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A New Model for Corrosion-induced Concrete Cracking

X. Xi and S. Yang
Department of Civil and Environmental Engineering, University of Strathclyde, Glasgow, UK

C. Li
School of Engineering, RMIT University, Melbourne, Australia

ABSTRACT

Corrosion of reinforced concrete is one of the major deterioration mechanisms which result in premature failure of the reinforced concrete structures. Due to the actual diffusion of chloride ingress, the corrosion products distribution is seldom uniform along the reinforcing bar. Recently, some non-uniform corrosion models have been proposed to investigate the corrosion-induced cracking mechanism of concrete. In this paper, a new corrosion model based on von Mises distribution is formulated and validated against experimental data. The developed model is then compared with the existing non-uniform models and the advantages are discussed. To demonstrate the application of the developed corrosion model, a concrete cover structure, containing aggregates, cement paste/mortar and ITZ, is simulated to predict the cracking phenomena of the concrete cover under different non-uniform coefficients in the developed corrosion model. It has been found that the non-uniform corrosion model can be used to simulate the realistic corrosion rust progression around the reinforcing bar, with the best accuracy. Moreover, parametric studies are conducted to investigate the effects of the basic factors formulated in the corrosion model on the surface cracking of the reinforced concrete structures.

Keywords: non-uniform corrosion, meso scale model, cohesive crack model, mixed mode fracture

1.0 INTRODUCTION

Reinforced concrete (RC) structures are widely used for civil structures and infrastructures, e.g., buildings, bridges, retaining walls and tunnels. Cracks induced by corrosion destroy the integrity of concrete cover, reduce the reliability of concrete and lead to premature failure of RC structures and infrastructure. Chlorides, as well as moisture and oxygen, diffuse into concrete and reach a threshold value at the surface of steel bar, before the passive layer on steel surface is destroyed and corrosion is initiated. However, it is rare to have a uniform corrosion around the reinforcing bar, due to different amount of chlorides, moisture and oxygen that are available on different sides of the reinforcement; for example, the side of a reinforcing bar facing concrete cover should have more sources to advance corrosion and hence more corrosion products accumulated on this side.

Recently, many researchers have started to model the cracking of concrete cover induced by non-uniform corrosion of reinforcement. Jang and Oh (2010) extended the experimental results in (1995) and designed a factor for the ratio of the maximum thickness of non-uniform corrosion layer to the thickness of uniform corrosion layer to express the non-uniform corrosion. Moreover, some researchers postulated corrosion products followed a linear decrease distribution along the circle of steel bar (Šavija *et al.*, 2013). Pan and Lu (2012) proposed a non-linear corrosion model with a quadratic expansion function to model the cracking of concrete induced by non-uniform corrosion. Further, Yuan and Ji (2009) conducted corrosion tests on RC samples by using artificial environmental chamber and found the corrosion products distribution around the reinforcement is in a semi-elliptical shape. Based on the semi-elliptical assumption, Yang *et al.* (2017) proposed an analytical model to calculate the time to cracking of concrete cover and Xi and Yang (2017) developed a numerical model to investigate cover cracking caused by corrosion of multiple reinforcing bars. In addition, Zhao *et al.* (2011a; 2011b; 2012; 2016) carried out corrosion tests on RC samples and proposed a Gaussian non-uniform corrosion model to quantitatively define the corrosion products distribution. Tran *et al.* (2011) and Qiao *et al.* (2016) proposed a non-uniform corrosion model by considering uniform corrosion for part of rebar and no corrosion for the other part, according to their results of experiments.

Once corrosion boundary model is established, the cracking of concrete caused by corrosion of reinforcement can be simulated. However, most studies considered concrete as a homogeneous material. The homogeneity assumption is only an approximation and, for more accurate prediction, concrete should be treated as a three-phase heterogeneous material at the mesoscale, consisting of cement paste/mortar, aggregate and interfacial transition zone (ITZ).

This paper attempts to develop a new rational model for concrete cover cracking induced by non-uniform corrosion of reinforcement. A novel non-uniform corrosion model is first formulated based on von Mises distribution and validated against test data. The corrosion model is then compared with existing non-uniform corrosion models in literatures, in terms of accuracy of fitting experimental data, and the merit of the developed corrosion model is discussed. Three parameters are modeled in the rust distribution function, i.e., the linear relationship to corrosion degree λ, the non-uniform coefficient k and the location of the maximum thickness of the corrosion rust layer μ. Under the expansion caused by corrosion of reinforcement, a heterogeneous discrete crack models is built to simulate the concrete cover cracking. The initial micro-cracking and the subsequent dominating discrete crack propagation, and the surface crack width development under various non-uniform corrosion coefficients are obtained.

2.0 A NEW CORROSION MODEL

Recently, some non-uniform corrosion models have been proposed to express the non-uniform corrosion products distribution. The thickness of corrosion rust layer $T_{cl}(\theta)$ consists of two parts as shown in Fig. 1, i.e., the corroded rebar with thickness $T_{co-st}(\theta)$ and the expansion displacement beyond the original rebar with thickness $T_d(\theta)$. Thus, $T_{cl}(\theta)$ can be expressed as follows:

$$T_{cl}(\theta) = T_{co-st}(\theta) + T_d(\theta) \tag{1}$$

For clarification, it should be mentioned that some of the existing non-uniform corrosion models are built based on the corrosion rust thickness distribution (i.e., $T_{cl}(\theta)$), e.g., (Qiao *et al.*, 2016; Tran *et al.*, 2011; Zhao *et al.*, 2011a), some based on the corroded steel bar (i.e., $T_{co-st}(\theta)$), e.g., (Jang and Oh, 2010; Yuan and Ji, 2009) and the others on the expansion displacement beyond the original bar (i.e., $T_d(\theta)$), e.g., (Pan and Lu, 2012).

According to the ratio α of rust expansion to corroded rebar, it can be approximately obtained as follows :

$$T_{cl}(\theta) = \alpha \times T_{co-st}(\theta) \tag{2}$$

By substituting Equation (2) into Equation (1), $T_d(\theta)$ can be derived as follows:

$$T_d(\theta) = (\alpha - 1) \times T_{co-st}(\theta) \tag{3}$$

By substituting Equation (3) into Equation (2), $T_{cl}(\theta)$ can be obtained as follows:

$$T_{cl}(\theta) = \frac{\alpha}{\alpha - 1} \times T_d(\theta) \tag{4}$$

By Equations (2-4), the non-uniform corrosion models can be linked up.

Von Mises distribution is a continuous probability distribution on a circle, which is the circular analogue of the normal distribution (Forbes *et al.*, 2010). Von Mises distribution was first used to study deviations of atomic weights from integer values and has become an important function in the statistical theory.

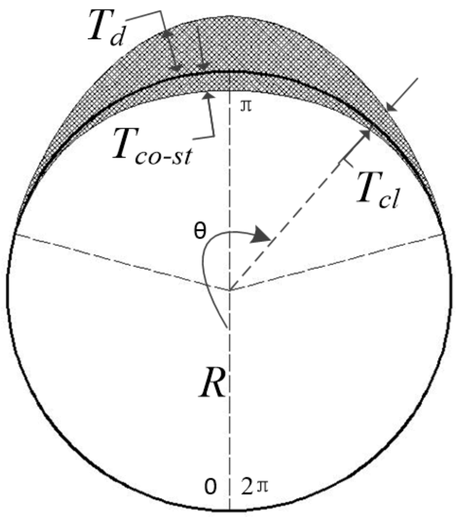

Fig. 1. Non-uniform corrosion model

By comparing all existing corrosion distribution data from experiments, we have found the von Mises distribution could be ideal to express the shape of corrosion progression. The corrosion rust layer thickness $T_{cl}(\theta)$ can be formulated as follows:

$$T_{cl}(\theta) = \lambda \frac{e^{k\cos(\theta-\mu)}}{2\pi I_0(k)} \tag{5}$$

where λ is a fitting parameter, μ is the location where the maximum thickness of corrosion layer appears, $I_0(k)$ is the modified Bessel function of order 0 and *k* is the concentration coefficient to define the level of non-uniform. The parameters will be discussed later in details.

The total amount of corrosion products *W* can be expressed as follows:

$$W = W_m + W_s \tag{6}$$

where is W_s the amount of rust replacing the corroded steel with thickness $T_{co-st}(\theta)$ and W_m is the amount of rust expansion from the circumference of the origin rebar with thickness $T_d(\theta)$. W_m can be obtained by an integration based on the radius of rebar:

$$W_m = \frac{1}{2}\int_0^{2\pi}(T_d(\theta)+R)^2 d\theta - \pi R^2 \quad (7)$$

Or,

$$W_m = \frac{1}{2}\int_0^{2\pi}(T_d(\theta)^2 + 2T_d R)d\theta \quad (8)$$

By neglecting the second order of small quantity, i.e., $T_d(\theta)^2$, W_m can be derived as follows:

$$W_m = \int_0^{2\pi} T_d(\theta) R d\theta \quad (9)$$

According to Equation (4), W_m can be rewritten as follows:

$$W_m = \int_0^{2\pi} \frac{\alpha-1}{\alpha} T_{cl}(\theta) R d\theta \quad (10)$$

By substituting Equations (5) into Equation (10), it becomes:

$$W_m = \int_0^{2\pi} \frac{\alpha-1}{\alpha} \lambda \frac{e^{k\cos(\theta-\mu)}}{2\pi I_0(k)} R d\theta \quad (11)$$

Further, it can be derived that:

$$W_m = \frac{\alpha-1}{\alpha} R\lambda \int_0^{2\pi} \frac{e^{k\cos(\theta-\mu)}}{2\pi I_0(k)} d\theta \quad (12)$$

The integral part becomes the cumulative function of von Mises distribution in a whole circle and its value is 1. W_m can be expressed as follows:

$$W_m = \frac{\alpha-1}{\alpha} R\lambda \quad (13)$$

The total amount of corrosion products W can be derived as follows:

$$W = \alpha W_s \quad (14)$$

By substituting Equations (13) into Equation (6), W can also be expressed as follows:

$$W = \frac{\alpha}{\alpha-1} W_m \quad (15)$$

According to Equations (13) and (15), the total amount of corrosion products W can be expressed as follows:

$$W = R\lambda \quad (16)$$

It can be postulated that λ reflects directly the amount of corrosion products. It should be noted that the formula is accurate when the radius loss of corroded rebar is relatively small compared with the radius of origin rebar, which is the case in most engineering practice. It has been reported a very small corrosion degree, i.e., 12 μm radius loss of rebar, can cause a visible crack at surface of concrete cover. Therefore, such a hypothesis of the von Mises model is well justified.

According to Equations (5) and (16), the corrosion layer thickness $T_{cl}(\theta)$ can be derived as follows:

$$T_{cl}(\theta) = \frac{W}{R} \frac{e^{k\cos(\theta-\mu)}}{2\pi I_0(k)} \quad (17)$$

The corrosion degree of reinforcement η can be expressed as follows:

$$\eta = \frac{W_s}{W_0} \times 100\% \quad (18)$$

where the W_0 is the original amount of the reinforcing steel.

According to Equations (14) and (17-18), the corrosion layer thickness $T_{cl}(\theta)$ can be written as a function of corrosion degree:

$$T_{cl}(\theta) = \alpha\pi R \eta \frac{e^{k\cos(\theta-\mu)}}{2\pi I_0(k)} \quad (19)$$

Therefore, the parameter λ has a linear relationship with corrosion degree η, and λ can be defined as the corrosion degree indicator.

3.0 VALIDATION AND PARAMETERS

To verify the proposed von Mises corrosion distribution model, test data on non-uniform corrosion development are searched and collected in a comprehensive manner. Fig. 2 shows the regression analysis of the proposed von Mises model with the test data (Qiao *et al.*, 2016; Yuan and Ji, 2009; Zhao *et al.*, 2011a). It should be mentioned that, the results from Qiao *et al.* (Qiao *et al.*, 2016) used corroded rebar thickness $T_{co-st}(\theta)$, while the data in other literature studies were based on the thickness of corrosion products layer $T_{cl}(\theta)$. The fitting parameters and R^2 (coefficient of determination) values for the test data are shown in Table1. It can be found that most values of R^2 are larger than 0.9. The R^2 of data 8 is 0.768, which is caused by significant crack width (i.e., 0.4mm). It can be proved that, prior to occurrence of any significant crack, the von Mises model can predict the non-uniform corrosion progression very well.

The average R^2 against the six groups of test data (i.e., $T_{cl}(\theta)$) for the existing non-uniform models (i.e., elliptical model (Yuan and Ji, 2009), linear decrease model (Jang and Oh, 2010), quadratic expansion model (Pan and Lu, 2012), Gaussian model (Zhao *et al.*, 2011a)) and the von Mises model are compared and shown in Fig. 3. It can be seen that the von Mises model has the best accuracy in fitting with the experimental data. In addition, the von Mises model has less number of parameters in formulating the non-uniform corrosion rust progression and these parameters also have direct physical meanings.

Table 1. Values of basic parameters formulated in the developed von Mises corrosion model

Test data	λ	*k*	*μ*	R^2
1	0.321	3.122	3.181	0.991
2	0.941	0.555	3.188	0.934
3	0.198	1.832	3.325	0.931
4	0.349	3.285	3.173	0.901
5	0.613	3.362	3.348	0.817
6	0.353	3.232	3.182	0.906
7	0.799	0.923	2.963	0.953
8	1.550	0.689	3.540	0.768

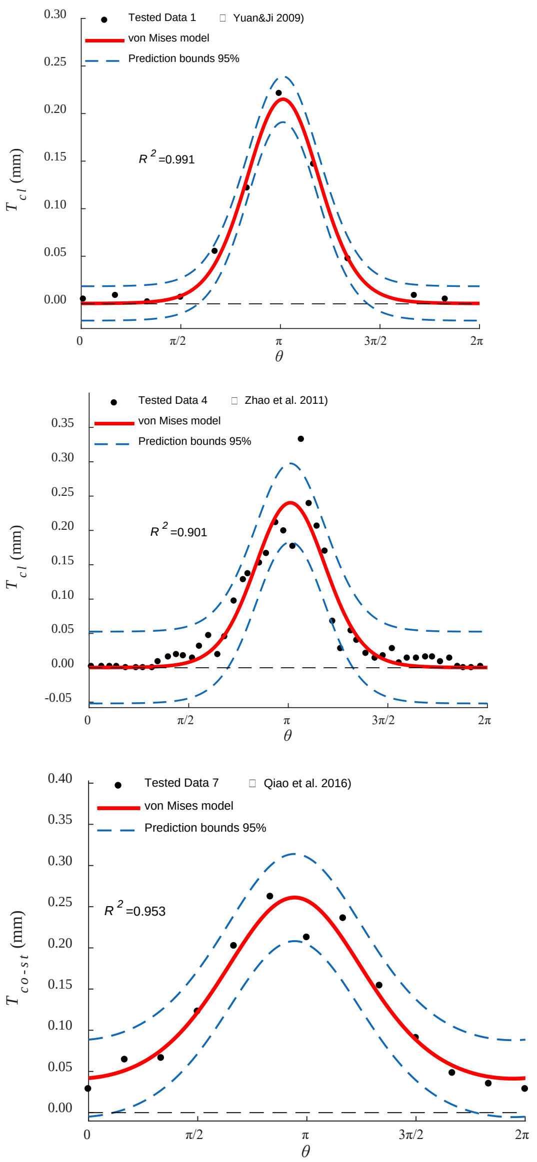

Fig. 2. Regression analysis of the developed von Mises model with test data

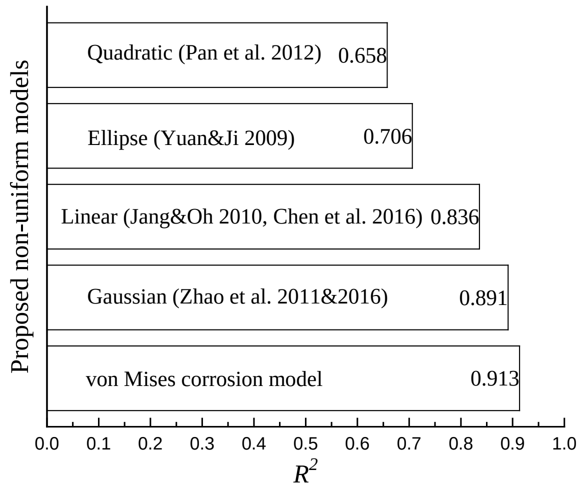

Fig. 3. Comparison of average value of R^2 for various corrosion models

As introduced in the experiments (Qiao *et al.*, 2016; Yuan and Ji, 2009; Zhao *et al.*, 2011a), the test data 4 and 6 were obtained from corrosion of corner rebar while the others were from corrosion of middle rebar. The polar coordinate system and the location of maximum thickness of corrosion layer in the experiments (Qiao *et al.*, 2016; Yuan and Ji, 2009; Zhao *et al.*, 2011a) are shown in Fig. 4. From fitting results by von Mises model, the value of μ is near π, which means the von Mises model can describe the location of maximum thickness of corrosion layer well. As a corrosion model used to analyse cracking of concrete cover induced by non-uniform corrosion of reinforcement, the parameter μ can be set as π in the polar coordinate system.

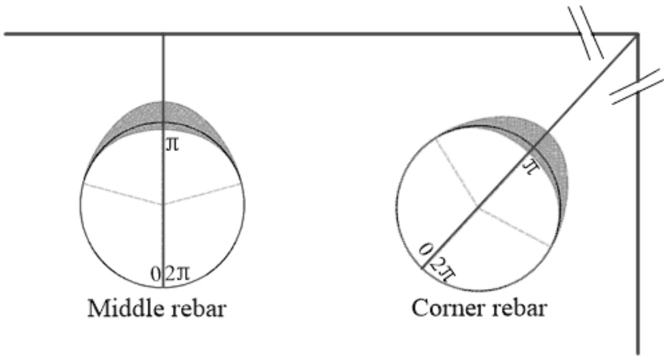

Fig. 4. Polar coordinate system defined in experiments (Qiao *et al.*, 2016; Yuan and Ji, 2009; Zhao *et al.*, 2011a) and the developed model

In the von Mises distribution *k* is the concentration coefficient and the smaller the *k* is, the distribution will be more close to uniform. From the fitting results with the test data, the value of *k* varies from 0.555 to 3.362. To investigate the effect of *k* on corrosion layer distribution in the von Mises corrosion model, the corrosion layer distribution under four values of k are produced and plotted in Fig. 5. It should be noted that the right figures in Fig. 5 show the front of the corrosion layer based on a circle representing the original rebar, not the expansion displacements of corrosion products. When *k* is zero, the corrosion layer thickness $T_{cl}(\theta)$ becomes a constant value and the von Mises corrosion model describes uniform corrosion. Therefore, *k*=0 is a special case which represents uniform corrosion while all other values of k (must be positive) define non-uniform corrosion.

In real corrosion propagation process, the non-uniform coefficient *k* may not be a constant but varying along time. There is very limited experimental and observational data available which cannot warrant a thorough understanding and elaboration about *k*. However, in this study, it has been found that for most existing data larger value of *λ* leads to smaller *k*. Moreover, it is considered that when corrosion progresses, the non-uniform coefficient will be smaller and smaller, i.e. the corrosion will become more close to uniform. Nonetheless, it has not been proved yet and hard to predict the varying value of *k* under different corrosion degrees. The value of *k* may be related to chloride content, geometry of RC structures, corrosion degree and surface crack of concrete. More researches are necessary, especially experimental results, to clarify the effects of different underlying parameters on *k* and then formulate an analytical function for *k.*

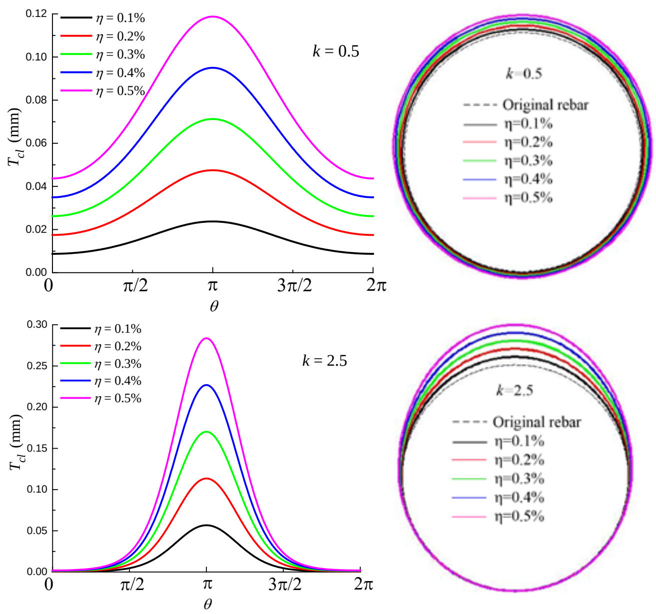

Fig. 5. Corrosion rust distributions based on the von Mises model under different values of *k* and corrosion degree (Note: the deformation scale for the right pictures is 10)

When the corrosion process starts, corrosion rusts first fill in the annular porous layer in concrete around the reinforcing bar, often referred to as "porous zone". This initial stage normally does not produce stresses in concrete. Taking into account the "porous zone" with thickness T_0 , the displacement boundary condition $T_d(\theta)$ can be modified as follows:

$$T_d(\theta)=\left\langle (\alpha-1)\pi R\eta\times\frac{e^{k\cos(\theta-\mu)}}{2\pi I_0(k)}-T_0\right\rangle \quad (20)$$

Where < > is the Macaulay bracket which means the displacement expansion boundary $T_d(\theta)$ will be regarded as zero if the value is less than zero.

4.0 MESO-SCALE MODELING OF CORROSION-INDUCED CONCRETE CRACKING

In this paper, concrete is modelled as a three-phase (i.e., consisting of mortar, aggregates and ITZ) material. The shape of aggregate is simplified to a random polygon with 3-7 sides. The aggregate size distribution can be represented by a grading curve, which is usually expressed in terms of cumulative percentage passing through a series of sieves with different opening sizes. For simplicity, only coarse aggregates larger than 2.4 mm are modelled in this study, while fine aggregates and cement are treated as mortar phase. Coarse aggregates generally occupy 40% of the whole volume of concrete.

Fig. 6 shows the mesh of the meso-scale RC cover structure with middle and corner rebars. The size of the RC structure is set 150×150 mm and the thickness of concrete cover is 40 mm. Two types of elements are employed in this study, i.e., a 4-node cohesive element for interfaces between the triangle elements and a 3-node plane strain element for the bulk mortar and aggregates. To model arbitrary cracking in concrete, the cohesive elements are embedded at the interfaces throughout the mesh; very fine mesh is produced to ensure random crack paths. Due to the lack of experimental data, the shear strength and Mode-II fracture energy were assumed to be the same as these of Mode-I (Ren *et al.*, 2015). The tensile strength of ITZ is regarded as 0.5 time of that of mortar in this paper. Due to the failure of ITZ is more brittle than mortar, the fracture energy of ITZ is assumed as 0.25 time of that of mortar. The values for all the basic numerical parameters are shown in Table 2. The displacement boundary condition is determined by Equation (20).

Crack width is an important parameter with regards to the durability of concrete structures. Upon measuring the distances between the nodes of specific cohesive elements of the surface cracks, the crack width evolution of the surface cracks as a function of corrosion degree for different non-uniform coefficients are shown in Fig. 8. It can be seen that the surface crack suddenly increases to about 0.03mm; after that, the surface crack width grows up almost linearly. A very small corrosion degree (less than 0.25%) can cause concrete cover cracking. Moreover, it has been found that the larger the non-uniform coefficient is, the smaller the corrosion degree at surface cracking is.

Table 2. Values for geometric and mechanical parameters for different phases

Description	Symbol	Values
Cover thickness	C	40 mm
Radius of steel bars	R	8 mm
Thickness of "porous zone"	T_0	12.5 μm
Expansion ratio of rust	α	3.83
Corrosion degree	η	0.26%
Young's modulus of aggregate	E_{Agg}	70 GPa
Young's modulus of mortar	E_{Mor}	25 GPa
Poisson's ratio of aggregate	υ_{Agg}	0.2
Poisson's ratio of mortar	υ_{Mor}	0.2
Tensile strength of mortar	$\sigma_{n,Mor}$	6 MPa
Tensile strength of ITZ	$\sigma_{n,ITZ}$	3 MPa
Fracture energy of mortar	$G_{I,Mor}$	60 N/m
Fracture energy of ITZ	$G_{II,ITZ}$	15 N/m

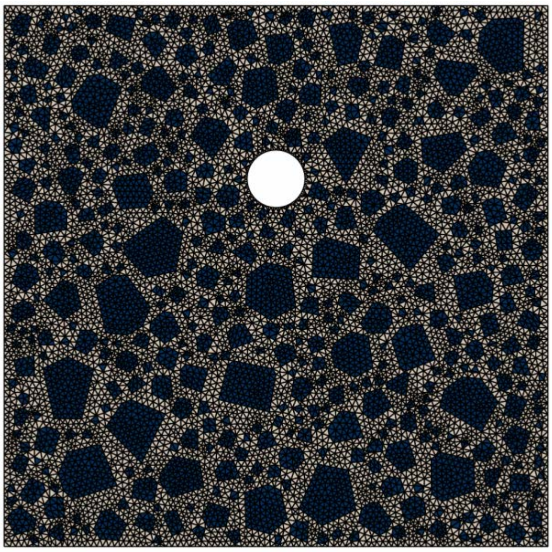

Fig. 6. Mesh for the worked example

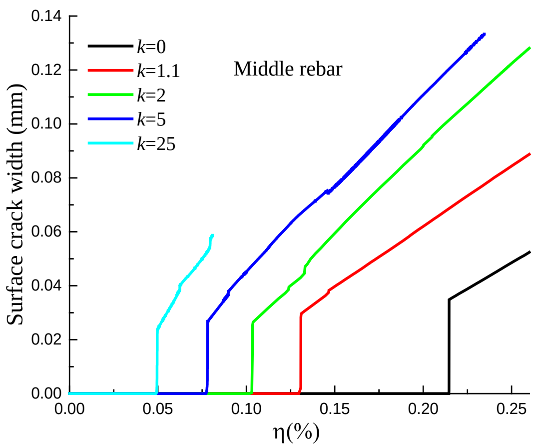

Fig. 7. Surface crack width induced by corrosion of middle rebar as a function of corrosion degree η under different non-uniform corrosion coefficient *k*

Although the final crack width for *k*=25 and *k*=0 is small, it can be predicted that more uniform corrosion can lead to smaller crack width under given corrosion degree. Most previous studies (Liu and Weyers, 1998) assumed uniform corrosion which overestimated the volume that is occupied by corrosion products and therefore underestimated the time to surface cracking and crack width development.

5.0 CONCLUSIONS

In this paper, a von Mises corrosion model was derived to formulate the corrosion expansion around the reinforcing bar. The model was simply composed of three parameters including the corrosion degree indicator λ, the non-uniform coefficient k and the location of the maximum thickness of the corrosion layer μ. The results predicted by the developed model have been compared with experimental results and a good agreement has been achieved. Compared with the existing models, the von Mises model has the best accuracy in fitting with the experimental data and fewer parameters with direct physical meanings. Moreover, the thickness of the "porous zone" between reinforcement and concrete has also been taken into account. A meso-scale model consisting of aggregates, mortar and ITZ was built for the cases of the middle rebar. The concrete cover cracking induced by corrosion of the middle and corner bars with five different non-uniform coefficients were simulated. Moreover, the crack width developments as a function of corrosion degree were obtained. It has been found that the larger the non-uniform coefficient is, the smaller the corrosion degree at surface cracking is.

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