

Numerical Simulation of the Biological Control of the Chestnut Gall Wasp with *T. Sinensis*

Carlos Balsa¹, Margaux Citerici², Isabel Lopes^{3,4} ✉, and José Rufino¹

¹ Research Centre in Digitalization and Intelligent Robotics (CeDRI), Instituto Politécnico de Bragança, Campus de Santa Apolónia, 5300-253 Bragança, Portugal
`{balsa,rufino}@ipb.pt`

² Université de Toulouse - Institut National Polytechnique de Toulouse, France
`margaux.citerici@etu.enseeiht.fr`

³ Applied Management Research Unit (UNIAG), Instituto Politécnico de Bragança, Campus de Santa Apolónia, 5300-253 Bragança, Portugal

⁴ Centro ALGORITMI, Escola de Engenharia - Universidade do Minho, Campus Azurém, 4800-058 Guimarães, Portugal
`isalopes@ipb.pt`

Abstract. Portugal is a country that produces chestnuts. However, in recent years, a *D. kuriphilus* plague has invaded chestnut trees and significantly affected the chestnut production. Studies in other countries, such as Japan or Italy, have shown that the *T. sinensis* parasitoid can achieve biological control and help to eradicate the invader. In this work, the evolution of the density of *D. kuriphilus* and *T. sinensis* eggs across time and space is studied through the numerical solution of mathematical models previously proposed. It is concluded that the biological control with *T. sinensis* operates and thus eradicates *D. kuriphilus* from the infected area. However, the simulations also show that biological control is not effective over time, as *D. kuriphilus* returns to the same area. Therefore, it would be necessary to reinject *T. sinensis* periodically into the infected zones for a sustained fight against *D. kuriphilus*.

Keywords: Biological control · Mathematical model · Numerical simulations · Chestnut · *Dryocosmus kuriphilus* · *Torymus sinensis*

1 Introduction

The chestnut gall wasp *Dryocosmus kuriphilus* (*D. kuriphilus*) disrupts the growth of chestnut trees and limits fruit production. Indeed, this species lays its eggs in the buds of chestnuts in early summer. The larva spends all winter in this bud. In the following spring, this induces the formation of galls on the buds and on the leaves of the tree (Fig. 1). Therefore, this disrupter has a significant impact on the production of chestnut, with further negative economical effects.

Originally from China, the pest spread to other Asian countries, like Japan, where it was first detected in 1941. In the early 2000s, it arrived in the United States and Europe, first in Italy, then in France and finally in Portugal, in

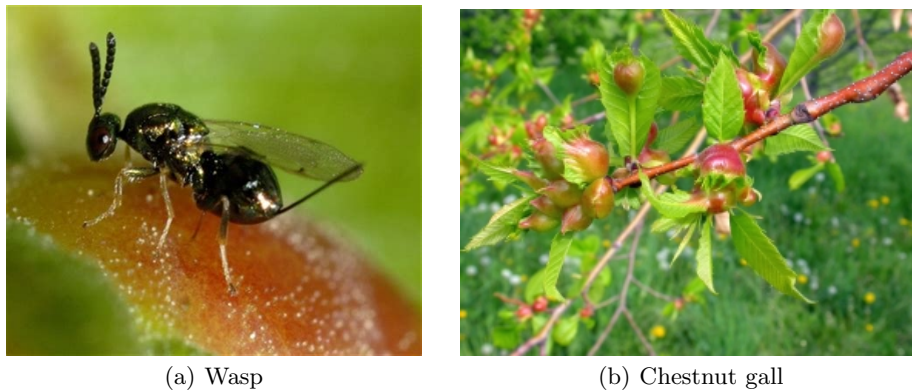


Fig. 1. Chestnut gall wasp *D. kuriphilus* (a) and buds infected with *D. kuriphilus* (b)

2014 [1]. Solving the problem is very important for the economy of this sector, specially for countries that are big producers, like Portugal, with an estimate of 25,000 to 30,000 tons of annual chestnut production.

Japan was the first country that tried to control the plague. Japanese researchers started by using pesticides, but this was found to be noneffective. Then they became interested in varieties of chestnut that could be resistant to *D. kuriphilus*. Finally, they made tests with several species of parasitoids and discovered that *Torymus sinensis* (*T. sinensis*), a parasite that also came from China, could be used to effectively control the plague: *T. sinensis* lays its eggs in the larvae of *D. kuriphilus*; this prevents them from developing and thus to later attack the chestnut tree and to further reproduce [7]. Besides Japan, experiments with the biological control conducted in France and Italy have confirmed that *D. kuriphilus* disappears entirely after a few years [9,5].

Researchers in Japan, Italy, and France began by studying this phenomenon in the laboratory and then on a specific area of the country, to be able to observe and study the different effects of *T. sinensis* on *D. kuriphilus*. These field studies often involve a considerable amount of time, human resources and logistics. In this regard, computational simulations provide an economic alternative to experimental testing or may be used to complement/validate experimental studies/results. The numerical solutions of the mathematical models representing the evolution of two interacting species allow to obtain important information that can be used to optimize the combat to plagues. In this context, carefully choosing the mathematical models to be adopted is of fundamental importance.

As far as the authors of this work know, the only mathematical models of the Biological Control of the Chestnut Gall Wasp by *T. sinensis* were developed by F. Paparella and co-authors [8]. The proposed models describe the dynamics of the wasp population in the presence of the parasite population. The interaction between the two species is a predator-prey type model based on the Lotka-Volterra equations (see, for instance, [4]). However, the interaction be-

tween the two species is not direct, but through eggs. The parasite lays its eggs inside the wasp larvae, thus causing its destruction. Therefore the mathematical models also contemplate the quantity of eggs of the two species.

The mathematical models proposed by F. Paparella and co-authors [8] are based on Ordinary Differential Equations (ODE) and Partial Differential Equations (PDE). In this work ODE and PDE are solved numerically by means of the Octave [2] open source software. Several simulations were carried out to analyze the evolutions of the two species over time and space. Finally, a study concerning the improvement of the biological control is also presented.

The paper is organized as follows. Section 2 introduces the mathematical models proposed by F. Paparella and co-authors [8] and presents the results of numerical simulations that enable the estimation of the density of the two species over the time. Section 3 is devoted to the mathematical models that include the spatial variation of the two densities over time. Section 4 presents the results of numerical simulations performed in order to evaluate the possibilities of improving the biological control. Section 5 wraps up the paper with some final considerations.

2 Density of the *D. kuriphilus* and *T. sinensis* Over Time

In this section, the mathematical models that take into account the separate evolution of *D. kuriphilus* and of *T. sinensis*, over time, are introduced, followed by the complete mathematical model for the joint evolution of the two species. These models are simplified because the spatial variation is not taken into account, *i.e.*, it is assumed that the two species do not change their territory (for details see [8]). This first approach enables to estimate the evolution of the density of the two species, and consequentially to know if the biological control works or does not.

2.1 Model for *D. kuriphilus*

Let U_n be the population of adult gall wasps carrying eggs during the summer of the year n . Let V_n be the density of eggs laid in the chestnut buds (this will allow to see the evolution of the population of *D. kuriphilus* over the years).

Considering the density β_{max} of chestnut buds and the maximum M of eggs to be laid per bud, then the maximum density of eggs laid is:

$$V_{max} = M\beta_{max} \quad (1)$$

It is also possible to predict the emergence rate of this species, its mortality rate, and its egg laying rate during one year. The expressions are next presented.

Let η be the survival rate during the overwintering; this will depend on the geolocation of the species (*e.g.*, it will not be the same in Japan and Portugal).

Let T_d be the length of the egg deposition season. The emergence rate is thus:

$$\text{emergence rate} = \frac{\eta V_{n-1}(T_d)}{T_d} \quad (2)$$

Let a be the adult life span. Therefore, the death rate is:

$$\text{death rate} = -\frac{U_n(t)}{a} \quad (3)$$

Finally, let r_d be the optimum condition where each *D. kuriphilus* lays all its eggs. This variable is worth N_d/a , with N_d being the maximum number of eggs that can be laid by an adult. The laying rate is proportional to the density of the laying rate of the location, which in the model is expressed as $M\beta_{max} - V_n(t)$:

$$\text{egg deposition rate} = r_d \frac{M\beta_{max} - V_n(t)}{V_{max}} U_n(t) \quad (4)$$

The combination of all these quantities results in the following formulation that describes the evolution of U_n and V_n during the season of the year n :

$$\begin{cases} \frac{\partial}{\partial t} U_n(t) = -\frac{1}{a} \frac{M\beta_{max} - V_n(t)}{V_{max}} U_n(t) - \frac{1}{a} U_n(t) + \eta \frac{V_{n-1}(T_d)}{T_d} \\ \frac{\partial}{\partial t} V_n(t) = \frac{N_d}{a} \frac{M\beta_{max} - V_n(t)}{V_{max}} U_n(t) \end{cases} \quad (5)$$

Non-dimensional variables are used in all the models of this work. For the model of this section such implies the following definitions: $u_n = U_n / (\eta V_{max})$, $v_n = V_n / V_{max}$. Moreover, the new t , μ and E_d are defined as $t = t/T_d$, $\mu = T_d/a$ and $E_d = \eta N_d$. Consequently, the new model to which the initial conditions are added is given by:

$$\begin{cases} \frac{\partial}{\partial t} u_n(t) = -\mu(2 - v_n(t))u_n(t) \\ \frac{\partial}{\partial t} v_n(t) = E_d\mu(1 - v_n(t))u_n(t) \\ u_n(0) = 0 \\ v_n(0) = 0 \end{cases} \quad (6)$$

2.2 Model for *T. sinensis*

To obtain the mathematical model for the evolution of *T. sinensis*, the same procedure that was previously used for the *D. kuriphilus* model is followed.

Initially, P_n is defined as the population of the egg-carrying *T. sinensis* females, of the year n , and Q_n as the density of eggs laid, of the same year (likewise, this allows to follow the evolution of the population of *T. sinensis* over the years).

Like before, it is possible to calculate the egg deposition rate. Let r_t be the optimum condition where each *T. sinensis* female lays all its eggs. This variable

evaluates to N_t/T_t , with N_t being the maximum number of eggs that can be laid by an adult female during its life span T_t . Thus, the egg laying rate is given by:

$$\text{egg deposition rate} = r_t \frac{\eta V_{n-1}(T_d) - Q_n(t)}{V_{max}} P_n(t) \quad (7)$$

Properly combined, these quantities support a formulation that describes the evolution of P_n and Q_n during the season of the year n :

$$\begin{cases} \frac{\partial}{\partial t} P_n(t) = -\frac{1}{T_t} \frac{\eta V_{n-1}(T_d) - Q_n(t)}{V_{max}} P_n(t) \\ \frac{\partial}{\partial t} Q_n(t) = r_t \frac{\eta V_{n-1}(T_d) - Q_n(t)}{V_{max}} P_n(t) \end{cases} \quad (8)$$

Inserting in the previous formulation the non-dimensional variables $p_n = P_n/(\eta\gamma V_{max})$ – with γ accounting for the sex ration of *T. sinensis* –, $q_n = Q_n/(\eta V_{max})$, $t = t/T_t$, $\tau = T_t/(\eta T_d)$, and adding the initial conditions, such results in the new model for *T. sinensis* described in Equation (9).

$$\begin{cases} \frac{\partial}{\partial t} p_n(t) = -\frac{1}{\tau} (v_{n-1}(1) - q_n(t)) p_n(t) \\ \frac{\partial}{\partial t} q_n(t) = \frac{E_t}{\tau} (v_n(1) - q_n(t)) p_n(t) \\ p_n(0) = q_{n-1}(\eta\tau) \\ q_n(0) = 0 \end{cases} \quad (9)$$

2.3 Complete Model

The complete mathematical model enables to describe the evolution of *D. kuriphilus* when *T. sinensis* is injected on the same territory. It allows measuring the impact of *T. sinensis* on *D. kuriphilus* and thus to see if it is possible to limit the evolution of *D. kuriphilus* thanks to biological control.

To have a complete model, it is necessary to introduce the effect of *T. sinensis* on *D. kuriphilus*. This effect will be that parasitized larvae of the gasp won't give rise to adults. This effect will have consequences on the emergence rate of *D. kuriphilus*, that now becomes:

$$\text{emergence rate} = \frac{\eta V_{n-1}(T_d) - Q_n(T_t)}{T_d} \quad (10)$$

This results in the complete model described by Equation (11), where initial conditions are added:

$$\left\{ \begin{array}{l}
 \frac{\partial}{\partial t} p_n(t) = -\frac{1}{\tau}(v_{n-1}(1) - q_n(t))p_n(t) \\
 \frac{\partial}{\partial t} q_n(t) = \frac{E_t}{\tau}(v_n(1) - q_n(t))p_n(t) \\
 \frac{\partial}{\partial t} u_n(t) = -\mu(2 - v_n(t))u_n(t) + v_{n-1}(1) - q_n(\eta\tau) \\
 \frac{\partial}{\partial t} v_n(t) = E_d\mu(1 - v_n(t))u_n(t) \\
 u_n(0) = 0 \\
 v_n(0) = 0 \\
 p_n(0) = q_{n-1}(\eta\tau) \\
 q_n(0) = 0
 \end{array} \right. \tag{11}$$

2.4 Numerical Simulations

The system of Ordinary Differential Equations (11) was solved numerically by the Octave built-in function `ode45`. This function combines the four and five order Runge-Kutta method (see [6], for instance).

Many model parameters are fixed. There are two, however, that depend on the geolocation of the species: the survival rate during overwintering of *D. kuriphilus* (η) and the accounts for the sex ration of *T. sinensis* (γ). Depending on the value of these parameters, several types of evolution are possible. In this work $\eta = 0.9$ and $\gamma = 0.45$. These values correspond to the reality of Portugal [8].

Fig. 2 depicts the evolution of the eggs density of the two species along the time, in years. It is verified that a cycle appears: the density of *D. kuriphilus* decreases for 2-3 years, which leads to the decline of *T. sinensis*. Then, the density of *D. kuriphilus* rises and, when it is worth 1, that of *T. sinensis* goes up there too. Notably, *D. kuriphilus* never disappears completely. To reduce the density of *D. kuriphilus* it would be necessary to inject *T. sinensis* into the environment, regularly. Indeed, *T. sinensis* lays its eggs in those of *D. kuriphilus*; thus, if *D. kuriphilus* disappears from the area, *T. sinensis* does not lay eggs and will also disappear from the area because it will not be able to reproduce. To sum it up, although *T. sinensis* controls the evolution of *D. kuriphilus*, this does not allow to definitely eradicate *D. kuriphilus*.

3 Two-Dimensional Model

Adding a spatial dimension to the previous complete model will take into account the fact that the two species move over time from one area to another. To accomplish that it is necessary to add a diffusivity term to the model, described by Equation (11).

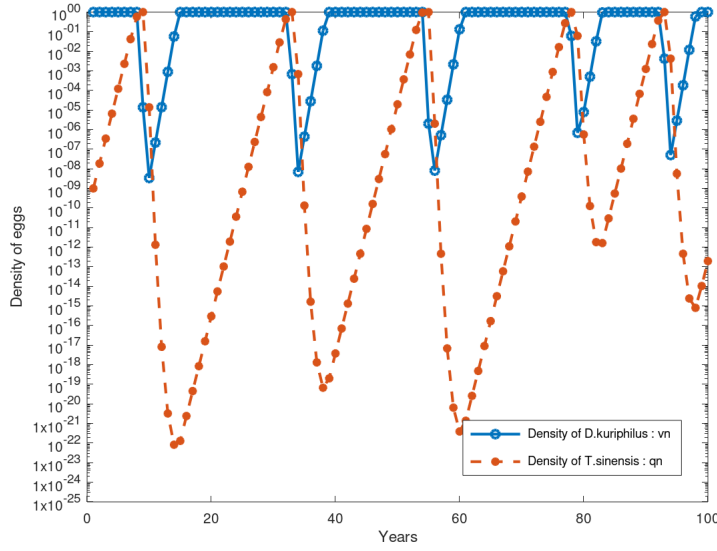


Fig. 2. Temporal evolution of *D. kuriphilus* and *T. sinensis* eggs density.

3.1 Mathematical Model

Let $\nabla^2 u_n$ be the diffusivity of u_n , $\nabla^2 p_n$ be the diffusivity of p_n , and δ represent the diffusivity ratio between the two species. It was observed that *D. kuriphilus* moves faster than the parasite and travels longer distances [3]. In this study, it is assumed $\delta = 0.2$, which corresponds to the speed of the *T. sinensis* front of about 6.4 km per season. Taking into account the new spacial-aware parameters, the complete mathematical model with spacial dimension is given by Equation (12):

$$\left\{ \begin{array}{l}
 \frac{\partial}{\partial t} p_n(x, y, t) = \delta \nabla^2 p_n(x, y, t) - \frac{1}{\tau} (v_{n-1}(x, y, 1) - q_n(x, y, t)) p_n(x, y, t) \\
 \frac{\partial}{\partial t} q_n(x, y, t) = \frac{E_t}{\tau} (v_n(x, y, 1) - q_n(x, y, t)) p_n(x, y, t) \\
 \frac{\partial}{\partial t} u_n(x, y, t) = \nabla^2 u_n(x, y, t) - \mu(2 - v_n(x, y, t)) u_n(x, y, t) + v_{n-1}(x, y, 1) - q_n(x, y, \eta\tau) \\
 \frac{\partial}{\partial t} v_n(x, y, t) = E_d \mu (1 - v_n(x, y, t)) u_n(x, y, t) \\
 u_n(x, y, 0) = 0 \\
 v_n(x, y, 0) = 0 \\
 p_n(x, y, 0) = q_{n-1}(x, y, \eta\tau) \\
 q_n(x, y, 0) = 0
 \end{array} \right. \tag{12}$$

3.2 Numerical Simulations

The numerical solution of the system of partial differential equations (12) implies the discretization of the diffusive term. Using the finite difference method, as stated in [6], the diffusivity of u_n is approached by

$$\begin{aligned} \nabla^2 u_n(x, y, t) \approx & \frac{u_n(x_{i+1}, y, t) - 2u_n(x_i, y, t) + u_n(x_{i-1}, y, t)}{\Delta x^2} \\ & + \frac{u_n(x, y_{i+1}, t) - 2u_n(x, y_i, t) + u_n(x, y_{i-1}, t)}{\Delta y^2} \end{aligned} \quad (13)$$

The diffusivity of p_n (approximation not shown) is discretized similarly. Once the diffusive terms are discretized, the resulting system of ODEs is solved with the built-in Octave function `ode45`, as in the non-spatial model. Moreover, after implementing the algorithm to solve this problem, it is possible to look at the evolution of the density of *D. kuriphilus* and *T. sinensis* over the years.

The results of the numerical solution of the model Equation (12), with $\delta = 0.2$, are presented in Fig. 3 for the years 3, 6, 9, 12, 15, 18, 21 and 23. The simulation considered that the initial density of *D. kuriphilus* is maximal everywhere except at a given place where it is zero, and the initial density of *T. sinensis* is zero everywhere except at a certain place where is maximal. This simulation allows to verify the impact of biological control once it is implanted in a given location.

In Fig. 3, it may be seen that when there is an area where the density of *D. kuriphilus* is maximal (Fig. 3 a), three years after its density is zero and *T. sinensis* has appeared on the same area (Fig. 3 b). Thus, it can be deduced that *T. sinensis* has eliminated the pest on that area. However, *D. kuriphilus* has moved and has invaded another place. It spreads like a wave.

Regarding Fig. 3, the same cyclic phenomenon previously observed in the non-spatial case is also observable. There is a cycle concerning the density of the two species: when *D. kuriphilus* is maximum, *T. sinensis* appears; the later controls *D. kuriphilus*, and therefore its density decreases; in turn, this leads to the decreasing of the density of *T. sinensis*; as a result *D. kuriphilus* reappears in the area. This cyclic phenomenon moves in the area over time.

So, biological control is a good way to control *D. kuriphilus* in a given area in a few years. But when that control is over (because both species have disappeared from the same area), *D. kuriphilus* returns and recolonizes that area. Thus, it is necessary to look for a way to control *D. kuriphilus* more durably. It is also important to know the time that *D. kuriphilus* and *T. sinensis* take to reappear.

3.3 Density of the Two Species Over Time in a Given Area

In this simulation, the density of the two species at the end of the year, in a given space (matching the site shown in Fig. 3), is traced over time. The initial conditions (year 0) sets the density of *T. sinensis* at its maximum ($q_0 = 1$) while that of *D. kuriphilus* is zero ($v_0 = 0$). Fig. 4 depicts the densities over 40 years.

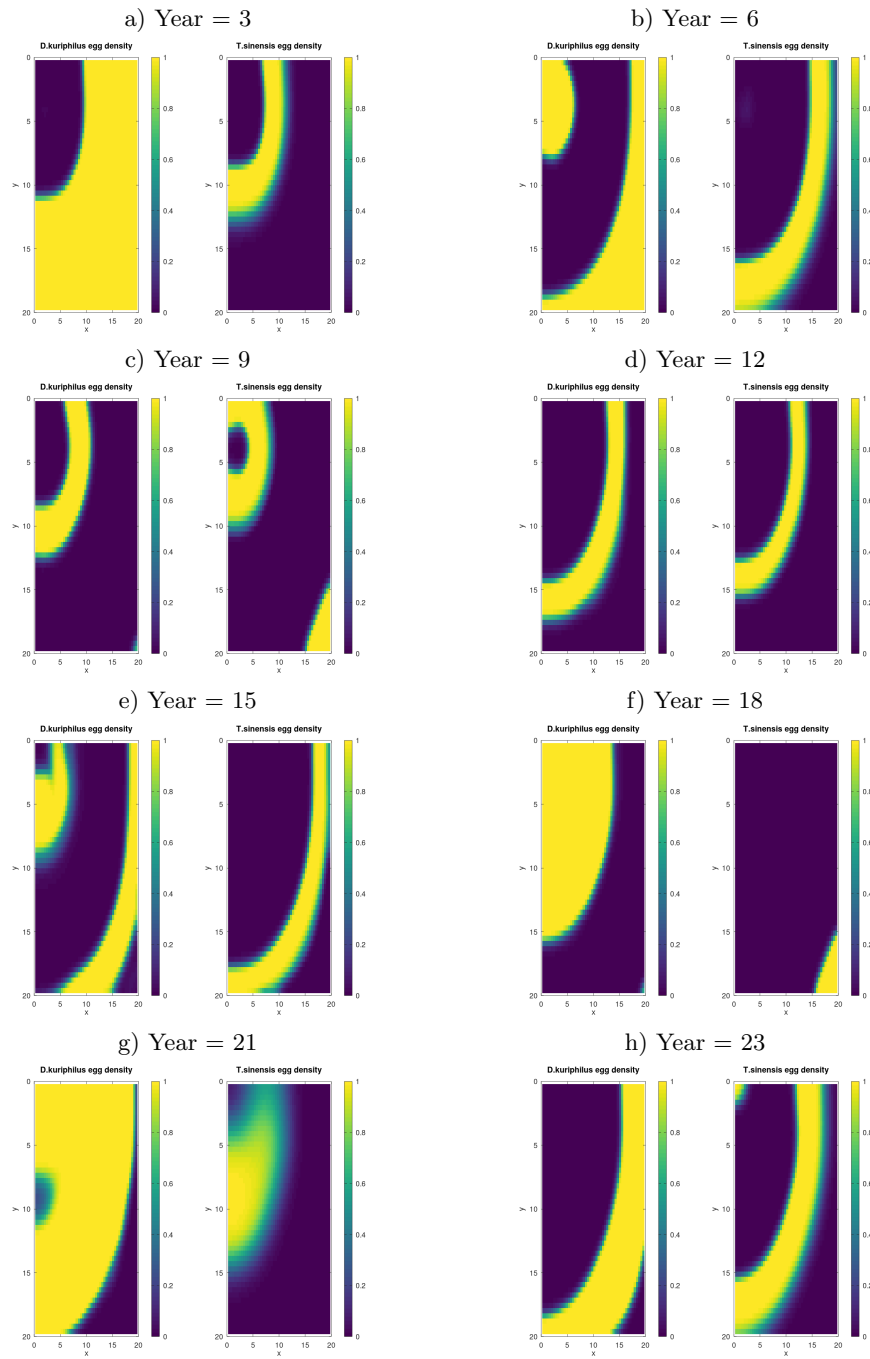


Fig. 3. *D. kuriphilus* and *T. sinensis* egg density after 3, 6, 9, 12, 15, 21, 23 years.

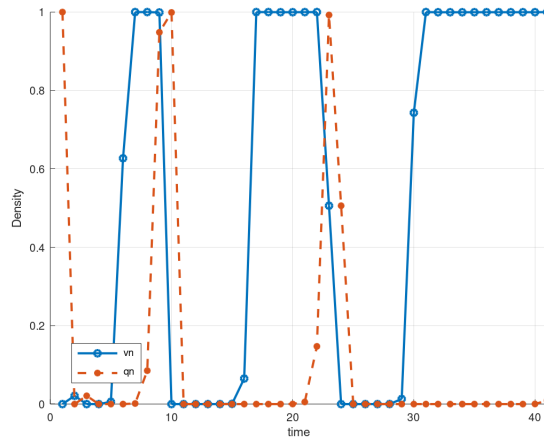


Fig. 4. Eggs density of *D. kuriphilus* (v_n) and *T. sinensis* (q_n) in a given space over the time

The observation of Fig. 4 permits to notice that a cycle appears. This is the same cycle already observed in the previous numerical experiments. The density of *D. kuriphilus* increases until it reaches a density of 1. Then it is the turn of the density of *T. sinensis* to increase to reach 1 as well. When *T. sinensis* reaches a certain density, 0.7 at the minimum, the density of *D. kuriphilus* begins to fall sharply, reaching 0 in 2 years (on average). Similarly, when the density of *D. kuriphilus* is very low, that of *T. sinensis* drops sharply so that it reaches 0. The cycle is therefore composed of 4 stages that are repeated.

It may also be observed in Fig. 4 that as the years pass, the second stage of the cycle takes more time to appear. Indeed, the more the cycles advance, the more the density of *D. kuriphilus* remains at 1 before that of *T. sinensis* begins to increase. For the 1st cycle the density of *D. kuriphilus* is 1 for 2.5 years. For the 2nd cycle, its density is 1 for 5 years. Then for the last cycle, it is worth 1 at least 10 years. So *T. sinensis* takes more and more time to reappear in the area.

Therefore, two questions concerning the release of *T. sinensis* must be answered: i) should *T. sinensis* be reintroduced after several years? ii) if so, when would it be necessary to reintroduce them to channel the density of *D. kuriphilus* so that does not reach 1?

4 Improving the Biological Control

Along with the different numerical studies of the previous sections, it was noticed that the biological control of *D. kuriphilus* by *T. sinensis* was effective, albeit not totally. Indeed, although biological control can neutralize *D. kuriphilus* and make it almost disappear from an area, the plague returns after a few years.

This happens for two reasons. The 1st is that *D. kuriphilus* moves from area to area naturally, as it was observed in the spatial simulations. The 2nd is that *T. sinensis* can not live without *D. kuriphilus*, with whom establishes a parasitic relationship; thus, when *D. kuriphilus* is absent, *T. sinensis* dies and disappears.

Let's briefly consider the two studies already presented. The first study looked at the evolution of the density of the two species as a function of time without taking into account the diffusive nature of the variables (Fig. 2). The last study also focused on the density of the two species over time, but then considering that the two species evolve in a two-dimensional space (Fig. 4).

On Fig. 2 and Fig. 4 the cycles mentioned earlier in this paper are easily identifiable. Fig. 2 shows that once the density of *D. kuriphilus* reaches 10^{-9} , it grows quickly, because *T. sinensis* has practically disappeared. In turn, Fig. 4 shows that *T. sinensis* takes more and more time to reappear in the area. This is the reason why the hypothesis of reintroducing *T. sinensis* into a given area, after a few years, was made in the previous section.

To verify the effect of reintroducing *T. sinensis*, the evolution of the density of the two species, over 15 years, is now studied. The initial conditions are modified so that, in the majority of the studied area, the density of *T. sinensis* is at 10^{-9} , as if it had been just released in this area and the density of *D. kuriphilus* is zero as if it had just disappeared. In the rest of the study area, the density of *D. kuriphilus* is maximal ($v_0 = 1$) and the density of *T. sinensis* is minimal ($q_0 = 0$). Several cases will be considered in order to find when it is necessary to reintroduce *T. sinensis* in function of *D. kuriphilus* density.

As a comparative baseline, that will allow to verify if injecting *T. sinensis* into the area is useful, it was considered the evolution over 15 years of the two densities, when *T. sinensis* is injected ($q_0 = 10^{-9}$) without *D. kuriphilus* inside the considered area ($v_0 = 0$). This baseline is presented in Fig. 5. This figure shows that the introduction of *T. sinensis* into the zone when $v_0 = 0$ gives rise to a period of maximum density of *D. kuriphilus* for at least 10 years. Several simulations were performed, with different initial values of the density of *D. kuriphilus* (v_0), and a *T. sinensis* density of $q_0 = 10^{-9}$. It was observed that the minimum period of maximum density of *D. kuriphilus* is reached with $v_0 = 10^{-9}$.

The evolution of the densities of eggs of the two species, corresponding to $v_0 = 10^{-9}$, is depicted in Fig. 6. This figure shows that reintroducing *T. sinensis* as soon as *D. kuriphilus* reaches its minimal value ($v_0 = 10^{-9}$, see Fig. 2) and is apparently extinct, reduces the years of the maximum density of *D. kuriphilus* to 4 years. This strategy is favorable for limiting the concentration of *D. kuriphilus* in this zone for too long years.

However, it seems impossible to eliminate the density of *D. kuriphilus*. In fact, it always comes to 1 after a while for several reasons. First, *D. kuriphilus* propagates better in space than *T. sinensis*. Moreover, *D. kuriphilus* is a female species, and so it can lay eggs much faster than *T. sinensis*, which is composed of males and females. Finally, with *T. sinensis* laying its eggs in those of *D. kuriphilus*, it depends on *D. kuriphilus* and therefore can not exist without it.

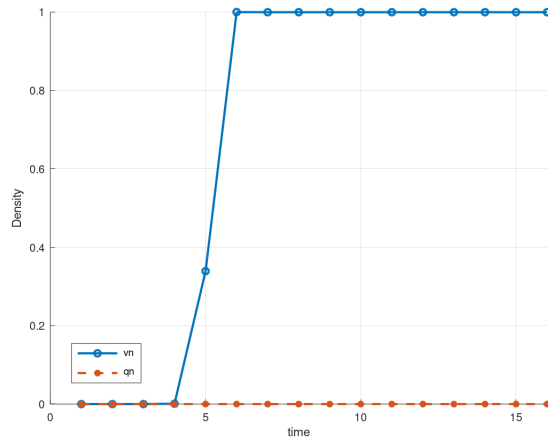


Fig. 5. Eggs density of *D. kuriphilus* and *T. sinensis* when $v_0 = 0$ and $q_0 = 10^{-9}$

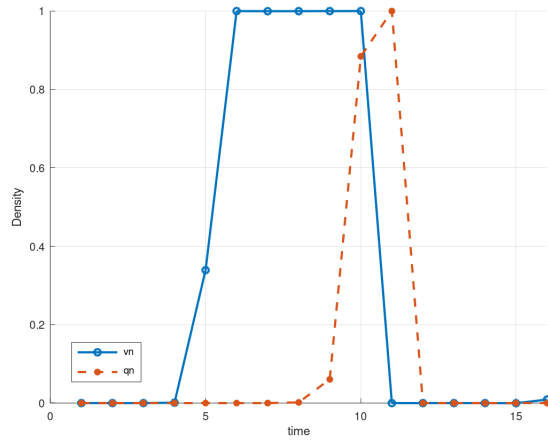


Fig. 6. Density of *D. kuriphilus* and *T. sinensis* eggs when $v_0 = 10^{-9}$ and $q_0 = 10^{-9}$

5 Final Considerations

The previous numerical simulations suggest that the biological control of *D. kuriphilus* with *T. sinensis* works. Indeed, *T. sinensis* takes a few years to establish itself on the territory. However, once this is done, the density of *D. kuriphilus* decreases drastically. Indeed, in 2 to 3 years *D. kuriphilus* almost disappears (only traces of its presence remain). However, it was also noticed that after such disappearance, *T. sinensis* also vanishes from the same zone because it lays its

eggs in those of *D. kuriphilus*, who is now absent. After a few years, *D. kuriphilus* recolonizes the same area, then *T. sinensis* resurges and a new cycle begins.

It was also observed that the more cycles pass, the more difficult it is for *T. sinensis* to reappear in the same area (it takes more and more time), in comparison to *D. kuriphilus*. This is due to several factors, such as the fact that for *D. kuriphilus* only females exist, and the diffusion rate of both species differs.

Another important observation upon the simulations is that in an area where *D. kuriphilus* and *T. sinensis* have disappeared, reinjecting *T. sinensis* from the moment *D. kuriphilus* disappears will reduce the number of years during which *D. kuriphilus* has a maximum density when it recolonizes that area. However, reinjecting *T. sinensis* into an area does not completely eliminate *D. kuriphilus*.

This study is based on mathematical models that yield very precise results, as to the value of the density of each species, of the order of 10^{-9} . But these models depend on parameters that describe the reality. Their estimation would be largely improved with the application of on-field instrumentation and sensors, like those provided by the Internet of Things platforms. An integration of the numerical simulations with these technologies would allow better tuning of the mathematical models, making them more realistic and thus more useful.

Acknowledgement

UNIAG, R&D unit funded by the FCT – Portuguese Foundation for the Development of Science and Technology, Ministry of Science, Technology and Higher Education. Project n.º UID/GES/4752/2019.

References

1. Borowiec, N., Thaon, M., Brancaccio, L., Cailleret, B., Ris, N., Vercken, E.: Early population dynamics in classical biological control : establishment of the exotic parasitoid *Torymus sinensis* and control of its target pest, the chestnut gall wasp *Dryocosmus kuriphilus*, in France. *Entomologia Experimentalis et Applicata* **vol.166** (2018)
2. Eaton, J.W., Bateman, D., Hauberg, S., Wehbring, R.: GNU Octave version 5.1.0 manual: a high-level interactive language for numerical computations (2019)
3. Gilioli, G., Pasquali, S., Tramontini, S., Riolo, F.: Modelling local and long-distance dispersal of invasive chestnut gall wasp in Europe. *Ecological Modelling* **263**, 281–290 (aug 2013)
4. Gillman, M.: *An Introduction to Mathematical Models in Ecology and Evolution: Time and Space*. Wiley-Blackwell (2009)
5. INRA: *Lutte biologique contre le cynips du chataigner en France*. Tech. rep., Institut National de la Recherche Agronomique (2013)
6. Linge, S., Langtangen, H.P.: *Programming for Computations - MATLAB/Octave*. Springer International Publishing (2016)
7. Moriya, S., Shiga, M., Adachi, I.: Classical biological control of the chestnut gall wasp in Japan. In: *Proceedings of the 1st international symposium on biological control of arthropods*. USDA Forest Service, Washington (2003)

8. Paparella, F., Ferracini, C., Portaluri, A., Manzo, A., Alma, A.: Biological control of the chestnut gall wasp with *T. sinensis*: A mathematical model. *Ecological Modelling* **vol.338** (2016)
9. Quacchia, A., Moriya, S., Bosio, G., Scapin, I., Alma, A.: Rearing, release and settlement prospect in Italy of *Torymus sinensis*, the biological control agent of the chestnut gall wasp *Dryocosmus kuriphilus*. *BioControl* **vol.53** (2008)