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Jaroslava Hlouskova, Ines Fortin, Panagiotis Tsigaris

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The consumption-investment decision of a prospect theory household: A two-period model with an endogenous second period reference level*

Jaroslava Hlouskova
Macroeconomics and Economic Policy, Institute for Advanced Studies, Vienna, Austria
Ecosystems Services and Management, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria

Ines Fortin
Macroeconomics and Economic Policy, Institute for Advanced Studies, Vienna, Austria

Panagiotis Tsigaris
Department of Economics, Thompson Rivers University, Kamloops, BC, Canada

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Abstract

In this paper we analyze the two-period consumption-investment decision of a household with prospect theory preferences and an endogenous second period reference level which captures habit persistence in consumption and in the current consumption reference level. In particular, we examine three types of household depending on how the household's current consumption reference level relates to a given threshold which is equal to the average discounted endowment income. The first type of household has a relatively low reference level (less ambitious household) and can avoid relative consumption losses in both periods. The second type of household (balanced household) always consumes exactly its reference levels. The third type of household has a relatively high reference level (more ambitious household) and cannot avoid to incur relative consumption losses, either now or in the future. Note that these households may act very differently from one another and thus there will often be a diversity of behavior. For all three types we examine how the household reacts to changes in: income (e.g., income fall caused by recession or taxation of endowment income), persistence to consumption, the first period reference level and the degree of loss aversion. Among others we find that the household increases its exposure to risky assets in good economic times if it is less ambitious and in bad economic times if it is more ambitious. We also find that in some cases more income can lead to less happiness. In addition, the less ambitious household and the more ambitious household with a higher time preference will be less happy with a rising persistence in consumption while the more ambitious household with a lower time preference will be happier if it sticks more to its consumption habits. Finally, the household's happiness decreases with an increasing consumption reference level and thus not comparing at all will lead to the highest level of happiness. In addition, the sensitivity of happiness with respect to the reference level gets larger the closer the household moves to the threshold level, and it is smaller for less ambitious households than for more ambitious households due to loss aversion.

Keywords: prospect theory, loss aversion, consumption-savings decision, portfolio allocation, happiness, income effects

JEL classification: G02, G11, E20

1 Introduction

One of the most important decisions households face is consumption today versus consumption in the future. Households transfer current consumption into the future by allocating their savings into different types of assets some of which are riskier than others. These decisions are done with the knowledge that the future is risky. The expected utility theory (EUT) has been the cornerstone model for exploring these household decisions. This research deviates from the EUT model and explores, in a two-period model, the behavior of households which are characterized by reference dependent preferences (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992) and by habit persistence (Abel, 1990; Alessie and Lusardi, 1997; Campbell and Cochrane, 1999; Constantinides, 1990; Flavin and Nakagawa, 2008; Pagel, 2017) when deciding on consumption, savings, and the portfolio allocation of savings. We explore the factors that influence a household's consumption, savings and portfolio decisions when the second period reference level is assumed to depend on first period consumption and the first period consumption reference level. Households have been observed to show a habit for consumption that persists into the future, and hence a habit persistence model combined with prospect theory preferences will provide new insights on such important life cycle decisions.

By incorporating prospect theory type of preferences and habit persistence we will be able to address a number of issues on consumption and risk taking behavior that have not been explored in the literature previously. How does a household make intertemporal decisions under these two behavioral traits? Does the optimal solution depend on avoiding relative losses or not? Does the optimal choice depend on whether the household is sufficiently loss averse? Is the choice dependent on the household being less or more ambitious on targets? How do the second period reference level, consumption, risk taking, and happiness change when the first period reference level changes? Do the responses depend on the household's level of ambition? What impact does the habit persistence in consumption have on consumption and portfolio choice? How will a household react to sudden income changes? Do happiness, current consumption and risk taking always increase when income increases? This paper will attempt to shed some light on the above questions.

The first reference levels ever used in economic research were developed by Stone (1954) and Geary (1950). The Stone-Geary utility preferences involve *reference dependent utility on subsistence levels of consumption* and thus subsistence levels can be considered as a special type of reference points. Under such preferences households derive utility from consumption in excess of a subsistence level. Achury et al. (2012) explored portfolio-savings decisions with subsistence consumption,[1] where they use a Stone-Geary expected utility model to explain the empirical findings that rich people observe a higher savings rate, a larger proportion of

[1]Merton (1969, 1971) used HARA preferences to examine savings and portfolio allocations in an infinite horizon expected utility model. Achury et al. (2012) added subsistence and also habit persistence to Merton's CRRA utility function (a subset of HARA preferences).

risky assets in their personal wealth, and a higher volatility in consumption than poor people.

Another model that has been used is *habit persistence*, where households are assumed to derive their utility from consumption relative to a reference level which depends on past consumption levels. Thus current consumption affects not only a household's current marginal utility but also its marginal utility in the next period, which may explain why the more a household consumes today the more it will want to consume tomorrow.[2] The macroeconomics and finance literature uses habit persistence models to explain many puzzles, e.g., the equity premium puzzle (Abel, 1990; Constantinides, 1990; Campbell and Cochrane, 1999), excess consumption smoothing (Lettau and Uhlig, 2000), asymmetric reactions due to income uncertainty (Bowman et al., 1999)[3] and many business cycle patterns (Boldrin et al., 2001; Christiano et al., 2005).

Reference levels are also used to compare one's own consumption levels to others (Falk and Knell, 2004; Hlouskova, Fortin and Tsigaris, 2017). Many households are influenced by the *self-enhancement* motive while others are determined by the *self-improvement* motive. The self-enhancement motive applies when people want to feel they are better than their peers and set their references at low levels possibly reflecting the wealth of poorer people. Others with a high reference level place importance to the self-improvement motive and compare themselves with the ones who are more successful. Hlouskova, Fortin and Tsigaris (2017) use a two-period life-cycle model with a sufficiently loss averse household to investigate the impact of these psychological traits on consumption, savings, portfolio decisions, as well as on welfare. They find that the optimal solution depends on whether the household's present value of the consumption reference levels is below, equal to, or above the present value of its endowment income. When reference levels are below the endowment income the authors associate this with the self-enhancement motive. Under this motive the household wants to avoid relative losses in consumption in any present or future state of nature (good or bad). Hence the degree of loss aversion does not affect optimal first period consumption and risky asset holdings. When reference levels are equal to the endowment income this is linked to the belonging motive (i.e., the sufficiently loss averse household belonging to a similar social class). They find that the sufficiently loss averse household's first period consumption is the exogenous reference consumption level and such households avoid playing the stock market. Finally, reference levels above the endowment income are connected with the self-improvement motive. Households with such high reference levels cannot avoid to consume below the reference level, either now or in the future. In this case loss aversion affects consumption and risky investment negatively. The current study differs from Hlouskova, Fortin and Tsigaris (2017) in that it incorporates habit persistence into the household's behavior.

[2]For a meta-analysis on the empirical evidence of the formation of habits in consumption see Havranek et al. (2017).

[3]The authors also consider loss aversion, and the observed asymmetric behavior is due to this feature of the household's utility.

Close to our work is also a recent paper by van Bilsen et al. (2017) who investigate optimal consumption and portfolio choice paths of a loss averse household with an endogenous reference level. The uncertainty arises from risky assets and it is assumed that the time is continuous. Mainly due to loss aversion, the household's behavior is geared towards protecting itself against bad states of nature to avoid or to reduce losses. Consumption choices are found to adjust slowly to financial shocks. In addition, welfare losses are found to be substantial given consumption and portfolio selections are suboptimal. Curatola (2017) also analyzes optimal consumption-savings decisions of a loss averse household with a time varying reference level in a continuous-time framework and finds that a loss averse household can consume below the reference level (to the subsistence level) in bad economic times. This is done in order to invest in risky assets and increase the likelihood that in the future consumption exceeds its reference level. This behavioral approach can explain why investors increase their exposure to risky assets during financial crises. In contrast, standard habit persistence models do not allow consumption to be below the reference level. Our research complements the work by van Bilsen et al. (2017) and Curatola (2017) in that it provides additional insights: as our model is a two-period life-cycle model we can derive closed-form solutions which allow us to conduct comparative static analysis to detect why certain adjustments happen and also to conduct a welfare analysis.

In this paper, we find closed-form solutions for consumption and risk taking of a loss averse household whose endogenous second period reference level depends on current consumption (habit persistence) and on reference consumption. Households who have a relatively low first period reference level are more conservative (less ambitious), which allows them to achieve relative gains in both periods in both states of nature. Households who have a relatively high first period reference level and a low discount factor are more adventurous (more ambitious) and will thus face relative losses in the bad state of nature in the second period while they will achieve relative gains in the first period and in the good state of nature in the second period. On the other hand more ambitious households who value future consumption relatively more will have first period consumption below the reference level but will maintain future consumption in both states of nature above the endogenous second period reference level. We then conduct comparative statics and examine how these different types of households react to income changes, to changes in the first period reference level, to changes in loss aversion, and to changes in habit persistence.

The main difference with respect to Hlouskova, Fortin and Tsigaris (2017), henceforth called HFT, is that this study considers also habit persistence. An increase in the consumption habit persistence will reduce current consumption but stimulate risk taking for less ambitious households, reduce both current consumption and risk taking for more ambitious households with a high time preference, and stimulate both current consumption and risk taking for more ambitious households with a low time preference. In addition, we analyze income effects, which are closely related to the effects of income taxes. Another difference between this study and HFT is that the response of first and second period consumption of less ambitious households to a change of the first period reference level is ambiguous. Finally, unlike in HFT we also consider here a scarcity constraint on consumption, i.e., the consumption in both periods can not fall below a certain value.

Note that the household's first period reference level may be interpreted to equal the first period consumption of a reference household, the Joneses. Then *following the Joneses*[4] means that an increase of first period consumption of the Joneses will also trigger an increase of this household's first period consumption.[5] In HFT the less ambitious household and the more ambitious household with a high time preference (low discount factor) do follow the Joneses, while the more ambitious household with a low time preference (high discount factor) does not. In this study the behavior of the more ambitious household is similar, while that of the less ambitious household may be similar or different, depending on the household's time preference: for a lower time preference (larger discount factor) the household does follow the Joneses (like in HFT), while for a higher time preference it does not. The rest of the results are somewhat similar to HFT in terms of the impact of the exogenous parameters on the choice variables but differ in terms of magnitude.

Another interesting result that was not elaborated in HFT is the reaction of the choice variables of the household to income changes. When focusing, for instance, on risk taking then less ambitious households reduce risk taking when their income falls while more ambitious households increase risk taking when their income shrinks, which is consistent with the observation that investors increase their exposure to risky assets during financial crises (see Curatola, 2017). Finally, the same finding as in HFT is that the highest utility is achieved for the lowest current consumption reference level (while keeping everything else unchanged). Thus, not comparing at all (e.g., to others) leads to the highest level of happiness.

In the next section we present the model and lay out the methodology used to find the solutions. Section 3 presents the main results with a discussion and investigates the impact of income taxation. Finally, we offer some concluding remarks.

2 The two-period consumption-investment model

2.1 Model set-up

Consider a household who decides on current and future consumption within a two-period model. In the first period it decides how to allocate a non-stochastic exogenous income,

[4]See Clark et al. (2008) and Falk and Knell (2004), among others.

[5]This will work through the household's first period reference level which is equal to the Joneses' first period consumption.

$Y_1 > 0$, to current consumption, C_1, risk-free investment, m, and risky investment, $\alpha \geq 0$:

$$Y_1 = C_1 + m + \alpha = C_1 + S \tag{1}$$

Savings are composed of the risk-free investment and the risky investment, i.e., $S = m + \alpha$. The net of the dollar return $r_f > 0$ represents the yield from the safe asset. The risky asset yields a stochastic net of the dollar return r. We assume two states of nature, good and bad. The good state of nature occurs with probability p while the bad state of nature occurs with probability $1 - p$. In the good state the risky asset yields net return r_g and in the bad state it yields net return r_b. Furthermore, it is assumed that $-1 < r_b < r_f < r_g$, $0 < p < 1$, and the expected return of the risky asset is greater than the return of the safe asset, namely $\mathbb{E}(r) = p\, r_g + (1 - p) r_b > r_f$. In the second period (e.g., retirement years in a two-period life-cycle model) the household consumes

$$C_{2s} = Y_2 + (1 + r_f)m + (1 + r_s)\, \alpha$$

where $Y_2 \geq 0$ is the non-stochastic income in the second period (e.g., government pension income) and $s \in \{b, g\}$. Note that $C_{2g} \geq C_{2b}$ as $\alpha \geq 0$ and $r_g > r_b$, where C_{2g} is the second period household's consumption in the good state of nature and C_{2b} in the bad state of nature. The household is allowed to consume the non stochastic future income Y_2 in the first period, as long as consumption exceeds its scarcity constraint in either period (i.e., $C_1 \geq C_L \geq 0$ and $C_{2s} \geq (1 + r_f)C_L$) and savings are negative. Hence, the household can partially borrow from the risk-free asset m against its future income. The earnings from total investments are equal to $(1 + r_f)m + (1 + r_s)\, \alpha$, $s \in \{b, g\}$. Based on this and (1) consumption in the second period for $s \in \{b, g\}$ is

$$C_{2s} = Y_2 + (1 + r_f)(Y_1 - C_1) + (r_s - r_f)\, \alpha \tag{2}$$

Preferences are described by the following reference based utility function

$$U(C_1, \alpha) = V(C_1 - \bar{C}_1) + \delta\, V(C_2 - \bar{C}_2) \tag{3}$$

$\bar{C}_1$ is the first period exogenous consumption reference (or comparison) level, which can be viewed, for instance, as the first period consumption of the Joneses (a reference household to which our household compares to) or their income or, alternatively, as a fraction of this household's income. The first two types of reference level are examples of an external reference level, which relates to, e.g., people in the same neighborhood, region or country, or people with distinct demographic features, while the third one is an example of an internal reference level, which depends on, e.g., one's own income or one's own past consumption, see Clark et al. (2008). $\bar{C}_2$ is the second period *endogenous* reference level given such that

$$\bar{C}_2 = (1 + r_f)\left[wC_1 + (1 - w)\bar{C}_1\right] \tag{4}$$

where $w \in [0, 1]$. Note that the second period endogenous reference level depends on the first period consumption and the first period consumption reference level. The weight w shows the influence of the current consumption upon the future reference level. A higher w implies a stronger dependence between the future reference level and the current consumption level. The weight w reflects thus the consumer's persistence to consumption habits. The weight $(1-w)$, on the other hand, determines the dependence of the second period consumption reference level on the first period consumption reference level. This can be seen as a habit persistence in consumption reference levels. The two weights are negatively related to each other, i.e., an increased habit persistence on current consumption implies a lower habit persistence on the first period reference level, and vice versa. The same habit-formation reference consumption level was used also in Fuhrer (2000). The assumption on the determination of the second period reference level is the main difference between this model and the one developed and analyzed in Hlouskova, Fortin and Tsigaris (2017) where the second period reference level was exogenous.

The δ is the discount factor, $0 < \delta < 1$, and will play an important role in the optimal solutions. A higher δ places more importance to the future relative to the presence, i.e., the household shows a lower time preference, while a smaller δ puts more weight to the presence, i.e., the household shows a higher time preference. The $V(\cdot)$ is a prospect theory (S-shaped) value function defined as

$$V(C_i - \bar{C}_i) = \left\{ \begin{array}{ll} \frac{(C_i - \bar{C}_i)^{1-\gamma}}{1-\gamma}, & C_i \geq \bar{C}_i \\ -\lambda \frac{(\bar{C}_i - C_i)^{1-\gamma}}{1-\gamma}, & C_i < \bar{C}_i \end{array} \right\} \tag{5}$$

for $i \in \{1, 2\}$, see Figure 1. Parameter $\lambda > 1$ represents the degree of loss aversion, while $\gamma \in (0, 1)$ represents diminishing sensitivity to consumption. Consumption in excess of the reference level represents a *(relative) gain* of the magnitude $C_i - \bar{C}_i$, while consumption below the reference level represents a *(relative) loss* of the magnitude equal to $\bar{C}_i - C_i$. Note that the value function is non-differentiable at the consumption reference level and is steeper in the domain of losses than in the domain of gains. This implies that there is a higher dissatisfaction from a reduction in consumption when the household is in the domain of losses than dissatisfaction from the same size of decline in consumption when the household is in the domain of gains. Finally, the household is risk averse in the domain of relative gains (i.e., the value function is concave when consumption exceeds the reference level) and risk seeking in the domain of relative losses (i.e., the value function is convex when consumption is below its reference level).

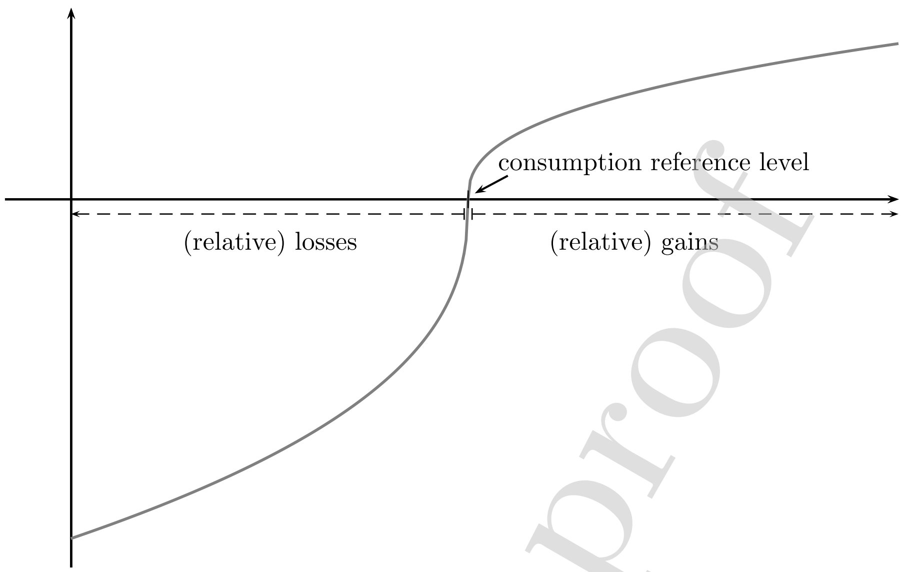

Figure 1: Prospect theory (S-shaped) value function

The household maximizes the following expected utility as given by (3) and (5)

$$\text{Max}_{(C_1,\alpha)}: \quad \mathbb{E}(U(C_1,\alpha)) = V(C_1 - \bar{C}_1) + \delta\, \mathbb{E}V(C_2 - \bar{C}_2)$$

$$\text{such that}: \quad C_1 \geq C_L,\ C_{2g} \geq C_{2b} \geq (1+r_f)C_L,\ \alpha \geq 0 \text{ and}$$
$$\bar{C}_2 = (1+r_f)\left[wC_1 + (1-w)\bar{C}_1\right]$$

where C_L and $(1+r_f)C_L$ determine the minimum first and second period consumption levels, so that the household does not starve ($C_L \geq 0$).[6] Based on this and (2) the household's

[6]Note that in Appendix B we provide the optimal solution for a problem with a more general second period reference level than specified by (4), namely $\bar{C}_2 = w_0 + w_1 C_1 + w_2 \bar{C}_1$, where $0 \leq w_0 \leq (1+r_f)Y_1 + Y_2$, $w_1, w_2 \geq 0$, $C_1 \geq C_{1L} \geq 0$ and $C_{2b} \geq C_{2L} \geq 0$. To reduce the complexity in the main text, however, we use a simpler way to determine the second period reference level, namely: $w_0 = 0$, $w_1 = (1+r_f)w$, $w_2 = (1+r_f)(1-w)$, $w \in [0,1]$, $C_{1L} = C_L \geq 0$ and $C_{2L} = (1+r_f)C_L$. Note that the model in Hlouskova, Fortin and Tsigaris (2017), which dealt with an exogenous second period consumption reference level, is imbedded in this general model, namely, when $w_1 = w_2 = 0$ and thus $\bar{C}_2 = w_0$, and $C_L = 0$.

maximization problem can be formulated as follows

$$\begin{aligned}
\mathrm{Max}_{(C_1,\alpha)}: \quad \mathbb{E}(U(C_1,\alpha)) \quad &= \quad V(C_1-\bar{C}_1)\\
&+ \quad \delta\,\mathbb{E}\,V \quad \big((1+r_f)(Y_1-(1-w)\bar{C}_1)+Y_2-(1+r_f)(1+w)C_1+(r-r_f)\alpha\big)\\
\text{such that}: \quad C_L \leq \; C_1 \quad &\leq Y_1+\tfrac{Y_2}{1+r_f}-C_L-\tfrac{r_f-r_b}{1+r_f}\,\alpha\\
0 \leq \; \alpha \quad &\leq \tfrac{(1+r_f)(Y_1-2C_L)+Y_2}{r_f-r_b}
\end{aligned} \tag{6}$$

Note that the upper bound on C_1 follows from $C_{2b} \geq (1+r_f)C_L$ and the upper bound on α follows from the imposition of the upper bound on C_1, which is at the same time larger than or equal to C_L, i.e., $Y_1 + \frac{Y_2}{1+r_f} - C_L - \frac{r_f-r_b}{1+r_f}\,\alpha \geq C_L$. Finally, the last inequality on α implies that

$$C_L \leq \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right) \tag{7}$$

which we will assume to hold. In addition we assume[7] that $C_L \leq \bar{C}_1$ and thus that

$$C_L \leq \min\left\{\frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right), \bar{C}_1\right\} \tag{8}$$

2.2 Different types of households

We consider three types of households based on their level of ambition as given by their first period consumption reference level $\bar{C}_1$. The following definition specifies the ambition level of households relative to their average discounted income.

Definition 1 *The household is:* (i) *less ambitious, if* $\bar{C}_1 < \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$, (ii) *neutral, if* $\bar{C}_1 = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$, *and* (iii) *more ambitious, if* $\bar{C}_1 > \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$.

The solution of (6) will depend on the household's level of ambition. The intuition behind these three ambition levels can be explained by offering different psychological motives well known in the (psychological) literature.[8] The choice of the reference level with respect to income endowment could be due to psychological motives such as self-enhancement (the need to feel good), in which case the (less ambitious) household compares itself to households that have a lower economic wealth, i.e., $\bar{C}_1 < \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$, where $\bar{C}_1$ coincides with the average discounted income of the other, in this case poorer, household. On the other hand, a (neutral) household could be driven by the belonging motive (similarity/attractive theory), in which case it selects to solve the problem where its average discounted endowment income

[7] This is required for the feasibility of certain solutions.

[8] See Falk and Knell (2004), Gaertner et al. (2012), Banaji and Prentice (1994) and Sedikides and Gregg (2008), among others.

is the same as others, i.e., $\bar{C}_1 = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$. The neutral household wants to associate with people that are from the same social class. Finally, the (more ambitious) household can be driven by the self-improvement motive (high aspirations), in which case it compares to a richer houshold, i.e., $\bar{C}_1 > \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$, where $\bar{C}_1$ is equal to the average present value of endowment income of the other, in this case richer, household.

We treat these motives as exogenous due to the household's psychological state of mind or due to its own income and/or the income of the Joneses to which it compares to.

2.3 Methodology

Prior to presenting the main results of the study we sketch the approach we chose to conduct the formal analysis, which requires the consideration of eight household consumption decision problems:

$$
\begin{array}{llllll}
(P1) & & \bar{C}_1 \le C_1, & & & \bar{C}_2 \le C_{2b} \le C_{2g} \\
(P2) & & \bar{C}_1 \le C_1, & (1+r_f)C_L \le C_{2b} \le & & \bar{C}_2 \le C_{2g} \\
(P3) & & \bar{C}_1 \le C_1, & (1+r_f)C_L \le C_{2g} \le & & \bar{C}_2 \le C_{2b} \\
(P4) & & \bar{C}_1 \le C_1, & (1+r_f)C_L \le C_{2b} \le C_{2g} \le & & \bar{C}_2 \\
(P5) & C_L \le C_1 \le & \bar{C}_1, & & & \bar{C}_2 \le C_{2b} \le C_{2g} \\
(P6) & C_L \le C_1 \le & \bar{C}_1, & (1+r_f)C_L \le C_{2b} \le & & \bar{C}_2 \le C_{2g} \\
(P7) & C_L \le C_1 \le & \bar{C}_1, & (1+r_f)C_L \le C_{2g} \le & & \bar{C}_2 \le C_{2b} \\
(P8) & C_L \le C_1 \le & \bar{C}_1, & (1+r_f)C_L \le C_{2b} \le C_{2g} \le & & \bar{C}_2
\end{array}
$$

These problems are formally presented in Appendix A and their significance consists in the fact that solving the main problem (6) is equivalent to solving these eight sub-problems and comparing their utility functions at the corresponding solutions. The one with the largest value of the utility function is determined to be the solution of problem (6). In more detail, in each of these problems we calculate at first potential maxima which are selected from local maxima (a global maximum was only present in problem (P1)) and potential candidates for maxima at the border, corner solutions.[9] Then we compare all potential maxima of all sub-problems among themselves and determine the global maximum for the main problem (6). Note that as $C_{2g} \ge C_{2b}$ any feasible solution for (P3) or (P7) satisfies $C_{2g} = C_{2b} = \bar{C}_2$. This implies that any solution feasible for (P3) is feasible also for (P1), (P2) and (P4), and any solution feasible for (P7) is feasible also for (P5), (P6) and (P8). Thus, problems (P3) and (P7) can be dropped from our analysis and we are left with six sub-problems.

The first four problems (P1)–(P4) assume that the household keeps current period consumption equal to or above the reference level experiencing a relative gain in the first period. In (P1) the household does not suffer from relative losses neither in the second period. In

[9] Finding corner solutions was tedious work as sometimes we needed to solve additional four or five optimization problems, as in cases (P2) and (P6).

problems (P2)–(P4), however, there are relative losses: in (P2) the relative losses occur in the bad state of nature while in (P4) the relative losses are observed in both states of nature. In the remaining problems (P5)–(P8) current consumption is below, or equal to, its reference level and thus the household experiences relative losses in the first period. In problem (P5) the household keeps future consumption above its reference level and suffers relative losses only in the first period. In (P6) there are losses if the bad state of nature occurs. In the last problem (P8) there are relative losses in both periods.

In what follows we show that (P1), no losses, (P2), losses in the second period in the bad state of nature, and (P5), losses only in the first period, have optimal interior solutions, and for certain conditions based on the degree of loss aversion, the size of the current reference consumption level $\bar{C}_1$, and/or the size of the discount factor, one of these solutions is the solution of our main problem (6). For higher values of the first period consumption reference level, which we do not explore further in this paper, some of the problems have solutions at the border of the set of feasible solutions. This concerns problems (P4), (P6) and (P8), whose utility functions at the maxima are exceeded by the utility functions at the maxima of problems (P1), (P2) and (P5), for certain (sufficiently low) first period consumption reference levels and for certain values of the discount factor. In more detail, the utility functions of problems (P4), (P6) and (P8) are exceeded by the utility function at the maximum of problem (P1) for $\bar{C}_1 \leq \bar{C}_1^{U,P1}$, where $\bar{C}_1^{U,P1} = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$; by the utility function at the maximum of problem (P2) for $\bar{C}_1^{U,P1} < \bar{C}_1 \leq \bar{C}_1^{U,P2}$ and a sufficiently low discount factor; and by the utility function at the maximum of problem (P5) for $\bar{C}_1^{U,P1} < \bar{C}_1 \leq \bar{C}_1^{U,P2}$ and a sufficiently high discount factor. See Appendix B and, in particular, the summary at the end of Appendix B.[10] Note again that the solution of main problem (6) depends (among other parameters) on the household's level of ambition, i.e., on the value of the first period consumption reference level $\bar{C}_1$.

Thus, throughout this paper we assume that

$$C_L \leq \bar{C}_1 \leq \bar{C}_1^{U,P2}$$

where

$$\bar{C}_1^{U,P2} = \frac{\frac{r_g - r_b}{r_g - r_f}\left(Y_1 + \frac{Y_2}{1+r_f}\right) - C_L}{1 + 2\,\frac{r_f - r_b}{r_g - r_f}} \tag{9}$$

Originally, we solved the main problem (6) for any value of the first period consumption reference level $\bar{C}_1$. However, for higher values of the reference level $\bar{C}_1$ (namely for $\bar{C}_1 > \bar{C}_1^{U,P2}$) the solution, which would be reached in either (P4) or in (P5) or in (P6) or in (P8), could

[10]This conclusion was made based on solving problems (P1)–(P8) and comparing the utilities at their optimal solutions with the ones of the preceding sub-problems. The comparisons were performed on overlapping sets of feasible solutions.

be expressed only in the implicit form[11] and in some cases the consumption would reach its lower bound. That is why we focus on reference levels such that $C_L \leq \bar{C}_1 \leq \bar{C}_1^{U,P2}$, where we obtain explicit, closed-form interior solutions. This range of the first period consumption reference level corresponds to the one used in Hlouskova, Fortin and Tsigaris (2017). The threshold values of the reference level, $\bar{C}_1^{U,P1} = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$ and $\bar{C}_1^{U,P2}$, are the largest possible values of $\bar{C}_1$ for which problems (P1) and (P2) are feasible (see Appendix A).

3 Main results

In Section 3.1, we show the optimal consumption and risky asset holdings to problem (6) for a *less ambitious* household. The household's solution is provided by problem (P1). The solution related to current consumption and risk taking exists for a sufficiently loss averse household with a relatively low reference level $\bar{C}_1$, namely below the average discounted income level, and is such that optimal consumption exceeds the corresponding reference consumption in both periods across both states of nature. By being less ambitious the household selects consumption and risk taking in such a way as to avoid relative losses today and in the future. In addition, the household needs to be sufficiently loss averse for an optimal solution to exist in (P1), even though the loss aversion parameter does not explicitly appear in the optimal solution for current consumption and risk taking. Proposition 1 shows the closed-form solution to consumption and risk taking.

In Section 3.2, we describe the optimal consumption and risk taking for a *balanced* household. This is a very special situation, where the household's first period reference level is equal to the average present value of its total wealth (*neutral* reference level) and hence consumption is exactly equal to its reference consumption in both periods. This can also be viewed as a comparison to a reference household with the same total wealth (comparison to *someone like me*).[12]

In Section 3.3 we show the optimal consumption and risky asset holdings to problem (6) for a *more ambitious* household. The solution is provided by problem (P2) or (P5). The optimal solution exists for a sufficiently loss averse household with a relatively high current reference level, namely above the threshold level, and is such that the optimal consumption is below its corresponding reference consumption in either the first or the second period. Proposition 2 shows the closed-form solution of (6) for a more ambitious household with a high time preference (or a sufficiently large probability of the good state of nature to occur), where the solution is the solution of problem (P2), while Proposition 3 presents the closed-form solution of (6) for a more ambitious household with a low time preference, where the solution is the solution of problem (P5). In the first case the household will achieve relative

[11]When we performed the implicit differentiation the effects of most parameters (habit persistence, loss aversion, etc.) could, in the vast majority of cases, not be determined.

[12]See Clark et al. (2008).

gains today and in the future in the good state of nature but will incur relative losses in the future in the bad state, while in the second case the household has to accept current relative losses but will achieve relative gains in both states in the future.

As we will show, the different types of households have very distinct solutions for current consumption and risk taking activity. Also their responses, as well as the responses of the indirect utility function (happiness), to exogenous changes in the loss aversion parameter, the first period reference level, the habit persistence and finally the income/wealth levels vary substantially.

In Sections 3.4 and 3.5 we summarize the income effects and other effects across the different types of household and investigate the impact of income taxation.

3.1 Low first period reference consumption: less ambitious households

In this section we consider a household with a relatively low first period reference consumption. This reference consumption is below a certain threshold[13] and is such that the household can consume above its reference levels in both the first and the second period, and may thus avoid any relative losses. We call a household with such a first period reference level *less ambitious*. This household's behavior is captured by problem (P1). Before proceeding further,

[13] Namely below the average present value of total wealth, see (12).

we introduce the following notation

$$\Omega = (1+r_f)Y_1 + Y_2 - 2\,(1+r_f)\,\bar{C}_1 \tag{10}$$

$$K_\gamma = \frac{(1-p)(r_f-r_b)^{1-\gamma}}{p\,(r_g-r_f)^{1-\gamma}} \tag{11}$$

$$\bar{C}_1^{U,P1} = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right) \tag{12}$$

$$\begin{aligned}\lambda^{P1-P2} &= \frac{\left[\left(\frac{r_f-r_b}{(1+r_f)(r_g-r_b+w(r_f-r_b))}\right)^{1-\gamma} + \delta p\right]\left[\Omega + (r_g-r_f)\alpha^{C_1=\bar{C}_1}_{C_{2b}=(1+r_f)C_L}\right]^{1-\gamma}}{\delta(1-p)\left[(1+r_f)(\bar{C}_1-C_L)\right]^{1-\gamma}} \\ &\quad - \frac{\Omega^{1-\gamma}[(1+r_f)(1+w)+M]^{\gamma}}{\delta(1-p)(1+r_f)(1+w)\left[(1+r_f)(\bar{C}_1-C_L)\right]^{1-\gamma}} \quad \text{for } \bar{C}_1 \le \bar{C}_1^{P1}\end{aligned} \tag{13}$$

$$\lambda^{P1-P5} = \left[\frac{k_2\left(1+K_\gamma^{\frac{1}{\gamma}}\right)}{(1+r_f)(1+w)}\right]^{\gamma} = \left[\frac{M}{(1+r_f)(1+w)}\right]^{\gamma} \tag{14}$$

$$k_2 = \left[\delta(1+r_f)(1+w)\,p\left(\frac{r_g-r_b}{r_f-r_b}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} \tag{15}$$

$$M = \left[\delta(1+r_f)(1+w)\,p\,\frac{r_g-r_b}{r_f-r_b}\right]^{\frac{1}{\gamma}} \frac{r_f-r_b+K_0^{\frac{1}{\gamma}}(r_g-r_f)}{r_g-r_b} \tag{16}$$

$$\alpha^{C_1=\bar{C}_1}_{C_{2b}=(1+r_f)C_L} = \frac{(1+r_f)(Y_1-\bar{C}_1-C_L)+Y_2}{r_f-r_b} \tag{17}$$

Note that $\bar{C}_1 < \bar{C}_1^{U,P1}$ is equivalent to $\Omega > 0$.[14] We present the optimal solution for first period consumption and risk taking of the less ambitious household in the following proposition.

Proposition 1 *Let* $\bar{C}_1 < \bar{C}_1^{U,P1}$ *and* $\lambda > \max\left\{\lambda^{P1-P2}, \lambda^{P1-P5}\right\}$*. Then problem (6) obtains*

[14] Note that HFT characterize the different types of household through Ω (being positive, equal to zero, or negative), while in this study we define the different types of household through their first period consumption reference levels (being smaller than, equal to, or larger than a threshold value), which we think makes more sense. However, we could equivalently describe our households through Ω.

a unique maximum at $(C_1^*, \alpha^*) = \left(C_1^{P1}, \alpha^{P1}\right)$, *where*

$$\begin{aligned} C_1^{P1} &= \bar{C}_1 + \frac{\Omega}{(1+r_f)(1+w)+M} \\ &= \frac{(1+r_f)Y_1 + Y_2 + [M-(1+r_f)(1-w)]\bar{C}_1}{(1+r_f)(1+w)+M} > \bar{C}_1 \end{aligned} \tag{18}$$

$$\alpha^{P1} = \frac{\left(1-K_0^{\frac{1}{\gamma}}\right)M}{r_f - r_b + K_0^{\frac{1}{\gamma}}(r_g - r_f)}\left(C_1^{P1} - \bar{C}_1\right) > 0 \tag{19}$$

Proof. See Appendix B. ■

The future relative gains, or excess consumption, are given by:

$$\left.\begin{aligned} C_{2g}^{P1} - \bar{C}_2 = &\ k_2 \tfrac{r_g - r_b}{r_f - r_b}\left(C_1^{P1} - \bar{C}_1\right) > 0 \\ C_{2b}^{P1} - \bar{C}_2 = &\ k_2 K_0^{\frac{1}{\gamma}} \tfrac{r_g - r_b}{r_f - r_b}\left(C_1^{P1} - \bar{C}_1\right) > 0 \end{aligned}\right\} \tag{20}$$

Current relative gains, $C_1^{P1} - \bar{C}_1$, are driving both the investment in the financial market as well as future excess consumption, see (19) and (20). The higher the relative gains in the first period the higher the investment in the financial market and the higher the relative gains (excess consumption) in the future. Note that the household invests positively in the risky asset. Total savings, however, which include both risky and risk-free assets, may be either positive or negative. The household's consumption and risk taking does not directly depend on the degree of loss aversion; however, the household needs to be sufficiently loss averse.[15] Thus the optimal consumption in both periods as well as the relative consumption in both periods, risk taking and happiness are insensitive to changes in the degree of loss aversion.

The effect of an increase in the first period consumption reference level on current and future consumption cannot be determined a priori, see

$$\frac{dC_1^{P1}}{d\bar{C}_1} = \frac{\frac{M}{1+r_f} - 1 + w}{\frac{M}{1+r_f} + 1 + w} \begin{cases} > 0, & \text{if} \quad \delta > \bar{\delta} \\ = 0, & \text{if} \quad \delta = \bar{\delta} \\ < 0, & \text{if} \quad \delta < \bar{\delta} \end{cases} \tag{21}$$

where

$$\bar{\delta} = \left(\frac{1-w}{1+K_\gamma^{1/\gamma}}\right)^{\gamma}\left(\frac{r_f - r_b}{r_g - r_b}\right)^{1-\gamma}\frac{1}{p\,(1+w)(1+r_f)^{1-\gamma}} \tag{22}$$

[15] As shown in Proposition 1, the loss aversion parameter needs to be sufficiently large, namely $\lambda > \max\left\{\lambda^{P1-P2}, \lambda^{P1-P5}\right\}$, to guarantee that the utility of (P1) at its maximum exceeds the potential maximum of (P2) at its border, $\lambda > \lambda^{P1-P2}$, as well as the potential maximum of (P5) at its border, $\lambda > \lambda^{P1-P5}$. Note that problem (P1) is a concave programming problem and its unique maximum does not depend on λ.

It depends on the household's time preference, i.e., on its discount factor, as follows: a relatively high discount factor (large weight placed to the future) will cause current consumption to increase with increasing $\bar{C}_1$, while a relatively low discount factor (small weight placed to the future) will cause current consumption to decrease.[16] However, the effect on optimal consumption in the second period is opposite: future consumption increases with a lower discount factor and shrinks with a higher discount factor. In addition, the sensitivity of second period consumption in the bad state to the first period reference consumption depends on the probability of the good state. Relative current and future consumption decreases with an increasing first period reference level and also risk taking decreases when the current consumption reference level increases. The latter happens because the increase in the current consumption reference level decreases the relative gains in the first period discouraging investment in the risky asset. Finally, an increase in the first period reference level will reduce the household's happiness and thus the highest possible level of happiness is achieved for the lowest possible current consumption reference level. This suggests that comparison does not make oneself happy, and indeed not comparing at all would be the best. Note that the sensitivity results with respect to the first period reference level are similar (in terms of sign) to the ones when the second period consumption reference level is exogenous (see Hlouskova, Fortin and Tsigaris, 2017), except for the sensitivity of first and second period consumption: if the second period reference level is exogenous then first period consumption always increases, and second period consumption in both states of nature always decreases, with a rising first period reference level.

As stated earlier habit persistence in consumption is determined by the parameter w. An increase in w reduces optimal first period consumption (and thus also the first period relative consumption) and the level of happiness, while it increases the investment in the risky asset. The effect of an increase in w on the second period reference level, however, is not unambiguous. It depends on the curvature, γ, the discount factor, δ, and on the level of habit persistence in consumption, w, itself. If the household is rather risk averse ($\gamma > 0.5$), however, then the effect of habit persistence on the second period reference level is always positive. Also the effect of w on the second period consumption in the bad state can be either positive or negative. Namely the second period consumption in the bad state increases with increasing habit persistence in the first period consumption when w is below a certain threshold and it decreases with increasing habit persistence in the first period consumption when w exceeds the threshold.[17] On the other hand, the impact of w on the second period consumption in the good state is always positive. Note that as habit persistence in consumption, w, relates negatively to habit persistence in the current consumption reference level, $1-w$, the reported dependencies hold with the opposite sign for habit persistence in the first period reference

[16] Note, however, that a larger persistence in consumption reduces the threshold of the discount factor, see (22), which makes it more plausible that first period reference consumption encourages current consumption.

[17] This threshold is a function of the parameters describing the financial market and on the curvature.

level.

Current consumption depends positively on income, i.e., it depends positively on both first period and second period income.[18] An increase in the first period income, as in good economic times, will increase current consumption by $(1+r_f)/[(1+r_f)(1+w)+M)]$, while an increase in the second period income (i.e., good future economic conditions) will increase current consumption by $1/[(1+r_f)(1+w)+M)]$. Note that the presence of habit persistence in consumption has reduced the impact of income upon current consumption relative to models without such a behavioral trait. Furthermore, an increase in income will increase second period consumption as well as the relative gains (excess consumption) in both periods, the second period reference level, the investment in the risky asset and the level of happiness. Note that a sudden reduction in income, caused by a recession or a loss of job (bad economic conditions) or by the introduction of an income tax, will cause the opposite effect and the household will thus reduce current consumption and risk taking. Note in addition that if the first period reference level is equal to a fraction of the present value of the total wealth, i.e., $\bar{C}_1 = c\left(Y_1 + \frac{Y_2}{1+r_f}\right)$ where $c \in \left(0, \frac{1}{2}\right)$,[19] then the sensitivity results will not change. This suggests that the direct income effect is stronger than the indirect effect of income through the first period consumption reference level. Table 1 summarizes the sensitivity results related to Proposition 1, which have been discussed above.

Finally, it can be shown that the expected utility evaluated at the optimal choices is determined by the relative gains in the first period:

$$(1-\gamma)\mathbb{E}\left(U\left(C_1^{P1}, \alpha^{P1}\right)\right) = \left[1 + \frac{M}{(1+r_f)(1+w)}\right]\left(C_1^{P1} - \bar{C}_1\right)^{1-\gamma} \tag{23}$$

The household will be more happy with a rising income, while it will be less happy with a larger first period reference level (as the first period relative consumption decreases) and a higher persistence in current consumption, see Table 1.

	$C_1^* = C_1^{P1}$ and $\alpha^* = \alpha^{P1}$								
	dC_1^*	dC_{2g}^*	dC_{2b}^*	$d\alpha^*$	$d\bar{C}_2$	$d(C_1^* - \bar{C}_1)$	$d(C_{2g}^* - \bar{C}_2)$	$d(C_{2b}^* - \bar{C}_2)$	$d\,\mathbb{E}(U^*)$
$d\lambda$	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$	$=0$
$d\bar{C}_1$	$\gtreqless 0$	$\lesseqgtr 0$	$\lesseqgtr 0$	<0	>0	<0	<0	<0	<0
dw	<0	>0	$\lesseqgtr 0$	>0	$\lesseqgtr 0$	<0	>0	>0	<0
dY_i	>0	>0	>0	>0	>0	>0	>0	>0	>0

Table 1: Sensitivity results for the less ambitious household with respect to λ, $\bar{C}_1$, w and Y_i, $i = 1, 2$.

[18] We say that some quantity depends positively (negatively) on income, if it depends positively (negatively) on both first period income and second period income.

[19] The fraction needs to be less than one half such that the household is less ambitious.

3.2 Neutral first period reference consumption: balanced households

This special case applies when the household is neither less ambitious (see the previous section) nor more ambitious (see the following section). The household is *balanced* in the sense that it consumes exactly its reference levels, in both the first and the second period. This requires that the household's first period reference level is equal to the threshold separating less ambitious from more ambitious households. The reference consumption is thus equal to the average of the discounted income, i.e., $\bar{C}_1 = \bar{C}_1^{U,P1} = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$. We call this reference level the *neutral* first period reference consumption. Note that the neutral reference level depends explicitly on the household's exogenous income. Note, in addition, that if the household's total income coincides with the total income of some reference household then this current reference consumption can be viewed as an *external* reference consumption, as the household compares itself to *someone like itself.*

The following corollary describes the solution of the balanced household.

Corollary 1 *Let* $\bar{C}_1 = \bar{C}_1^{U,P1}$ *and* $\lambda > \max\left\{\lambda^{P1-P2}, \lambda^{P1-P5}\right\}$. *Then problem (6) obtains its unique maximum at* (C_1^*, α^*), *where*

$$\begin{aligned} C_1^* &= \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right) = \bar{C}_1^{U,P1} \\ \alpha^* &= 0 \end{aligned}$$

Proof. See Appendix B. ■

The sufficiently loss averse balanced household will consume exactly its consumption reference level in the first period, which is equal to half the current value of total income. In addition, it will not invest in the financial market even though the expected return from the risky asset is greater than the return from the safe asset. This phenomenon can help to explain the equity premium puzzle as it indicates that the risk premium is not sufficient to induce the household to invest in the risky asset. The savings will thus consist only of the risk-free investment, which can be positive, zero or negative, based on how the first period income and the discounted second period income relate to each other:

$$S = m = \frac{1}{2}\left(Y_1 - \frac{Y_2}{1+r_f}\right) \begin{cases} > 0 & \text{if } Y_1 > \frac{Y_2}{1+r_f} \\ = 0 & \text{if } Y_1 = \frac{Y_2}{1+r_f} \\ < 0 & \text{if } Y_1 < \frac{Y_2}{1+r_f} \end{cases} \tag{24}$$

Note, in addition, that also in the second period in both states of nature the household consumes exactly its consumption reference level, i.e., $\bar{C}_2 = C_{2g}^* = C_{2b}^* = \frac{1}{2}[(1+r_f)Y_1 + Y_2] = (1+r_f)\bar{C}_1^{U,P1} = (1+r_f)\bar{C}_1 = (1+r_f)C_1^*$, which can be viewed as perfect consumption smoothing. This implies that the solution is feasible for all sub-problems (P1)–(P8) and thus can be considered a threshold solution, where the household achieves no relative gains and no relative losses in either period.

If the household's income increases either in the first period and/or in the second period, while other parameters remain unchanged, including $\bar{C}_1$, then the household's upper bound $\bar{C}_1^{U,P1}$ will also increase and as a result the household will become relatively less ambitious since now $\bar{C}_1 < \bar{C}_1^{U,P1}$. Thus, the household will be able to avoid relative losses in both periods. If on the other hand, the household's income falls unexpectedly, while other parameters remain unchanged, then this will reduce the household's threshold level $\bar{C}_1^{U,P1}$ and thus the first period reference level will be above this new upper bound $\bar{C}_1^{U,P1}$. As a result the household will become more ambitious in order to make up for the lost income. In this case its optimal consumption will be below the reference level either in the second period in the bad state of nature, problem (P2), or in the first period, problem (P5). We will discuss these cases in the next section.

Suppose the household has initially a current consumption reference level below the threshold level and hence is less ambitious. Then it is hit by a sudden reduction in income, e.g., due to a loss of job in bad economic times, which triggers a decrease of the threshold level such that the household's (constant) reference level is above the new threshold, and hence the household is more ambitious. This switch from the less ambitious (across the balanced) to the more ambitious type will change, for example, its sensitivity of risk taking with respect to income: while before the drop in income the household (which is less ambitious) takes on less risk with decreasing income, it will be eager to take on more risk with a decreasing income – with the hope to make up for the lost income – after the drop in income (when it will be more ambitious).[20]

Note that consumption in both periods (as well as the relative consumption in both periods), risk taking and happiness are unaffected by changes in the level of loss aversion, as well as by changes in the persistence level in current consumption.

3.3 High first period reference consumption: more ambitious households

If the first period reference level exceeds the threshold level which is equal to the average of the discounted income, i.e., if $\bar{C}_1 > \bar{C}_1^{U,P1} = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$, then the household cannot consume above its reference levels in both periods. In either the first or the second period the household will have to consume below its reference consumption, and thus will incur relative losses. A household with such a high first period reference level is called *more ambitious*. The optimal consumption of the more ambitious household will be either below its consumption reference level in the second period in the bad state of nature, problem (P2), or in the first period, problem (P5). Which case occurs, problem (P2) or (P5), depends on the household's time preference, i.e., on its discount factor, and on the probability of the good state to occur. If the sufficiently loss averse household is relatively time impatient and assigns a low weight

[20] See the sensitivity results in Tables 1 and 2.

to future consumption (i.e., it has a small discount factor, or a high time preference) then the optimal solution of (6) for optimal consumption and risk taking coincides with the optimal solution of problem (P2). In this problem the optimal consumption in the first period is above its reference level, as in problem (P1). However, in the second period the household cannot avoid relative losses in the bad state of nature. Proposition 2 provides the optimal solution for this case. This case also applies if the probability of the good state of nature is sufficiently large (irrespective of the household's time preference). On the other hand, if the discount factor is relatively large (i.e, future consumption is valued high), and the probability of the good state is not too high, then the sufficiently loss averse household will find a solution where first period consumption is below the first period reference level (suffering relative losses in the first period) but will keep future consumption above the endogenous reference level in both states of nature. The solution for this case is presented in Proposition 3. The first period reference level cannot be arbitrarily large, however. It needs to be smaller than a certain threshold, $\bar{C}_1^{U,P2}$.

To summarize, if the more ambitious household values first period consumption relatively high (lower discount factor), then it focuses on avoiding relative losses in the first period and thus first period consumption is above its reference level. If, however, the more ambitious household values second period consumption relatively high (larger discount factor), then it wants to prevent relative losses in the second period and consequently second period consumption exceeds its reference level. This is only true, however, if the probability of the good state is not too large. If it is larger than a certain threshold then only the first case applies, where relative losses occur in the second period in the bad state, irrespective of the household's time preference.[21]

[21]Note that for better readability we will often omit the information on the large (small) enough probability of the good state of nature in identifying the type of household, and simply call a household that finds it optimal solution in problem (P2) "more ambitious with a high time preference", and a household that finds its optimal solution in problem (P5) "more ambitious with a low time preference".

Before proceeding further, we introduce the following notation

$$\bar{C}_1^{U,P5} = \frac{Y_1 + \frac{Y_2}{1+r_f} - (1+w)C_L}{1-w} \tag{25}$$

$$k = \left[\delta(1+r_f)(1+w)(1-p)\left(\frac{r_g - r_b}{r_g - r_f}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} \tag{26}$$

$$M(\lambda) = \left[\delta(1+r_f)(1+w)\,p\left(\frac{r_g - r_b}{r_f - r_b}\right)^{1-\gamma}\right]^{\frac{1}{\gamma}} \left[(\lambda K_\gamma)^{1/\gamma} - 1\right] \tag{27}$$

$$C_L^U = \frac{r_g - r_b}{r_g - r_f}\left(Y_1 + \frac{Y_2}{1+r_f}\right) - \left(1 + 2\,\frac{r_f - r_b}{r_g - r_f}\right)\bar{C}_1 \tag{28}$$

$$\lambda^{P2} = \left[\frac{(1+r_f)(1+w)}{k} + \left(\frac{1}{K_\gamma}\right)^{1/\gamma}\right]^{\gamma} \left[1 + \frac{\frac{r_f - r_b}{r_g - r_f} + \frac{1+r_f+k_2}{(1+r_f)(1+w)+k_2}}{C_L^U - (1+r_f)C_L}(-\Omega)\right]^{\gamma} \tag{29}$$

$$\text{for} \quad C_L < \frac{C_L^U}{1+r_f} \quad \text{and} \quad \bar{C}_1^{U,P1} < \bar{C}_1 < \bar{C}_1^{U,P2}$$

$$\lambda^{P2-P2} = \frac{\left[\left(\frac{r_f - r_b}{(1+r_f)\left(r_g - r_b + w(r_f - r_b)\right)}\right)^{1-\gamma} + \delta p\right]\left[\Omega + (r_g - r_f)\alpha^{C_1=\bar{C}_1}_{C_{2b}=(1+r_f)C_L}\right]^{1-\gamma}}{\delta(1-p)\left[\left((1+r_f)(\bar{C}_1 - C_L)\right)^{1-\gamma} - \left(\frac{r_g - r_b}{r_g - r_f}\right)^{1-\gamma}(-\Omega)^{1-\gamma}\right]} \tag{30}$$

$$\lambda^{P4} = \frac{1}{(1+r_f)^{1-\gamma} w\,\delta}\left[\frac{w\left(Y_1 + \frac{Y_2}{1+r_f}\right) + (1-w)\bar{C}_1 - (1+w)C_L}{\alpha^{C_1=\bar{C}_1}_{C_{2b}=(1+r_f)C_L} - \alpha^{C_{2g}=\bar{C}_2}_{C_{2b}=(1+r_f)C_L}}\,\frac{1+r_f}{r_f - r_b}\right]^{\gamma} \tag{31}$$

$$\text{for} \quad \bar{C}_1 < \bar{C}_1^{U,P2}$$

$$\lambda^{P5} = \lambda^{P1-P5}\left[\frac{(1+w)(\bar{C}_1 - C_L)}{(1-w)\left(\bar{C}_1^{U,P5} - \bar{C}_1\right)}\right]^{\gamma} \tag{32}$$

$$\lambda^{P2-P6} = \frac{\delta p(1+r_f)^2\left(\frac{r_g - r_b}{r_f - r_b} + w\right)^2 (\bar{C}_1 - C_L)^{1+\gamma}}{[1 + \delta(1-p)(1+r_f)^{1-\gamma}w^2]\left[\Omega + (r_g - r_f)\alpha^{C_1=\bar{C}_1}_{C_{2b}=(1+r_f)C_L}\right]^{1+\gamma}} \tag{33}$$

$$\delta^{P2-P5} = \frac{1}{1-p}\left[\frac{r_g - r_f}{(1+r_f)(1+w)(r_g - r_b)}\right]^{1-\gamma} \tag{34}$$

$$\alpha^{C_{2g}=\bar{C}_2}_{C_{2b}=(1+r_f)C_L} = \frac{(1+r_f)(\bar{C}_1 - C_L) + w\left[(1+r_f)(Y_1 - \bar{C}_1 - C_L) + Y_2\right]}{r_g - r_b + w\,(r_f - r_b)} \tag{35}$$

The optimal solution for first period consumption and risk taking is given in the next proposition.

Proposition 2 *Let* $\bar{C}_1^{P1} < \bar{C}_1 < \bar{C}_1^{U,P2}$, $\lambda > \max\left\{\lambda^{P2}, \lambda^{P2-P2}, \lambda^{P4}, \lambda^{P5}, \lambda^{P2-P6}\right\}$, $\delta \leq \delta^{P2-P5}$ *and* $C_L < C_L^U$. *Then problem (6) obtains a unique maximum at* $(C_1^*, \alpha^*) = \left(C_1^{P2}, \alpha^{P2}\right)$, *where*

$$C_1^{P2} = \bar{C}_1 - \frac{\Omega}{M(\lambda) - (1+r_f)(1+w)} > \bar{C}_1 \tag{36}$$

$$\alpha^{P2} = \frac{\left[\left(\frac{1}{K_0}\right)^{1/\gamma} + \lambda^{1/\gamma}\right] k}{r_g - r_f} \left(C_1^{P2} - \bar{C}_1\right) > 0 \tag{37}$$

Proof. See Appendix B. ■

Note that for a sufficiently large probability of the good state[22] the threshold value of the discount factor is larger than one ($\delta^{P2-P5} > 1$) and is thus not binding. In that case Proposition 2 applies, irrespective of the household's time preference. The reason is that the household is rather willing to accept a relative loss in the bad state of nature, which occurs with a small enough probability, than to face a relative loss in the first period, which occurs with certainty.

Future relative gains (in the good state of nature) and losses (in the bad state of nature) are given by

$$\left.\begin{array}{lll} C_{2g}^{P2} - \bar{C}_2 = & k\frac{r_g - r_b}{r_g - r_f}\left(\frac{1}{K_0}\right)^{\frac{1}{\gamma}}\left(C_1^{P2} - \bar{C}_1\right) & > 0 \\ \bar{C}_2 - C_{2b}^{P2} = & k\frac{r_g - r_b}{r_g - r_f}\lambda^{\frac{1}{\gamma}}\left(C_1^{P2} - \bar{C}_1\right) & > 0 \end{array}\right\} \tag{38}$$

In problem (P1) the loss aversion parameter does not affect the optimal choices but here loss aversion plays a significant role. An increase in the degree of loss aversion will result in a decline in the first period consumption, a decline in the future consumption in the good state of nature, and a decline in the endogenous second period consumption reference level, but will increase future consumption in the bad state of nature. An increase in loss aversion will also reduce relative gains in the good state of nature in the second period because of the decline in relative gains in the first period. In addition, an increase in loss aversion will reduce relative losses in the bad state of nature in the second period. Even though there are two opposite effects on relative losses in the second period arising from an increase in loss aversion it can be shown that the indirect effect from the decline in $C_1^{P2} - \bar{C}_1$ overpowers the direct impact from increasing the loss aversion parameter. Finally, an increase in loss aversion will reduce the exposure to the stock market and reduce the happiness level.

[22] Namely for $1 > p > 1 - \left[\frac{r_g - r_f}{(1+r_f)(1+w)(r_g - r_b)}\right]^{1-\gamma}$. Note that p must also be larger than $\frac{r_f - r_b}{r_g - r_b}$, which is implied by the assumption $\mathbb{E}(r) > r_f$.

Contrary to problem (P1), an increase in the first period reference level will increase first period consumption, see (36), which is in line with the assumption on preferring the presence to the future. Also it will increase second period consumption in the good state of nature, the second period reference level, and the investment in the financial market because the increase in $\bar{C}_1$ increases relative gains $C_1^{P2} - \bar{C}_1$. However, an increase in $\bar{C}_1$ will reduce future consumption in the bad state of nature as well as savings and investment in the risk-free asset. Relative gains of consumption in the first period will increase, and so will future relative gains in the good state of nature by having higher future relative losses in the bad state of nature. Similarly as in problem (P1), an increase in the first period reference level will decrease the level of happiness, i.e., not comparing at all makes one the happiest. Note that the sensitivities of the solutions (in terms of signs) with respect to loss aversion and the first period consumption reference level are the same as in the case of an exogenous second period reference level, as reported in Hlouskova, Fortin and Tsigaris (2017).

An increase in the habit persistence in consumption reduces the current consumption, the relative consumption in both periods, risk taking, as well as the happiness level. Finally, the increase in the habit persistence in current consumption reduces also the second period endogenous reference level, $\bar{C}_2$, and future consumption in the good state of nature, C_{2g}^{P2}, for a sufficiently large habit persistence level (where the threshold depends on the curvature parameter which is binding only for $\gamma \leq 0.5$), while it increases both $\bar{C}_2$ and C_{2g}^{P2} when the household is sufficiently loss averse and at the same time exhibits a lower level of habit persistence in consumption. Note that the opposite dynamics hold when we consider the effect of the habit persistence in the consumption *reference level*. Finally, note that the dynamics of the current consumption, current relative consumption, second period endogenous reference level and the happiness level with respect to the habit persistence are in line with the dynamics of the less ambitious households.

A change in income here has profoundly different effects from those related to the less ambitious household. An unexpected decrease in income, due to, e.g., a loss of job in bad economic times, will increase first period consumption, second period consumption in the good state of nature, investment in the financial market and also the endogenous second period consumption reference level. In addition, a decrease of income increases the relative consumption in both periods. On the other hand, the second period consumption in the bad state of nature will decrease when income decreases, and so will the happiness level. These effects are opposite (in terms of sign) with respect to those reported for the less ambitious household, with the exception of the future consumption in the bad state of nature and the happiness level, which both decrease with a falling income. The reason is probably related to the fact that the more ambitious household cannot consume above its consumption reference levels at all times while the less ambitious household can always do that. Total savings actually decrease with a falling income. Note finally that if the first period reference consumption level is equal to a fraction of the present value of total wealth, i.e., $\bar{C}_1 = c\left(Y_1 + \frac{Y_2}{1+r_f}\right)$ where $c > \frac{1}{2}$,[23] then the vast majority of sensitivity results become opposite in sign, including the happiness level. This suggests that the indirect effect of income through the first period consumption reference level is stronger than the direct income effect. Thus, in this case the happiness decreases with an increasing income, which is not entirely inconsistent with the literature which finds that as income moves beyond the levels associated with happiness, overall life satisfaction actually decreases, see Jebb et al. (2018).[24] All the sensitivities with respect to problem (P2), which we discussed above, are presented in Table 2.

Note, that the value of the expected utility at the optimum (the level of happiness) is determined by the relative gains in the first period, like in problem (P1):

$$(1-\gamma)\mathbb{E}\left(U\left(C_1^{P2}, \alpha^{P2}\right)\right) = -\left[\frac{k}{(1+r_f)(1+w)}\left(\lambda^{\frac{1}{\gamma}} - \left(\frac{1}{K_\gamma}\right)^{\frac{1}{\gamma}}\right) - 1\right]\left(C_1^{P2} - \bar{C}_1\right)^{1-\gamma} \tag{39}$$

The more ambitious household will be happier with a larger income, while it will be less happy with an increasing first period reference level (even though first period relative gains increase, but also relative losses in the second period in the bad state rise) and a higher persistence in current consumption. These effects are the same as those for the less ambitious household. In addition a larger degree of loss aversion affects the happiness negatively, see Table 2.

Before proceeding further let us introduce the following notation

$$\lambda^{P5-P2} = \left[\frac{\frac{k}{(1+r_f)(1+w)}\frac{1}{K_\gamma^{1/\gamma}} + 1}{\frac{k}{(1+r_f)(1+w)} - 1}\right]^\gamma \quad \text{for} \quad \delta > \delta^{P2-P5} \tag{40}$$

The next proposition shows the case where the household is again more ambitious (i.e., it cannot avoid relative losses at all times) but values future consumption higher (i.e., has a larger discount factor) than the household described by Proposition 2. This is why it strives to avoid relative losses in the second period but has to accept them in the first period. For this to hold, the probability of the good state of nature must be small enough. If it is larger,[25] then the household can avoid relative losses in the first period but has to accept them in the second period in the bad state (which occurs with a small enough probability), i.e., it always solves problem (P2), irrespective of its time preference.

Proposition 3 *Let* $\bar{C}_1^{U,P1} < \bar{C}_1 < \bar{C}_1^{U,P2}$, $\lambda > \max\left\{\lambda^{P1-P5}, \lambda^{P2}, \lambda^{P2-P2}, \lambda^{P5}, \lambda^{P5-P2}, \lambda^{P2-P6}\right\}$ *and* $\delta > \delta^{P2-P5}$. *Then problem (6) obtains a unique maximum at* $(C_1^*, \alpha^*) = \left(C_1^{P5}, \alpha^{P5}\right)$

[23] The fraction needs to be larger than one half such that the household is more ambitious.

[24] Jebb et al. (2018) find that the ideal income point when money no longer increases an individual's happiness is $95,000 for overall satisfaction with life, and $60,000 to $75,000 for emotional well-being. They use a collection of survey responses from over 1.7 million people spanning 164 countries.

[25] For the precise threshold see Footnote 22.

where

$$C_1^{P5} = \bar{C}_1 - \frac{\lambda^{1/\gamma}}{\lambda^{1/\gamma} - (\lambda^{P1-P5})^{1/\gamma}} \times \frac{-\Omega}{(1+r_f)(1+w)}$$
$$= \frac{\lambda^{1/\gamma}\left[Y_1 + \frac{Y_2}{1+r_f} - (1-w)\bar{C}_1\right] - \left(\lambda^{P1-P5}\right)^{1/\gamma}(1+w)\bar{C}_1}{\left[\lambda^{1/\gamma} - \left(\lambda^{P1-P5}\right)^{1/\gamma}\right](1+w)} < \bar{C}_1 \quad (41)$$

$$\alpha^{P5} = \frac{1-K_0^{1/\gamma}}{r_f - r_b + K_0^{1/\gamma}(r_g - r_f)} \times \frac{\left(\lambda^{P1-P5}\right)^{1/\gamma}}{\lambda^{1/\gamma} - \left(\lambda^{P1-P5}\right)^{1/\gamma}} \times (-\Omega) > 0 \quad (42)$$

Proof. See Appendix B. ■

Here, too, the degree of loss aversion enters the solution, as in Proposition 2. However, loss aversion has a different impact on the consumption pattern. An increase in loss aversion will increase the first period consumption (and thus reduce the relative consumption losses in the first period) as well as the second period consumption reference level. On the other hand an increase in loss aversion will reduce the second period consumption in the good state of nature and also the relative reference consumption in both states of nature. Higher loss aversion will also reduce investment in the financial market and the happiness level, like in problem (P2). Finally, the second period consumption in the bad state of nature will be reduced with higher loss aversion if the habit persistence in consumption is sufficiently low, it will be enhanced if the habit persistence in consumption is sufficiently large, and it will remain the same if the habit persistence in consumption equals some threshold depending on the parameters describing the financial market.[26] Note that in the case with an exogenous second period consumption reference level C_{2b}^{P5} is decreasing with an increasing λ, while the other effects remain the same (in terms of sign), see Hlouskova, Fortin and Tsigaris (2017).

The effect of the first period reference level on the first period consumption is negative, while it is positive for the second period consumption in the good state, and mixed[27] for the second period consumption in the bad state. Also the effect of the first period reference level on the second period endogenous reference level is not unambiguous. For sufficiently loss averse households the effect of an increasing first period reference level is positive, while for households with a smaller loss aversion it is negative. Moreover, an increasing first period reference level enhances current relative losses and future relative gains in both states of nature. Also risk taking increases with a higher first period reference level. Finally, the happiness level shrinks when the first period consumption reference increases. Note that the sensitivities with respect to the first period reference level coincide (in terms of signs) with those related to an exogenous second period reference level, see Hlouskova, Fortin and Tsigaris

[26]This threshold value is equal to $\frac{r_g - r_b}{r_f - r_b} \frac{K_0^{1/\gamma}}{1-K_0^{1/\gamma}}$, and it is smaller than one, i.e., binding, only for a sufficiently large probability of the good state of nature, p.

[27]The effect is positive if the household is sufficiently loss averse.

(2017).

The effect of habit persistence in consumption, w, on the second period reference level is negative, while its effect on the investment in the risky asset, relative gains in the second period as well as on the happiness level is positive. The effects on consumption in the first and second periods and on relative losses in the first period cannot be determined unambiguously. For sufficiently loss averse households, however, the effect of w on the first period consumption is positive (and hence negative on current relative losses), while it is negative for the second period consumption in both states of nature (and vice versa).

Regarding the sensitivity analysis with respect to income, an increase in income increases first period consumption, the second period reference level and the happiness level, while it decreases risk taking and relative consumption in both periods. The income effect on consumption in the second period (in the good and in the bad states) cannot be determined unambiguously, it can be either positive or negative. The effect is positive if the household is sufficiently loss averse, while it is negative if the household is not that loss averse.[28] These income effects are partially different from those related to the less ambitious household: while the effect on the first period consumption is the same (in terms of sign), the effect on risk taking is opposite, and the effect on second period consumption can be either the same (if the household is sufficiently loss averse) or opposite. The reason for the difference is that the more ambitious household cannot consume above its reference levels at all times and incurs relative losses in the first period. Consequently, an extra amount of income is rather used to increase consumption in the first period, in order to decrease relative losses in the first period, than to increase consumption in the second period when it is anyway above the reference level. In addition, the less ambitious household increases its risk taking in the financial market with increasing income, while the more ambitious household reduces its risk taking. Finally, the main difference between the more ambitious household with a lower time preference, (P5), and the more ambitious household with a higher time preference, (P2), is that the household with a lower time preference increases its first period consumption and the second period reference consumption when income increases while the household with a higher time preference decreases its first period consumption and the second period reference consumption with a growing income. Note finally that if the first period reference consumption level is equal to a fraction of the present value of the total wealth, i.e., $\bar{C}_1 = c\left(Y_1 + \frac{Y_2}{1+r_f}\right)$ where $c > \frac{1}{2}$,[29] then, as in the case with a more ambitious household with a higher time preference, the majority of sensitivity results become opposite in sign. This suggests again that the indirect effect of income through the first period consumption reference level is stronger than the

[28] Note again that an unexpected reduction in income can change the household's degree of ambition. Let the household be originally less ambitious. Then a drop of income (while keeping the first period reference level constant) will also decrease the threshold $\bar{C}_1^{U,P1}$, which may change the household to be more ambitious, as the first period reference level might then exceed its threshold ($\bar{C}_1 > \bar{C}_1^{U,P1}$) and the household will face losses in the first period.

[29] The fraction needs to be larger than one half such that the household is more ambitious.

direct income effect. Thus, in this case the happiness will again decrease with an increasing income.

Table 2 summarizes the sensitivity results related to the solutions of problem (P5), which we discussed above.

	$C_1^* = C_1^{P2}$ and $\alpha^* = \alpha^{P2}$								
	dC_1^*	dC_{2g}^*	dC_{2b}^*	$d\alpha^*$	$d\bar{C}_2$	$d(C_1^* - \bar{C}_1)$	$d(C_{2g}^* - \bar{C}_2)$	$d(\bar{C}_2 - C_{2b}^*)$	$d\,\mathbb{E}(U^*)$
$d\lambda$	< 0	< 0	> 0	< 0	< 0	< 0	< 0	< 0	< 0
$d\bar{C}_1$	> 0	> 0	< 0	> 0	> 0	> 0	> 0	> 0	< 0
dw	< 0	$\lesseqgtr 0$	> 0	< 0	$\lesseqgtr 0$	< 0	< 0	< 0	< 0
dY_i	< 0	< 0	> 0	< 0	< 0	< 0	< 0	< 0	> 0
	$C_1^* = C_1^{P5}$ and $\alpha^* = \alpha^{P5}$								
	dC_1^*	dC_{2g}^*	dC_{2b}^*	$d\alpha^*$	$d\bar{C}_2$	$d(\bar{C}_1 - C_1^*)$	$d(C_{2g}^* - \bar{C}_2)$	$d(C_{2b}^* - \bar{C}_2)$	$d\,\mathbb{E}(U^*)$
$d\lambda$	> 0	< 0	$\lesseqgtr 0$	< 0	> 0	< 0	< 0	< 0	< 0
$d\bar{C}_1$	< 0	> 0	$\gtreqless 0$	> 0	$\gtreqless 0$	> 0	> 0	> 0	< 0
dw	$\gtreqless 0$	$\lesseqgtr 0$	$\lesseqgtr 0$	> 0	< 0	$\lesseqgtr 0$	> 0	> 0	> 0
dY_i	> 0	$\gtreqless 0$	$\gtreqless 0$	< 0	> 0	< 0	< 0	< 0	> 0

Table 2: Sensitivity results for the more ambitious household with respect to λ, $\bar{C}_1$, w, Y_i, $i = 1, 2$.

Finally, the value of the expected utility at the optimum can be determined by the relative losses in the first period:

$$\begin{aligned}(1-\gamma)\mathbb{E}\left(U\left(C_1^{P5}, \alpha^{P5}\right)\right) &= -\left[\frac{-\Omega}{(1+r_f)(1+w)}\right]^{1-\gamma}\left[\lambda^{1/\gamma} - (\lambda^{P1-P5})^{1/\gamma}\right]^{\gamma} \\ &= -\lambda\left[1 - \left(\frac{\lambda^{P1-P5}}{\lambda}\right)^{1/\gamma}\right](\bar{C}_1 - C_1^{P5})^{1-\gamma} \qquad (43)\end{aligned}$$

The effect of income on happiness is positive, while loss aversion and the first period reference level impact the level of happiness negatively, see Table 2. These results are the same (in terms of signs) as those for problem (P2). The only difference (between the two more ambitious households differing in the rate of time preference) is in the effect of the persistence of current period consumption on the happiness level. It is positive for the more ambitious household with a lower time preference, while it is negative for the more ambitious household with a higher time preference. The positive effect of income on happiness and the negative effect of the first period reference level are also the same as for the less ambitious household.

Before proceeding further let us introduce the following notation:

$$\lambda^{P2-P6}_{\bar{C}_1=\bar{C}_1^{U,P2}} = \frac{(1+r_f)p}{\frac{1}{\delta}-\frac{1}{\delta^{P2-P6}}}\left(\frac{r_g-r_b}{r_f-r_b}+w\right)^{1-\gamma} \quad (44)$$

$$\delta(\lambda)^{P2-P5}_{\bar{C}_1=\bar{C}_1^{U,P2}} = \delta^{P2-P5}\left[1-\left(\frac{\lambda^{P1-P5}}{\lambda}\right)^{1/\gamma}\right]^{\gamma} \quad (45)$$

$$\delta^{P2-P6} = \frac{\left[\frac{r_g-r_b}{r_f-r_b}\frac{Y_1-\bar{C}_1-C_L+\frac{Y_2}{1+r_f}}{\bar{C}_1^{U,P2}-C_L}-w\right]^{\gamma}}{(1-p)\,w} \quad (46)$$

The following corollary describes the results when the current consumption reference level reaches its upper bound. Note that, similarly as in Corollary 1, the first period reference level is a function of the household's income, but also its minimum consumption in the first period and the (risk-free and risky) financial returns are part of the solution.

Corollary 2 *Let* $\bar{C}_1 = \bar{C}_1^{U,P2}$, $\lambda > \max\left\{\lambda^{P2-P4}, \lambda^{P5}, \lambda^{P2-P6}_{\bar{C}_1=\bar{C}_1^{U,P2}}\right\}$ *and* $\delta < \min\left\{\delta(\lambda)^{P2-P5}_{\bar{C}_1=\bar{C}_1^{U,P2}}, \delta^{P2-P6}\right\}$. *Then problem (6) obtains a unique maximum at* (C_1^*, α^*), *where*

$$C_1^* = \bar{C}_1 = \bar{C}_1^{U,P2}$$

$$\alpha^* = \frac{(1+r_f)\left(Y_1-\bar{C}_1^{U,P2}-C_L\right)+Y_2}{r_f-r_b} > 0$$

Proof. See Appendix B. ■

Note that in this case, $C_1 = \bar{C}_1 = \bar{C}_1^{U,P2}$, $C_{2g} = \bar{C}_2 = (1+r_f)\bar{C}_1^{U,P2}$ and $C_{2b} = (1+r_f)C_L$. The solution implies that the household can only consume its minimum level in the second period in the bad state, and it consumes exactly the reference level in the first period and in the second period in the good state. This household, like the other ones but unlike the balanced household, engages positively in the stock market. Any further increase in the first period reference level, while keeping the other parameters constant (such as income, the scarcity constraint and/or returns of the safe and risky assets)[30], i.e., $\bar{C}_1 > \bar{C}_1^{U,P2}$, will result in a state where the household faces relative losses either in the second period (under both states of nature) while keeping gains in the first period, (P4), or where it faces relative losses only in the first period while keeping sure gains in the second period (in both states of nature), (P5).[31]

[30] Or any reduction in income while keeping the other parameters (such as the first period reference level, the scarcity constraint and/or returns of the safe and risky assets) constant.

[31] The solutions actually depend on the following threshold levels for the current reference consumption, namely on $\bar{C}_1^{U,P2} < \bar{C}_1^{U,P4} < \bar{C}_1^{U,P5} < \bar{C}_1^{U,P6}$. For $\bar{C}_1^{U,P2} < \bar{C}_1 \leq \bar{C}_1^{U,P4}$ the household that values future

3.4 Income and other effects – a summary

In this section we summarize the income effects and other effects on happiness, (relative) consumption and risk taking across less ambitious and more ambitious households.

Income effects

All other things equal, more income, as in good economic times, is good: for both less ambitious and more ambitious sufficiently loss averse households with a constant first period consumption reference level a higher income increases the household's happiness. For the less ambitious household, which always consumes above its reference levels, this works through higher current relative gains, see (23). For the more ambitious household which places less weight to the future and thus faces relative losses in the bad state of nature in the second period, this works through smaller current relative gains, see (39),[32] while for the more ambitious household which places more weight to the future and thus faces relative losses in the first period, this works through a reduction of the current relative losses, see (43). Note that the relation "more income – more happiness" is not true for the more ambitious household, when its first period reference level depends on income, e.g., when it coincides with a fraction (that exceeds one half) of the present value of its wealth. In this case more income will imply a larger first period consumption reference level and will actually lead to less happiness, which is due to the indirect effect through the first period reference level outweighing the direct effect of income.

In addition, the less ambitious household increases its investment in the risky asset with good economic times as income increases, while the more ambitious household decreases its exposure to risky financial markets in good economic times and increases its exposure in bad economic times.[33] See Tables 1 and 2 for the described sensitivity results with respect to the first and second period income.

Note that the effects of an increase of income, as discussed above, are the same as the effects of a decrease of the tax rate when we assume a taxation of income endowment. See Section 3.5 for more details.

Note finally that an increase of current income increases savings for both less and more ambitious households.[34] The effect of second period income on savings is opposite to the effect

consumption more, faces relative losses only in the current period, (P5), while the household that values current consumption more achieves relative gains only in the current period, (P4). For $\bar{C}_1^{U,P4} < \bar{C}_1 \leq \bar{C}_1^{U,P5}$ the household faces relative losses only in the current period, (P5). For even higher reference consumption, $\bar{C}_1^{U,P5} < \bar{C}_1 \leq \bar{C}_1^{U,P6}$, the household achieves relative gains only in the good state in the second period, (P6), and finally for the largest values of reference consumption, namely $\bar{C}_1 > \bar{C}_1^{U,P6}$, the household faces relative losses in both periods, (P8).

[32] Note that these smaller relative gains in the first period go hand in hand with smaller relative losses in the second period in the bad state.

[33] Technically speaking, we must consider the effects of larger first and second period income separately. As they are always the same (in terms of signs), however, we simply talk about the effects of income.

[34] Only more ambitious households with a low time preference need to be sufficiently loss averse. If the

of second period income on current consumption for both types of households. Thus, savings are discouraged for less ambitious households and for more ambitious households with a low time preference (that achieve relative gains in the second period), while they are encouraged for more ambitious households with a high time preference (that face relative losses in the bad state of nature in the second period).

Effects of the first period consumption reference level

Ceteris paribus, a higher first period consumption reference level is bad: both the less ambitious and the more ambitious households will be less happy with a larger first period reference level, i.e., a higher comparison level decreases happiness (see Figure 2, the second graph in the bottom row). Thus, the household seems to be happiest when it does not compare itself to anybody at all. For the less ambitious household being less happy with a rising first period reference level works through lower relative gains in both periods as the relative gains shrink with an increasing first period reference level. However, for the more ambitious household, this works only through larger relative losses, enhanced by the penalty on losses, as both relative gains and losses increase with an increasing first period consumption reference level. Note that the shape of indirect utility (happiness) as a function of the first period consumption reference level somehow *mirrors* the value function, where the threshold level of the first period reference consumption (namely the average total discounted income) corresponds to the consumption reference level. Hence the indirect utility function is decreasing with increasing current reference consumption, is concave for reference consumption below the neutral level, is convex for values above this threshold and is non-differentiable at the threshold consumption reference level. This implies that the sensitivity of happiness with respect to the reference level increases with an increasing reference level for less ambitious households (i.e., towards the threshold level), decreases with an increasing reference level for more ambitious households (i.e., when reaching and moving away from the threshold level) and is smaller for less ambitious households than for more ambitious households which follows from the loss aversion. Thus, households with current consumption reference levels around the threshold are the most sensitive (or unstable) with respect to changes of their reference levels.

The reaction of the less ambitious household's consumption to an increase in the first period reference level is ambiguous: A household with a smaller weight placed to the future will decrease its current consumption and increase its future consumption while the opposite happens for a household with a larger weight placed to the future. In the case of a more ambitious household with a higher time preference, the current consumption as well as the second period consumption in the good state of nature increase with increasing $\bar{C}_1$ while the second period consumption in the bad state shrinks. On the other hand, in the case

degree of loss aversion of these households, (P5), is not too large then savings will decrease with an increasing current income. This may happen for households with a small persistence to current consumption.

of a more ambitious household with a lower time preference, the increase of the first period reference level will cause a reduction in the current consumption but an increase in the future consumption.[35] Thus, for instance, if $\bar{C}_1$ is equal to the consumption of a reference household (the Joneses) then our household is following the Joneses[36] when it is either less ambitious with a low time preference or when it is more ambitious with a high time preference. Note that in the case of an exogenous second period consumption reference level, see Hlouskova, Fortin and Tsigaris (2017), the less ambitious household follows the Joneses irrespective of its time preference.

A larger first period reference level also implies a larger second period reference level, except for the more ambitious household with a low rate of time preference (high discount factor), where the effect can be positive or negative. Finally, the effect of a rising first period reference level upon the household's investment in the risky asset is negative for the less ambitious household, and positive for the more ambitious household. Thus, the risk taking decreases for less ambitious households with an increasing current consumption reference level while it increases for more ambitious households when the current reference level increases. See Tables 1 and 2 for the sensitivity results with respect to the first period consumption reference level.

Note that if the current consumption reference level of any household (less or more ambitious) approaches the threshold value $\bar{C}_1 = \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r_f}\right)$ then the household moves towards smoothing consumption across periods, as $\frac{C^*_{2s}}{(1+r_f)C^*_1}$ tends to unity. The balanced household perfectly smoothes consumption across periods (see Figure 2, the first graph in the top row).

The balanced household does not incur any risk with respect to its second period consumption as consumption is the same across both states of the world. If, however, the household's first period reference level moves away from its threshold level (in either direction) then the difference between consumption in the good state and consumption in the bad state will increase, and so will the consumption risk (see Figure 2, the first graph in the bottom row, where the risk of second period consumption is measured by its standard deviation).[37] Note that the Sharpe ratio of second period consumption with respect to the second period reference consumption[38] does not depend on the first period reference level, neither for the less ambitious nor for the more ambitious household (it depends only on whether it is less or more

[35]This holds also for the second period consumption in the bad state of nature, when the household is sufficiently loss averse.

[36]The household is *following the Joneses* when the increase, or decrease, of the first period consumption of a reference household (the Joneses) impacts this household such that its current consumption will change in the same way as the one of the Joneses, i.e., it will increase if the current consumption of the Joneses increases and vice versa. Note that in this context the household's first period reference consumption is equal to the current consumption of the Joneses.

[37]Note that a given effect on consumption risk in the second period, $std(C_2)$, is the same (in terms of signs) as that effect on risk taking, α, as $std(C_2) = \sqrt{p\,(1-p)}\,(C_{2g} - C_{2b}) = \sqrt{p\,(1-p)}\,(r_g - r_b)\,\alpha$.

[38]This Sharpe ratio is $SR_{target}(C_2) = (\mathbb{E}(C_2) - \bar{C}_2)/std(C_2)$.

ambitious). On the other hand, the Sharpe ratio without any target[39] increases when the first period reference level approaches its threshold level (from both sides), as the consumption risk tends to zero.

We show some of the above findings graphically by assuming certain parameter values describing the household and the financial market, see Figure 2. All results are shown as functions of the first period reference level, where we move (from left to right) from less ambitious households to the balanced household to more ambitious households. We only present a selected region of the household's first period reference level around the neutral level. Note that the more ambitious household in our example is the type with a high time preference which faces losses in the bad state of nature. While most of the graphical illustrations show general properties of the household's solution, this is not true for the savings and the risk-free asset in the second graph in the top row. Savings may also increase with a rising first period reference level for less ambitious households (different from the graph) and savings do actually increase with the first period reference level for more ambitious households with a low time preference (however, the latter is not presented in Figure 2).

Effects of loss aversion

All other things equal, the degree of loss aversion does not have any effects on the less ambitious household's happiness, nor on its consumption or its investment in the risky asset.[40] On the other hand, the more ambitious household is less happy with an increasing level of loss aversion, which is triggered solely by shrinking relative gains (whenever there are gains). An increasing level of loss aversion shows opposite effects (in terms of signs) on the second period reference level, for different time preferences. The effect is negative for a high time preference, and it is positive for a low time preference. Technically speaking, this works through the impact of loss aversion on first period consumption (which is negative for a high time preference, and positive for a low time preference). Finally, a higher degree of loss aversion implies a lower investment in the risky asset for the more ambitious household, which is what one would probably expect. See Tables 1 and 2 for the sensitivity results with respect to loss aversion.

Effects of the habit persistence in consumption

The effect of persistence in consumption on happiness depends on whether the household's first period optimal consumption is above or below the reference level. Whatever is smaller – either consumption or the reference level – should be followed more intensely (in the formation of the second period reference level) in order to increase happiness. If first period consumption

[39] This Sharpe ratio is $SR(C_2) = E(C_2)/std(C_2)$.

[40] The assumption on the degree of loss aversion to be sufficiently large is to guarantee that the maximum of (P1) exceeds the potential maxima of (P2) and (P5).

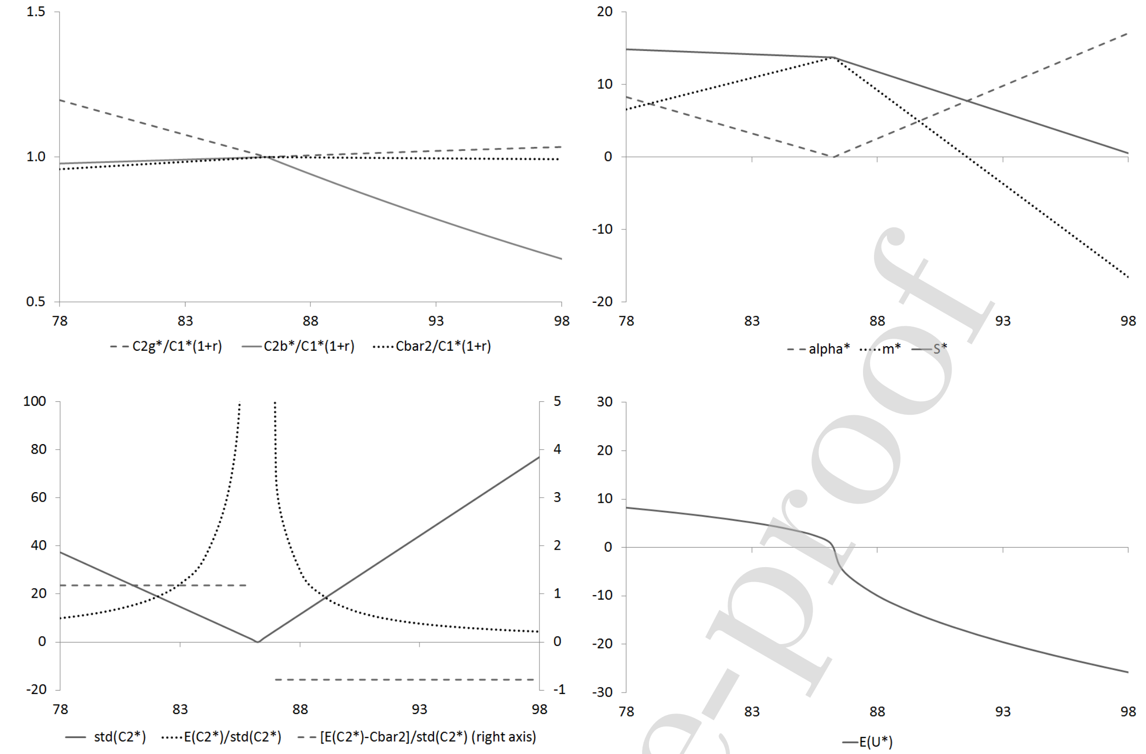

Figure 2: Analysis with respect to the first period reference level
The example is based on the following parameter values characterizing the household and the financial market: $r_f = 3$, $r_g = 10$, $r_b = 1$, $p = 0.5$, $w = 0.5$, $\gamma = 0.5$, $\delta = 0.25$, $\lambda = 10$, $Y_1 = 100$, $Y_2 = 290$, $C_L = 0$. Assuming that one period covers, e.g., 40 years the financial market rates correspond to annual returns of 3.5% (for the risk-free asset), 6.2% (for the risky asset in the good state) and 1.7% (for the risky asset in the bad state), respectively. The discount factor is chosen such that the corresponding discount rate is equal to the risk-free interest rate. The threshold level of the first period reference consumption $\bar{C}_1^{U,P1}$ is equal to 86.25 and the more ambitious household is the one with a high time preference ($\delta \leq \delta^{P2-P5} = 0.72$). The graphs in the top row display the household's consumption smoothing and investment, the graphs in the bottom row present consumption risk in the second period and happiness, where everything is shown as a function of the first period reference level.

is above the reference level, then increasing habit persistence in consumption makes the household less happy while increasing persistence in the consumption reference level makes it happier. Thus the household should intensify its persistence on the consumption target. This situation applies to the less ambitious household and the more ambitious household with a higher time preference. On the other hand, if first period consumption is below the reference level, then growing habit persistence on consumption leads to more happiness while increasing persistence on the consumption reference level results in less happiness. Hence the household should stick more to its consumption habits. This applies to the more ambitious household with a lower time preference.

For the less ambitious household the decrease in happiness materializes only through a decline of the first period consumption (or, equivalently, through a decline of the first period relative gains), as the second period relative gains actually increase with an increasing persistence in consumption, see Table 1. On the other hand, for the more ambitious household with a sufficiently small discount factor the decrease in happiness is triggered by a decrease of both the first period relative gains and the second period relative gains when the good state of nature occurs. Finally, for the more ambitious household with a sufficiently large discount factor the increase in happiness is caused by a decrease of the relative losses in the first period (for a sufficiently loss averse household) as well as by an increase of the relative gains in the second period.

3.5 Implications for income taxation

Our analysis has important implications in terms of how a household responds to income reductions due to the impact of taxation of endowment income.[41] In fact an increase in the tax rate is equivalent to a decrease in income in the model without taxes. The effects of taxation will depend on whether the household has a low or a high first period consumption reference level, hence on the type of household. Suppose suddenly income is taxed and the household is less ambitious such that it only experiences relative gains. Then increased taxation will reduce current consumption, future consumption in both states of nature, risk taking, second period reference level, relative gains and happiness.

Suppose, then, the household is more ambitious with a high time preference (i.e., it values more current consumption) and thus experiences relative losses in the bad state of nature in the second period. Increased taxation of income in this case will increase current consumption, risk taking, consumption in the good state of nature, the second period reference level, current relative gains, second period relative gains in the good state of nature and second period relative losses in the bad state of nature, which is opposite to the response of a less ambitious household towards taxation of income. On the other hand, increased taxation will reduce consumption in the bad state of nature as well as happiness.

Suppose, further, the household is more ambitious with a low time preference (i.e., it values more future consumption) and is thus willing to experience relative losses in the first period, then increased taxation will discourage current consumption but stimulate risk taking while the direction of future consumption is ambiguous and happiness will decrease. In terms of relative gains and losses increased taxation of income will increase relative losses in the first period as well as relative gains in the future.

[41] The corresponding model set-up is the same as presented by (6), only income is replaced by after-tax income in all formulations, propositions and corollaries. I.e., Y_1 is replaced by $(1-\tau)Y_1$ and Y_2 is replaced by $(1-\tau)Y_2$, where $\tau \in (0,1)$ is the tax rate of income.

Finally, if the household is at the threshold level, i.e., if it is balanced, then a sudden increase in taxation (while keeping all other parameters unchanged) will reduce the present value of after-tax income and thus the threshold level of the current reference consumption $\bar{C}_1^{U,P1}$ will shrink, which in turn makes the household more ambitious.

For the discussion of the effects of income taxes on savings it is reasonable to assume that the tax rates on current and future income are not independent. For simplicity we assume that they are the same. Then a higher income tax (which induces lower disposable income) discourages savings for the more ambitious household with a high time preference, and it also discourages savings for the less ambitious household and the more ambitious household with a low time preference provided second period income is small enough.[42] On the other hand, if second period income is larger than the threshold then a higher tax rate stimulates savings for the less ambitious household and the more ambitious household with a low time preference. Note that if the household has a sufficiently large persistence in current consumption and the second period is the retirement period then more plausible is the case when second period income does not exceed its threshold.[43]

A particularly interesting result is the impact of taxation on risk taking. Taxation of income will discourage risk taking for less ambitious households, while for more ambitious households, irrespective of their rate of time preference, taxation will increase risk taking. Finally, taxation makes a household less happy irrespective of its first period reference level.

$d\tau$	dC_1^*	dC_{2g}^*	dC_{2b}^*	$d\alpha^*$	$d\bar{C}_2$	$d\lvert C_1^* - \bar{C}_1\rvert$	$d\lvert C_{2g}^* - \bar{C}_2\rvert$	$d\lvert C_{2b}^* - \bar{C}_2\rvert$	dS^*	$d\,\mathbb{E}(U^*)$
$\bar{C}_1 < \bar{C}_1^{U,P1}$	< 0	< 0	< 0	< 0	< 0	< 0	< 0	< 0	$\lesseqgtr 0$	< 0
$\bar{C}_1 > \bar{C}_1^{U,P1}$, $\delta \leq \delta^{P2-P5}$	> 0	> 0	< 0	> 0	> 0	> 0	> 0	> 0	< 0	< 0
$\bar{C}_1 > \bar{C}_1^{U,P1}$, $\delta > \delta^{P2-P5}$	< 0	$\lesseqgtr 0$	$\lesseqgtr 0$	> 0	< 0	> 0	> 0	> 0	$\lesseqgtr 0$	< 0

Table 3: Sensitivity results for less ambitious and more ambitious households with respect to the income tax.

4 Concluding remarks

In this paper we analyze the two-period consumption-investment decision of a household with prospect theory preferences and an endogenous second period reference level which captures habit persistence in consumption and in the current consumption reference level. We find that the optimal solution of a sufficiently loss averse household depends on how its first period consumption reference level relates to a given threshold which is equal to the average discounted endowment income. The reference level may be below, equal to, or above this threshold and hence households can be of three types. These three types are characterized

[42] In the latter case the household, in addition, needs to be sufficiently loss averse.
[43] The threshold is given by $(1 + r_f)wY_1 + \tilde{k}$, where $\tilde{k} \geq 0$.

by how their optimal consumption relates to their reference consumption. First there are households with a relatively low reference level (less ambitious households), which can avoid relative consumption losses in both periods. This means that they always consume above their reference levels. Second there are balanced households with a neutral reference level, which always consume exactly their reference levels. This type of household, however, is very special and can only occur when its first period reference level is equal to the average of the discounted income. Third there are households with a relatively high reference level (more ambitious households), which cannot avoid to incur relative consumption losses, either now or in the future. More precisely, a more ambitious household with a lower discount factor (high time preference) will face relative losses in the second period in the bad state of nature while a more ambitious household with a higher discount factor (low time preference) incurs relative losses in the first period. Note that the three types of household sometimes act very differently from one another and thus there is a diversity of behavior resulting from the different levels of comparison.

We observe the following effects of habit persistence in consumption. A less ambitious household will be less happy with a rising persistence in consumption, but at the same time it will be happier with a rising persistence in the reference consumption. Hence it is better to stick to one's exogenously given consumption target than to one's consumption habits. The same applies to the more ambitious household with a high time preference. However, the situation is reverse for the more ambitious household with a low time preference: it will be happier if it sticks more to its consumption habits than to its target, i.e., if it intensifies its consumption habits. In addition clinging to one's consumption habits decreases current consumption for the less ambitious household and the more ambitious household with a relatively high time preference, while evidence is mixed for the more ambitious household with a relatively low time preference.

It is always true that more income is better, i.e., the larger the income, the happier the household – provided the first period reference level does not depend on income. However, if the reference level depends on income in the sense that it is equal to a fraction of the present value of total wealth and the household is more ambitious, then a higher income reduces happiness. This is due to the fact that in this case the indirect effect of income (through the first period reference level) outweighs the direct effect of income. We also observe that less ambitious households increase their exposure to risky assets during good economic times (i.e., when their income increases) while the more ambitious households increase their exposure to risky assets during bad economic times (i.e., when their income decreases).

Finally, we obtain the same findings as in Hlouskova, Fortin and Tsigaris (2017) related to the dependence of happiness upon the current consumption reference level: the highest utility is achieved for the lowest current consumption reference level. Thus, not comparing at all (e.g., to others) leads to the highest level of happiness. In addition, the sensitivity of happiness with respect to the reference level increases with an increasing reference level for less ambitious households, decreases with an increasing reference level for more ambitious households and is smaller for less ambitious households than for more ambitious households due to loss aversion. The highest sensitivity (instability) occurs around the threshold reference consumption.

We also discuss the effects of taxation of endowment income: increasing the tax rate in a model with income taxes is actually equivalent to decreasing income in a model without taxes. An interesting extension would certainly be to examine the household's optimal consumption-investment behavior if also capital income is taxed.

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