

# Nonparametric identification of unobserved technological heterogeneity in production



## Working Paper Research

by Laurens Cherchye, Thomas Demuynck,  
Bram De Rock and Marijn Verschelde

February 2018 No 335

---

**Editor**

Jan Smets, Governor of the National Bank of Belgium

**Statement of purpose:**

The purpose of these working papers is to promote the circulation of research results (Research Series) and analytical studies (Documents Series) made within the National Bank of Belgium or presented by external economists in seminars, conferences and conventions organised by the Bank. The aim is therefore to provide a platform for discussion. The opinions expressed are strictly those of the authors and do not necessarily reflect the views of the National Bank of Belgium.

**Orders**

For orders and information on subscriptions and reductions: National Bank of Belgium,  
Documentation - Publications service, boulevard de Berlaimont 14, 1000 Brussels

Tel +32 2 221 20 33 - Fax +32 2 21 30 42

The Working Papers are available on the website of the Bank: <http://www.nbb.be>

© National Bank of Belgium, Brussels

All rights reserved.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

ISSN: 1375-680X (print)

ISSN: 1784-2476 (online)

## Abstract

We propose a novel nonparametric method for the structural identification of unobserved technological heterogeneity in production. We assume cost minimization as the firms' behavioral objective, and we model unobserved heterogeneity as an unobserved productivity factor on which firms condition the input demand of the observed inputs. Our model of unobserved technological differences can equivalently be represented in terms of unobserved "latent capital" that guarantees data consistency with our behavioral assumption, and we argue that this avoids a simultaneity bias in a natural way. Our empirical application to Belgian manufacturing data shows that our method allows for drawing strong and robust conclusions, despite its nonparametric orientation. For example, our results pinpoint a clear link between international exposure and technological heterogeneity and show that primary inputs are in the considered sectors substituted for materials rather than for technology.

JEL classification: C14, D21, D22, D24.

Key words: production behavior, unobserved heterogeneity, cost minimization, nonparametric identification, simultaneity bias, latent capital, manufacturing

### **Authors:**

Laurens Cherchye, Department of economics, University of Leuven. E. Sabbelaan 53, 8500 Kortrijk, Belgium – e-mail: [laurens.cherchye@kuleuven.be](mailto:laurens.cherchye@kuleuven.be).

Laurens Cherchye gratefully acknowledges the European Research Council (ERC) for his Consolidator Grant 614221. Part of this research is financed by the Fund of Scientific Research Flanders (FWO-Vlaanderen).

Thomas Demuynck, ECARES, Université Libre de Bruxelles. avenue F. D. Roosevelt 50, CP 114, 1050 Brussels, Belgium – e-mail: [thomas.demuynck@ulb.ac.be](mailto:thomas.demuynck@ulb.ac.be).

Bram De Rock, ECARES, Université Libre de Bruxelles, and Department of Economics, University of Leuven (KULeuven). Avenue F. D. Roosevelt 50, CP 114, 1050 Brussels, Belgium. e-mail: [bderock@ulb.ac.be](mailto:bderock@ulb.ac.be).

Bram De Rock gratefully acknowledges FWO and FNRS for their financial support.

Marijn Verschelde, Department of Economics and Quantitative Methods, IÉSEG School of Management, LEM (UMR-CNRS 9221), and Department of Economics, KULeuven. e-mail: [m.verschelde@ieseg.fr](mailto:m.verschelde@ieseg.fr).

The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium or any other institution to which the authors are affiliated.



## TABLE OF CONTENTS

<b>1. Introduction</b> .....	<b>1</b>
<b>2. Methodology</b> .....	<b>3</b>
2.1. Production with technological heterogeneity .....	3
2.2. Cost minimizing production behavior .....	4
2.2. Bringing our model to data.....	7
<b>3. Application set-up and data</b> .....	<b>8</b>
<b>4. Empirical results</b> .....	<b>11</b>
4.1. Latent capital estimates: a first look .....	12
4.2. Latent capital, sourcing and international exposure .....	14
4.3. Substitution between latent capital and observed inputs.....	16
<b>5. Conclusion</b> .....	<b>20</b>
References.....	21
Appendixes.....	25
National Bank of Belgium - Working papers series .....	37



# 1 Introduction

The increasing prevalence of global sourcing (Antras and Helpman, 2004) and changing input cost shares (Autor et al., 1998, 2003) lies at the heart of the industrial policy debate.<sup>1</sup> Paradoxically, these phenomena are excluded by construction under the assumption of Hicks neutral technical change, which is usually made in existing methods for empirical production analysis. The few empirical production studies that do relax this assumption of Hicks neutrality typically rely on a specific parameterization of the production technology or impose a common structure on the factor bias across firms.<sup>2</sup> However, empirical evidence and economic theory show that there can be firm heterogeneity in factor biased technical change (see, for example, Acemoglu et al. (2015) and references therein). This makes an a priori parametrization difficult.

In a series of seminal papers, Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984) proposed an intrinsically nonparametric approach to address the identification of production functions.<sup>3</sup> It recovers the production possibilities directly from the data and avoids functional specification bias by not imposing any (nonverifiable) parametric structure on the production technology. Its identifying power comes from a structural specification of the firms' objectives that underlie the observed production behavior.

Despite this conceptually appealing starting point, the more recent literature on the identification and estimation of production functions has largely ignored this nonparametric alternative. We take it that this lack of attention principally originates from the fact that the existing nonparametric methods are unable to deal with unobserved technological heterogeneity. The importance of effectively dealing with unobserved technological heterogeneity is by now well-established in the literature (see, for example, the recent review of Syverson (2011)). Basically, incorporating unobserved heterogeneity in the empirical analysis is a prerequisite to account for endogeneity between input choice and unobserved productivity. This endogeneity issue was first pointed out by Marschak and Andrews (1944), and originates from the fact that a firm's productivity transmits to its optimal input choices. It implies that standard OLS-type estimation techniques will suffer from a simultaneity bias (see also Olley and Pakes (1996) and Griliches and Mairesse (1998)).<sup>4</sup>

---

<sup>1</sup>See, for example, OECD (2012), Dall'Olio et al. (2013) and CompNet Task Force (2014) for empirical studies of manufacturing firms with a specific focus on policy implications.

<sup>2</sup>See, for example, the recent study of Doraszelski and Jaumandreu (Forthcoming), which parametrizes the labor augmenting technological change next to the Hicks neutral technological change in a constant elasticity of substitution (CES) framework.

<sup>3</sup>We refer to Grifell-Tatjé et al. (Forthcoming) (and references therein) for a recent review of alternative approaches of productivity measurement that have been proposed in the Economics and OR/MS literature.

<sup>4</sup>The literature on the estimation and identification of production functions has paid considerable attention to developing techniques that address this endogeneity problem. Notable examples include Olley and Pakes (1996); Levinsohn and Petrin (2003); Wooldridge (2009); Akerberg et al. (2015); Gandhi et al. (2017). A main difference with our nonparametric approach is that the empirical implementation of these existing approaches requires a (semi)parametric specification of the production technology.

The principal aim of the current paper is to re-establish the nonparametric approach as a full-fledged alternative for empirical production analysis. To this end, we present a methodology that uses minimal assumptions to address identification of unobserved technological heterogeneity across firms. Specifically, we assume cost minimization as the firms' behavioral objective and we model unobserved heterogeneity as an unobserved productivity factor, on which we condition the input demand of observed inputs. This avoids the endogeneity bias in a natural way, by explicitly accounting for the simultaneity between technological productivity and input decisions in our structural specification of the firm's optimization problem. We also provide a novel and intuitive way to quantify unobserved heterogeneity in terms of "*latent capital*". Our method allows us to analyze cost share changes of both observed costs and (unobserved) latent capital. For example, we can investigate to what extent observed primary manufacturing inputs are substituted over time for other observed inputs and/or unobserved technology. This unique feature is intrinsic to the nonparametric nature of our methodology, which avoids imposing particular functional structure on the (changing) production technology (such as Hicks neutrality).

An attractive feature of our method is that it can be operationalized through linear programming, which makes it easy to apply in practice. We demonstrate this through an empirical application that studies technological heterogeneity at the firm-year level in the Belgian manufacturing sector for the period 1997-2007. Our application shows that our method does allow for drawing strong and robust conclusions, despite its nonparametric orientation. For the period under study, we confirm the well-established connection between international exposure and technological heterogeneity. Generally, the cost share of latent capital remains constant over time, which is in accordance with the well-documented productivity slowdown in manufacturing since the early 2000s (see, for example, Syverson (2017)). Further, we document that Belgian manufacturing firms substitute labor and capital for domestic and foreign materials (i.e., outsourcing), rather than for latent capital (i.e., technology), and that this substitution pattern is more pronounced for large firms. We also show that our results are robust for altering our revenue based definition of output to a produced value based definition that excludes servicing and carry-along trade (see Bernard et al. (Forthcoming)). We see all this as strong empirical evidence against the assumption of Hicks neutrality.<sup>5</sup>

The remainder of this paper unfolds as follows. Section 2 presents our novel methodology for nonparametric production analysis with unobserved technological heterogeneity. We also introduce our concept of latent capital to empirically quantify unobserved technological heterogeneity, and we indicate how to bring our model to data. Section 3 motivates our

---

<sup>5</sup>Our empirical application shows general patterns of input cost share changes, hereby specifically concentrating on the role of latent capital. Our results on observed cost share changes fall in line with those reported by Vershelde et al. (2014), who focused on changes of output elasticities over time for a closely similar dataset of Belgian manufacturing firms. More recently, Dewitte et al. (2017) provided a detailed study of heterogeneity in factor biased technological change for Belgian manufacturing firms. These authors considered firm-level changes of output elasticities by applying a nonparametric kernel regression with time-varying fixed effects.



application to Belgian manufacturing firms, and discusses the input and output data that we use. Section 4 presents our main empirical findings. Section 5 concludes and discusses possible avenues for follow-up research.

## 2 Methodology

We begin this section by presenting our specification of the firm’s optimization problem under technological heterogeneity, to subsequently establish the associated nonparametric characterization of optimizing firm behavior. This will pave the way for introducing our concept of latent capital to empirically quantify technological differences between firms. We conclude by showing how to bring our model to data. We will explain how we can account for (small) deviations from “exactly” optimizing behavior in empirical applications, by using a nonparametric measure of goodness-of-fit.

### 2.1 Production with technological heterogeneity

Firms’ production levels depend on observed inputs, as well as some unobserved heterogeneity/productivity factor. Formally, we assume a production function  $f$  that defines

$$y = f(\mathbf{q}, \varepsilon),$$

for  $y \in \mathbb{R}_+$  the output level,  $\mathbf{q} \in \mathbb{R}_+^n$  an  $n$ -dimensional vector of observed inputs, and  $\varepsilon \in \mathbb{R}_+$  a single-dimensional measure of the unobserved heterogeneity in the production process across firms. The assumption that unobserved technological differences are one-dimensional follows the standard practice in the literature (see, for example, Olley and Pakes (1996); Levinsohn and Petrin (2003); Wooldridge (2009); Akerberg et al. (2015); Gandhi et al. (2017)). A useful implication is that it allows for a transparent empirical analysis of technological heterogeneity patterns, as we will demonstrate in our empirical application in Section 4.<sup>6</sup>

Generally, we can interpret the unobserved  $\varepsilon$  in two ways.<sup>7</sup> In the first interpretation,  $\varepsilon$  falls beyond the firms’ control and stands for external drivers of productivity variation and random productivity shocks. For example, firms with higher  $\varepsilon$  have access to better technologies, thereby increasing their output  $f(\mathbf{q}, \varepsilon)$  for the same level of observed inputs  $\mathbf{q}$ . Alternatively, we can also interpret  $\varepsilon$  as an unobserved input, which implies that it is optimally chosen by the firm. This interpretation includes all factors under the control of

---

<sup>6</sup>Multi-dimensional (preference) heterogeneity has been considered in the nonparametric analysis of consumer behavior. See, for example, Hoderlein and Stoye (2014), who use the random utility framework of McFadden and Richter (1991). Different from our current study, the principal motivation of this other research is (*only*) to structurally *model* heterogeneity in the empirical analysis of consumer behavior, and not to (*also*) *identify* the level of heterogeneity as such.

<sup>7</sup>See Syverson (2011) for a general discussion on alternative interpretations of unobserved technological heterogeneity (i.e., productivity differences) that appeared in the literature.

the firm that influence productivity, such as managerial input, information technology and R&D. Importantly, while the two interpretations are clearly distinct, we will show that the associated models of optimizing firm behavior are empirically equivalent in terms of their nonparametric testable implications. As a result, our following characterization of optimizing behavior does not depend on the specific meaning that is attached to  $\varepsilon$ .

Throughout, we assume that the function  $f$  is strictly monotonic, continuous and jointly concave in  $(\mathbf{q}, \varepsilon)$ . In addition, we postulate that the production technology is characterized by constant returns-to-scale (CRS), which means that, for all numbers  $t > 0$ ,

$$f(t\mathbf{q}, t\varepsilon) = tf(\mathbf{q}, \varepsilon).$$

In our main empirical analysis, we only impose CRS within a specific firm size category, as technological heterogeneity is analyzed for each firm size category separately. This effectively implies that we (only) assume CRS to hold “locally” (i.e., for the given firm size), so avoiding the “global” CRS postulate, which –admittedly– may seem overly strong in many practical settings.

Usually, the CRS assumption is motivated by a replication argument: if one doubles all the inputs, one can always double the output. Implicitly, this assumes that all inputs are taken into account. From this perspective, we can effectively motivate the CRS assumption in our context by interpreting  $\varepsilon$  as an unobserved input factor. That is, we assume constant returns to scale of the production function  $f(\mathbf{q}, \varepsilon)$ , which takes into account both the observable and unobservable inputs. Some applications assume decreasing instead of constant returns-to-scale in terms of observed inputs. The main argument for using decreasing returns-to-scale is that the firm has a CRS technology but some (unobserved) input is fixed in the short run. From that perspective, the heterogeneity term  $\varepsilon$  can also be seen as such a fixed unobserved input.

## 2.2 Cost minimizing production behavior

Throughout we assume that firms are price takers in the input market and we impose no structure on the form of the output market. As shown by Carvajal et al. (2013, 2014), it is possible to impose alternative (for example, Cournot or Bertrand) structures on the output market in our advocated nonparametric framework. In our following analysis, we purposely do not impose any such assumption, so showing that our identification results are independent of the output market form.

Let  $\mathbf{w} \in \mathbb{R}_{++}^n$  be the price vector for the observed inputs. Our above two interpretations of the heterogeneity factor  $\varepsilon$  yield two different models of optimizing firm behavior. First, if we assume that  $\varepsilon$  is beyond the firm’s control, then the firm solves the optimization problem

$$\min_{\mathbf{q}} \mathbf{w}\mathbf{q} \text{ s.t. } f(\mathbf{q}, \varepsilon) \geq y_0 \quad (O.P.I).$$

That is, the firm's input choice  $\mathbf{q}$  is conditional on the unobserved factor  $\varepsilon$ . Second, if  $\varepsilon$  is an unobserved input factor that is chosen by the optimizing firm, then this firm solves

$$\min_{\mathbf{q}, \varepsilon} \mathbf{w}\mathbf{q} + \tau\varepsilon \text{ s.t. } f(\mathbf{q}, \varepsilon) \geq y_0, \quad (OP.II)$$

for  $\tau \in \mathbb{R}_{++}$  the unobserved price of  $\varepsilon$ . In both scenarios, the simultaneity bias is absent by construction, because either the (observed) inputs  $\mathbf{q}$  are optimally chosen conditionally on the unobserved  $\varepsilon$  or, alternatively, these inputs are defined simultaneously with the unobserved input  $\varepsilon$ .

We demonstrate the empirical equivalence of optimizing behavior in terms of (OP.I) and (OP.II) by establishing the associated testable implications. To this end, we assume to observe a dataset

$$S = \{\mathbf{w}_t, \mathbf{q}_t, y_t\}_{t \in T},$$

with  $\mathbf{w}_t$  the observed input prices,  $\mathbf{q}_t$  the observed input levels, and  $y_t$  the observed output levels for a set of  $T$  firm observations. The data set can be a cross-section, a time-series or, as in our own empirical application, a panel with firm observations specified at the firm-year level. The set  $S$  contains all information on observed production behavior that is used by the empirical analyst. In principle, it is possible to integrate in our set-up extra information on indicators that are (assumed to be) correlated with the unobserved technological heterogeneity (e.g., R&D investments). Again, we intentionally restrict to our minimalistic setting to show the generality of our identification results.

The functional form of the production function  $f$  is unknown to the empirical analyst. Our nonparametric method basically checks whether there exists at least one specification of  $f$  that represents the observed firm behavior in terms of the optimization problems (OP.I) and (OP.II). If such a function exists, we say that the dataset  $S$  is rationalizable in terms of (OP.I) and (OP.II).

**Definition 1.** *Let  $S = \{\mathbf{w}_t, \mathbf{q}_t, y_t\}_{t \in T}$  be a given dataset.  $S$  is (OP.I)-rationalizable if there exist heterogeneity numbers  $\varepsilon_t \in \mathbb{R}_+$  and a production function  $f : \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+$  such that, for all firm observations  $t \in T$ ,*

$$\mathbf{q}_t \in \arg \min_{\mathbf{q}} \mathbf{w}_t \mathbf{q} \text{ s.t. } f(\mathbf{q}, \varepsilon_t) \geq y_t.$$

*The dataset  $S$  is (OP.II)-rationalizable if, in addition, there exist prices  $\tau_t \in \mathbb{R}_+$  such that, for all firm observations  $t \in T$ ,*

$$(\mathbf{q}_t, \varepsilon_t) \in \arg \min_{\mathbf{q}, \varepsilon} \mathbf{w}_t \mathbf{q} + \tau_t \varepsilon \text{ s.t. } f(\mathbf{q}, \varepsilon) \geq y_t.$$

In Appendix A.1 we prove that (OP.I)-rationalizability and (OP.II)-rationalizability generate exactly the same nonparametric testable implications for a given dataset  $S$ . This conclusion is summarized in the following proposition.

**Proposition 1.** Let  $S = \{\mathbf{w}_t, \mathbf{q}_t, y_t\}_{t \in T}$  be a given dataset. The following statements are equivalent:

- (i) The dataset  $S$  is (OP.I)-rationalizable;
- (ii) The dataset  $S$  is (OP.II)-rationalizable;
- (iii) There exist  $\varepsilon_t \in \mathbb{R}_+$  and  $\gamma_t \in \mathbb{R}_{++}$  that satisfy, for all  $t, v \in T$ , the inequalities

$$\frac{y_t}{y_v} \leq \frac{\gamma_v \mathbf{w}_v \mathbf{q}_t + \varepsilon_t}{\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v}.$$

In the next section we show how the inequalities in the third statement of Proposition 1 can be brought to the data by means of linear programming, which will allow us to specify values of  $\varepsilon_t$  that rationalize the dataset  $S$ . Moreover, when interpreting these numbers  $\varepsilon_t$  as representing unobserved input quantities, the associated numbers  $\gamma_t$  give the inverse of the corresponding shadow input prices ( $1/\tau_t$ ). Interestingly, we can use this to nonparametrically quantify technological heterogeneity in terms of unobserved input cost, which we refer to as “*latent capital*”,

$$\text{LC} = \tau \varepsilon.$$

It readily follows from our above discussion that this LC measure has a direct interpretation as capturing productivity differences. All else equal, higher LC values indicate that the same output can be produced with less observed costs, which effectively reveals a higher (unobserved) productivity level. In our empirical analysis, we will not only focus on latent capital levels, but also on “*cost shares of latent capital*”,

$$\text{CSLC} = \frac{\tau \varepsilon}{\mathbf{w} \mathbf{q} + \tau \varepsilon},$$

which expresses the firm’s latent capital as a fraction of the total (observed plus unobserved) cost. This measure is naturally bounded between zero and one, and a higher CSLC value indicates a greater importance of the unobserved input relative to the other (observed) inputs. As we will show in our empirical application, we can use the CSLC measure to investigate substitution patterns between the observed inputs and the unobserved technology.

As a concluding note, Appendix A.2 presents a numerical example that illustrates the testable implications in Proposition 1. It shows that our empirical conditions for cost minimization with unobserved heterogeneity can be rejected (i.e., have empirical content) even in a minimalistic setting with only two firm observations and two observed inputs. Generally, the empirical bite of the conditions will increase with the number of observations and observed inputs.

## 2.3 Bringing our model to data

The rationalizability conditions in Proposition 1 are strict: either the dataset  $S$  satisfies them “exactly” or it does not. In practice, it is often useful to allow for small deviations from exactly rationalizable behavior. Such deviations may be due to (small) optimization errors by the firms or, alternatively, data imperfections (for example, ill-measured input/output quantities and/or input prices).<sup>8</sup> To include these possibilities, we define a nonparametric goodness-of-fit parameter that has an intuitive economic interpretation in terms of departures from the cost minimization hypothesis that we maintain as our core identifying assumption (see Afriat (1972) and Varian (1990)).<sup>9</sup> By fixing our goodness-of-fit parameter at a value close to (but different from) one, we can take account of observed behavior that is close to (but not exactly) rationalizable in the sense of Definition 1.

More precisely, we increase the right hand sides of the inequality requirements in Proposition 1 by using the goodness-of-fit parameter  $\theta$  (with  $0 \leq \theta \leq 1$ ) to specify

$$\frac{y_t}{y_v} \leq \frac{\gamma_v \mathbf{w}_v \mathbf{q}_t + \varepsilon_t}{\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v} + (1 - \theta). \quad (1)$$

Obviously  $\theta = 1$  obtains the exact conditions in Proposition 1, while lower values for  $\theta$  weaken the rationalizability requirements. To further interpret our goodness-of-fit measure, we formally show in Appendix A.3 that adding  $(1 - \theta)$  is equivalent to equiproportionally contracting the inputs  $(\mathbf{q}_v, \varepsilon_v)$  to  $(\theta \mathbf{q}_v, \theta \varepsilon_v)$ . This in turn corresponds to lowering the cost level  $(\mathbf{w}_v \mathbf{q}_v + \tau_v \varepsilon_v)$  by the same degree and, therefore, implies a weaker criterion of “nearly” (instead of “exactly”) optimizing behavior. In our following empirical application, our main focus will be on  $\theta = 0.95$  (which, intuitively, decreases firm  $v$ ’s total cost level  $(\mathbf{w}_v \mathbf{q}_v + \tau_v \varepsilon_v)$  by 5 percent). In Appendix C, we also check robustness of our main results for alternative  $\theta$ -values.

To bring our inequalities to the data, we reformulate (1) as

$$y_t(\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v) - y_v(\gamma_v \mathbf{w}_v \mathbf{q}_t + \varepsilon_t) \leq (1 - \theta)y_v(\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v). \quad (2)$$

For a fixed value of  $\theta$ , this defines restrictions that are linear in the unknowns  $\gamma_t$  and  $\varepsilon_t$ . We can use simple linear programming tools to check if there exists a solution of (2) and, thus, to conclude if the dataset is exactly rationalizable (when using  $\theta = 1$ ) or nearly rationalizable (when using  $\theta < 1$ ).

Finally, the linear restrictions (2) will generally define a multitude of feasible specifications of  $\gamma_t$  and  $\varepsilon_t$  (and, thus, of our latent capital measures LC and CSLC). To empirically

---

<sup>8</sup>In fact it is also possible to explicitly account for measurement errors in prices and quantities in our nonparametric analysis. For example, we may use the procedure suggested by Varian (1985), which is fairly easily adjusted to our setting. To facilitate our exposition, we will not consider this extension in the current paper.

<sup>9</sup>In a similar spirit, Varian (1990) argues that such a nonparametric goodness-of-fit measure can also be interpreted in terms of “economic significance” of departures from optimization, which is to be distinguished from the more standard notion of statistical significance.

evaluate the importance of firm heterogeneity, a natural choice is to use the specification that minimizes the cost shares of latent capital that are required for rationalizability (as characterized in (2)) or, equivalently, that maximizes the role played by the observed inputs. This corresponds to solving the program

$$\min \sum_t \text{CSLC}_t = \min \sum_t \frac{\tau_t \varepsilon_t}{\mathbf{w}_t \mathbf{q}_t + \tau_t \varepsilon_t} = \min \sum_t \frac{(1/\gamma_t) \varepsilon_t}{\mathbf{w}_t \mathbf{q}_t + (1/\gamma_t) \varepsilon_t} \quad (3)$$

subject to the linear restrictions (2). In Appendix A.4, we discuss the technical issue that the objective function in (3) is not linear in the unknowns  $\gamma_t$  and  $\varepsilon_t$ . Given this, we replace this objective function with a linearized version, which conveniently allows us to use standard linear programming techniques in our empirical analysis.

### 3 Application set-up and data

We demonstrate the empirical usefulness of our novel nonparametric method by applying it to production data drawn from the Central Balance Sheet Office database, which provides annual information on the financial accounts of Belgian firms. We link this database with firm-year level international trade data of the National Bank of Belgium to include export information (dummies per export region) and import information (dummies and shares per import region) into our analysis.<sup>10</sup>

Before describing our empirical application in more detail, we remark that our dataset shares the characteristics and limitations of many large-scale datasets that have been used in other productivity analyses based on recently developed production function estimators (see, for example, Olley and Pakes (1996); Levinsohn and Petrin (2003); Wooldridge (2009); Akerberg et al. (2015); Gandhi et al. (2017)). We pool single-product and multi-product firms, and we use industry-wide deflators to approximate firm-level prices. This implies that our measure of productivity (in terms of latent capital) does not only include the pure technological features of the firm (for example, innovation, intangibles and managerial quality), but also potential influences from firm-level price setting behavior in the output market (Klette and Griliches, 1996; Foster et al., 2008; De Loecker, 2011; De Loecker and Warzynski, 2012), differences in accounting practices, and/or differences (e.g., across products) in production structures (Diewert, 1973; Panzar and Willig, 1981; Bernard et al., 2010, 2011; De Loecker, 2011; Dhyne et al., 2014; De Loecker et al., 2016).

For our main analysis, we include as output the deflated revenue and as inputs the number of employees in full time equivalents (FTE), deflated tangible fixed assets and deflated (domestic and foreign) materials use (i.e., raw materials, consumables, services and other

---

<sup>10</sup>Import shares have been computed by the National Bank of Belgium at the firm (and group of countries) level by merging data on import from the Transaction Trade dataset and data on material inputs purchases from the VAT database. No distinction is made between final and intermediate products in either database. See, for example, Mion and Zhu (2013) for a detailed discussion.

goods). For the input prices, we use the price of labor, and the nace 2-digit deflators of intermediary inputs and tangible fixed assets.<sup>11</sup> The firm-year level price of labor is obtained from dividing labor cost by labor numbers in full time equivalents. We estimate the unobserved heterogeneity/productivity in manufacturing production at the firm-year level for the eight largest nace (rev.1.1) 2-digit sectors for the time horizon of 1997 to 2007 (see Table 1 for more details about our dataset). We thus restrict the sample to before the 2008 financial crisis.<sup>12</sup> As explained in Section 2.1, we (only) assume that the CRS assumption holds “locally” (i.e., for the given firm size), so avoiding the more debatable “global” CRS postulate. For that reason, we split up the sample according to firm size: small firms (Labor in FTE from 10 to 50; 10,680 observations), medium firms (Labor in FTE from 50 to 250; 8,505 observations), and large firms (Labor in FTE larger than 250; 2,365 observations). We conduct our nonparametric analysis for each firm size group separately.

**Table 1:** Included sectors

Nace rev.1.1. sector	Obs.	Firms
Nace 15: Manufacture of food products and beverages	4,480	755
Nace 17: Manufacture of textiles, manufacture of articles of straw and plaiting materials	2,326	421
Nace 22: Publishing, printing and reproduction of recorded media	1,854	390
Nace 24: Manufacture of chemicals and chemical products	2,611	426
Nace 25: Manufacture of rubber and plastic products	1,992	337
Nace26: Manufacture of other non-metallic mineral products	2,176	370
Nace 28: Manufacture of fabricated metal products, except machinery and equipment	3,769	808
Nace 29: Manufacture of machinery and equipment n.e.c.	2,342	454
Total	21,550	3,875

Revenue as included in balance sheets not only involves in-house production of manufacturing goods, but often also includes servicing (see, for example, Pilat et al. (2006) for a policy-oriented discussion) and reselling of products that are not produced by the firm.<sup>13</sup> As these decisions are closely related to any make-or-source decision, we will verify whether our empirical results are robust for altering the definition of firm output to the deflated sales of produced manufacturing goods by the firm. To this end, we make use of the sub-sample of Belgian firms that participate to the Prodcom survey of Eurostat, which allows

<sup>11</sup>Deflators are based on EU KLEMS and measured as described in Merlevede et al. (2015, p.8).

<sup>12</sup>To avoid extreme outliers, we limit our sample to observations of firms with at least ten employees. We changed the flows to a number of months in a book year equal to 12 and removed observations with book periods shorter than 6 months and longer than 24 months. We removed the highest and lowest percentiles of the growth rates, at the sector-year level, for the output, observed inputs, the price of labor and the share of materials in observed costs. We also removed clear erroneous reporting by limiting the sample to input-output observations with values over 1,000 euro and labor price with values over 10,000 euro. Smaller firms (either having on average less than 100 employees during the year or not exceeding two of the following three criteria: annual average of 50 employees, annual revenue of 7,300,000 euro or a balance-sheet total of 3,650,000 euro) can report their annual accounts using an abbreviated model with the possibility of no separation between gross revenue and input use. These smaller firms have a higher probability to be excluded from the analysis due to missing values.

<sup>13</sup> Bernard et al. (Forthcoming) document widespread exportation of manufacturing products that are not produced by the firm and label this carry-along trade (CAT). They show that CAT relates positively with productivity.

us to use deflated produced value as output.<sup>14</sup> A main motivation of Eurostat to initiate the Prodcom survey was exactly to obtain comparable statistics on manufacturing at the product level across the European Union. Participation to the Belgian Prodcom survey is mandatory for the firms that operate above a given threshold of operation size.<sup>15</sup> Recent studies that make use of the Belgian Prodcom database include De Loecker et al. (2014), Dhyne et al. (2014), Forlani et al. (2016) and Bernard et al. (Forthcoming).

Interestingly, by using this Prodcom database we can also show the applicability of our approach to estimate pure technological heterogeneity among single-product firms, by using output quantity data for the tightly defined sector of ready mixed concrete producers (8 digit-level product, used in numerous studies, including Syverson (2004) and Foster et al. (2008)). Syverson (2004) argues that proxy variable approaches such as the Olley and Pakes (1996) routine are not appropriate for this specific sector, as local demand states may influence input and investment decisions, which makes the assumption of a one-to-one relation between unobserved productivity and observable investment difficult to maintain. Because our routine does not rely on (semi-)parametric structuring of the simultaneity issue, it remains well applicable to such sectors that fall beyond the reach of proxy variable approaches. In Appendix B, we show that our results based on the very small sample of Belgian ready mixed concrete producers (using quantity based, revenue based and produced value based estimations) largely confirm our main conclusions on the evolution of cost shares over time.

Measured productivity differences usually relate to firm-level heterogeneity in observable characteristics (Syverson, 2011). Included firm characteristics that are expected to relate to our measure of latent capital are firm size (Haltiwanger et al., 1999; Van Biesebroeck, 2005; Forlani et al., 2016), international exposure (Bernard and Jensen, 1995; Bernard et al., 2003, 2010, Forthcoming; Melitz, 2003; Antras and Helpman, 2004; Egger et al., 2015), firm age (Wagner, 1994), and firm entry and exit (Olley and Pakes, 1996; Melitz and Polanec, 2015). Table 2 shows some descriptive statistics on these variables for the eight sectors that we consider, comprising 21,550 observations of 3,875 firms.

As Belgium is a small open economy, international exposure is usually high. In our sample of manufacturing firms, only 12 percent of the firm observations shows no exporting behavior, and 68 percent exports to non-EU countries. Export to distant countries is thus the rule rather than the exception. Production processes are generally disintegrated, with the average share of materials in observed costs amounting to 64 percent. We label this material share in observed costs as *sourcing* (see, for example, Arvantis and Loukis (2013) for a review of empirical studies that use material shares as proxies for outsourcing). The vast majority (94 percent) of observations indicate to import intermediary inputs, yet the domestic component of disintegrated activities is 2.28 times the foreign component. While

---

<sup>14</sup>We cleaned our production data by using the same criteria as for the main analysis.

<sup>15</sup>For our considered time period the threshold was 10 employees and a specific revenue threshold in a given year.



66 percent import from outside the EU, the average share of materials from outside the EU in observed costs equals only 3 percent. 16 percent of the sampled firms source from China, and this percentage is increasing over time (descriptive statistics available upon request; see also Mion and Zhu (2013) for a detailed analysis). Finally, we proxy 1 percent of our observed firms as entering firms, 6 percent as starting firms (i.e., firm age at most 5), 13 percent as young firms (i.e., firm age between 5 and 10), 80 percent as mature firms (i.e., firm age higher than 10), and 1 percent as exiting firms.<sup>16</sup>

**Table 2:** Summary statistics

	Mean	St.Dev.	Min.	25%	Med.	75%	Max.
Deflated revenue (output)	37.56	125.45	0.09	5.34	10.89	28.35	4584.31
Deflated produced value (Prodcom-based; 18,757 obs.)	29.54	107.72	0.01	4.52	9.57	23.68	4488.67
Output price	1.06	0.09	0.89	1.00	1.04	1.08	1.35
Labor in FTE	127.22	282.60	10.00	28.20	50.60	113.00	5686.00
Deflated tangible fixed assets	5.89	22.79	0.00	0.61	1.54	4.00	664.20
Deflated material costs	28.30	96.49	0.00	3.41	7.50	20.43	3492.81
Labor price	0.04	0.01	0.01	0.03	0.04	0.05	0.18
Capital price	1.12	0.08	1.01	1.07	1.11	1.16	1.38
Intermediates price	1.08	0.09	0.95	1.02	1.06	1.12	1.36
Exporting (dummy)	0.88	0.33	0.00	1.00	1.00	1.00	1.00
Exporting to Eastern Europe (dummy)	0.49	0.50	0.00	0.00	0.00	1.00	1.00
Exporting outside the EU (dummy)	0.68	0.47	0.00	0.00	1.00	1.00	1.00
Foreign sourcing (dummy)	0.94	0.24	0.00	1.00	1.00	1.00	1.00
Sourcing from outside the EU (dummy)	0.66	0.47	0.00	0.00	1.00	1.00	1.00
Sourcing from Eastern Europe (dummy)	0.30	0.46	0.00	0.00	0.00	1.00	1.00
Sourcing from China (dummy)	0.16	0.37	0.00	0.00	0.00	0.00	1.00
Sourcing (share)	0.64	0.16	0.00	0.54	0.66	0.76	0.99
Domestic sourcing (share)	0.45	0.17	0.00	0.32	0.43	0.56	0.98
Starting	0.06	0.24	0.00	0.00	0.00	0.00	1.00
Young	0.13	0.33	0.00	0.00	0.00	0.00	1.00
Mature	0.80	0.40	0.00	1.00	1.00	1.00	1.00
Entry	0.01	0.11	0.00	0.00	0.00	0.00	1.00
Exiting	0.01	0.10	0.00	0.00	0.00	0.00	1.00

Note: Deflated revenue, deflated produced value, deflated tangible fixed assets, deflated material costs and labor price are expressed in millions of euro. Eastern Europe countries: Bulgaria, Czech Republic, Cyprus, Estonia, Croatia, Hungary, Lithuania, Latvia, Malta, Poland, Romania, Slovenia, Slovakia.

## 4 Empirical results

In this section, we first present some descriptive statistics on our estimates of (the cost share of) latent capital (LC and CSLC).<sup>17</sup> Next, we relate our nonparametric productivity estimates to observable firm characteristics. This will demonstrate that our estimates effectively replicate stylized findings in the literature. We conclude by analyzing the evolution of cost shares (of observed inputs and latent capital) over time. In particular, we assess

<sup>16</sup>A firm is considered to enter in the first year for which employment is strictly positive, provided that the firm is not older than five years (based on its year of incorporation). Next, a firm is considered to exit in the year for which employment is no longer reported after previous year(s) with strictly positive employment, insofar the number of years to the declared exit date does not exceed five.

<sup>17</sup>Throughout the paper we express latent capital in millions of euro. We excluded one observation ex post with a cost share of latent capital above 0.999.

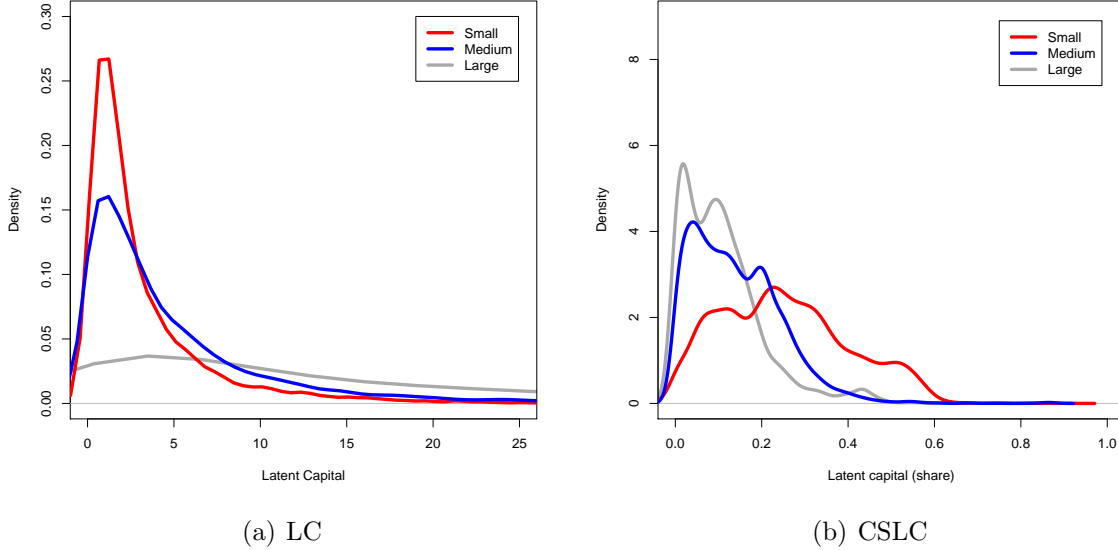
to what extent observed primary manufacturing inputs are substituted for other observed inputs and/or unobserved technology. Our methodology allows us to address this question in a fully nonparametric fashion, without imposing a priori assumptions of Hicks neutrality or any other functional structure for the unknown production technology.

Two remarks are in order before discussing our results. First, our main analysis will be on the aggregate of all eight nace 2-digit sectors for which we solved a linear program with objective function (5), given the constraints as formulated in (2). Evidently, each sector has its own particularities (related to input use and output production), but our principal findings turn out to be robust across sectors. In Appendix D.1, we provide figures that show the evolution over time of the cost share of latent capital and observed inputs for the eight individual sectors.

Second, we present two additional robustness checks in Appendices B and C. As motivated above, in Appendix B we demonstrate the possible application of our method to a sample of single-product producers (of ready mixed concrete). Next, as discussed in Section 2.4, in our empirical analysis we will use the goodness-of-fit parameter  $\theta = 0.95$  to account for (small) deviations of observed firm behavior from exact rationalizability (i.e., data consistency with the strict cost minimization conditions in Proposition 1). In Appendix C, we show that our main conclusions are robust for alternative specifications of the  $\theta$ -parameter.

## 4.1 Latent capital estimates: a first look

Figure 1 depicts the distributions of our latent capital (LC) and cost shares of latent capital (CSLC) estimates (see Table 8 in Appendix D.2 for additional descriptives). We clearly observe that accounting for technological heterogeneity is required to rationalize the observed firm behavior in terms of our cost minimization hypothesis. This provides strong nonparametric evidence against any framework that is based on a representative firm and a sector aggregate production function.



**Figure 1:** Latent capital and firm size

Larger firms are generally characterized by higher productivity levels, which falls in line with a common finding in the literature. However, this does not mean that larger firms also have higher cost shares of latent capital. For example, the cost share of latent capital is above 0.24 for half of the small firms, whereas it exceeds the same cut-off level for less than ten percent of the large firms. On average, latent capital accounts for approximately 25, 14 and 11 percent of the total costs of small, medium and large firms, respectively. Finally, we observe that smaller firms generally show more variation in their cost shares of latent capital. This indicates that smaller firms are not only more heterogeneous in terms of their observable characteristics (summary statistics available upon request), but also in terms of their unobservable input.

Without further information, we cannot directly disentangle whether these differences across firm sizes are effectively driven by actual differences in the intra-group distributions of unobserved inputs or, rather, by inter-group differences in the precision of measurement of the observable characteristics. Therefore, in what follows we will analyze our latent capital estimates for each firm group separately, and largely abstain from making statements that compare firm size groups. We assume that, within a given firm size group, there are no systematic differences in the precision of measurement of the observable characteristics.

In Table 3, we report correlation results that further validate our interpretation of latent capital as measuring productivity. First, we find that latent capital relates strongly and positively to labor productivity as measured by dividing deflated revenues by the number of employees in FTE. The Spearman correlation increases with firm size: it equals 0.44 for small firms, and it amounts to 0.56 for large firms. Second, the correlations with a one-

year lag of latent capital are also large and positive for the different firm size groups. The Spearman correlation over all firms equals 0.89 and is above 0.86 for all firm size groups. We conclude that our latent capital estimates robustly confirm the documented stylized fact of huge and persistent differences across producers in terms of measured productivity (see, for example, Syverson (2011)).

**Table 3:** Spearman correlations

	All	Small	Medium	Large
Labor productivity	0.46	0.44	0.52	0.56
Lagged LC	0.89	0.91	0.86	0.89

## 4.2 Latent capital, sourcing and international exposure

As an additional validation of our interpretation of latent capital as representing unobserved firm productivity, we next study the relation between latent capital, sourcing (i.e. total amount of domestic and foreign material inputs) and international exposure. As argued above, the empirical and theoretical literature shows a generally positive correlation between productivity, international exposure and foreign outsourcing. This not only reveals a direct impact of internationalization on productive efficiency, it is also related to quality differences between intermediates of different origin, and to differences in the variety of intermediates used together with a taste for variety in the production process (see, for example, Goldberg et al. (2009, 2010) and Halpern et al. (2015)). Next, the literature on export behavior of firms documents a positive correlation between measured productivity and export as a stylized fact.

The left hand side of Table 4 shows the relation between (logged) latent capital, sourcing and international exposure within the three firm size groups. Following our discussion in Section 3, in all regressions we include nace 2-digit and year fixed effects as well as dummies controlling for firm age (starting, young, mature), entry and exit.<sup>18</sup> In the specific case of Belgian manufacturers, sourcing almost always implies some sort of international exposure (94 percent of the sampled firms use foreign sourcing). Thus, we can expect multicollinearity to impede disentangling the effects of foreign and domestic sourcing.

Our regression results support the widespread findings from the productivity literature. Overall, we observe a significantly positive relationship between latent capital and international exposure for all firm size groups. More specifically, for small firms this significant positive relationship applies to both foreign sourcing and exporting, with the correlation being higher when sourcing is from outside the EU and exporting is to Eastern Europe.

<sup>18</sup>Foster et al. (2008) find that firm age, entry and exit relate to idiosyncratic demand shocks and firm-specific output prices. Therefore, we include these variables as control variables to mitigate confounding influences. Some caution is needed when interpreting our results on export, as exporting is known to imply product-specific pricing.

Medium firms show a significant positive relation between latent capital and sourcing from outside the EU and exporting outside the EU. Large firms show a significantly positive relation with sourcing from Eastern Europe. Further, our regressions also reveal a significantly positive correlation between latent capital and the share of sourcing for the three firm size groups. The significant negative relations for exporting outside the EU (small firms), exporting (medium firms) and exporting to Eastern Europe (large firms) are not robust for altering the output definition (see below). In sum, our latent capital estimates confirm that, more disintegrated, international production processes are positively associated with measured productivity.

Next, as discussed in Section 3, our main analysis considers (deflated) revenue of the firm as output (i.e., estimates are revenue based), pooling together multiple products, but also servicing and carry-along trade (see Bernard et al. (Forthcoming)). To verify whether our results are robust for influences of servicing and resale of out-house production, we redefined firm output as deflated sales of produced goods (reported in the Prodcom database). Summary statistics are provided in Table 8 in Appendix D.2. As for the connection between latent capital, sourcing and international exposure, the right hand side of Table 4 confirms the positive relationship that we found before. In fact, when using in-house production to measure output (yielding produced value based estimates of latent capital), for all firm size groups we find a significant positive relation between latent capital and both the foreign sourcing and exporting aspect of internationalization. Overall, we may safely conclude that our principal qualitative conclusions are largely robust to the chosen output definition.

**Table 4:** Latent capital and international exposure: a truncated regression analysis

	Revenue based			Produced value based		
	Small	Medium	Large	Small	Medium	Large
Sourcing (share)	2.364*** (0.152)	4.088*** (0.225)	3.378*** (0.791)	3.692*** (0.212)	3.921*** (0.273)	4.224*** (0.724)
Foreign sourcing (dummy)	0.465*** (0.0581)	0.0162 (0.127)	0.0500 (0.491)	0.422*** (0.0707)	0.0105 (0.195)	0.925*** (0.348)
Sourcing from Eastern Europe (dummy)	0.0526 (0.0372)	0.0384 (0.0463)	0.372*** (0.134)	0.0789 (0.0580)	0.0525 (0.0518)	0.0474 (0.135)
Sourcing from outside the EU (dummy)	0.0820** (0.0371)	0.301*** (0.0557)	0.410 (0.269)	0.0190 (0.0501)	0.167*** (0.0601)	0.308 (0.235)
Sourcing from China (dummy)	0.0231 (0.0458)	0.00631 (0.0578)	0.203 (0.141)	-0.0282 (0.0915)	0.0700 (0.0657)	0.193 (0.155)
Exporting (dummy)	0.230*** (0.0502)	-0.355*** (0.103)	-0.613 (0.417)	0.279*** (0.0742)	-0.125 (0.122)	-0.166 (0.333)
Exporting to Eastern Europe (dummy)	0.196*** (0.0362)	0.107* (0.0559)	-0.435*** (0.187)	0.218*** (0.0523)	0.116* (0.0647)	-0.130 (0.186)
Exporting outside the EU (dummy)	-0.0930** (0.0408)	0.135** (0.0645)	0.414* (0.227)	0.0130 (0.0537)	0.206*** (0.0783)	0.540** (0.240)
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Control variables	Yes	Yes	Yes	Yes	Yes	Yes
Year effects	Yes	Yes	Yes	Yes	Yes	Yes
Nace 2-digit effects	Yes	Yes	Yes	Yes	Yes	Yes
Non-truncated Observations	10,567	8,324	2,037	8,201	7,172	1,897
Observations	10,679	8,505	2,365	8,397	7,417	2,006
Firms	2,591	1,445	349	2,031	1,279	316

Note: The dependent variable is the log of latent capital. Marginal effects of left-truncated regressions shown. Robust standard errors in parentheses with clustering at the firm level.\*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$

### 4.3 Substitution between latent capital and observed inputs

As motivated above, a main distinguishing feature of our methodology to identify unobserved technological heterogeneity is that it deals with the simultaneity bias while naturally relaxing the Hicks neutrality assumption. We do not need to assume that input cost shares are constant over time, and we do not have to impose a common structure on factor biased technological change. As a final investigation, we exploit this unique aspect by considering variation in (observable and unobservable) input substitution patterns over time. Figure 2(a) depicts the evolution of average cost shares (of latent capital, tangible fixed assets (TFA), foreign materials, domestic materials and labor) defined over our full sample of firms. Table 9 in Appendix D.2 reports the associated descriptive statistics.

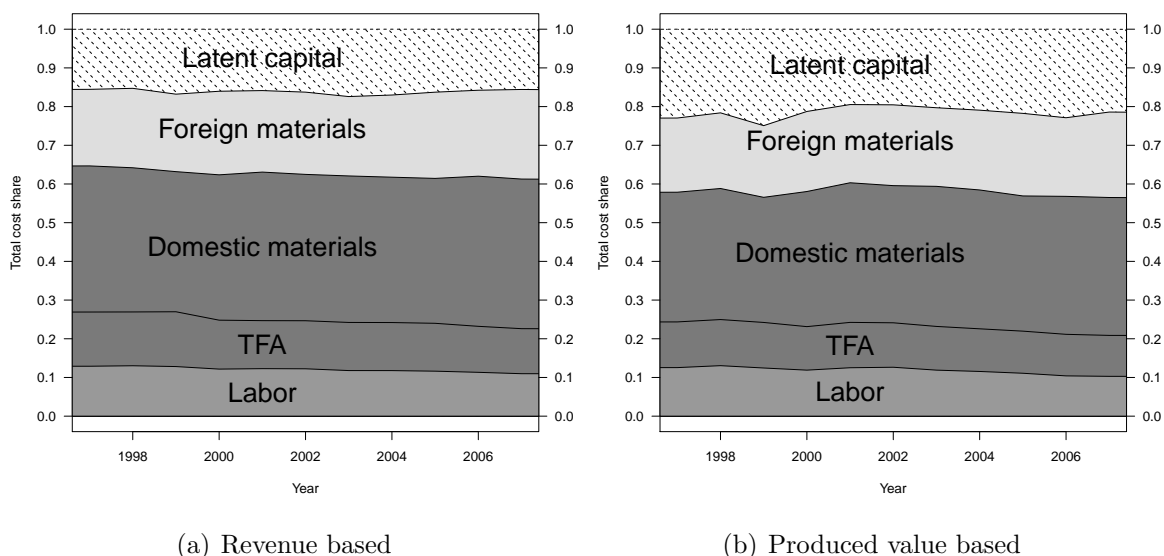
A first observation is that labor gradually loses ground. The labor cost share decreases from 13 percent in 1997 to 11 percent in 2007, even though the relative price of labor is non-decreasing relative to the price of the other observed inputs (summary statistics available upon request). This picture confirms the well-documented loss in labor shares (OECD, 2012), now explicitly taking into account productivity differences. For our observational setting, it provides robust evidence against the often made assumption of Hicks neutral technical change.

Further, Figure 2(a) reveals that the cost share of tangible fixed assets (TFA) is decreasing

rather than increasing. The average TFA cost share is still 14 percent in 1997, but goes down to only 12 percent in 2007. Stated differently, our within-industry estimates provide no empirical support for the argument that technological change was detrimental for labor and favorable for TFA. We find that both primary inputs are substituted for other inputs in the Belgian manufacturing sector.

By contrast, we do observe steadily increasing cost shares of materials. Material cost shares have gone up by 4 percentage points between 1997 and 2007 (i.e., from 0.58 to 0.62). This comprises an increase in both domestic and foreign materials of respectively 1 and 3 percentage points. The cost share of latent capital (CSLC) remains constant over the time horizon under investigation, supporting the idea of a productivity stagnation in the manufacturing sector. Taken together, the patterns in Figure 2(a) suggest that primary inputs are overall substituted for more use of materials (i.e., increased prevalence of both domestic and international disintegration) rather than for latent capital (i.e., technology). This confirms that production processes have become less integrated within firms and more international, while being characterized by a productivity stagnation.

### All firms



**Figure 2:** Cost shares

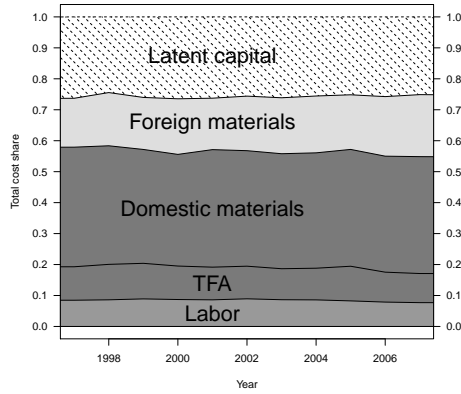
As a following step, Figure 3(a) shows that the patterns of input substitution differ between small, medium and large firms. Once more, Table 9 in Appendix D.2 contains the associated descriptive statistics. The substitution of primary inputs for material is most pronounced for large firms: for these firms, primary inputs (i.e., labor and TFA together) lose 6 percentage points in terms of cost shares, while the cost shares of domestic and foreign materials increase with 2 and 3 percentage points, respectively. Latent capital increases with 1 percentage point for large firms. Thus, large firms substitute primary inputs to a greater extent for outsourced activities and to

a lesser extent for technology. A less pronounced, but similar substitution pattern applies to medium sized firms. However, the picture is different for small firms. Here, we find substitution of tangible fixed assets, latent capital and domestic materials for foreign materials. For all firm size groups, we actually observe that the average cost share of latent capital stagnates after 2000 meaning that firms suffered from productivity stagnation since the early 2000s.

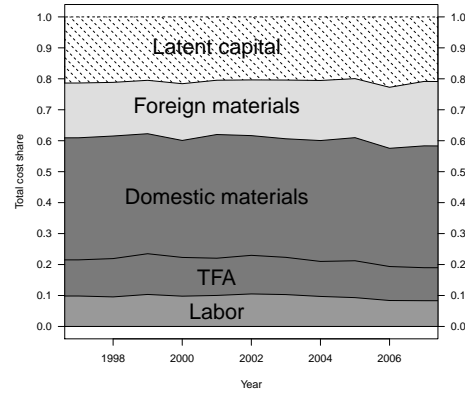
Similar to before, we check robustness of our findings by redefining firm output as deflated sales of produced goods. Figures 2(b) and 3(b),(d),(e) show the evolution of input cost shares for this alternative output definition, and Table 10 in Appendix D.2 contains the corresponding descriptive statistics. Generally, Figure 2(b) reveals the same patterns of input substitution as Figure 2(a), but shows more volatility of the cost shares over time (potentially due to a higher level of misreporting in the Prodcum survey, as discussed above). Primary inputs are substituted for materials and the cost share of latent capital shows no increasing pattern over time. Similarly, Figures 3(b),(d),(e) show, for all firm size groups, that the cost share of materials is increasing over time, while the cost share of both primary inputs is steadily decreasing over time. For large firms, the substitution against primary inputs is again most pronounced.



### Small firms

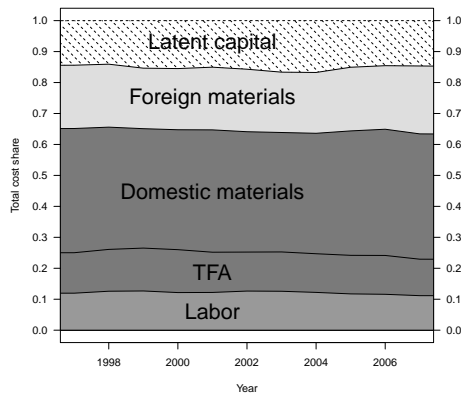


(a) Revenue based

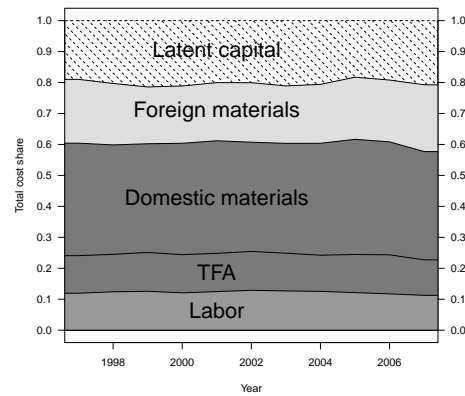


(b) Produced value based

### Medium firms

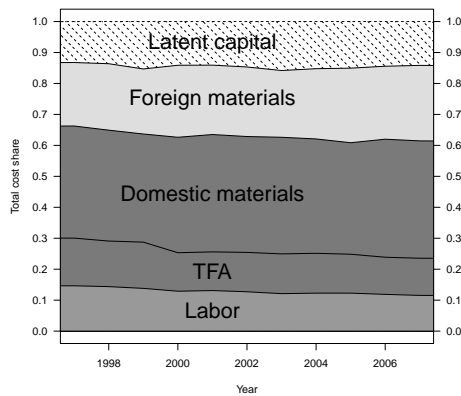


(c) Revenue based

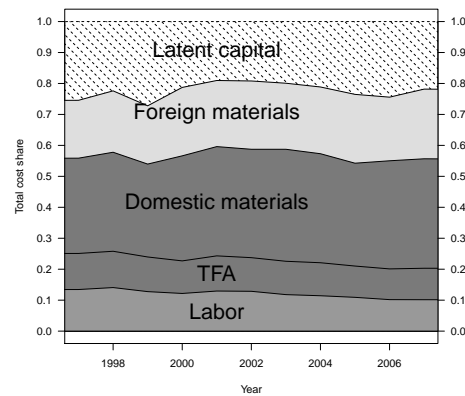


(d) Produced value based

### Large firms



(e) Revenue based



(f) Produced value based

**Figure 3:** Cost shares and firm size

## 5 Conclusion

We have developed a novel structural method for production analysis that identifies unobserved technological heterogeneity in a fully nonparametric fashion. We model unobserved technological heterogeneity as an unobserved productivity factor on which we condition the demand of the observed inputs. Our method deals with the simultaneity bias in a natural way, and it empirically quantifies technological differences across firms in terms of differences in (unobserved) latent capital. Our nonparametric methodology is easy to implement as it merely requires the use of linear programming techniques. It allows for a powerful identification analysis, while avoiding (nonverifiable and often debatable) assumptions of functional form regarding the relationship between inputs and outputs (including the hypothesis of Hicks neutral technical change).

Our empirical application has shown that the method does allow for drawing strong empirical conclusions, despite its nonparametric nature. For a set of Belgian manufacturing firms, we have recovered technological heterogeneity at the firm-year level over the period 1997-2007 for broad industry categories. Consistent with the well-established literature on international trade, we find that disintegrated firms with international sourcing are more productive. Further, we find that primary inputs (labor and tangible fixed assets) are substituted over time for (domestic and foreign) outsourcing, but usually not for greater use of technology. For large firms, this substitution is more pronounced. Overall, we provide robust empirical evidence against the assumption of Hicks neutrality for the setting at hand.

We emphasize that we see the current paper primarily as providing a promising starting ground, rather than a complete toolkit for nonparametric production analysis with unobserved technological heterogeneity. Most notably, we have focused on a single-output setting throughout. As discussed in De Loecker et al. (2016), a multiproduct framework (also involving the identification of input allocations across products) is warranted to obtain a more detailed insight into influences of exogenous trade or cost shocks. To develop this multi-output version of our methodology, a useful starting point is the study of Cherchye et al. (2014), who presented a nonparametric framework (abstracting from technological heterogeneity issues) for the analysis of firms producing multiple products. A closely related issue concerns dealing with non-competitive output markets. In this respect, Carvajal et al. (2013, 2014) show how to analyze alternative (for example, Cournot or Bertrand) structures on output markets in the advocated nonparametric framework. In our opinion, integrating these authors' insights with our newly developed methodology may constitute another fruitful avenue for follow-up research.

## References

- Acemoglu, D., Gancia, G., Zilibotti, F., 2015. Offshoring and Directed Technical Change. *American Economic Journal: Macroeconomics* 7 (3), 84–122.
- Akerberg, D., Caves, K., Frazer, G., 2015. Identification properties of recent production function estimators. *Econometrica* 83 (6), 2411–2451.
- Afriat, S., 1972. Efficiency estimation of production functions. *International Economic Review* 13 (3), 568–598.
- Antras, P., Helpman, E., 2004. Global Sourcing. *Journal of Political Economy* 112 (3), 552–580.
- Arvantis, S., Loukis, E., 2013. Outsourcing and firm performance - a comparative study of Swiss and Greek firms. *Industrial and Corporate Change* 22 (3), 771–806.
- Autor, D. H., Katz, L., Krueger, A., 1998. Computing inequality: Have computers changed the labor market? *The Quarterly Journal of Economics* 113 (4), 1169–1213.
- Autor, D. H., Levy, F., Murnane, R. J., 2003. The Skill Content of Recent Technological Change: An Empirical Exploration. *The Quarterly Journal of Economics* 118 (4), 1279–1333.
- Bernard, A., Eaton, J., Jensen, J., Kortum, S., 2003. Plants and productivity in international trade. *American Economic Review* 93 (4), 1268–1290.
- Bernard, A., Jensen, B., 1995. Exporters, Jobs, and Wages in US Manufacturing: 1976-87. *Brookings Papers on Economic Activity: Microeconomics*, 67–112.
- Bernard, A., Redding, S., Schott, P., 2010. Multi-product firms and product switching. *American Economic Review* 100 (1), 70–97.
- Bernard, A., Redding, S., Schott, P., 2011. Multiproduct firms and trade liberalization. *The Quarterly Journal of Economics* 126 (3), 1271–1318.
- Bernard, A. B., Blanchard, E. J., Beveren, I. V., Vandenbussche, H. Y., Forthcoming. Carry-along trade. *Review of Economic Studies*.
- Carvajal, A., Deb, R., Fenske, J., Quah, K.-H., 2013. Revealed preference tests of the cournot model. *Econometrica* 81 (6), 2351–2379.
- Carvajal, A., Deb, R., Fenske, J., Quah, K.-H., 2014. A Nonparametric Analysis of Multi-Product Oligopolies. *Economic Theory* 57, 253–277.
- Cherchye, L., Demuynck, T., De Rock, B., De Witte, K., 2014. Nonparametric analysis of multi-output production with joint inputs. *Economic Journal* 124 (577), 735–775.
- CompNet Task Force, 2014. Micro-based evidence of EU competitiveness: The CompNet database. *ECB Working Paper Series* 1634.
- Dall’Olio, A., Iooty, M., Kanehira, N., Saliola, F., 2013. Productivity growth in Europe. *Policy Research Working Paper* 6425, Word Bank.

- De Loecker, J., 2011. Product differentiation, multiproduct firms, and estimating the impact of trade liberalization on productivity. *Econometrica* 79 (5), 1407–1451.
- De Loecker, J., Fuss, C., Van Biesebroeck, J., 2014. International competition and firm performance : Evidence from Belgium. Tech. Rep. 269, National Bank of Belgium Working Paper Series.
- De Loecker, J., Goldberg, P., Khandelwal, A., Pavcnik, N., 2016. Prices, markups, and trade reform. *Econometrica* 84 (2), 445–510.
- De Loecker, J., Warzynski, F., 2012. Markups and firm-level export status. *American Economic Review* 102 (6), 2437–2471.
- Dewitte, R., Dumont, M., Merlevede, B., Rayp, G., Vershelde, M., 2017. Firm heterogeneous factor-biased technological change, mimeo.
- Dhyne, E., Petrin, A., Smeets, V., Warzynski, F., et al., 2014. Import competition, productivity and multi-product firms. Working Paper 268, National Bank of Belgium.
- Diewert, W., 1973. Functional forms for profit and transformation functions. *Journal of Economic Theory* 6 (3), 284–316.
- Diewert, W., Parkan, C., 1983. Linear programming tests of regularity conditions for production frontiers. In: Eichhorn, W., Henn, R., Neumann, K., Shephard, R. (Eds.), *Quantitative Studies on Production and Prices*. Physica-Verlag, Würzburg.
- Doraszelski, U., Jaumandreu, J., Forthcoming. Measuring the bias of technological change. *Journal of Political Economy*.
- Egger, H., Kreickemeier, U., Wrona, J., 2015. Offshoring domestic jobs. *Journal of International Economics* 97 (1), 112–125.
- Forlani, E., Martin, R., Mion, G., Muûls, M., 2016. Unraveling firms: Demand, productivity and markups heterogeneity. Tech. rep., National Bank of Belgium Working Paper Series, No. 293.
- Foster, L., Haltiwanger, J., Syverson, C., 2008. Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review* 98 (1), 394–425.
- Gandhi, A., Navarro, S., Rivers, D., 2017. On the Identification of Production Functions: How Heterogeneous is Productivity? *Journal of Political Economy* forthcoming.
- Goldberg, P., Khandelwal, A., Pavcnik, N., Topalova, P., 2009. Trade liberalization and new imported inputs. *American Economic Review* 99 (2), 494–500.
- Goldberg, P., Khandelwal, A., Pavcnik, N., Topalova, P., 2010. Imported intermediate inputs and domestic product growth: evidence from indianomics. *The Quarterly Journal of Economics* 125 (4), 1727–1767.
- Grifell-Tatjé, E., Lovell, C., Sickles, R. (Eds.), Forthcoming. *The Oxford Handbook of Productivity Analysis*. Oxford University Press.

- Griliches, Z., Mairesse, J., 1998. Production functions: The search for identification. In: *Econometrics and Economic Theory in the Twentieth Century: The Ragnar Frisch Centennial Symposium*. New York: Cambridge University Press, pp. 169–203.
- Halpern, L., Koren, M., Szeidl, A., 2015. Imported inputs and productivity. *American Economic Review* 105 (12), 3660–3703.
- Haltiwanger, J., Lane, J., Spletzer, J., 1999. Productivity differences across employers: The roles of employer size, age, and human capital. *American Economic Review* 89 (2), 94–98.
- Hanoch, G., Rothschild, M., 1972. Testing assumptions of production theory: A nonparametric approach. *Journal of Political Economy* 80 (2), 256–275.
- Hoderlein, S., Stoye, J., 2014. Revealed preferences in a heterogeneous population. *Review of Economics and Statistics* 96, 197–213.
- Klette, T. J., Griliches, Z., 1996. The inconsistency of common scale estimators when output prices are unobserved and endogenous. *Journal of Applied Econometrics* 11 (4), 343–361.
- Levinsohn, J., Petrin, A., 2003. Estimating production functions using inputs to control for unobservables. *Review of Economic Studies* 70 (2), 317–341.
- Marschak, J., Andrews, W., 1944. Random simultaneous equations and the theory of production. *Econometrica* 12 (3-4), 143–205.
- McFadden, D., Richter, K., 1991. Stochastic rationality and revealed stochastic preference. *Preferences, Uncertainty and Rationality*, ed. by J. Chipman, D. McFadden, and M.K. Richter. Boulder: Westview Press.
- Melitz, M., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica* 71 (6), 1695–1725.
- Melitz, M., Polanec, S., 2015. Dynamic Olley-Pakes productivity decomposition with entry and exit. *RAND Cooperation* 46 (2), 362–375.
- Merlevede, B., De Zwaan, M., Lenaerts, K., Purice, V., 2015. Multinational networks, domestic, and foreign firms in Europe. *Ghent University Working Paper Series D/2015/7012/02*.
- Mion, G., Zhu, L., 2013. Import competition from and offshoring to china: A curse or blessing for firms? *Journal of International Economics* 89, 202–215.
- OECD, 2012. *Employment Outlook 2012*. Tech. rep., Organisation of Economic Cooperation and Development, Paris.
- Olley, G. S., Pakes, A., 1996. The dynamics of productivity in the telecommunications equipment industry. *Econometrica* 64 (6), 1263–1297.
- Panzar, J., Willig, R., 1981. Economies of Scope. *American Economic Review* 71 (2), 268–272.

- Pilat, D., Cimper, A., Olsen, K., Webb, C., 2006. The changing nature of manufacturing in oecd economies. Tech. rep., OECD Science, Technology and Industry Working Paper 2006 no. 9, Paris.
- Syverson, C., 2004. Market structure and productivity: A concrete example. *Journal of Political Economy* 112 (6), 1181–1222.
- Syverson, C., 2011. What determines productivity? *Journal of Economic Literature* 49 (2), 326–365.
- Syverson, C., 2017. Challenges to mismeasurement explanations for the US productivity slowdown. *Journal of Economic Perspectives* 31 (2), 165–186.
- Van Biesebroeck, J., 2005. Firm size matters: Growth and productivity growth in African manufacturing. *Economic Development and Cultural Change* 53 (3), 545–583.
- Varian, H., 1984. The nonparametric approach to production analysis. *Econometrica* 52 (3), 579–598.
- Varian, H., 1985. Non-parametric analysis of optimizing behavior with measurement error. *Journal of Econometrics* 30 (1-2), 445–458.
- Varian, H., 1990. Goodness-of-fit in optimizing models. *Journal of Econometrics* 46 (1-2), 125–140.
- Vershelde, M., Dumont, M., Merlevede, B., Rayp, G., 2014. A constrained nonparametric regression analysis of factor-biased technical change and TFP growth at the firm-level. Working Paper 266, National Bank of Belgium.
- Wagner, J., 1994. The post-entry performance of new small firms in german manufacturing-industries. *Journal of Industrial Economics* 42 (2), 141–154.
- Wooldridge, J. M., 2009. On estimating firm-level production functions using proxy variables to control for unobservables. *Economics Letters* 104 (3), 112–114.

# Appendix A: technical results

## A.1 Proof of Proposition 1

**Necessity of condition (iii).** The CRS assumption implies that we can use Euler's theorem to obtain

$$\sum_i \frac{\partial f(\mathbf{q}, \varepsilon)}{\partial q^i} q^i + \frac{\partial f(\mathbf{q}, \varepsilon)}{\partial \varepsilon} \varepsilon = f(\mathbf{q}, \varepsilon).$$

The first order conditions for the cost minimization problem (both OP.I and OP.II) imply

$$w_t^i = \lambda_t \frac{\partial f(\mathbf{q}_t, \varepsilon)}{\partial q^i}.$$

From the concavity of  $f$ , we also have that,

$$f(\mathbf{q}_t, \varepsilon_t) - f(\mathbf{q}_v, \varepsilon_v) \leq \sum_i \frac{\partial f(\mathbf{q}_v, \varepsilon_v)}{\partial q^i} (q_t^i - q_v^i) + \frac{\partial f(\mathbf{q}_v, \varepsilon_v)}{\partial \varepsilon} (\varepsilon_t - \varepsilon_v).$$

Substituting  $y_t = f(\mathbf{q}_t, \varepsilon_t)$ ,  $y_v = f(\mathbf{q}_v, \varepsilon_v)$  and using the above then gives

$$y_t - y_v \leq \frac{1}{\lambda_v} (\mathbf{w}_v \mathbf{q}_t + \tau_v \varepsilon_t) - y_v,$$

where  $\tau_v = \frac{\partial f(\mathbf{q}_v, \varepsilon_v)}{\partial \varepsilon} \lambda_v$ . If *OP.I* is used, then  $\tau_v$  is the shadow price of  $\varepsilon_v$ , while if *OP.II* is used then this equation follows from the first order conditions. Thus,

$$y_t \leq \frac{1}{\lambda_v} (\mathbf{w}_v \mathbf{q}_t + \tau_v \varepsilon_t).$$

From the first order conditions (and definition of  $\tau_v$ ) we also have that,

$$y_v = \sum_i \frac{\partial f(\mathbf{q}_v, \varepsilon_v)}{\partial q^i} q_v^i + \frac{\partial f(\mathbf{q}_v, \varepsilon_v)}{\partial \varepsilon} \varepsilon = \frac{1}{\lambda_v} (\mathbf{w}_v \mathbf{q}_v + \tau_v \varepsilon_v).$$

Thus,

$$\frac{y_t}{y_v} \leq \frac{\mathbf{w}_v \mathbf{q}_t + \tau_v \varepsilon_t}{\mathbf{w}_v \mathbf{q}_v + \tau_v \varepsilon_v}.$$

Dividing numerator and denominator by  $\tau_v$  and defining  $\gamma_v = 1/\tau_v$  gives,

$$\frac{y_t}{y_v} \leq \frac{\gamma_v \mathbf{w}_v \mathbf{q}_t + \varepsilon_t}{\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v}.$$

**Sufficiency of condition (iii).** Assume that numbers  $\varepsilon_t, \gamma_t$  exist that satisfy the inequalities and define  $f(\mathbf{q}, \varepsilon) = \min_t y_t \frac{\gamma_t \mathbf{w}_t \mathbf{q} + \varepsilon}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t}$ . It is easy to verify that his function is concave, homogeneous of degree one and continuous. Moreover the inequality conditions imply that  $f(\mathbf{q}_t, \varepsilon_t) = y_t$ . To

verify that  $\mathbf{q}_t$  solves *OP.I*, assume, towards a contradiction that there is an input bundle  $\mathbf{q}$  such that  $\mathbf{w}_t\mathbf{q} < \mathbf{w}_t\mathbf{q}_t$  and  $f(\mathbf{q}, \varepsilon_t) \geq y_t$ . Then, we have

$$\begin{aligned} y_t &\leq f(\mathbf{q}, \varepsilon_t) \\ &\leq y_t \frac{\gamma_t \mathbf{w}_t \mathbf{q} + \varepsilon_t}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t} \\ &< y_t \frac{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t} \\ &= y_t. \end{aligned}$$

Similarly, for *OP.II* define  $\tau_t = 1/\gamma_t$ . Now, if, towards a contradiction, there is an input bundle  $(\mathbf{q}, \varepsilon)$  such that  $\mathbf{w}_t\mathbf{q} + \tau_t\varepsilon < \mathbf{w}_t\mathbf{q}_t + \tau_t\varepsilon_t$  and  $f(\mathbf{q}, \varepsilon) \geq y_t$ , then we have

$$\begin{aligned} y_t &\leq f(\mathbf{q}, \varepsilon) \\ &\leq y_t \frac{\gamma_t \mathbf{w}_t \mathbf{q} + \varepsilon}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t} \\ &< y_t \frac{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t} \\ &= y_t. \end{aligned}$$

## A.2 Testability: a numerical example

The following example illustrates the testable implications in Proposition 1. It shows that these implications can be rejected even in a minimalistic setting with only two firm observations and two observed inputs.

Consider a dataset  $S$  with input prices  $\mathbf{w}_1 = (1, 2)$  and  $\mathbf{w}_2 = (2, 1)$  and input quantities  $\mathbf{q}_1 = (1, 2)$  and  $\mathbf{q}_2 = (2, 1)$ . Proposition 1 requires

$$\begin{aligned} \frac{y_1}{y_2} &\leq \frac{\gamma_2 4 + \varepsilon_1}{\gamma_2 5 + \varepsilon_2}, \\ \frac{y_2}{y_1} &\leq \frac{\gamma_1 4 + \varepsilon_2}{\gamma_1 5 + \varepsilon_1}. \end{aligned}$$

Reformulating these inequalities obtains

$$\begin{aligned} (y_1\varepsilon_2 - y_2\varepsilon_1) &\leq (4y_2 - 5y_1)\gamma_2 \text{ and} \\ (y_1\varepsilon_2 - y_2\varepsilon_1) &\geq (5y_2 - 4y_1)\gamma_1, \end{aligned}$$

which implies that  $(5y_2 - 4y_1)\gamma_1 \leq (4y_2 - 5y_1)\gamma_2$ . If we then assume that the (observed) output levels  $y_1$  and  $y_2$  are such that

$$\frac{4}{5} < \frac{y_2}{y_1} < \frac{5}{4},$$

we obtain that there can never exist strict positive  $\gamma_1$  and  $\gamma_2$  that satisfy this inequality restriction (since  $4y_2 - 5y_1 < 0$  and  $5y_2 - 4y_1 > 0$ ).



### A.3 Goodness-of-fit parameter $\theta$

We start from the rationalizability requirements in Proposition 1 and define  $r_v = (\gamma_v y_v)/(\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v)$ , which allows us to rewrite the inequality restrictions as

$$y_t - y_v \leq r_v (\mathbf{w}_v (\mathbf{q}_t - \mathbf{q}_v) + \tau_v (\varepsilon_t - \varepsilon_v)).$$

We can weaken these requirements by equiproportionally contracting the inputs  $(\mathbf{q}_v, \varepsilon_v)$ , which corresponds to lowering the cost level  $(\mathbf{w}_v \mathbf{q}_v + \tau_v \varepsilon_v)$  by the same degree. To do so, we use  $\theta \leq 1$  and obtain

$$y_t - y_v \leq r_v (\mathbf{w}_v (\mathbf{q}_t - \theta_v \mathbf{q}_v) + \tau_v (\varepsilon_t - \theta_v \varepsilon_v)).$$

Generally, lower values of  $\theta$  imply weaker rationalizability restrictions. Our optimization model provides a better (economic) fit of the dataset  $S$  if this set  $S$  satisfies the restrictions for a higher value of  $\theta$ .

By using that  $r_v = (\gamma_v y_v)/(\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v)$ , we can also include the goodness-of-fit measure  $\theta$  in the original inequality requirements that appeared in Proposition 1. Specifically, this obtains

$$\frac{y_t}{y_v} \leq \frac{\gamma_v \mathbf{w}_v \mathbf{q}_t + \varepsilon_t}{\gamma_v \mathbf{w}_v \mathbf{q}_v + \varepsilon_v} + (1 - \theta),$$

which gives equation (1) in the main text

### A.4 Reformulating the objective function in program (3)

Computing the objective  $\min \sum_{t \in T} \text{CSLC}_t$  in program (3) is equivalent to computing

$$\max \sum_{t \in T} (1 - \text{CSLC}_t) = \max \sum_{t \in T} \frac{\mathbf{w}_t \mathbf{q}_t}{\mathbf{w}_t \mathbf{q}_t + \tau_t \varepsilon_t} = \max \sum_{t \in T} \frac{\gamma_t \mathbf{w}_t \mathbf{q}_t}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t}. \quad (4)$$

This objective is nonlinear in the unknowns  $\gamma_t$  and  $\varepsilon_t$ , which makes it difficult to compute. Therefore, in our empirical analysis we replace (4) by the objective

$$\max \sum_{t \in T} (\gamma_t \mathbf{w}_t \mathbf{q}_t - \varepsilon_t), \quad (5)$$

which is linear in unknowns.

To see the connection between objective (5) instead of (4), let us consider

$$\begin{aligned} \frac{\gamma_t \mathbf{w}_t \mathbf{q}_t}{\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t} &\geq \rho \\ \Leftrightarrow \gamma_t \mathbf{w}_t \mathbf{q}_t &\geq \rho (\gamma_t \mathbf{w}_t \mathbf{q}_t + \varepsilon_t) \\ \Leftrightarrow (1 - \rho) \gamma_t \mathbf{w}_t \mathbf{q}_t - \rho \varepsilon_t &\geq 0. \end{aligned}$$

Thus, larger differences in  $(\gamma_t \mathbf{w}_t \mathbf{q}_t - \varepsilon_t)$  relates to setting a higher  $\rho$  (which corresponds to a higher value of  $(1 - \text{CSLC}_t)$ ). As a result, higher values of  $\sum_t (\gamma_t \mathbf{w}_t \mathbf{q}_t - \varepsilon_t)$  lead to higher values of  $\sum_t (1 - \text{CSLC}_t)$ .

## Appendix B: Quantity based estimates of latent capital for ready mixed concrete producers

Syverson (2004) argues that proxy variable approaches, such as the Olley and Pakes (1996) routine, are not appropriate to empirically analyze the sector of ready mixed concrete producers. Local demand states may influence input and investment decisions, which makes the assumption of a one-to-one relation between unobserved productivity and observable investment difficult to maintain. Interestingly, because our routine does not rely on (semi-)parametric structuring of the simultaneity issue, it remains well applicable to this sector. More generally, as instruments to deal with the simultaneity bias are not always easily available, our methodology broadens the reach of available empirical methods to analyze productivity variation.

In our analysis of ready mixed concrete producers, we make use of (only) 118 firm-year observations on 30 small firms (see Table 5 for summary statistics). To deal with the large heterogeneity in production quantities, we set our goodness-of-fit parameter  $\theta$  equal to 0.90 for this particular setting.<sup>19</sup> The very small sample size indicates that caution is needed when interpreting the results. Medium and large firms were not considered because of the small number of observations available. Our estimates of (the cost shares) of latent capital are summarized in Table 6. The Spearman correlations with a one-year lag are above 0.77 for our three measures of latent capital (i.e., revenue based, produced value based and quantity based), which confirms the well established finding of persistent technological heterogeneity in narrowly defined industries. The Spearman correlation between our quantity based indicators and produced value based indicators is positive, but moderate (0.70). This demonstrates once more that value based estimation results may differ substantially from quantity based results at the level of individual firm observations. The Spearman correlation between revenue based latent capital and produced value based latent capital is 0.54 and the Spearman correlation with quantity based latent capital equals only 0.39. This last result reveals that a general indicator of latent capital captures more than the pure technological manufacturing features of the firm. On average, the quantity based and produced value based cost shares of latent capital amount to respectively 11 and 17 percent. The revenue based estimates are on average 6 percent.

Figure 4 presents the evolution of the input cost shares. Due to the small sample size, the evolution patterns should be considered with caution as they are subject to changing sample compositions over the period under study. Still, all three latent capital estimates show a similar evolution over time of the input cost shares. Cost shares are evolving in favor of domestic materials and tangible fixed assets and against foreign materials and latent capital, while labor cost shares are fairly constant over time. Stated differently, regardless of the how we measure latent capital, also for this well defined industry we find strong empirical evidence against Hicks neutrality.

---

<sup>19</sup>We cleaned the data in a similar manner as for our main analysis, but add the output value and output quantity as variables that require cleaning. We define firm-year observations as representing single-product firms if the value of one 8-digit product is over 90 percent of the production value.

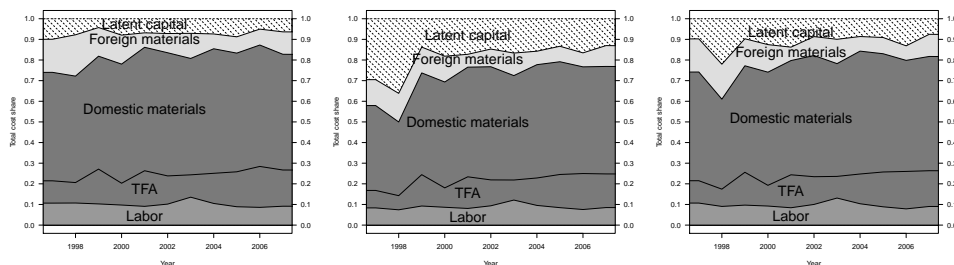
**Table 5:** Summary statistics – ready mixed concrete sector

	Mean	St.Dev.	Min.	25 perc.	Median	75 perc.	Max.
Deflated revenue	7.12	3.91	0.63	4.74	6.23	8.20	22.78
Deflated produced value	5.69	2.46	0.76	4.36	5.12	7.02	13.71
Output quantity	245.26	104.39	29.70	177.16	235.80	292.14	634.04
Nace 2-digit output price	1.16	0.09	1.02	1.07	1.17	1.22	1.31
Labor in FTE	22.67	11.26	10.00	13.42	19.75	30.67	49.70
Deflated tangible fixed assets	1.35	1.23	0.04	0.61	0.98	1.64	6.83
Deflated material costs	5.95	3.46	0.66	3.84	5.29	6.72	20.14
Labor price	0.04	0.01	0.03	0.04	0.04	0.05	0.06
Capital price	1.14	0.07	1.03	1.08	1.11	1.18	1.26
Intermediates price	1.14	0.09	1.01	1.03	1.13	1.20	1.28
Sourcing (share)	0.72	0.11	0.30	0.67	0.72	0.79	0.94

Note: Deflated revenue, deflated produced value, deflated tangible fixed assets, deflated material costs and labor price are expressed in millions of euro. Output quantity is expressed in millions of kilogram.

**Table 6:** Summary statistics of latent capital

	Mean	St.Dev.	Min.	25 perc.	Median	75 perc.	Max.
<b>Ready mixed concrete producers</b>							
LC (quantity based)	1.18	1.86	0.00	0.16	0.48	1.62	15.29
LC (produced value based)	2.08	2.91	0.00	0.41	1.25	2.64	20.25
LC (revenue based)	0.71	0.94	0.00	0.08	0.45	0.93	5.55
CSLC (quantity based)	0.11	0.12	0.00	0.02	0.05	0.17	0.66
CSLC (produced value based)	0.17	0.13	0.00	0.06	0.15	0.25	0.66
CSLC (revenue based)	0.06	0.06	0.00	0.01	0.05	0.10	0.32



(a) Revenue based      (b) Produced value based      (c) Quantity based

**Figure 4:** Cost shares

## Appendix C: Alternative values for the goodness-of-fit parameter $\theta$

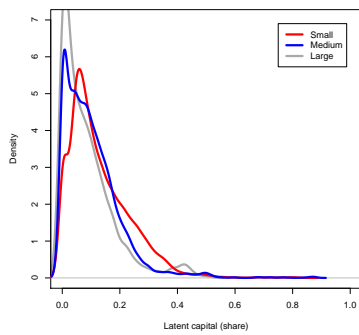
Our main empirical findings are not sensitive to altering the value of the goodness-of-fit parameter  $\theta$ . To show this, we replicated our complete analysis for values of  $\theta$  equal to 0.925 and 0.900. Results are highly robust for altering the value of  $\theta$ . Table 7 shows that the Spearman correlation between our measures of latent capital with  $\theta = 0.950$  and  $\theta = 0.925$  is more than 0.8. Further, from Figure 5 we learn that the distribution of (the cost share of) latent capital for the three firm

size groups is highly similar for different  $\theta$ -values. All main findings on the evolution of input cost shares are robust for changing  $\theta$  to 0.925 or 0.900. The same applies to the associations between international exposure, sourcing and latent capital (results available upon request).

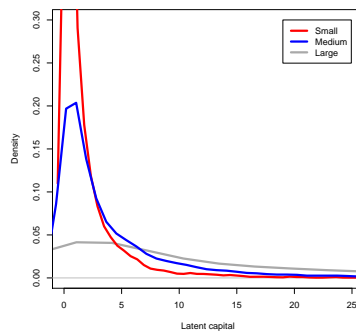
**Table 7:** Spearman correlations between latent capital estimates for different values of the goodness-of-fit parameter

	All	Small	Medium	Large
$\theta = 0.925$	0.81	0.77	0.80	0.93
$\theta = 0.900$	0.70	0.52	0.76	0.94

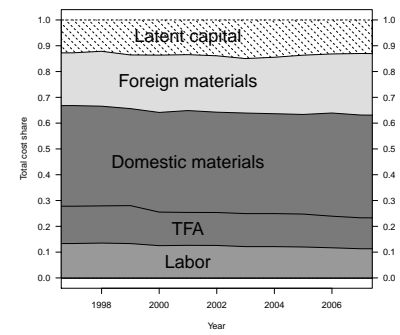
$\theta = 0.925$



(a) CSLC

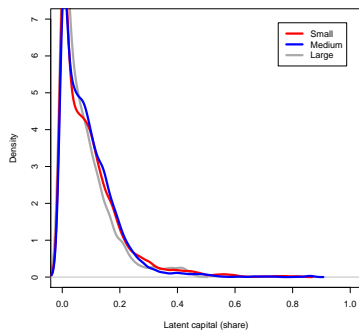


(b) LC

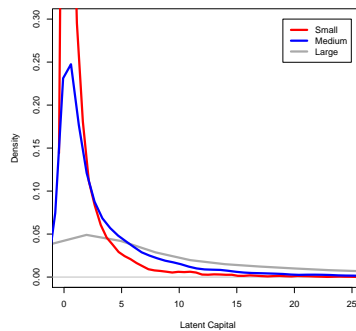


(c) Cost shares

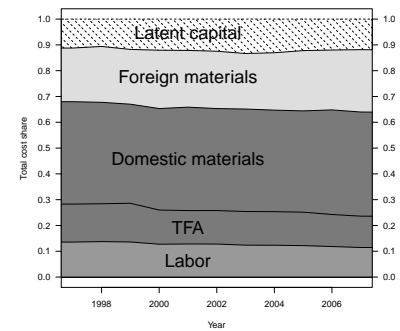
$\theta = 0.900$



(d) CSLC



(e) LC



(f) Cost shares

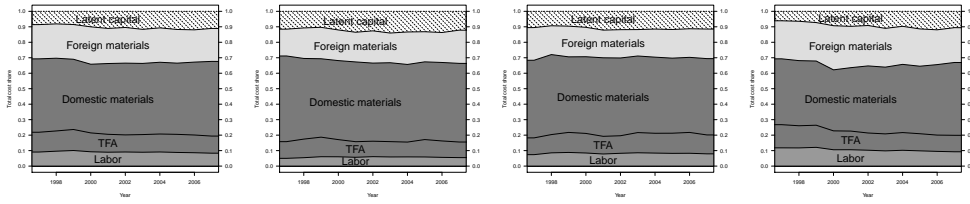
**Figure 5:** Distribution LC and CSLC, and cost shares for alternative values of  $\theta$

## Appendix D: Additional results

### D.1 Cross-sectoral variation and firm characteristics

Figures 6 and 7 show that our empirical findings on the evolution of cost shares (summarized in Figures 2 and 3 in the main text) are not specific to one manufacturing sector. The reported patterns are also not sensitive to including additional information on the firm's age and exporting status, or to applying a more detailed definition of the sector. Specifically, Figure 8 confirms that the general picture of input cost share changes against primary inputs equally applies to mature firms, non-exporting and exporting firms.

## Nace 15: Food and beverages



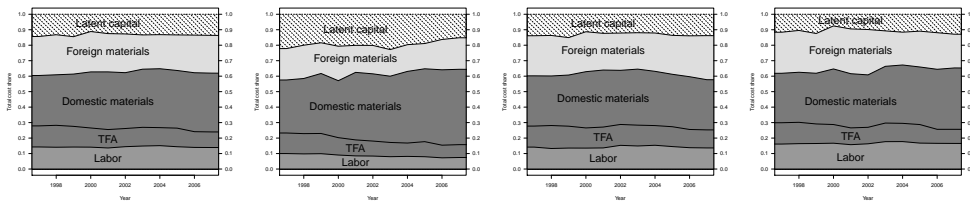
(a) All

(b) Small

(c) Medium

(d) Large

## Nace 17: Textiles



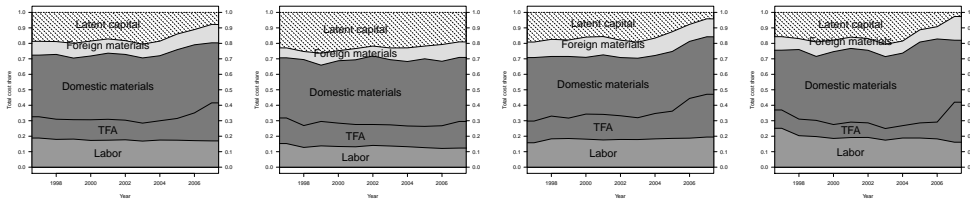
(e) All

(f) Small

(g) Medium

(h) Large

## Nace 22: Publishing



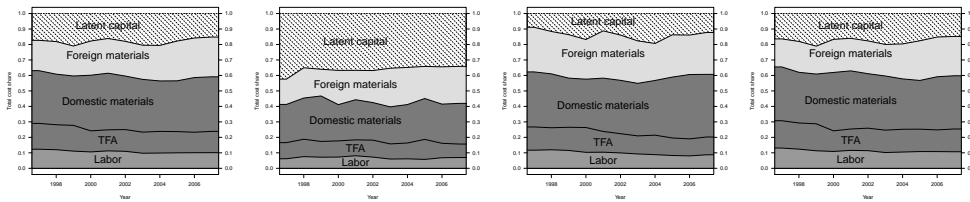
(i) All

(j) Small

(k) Medium

(l) Large

## Nace 24: Chemicals and chemical products



(m) All

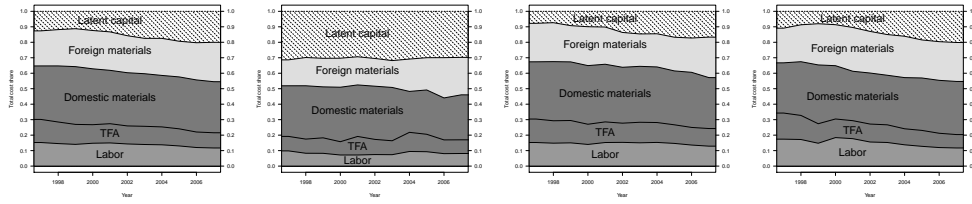
(n) Small

(o) Medium

(p) Large

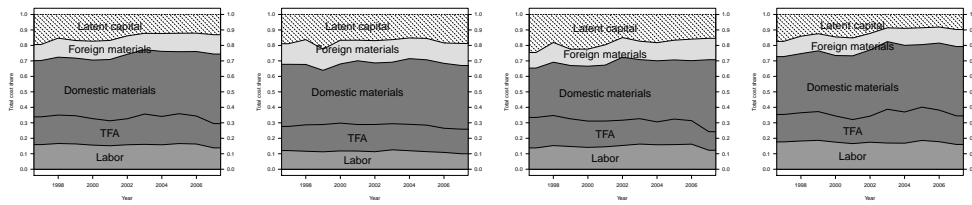
**Figure 6: Evolution of cost shares**

### Nace 25: Rubber and plastic



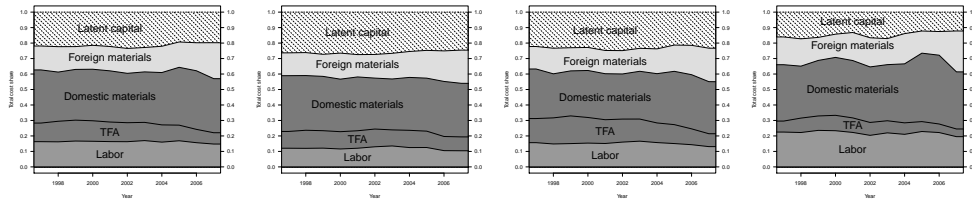
(a) All      (b) Small      (c) Medium      (d) Large

### Nace 26: Other non-metallic minerals products



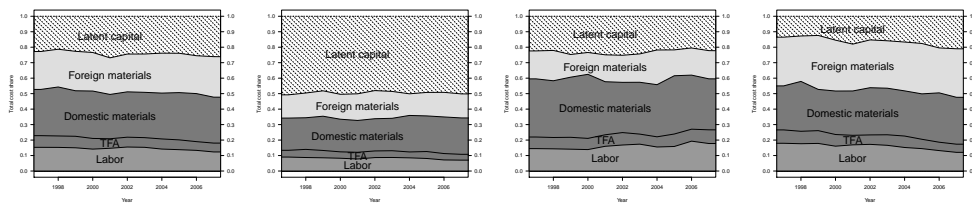
(e) All      (f) Small      (g) Medium      (h) Large

### Nace 28: Manufacture of fabricated metal products, except machinery and equipment



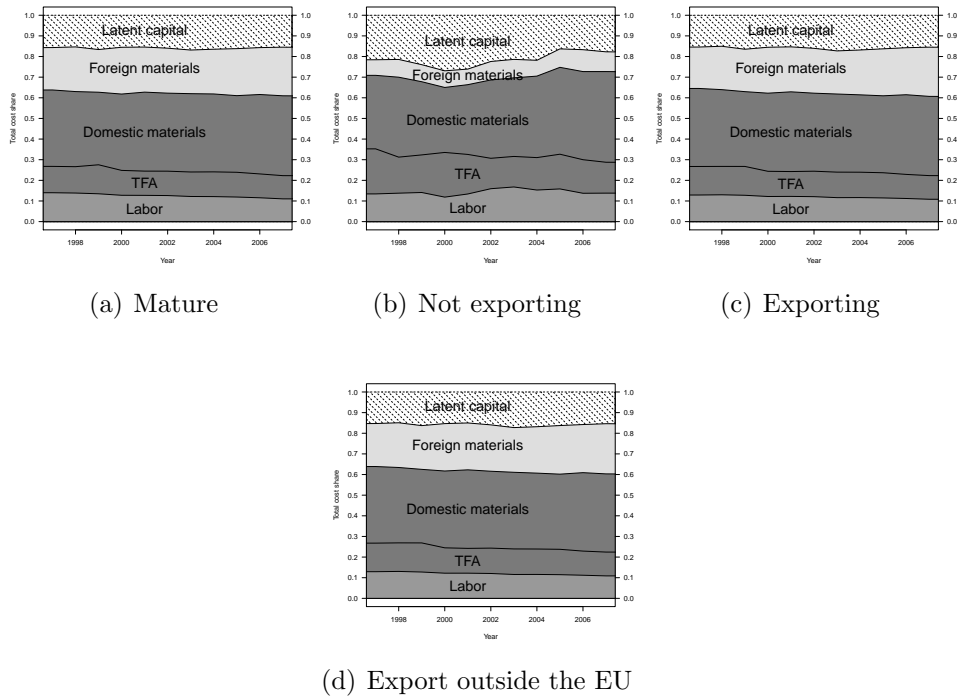
(i) All      (j) Small      (k) Medium      (l) Large

### Nace 29: Machinery and equipment n.e.c.



(m) All      (n) Small      (o) Medium      (p) Large

**Figure 7: Evolution of cost shares**



**Figure 8:** Evolution of cost shares for different categories of firms

## D.2 Summary statistics of latent capital

Table 8 provides summary statistics for our (cost share of) latent capital estimates (both revenue based and produced value based).<sup>20</sup> Tables 9 and 10 provide the evolution over time of the cost shares of latent capital (again both revenue based and produced value based).

<sup>20</sup>Our estimates of latent capital (based on produced value) contain a small proportion of unrealistic values for some specific firms. Therefore, we exclude the observations that belong to the 5 percent highest values of our estimated cost share of latent capital. Results available upon request show that our main results are robust for including these observations. A potential explanation for this difference may be that there is a higher level of misreporting in the Prodcum survey than in the financial accounts contained in the Central Balance Sheet Office database.



**Table 8:** Summary statistics of latent capital

	Mean	St.Dev.	Min.	25 perc.	Median	75 perc.	Max.
<b>Revenue based – all sectors</b>							
LC	8.49	38.06	0.00	1.04	2.51	6.19	1573.93
LC (small firms)	3.45	6.03	0.00	0.89	1.83	4.02	281.56
LC (medium firms)	5.89	10.97	0.00	1.15	2.99	6.69	307.79
LC (large firms)	40.60	106.94	0.00	3.51	11.15	32.02	1573.93
CSLC	0.19	0.14	0.00	0.08	0.17	0.27	0.91
CSLC (small firms)	0.25	0.15	0.00	0.13	0.24	0.35	0.91
CSLC (medium firms)	0.14	0.10	0.00	0.06	0.13	0.21	0.88
CSLC (large firms)	0.11	0.09	0.00	0.03	0.09	0.15	0.63
<b>Produced value based – all sectors</b>							
LC	11.89	70.67	0.00	0.75	2.15	6.39	2835.74
LC (small firms)	2.51	5.26	0.00	0.46	1.22	2.59	156.07
LC (medium firms)	7.56	16.56	0.00	1.22	3.37	8.51	695.44
LC (large firms)	67.18	199.40	0.00	4.49	17.54	49.05	2835.74
CSLC	0.17	0.13	0.00	0.08	0.16	0.25	0.74
CSLC (small firms)	0.18	0.12	0.00	0.09	0.17	0.26	0.74
CSLC (medium firms)	0.17	0.13	0.00	0.08	0.15	0.24	0.73
CSLC (large firms)	0.17	0.15	0.00	0.04	0.14	0.25	0.73

**Table 9:** Revenue based cost shares and firm size

	97	98	99	00	01	02	03	04	05	06	07
<b>All firms</b>											
Labor	0.13	0.13	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.11	0.11
Tangible fixed assets	0.14	0.14	0.14	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12
Domestic materials	0.38	0.37	0.36	0.38	0.38	0.38	0.38	0.38	0.37	0.39	0.39
Foreign materials	0.20	0.21	0.20	0.22	0.21	0.21	0.21	0.21	0.22	0.22	0.23
Latent capital	0.16	0.15	0.17	0.16	0.16	0.16	0.17	0.17	0.16	0.16	0.16
<b>Small firms</b>											
Labor	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08
Tangible fixed assets	0.11	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.11	0.10	0.09
Domestic materials	0.39	0.38	0.37	0.36	0.38	0.37	0.37	0.37	0.38	0.37	0.38
Foreign materials	0.16	0.17	0.17	0.18	0.17	0.18	0.18	0.18	0.18	0.19	0.20
Latent capital	0.26	0.24	0.26	0.26	0.26	0.26	0.26	0.26	0.25	0.26	0.25
<b>Medium firms</b>											
Labor	0.12	0.13	0.13	0.12	0.12	0.13	0.13	0.12	0.12	0.12	0.11
Tangible fixed assets	0.13	0.14	0.14	0.14	0.13	0.13	0.13	0.12	0.12	0.13	0.12
Domestic materials	0.40	0.39	0.39	0.39	0.40	0.39	0.39	0.39	0.40	0.41	0.40
Foreign materials	0.20	0.20	0.20	0.20	0.20	0.20	0.19	0.20	0.21	0.21	0.22
Latent capital	0.14	0.14	0.15	0.15	0.15	0.16	0.17	0.17	0.15	0.15	0.15
<b>Large firms</b>											
Labor	0.15	0.14	0.14	0.13	0.13	0.13	0.12	0.12	0.12	0.12	0.12
Tangible fixed assets	0.15	0.15	0.15	0.12	0.12	0.13	0.13	0.13	0.13	0.12	0.12
Domestic materials	0.36	0.36	0.35	0.37	0.38	0.37	0.38	0.37	0.36	0.38	0.38
Foreign materials	0.21	0.21	0.21	0.23	0.22	0.22	0.22	0.23	0.24	0.24	0.24
Latent capital	0.13	0.14	0.15	0.14	0.14	0.15	0.16	0.15	0.15	0.14	0.14

**Table 10:** Produced value based cost shares and firm size

	97	98	99	00	01	02	03	04	05	06	07
	<b>All firms</b>										
Labor	0.13	0.13	0.12	0.12	0.13	0.13	0.12	0.12	0.11	0.10	0.10
Tangible fixed assets	0.12	0.12	0.12	0.11	0.12	0.11	0.11	0.11	0.11	0.11	0.11
Domestic materials	0.34	0.34	0.32	0.35	0.36	0.35	0.36	0.36	0.35	0.36	0.36
Foreign materials	0.19	0.20	0.19	0.21	0.20	0.21	0.20	0.21	0.21	0.20	0.22
Latent capital	0.23	0.22	0.25	0.21	0.19	0.20	0.20	0.21	0.22	0.23	0.21
	<b>Small firms</b>										
Labor	0.10	0.10	0.10	0.10	0.10	0.11	0.10	0.10	0.09	0.08	0.08
Tangible fixed assets	0.12	0.12	0.13	0.13	0.12	0.12	0.12	0.11	0.12	0.11	0.11
Domestic materials	0.39	0.40	0.39	0.38	0.40	0.39	0.38	0.39	0.40	0.38	0.39
Foreign materials	0.18	0.17	0.17	0.18	0.18	0.18	0.19	0.19	0.19	0.20	0.21
Latent capital	0.21	0.21	0.20	0.22	0.20	0.20	0.20	0.21	0.20	0.23	0.21
	<b>Medium firms</b>										
Labor	0.12	0.12	0.13	0.12	0.12	0.13	0.13	0.13	0.12	0.12	0.11
Tangible fixed assets	0.12	0.12	0.13	0.12	0.12	0.13	0.12	0.12	0.12	0.13	0.11
Domestic materials	0.36	0.35	0.35	0.36	0.36	0.35	0.35	0.36	0.37	0.36	0.35
Foreign materials	0.21	0.20	0.18	0.19	0.19	0.19	0.19	0.19	0.20	0.20	0.22
Latent capital	0.19	0.20	0.21	0.21	0.20	0.20	0.21	0.21	0.18	0.19	0.21
	<b>Large firms</b>										
Labor	0.13	0.14	0.13	0.12	0.13	0.13	0.12	0.11	0.11	0.10	0.10
Tangible fixed assets	0.12	0.12	0.11	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.10
Domestic materials	0.31	0.32	0.30	0.34	0.35	0.35	0.36	0.35	0.33	0.35	0.35
Foreign materials	0.19	0.20	0.19	0.22	0.21	0.22	0.21	0.22	0.22	0.21	0.23
Latent capital	0.25	0.22	0.27	0.21	0.19	0.19	0.20	0.21	0.24	0.24	0.22

## NATIONAL BANK OF BELGIUM - WORKING PAPERS SERIES

The Working Papers are available on the website of the Bank: <http://www.nbb.be>.

290. "Predicting Belgium's GDP using targeted bridge models", by Ch. Piette, *Research series*, January 2016.
291. "Did export promotion help firms weather the crisis?", by J. Van Biesebroeck, J. Konings and C. Volpe Martincus, *Research series*, January 2016.
292. "On the role of public policies and wage formation for business investment in R&D: A long-run panel analysis", by T. Buysse, F. Heylen and R. Schoonackers, *Research series*, January 2016.
293. "Unraveling firms: Demand, productivity and markups heterogeneity", by E. Forlani, R. Martin, G. Mion and M. Muûls, *Research series*, February 2016.
294. "Unemployment risk and over-indebtedness: A micro-econometric perspective", by Ph. Du Caju, F. Rycx and I. Tojerow, *Research series*, February 2016.
295. "International shocks and domestic prices: How large are strategic complementarities?", by A. Amiti, O. Itskhoki and J. Konings, *Research series*, March 2016.
296. "The supplier network of exporters: Connecting the dots", by E. Dhyne and S. Rubínová, *Research series*, May 2016.
297. "Does one size fit all at all times? The role of country specificities and state dependencies in predicting banking crises" by S. Ferrari and M. Pirovano, *Research series*, May 2016.
298. "Competition and product mix adjustment of multi-product exporters: Evidence from Belgium", by K. Breemersch, *Research series*, June 2016.
299. "Flemish maritime ports, Liège port complex and the port of Brussels – Report 2014", by G. Van Gastel, *Document series*, June 2016.
300. "Misalignment of productivity and wages across regions? Evidence from Belgian matched panel data", by F. Rycx, Y. Saks and I. Tojerow, *Research series*, July 2016.
301. "The European Payments Union and the origins of Triffin's regional approach towards international monetary integration", by I. Maes and I. Pasotti, *Research series*, September 2016.
302. "The transmission mechanism of credit support policies in the Euro Area", by J. Boeckx, M. de Sola Perea and G. Peersman, *Research series*, October 2016.
303. "Bank capital (requirements) and credit supply: Evidence from pillar 2 decisions", by O. De Jonghe, H. Dewachter and S. Ongena, *Research series*, October 2016.
304. "Monetary and macroprudential policy games in a monetary union", by R. Dennis and P. Ilbas, *Research series*, October 2016.
305. "Forward guidance, quantitative easing, or both?", by F. De Graeve and K. Theodoridis, *Research series*, October 2016.
306. "The impact of sectoral macroprudential capital requirements on mortgage loan pricing: Evidence from the Belgian risk weight add-on", by S. Ferrari, M. Pirovano and P. Rovira Kaltwasser, *Research series*, October 2016.
307. "Assessing the role of interbank network structure in business and financial cycle analysis", by J-Y Gnabo and N.K. Scholtes, *Research series*, October 2016.
308. "The trade-off between monetary policy and bank stability", by M. Lamers, F. Mergaerts, E. Meuleman and R. Vander Vennet, *Research series*, October 2016.
309. "The response of euro area sovereign spreads to the ECB unconventional monetary policies", by H. Dewachter, L. Iania and J-C. Wijnandts, *Research series*, October 2016.
310. "The interdependence of monetary and macroprudential policy under the zero lower bound", by V. Lewis and S. Villa, *Research series*, October 2016.
311. "The impact of exporting on SME capital structure and debt maturity choices", by E. Maes, N. Dewaelheynes, C. Fuss and C. Van Hulle, *Research series*, October 2016.
312. "Heterogeneous firms and the micro origins of aggregate fluctuations", by G. Magerman, K. De Bruyne, E. Dhyne and J. Van Hove, *Research series*, October 2016.
313. "A dynamic factor model for forecasting house prices in Belgium", by M. Emiris, *Research series*, November 2016.
314. "La Belgique et l'Europe dans la tourmente monétaire des années 1970 – Entretiens avec Jacques van Ypersele", by I. Maes and S. Péters, *Research series*, December 2016.
315. "Creating associations to substitute banks' direct credit. Evidence from Belgium", by M. Bedayo, *Research series*, December 2016.
316. "The impact of export promotion on export market entry", by A. Schminke and J. Van Biesebroeck, *Research series*, December 2016.
317. "An estimated two-country EA-US model with limited exchange rate pass-through", by G. de Walque, Ph. Jeanfils, T. Lejeune, Y. Rychalovska and R. Wouters, *Research series*, March 2017.

318. Using bank loans as collateral in Europe: The role of liquidity and funding purposes", by F. Koulischer and P. Van Roy, *Research series*, April 2017.
319. "The impact of service and goods offshoring on employment: Firm-level evidence", by C. Ornaghi, I. Van Beveren and S. Vanormelingen, *Research series*, May 2017.
320. "On the estimation of panel fiscal reaction functions: Heterogeneity or fiscal fatigue?", by G. Everaert and S. Jansen, *Research series*, June 2017.
321. "Economic importance of the Belgian ports: Flemish maritime ports, Liège port complex and the port of Brussels - Report 2015", by C. Mathys, *Document series*, June 2017.
322. "Foreign banks as shock absorbers in the financial crisis?", by G. Barboni, *Research series*, June 2017.
323. "The IMF and precautionary lending: An empirical evaluation of the selectivity and effectiveness of the flexible credit line", by D. Essers and S. Ide, *Research series*, June 2017.
324. "Economic importance of air transport and airport activities in Belgium – Report 2015", by S. Vennix, *Document series*, July 2017.
325. "Economic importance of the logistics sector in Belgium", by H. De Doncker, *Document series*, July 2017.
326. "Identifying the provisioning policies of Belgian banks", by E. Arbak, *Research series*, July 2017.
327. "The impact of the mortgage interest and capital deduction scheme on the Belgian mortgage market", by A. Hoebbeck and K. Inghelbrecht, *Research series*, September 2017.
328. "Firm heterogeneity and aggregate business services exports: Micro evidence from Belgium, France, Germany and Spain", by A. Ariu, E. Biewen, S. Blank, G. Gaulier, M.J. González, Ph. Meinen, D. Mirza, C. Martín and P. Tello, *Research series*, September 2017.
329. "The interconnections between services and goods trade at the firm-level", by A. Ariu, H. Breinlichz, G. Corcosx, G. Mion, *Research series*, October 2017.
330. "Why do manufacturing firms produce services? Evidence for the servitization paradox in Belgium", by P. Blanchard, C. Fuss and C. Mathieu, *Research series*, November 2017.
331. "Nowcasting real economic activity in the euro area: Assessing the impact of qualitative surveys", by R. Basselier, D. de Antonio Liedo and G. Langenus, *Research series*, December 2017.
332. "Pockets of risk in the Belgian mortgage market: Evidence from the Household Finance and Consumption Survey (HFCS)", by Ph. Du Caju, *Research series*, December 2017.
333. "The employment consequences of SMEs' credit constraints in the wake of the great recession" by D. Cornille, F. Rycx and I. Tojerow, *Research series*, December 2017.
334. "Exchange rate movements, firm-level exports and heterogeneity", by A. Berthou and E. Dhyne, *Research series*, January 2018.
335. "Nonparametric identification of unobserved technological heterogeneity in production", by L. Cherchye, T. Demuyne, B. De Rock and M. Vershelde, *Research series*, February 2018.

National Bank of Belgium  
Limited liability company  
RLP Brussels – Company's number: 0203.201.340  
Registered office: boulevard de Berlaimont 14 – BE-1000 Brussels  
[www.nbb.be](http://www.nbb.be)

Editor

**Jan Smets**

Governor of the National Bank of Belgium

© Illustrations: National Bank of Belgium

Layout: Analysis and Research Group  
Cover: NBB AG – Prepress & Image

Published in February 2018