MODELING AND CONTROL OF LANE KEEPING SYSTEM FOR AUTONOMOUS VEHICLE

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MODELING AND CONTROL OF LANE KEEPING SYSTEM FOR AUTONOMOUS VEHICLE

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TABLE OF CONTENTS

Page

Acknowledgements ii

Table of Contents iii

List of Tables vii

List of Figures viii

List of Plates xiv

List of Abbreviations xv

List of Symbols xvi

Abstrak xxiv

Abstract xxv

CHAPTER 1 – INTRODUCTION

1.1 Autonomous Ground Vehicle 1

1.2 Motivation and Problem statement 4

1.2.1 Theoretical Aspects 4

1.2.1(a) Mathematical Modeling 5

1.2.1(b) Control Methods 7

1.2.2 Practical Implication 9

1.2.2(a) Road Accident Statistics in Malaysia 9

1.2.2(b) Traffic Jam in Malaysia 10

1.3 Objective of Research 11

1.4 Research Findings 12

1.5 Scope of Research 13

1.6 Organization of Thesis 13

CHAPTER 2 – LITERATURE REVIEW

2.1 Background of Autonomous Ground Vehicle 16

2.2 History of Lane Keeping of Ground Vehicle 17

2.3 Background of Mathematical Modeling of Ground Vehicle..... 19

2.4 Representation of Mathematical Model 22

2.4.1 Literature Review on Analysis of State-Space System..... 25

2.4.1(a) Controllability 26

2.4.1(b) Observability 27

2.4.2 Background on Analysis of System Stability 27

2.5 Review on Control of Autonomous Ground Vehicle 32

2.5.1 PID Control Theory and Background 32

2.5.2 PID Control in Lateral Dynamics Control of Ground Vehicle 34

2.5.3 MPC Control Theory Background 36

2.5.4 Advantages of MPC..... 39

2.5.5 MPC Control in Controlling Lateral Dynamics of Ground Vehicle 40

2.6 Background of State Estimation 41

2.6.1 Kalman Filter as Estimator 43

2.6.2 Kalman Filter Algorithm and Formulation 43

2.7 Background of Parametric Uncertainty and Robustness Analysis of Closed Loop Control System 45

2.7.1 Effects of Uncertainty..... 45

2.7.2 Uncertainty in Dynamic System..... 45

2.7.3 Parametric Uncertainty 48

2.7.4 Robustness Analysis 50

2.8 Summary 54

CHAPTER 3 – METHODOLOGY

3.1 Vehicle Modeling 56

3.1.1 Dynamics of Ground Vehicle 56

3.1.2 Linear Tire Model 62

3.1.3 Linear Bicycle Model 66

3.1.4 Road Curvature 69

3.1.5 Error Variables in terms of Road Curvature 70

3.1.6 State-space Representation of Lane Keeping Ground Vehicle 72

3.2 PID Control of Lane Keeping Ground Vehicle 75

3.2.1 Classical Predictive PID controller 76

3.2.2 PID Controller Formulation 77

3.2.3 Regulator Formulation 84

3.3 Model Predictive Control of Vehicle 85

3.3.1 Vehicle Model Discretization and Summarization 86

3.3.2 Constraints for Safe Driving and Passenger Comfort 87

3.3.3 Optimal Control and Performance Index Formulation for Lane Keeping 94

3.3.4 Model Predictive Controller Formulation 97

3.3.5 Kalman Filter Formulation for State Estimation of MPC 98

3.4 Parametric Uncertainty in Vehicle 100

3.4.1 Uncertain Parameters for Lane Following vehicle 100

3.4.1(a) Uncertain Road Tire Friction Coefficient 101

3.4.1(b) Uncertain Look Ahead Distance 102

3.5 Summary 104

CHAPTER 4 – RESULT AND DISCUSSION

4.1 Analysis of Vehicle Model with Real Car Parameters 105

4.1.1 Analysis of Uncompensated Vehicle Model 106

4.1.2 Simulation Result for Compensated System 108

4.2 Analysis of Closed Loop Vehicle Model with PID Controller 112

4.2.1 Simulation Result of Closed Loop PID at Dry Surface 114

4.2.2 Simulation Result of Closed Loop PID at Wet Surface 124

4.3 Analysis of Closed Loop Vehicle Model with MPC 132

4.3.1 Calculation of Output Constraint 133

4.3.2 Calculation of Input Constraint 133

4.3.3 Simulation Result of Closed Loop MPC at Dry Surface 134

4.3.4 Simulation Result of Closed Loop MPC at Wet Surface 143

4.4 Robustness Analysis in the Presence of Uncertainty 149

4.4.1 Simulation Result of Closed loop PID with Parametric Uncertainty 150

4.4.2 Simulation Result of MPC Controlled Parametric Uncertain Model 152

4.4.2(a) Robustness Analysis 153

4.4.2(b) Robust Stability Analysis 161

4.4.2(c) Robust Performance Analysis 164

4.4.2(d) Robustness Analysis for Input Disturbance 166

CHAPTER 5 – CONCLUSION

5.1 Conclusion of Thesis 174

5.2 Future Work 176

References 177

Appendices

List of Publications

LIST OF TABLES

		Page
Table 1.1	Road accidents in various countries in recent years	9
Table 4.1	Real car parameters	106
Table 4.2	Poles of uncompensated vehicle	108
Table 4.3	Poles of compensated vehicle	109
Table 4.4	PID controller values	113
Table 4.5	PID controller results at dry surface	121
Table 4.6	PID controller results at wet road surface	130
Table 4.7	MPC controller results at dry surface	144
Table 4.8	MPC controller results at wet surface	148
Table 4.9	Robust stability analysis results	163
Table 4.10	Robust performance analysis results	166

LIST OF FIGURES

		Page
Figure 1.1	Control system architecture of autonomous vehicle	3
Figure 1.2	Classification of dynamic system	6
Figure 1.3	Research tasks	14
Figure 2.1	Research flowchart	30
Figure 2.2	Pole placement technique	31
Figure 2.3	PID controller	33
Figure 2.4	MPC controller flowchart	37
Figure 2.5	Model predictive controller elements	38
Figure 2.6	Model predictive control	39
Figure 2.7	Kalman filter algorithm	44
Figure 2.8	Closed loop dynamic system with uncertainty	46
Figure 2.9	Closed loop dynamic system with possible uncertainties	47
Figure 2.10	Closed loop dynamic system with uncertainty including controller	47
Figure 2.11	Stability domains in the space of parametric uncertainties	49
Figure 2.12	Closed loop plant with uncertainty	53
Figure 2.13	Closed loop plant with uncertain block	53
Figure 3.1	Research flowchart	55
Figure 3.2	Coordinate system of ground vehicle	57
Figure 3.3	Road curvature based global coordinate system	59
Figure 3.4	Linear approximation of road centerline	60
Figure 3.5	Track based coordinate system of vehicle	61
Figure 3.6	Side slip angle	63
Figure 3.7	Linear relation of side slip angle and lateral tire force	63

Figure 3.8	Relation among heading angle, steering angle with side slip angle	64
Figure 3.9	Bicycle model of ground vehicle	67
Figure 3.10	Oscullating circle and curvature	70
Figure 3.11	Bicycle vehicle model	71
Figure 3.12	PID controller flowchart	75
Figure 3.13	PID controller	76
Figure 3.14	Predictive nature of PID control	78
Figure 3.15	SITO feedback system	79
Figure 3.16	Transfer function variables	80
Figure 3.17	Closed loop vehicle system	81
Figure 3.18	Overall close loop vehicle with PID controller	84
Figure 3.19	Ackerman geometry for steering angle of ground vehicle	89
Figure 3.20	Under-steer and over-steer of ground vehicle	90
Figure 3.21	Calculation of output constraint: vehicle at desired position	92
Figure 3.22	Calculation of output constraint: vehicle at deviated position	93
Figure 3.23	Calculation of output constraint: vehicle at maximum deviated position	93
Figure 3.24	Closed loop model predictive control system for vehicle	98
Figure 3.25	Model predictive controller work diagram including controller state estimation	99
Figure 3.26	Usual range of μ on different road condition	101
Figure 3.27	Range of look ahead distance for different road curvature	103
Figure 4.1	Step response of uncompensated vehicle model	108
Figure 4.2	Pole zero map of uncompensated vehicle model	109
Figure 4.3	Step response of compensated vehicle model	110
Figure 4.4	Pole zero map of compensated vehicle model	111
Figure 4.5	Results of PID loop with high PID values	113

Figure 4.6	Results of PID loop with higher PID values	114
Figure 4.7	Output variables and input variables for PID controlled low speed vehicle on dry surface with 0.01 m road curvature	115
Figure 4.8	Output variables and input variables for PID controlled low speed vehicle on dry surface with 0.004 m road curvature	116
Figure 4.9	Output variables and input variables for PID controlled low speed vehicle on dry surface with 0.002 m road curvature	117
Figure 4.10	Output variables and input variables for PID controlled medium speed vehicle on dry surface with 0.01 m road curvature	118
Figure 4.11	Output variables and input variables for PID controlled medium speed vehicle on dry surface with 0.004 m road curvature	119
Figure 4.12	Output variables and input variables for PID controlled medium speed vehicle on dry surface with 0.002 m road curvature	120
Figure 4.13	Output variables and input variables for PID controlled high speed vehicle on dry surface with 0.01 m road curvature	121
Figure 4.14	Output variables and input variables for PID controlled high speed vehicle on dry surface with 0.004 m road curvature	122
Figure 4.15	Output variables and input variables for PID controlled high speed vehicle on dry surface with 0.002 m road curvature	123
Figure 4.16	Output variables and input variables for PID controlled low speed vehicle on wet surface with 0.01 m road curvature	124
Figure 4.17	Output variables and input variables for PID controlled low speed vehicle on wet surface with 0.004 m road curvature	125
Figure 4.18	Output variables and input variables for PID controlled low speed vehicle on wet surface with 0.002 m road curvature	126
Figure 4.19	Output variables and input variables for PID controlled medium speed vehicle on wet surface with 0.01 m road curvature	127
Figure 4.20	Output variables and input variables for PID controlled medium speed vehicle on wet surface with 0.004 m road curvature	127
Figure 4.21	Output variables and input variables for PID controlled medium speed vehicle on wet surface with 0.002 m road curvature	128
Figure 4.22	Output variables and input variables for PID controlled high speed vehicle on wet surface with 0.01 m road curvature	129

Figure 4.23 Output variables and input variables for PID controlled high speed vehicle on wet surface with 0.004 m road curvature 130

Figure 4.24 Output variables and input variables for PID controlled high speed vehicle on wet surface with 0.002 m road curvature 131

Figure 4.25 Output variables and input variables for MPC controlled low speed vehicle on dry surface with 0.01 m road curvature 135

Figure 4.26 Output variables and input variables for MPC controlled low speed vehicle on dry surface with 0.004 m road curvature 136

Figure 4.27 Output variables and input variables for MPC controlled low speed vehicle on dry surface with 0.002 m road curvature 137

Figure 4.28 Output variables and input variables for MPC controlled medium speed vehicle on dry surface with 0.01 m road curvature 138

Figure 4.29 Output variables and input variables for MPC controlled medium speed vehicle on dry surface with 0.004 m road curvature 139

Figure 4.30 Output variables and input variables for MPC controlled medium speed vehicle on dry surface with 0.002 m road curvature 140

Figure 4.31 Output variables and input variables for MPC controlled high speed vehicle on dry surface with 0.01 m road curvature 141

Figure 4.32 Output variables and input variables for MPC controlled high speed vehicle on dry surface with 0.004 m road curvature 142

Figure 4.33 Output variables and input variables for MPC controlled high speed vehicle on dry surface with 0.002 m road curvature 143

Figure 4.34 Output variables and input variables for MPC controlled low speed vehicle on wet surface with 0.01 m road curvature 145

Figure 4.35 Output variables and input variables for MPC controlled low speed vehicle on wet surface with 0.004 m road curvature 146

Figure 4.36 Output variables and input variables for MPC controlled low speed vehicle on wet surface with 0.002 m road curvature 147

Figure 4.37 Output variables and input variables for MPC controlled medium speed vehicle on wet surface with 0.01 m road curvature 148

Figure 4.38 Output variables and input variables for MPC controlled medium speed vehicle on wet surface with 0.004 m road curvature 149

Figure 4.39	Output variables and input variables for MPC controlled medium speed vehicle on wet surface with $0.002\,m$ road curvature	150
Figure 4.40	Output variables and input variables for MPC controlled high speed vehicle on wet surface with $0.01\,m$ road curvature	151
Figure 4.41	Output variables and input variables for MPC controlled high speed vehicle on wet surface with $0.004\,m$ road curvature	152
Figure 4.42	Output variables and input variables for MPC controlled high speed vehicle on wet surface with $0.002\,m$ road curvature	153
Figure 4.43	Road Curvature as input disturbance for PID controlled uncertain vehicle system	154
Figure 4.44	Steering angle at the presence of parametric uncertainty for PID controlled system	154
Figure 4.45	Lateral error variable at the presence of parametric uncertainty for PID controlled system	155
Figure 4.46	Angular error variable at the presence of parametric uncertainty for PID controlled system	155
Figure 4.47	Step response of closed-loop nominal plant for low speed vehicle	156
Figure 4.48	Step response of closed loop uncertain plant for low speed vehicle	157
Figure 4.49	Pole zero map of uncertain closed loop plant for low speed vehicle	157
Figure 4.50	Step response of closed loop nominal plant for medium speed vehicle	158
Figure 4.51	Step response of closed loop uncertain plant for medium speed vehicle	158
Figure 4.52	Pole zero map of uncertain closed loop plant for medium speed vehicle	159
Figure 4.53	Step response of closed loop nominal plant for high speed vehicle	160
Figure 4.54	Step response of closed loop uncertain plant for high speed vehicle	160
Figure 4.55	Pole zero map of uncertain closed loop plant for high speed vehicle	161
Figure 4.56	Robust stability analysis in terms of singular value for low speed	162

Figure 4.57	Robust stability analysis in terms of singular value for medium speed	162
Figure 4.58	Robust stability analysis in terms of singular value for high speed	163
Figure 4.59	Robust performance analysis in terms of singular value for low speed	165
Figure 4.60	Robust performance analysis in terms of singular value for medium speed	165
Figure 4.61	Robust performance analysis in terms of singular value for high speed	166
Figure 4.62	Road curvature as input disturbance	167
Figure 4.63	Lateral error variable output of uncertain system in the presence of input disturbance at low speed	168
Figure 4.64	Angular error variable output of uncertain system in the presence of input disturbance at low speed	169
Figure 4.65	Steering angle input of uncertain system in the presence of input disturbance at low speed	170
Figure 4.66	Lateral error variable output of uncertain system in the presence of input disturbance at medium speed	170
Figure 4.67	Angular error variable output of uncertain system in the presence of input disturbance at medium speed	171
Figure 4.68	Steering angle input of uncertain system in the presence of input disturbance at medium speed	171
Figure 4.69	Lateral error variable output of uncertain system in the presence of input disturbance at high speed	172
Figure 4.70	Angular error variable output of uncertain system in the presence of input disturbance at high speed	172
Figure 4.71	Steering angle input of uncertain system in the presence of input disturbance at high speed	173

LIST OF PLATES

Page

Plate 1.1	Lane keeping assist system in Ford and Google driver-less car	2
Plate 1.2	Traffic accidents due to lane change	10
Plate 1.3	Traffic congestion due to lane change	10

LIST OF ABBREVIATIONS

ASM	Active Set methods
ASTM	American Society for Testing and Materials
AHS	Automated Highway System
CG	Center of Gravity
CCD	Charge Coupled Devices
CMOS	Complementary Metal Oxide Semiconductor
DARPA	Defense Advanced Research Projects Agency
Dist	Disturbance
GM	General Motor
KF	Kalman Filter
LDWS	Lane Departure Warning System
LKAS	Lane Keeping Assista System
lim	limit
MPC	Model Predictive Controller
min	Minimum
max	Maximum
MIMO	Multi Input Multi Output
MV	Manipulated Variable
PI	Proportinal Integral
PO	Percentage of Overshoot
PID	Proportinal Integral and Derivative
QP	Quadratic Programming
Ref	Reference
SSV	Structured Singular Value
SISO	Single Input Single Output
SITO	Single Input Two Output Motor
SAE	Society of Automotive Engineers
sup	Supremum
TISO	Two Input Single Output

LIST OF SYMBOLS

A	System matrix
a_y	Inertial acceleration of the vehicle
A_k	System matrix at time k
A_o	Nominal system matrix
A_i	Uncertain system matrix
A_c	Real matrix
B_o	Nominal input matrix
B_i	Uncertain input matrix,
B	Input matrix
b	Real vector
B_k	Input matrix at time k
C	Output matrix
C_o	Nominal output matrix
C_i	Uncertain output matrix
$\mathscr{C}$	Observability matrix
c^T	Transpose of vector c
C_α	Cornering stiffness
C_{α_f}	Cornering stiffness of the fornt tire
C_{α_r}	Cornering stiffness of the rear tire
C_m	Output matrix for measured variable
D_o	Nominal feedthrough matrix
D_i	Uncertain feedthrough matrix
D_m	Feedthrough matrix for measured variable
d_{o_1}	Disturbance to y_1
d_{o_2}	Disturbance to y_2
d_1	Disturbance to input
D	Feedthrough matrix
$d(s)$	Disturbance of plant in s-domain

D_s	Denominator of the transfer function of a system
e	Exponential
$\dot{e}_y$	Derivative of lateral error variable of the vehicle
$\dot{e}_p$	Derivative of angular error of the vehicle
e_1	Error for $output_1$
e_2	Error for $output_2$
$e_{y_{min}}$	Lower limit of lateral error variable constraint
$e_{y_{max}}$	Upper limit of lateral error variable constraint
e_p	Angular deviation of the vehicle with respect to lane centerline
e_y	Lateral deviation of the vehicle with respect to lane centerline
$e(t)$	Error variable vector at time t
f	Vector function
F_x	Longitudinal force acting on the tire
F_x	Lateral force on the tires
F_y	Lateral force acting on the tire
F_{x_r}	Longitudinal force acting on the rear tire
F_{y_r}	Lateral force acting on the rear tire
F_{x_f}	Longitudinal force acting on the front tire
F_{y_f}	Lateral force acting on the front tire
f_c	Continuous time domain vector function
f_d	Discrete time domain vector function
$F_t(s)$	Transfer function of a system
g	Vector function
G_p	Plant transfer function
G_c	Controller transfer function
G_d	Output to disturbance transfer function
G_y	output to control input transfer function
G_{y_1}	$Output_1$ to control input transfer function

G_{y_2}	$Output_2$ to control input transfer function
$G(s)$	Overall transfer function of plant
$G_{d_1}(s)$	$Output_1$ to disturbance input transfer function
$G_{d_2}(s)$	$Output_2$ to disturbance input transfer function
G_r	Transfer function of regulator
H	Hessian matrix
H_p	Prediction horizon
H_c	Control horizon
h	Vector function
H_k	Hessian matrix at time k
i	Lower limit of constraint
I_{zz}	Moment of inertia of the vehicle
$\tilde{I}_{zz}$	Normalized moment of inertia of the vehicle
I	Identity matrix
J	Cost function
j	Laplace variable
K_d	Controller
K	Compensation matrix for pole placement
K_k	Kalman gain at time k
K_p	Proportional coefficient
K_c	Integral coefficient
K_d	Derivative coefficient
L_i	Uncertain look ahead distance
$-\bar{L}_i$	Lower limit of interval of parameter L_i
$\bar{L}_i$	Upper limit of interval of parameter L_i
l_x	Lower limit of constraint
l_f	Distance between the vehicle CG from the front axle of the vehicle
l_r	Distance between the vehicle CG from rear axle of the vehicle

L	Look ahead distance
m_1	Dimension of output matrix
m	Mass of vehicle
$\tilde{m}$	Normalized mass of vehicle
M	Nominal closed loop system
M_z	Rotating moment of the vehicle around z axis
$\mathscr{M}$	Matrix containing all uncertainties
n_{y_1}	Noise input to y_1
n_{y_2}	Noise input to y_2
N_z	Normal force acting on tire
n_1	Dimension of matrix
n	Number of dimensional space
N_s	Nominator of the transfer function of a system
$\mathscr{O}$	Controllability matrix
P	Point
P_d	Interconnection matrix
p_1	Dimension of input matrix
P_k	Predicted estimated covariance matrix at time k
P_o	Predicted estimated covariance matrix at time 0
$P_{k\|k-1}$	Predicted estimated covariance matrix
p_n	Poles of a system
Q	Upper limit of constraint
Q_k	Process noise covariance matrix at time k
R	Road radius
R_1	Lower limit of constraint
R_2	Upper limit of constraint
$\dot{r}$	Derivative of yaw rate of the vehicle
r_1	Reference signal $output_1$
r_2	Reference signal $output_2$

r	Yaw rate
r_{des}	Desired yaw rate
$\mathbb{R}$	Set of all real numbers
R_k	Measured noise covarianlce at time k
S_k	Innovation covariance matrix at time k
s_1	Path length coordinate
S	Frequency domain
S_I	Input sensitivity
S_f	Scale factor
T_s	Settling time
t	Time
T_i	Time unit for integral control
t_0	Initial time
T_d	Time unit for derivation control
T_s	Discretization time interval
t_k	Time step
t_w	Distance in betwwen steering wheels of ground vehicle
u	Control input
u_x	Upper limit of constraint
u_{min}	Longitudinal force acting on the rear tire
u_{max}	Lateral force acting on the rear tire
$u(t_k+1)$	Discrete time domain input vector function at time t_k+1
$u(t_k)$	Discrete time domain input vector function at time t_k
U_k	Control input at time k
$u(s)$	Control input of plant in s-domain
$\mathbf{u}$	Input vector
v	Feedback signal vector
V_x	Longitudinal velocity of vehicle
V_y	Lateral velocity of vehicle

w	Noise and disturbances
w	White gaussian noise
w_1	Noise $output_1$
w_2	Noise $output_2$
$w(k)$	White noise at time k
x	Horizontal axis
x_0	State vector at initial time
$x(t)$	State vector at time t
$x_c(k+1)$	Controller state at time $k+1$
$x_c(k)$	Controller state at time k
x_n	State vector of dimension n
$\hat{x}_k$	Estimation of vector x at time k given all measurements available
$\hat{x}_{k\|k-1}$	Estimation of vector x at time k given first $k-1$ measurements
$\hat{x}_{k-1\|k-1}$	Estimation of vector x at time $k-1$ given first $k-1$ measurements
$\hat{x}_o$	State vector x at time 0
x_l	Lane width
x^T	Transpose of vector x
$\hat{x}$	State vector
X	Global Cordinate along horizon
$\dot{x}(t)$	Derivative of state vector at time t
y	Output vector
y_1	$Out\,put_1$
y_2	$Out\,put_2$
$y(t)$	Output vector at time t
$y_{meas}(k)$	Measured plant output at time k
$y(s)$	Output of plant in s-domain
y_l	Car width

$\hat{y}_k$	Innovated output variable at time k
$\dot{y}$	Lateral velocity in temrs of lateral position of the vehcile
y	Verticle axis
Y	Global corodinate along verticle
z_r	Vector including reference signal, controlled signal and error signal
z	Plane
z_l	Avaibale space in the lane
z	Right angle axis
Z	Global corodinate along right angle
$\hat{z}_k$	Measurements at time k
z_n	Zeros of a system
α_d	Disturbance attenuation coefficient
β_f	Side-slip angle of the front tire of the vehicle
$\dot{\beta}$	Derivative of the side-slip angle of the vehicle
β_m	Real Number
β	Side-slip angle
δ	Steering angle
Δ	Uncertain transfer matrix
Δ_i	Uncertain element
δ_{i_L}	Uncertain element for L
δ_{i_μ}	Uncertain element for μ
Δu_{min}	Lower limit of rate of change of control input constraint
Δu_{max}	Upper limit of rate of change of control input constraint
δ_i	Inner wheels steering angle
δ_o	Outer wheels steering angle
δ_1	Output of PID_1
δ_1	Output of PID_1
δ_2	Output of PID_2

Δ_{u_p}	Optimization variable
$\eta(t_k+1)$	Discrete time domain output vector function at time t_k+1
$\eta(t_k)$	Discrete time domain output vector function at time t_k
η	Output vector
η_d	Uncertainty vector
$\eta_{t+i,t}$	Lower limit of lateral error variable constraint
$\eta_{ref_{t+i,t}}$	Upper limit of lateral error variable constraint
$\in$	Is an element of
ε	Vector of cotinuous time domain
$\dot{\gamma}$	Parameterized curve
μ_i	Uncertain road-tire friction coefficient
$-\bar{\mu}_i$	Lower limit of interval of parameter μ_i
$\bar{\mu}_i$	Upper limit of interval of parameter μ_i
μ_Δ	Analysis parameter for measuring robustness
μ	Road tire friction coefficient
ω	Laplace variable
ω_n	Natural frequency
ψ	Orientation of the vehicle
Π	Uncertainty model set
ρ	Road curvature
τ	Time coordinate
θ	Orientation angle
θ_i	Parameter of a system
$-\bar{\theta}_i$	Lower limit of interval of parameter θ_i
$\bar{\theta}_i$	Upper limit of interval of parameter θ_i
θ_{v_f}	Velocity vector angle of front tire
θ_{v_r}	Velocity vector angle of rear tire
ζ	Damping ratio