Predictive Control Approach for Restricted Areas Avoidance of Autonomous System

Sutrisno
Department of Mathematics
Diponegoro University
Semarang, Indonesia
tresno.math@live.undip.ac.id

Widowati
Department of Mathematics
Diponegoro University
Semarang, Indonesia
widowati_math@undip.ac.id

Sunarsih
Department of Mathematics
Diponegoro University
Semarang, Indonesia
narsih_pdil@yahoo.com

Kartono
Department of Mathematics
Diponegoro University
Semarang, Indonesia
kartonoundip@gmail.com

***Abstract*—In this paper, we have formulated and simulated a hybrid dynamics in a piecewise affine (PWA) model of an autonomous system for several restricted areas avoiding purposes. The scenario of the restricted area avoidance is formulated by labeling the normal (unrestricted) area and the restricted area as mode 0 and mode 1 respectively. The dynamics of the autonomous system is formulated as a PWA model governed by these two modes. We simulate dynamics of the given autonomous system as follows. The given autonomous system is initially located at some point/position and it have to reach some given final/target point/position with optimal condition by minimizing the trajectory and effort. To determine the optimal trajectory, we applied the model predictive control method to generate the optimal input so that the autonomous system avoids some given restricted areas. From the simulation results, the given autonomous system reached the target position and avoids the given restricted areas with optimal trajectory generated by the predictive controller.**

Keywords—Autonomous system, hybrid dynamical system, several restricted areas avoidance.

I. Introduction

Suppose that an autonomous system, located at its initial state or position, has to move or maneuver to some final or target state with optimal trajectory i.e. it has to reach the final state with minimal effort. Unfortunately, the movement's area/space contains several restricted areas that has to be avoided. To avoid these restricted areas, the controller have to generate the optimal input to obtain the optimal trajectory so that this autonomous system reaches the final state. Some researchers call the scenario of avoiding a restricted area as obstacle avoidance. If there is only one restricted area, it can be done by modeling it's dynamic as hybrid model in the piecewise-affine form by defining the obstacle area and normal area [1]. In the previous works, some researchers were applied the obstacle avoidance scenario with single obstacle on several autonomous systems like unmanned aerial vehicle of small scale helicopter, undersea unmanned vehicle, and mobile robot with dynamic environment [2]–[4]. The optimal trajectory for the obstacle avoidance scenario can be determined by using some optimization algorithm. Several optimization methods were used to solve the corresponding optimization problem of an optimal control problem for the obstacle avoidance scenario like particle swarm optimization [5], [6]. In the other hand, some researchers were developing the obstacle avoidance problem with several unmanned system by coordinating and controlling them by some coordination control method such as the flocking algorithm approach was used to coordinating multi agent dynamic system for single obstacle avoidance scenario [7], [8].

Piecewise-affine (PWA) model is one of several forms of hybrid dynamical systems. It consists of the dynamic of real the discrete and real valued variables, and their interaction. PWA model can be converted or transformed into equivalent mixed logical dynamic (MLD) form which is more suitable to design its controller [10]. This conversion can be done by using hybrid system toolbox for MATLAB by typing the PWA model in HYSDEL programming language and then convert it using function *mld* [9]. To control the MLD model, we can use model predictive control (MPC) for hybrid systems. MPC for hybrid system that was formulated based on classic MPC with some modifications such as the corresponding objective function [10], [11]. One of some reasons that we chose the predictive control method is the good performance that have been reported in many application areas. Some latest research reports were said that predictive control is a good tools in agriculture control systems [12], [13], mechanical vehicle roll-over maneuver control problem [14], boiler-turbine control problem [15] and spacecraft control systems [16].

In this paper, we formulate a hybrid mathematical model of an autonomous system in a discrete linear time invariant dynamics in two dimensional space with two restricted areas. This hybrid model will be written in a PWA form and its equivalent MLD form. By giving some initial state and target state, we calculate the optimal input of this autonomous system by using MPC control method for hybrid system embedded in MATLAB as hybrid system toolbox so that this system reaches the target state with optimal trajectory or minimal effort. To observe how this trajectory looks like, we will give some numerical simulations and their visualizations.

978-1-5386-5529-0/18/$31.00 ©2018 IEEE

II. Dynamical System

Suppose that an autonomous system has a linear time invariant state space described as follows

$$\left.\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ y(k) &= Cx(k)+Du(k) \end{aligned}\right\} \quad (1)$$

where k is time instant or time period, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$ and $y \in \mathbb{R}^m$ are state, input and output of the this autonomous system respectively. The matrices *A, B, C,* and *D* are real constant with appropriate dimension. Assumed that (1) is controllable and observable. The problem is this autonomous system has to move from its initial position to some target position with optimal trajectory (i.e. minimal effort) by avoiding several restricted areas, in this paper, in the rectangle shapes. Let the initial position of this autonomous system is $x_0 = x(0)$ and the target position or final position is x_f. Assume that there are several restricted areas for this autonomous system, for two dimensional states, illustrated by Fig. 1 with two restricted areas. Fig. 1 illustrates an example of optimal trajectory of an autonomous system with two restricted states in the rectangle shapes. This optimal trajectory will be generated by using MPC control design. In this paper, w.l.o.g., we formulate the model for two dimensional states with two restricted areas. The normal area is defined as the area that corresponds to the dynamic of the autonomous system for normal condition and the restricted area is defined as the area that has to be avoided. The restricted areas is defined by labeling the area that has to be avoided illustrated by Fig. 2.

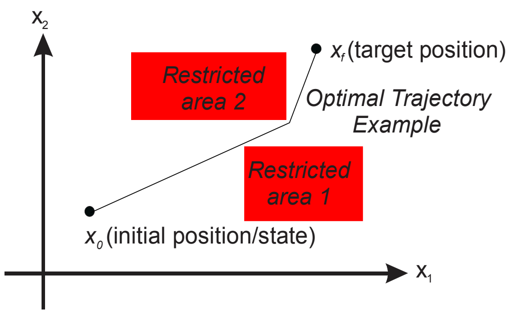

Fig. 1. Illustration for optimal trajectory of an autonomous system with two restricted areas

Fig. 2 illustrates the restricted area labeling by defining the normal area and restricted areas as follows

$$\begin{cases} \text{Restricted area 1} : \{(x_1,x_2) : a_1 \le x_1 \le b_1, c_1 \le x_2 \le d_1\} \\ \text{Restricted area 2} : \{(x_1,x_2) : a_2 \le x_1 \le b_2, c_2 \le x_2 \le d_2\} \\ \text{Normal area} \quad : \text{otherwise.} \end{cases}$$

This restricted area labeling can be used to define the restricted area for three or more restricted areas or three or more dimensional state. The dynamic corresponds to the restricted area is defined as follows

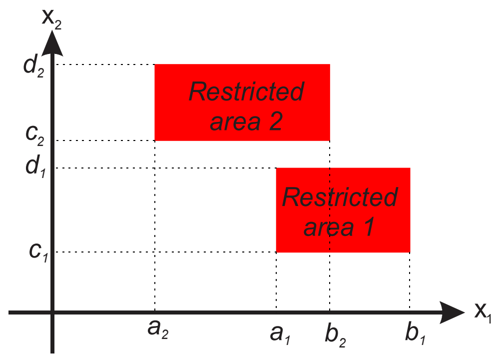

Fig. 2. Illustration for labeling the restricted areas

$$x(k+1) = Ix(k) \quad (2)$$

$$y(k) = Cx(k). \quad (3)$$

where *I* is identity matrix with same dimension as *A*. It means that if the autonomous system reach the restricted area then it will be located on the restricted area forever and never reach the target position. The PWA model for this restricted area avoidance scenario is defined as follows

$$x(k+1) = \begin{cases} Ix(k), & \text{if } x \text{ is on restricted area} \\ Ax(k)+Bu(k), & \text{if } x \text{ is on normal area} \end{cases} \quad (4)$$

$$y(k) = Cx(k) \quad (5)$$

The above PWA model can be converted into equivalent MLD model by typing it in HYSDEL programming language then generate it to obtain the MLD model by using function *mld* in hybrid system toolbox for MATLAB. We convert the above PWA model into MLD model in order to simplify the controller designing by using MPC. This MLD model will be in the form

$$\left.\begin{aligned} & x(k+1) = Ax(k)+B_1u(k)+B_2\delta(k)+B_3z(k) \\ & y(k) = Cx(k)+D_1u(k)+D_2\delta(k)+D_3z(k) \\ & E_2\delta(k)+E_3z(k) \le E_1u(k)+E_4x(k)+E_5 \end{aligned}\right\} \quad (6)$$

where $x(k)$ is the state vector, $y(k)$ is the output vector, $u(k)$ is the input vector, $z(k)$ is auxiliary variable and $\delta(k)$ is binary auxiliary variable. A, B_i, C, D_i and E_i are real constant matrices, E_5 is a real vector.

We will generate the optimal input for MLD model (6) so that the final state equals to the given target state with minimum effort for some time horizon hence it is a terminal state control problem. This optimal input will generate the optimal trajectory from its initial state to the target state. We use model predictive control method to solve this terminal state control problem by predicting the state, input and output vectors of the model, substituting them into an objective function in the quadratic form then optimizing it by using mixed integer quadratic programming (miqp). This problem can be formulated as the following optimal control problem

$$\min_{[u,\delta,z]^T} J = \sum_{t=0}^{T-1} \Big[\|u(k)-u_f\|_{Q_2}^2 + \|\delta(k)-\delta_f\|_{Q_3}^2 + \|z(k)-z_f\|_{Q_4}^2 + \|x(k)-x_f\|_{Q_1}^2 \Big] \quad (7)$$

subject to :

$$x(T) = x_f$$
$$x(k+1) = Ax(k) + B_1 u(k) + B_2 \delta(k) + B_3 z(k)$$
$$-E_4 x(k) - E_1 u(k) + E_2 \delta(k) + E_3 z(k) \le E_5$$

where T denotes the prediction horizon time, Q_1, Q_2, Q_3, and Q_4 are weighting matrices which are symmetric and positive definite, x_f, u_f, δ_f, and z_f are the final/target state, final input, final auxiliary binary variable value of δ, final auxiliary variable value of z respectively where $\|v\|_Q^2 = v^T Q v$. By following [1], the corresponding miq optimization for (7) is

$$\min_{\mathbf{R}} \quad \mathbf{R}' S_1 \mathbf{R} + 2(S_2 + x_0' S_3)\mathbf{R} \quad (8)$$

subject to :

$$F_1 \mathbf{R} \le F_2 + F_3 x_0$$
$$\mathbf{A}\,\mathbf{B}\,\mathbf{R} = x_f - A^T x_0,$$
$$\mathbf{R} = \left[u(0),...,u(T-1),...,\delta(0),..., \delta(T-1), z(0),...,z(T-1)\right]^T$$

where S_1, S_2, and S_3 are real constant matrices where the matrices F_1, F_2, F_3, **A** and **B** are real matrices. We solve (8) by using *miqp* in MATLAB functions which was embedded in hybrid toolbox. The optimal values for u generated by (8) will be applied to the system (6) to generate the optimal trajectory of the system from its initial position to the target position.

III. Computational Simulation

Firstly, we simulate an autonomous system with two dimensional state with two restricted areas in the rectangle shape with some randomly generated data. Suppose that an autonomous system has vector state $x(k) = [x_1(x), x_2(k)]' \in \mathbb{R}^2$ which presenting the position of the system in the Cartesian coordinate and $y(k) = x(k)$ is presenting the output of the system i.e. the position for every time step $k = 1,2,3,...$. Suppose that the initial position of the system is $x_0 = [1,1]'$. The target position is defined as $x_f = [8,7]'$. The dynamic of this autonomous system in normal area is

$$\left.\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k) \\ y(k) &= x(k) \\ x(0) &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \end{aligned}\right\} \quad (9)$$

and there are two restricted areas in the rectangle shape defined as follow

$$\begin{cases} \text{Restricted area-1 : } \{(x_1,x_2): 3 \le x_1 \le 5, 3 \le x_2 \le 4\} \\ \text{Restricted area-2 : } \{(x_1,x_2): 5 \le x_1 \le 7, 5 \le x_2 \le 8\} \\ \text{Normal area : otherwise.} \end{cases}$$

The hybrid dynamic of this autonomous system in the PWA form for this restricted area avoidance scenario is

$$\left.\begin{aligned} x(k+1) &= \begin{cases} Ix(k), & \text{if } x \text{ on the restricted area} \\ Ax(k) + Bu(k), & \text{if } x \text{ on the normal area} \end{cases} \\ y(k) &= Cx(k) \end{aligned}\right\} \quad (10)$$

where $A = I, B = [1,1]', C = I$ and I is an identity matrix with dimension 2×2. By writing this PWA model into HYSDEL programming language appeared in Listing Code 1 and converting it by using function *mld* embedded in hybrid toolbox for MATLAB, we obtain the equivalent MLD model of (10) as (6) with the matrices

$$A = B_1 = D_2 = D_3 = zeros(2,2),$$
$$B_2 = D_2 = zeros(2,10), B_3 = C = I_2,$$

$$E_1 = \begin{bmatrix} zeros(18,2), & \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}, & zeros(38,2) \end{bmatrix}'$$

```
SYSTEM pwa_model_1 {
INTERFACE { STATE { REAL x1 [0,10]; REAL x2 [0,10];}
        INPUT { REAL u1 [-3,3]; REAL u2 [-3,3]; }
        OUTPUT{ REAL y1,y2; }
        PARAMETER REAL a1,a2,b1,b2,c1,c2,d1,d2; } }
IMPLEMENTATION { AUX { REAL z1,z2;
        BOOL a1,da2,db1,db2,dc1,dc2,dd1,dd2;}
        AD  {   da1 = x1>=3;     da2 = x1>=5;
                db1 = x1>=5;     db2 = x1>=7;
                dc1 = x2>=3;     dc2 = x2>=5;
                dd1 = x2>=4;     dd2 = x2>=8; }
        DA  { z1 = {IF
((da1&~db1)&(dc1&~dd1))|((da2&~db2)&(dc2&~dd2)) THEN
x1 ELSE x1+u1 };
        z2 = {IF
((da1&~db1)&(dc1&~dd1))|((da2&~db2)&(dc2&~dd2)) THEN
x2 ELSE x2+u2 };}
        CONTINUOUS {     x1 = z1;x2 = z2; }
        OUTPUT {         y1 = x1;y2 = x2;  } } }
```

Listing Code 1. PWA model (10) with two restricted areas in HYSDEL

and we do not append the matrices E_2, E_3, E_4 and E_5 since their dimension are relatively large, E_1 is 60×10, E_2 is 60×2, E_4 is 60×2 and E_5 is 60×1. By applying the optimal input values that generated by (8), we simulate the maneuver of this autonomous system as illustrated by Fig. 3.

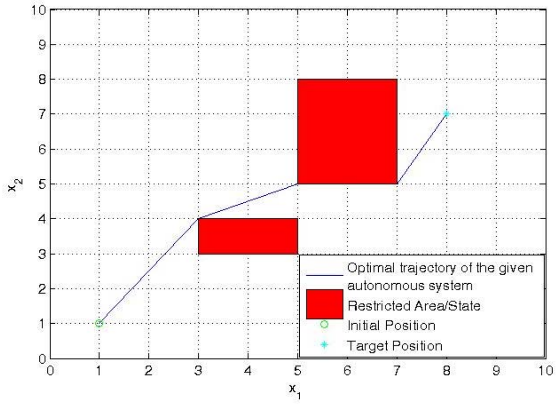

Fig. 3. Optimal trajectory of autonomous system (9) with two restricted areas

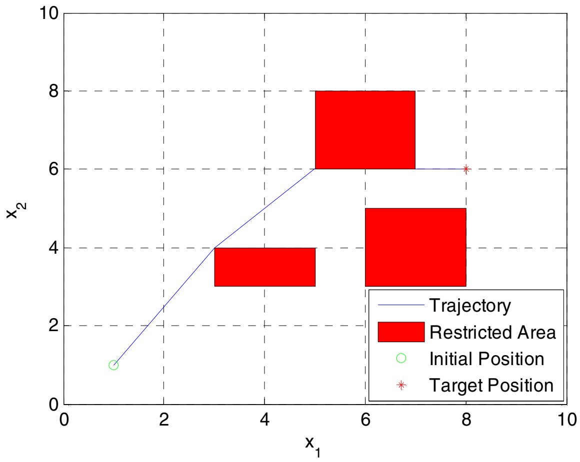

Fig. 4. Optimal trajectory of autonomous system with three restricted areas in rectangle of shape

Fig. 3 shows the optimal trajectory from the initial/start position/point to the target/final position/point of this autonomous system and its restricted areas in the rectangle shape. From this simulation, we can observe that this autonomous system moves from its initial position and avoids the given restricted areas and reaches the target position with minimal effort. We also give the simulation of system (9) with initial position (1,1) and target position (8,6) with three restricted areas in the rectangle shape. The simulation result for this problem is shown by Fig. 4. From Fig. 4, it can be seen that this system moves from its initial position and avoids the given restricted areas and reaches the target position. If we focus on position (6,6), geometrically, the system is entering the restricted area. But, the values of the output i.e. the position of this system was not entering the restricted area but it was sufficiently closed to the restricted area.

IV. Concluding Remarks And Future Researches

In this paper, we have formulated a hybrid model of an autonomous system to avoid several restricted areas in the rectangle shape. The model is formulated in the PWA and MLD forms. The optimal input has been generated by applying model predictive control method. From simulation results, we concluded that the optimal trajectory of this system got avoided in the given restricted areas.

In our future works, we will simulate this problem with various shapes in the restricted area and dynamic restricted areas where the results will be compared to other control methods like Virtual Force Field, Dynamic Windows Approach, etc. in term of the performance, efficiency and computational aspects. Furthermore, we will simulate this restricted areas avoidance scenario in three or more dimensional state and applying it for some autonomous systems like unmanned vehicle, simple robot or the others.

References

[1] J. Keij, "Obstacle Avoidance with Model Predictive Control in a Hybrid Controller," Tokyo Institute of Technology, 2002.

[2] Sutrisno, Salmah, E. Joelianto, A. Budiyono, I. E. Wijayanti, and N. Y. Megawati, ""Model Predictive Control for Obstacle Avoidance as Hybrid Systems of Small Scale Helicopter," in *3rd International Conference on Instrumentation Control and Automation (ICA)*, 2013, pp. 127–132.

[3] Z. Yan, Y. Zhao, S. Hou, H. Zhang, and Y. Zheng, "Obstacle Avoidance for Unmanned Undersea Vehicle in Unknown Unstructured Environment," *Mathematical Problems in Engineering*, vol. 2013, pp. 1–12, 2013.

[4] R. Wang, M. Wang, Y. Guan, and X. Li, "Modeling and Analysis of the Obstacle-Avoidance Strategies for a Mobile Robot in a Dynamic Environment," *Mathematical Problems in Engineering*, vol. 2015, pp. 1–11, 2015.

[5] T. Yang, Z. Liu, H. Chen, and R. Pei, "Robust Tracking Control of Mobile Robot Formation with Obstacle Avoidance," *Journal of Control Science and Engineering*, vol. 2007, pp. 1–10, 2007.

[6] G. S. Chyan and S. G. Ponnambalam, "Robotics and Computer-Integrated Manufacturing Obstacle avoidance control of redundant robots using variants of particle swarm optimization," *Robotics and Computer Integrated Manufacturing*, vol. 28, no. 2, pp. 147–153, 2012.

[7] J. Wang, H. Zhao, Y. Bi, S. Shao, Q. Liu, X. Chen, R. Zeng, Y. Wang, and L. Ha, "An Improved Fast Flocking Algorithm with Obstacle Avoidance for Multiagent Dynamic Systems," *Journal of Applied Mathematics*, vol. 2014, 2014.

[8] Z. Chen, L. Ding, K. Chen, and R. Li, "The Study of Cooperative Obstacle Avoidance Method for MWSN Based on Flocking Control," *The Scientific World Journal*, vol. 2014, pp. 1–12, 2014.

[9] A. Bemporad, "Hybrid Toolbox User's Guide," 2012.

[10] F. Borrelli, A. Bemporad, and M. Morari, *Predictive Control for linear and hybrid systems*. Cambridge, UK: Cambridge University Press, 2014.

[11] J. M. Maciejowski, *Predictive Control with Constrains*. USA: Prentice Hall, 2001.

[12] Y. Ding, L. Wang, Y. Li, and D. Li, "Model predictive control and its application in agriculture: A review," *Computers and Electronics in Agriculture*, vol. 151, pp. 104–117, 2018.

[13] L. Chen, S. Du, Y. He, M. Liang, and D. Xu, "Robust model predictive

control for greenhouse temperature based on particle swarm optimization," *Information Processing in Agriculture*, vol. 5, no. 3, pp. 329–338, 2018.

[14] M. Jalali, E. Hashemi, A. Khajepour, S. Chen, and B. Litkouhi, "Model predictive control of vehicle roll-over with experimental verification," *Control Engineering Practice*, vol. 77, pp. 95–108, 2018.

[15] X. Liu and J. Cui, "Economic model predictive control of boiler-turbine system," *Journal of Process Control*, vol. 66, pp. 59–67, 2018.

[16] P. Li and Z. H. Zhu, "Model predictive control for spacecraft rendezvous in elliptical orbit," *Acta Astronautica*, vol. 146, pp. 339–348, 2018.