CHAPTER
Quantum Mechanical /Molecular Mechanical (QM/MM) Approaches: Flawed Perfection

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1 Introduction

Computational modelling techniques are now standard tools in solid state science. They are used routinely to model and predict structures, to investigate defect, transport and spectroscopic properties of solids, to simulate sorption and diffusion, to develop models for nucleation and growth of solids and increasingly to model and predict reaction mechanisms. They are applied to bulk solids, surfaces and nano-structures and successful applications are reported for all major classes of solid: metals, semiconductors, inorganic and ceramic materials and molecular crystals. Modelling methods are now indeed tools that are used to guide, interpret and predict experiment (Butler et al., 2016; Moniz et al., 2015; Woodley, 2016).

The solid state modeller now has a rich and powerful range of computational tools. Methods based on interatomic potentials have a long and successful history in the field and have been used to great effect in structure modelling and prediction in modelling defects and in simulating molecular and ionic diffusion. These methods still have an important role in the field, but they are inevitably limited by their exclusion of any explicit information on electronic structure. Electronic structure methods, which solve the Schrödinger equation at some level of approximation, give such information but need substantially greater computational resources, limiting them to smaller systems and, in dynamic simulations, shorter time scales. However, with the continuing exponential growth in computer power and with theoretical, technical and algorithmic developments, the range and scope of such methods has expanded enormously in recent decades. In particular, there has been an explosion in the use of density functional theory (DFT) based methods – see Chapter 1 and 22 of the current volume, which evaluate exchange and correlation energies by using an approximate functional of the electron density, and scale with the system size less drastically than the Hartree-Fock (HF) methods. DFT has become an increasingly standard tool in computational solid state science.

In modelling crystalline solids, the most obvious procedure is to use three dimensional periodic boundary conditions, by repeating the crystallographic unit cell (or a supercell of the latter) infinitely in three dimensions. Such approaches may also be adapted to model surfaces, where a two dimensional slab is used with periodicity applied in the third dimension perpendicular to the slab leaving a large gap between slabs in neighbouring cells. Three dimensional periodicity has many substantial technical advantages, including the ability to represent the wave function with plane-wave basis sets.

Historically, a number of inventive schemes had been advanced, introducing the two-dimensional analogues of this approach and various central cell corrections (Bennett et al., 1971; Evarestov, 1975; Janetzko et al., 2002; Stoneham, 1975; Zhang et al., 2013). With the advent of high-power modern computers, however, the methodological complexity of these techniques has mostly been replaced by less efficient but more flexible, easy-to-implement schemes. Modelling using 3D periodicity is facilitated by a number of excellent and versatile computer codes. Examples include the VASP (Kresse and Furthmüller, 1996) and CASTEP codes (Clark et al., 2005), which employ plane-wave basis sets, and CRYSTAL (Dovesi et al., 2014), which uses atomic centred basis sets in common with molecular quantum chemistry. These and other computer codes have been very widely used and have given very extensive insight into the structure and bonding, as well as physical and chemical properties, of a wide range of solids and their surfaces (Freysoldt et al., 2014; Groß, 2008; Lany and Zunger, 2008; Nieminen, 2007; Pacchioni, 2015). There are, however, inherent limitations in this approach, the most obvious being the treatment of localised states such as defects, polarons and adsorbed molecules. Here, the only strategy within the context of 3D periodicity is to embed these states within a supercell, to which the periodicity is applied. This approach, while in widespread usage, inevitably includes interactions between the defects or adsorbed species which may give uncertain and in some case spurious contributions to the calculated energy and properties. A related practical problem is that, to minimise interactions between the species of interest in neighbouring cells, it may be necessary to use very large supercells with a consequent increase in the computational requirements. A second and subtler (and less well documented) problem is that, with an infinite 3D periodic system employing atom-centred pseudopotentials, there is no clear definition of the vacuum energy level. This difficulty is a consequence of the strategy adopted to resolve singularities that arise in the Fourier transforms of the periodic Coulomb potential and atomic pseudopotentials (specifically the so-called **G**=0 term), which involves the introduction of a problem- and implementation-specific constant (Zunger and Cohen, 1979). As a common reference is therefore not available, calculating ionisation potentials or defects in different charge states consistently is problematic.

For these reasons, alternative methods are needed and the solid state modeller can have recourse to a long standing approach widely used in molecular quantum chemistry and in biomolecular modelling, that is to represent the region of interest – the defect, localised electronic state, adsorbed molecule and the surrounding species – by a molecular cluster. In earlier work, a simple cluster was often used, but it became apparent that such a strategy was severely limited as, in the majority of cases, the effects of long range interactions could not be ignored, and the description of the species at the outermost parts of the cluster would be particularly problematic. For this reason, the “QM/MM” approach was developed in which the quantum mechanical (QM) cluster is embedded in an approximate representation of the surrounding lattice which is modelled using a “molecular mechanics” (MM) or interatomic-potential-based technique. Calculations involving interatomic potentials are computationally far cheaper than QM calculations, so the embedding region can be large and can effectively represent the electrostatic effects and steric constraints of the surrounding lattice. A number of methods are used for describing the interactions between the two regions and for handling the crucial issue of the interface between them, as will be discussed in later sections of this chapter. It is, however, clear that, given care in the choice of cluster size and embedding methodology and potentials, these methods can yield accurate results and are indeed the method of choice for several problems in condensed matter physics and chemistry. This chapter will therefore, first outline the methodological aspects of the field before giving illustrations of the power of the techniques in modelling defects, electronic states and the reactivity of solids. Examples given demonstrate the capability of the hybrid QM/MM method to treat problems extending to the boundaries of contemporary solid state chemistry and physics.

2 The Hybrid Quantum Mechanical /Molecular Mechanical approach

As outlined above, the basic idea of a hybrid QM/MM method is that an extended system, such as a crystal, surface, nanowire or large nanoparticle, can be split into two or more regions, each described with different levels of theory. A site of interest can then be modelled with high accuracy using quantum mechanical methods at amenable computational costs, whereas the surroundings will be included in a more approximate manner employing cheaper MM techniques. Development of an appropriate hybrid QM/MM model poses the following key questions:

- What is the nature of *chemical bonding* in the system of interest (see Chapter 13 of this volume)? The answer should tell us how electrons are distributed between the QM and MM regions.
- What is the *embedding potential* experienced by both nuclei and electrons in the QM region?
- What is the *polarising potential* experienced by the surroundings?
- What is the master equation that will control the behaviour of the model's components?

Below we will briefly outline the commonly used approaches to address these questions.

2.1 Background

2.1.1 One electron problem

In solid state theory, quantum mechanical / molecular mechanical models have been developed, to understand and predict defect properties of bulk materials, commonly in their crystalline form, and the chemical reactivity of their surfaces. Ideal extended systems (without defects) are most often described using one-electron wave functions, or orbitals, that are used as the basis of many-electron wave functions, charge densities or Green's functions in HF, DFT or many-body theories (see Chapters 1, 5, 7, 8, 9, 10, 11 of this volume). Such orbitals are solutions of corresponding one-electron Schrödinger equations with a periodic potential and, therefore, are normally taken as fully delocalised Bloch waves, which, however, can be recast as localised Wannier functions (Brouder et al., 2007); both types of functions are orthonormal and complete by design and can be transformed into each other by a unitary transformation. For metals, the transformation can present certain difficulties due to the fractionally occupied bands at the Fermi level (Souza et al., 2001), and it has not been proven mathematically that the resultant set is fully localised (although this result has been frequently obtained in practice, e.g. (Sporkmann and Bross, 1994)). A local perturbation in the potential, due to a defect in bulk or an adsorbate on the surface, destroys the periodicity. The resulting orbitals for an extended system with a perturbation can either localise around the defect, with their energies below a certain threshold (or within a certain range, supporting bound states), or retain their delocalised character, while experiencing only a small fluctuation around the defect (scattered states). Both bound and scattered states can be expanded using the perfect periodic solutions as a basis within perturbation theory or semi-classical approaches (Koster and Slater, 1954). In either case, the long range character of realistic perturbations requires further approximations in how (a) perturbed orbitals are combined in the many-electron solutions self consistently; (b) the perturbed nuclear potential energy is calculated around the defect; and (c) the whole electron-nuclear system is brought into equilibrium. Following (Pisani et al., 1983), “perturbed crystal” techniques consider solutions for the whole crystal, describing the effect of a local perturbation in an approximate manner. Alternatively, for well localised perturbations, one can adopt a “perturbed cluster” approach that focuses on the bound states perturbed by the crystalline environment in which the cluster is embedded. This environment can be represented as an external potential, continuous medium, or by special boundary conditions. These perturbed cluster approaches can be constructed in an *ad hoc* manner, or derived following certain approximations, from the full defected-crystal solutions, with a net effect that the external potential acting on the cluster orbitals is nonlocal and, in its simplest form, energy-dependent (Pisani et al., 1990; Pisani et al., 2000; Pisani et al., 1983). The majority of contemporary quantum mechanical / molecular mechanical approaches make a further approximation, replacing the fully nonlocal potentials with atom-centred local or semilocal energy-independent approximate forms. Such embedding pseudopotentials take the same form and are derived in a similar spirit to effective core and / or model pseudopotentials employed in molecular and periodic calculations (see Chapter 3 of this volume).

Perturbed, embedded clusters present a significant improvement on simple molecular clusters. With these methods one can make use of numerous quantum chemical techniques that account for local and semi-local electron correlation effects on the bound, ground, and excited states, spectroscopic properties of the defect, and are implemented in widely available molecular quantum mechanical codes. Despite obvious limitations, a molecular cluster approach has been successfully used to calculate binding energies and frequency shifts of adsorbed molecules on the surfaces of ionic materials (Pelmenschikov et al., 1995, 1998). This success has, however, been clearly limited to the cases of neutral and weakly dipolar species such as CO, which coordinate to a local adsorption site. The long-range Coulomb interactions and electron exchange prove to be essential for the treatment of charged defects, charge transfer and chemical reactivity, which are sensitive to the electrostatic field – see, e.g., (Johnson and Truong, 1999; Kachurovskaya et al., 2002).

2.1.2 Many electron problem, electron groups, localisation and structural elements

When we define the central QM region of an extended system, it is obvious how we select nuclei and the associated core electrons but not so for the valence electrons. In general, we would like to reduce the difficult problem of a very large number of N electrons (assumed infinite in periodic models) to explicit treatment of only a small number of n electrons (the QM group) and an approximate implicit tackling of the (still very large) N-n number of electrons in the surroundings (the MM group). In the context of quantum mechanics, we should account for the interaction between electrons in these two groups. Such interactions can be split into the classical Coulomb term and quantum mechanical exchange and correlation terms. Whereas the former is straightforward, although the subject of many inventive optimisation schemes, and can be described using the charge densities of the two regions (e.g. see (Laino et al., 2006; Sherwood et al., 1997; Sushko and Abarenkov, 2010)), the latter are nontrivial. The exchange term can be formally set by choosing an antisymmetric ansatz for the wave function of the whole (McWeeny, 1959, 1960; Tolpygo, 1950), and often the correlation term is neglected or included in an *a posteriori* fashion using an MM level of theory, which has underpinned a number of theoretical developments. This has been the basis of many embedding concepts for systems with well localised electrons such as molecular solids. This class of material, however, forms a minority within the broad range of systems in the solid state. Usually, in materials of interest for chemical and physical applications, one should employ strong electron localisation procedures to make a link from the standard molecular orbital type of theories, either based on the HF approximation or DFT, to such useful objects as ionic closed shells, lone pairs, or bonds, suggested by chemical intuition, and widely employed from early hybrid quantum mechanical / molecular mechanical work on solvents, organic macromolecules, enzymatic reactions, *etc*. (Åqvist and Warshel, 1993; Lennard-Jones, 1937; Warshel and Karplus, 1972; Warshel and Levitt, 1976). To perform such a strong electron localisation, one should remove the orthonormal constraint on the wave functions, which necessitates the switch from canonical Hartree-Fock (and correspondingly Kohn-Sham in DFT) equations and their orbital solutions to noncanonical orbitals, subject to Adams-Gilbert equations (Danyliv and Kantorovich, 2004; Danyliv et al., 2007). Much effort has, therefore, concentrated on the derivation of appropriate embedding schemes (Abarenkov et al., 2011; Abarenkov and Tupitsyn, 2001; Antes and Thiel, 1999; Danyliv and Kantorovich, 2004; Danyliv et al., 2007; Guo et al., 2012; He et al., 2009; Hégely et al., 2016; Huzinaga et al., 1987; Kantorovich, 1983, 1988; Laino et al., 2006; Manby et al., 2012; Milov et al., 2015; Seijo and Barandiarán, 1996; Shidlovskaya, 2002; Tchougréeff, 1999, 2016; Wesolowski et al., 2015). In this respect, the external potential can originate from the surroundings, represented by their charge density, giving rise to the family of so-called Density-Functional Embedding methods – see (Fabiano et al., 2014; Huang et al., 2011; Peng et al., 2008; Stefanovich and Truong, 1997; Wesolowski et al., 2015) and references therein, which are however beyond the scope of the current review.

Following (Kantorovich, 1983, 1988, 2000a, b), in a construction of a quantum mechanical / molecular mechanical model it is useful to consider the whole extended system as a set of structural elements, each of which is associated with an electronic group, which, as mentioned, can be realised as an ion or a chemical bond. Such structural elements may be polarised or locally excited (transferred into an excited state) by external fields but remain intact, if the fields are sufficiently weak. In the core defect region, however, the structural elements may be destroyed by charge transfer, chemical bonding or atoms (ions) added or removed from the system. Ideally, therefore, separation of the whole system into the QM cluster of interest and the surroundings described with an MM method should avoid cutting through structural elements, which allows one to associate with the QM cluster a given integral number of electrons, subject to treatments with standard methods of molecular quantum chemistry.

2.1.3 Electronic structure of the defect containing system and the choice of the method

Considering the electronic structure, materials are classed as insulators or semiconductors – where all electrons can be localised in a relatively small volume – or metals, semi-metals and degenerate semiconductors – where localised electronic states still extend over many hundreds atoms and beyond. In practice, quantum mechanical / molecular mechanical methods are immediately applicable to a particular material, if the characteristic Bohr radius of the most extended occupied localised state is below that of a computationally tractable QM cluster. This approach is therefore suitable and practical for all strongly ionic (*e.g*., NaCl, Al_2O_3, or GaN) and covalent compounds (*e.g*., diamond C, Si, Ge, saturated carbon chain and silicone polymers, *etc*.), but also for intermediate heteropolar materials (*e.g*. SiO_2, SiC, MoS_2, *etc.*) and molecular solids (e.g. elemental arsenic, selenium or sulphur, fullerenes, ice, hydrocarbons, etc.). Materials with naturally extended states would be more appropriately modelled with alternative approaches – where the electron separation into groups is performed, for example in reciprocal rather than real space, which is, however, beyond the scope of this chapter. Alternatively, a real space approach to embedding, which allows for nonlocal perturbed solutions to be constructed, usually within a Green's Functions formalism, may be employed, which, however, poses the problem of how to account for atomic relaxations –*cf*. *e.g*. (Grimley and Pisani, 1974; Inglesfield, 1981; James and Woodley, 1996; Pisani, 1978; Pisani et al., 1979; Pisani et al., 1983; Woodley, 1997) and the perturbed crystal approaches mentioned above. If the perturbation of an extended system is assumed to be localised within a small region, then one of the above approaches can still be applied, and the relaxation performed within the cluster, *cf*. (Evarestov et al., 1989; Gao and Wang, 1990; Kruger and Rösch, 1994). The problem of self-consistent mutual polarisation of the surroundings and the inner QM cluster has in turn been addressed in (Harding et al., 1985; Kantorovich, 1983; Pascual and Seijo, 1995; Pisani et al., 1994).

A number of alternative approaches have been developed and implemented in computer codes to treat point defects in ionic solids, with ICECAP (Harding et al., 1985; Kunz and Vail, 1988; Vail, 1990), CLUSTER (Kantorovich, 1988; Shluger et al., 1993; Shluger et al., 1986), GUESS (Mukhopadhyay et al., 2004; Sulimov et al., 2002; Sushko et al., 2000), and AIMP (Pascual et al., 2009; Pascual and Seijo, 1995; Seijo and Barandiarán, 1996, 1999), with extensions to covalent and other materials and types of application reported for example by (Åqvist and Warshel, 1993; Bakowies and Thiel, 1996b; Bredow et al., 1996; Greatbanks et al., 1994; Johnson and Truong, 1999; Kachurovskaya et al., 2002; Nasluzov et al., 2003; Pelmenschikov et al., 1995, 1998; Sierka and Joachim Sauer, 1997; Tchougréeff, 1999). These implementations differ in:

- The boundary conditions on the cluster – *cf*. the perturbed cluster approach (Pisani, 1993; Pisani et al., 1994; Pisani et al., 2000) with the embedded and cyclic cluster approaches (Bredow et al., 1996; Janetzko et al., 2002) and the localisation procedure – *cf*. (Huzinaga et al., 1987; Pascual and Seijo, 1995; Seijo and Barandiarán, 1996) employing the strong orthogonality approximation with the more general approach in (Shidlovskaya, 2002);
- The cluster termination employed for semi-covalent materials – *cf*. the popular hydrogen termination with the semiempirical link atom and *ab initio* separable potential operator approaches (Abarenkov et al., 2011; Abarenkov and Tupitsyn, 2001; Antes and Thiel, 1999; Nasluzov et al., 2003; Sushko et al., 2000) and the use of the anti-symmetrised product of strictly localized geminals (Tchougréeff, 1999);
- The level of coupling between the cluster and its environment, which is particularly important in the context of semi-covalent and biological materials – *cf*. (Bakowies and Thiel, 1996b; Field et al., 1990; Greatbanks et al., 1994; Sierka and Joachim Sauer, 1997).

Hybrid quantum mechanical / molecular mechanical techniques have a close relationship with fragmentation methods that simulate a large system of interest as a set of small interacting fragments, which, in fact, may even overlap with each other, reminiscent of divide-and-conquer geometry optimisation strategies (Brinkmann et al., 2014; Fabiano et al., 2014; Gordon et al., 2012; Guo et al., 2012; He et al., 2009; Yang, 1991).

Now we will focus on the hybrid quantum mechanical / molecular mechanical approaches that have been implemented in the computational chemistry software ChemShell (Sherwood et al., 2003) with exemplar applications highlighted in Section 3.

2.2 ChemShell quantum mechanical / molecular mechanical coupling and master equations

ChemShell, a particularly effective package for quantum mechanical / molecular mechanical modelling that implements a number of alternative additive and subtractive approaches, has been developed by Sherwood and co-workers (Keal et al., 2011; Metz et al., 2014; Sherwood et al., 2003). Indeed, this package has been widely used in both biomolecular and solid state modelling.

Quantum mechanical / molecular mechanical schemes implemented in ChemShell can be classified by the nature of the total energy expression (Metz et al., 2014; Sherwood, 2000; Sherwood et al., 2003). The most physically intuitive, “additive” approach is to construct a total QM/MM energy expression as a sum of the QM, MM and coupling terms:

$$E=E(Inner,QM)+E(Outer,MM)+E(Couple,QM/MM) \tag{1}$$

where, E(Couple,QM/MM) accounts for all interactions between the two regions, for example, classically handled bonding and van der Waals interactions and modifications to the QM Hamiltonian to reflect the electrostatic influence of some or all of the atoms in the outer region. Link atoms, used to saturate cluster terminating bonds in homo- or hetero-polar covalent materials, need to be tackled carefully in this approach, to avoid unwanted QM/MM interactions. Indeed, in the additive zeolite model of (Sherwood et al., 1997), employed in the application work highlighted below, the link atoms are the intrinsic part of the Inner region. Both QM and MM centres enter bending and torsional, 3- and 4-body MM terms, but due to the electrostatic coupling between MM and QM regions they are also accounted for by the QM term. To exclude such unphysical double counting, the corresponding MM terms are omitted in the calculation of the total energy and its derivatives (forces).

An alternative “subtractive” approach applies the MM scheme to the whole (Inner+Outer) system and then corrects for the double counting by subtracting the energy (computed at the classical level) of the Inner system:

$$E=E(Inner+Link,QM)+E(Inner+Outer,MM)-E(Inner+Link,MM) \quad . \tag{2}$$

Thus, the double counting of the Inner region energy and the link atom contributions (that are a model artefact) are removed. This approach relies on the MM energy closely reproducing the QM energy landscape for the Inner and Link regions. Many subtractive approaches, perhaps most notably of the multi-shell ONIOM type – *cf*. (Guo et al., 2012; Svensson et al., 1996), include all coupling between the inner and outer regions at the MM level of theory, and the QM calculation is performed effectively as a gas-phase cluster calculation.

The additive and subtractive schemes are two extreme cases, and a number of intermediate approaches are possible. If both of the MM terms in Eq. (2) are redefined to include only an outer shell of the Inner region including link atoms, the requirement of a close match between MM and QM landscapes for the core Inner region is lifted. The subtractive approach can still be exploited to delete the terms associated with the link atoms. In the limit of excluding all Inner atoms one arrives at an additive expression with a link atom correction:

$$E=E(Inner+Link,QM)+E(Outer,MM)-E(Couple,QM/MM)-E(Link,MM) \quad \textbf{(3)}$$

.

A careful analysis of these approaches using such artificial objects as link atoms and their foundations can be found in (Tchougréeff, 2008).

Further, the quantum mechanical / molecular mechanical coupling can be classified depending on how the MM centres affect the electrons that are described with the QM terms and *vice versa*. Following (Bakowies and Thiel, 1996a; Sherwood, 2000; Sherwood et al., 2003), we distinguish: (A) mechanical embedding, where MM centres are not present in the QM calculation, and affect the QM centres only via interatomic potentials (a force field); (B) electrostatic embedding, where the charge distribution of the surroundings that are represented via MM centres is included in the QM Hamiltonian; (C) polarised embedding, in which MM centres respond to the electrostatic field of the QM region but their response in not included fed back into QM Hamiltonian; and (D) self-consistent polarised embedding, where both the electrostatic embedding and polarisation effects are treated self-consistently. Only self-consistent schemes (D) truly minimise the total QM/MM energy (1) with respect to both electronic and nuclear degrees of freedom in the entire system. As the response by MM centres, in most interesting cases of charged and strongly dipolar defects, reactive centres or adsorbates is nonlinear (the system is essentially anharmonic), the calculation of the total energy and forces has to be iterative, with a number of improvements already implemented or under development in ChemShell (Kantorovich, 1983; Kästner et al., 2009; Kästner et al., 2007; Metz et al., 2014; Sokol et al., 2004).

In our work on ionic systems, including insulating metal oxides and wide-gap semiconductors, we have employed a so-called solid-state embedding scheme, which places semi-local pseudopotentials around the QM cluster, typically on cations, which constitutes an Interface region of the QM/MM model. The primary rôle of such pseudopotentials is to provide an appropriate account for short-range exchange and correlation embedding potential between the QM electrons group and the surroundings. In the case of cations in low oxidations states, standard quantum-chemical effective large-core potential parameterisations are commonly employed, which however proves often too crude, and, typically, for cations is high oxidation states and / or transition metal ions, a customary parameterisation is necessary.

For zeolites, a hetero-polar covalent binding model is assumed for the siliceous framework of the material, where each pair of Si and O atoms is linked by a σ bond, which is a linear combination of sp^3 hybrids residing on both atoms. The preferential occupation of O-centred states, makes it anionic with 2 lone pairs and two σ bonds per centre, whereas the occupation of Si-centred states renders it cationic supporting four σ bonds in a nearly perfect tetrahedral coordination. Using the link atom terminating scheme, at the boundary of the QM region any terminating dangling bond is saturated by hydrogen placed at an appropriate O–H or Si–H distance away from the QM bonded centre along the corresponding QM–MM bond. The counterpart MM atom in this case is removed from the MM model and its charge is withdrawn onto further nearest MM neighbours. The resulting local bond dipoles induced by this termination procedure are compensated by adding to the model one pair of point charges forming opposite dipoles as described in (Sherwood et al., 1997).

As mentioned in the Introduction, a particular strength of quantum mechanical / molecular mechanical approaches is their ability to describe systems in different charge states with respect to the same reference level. For charged species, polarisation outside the core defect region can be efficiently and accurately described in the continuous medium (dielectric continuum) approximation, which we employ to describe the contribution from the system surroundings beyond a certain cut-off radius, R. In this case, the total energy of the system can be shown to include the energy of the charged defect in the Madelung potential of the whole system, the self-energy of the defect, both of which are calculated via (1), and the polarisation energy included *a posteriori*. Following (Jost, 1933), this correction can be obtained from a continuum approximation to a system of interest beyond a certain cut-off radius; it has, in fact, been first derived for bulk systems in (Born, 1920) as the heat of hydration of an ion, and reads as follows:

$$E^{Born} = \frac{-Q^2}{2R}\left(1 - \frac{1}{\epsilon}\right) \quad , \qquad \textbf{(4)}$$

where *Q* is the charge, and ϵ is the effective isotropic dielectric constant. This formula was later complemented with a similar result for the solvation of a dipole, μ (Onsager, 1936):

$$E^{Onsager} = \frac{-2\mu^2}{R^3}\frac{\epsilon - 1}{2\epsilon + 1}, \qquad \textbf{(5)}$$

which can be readily extended to higher multipoles, with the leading quadrupolar term decaying as R^{-5} and effectively vanishing at typical cut-off radii in modern simulations. In vertical (optical) processes, the ionised system should be described using the high-frequency constant and in adiabatic processes the static constant should be employed instead.

2.3 Practicalities of setting up a quantum mechanical / molecular mechanical calculation

A quantum mechanical / molecular mechanical calculation should be preceded by the set-up procedure, a nontrivial process that typically includes the following steps:

- Geometry optimization of the extended system, which may need to be preceded by system preparation; for example, solid solutions may require selection of a representative sample and polar surfaces have to be reconstructed;
- An active, or defect site of interest is chosen in the extended system and a cluster is carved, for example based on a cut-off radius – spherically shaped clusters are generated for bulk systems and interfaces while hemi-spherical for surfaces;
- Atomic centres within the cut cluster are designated for QM, MM, and QM/MM calculations according to the hybrid QM/MM coupling scheme. In particular, it is important to identify atoms around the active site, which should be treated at a QM level of theory, and introduce relevant chemical modifications in the model, *i.e.*, point defects and adsorbates;
- In the case of electrostatically coupled models, the terminating centres have their charge modified and extra point charges are placed around the carved cluster to simulate the Madelung field of the extended system within the cluster inner core.

These steps are illustrated in Figure 1 for an ionic system. We highlight regions in the model within which atomic centres play different rôles (five in total): (a) QM region—all centres are treated quantum mechanically; (b) Interface region — all atoms bear pseudopotentials, which provide a localising embedding potential for the QM region, but interact with each other and MM centres by classical interatomic potentials; (c) active MM region – all centres are fully relaxed; (d) inactive, or frozen MM region, – all centres are fixed; and (e) terminating point charges maintaining the Madelung potential within the active region.

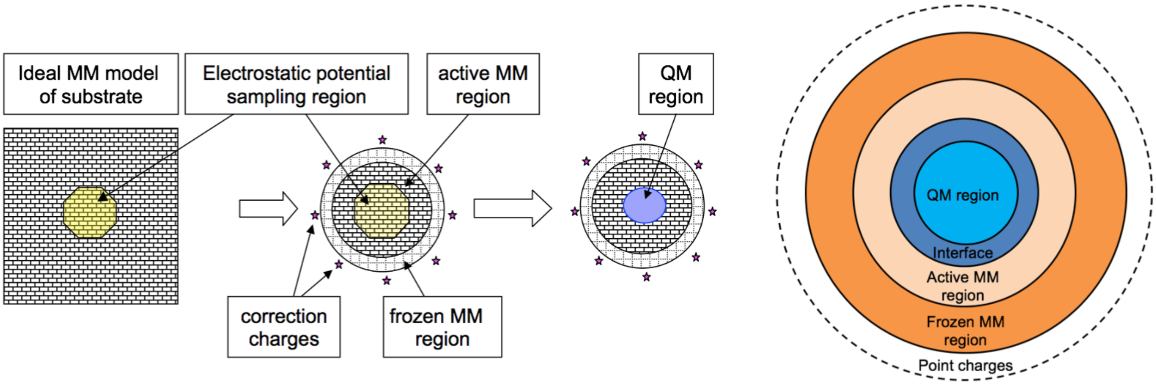

Figure 1. A sketch of a quantum mechanical / molecular mechanical model. A model generation is illustrated on the left with QM and MM regions highlighted, rendered from Figure 2 of (Sherwood et al., 2003). A representative 5-region model employed in the study of defects in ionic compounds is shown on the right.

This procedure is implemented in the code ChemShell (Metz et al., 2014; Sherwood et al., 2003; Sherwood et al., 1997; Sokol et al., 2004); other software packages also implement this method or other closely related procedures.

In the next section we present applications of this methodology to physical and chemical problems arising in the study of catalytic and energy materials.

3 Recent applications of quantum mechanical / molecular mechanical approaches

Quantum mechanical / molecular mechanical techniques are increasingly becoming the methods of choice in modelling localised states in solids. We now illustrate the range of applications by examining a number of key areas relating to electronic structure, to defect structures energies and properties, including processes of importance in energy materials and to reactivity and catalysis.

First we focus on the calculation of ionisation and redox potentials that, as argued in the Introduction, raises problems with 3D periodic methods which are absent within the quantum mechanical / molecular mechanical.

3.1 Ionisation potentials

In many applications, and to provide a fundamental view of materials, the absolute position of electronic energy bands is of primary interest. Measurements of the ionisation potentials, work functions and Fermi levels on variously prepared surfaces and interfaces require computational input for interpretation, analysis and general prediction and design. As mentioned in the Introduction, energies of processes that involve a change in the charge of a local centre are exceptionally problematic for supercell approaches. In contrast, the additive hybrid QM/MM techniques with electrostatic coupling between the QM cluster and its surroundings, including polarisation effects, are well suited to tackle this challenge (Sherwood et al., 2003). Explicit simulations of charged models with a common reference, in bulk or on surfaces, provide a unified approach to the modelling of both vertical (or optical, i.e. involving relaxation only of the electronic subsystem) and adiabatic (or thermodynamic, involving relaxation of ions and electrons) processes that are characterised by ionisation potentials and/or electron affinities. We note that such simulations are also key for modelling chemical processes with charged intermediates, such as deprotonation.

The vertical ionisation potential (or vertical ionisation energy, I_v), can be computed as:

$$\begin{gathered} +¿ \\ system^{¿} \\ ¿ \\ -¿ \\ e^{¿} \\ I_v = E¿ \end{gathered} \quad , \qquad \textbf{(6)}$$

where $E\left(system^0\right)$ is the total energy of the relaxed neutral system of interest, $\begin{gathered} +¿ \\ system^{¿} \\ E¿ \end{gathered}$ the total energy of the singly (positively) charged system with the same atomic configuration, and $\begin{gathered} -¿ \\ e^{¿} \\ E¿ \end{gathered}$ is the energy of an isolated electron (at the reference vacuum level). The equation for I_v can be applied to a model with any dimensionality, making it suitable for application to bulk and surface models. This approach has been adopted, for example, by Scanlon *et al.* to determine the energy band alignment between the two most abundant and technologically important TiO_2 polymorphs, rutile and anatase (Scanlon et al., 2013). The band alignment was shown to be of Type II – staggered – with the anatase bands lying below those of rutile, in contrast to the previous, widely cited work of (Kavan et al., 1996) and in agreement with accompanying experiments. This result has rationalised the improved charge carrier separation in mixed phase TiO_2 crystals. Buckeridge *et al.* then determined the band alignment of a wide range of TiO_2 polymorphs (illustrated in Figure 2), showing how the photocatalytic performance of mixed-phase samples could be improved with novel combinations of different crystal structures (Buckeridge et al., 2015a).

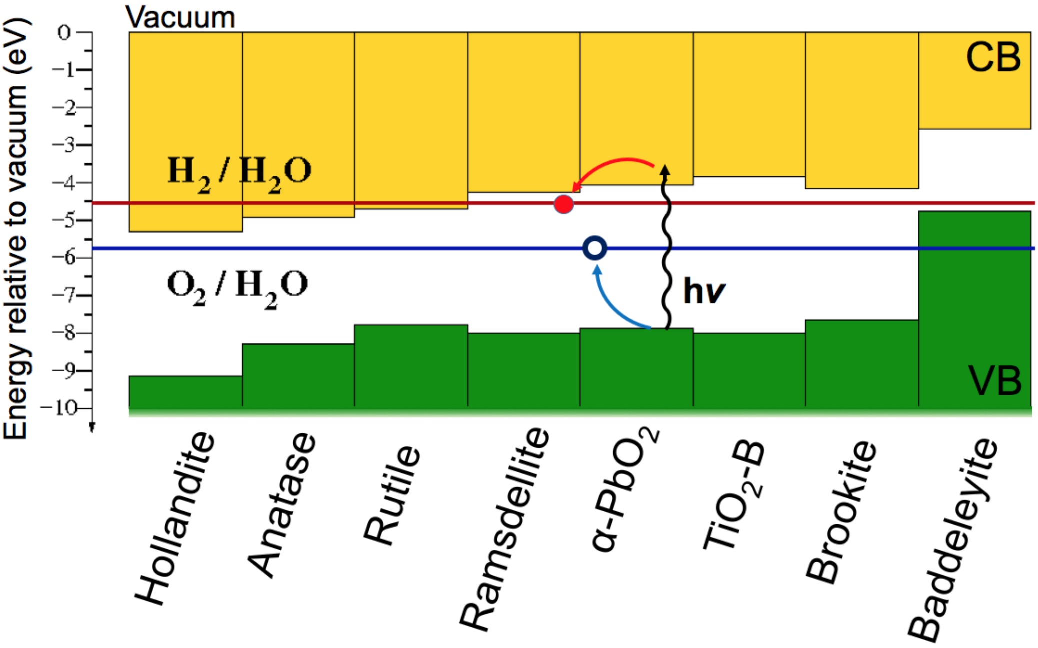

Figure 2. Quantum mechanical / molecular mechanical calculations using ChemShell showing the conduction band (CB) and valence band (VB) positions relative to the vacuum level for the various TiO2 polymorphs, shown in comparison with the H_2 and O_2 redox potentials.

The study of the polymorphs of titania followed the initial investigation into the electron binding energies of other binary wide-gap semiconducting compounds including ZnO and GaN, which allowed the authors to determine their band offset (Walsh et al., 2011). Bulk studies are, however, rare due to the poor mathematical definition and inaccessibility of the bulk ionisation potential to experiment. A step towards linking the bulk and surface ionisation potentials has been reported in the investigation of the electrostatic potential in SnO_2 (Butler et al., 2014). The direct measurability of surface ionisation potentials and electron work functions has motivated several quantum mechanical / molecular mechanical studies, a representative sample of which we will now discuss.

The most tested material in this regard is MgO, the rock-salt structure of which exposes a very stable (100) surface. Initial calculations gave an ionisation potential of 6.7 eV (Kantorovich et al., 1999; Sushko et al., 2000), which was consistent with the experimental value of 6.7 ± 0.4 eV (Kantorovich et al., 1999). In that study, however, the authors did not consider the long-range surface polarisation. Subsequent work including this effect has resulted in I_v=6.46 eV, still in very good agreement with experiment (Downing et al., 2014b). Complementary to I_v one can compute the adiabatic ionisation potential, I_a, which also allows the surrounding ions to respond to a charged defect. (Downing et al., 2014b) have determined I_a=5.52 eV, *i.e.* roughly 1 eV lower than I_v.

Whilst pristine surfaces are fundamentally important, more commonly it is intrinsic surface defects that offer reactivity for chemical applications including catalysis. Following the standard theory of chemical (acid, base and redox) reactivity, the active sites are characterised by their electronegativity and hardness (or softness) that are, in turn, determined by the electron (or proton) ionisation potential and affinity. For metal oxides, a well-studied example of such a site is the neutral F^0 centre, the formation of which leaves two electrons trapped in the vacant oxygen lattice site. On reaction with electron deficient species, this site can be ionised, resulting in an F^+ centre. Using embedded-cluster quantum mechanical / molecular mechanical techniques, one can calculate such ionisation energies (often referred to as defect levels) accurately: (Ferrari and Pacchioni, 1995) reported the I_v of an F^0 and F^+ centre on (100) MgO as 4.71 and 9.69 eV, respectively. A subsequent study that included long-range polarisation (Downing et al., 2014b) reduced these levels to 3.05 and 4.55 eV, respectively, which indicated that both defect levels sit in the middle of the band gap. Pacchioni *et al.* demonstrated that I_v depends on the surface coordination of the defect site: for an edge vacancy (i.e. lower coordinated), their calculated I_v for the F^0 centre reduced from 4.71 to 3.81 eV, a result confirmed by further studies (Ferrari and Pacchioni, 1995; Norgett, 1971; Paganini et al., 1999; Sousa et al., 1999). The strong dependence of the defect levels on the surrounding electrostatic environment has further been highlighted in a quantum mechanical / molecular mechanical study of reconstructed ZnO surface sites (Sokol et al., 2004), the authors of which found a significant destabilisation of electron levels in isovalent vacant oxygen interstitial surface sites compared to “true” surface vacancies by *ca*. 1.57 eV.

3.2 Modelling point defects with hybrid quantum mechanical / molecular mechanical methods

The hybrid quantum mechanical / molecular mechanical embedded cluster technique is particularly suited to modelling point defects and defect clusters in ionic solids. The methodology is based on the two-region approach of Mott and Littleton (Catlow and Mackrodt, 1982; Lidiard, 1989; Mott and Littleton, 1938; Norgett et al., 1974), whereby the system is divided into a central region that includes the defect, in which ionic and electronic relaxations are treated explicitly, and a surrounding region where the dielectric response of the infinite solid is calculated using a more simplified relaxation technique combined with a continuum picture, allowing one to model defects at the true dilute limit. With hybrid quantum mechanical / molecular mechanical methods, the local relaxations around a defect can be determined to a high degree of accuracy using density functional theory or beyond within the QM part of the calculation, while the polarization of the surrounding infinite system can be computed as precisely as the interatomic force field used in the MM part of the calculation allows (which can be very accurate if the force field reproduces well the structural, elastic and dielectric properties of the bulk material). This approach builds upon the earlier work of (Norgett, 1971), who modelled a vacancy in binary metal oxides using a set of Gaussian basis functions to describe the wave function, embedded within a lattice of polarisable ions represented by point charges. The boundary conditions allow one to access an absolute reference energy, from which ionisation energies can be calculated consistently (see Section 3.1), while two-dimensional (Berger et al., 2014; Berger et al., 2015; Downing et al., 2014a, b; Dutta et al., 2012; Goumans et al., 2008; Herschend et al., 2004; Keal et al., 2011; Logsdail et al., 2016; Sokol et al., 2004; Stefanovich and Truong, 1997) and one-dimensional (Buckeridge et al., 2013; Oliva et al., 2008) systems pose no significant problems to the technique.

The advantages of this approach, we reiterate, over the commonly used supercell method employing a plane-wave basis set are clear: no spurious interactions between periodic images of charged defects; access to a common reference; and ease of treatment of charged centres on surfaces and in wires. A drawback is the difficulty in treating defects with associated diffuse electron charge densities, such as shallow centres, where the density extends far beyond the confines of the QM region, although it is noteworthy that this difficulty also arises in plane-wave calculations, due to the limited size of the supercell used.

In the rest of this section we will first present an application of hybrid quantum mechanical / molecular mechanical methods to treat localised defects in wide-gap semiconductors, where we demonstrate the power of the technique, then discuss an approach to treating diffuse defect-related electron states. The interested reader can find alternative examples of the theory and application of hybrid quantum mechanical / molecular mechanical approaches to treat defects in solids in (Chen and Ortner, 2015a, b; Fang et al., 2015; Fang et al., 2014; Peng et al., 2008; Peng et al., 2010; Vörös et al., 2012; Zhang and Lu, 2008, 2010; Zhang et al., 2012).

3.2.1 Defect chemistry

Access to a common reference allows one to calculate defect ionizations, both vertical and adiabatic, in a consistent manner (Buckeridge et al., 2015b, c; Sokol et al., 2007). In predominantly ionic systems such as wide-gap semiconductors, most point defects of interest are localized in nature, and hence can be modelled quite well using hybrid quantum mechanical / molecular mechanical methods. In particular, reactions involving the formation and ionisation of point defects balanced by charge carriers can be determined accurately, as one can calculate the ionisation potential, giving the formation energy of a hole, and use a scissors operator approach, including the experimental band gap, to determine the electron affinity and therefore electron formation energies.

Such reactions and ionisations are key to understanding the defect properties of a material. In conventional semiconductors such as Si or GaAs, where the band gap is of the order of 1 eV, electronic disorder governs the defect chemistry allowing the materials to be easily *n*- or *p*-doped with concentrations of free carriers controlled by the doping level. At the other extreme, in insulators, ionic disorder dominates via the formation of compensating sets of defects such as Frenkel pairs or stoichiometric vacancies (Schottky defects). Wide-gap semiconductors fall somewhere in between these two extremes (see Figure 3). Whether ionic or electronic disorder dominates is not immediately clear and needs to be carefully worked out using the relevant defect reactions and ionisations.

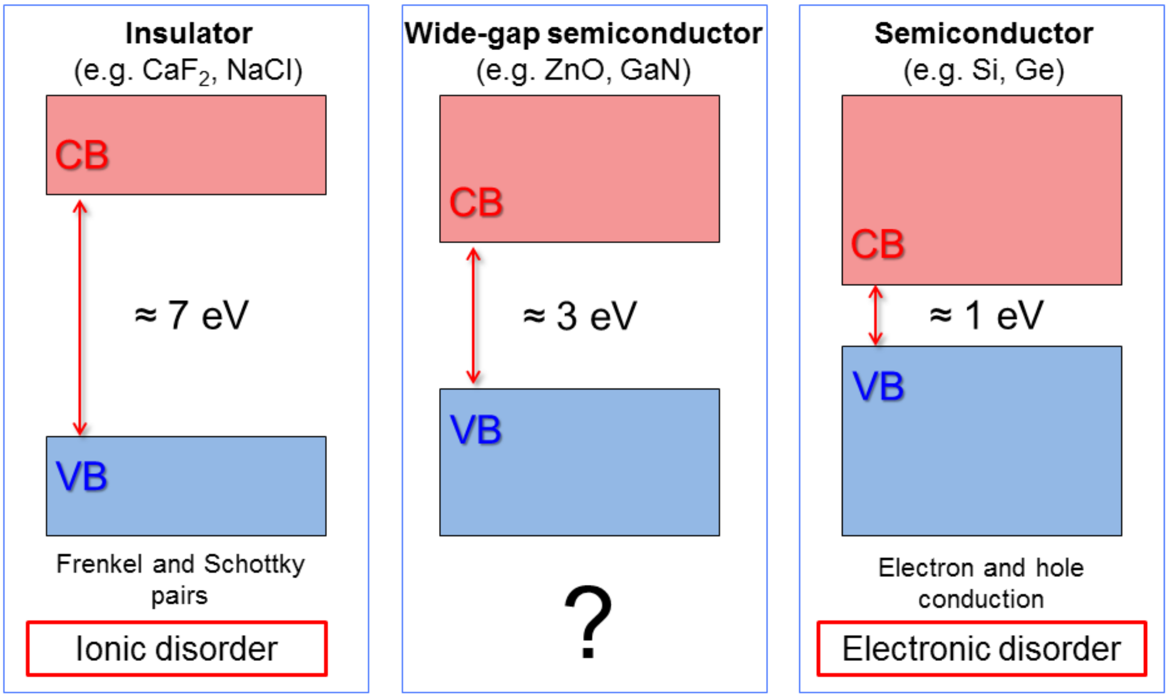

Figure 3. Cartoon depicting ionic *vs.* electronic disorder in conventional semiconductors, wide-gap semiconductors and insulators. Conduction bands (CB) are in red, valence bands (VB) in blue.

In the following we take ZnO and GaN as representative wide-gap semiconductors (Catlow et al., 2011; Walsh et al., 2013; Walsh et al., 2011). ZnO (band gap 3.44 eV) is a widely used transparent conducting oxide that is easily *n*-doped but cannot be reliably *p*-doped (Ellmer, 2008). GaN (band gap 3.5 eV) is an important material for high-powered microelectronics and solid state lighting which is intrinsically *n*-type, but can only be *p*-doped by introducing very high concentrations of Mg close to the solubility limit (Morkoç, 2009). Modifying concentrations of electrons and holes in such systems can be achieved by changing the stoichiometry or by introducing aliovalent dopants, compensated by charge carriers. For example, one can introduce a monovalent cation dopant on the Zn site in ZnO, such as Li^+, which has an effective negative charge and can be compensated by the formation of a hole, while a monovalent anion dopant on the O can be compensated by an electron. Compensation by charged point defects is, however, always in principle possible. In the case of *p*-doping in ZnO, an acceptor-introduced hole can be compensated by the formation of a positively-charged O vacancy, V_O, using the standard Kröger-Vink notation (Kröger and Vink, 1956):

$$h^{\cdot} \rightleftharpoons \frac{1}{2} V_O^{\cdot\cdot} + \frac{1}{4} O_2 \quad , \tag{7}$$

or by a Zn interstitial (Zn$_i$):

$$h^{\cdot} + \frac{1}{2} Zn(s) \rightleftharpoons \frac{1}{2} Zn_i^{\cdot\cdot}; \tag{8}$$

while for *n*-doping the relevant equations are (V$_{Zn}$ is a Zn vacancy and O$_i$ is an O interstitial):

$$e' + \frac{1}{4} O_2 \rightleftharpoons \frac{1}{2} O_i''; e' \rightleftharpoons \frac{1}{2} V_{Zn}'' + Zn(s) \quad . \tag{9}$$

If the energies of these reactions are appreciatively negative, then the charge carrier in question will be thermodynamically unstable with respect to the formation of the corresponding point defect.

The defect formation energies can be calculated using the relevant defect reaction. For example, the formation of a doubly ionized oxygen vacancy in ZnO is governed by the equation:

$$O_O^{\times} \rightleftharpoons V_O^{\cdot\cdot} + 2e' + \frac{1}{2} O_2 \quad , \tag{10}$$

where the reaction occurs in equilibrium with oxygen gas (termed 'anion-rich' conditions) and exchange of electrons at the Fermi level. Equilibrium with ZnO and zinc metal may also be assumed:

$$O_O^{\times} + Zn(s) \rightleftharpoons V_O^{\cdot\cdot} + 2e' + ZnO(s) \quad , \tag{11}$$

which corresponds to 'anion-poor' conditions. Such environmental conditions may also be applied to the defect reactions above governing the stability of electrons and holes. The corresponding equations for oxygen interstitials and metal defects are obvious, as is the application of this analysis to GaN.

The calculated energies of these reactions, determined using hybrid quantum mechanical / molecular mechanical methods, are presented in Table 1. In both materials, electron carriers are thermodynamically stable, while hole carriers are not, with respect to point defect formation. These results explain the observed ease with which both GaN and ZnO are *n*-doped and difficulty with which they are *p*-doped. Such analysis is much more challenging when using plane-wave supercell methods, due to the lack of a common reference and the problem of image charge interaction (we note that several schemes have been proposed to treat this image charge interaction problem, see e.g. (Buckeridge and Sokol, 2016; Farrow and Probert, 2011; Freysoldt et al., 2009; Gavartin et al., 2005; Kantorovich, 1999; Lany and Zunger, 2008; Lento et al., 2002; Leslie and Gillan, 1985; Makov and Payne, 1995; Murphy and Hine, 2013; Schultz, 2000; Wilson et al., 2008)).

ZnO	**ΔE_f (kJmol^{-1})**		**GaN**	**ΔE_f (kJmol^{-1})**	
	Anion poor	**Anion rich**		**Anion poor**	**Anion rich**
$h^{\cdot} \rightarrow \frac{1}{2} V_O^{\cdot\cdot}$	-250	-71	$h^{\cdot} \rightarrow \frac{1}{3} V_N^{\cdot\cdot\cdot}$	-114	-74
$h^{\cdot} \rightarrow \frac{1}{2} Zn_i^{\cdot\cdot}$	-227	-48	$h^{\cdot} \rightarrow \frac{1}{3} Ga_i^{\cdot\cdot}$	-50	-11
$e^{'} \rightarrow \frac{1}{2} O_i^{''}$	251	167	$e^{'} \rightarrow N_i^{'}$	567	450
$e^{'} \rightarrow \frac{1}{2} V_{Zn}^{''}$	345	72	$e^{'} \rightarrow \frac{1}{3} V_{Ga}^{'''}$	173	134

Table 1. Calculated reaction energies (ΔE_f) for processes in which charge carriers are compensated by point defect formation. Results are taken from (Walsh et al., 2013).

Of course, these reactions assume the condition of thermodynamic equilibrium with reservoirs of the relevant metals, gases and metal oxides/nitrides. One can always use experimental techniques to overcome these doping limits in wide-gap semiconductors, such as applying capping layers or epitaxially growing materials far from equilibrium to ‘freeze in’ defects with kinetic barriers. In doing so, however, one is constantly battling against thermodynamics. The ease with which such analysis can be carried out demonstrates the power of the hybrid quantum mechanical / molecular mechanical embedded cluster technique when applied to study defects in wide-gap systems. First-principles calculations of thermochemical properties are discussed in detail in Chapter 14.

3.2.2 Diffuse states

As mentioned above, the major limitation of hybrid quantum mechanical / molecular mechanical methods when studying defect structures is in treating defects with diffuse charge densities. A method to overcome this difficulty takes advantage of the two region approach: using hydrogenic eigenfunctions as the basis set, within the QM region one can determine a perturbing potential, or more simply apply a scissors operator approach to determine the defect energy, accounting for the quantum confinement effect, while outside a hydrogenic donor potential is assumed. The extent of the diffuse charge carrier within the two regions can be calculated semiclassically using effective mass theory. With this approach, more accurate defect levels can be calculated, as has been shown for the case of Si and O doping in GaN (Xie et al., 2016). The resulting thermal ionisation energies for the +/0 transition for both impurities, determined using two different exchange and correlation functionals, B97-2 (Wilson et al., 2001) and BB1k (Zhao et al., 2004), are shown in Table 2, compared with the experimentally determined values (Götz et al., 1996; Zolper et al., 1996). The agreement between theory and experiment is unprecedented for first-principles calculations of such ionisation levels. Further developments of this approach are underway within the hybrid QM/MM community.

	B97-2	**BB1k**	**Experiment**
Si_{Ga}	0.046	0.037	0.022±0.004
O_N	0.056	0.041	0.029

Table 2. Thermal ionisation energies of the +/0 transition for Si and O dopants in GaN, determined using the B97-2 and BB1k hybrid density functionals and compared with experiment (Götz et al., 1996; Zolper et al., 1996). Energies are in eV. Results are taken from (Xie et al., 2016).

3.3 Active sites in zeolites

Having discussed the application of quantum mechanical / molecular mechanical approaches to study the electronic properties of bulk, surface and defect structures in wide-gap systems, we turn our attention to quantum mechanical / molecular mechanical investigations of chemical reactivity in nanoporous materials, a topic of fundamental importance for catalytic science.

3.3.1 Deprotonation energies as a measure of site reactivity

While electron ionisation energies are key indices of Lewis acidity or basicity, the index of Brønsted acidity or basicity, *i.e.* the ability to donate protons, is of primary interest in catalysis over porous siliceous and aluminium silicate materials (zeolites). When SiO_2 frameworks are doped with cationic species, such as Al^{3+} replacing Si^{4+}, counterions must be introduced onto the framework to ensure charge neutrality. In basic zeolites, extra framework Li^+ or Na^+ ions (Lewis bases) are typically used as counterions, while in acidic, H^+ – forming framework hydroxyl groups – are found (Brønsted acids). The energy, E_{DP}, required to separate these protons from the framework, a key index of the acidity of the material, is given by:

$$\begin{gathered}-¿\\ Z-O^{¿}\\ ¿\\ +¿\\ H^{¿}\\ ¿\\ E_{DP}(Z-OH)=E¿\end{gathered} \qquad (12)$$

here Z – OH represents the zeolite material and Z – O^{-} is the deprotonated model with an overall charge of minus one, where an electron has been left on the framework oxygen species. $\begin{gathered}+¿\\ H^{¿}\\ E¿\end{gathered}$ is the energy of a proton at the reference vacuum level.

Early investigations of deprotonation energies for zeolites using a quantum mechanical / molecular mechanical approach were carried out by the groups of Sauer (Brändle and Sauer, 1998; Brändle et al., 1998; Eichler et al., 1997; Haase and Sauer, 2000; Haase et al., 1997; Sierka et al., 1998; Sierka and Joachim Sauer, 1997) and Sherwood (de Vries et al., 1999; Greatbanks et al., 1994; Sherwood et al., 1997). Extensive investigations were made to show how the type of the zeolitic framework influenced the deprotonation energy *as well as* the crystallographic position of the counterion within each framework (de Vries et al., 1999). Initial calculations (Brändle and Sauer, 1998) suggested that the deprotonation energies for the frameworks Y, CHA, MOR and ZSM-5 were ranked as Y (1171/kJ/mol) > CHA (1190 kJ/mol) > MOR (1195 kJ/mol) > ZSM-5 (1200 kJ/mol), with further tests of increasing concentrations of aluminium strongly correlated to increases in $E_{DP}(Z-OH)$ (Sierka et al., 1998). Following these studies, improvements in the description of long-range polarisation effects within the embedding environment (Ivanova Shor et al., 2005; O'Malley et al., 2016) have resulted in a reduction in deprotonation energies by ~10-20 kJ/mol. A recent study investigated two-dimensional zeolitic materials, showing that the deprotonation energy is reduced, relative to the three-dimensional case, due to the alteration of the long-electrostatic environment (Rybicki and Sauer, 2015); effected by the smaller dielectric constant. However, lower interaction energies between protonated molecules and negatively charged surfaces compensate for the reduced dimensionality of the substrate. As a consequence, overall adsorption energies for reaction intermediates are similar to those in bulk (Sauer, 2016).

A key application of QM/MM-calculated deprotonation energies can be found in an extensive study of the zeolite catalysed conversion of methanol to hydrocarbons (MTH) (O'Malley et al., 2016), a process that has been researched significantly since its initial discovery by Chang and Silvestri (Chang and Silvestri, 1977) and first commercialised in 1985 (Maiden, 1988). The authors presented a comparison of the deprotonation, methanol adsorption (E_{ads}), and methoxylation energies (E_{methox}) in H-ZSM-5 and H-Y zeolite frameworks, in an attempt to investigate the effect of framework topology and active site location on reactive sorbate behaviour. The main results are summarised in Table 3, and an example of their computed adsorption structures is shown in Figure 4.

System	**E_{methox}**	**E_{ads} - E_{methox}**
Me-Y + H_2O	-162.0	+17.6
Me-ZSM-5 (I2) + H_2O	-150.4	+27.1
Me-ZSM-5 (M7) + H_2O	-138.6	+56.2
Me-ZSM-5 (Z6) + H_2O	-118.0	+18.6

Table 3 Methoxylation energies in (kJ mol^{-1}) of the four zeolite systems and energy difference between the adsorbed state and the methoxylated state.

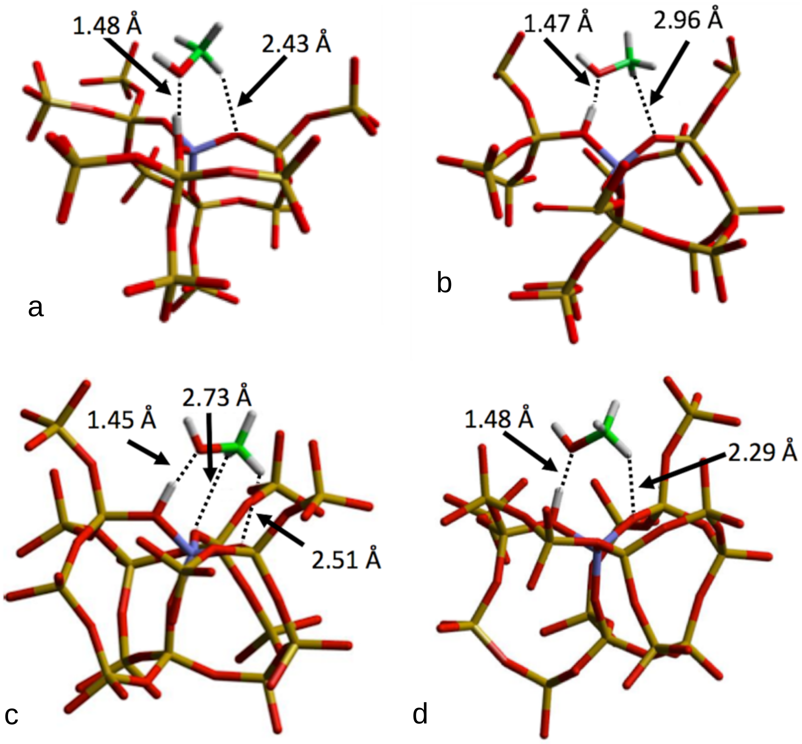

Figure 4. Optimised configuations of methanol adsorbed in a) H-Y, b) H-ZSM-5 (I2), c) H-ZSM-5 (M7), and d) the H-ZSM-5 (Z6). The shortest distances between methanol and host framework atoms are highlighted by dotted lines.

The order of methoxylation energy proved to be H-Y < H-ZSM-5 (Z6) < H-ZSM-5 (M7) < H-ZSM5-I2 (I2), which was in contrast to the order of methanol adsorption energies (H-ZSM-5 (M7) < H-Y < H-ZSM-5 (I2) < H-ZSM-5 (Z6)), at least when only one methanol molecule is present in the immediate environment. The study highlighted the importance of highly coordinated structures in stabilising the system when considering the interaction of sorbates such as methanol with the active sites of acidic zeolite catalysts.

3.3.2 REDOX potentials

An alternative measure of chemical reactivity is a material's redox potential that is determined as the energy of (de)hydrogenation, given by:

$$E_{DH}(Z-OH)=E(Z-O)-E(Z-OH)+\frac{1}{2}E(H_2) \quad , \tag{13}$$

in analogy with the deprotonation energy described by Equation (12). In this case, the system of interest is neutral, which implies that, in principle, it would be possible to calculate E_{DH} using more common supercell methods; however, the typical size of siliceous unit cells, which can include hundreds of atoms, means that higher levels of theory are only affordable employing quantum mechanical / molecular mechanical models. Rather than the deprotonation energy considered above, for transition metal doped zeolites (and other related materials) that exhibit high sustainable redox activity, the dehydrogenation energy provides an effective tool for characterisation, as the process results in the oxidation of the corresponding cation.

Rather than investigating full reactions with quantum mechanical / molecular mechanical methods, redox potentials can be calculated to provide a useful index for the reactivity of a system. (Berger et al., 2014) showed that Fe(II) is more stable than Fe(III) when doped into the MFI siliceous framework, although the computed dehydrogenation energy varied with the level of theory used: with standard density functional theory, different sites were found to have dehydrogenation energies of 0.16 – 0.39 eV; this range decreased to -0.04 to 0.09 eV when hybrid density functionals were employed, i.e. the energy difference between the Fe(II) and Fe(III) states was insignificant. However, more sophisticated many body wave function theory calculations, that include unoccupied orbitals in their description of electron correlation, stabilised the Fe(II), resulting in oxidation energies of 0.18 – 0.34 eV, but with different ordering to that of standard DFT.

In contrast, Ti(IV) has been shown to be the most stable oxidation state for TS-1, an MFI framework doped with Ti. (To et al., 2006b) studied a range of oxidation states and geometries for such Ti dopants, concluding that reduction from Ti(IV) to Ti(III) is easiest on a low-coordinated site. The reduction of a tripodal (3-coordinate) Ti(IV) cation to Ti(III) was found to be endothermic by 120-175 kJ/mol, while for a 4-coordinate species, *i.e.* part of the standard MFI framework, the Ti(III) species was an additional 290 kJ/mol higher in energy. These results showed that an inverted configuration, with the Ti dopant extending into the framework's porous channels, was considerably more favourable for reactivity than sites within the framework, as expected. This point was further confirmed when considering 2-coordinated Ti cations, whereby the oxidation of Ti(III) to Ti(IV) was reduced by 70 kJ/mol, compared with 3-coordinated sites. The results of this study are discussed in another context in the next section.

3.3.3 Coordination of metal centres

Incorporation of metal sites in zeolitic frameworks has been the subject of numerous investigations, some of which we discuss here. The material we introduced above, a Ti doped silicalite-1 (TS-1) (Taramasso et al., 1983) is a siliceous counterpart of ZSM-5, which has been of great interest, since it is used with aqueous H_2O_2 as the primary oxidant in a large number of important reactions such as the epoxidation of linear olefins, hydroxylation of aromatics, and oxidation of alcohols, amines, linear alkenes, sulphur compounds and ethers (Bordiga et al., 2001).

(French et al., 2004; To et al., 2006a; To et al., 2006b) have proposed a number of structural models for a Ti species that would result from the framework hydrolysis and site inversion that takes place during synthesis, post-synthetic treatment and / or, depending on the catalytic process, during the catalytic transformation. The structures of interest are shown in Figure 5.

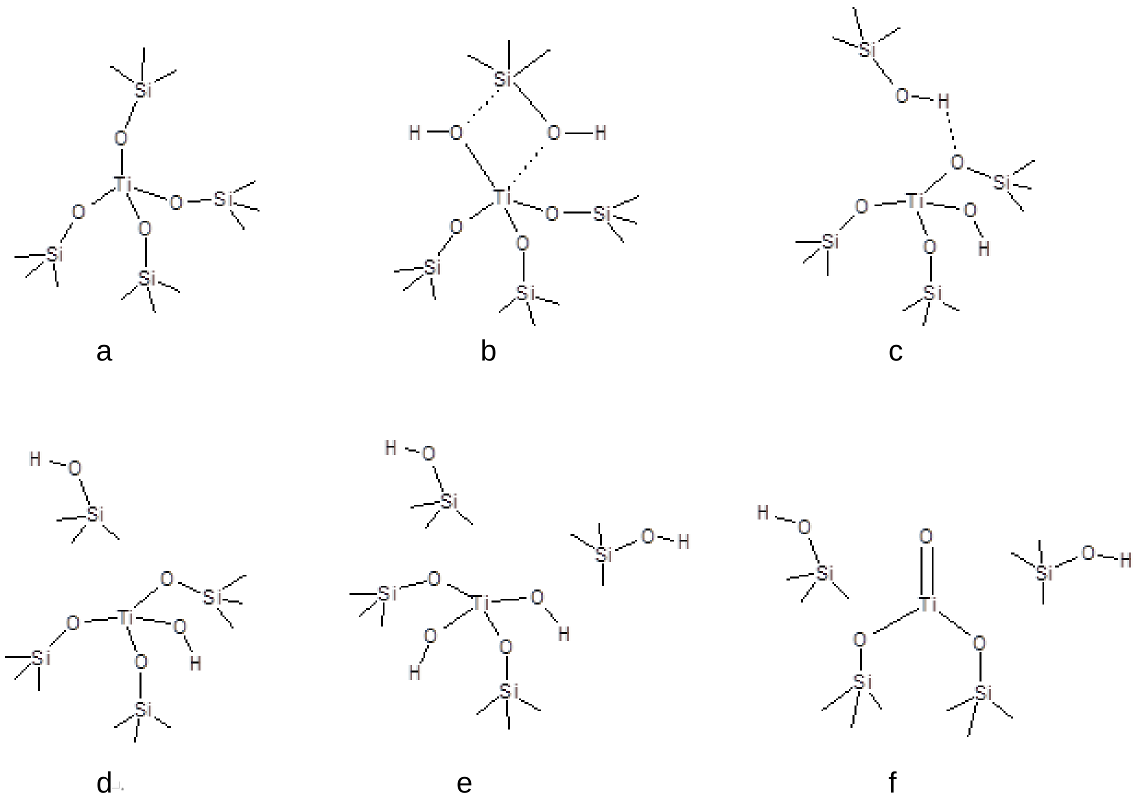

Figure 5 The six models used to represent the active Ti sites in TS-1: (a) tetrapodal, (b) 2MR, (c) tripodal (1I), (d) tripodal (2I), (e) bipodal, and (f) titanyl.

As the calculated geometries of the tripodal sites were in excellent agreement with experimental extended X-ray absorption fine structure (EXAFS) data (Gleeson et al., 2000), they helped rationalise the difference in the observed spectroscopic signature of metal centres in differently treated samples. The analysis of site transformation in the presence of water (Danyliv et al., 2007) resulted in the crucial conclusion that the catalytically active sites in TS-1 are hydrated tripodal rather than tetrapodal, a strikingly similar situation to that of other Ti-doped mesoporous materials previously explored using simple cluster models (Barker et al., 2002).

By considering the site reactivity towards H_2O_2 in anhydrous and hydrous conditions, (To et al., 2008) confirmed the close competition between η^1 and η^2 complexes as candidates for the oxygen donating species in TS-1. Curiously, from electron paramagnetic resonance (EPR) studies, a much higher energy (by *ca*. 400-450 kJ mol^{-1}) superoxide radical species had been reported to emerge as a result of interaction of aqueous H_2O_2 and non-aqueous urea H_2O_2 (Shetti et al., 2003). By calculating the related spectroscopic signature, (To et al., 2008) unambiguously identified the superoxide supported on tripodal Ti as the magnetically active centre (see Table 4, where the calculated components of the **g** tensor are compared to experimental values, which are expected to be accurate to at least the third decimal place), which allowed them to argue in support of the tetrahedral site inversion mechanism as essential for promoting the catalytic activity of these systems. In the absence of definitive structural experimental characterisation, this combination of spectroscopic evidence and supporting computational results is key to unravelling the mechanisms of catalysis in atomistic and electronic detail (for more examples of such combinations see Chapters 9, 19, 21 of this volume).

g tensor	Calculated (B3LYP) at BB1K geometry	Experiment[a]	
		aqueous H_2O_2	urea-H_2O_2
g_x	2.003	2.002	2.004
g_y	2.010	2.009	2.010
g_z	2.025	2.026	2.028

Table 4 EPR spin Hamiltonian parameters of the Ti(OO·) radical species. a Ref. (Shetti et al., 2003)

Clearly, it can be seen that the quantum mechanical / molecular mechanical embedded-cluster methodology is beneficial in the above cases due to its ability to (a) deal with charged systems and calculate energies with a common reference, and (b) reduce the size of the system of interest to that of a finite fragment, while ensuring high accuracy is maintained. As a consequence, complex reactions using the substrate material as a heterogeneous catalyst can be studied effectively.

4 Final remarks

The applications discussed above clearly show the range and power of the hybrid quantum mechanical / molecular mechanical technique. Furthermore, as we have argued, for many problems in the science of defects and in catalysis this technique is increasingly the method of choice. The approach offers the possibility of achieving high levels of precision by the use of highly accurate QM techniques in the inner region as elegantly illustrated by the recent work of Piccini *et al*. (Piccini et al., 2016) on catalytic reactions in zeolites (see also (Catlow, 2016) for a discussion of the implications of this work). Applications in catalytic science are expected to be particularly fruitful in future work. The method also offers the possibility of enhanced accuracy in modelling defects in key materials such as those used as transparent conducting oxides. The biggest practical challenges relate to the development of viable interatomic potentials for new materials and to improvements in the usability of the techniques. However, with the growing sophistication of the software for quantum mechanical / molecular mechanical modelling, the range and impact of the applications of the techniques in solid state science will rapidly grow.

Acknowledgements

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