Yanagida [1], and Malaney, Starkman and Tremaine[2].

Three of us later investigated a similar system, in which the scalar particle has a mass of order 1/a.u., which leads to neutrino clustering[3].

Following the observation of the re-acceleration of the expansion of the Universe [4, 5], which led to the concept of dark energy (for a recent review see ref. [6]). Fardon, Nelson and Weiner [7] noted that the inferred energy density of dark energy, of order $(2.4 \text{ meV})^4$ [6], was, remarkably, of the order of the experimentally inferred value of neutrino masses. They also utilized the concept of a scalar field interacting with the neutrinos to obtain the desired effect.

It is worth emphasizing that negative pressure is not a strange feature in physics, in that any self bound system has an equilibrium density, and will have a higher energy per particle as the density is decreased, and thus a negative pressure in this region. Because this feature exists in our neutrino clustering model, we realized that it could produce negative values of w and was thus relevant to the dark energy problem. We provided some early comments [8], and then showed how it led to a system with $w \approx -1$ in a restricted range of the development of the universe [9], with $w \to 1/3$ at very early times, and $w \to 0$ near the present. In this paper we will be elaborating our model of dark energy.

To obtain the energy density of the neutral fermion – scalar field system as a function of z one needs to make assumptions about parameters. Typically w has a minimum as a

function of z, and the position of the minimum may be moved by a choice of parameters, but a rapid variation of w between the minimum and the present is characteristic of our model. This lends support to attempts to probe the z dependence of dark energy in more detail.

It is worth noting that Mota et al[10, 11] have recently introduced the possibility of neutrino clustering in the neutrino – scalar field model to obtain a dependence of the dark energy density on z. Our work shows that a variation of w and the dark energy density ρ_E with epoch can be obtained in a homogeneous model.

After outlining the model, we display our results for w as a function of the density of the neutral fermions, and then discuss how to choose the parameters of the model to obtain results for w as a function of z, and discuss the implications of the results.

THE NEUTRAL FERMION - SCALAR FIELD MODEL

Following ref. [3], the equation of motion for a neutral fermion field, ψ, interacting with a scalar field, ϕ, is

$$\left[\partial^2 + m_s^2\right]\phi = g\overline{\psi}\psi \tag{1}$$

$$\left[i\slashed{\partial} - m_n^{(0)}\right]\psi = -g\phi\psi, \tag{2}$$

with $\hbar = c = 1$. The nonlinear scalar selfcouplings are omitted here, even though they are required to exist by field theoretic selfconsistency [8], as they may consistently be assumed to be sufficiently weak as to be totally irrelevant. The parameter $m_n^{(0)}$ is the renormalized vacuum mass of the isolated neutral fermion, and takes into account the contributions from the electroweak theory, as well as contributions from the vacuum expectation value of the new scalar field ϕ.

We look for solutions of these equations in infinite matter which are static and translationally invariant. In the Freidman equation description of the evolution of the universe we use the adiabatic approximation, assuming that the scalar field expectation value takes the value determined by the fermion density at the appropriate time[1]. Equations (1, 2) then lead to an effective mass for the neutral fermion of

$$m_n^* = m_n^{(0)} - \frac{g^2}{m_s^2}\overline{\psi}\psi. \tag{3}$$

These equations are simply the equations of Quantum Hadrodynamics [12].

As in [3], we act with these operator equations on state which is a filled Fermi sea of neutral fermions, with a fermion number density ρ_n and Fermi momentum k_F, related, for Majorana particles (which we assume henceforth), by $\rho_n = k_F^3/(3\pi^2)$. We introduce the parameter $K_0 = \left(g^2(m_n^{(0)})^2\right)/\left(\pi^2 m_s^2\right)$, and the variables $y = m_n^*/m_n^{(0)}$,

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[1] We have studied the corrections to the adiabatic approximation in the fermion-scalar field model and found them to be small, but have not yet investigated the corrections to this approximation when that model is coupled to the Freidman equation.

$x = k/m_n^{(0)}$, $x_F = k_F/m_n^{(0)}$, and $e_F = \sqrt{x_F^2 + y^2}$. Equation (3) becomes an equation for y:

$$y = 1 - \frac{yK_0}{2}\left[e_F x_F - y^2 \ln\left(\frac{e_F + x_F}{y}\right)\right],\tag{4}$$

The total energy of the system, $E = em_n^{(0)}N$, with N the number of fermions, and

$$e = \frac{3}{4}\left[e_F + \frac{1}{K_0 x_F^3}(2 - y)(1 - y)\right].\tag{5}$$

As x_F goes from 0 to ∞, y varies monotonically from 1 to 0, behaving as $y \approx 1 - (K_0 x_F^3)/3$ for small x_F, and $y \approx 2/(K_0 x_F^2)$ for large x_F.

For the fermion system to be bound, the minimum of e must be less than 1. We find that for sufficiently large K_0 the neutral fermion – scalar field system is bound [3], and the total energy displays the characteristic behaviour of a self-bound system noted above, suggesting that we should see negative pressures in this system. We now go on to compute the equation of state parameter.

DETERMINING w

The Equation of State, or the equation for the pressure, P, as a function of the other variables of the system, is represented through the equation of state parameter, w, which is defined by the proportionality between P and ρ_E, the energy density

$$P = w\rho_E.\tag{6}$$

As is well known, for radiation, $w = \frac{1}{3}$, for cold non-interacting matter, $w = 0$, and, for a Cosmological constant, $w = -1$. Any $w < 0$ describes a system with negative pressure.

Using the equations of the Friedman-Lemaître-Robertson-Walker (FLRW) metric, with a scale parameter a, the equation of state parameter is shown to satisfy

$$1 + w = -\frac{1}{3}\frac{\partial \ln \rho_E}{\partial \ln a}.\tag{7}$$

In these equations ρ_E is the *total* energy density. Assuming for this paper that the only source of this energy is the coupled fermion-scalar system, then

$$\rho_E = em_n^{(0)}\rho_n, \qquad \rho_n \propto x_F^3 \quad \text{and} \quad x_F \propto a^{-1},\tag{8}$$

and we obtain

$$w = \frac{1}{3}\frac{\partial \ln(e)}{\partial \ln(x_F)} = \frac{1}{3}\frac{x_F}{e}\frac{\partial e}{\partial x_F}\tag{9}$$

This is the basic equation for computing w in our model, and it gives

$$w = \frac{1}{3}\frac{e_F K_0 x_F^3 - 3(2 - y)(1 - y)}{e_F K_0 x_F^3 + (2 - y)(1 - y)}\tag{10}$$
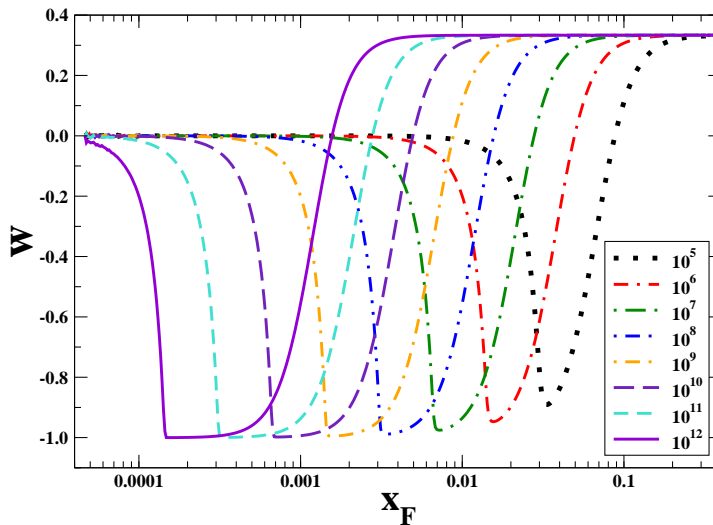
FIGURE 1. w vs. $\log(x_F)$ for 8 values of K_0.

It follows immediately that $w > -1$ in our model.

From equations (4,10), we can compute w as a function of x_F, given K_0. Figure 1 shows some results using a log scale for x_F.

Note that w approaches close to -1 as the density decreases (remember the direction of decreasing density is the direction of expansion of the universe) and then departs sharply towards zero, At large x_F, it is clear that w approaches $+1/3$ as it should for a relativistic gas of fermions. The value goes to zero at zero density.

For large values of K_0 the minimum value of w is very close to -1 and is reached near the value $x_F = x_{F,1} = (3/K_0)^{1/3}$, which can be obtained from an analytic approximation. More precisely, and empirically $x_{F,m} \approx (3.82/K_0)^{1/3}$.

TABLE 1.   The minimum value of w, the point $x_{F,m}$ at which this is achieved, and the approximate location $x_{F,1}$

$\log K_0$	$\log x_{F,m}$	$\log x_{F,1}$	w
6	-1.809	-1.841	-0.94709
9	-2.806	-2.841	-0.99453
12	-3.806	-3.841	-0.99945

The minimum value of w approaches -1 more closely as K_0 increases, and the low density recovery of w to zero becomes steeper as K_0 increases.

It is clear from these results that we have a model in which the value of w is a function of the scale parameter a or the red-shift z, i.e. that we have an epoch dependent dark energy. But these results are in terms of dimensionless parameters and variables. To connect to the real world we must link these dimensionless numbers to the dimensionful

numbers which characterize it. As was shown in ref. ([3]), there are only a few, weak constraints on the actual parameter values. Moreover, very large values of K_0 are possible even for very small values of g^2 if the range of the scalar is very large, corresponding to very small values of m_s. Even if long-ranged, such weak interactions between fermions, especially neutrinos or those outside of the Standard Model altogether (such as the LSP) are exceptionally difficult to constrain by any laboratory experiments.

CONNECTING WITH DARK ENERGY, AND FIXING PARAMETERS

The energy density of dark energy is quoted as $\rho_{DE} = (3.20 \pm 0.4) \times 10^{-47} \text{Gev}^4$ [6] If we define $\rho_{DE} = m_E^4$, then $m_E = 2.4$ meV. It is important to remember that the conventional value of the dark energy density is derived on the assumption that it is constant during the evolution of the universe, and this is not the case in our model. Pending a study of the development of the universe using our model we use the present estimate of ρ_E and apply it to the region $w \approx -1$ to estimate the relevant parameters in our model.

$$\rho_E = m_n^{(0)} e \rho_n = \frac{(m_n^{(0)})^4 e x_F^3}{3\pi^2} \tag{11}$$

To use this equation to estimate $m_n^{(0)}$, first note that near $w \approx -1$, for large K_0, x_F is small and we can use the appropriate approximations to get

$$\rho_E \approx \frac{(m_n^{(0)})^4 x_F^3}{6\pi^2} \approx \frac{(m_n^{(0)})^4}{2\pi^2 K_0}, \tag{12}$$

where, in the last equation, we use $x_F \approx x_{F,1}$ at the minimum. K_0 determines the relationship between $m_n^{(0)}$ and m_E, $m_n^{(0)} = \left[2\pi^2 K_0\right]^{1/4} m_E$. We choose K_0, taking care that the implied values of g^2 and m_s are reasonable. For $K_0 = 10^6$, $m_n^{(0)} = 150$ meV, in the neutrino range, and for $K_0 = 10^{54}$, $m_n^{(0)} = 150$ GeV, in the range expected for neutralinos. What are the implications for the other parameters? One can immediately estimate the density of the neutral fermions as 47×10^3 cm^{-3} and 3.5×10^{-8} cm^{-3} for these two cases.

Because we are assuming that the neutral fermions form a homogeneous background, it is appropriate to set the range of the scalar field to be the scale parameter of the universe at the relevant time, which we will take to be $z = 1$, i.e. $m_s \sim \left(7 \times 10^9 \text{ lightyears}\right)^{-1} \sim 3 \times 10^{-30}$ meV. With this value of m_s we can obtain the implied value of $g^2/(4\pi)$: 3×10^{-58} and 3×10^{-54} in these two cases. Even the largest coupling is far too weak to be constrained by terrestrial experiments.

An alternative way to proceed would be to assume a present density of the neutral fermions, ρ_0, and thus a present value of the Fermi momentum, k_{F0}, at a scale parameter a_0, use $ak_F = a_0 k_{F0}$ and $a_0 = (1+z)a$. Equation (11) then gives

$$m_n^{(0)} = \frac{2}{(1+z)^3} \frac{\rho_E}{\rho_0} \tag{13}$$

To give an explicit example, assume that the present density of neutral fermions is 100 cm^{-3}, characteristic of neutrinos, and extrapolate this to a density of 800 cm^{-3} at $z = 1$. This gives a value $m_n^{(0)} = 9.2\text{eV}$, and $x_F = 3 \times 10^{-5}$, $K_0 = 9.7 \times 10^4$, and the even smaller coupling constant, $g^2/(4\pi) = 9 \times 10^{-64}$.

At this value of K_0, the minimum value of w is -0.8, at the 1.5σ level from the value -1.05 ± 0.18 of the ESSENCE supernova survey[13], so even these parameters are not excluded.

It is impossible to make further progress without finding additional ways to constrain the parameters. Given their extreme values the most promising approach is to use additional theoretical input, which we leave as a challenge for future work, by us and others.

RESULTS — z DEPENDENCE OF w

Now that we have some parameters we can convert the determination of w as a function of the dimensionless parameter x_F to a dependence of w on the red shift z. To illustrate the results we have selected the values 10^6 and 10^9 for K_0, and set the minimum of w to occur at $z = 1$.
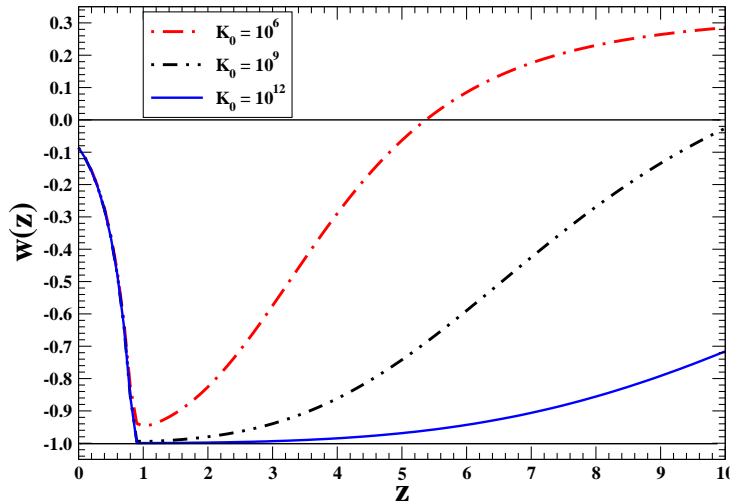
These results are illustrated in figure 2.

FIGURE 2. w vs. z for $K_0 = 10^6$, $K_0 = 10^8$ and $K_0 = 10^{12}$.

Note that there is a slow variation of w with z at epochs earlier than z_{min} at which the minimum occurs, but a rapid change for z between the present epoch ($z = 0$) and that of the minimum.

With the present uncertainty in data we can choose to place the value of z_{min} at any value of z between, say $z = 0.5$ and $z = 1.2$ without being in conflict with the present

data, and have chosen to show the results at $z_{min} = 1$ as illustrative results.

CONCLUSIONS

In an ideal future world one could imagine that $w(z)$ is indeed observed to have a minimum value at some z_{min}, and the the value of w_{min}, close to but greater than -1 is accurately known. The fact that the minimum value of w is near -1 shows that $K_0 > 10^{4.5}$, and the actual value of w_{min}, if known accurately enough, would determine the value of K_0. Knowing the value of K_0 allows us to predict a value of ρ_E at the minimum w, which provides a good test on the model. Then a selection of parameters can place that minimum near an appropriate z value, z_{min}.

Our prediction of a characteristic z dependence of w and ρ_E leaves us vulnerable to developments in the precision of measurements and the extension of observations to larger and (with more difficultly) to smaller z, and we await with interest the results of the proposed experiments, such as those described in ref. [6].

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