Geometric Errors and CAD Model Integration Method for Precision Assembly

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Abstract**—***It is difficult to integrate the complete geometric errors into the three-dimensional (3D) solid model in the present computer-aided design (CAD) system, which leads to the fact that virtual assembly technology based on the ideal CAD model cannot predict the influence of geometric errors on the assembly quality. The construction of the realistic geometric solid model is one of the key technologies to promote the practical application of virtual assembly technology. The dimension information and the geometric errors information of the parts are all expressed in the CAD model through the proposed integration method. The realistic geometric solid model is expressed by boundary representation (B-rep), and is reconstructed based on SAT file. C language programming is used to achieve the above functions. This method realizes the underlying data integration between the ideal CAD model and the geometric errors surface model, and improves the credibility of virtual assembly.*

Keywords-Precision assembly; Geometric errors; CAD; Solid model

I. Introduction

In modern industrial manufacturing, the assembly accounts for 45%-60% of the total workload, and the cost is 20%-30% or more of the total manufacturing cost [1]. Assembly directly affects the quality performance, life cycle and manufacturing costs of mechanical products. For a long time, designers try to find and solve potential assembly problems as early as possible in the overall design stage to improve assembly quality and success rates. With the development of computer integrated manufacturing system (CIMS), virtual assembly technology provides a new assembly approach for traditional assembly processes [2]. Based on computer-aided design (CAD), virtual assembly technology is used to simulate the assembly process. However, its application in industrial manufacturing depends on the authenticity of models.

The basis of virtual assembly is the 3D solid models. The current CAD systems focus on the ideal model with tolerance, and are difficult to establish the model with irregular form errors. A 3D solid model is defined as the Realistic Geometric Solid Model if the model contains all available information of actual parts such as dimensions, orientation, position and form. Figure 1 illustrates the actual assembly condition, the nominal models, realistic geometric solid models and the simplified models without form errors. Most research only considers the effect of part dimension deviations (Fig. 1b) or orientation deviation on the assembly (Fig. 1d), but do not pay enough attention to form errors. Actually, there are a number of peaks and valleys in different sizes and shapes, which distribute non-uniformly on the surface. As shown in Figure 2, due to form errors, the mating surfaces are in contact by a limited number of contact points, which leads to the parts deviated from the ideal assembly position. In addition, form errors can propagate and accumulate through assembly parts, and even very small form errors will further lead to variation in assembly accuracy and decrease in assembly stability [3]. Therefore, there will be some differences between CAD models and actual parts if lack of form errors, resulting in lowering the accuracy of subsequent simulations. The geometric errors representation considering form errors is one of the contributions in this work.

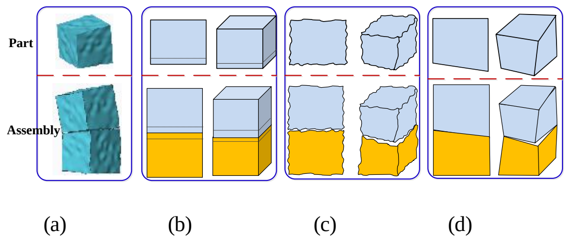

Figure 1. The impact of different models on assembly analysis; (a) actual assembly; (b) nominal models; (c) realistic geometric solid model; (d) models with simplified geometric errors.

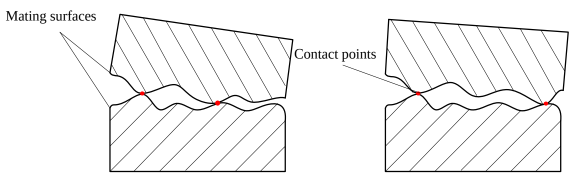

Figure 2. Assembled parts with form errors

II. Related Work

Requicha proposed an offsetting model and laid the theoretical basis for the integration of geometric error and solid model [4]. Desrochers established a technologically and topologically related surface (TTRS), which represented three-dimensional parts as tree structure consisting of surfaces, and the tolerances information as tree nodes [5]. Teck et al. used the small displacement torsor (SDT) to describe the small variation of the actual characteristic relative to the ideal characteristic for the planar feature, and proposed a new mathematical definition of the plane tolerance zone [6]. The tolerance zone was determined by calculating the range of the flatness parameter. On this basis, a 3D modelling environment of UG/Open API was developed, and a solid model containing tolerance information was established. For the polyhedral model formed by planar elements, Roy proposed representation and interpretation of geometric tolerances for polyhedral objects [7]. Based on degrees of freedom (DOF) variation analysis, the surface mathematical model was reconstructed according to the tolerance information or the actual measurement data, and the B-rep data of the solid model was modified to that which contained the dimension, orientation, position and shape deviation. Louhichi et al. proposed a model method with the consideration of geometric deviations [8]. According to the tolerance information, SDT vector and numerical perturbation method were used to modify the 3D model to realize the vector correction of the geometric boundary. On this basis, the model dimension information was reconfigured, and the constraint information was updated to generate the assembly simulation model considering geometric deviation. Franciosa et al. proposed a method of transforming the tolerance into a variational feature using a homogeneous transformation matrix, and a 3D model containing variational features was established [9]. Liu et al. proposed a unified tolerance modeling method based on the mathematical definition and SDT [10]. On the basis of mathematical model of hole, a solid model of variable hole was constructed. Based on the kinematic model of variational geometric constraints network, Hu and Wu studied the automatic generation rules of computer-aided tolerances, and established the variable solid model [11]. Q. Zhang et al.[12,13] proposed a virtually assembly method based on constrained surface registration and carried on simulation contact analysis of solid model with form errors, but the modelling method was not described in detail.

From the above research, the dimension and position deviation were considered in the representation of geometric errors while the form errors were ignored. That is, the 3D model established by the above method did not contain irregular form errors. This paper presents a geometric errors and CAD model integration method, which enables the new model can express and analyze the complete geometric characteristics with form errors for precision assembly. This paper is organized as follows: section III describes the surface model based on IGES file; section IV gives the expression of realistic geometric solid model; section V proposes the NURBs-B-Rep geometric solid model and their data structure; the paper is concluded in section VI.

III. Surface Model Based on IGES File

The prerequisite for integration between geometric errors and solid model is that the mathematical models of the surface with geometric errors are accurately expressed in the CAD system. Non-uniform rational B-spline (NURBS) is suitable to represent curves and surfaces [3]. NURBS is commonly used in computer-aided design, manufacturing, and engineering and part of numerous industry wide standards. Among them, IGES (Initial Graphics Exchange Specification) is a vendor-neutral file format that allows the digital exchange of information among CAD systems. [14,15]. The IGES file contains the necessary information for creating NURBS surface, including control points, degree of B-spline basis functions, knot vectors and weights. Based on the measurement data of surfaces, the mathematical models are established. After analyzing the IGES standard structure, the NURBS mathematic model parameters are written in the IGES file. And then IGES file is imported into the CAD system to generate the surface model with geometric errors. Figure 3 shows the surface models of a cylinder and a plane in CAD system.

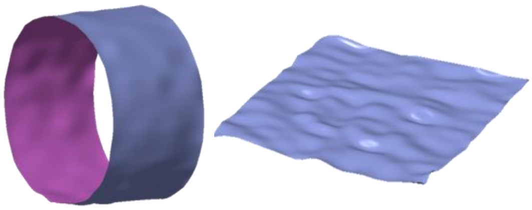

Figure 3. Examples of NURBS surface model

IV. Expression of Realistic Geometric Solid Model

A wireframe model, a surface model, or a solid model can represent the shape of a three-dimensional object, wherein the solid model can represent it completely and unambiguously. Constructive solid geometry (CSG) and boundary representation (B-rep) are commonly used solid modelling methods. CSG means that any complex shape can be represented by a Boolean operation of several simple objects. It can record the original feature parameters of the solid, but cannot store the details such as boundaries and vertices. B-rep is a method to represent shapes using the limits. B-rep can record the geometric information and topology information of model in detail, while CSG can better describe the construction process of solid. The research purpose is to integrate non-uniform geometric information into the solid rather than concerning the construction process, so B-rep is more suitable for describing the model.

The B-rep data includes geometric information and topology information. Geometric information refers to the information of the basic geometric elements such as points, curves and surfaces. Figure 4 shows the types of geometric elements in CAD system, where the point is the most basic element, represented by coordinates. The geometric element parameters are expressed as follows. Figure 5 and 6 shows the parameterization of common curve and surface.

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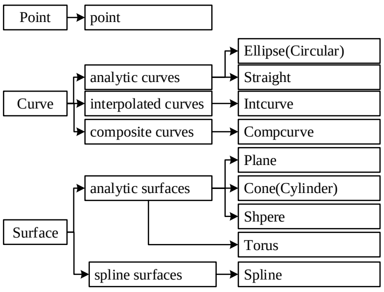


Figure 4. Geometric elements in the CAD system

A. Point

Points are usually expressed by coordinates, i.e., P=(x, y, z).

B. Curve

When the curve is straight,

$$P = S + t*(E - S) \tag{1}$$

Where, S is the starting point of a line; E is the ending point of a line; t, ranging [0, 1].

When the curve is an ellipse,

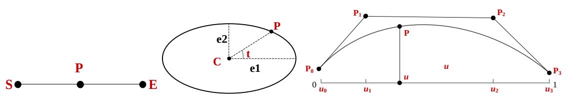
$$P = C + M*\cos(t) *e1 + N*\sin(t)*e2 \tag{2}$$

Where, C is the center point; e1 is the unit vector of major axis; e2 is the unit vector of minor axis; M is the major axis length; N is the minor axis length; t, ranging [0, 2π]. When M is equal to N, the curve is a circle.

When the curve is NURBS,

$$\begin{cases} P(u) = \dfrac{\sum_{i=0}^{n} w_i P_i B_{i,p}(u)}{\sum_{i=0}^{n} w_i B_{i,p}(u)}, 0 \le u \le 1 \\ B_{i,0}(u) = \begin{cases} 1 & if\ u_i \le u \le u_{i+1} \\ 0 & otherwise \end{cases} \\ B_{i,p}(u) = \dfrac{u - u_i}{u_{i+p} - u_i} B_{i,p-1}(u) + \dfrac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} B_{i+1,p-1}(u) \end{cases} \tag{3}$$

Where, P is control points; w is weights; u is knot vector; B is basis functions; p is the degree of basis function.



(a) (b) (c)

Figure 5. Parameterization of curve; (a) straight line; (b) ellipse and circle; (c) NURBS curve.

C. Surface

The plane is expressed as,

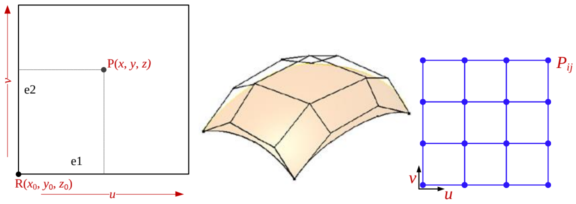
$$P = R + u* e1+ v* e2 \tag{4}$$

Where, R is the origin of the plane; e1 is the unit vector of u direction; e2 is the unit vector of v direction.

NURBS surface is expressed as,

$$S(u,v) = \frac{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) w_{i,j} P_{i,j}}{\sum_{i=0}^{m}\sum_{j=0}^{n} N_{i,p}(u) N_{j,q}(v) w_{i,j}}, 0 \le u, v \le 1 \tag{5}$$

The mathematical model of NURBS surface is determined by parameters such as control points $P_{i,j}$, node vectors *U* and *V*, and weight of control points $w_{i,j}$, as well as degrees *p* and *q* of B-spline basis functions.



(a) (b)

Figure 6. Parameterization of curve; (a) plane; (b) NURBS surface.

The topology information describes the connection relationship between the geometric information by 'Body-Lump-Shell-Face-Loop-Coedge-Edge-Vertex'. It is like the topological information constitutes the 'skeleton' of the real geometric solid model, and the geometric information is like the 'muscle' attached to the 'skeleton'. Figure 7 shows the basic topology elements and the relationship between geometric information and topology information. The faces, edges, and vertex in the topological element correspond to the surface, curve, and points in the geometric element, respectively.

In order to effectively store and manage geometric element data and its topological relationships in the computer, Baumgart et al. [16] proposed the Winged Edge Data Structure (WED) in 1972. The data structure organizes B-rep data with edges as the core to determine the information of connected vertices and faces. Geometric information and topology information can be separated, and the geometric element table and the topology element table are respectively established by the doubly linked list (Table 1 and Table 2). Thus, the solid model can be operated partially. Partial operation means that the overall topological information is not changed, and only the geometric information of surface is partially modified.

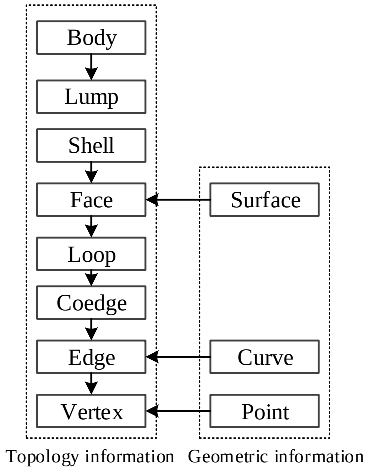


Figure 7. Relationship between geometric information and topology information in B-rep data

TABLE1. B-REP GEOMETRY ELEMENT DATA STRUCTURE

Name	Data structure and members
Point	Struct PointSt {long double axisX; long double axisY; long double axisZ;}
Curve	Struct LumpSt {Curvetype; *Curve; Domain;}
Surface	Struct SurfaceSt {Surfacetype; *Surface; Domain;}

TABLE2. B-REP TOPOLOGY ELEMENT DATA STRUCTURE

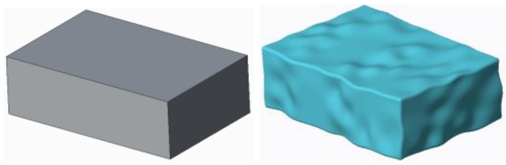
Name	Data structure and members
Body	Struct BodySt {Struct Lump *first_lump; Struct Transstruct *Trans;}
Lump	Struct LumpSt {Struct LumpSt *next_lump; Sruct Shell *first_shell; Struct BodySt *body;}
Shell	Struct ShellSt {Struct Shell *next_shell; Struct FaceSt * first_face; Struct LumpSt * lump;}
Face	Struct FaceSt {Struct FaceSt *next; Struct LoopSt *first_loop; Struct Shell *shell; Struct SurfaceSt *surface int dir;}
Loop	Struct LoopSt { Struct LoopSt *next_Loop Struct CoedgeSt *first_coedge; Struct FaceSt *face;}
Coedge	Struct CoedgeSt {Struct CoedgeSt *next; Struct CoedgeSt * previous; Struct CoedgeSt * partner; Struct EdgeSt*edge; Struct LoopSt *loop; int dir;}
Edge	Struct EdgeSt {Struct VertexSt *start_vertex; Struct VertexSt *end_vertex; Struct CoedgeSt *coedge; Struct CurveSt *straight_curve int dir;}
Vertex	Struct VertexSt {Struct EdgeSt *edge; Struct PointSt *point; };

Similarly, a realistic geometric solid model includes topological layer and geometric error layer. Topology layer can be obtained from the ideal model built in CAD systems. And the geometric error layer is from the actual measurement, including dimension, position and form errors.

V. REALISTIC GEOMETRIC SOLID MODEL BASED ON NURBS-B-REP

The construction of realistic geometric solid model based on NURBS-B-rep includes two steps. Firstly, the topological information is extracted from the ideal designed model, while the geometric information from the NURBS surface model. The relationship between the topological layer and the geometric information layer is implemented by the struct and pointer in C programming language. Then, the realistic geometric solid model is reconstructed with SAT file.

For example, for a solid model as shown in Figure 8, there are the same topological information and different surface geometric information between the ideal solid model and the realistic geometric solid model. In Figure 8(a), the geometric boundary of the ideal solid models include the point, straight and plane; and the geometric errors are not included. In Figure 8(b), the geometric boundary of the realistic geometric solid models includes NURBS curves and surfaces, and contains the geometric errors information.



(a) (b)

Figure 8. Models with same topology and different geometric information; (a) ideal solid model; (b) realistic geometric solid model

SAT is a standard storage file for model data. The SAT file can be processed by the CAD system based on the ACIS kernel. The model information can be easily extracted and the solid model is constructed using different CAD software. SAT files record and store the topological information and geometric information completely. Taking Creo system as an example, the structure of SAT file is as follows.

File Header

```
500 0 1 0
58 CREO BY PARAMETRIC TECHNOLOGY
CORPORATION, 2015290 8 ACIS 7.0 24
Sun Mar 12 19:38:07 2017
1 0.036999890109074132 1e-10
```

The file header contains the encoded version number, the number of stored solid models, ACIS version information, generation time, unit length of the model, and calculation accuracy.

Entity Records

```
-0 body $-1 $1 $-1 $2 #
-1 lump $-1 $-1 $3 $0 #
-2 transform $-1 1 0 0 0 1 0 0 0 1 0 0 0 1 rotate
no_reflect no_shear #
-3 shell $-1 $-1 $-1 $4 $-1 $1 #
```

```
-4 face $-1 $5 $6 $3 $-1 $7 reversed single #
-5 face $-1 $8 $9 $3 $-1 $10 forward single #
-6 loop $-1 $-1 $11 $4 #
-7 plane-surface $-1 0 0 0 0 0 1 1 0 0 forward_v F
-2 F 102 F
```

Each entity record consists of a sequence number, an entity type identifier, entity data and a terminator. The sequence number is a continuous number starting from 0. '$' means a pointer operation. For example, '$1' points to the record with sequence number 1 (the lump), and '$-1' indicates a null pointer. The terminator is '#'. And the end marker of SAT file is 'End-of-ACIS'.

Figure 9 describes the data structure of SAT file. Keep the topological information unchanged, remove the ideal geometric data, and insert the geometry data of NURBS surfaces. Then the topological information and the modified geometric information are written in the SAT file. when the SAT file is imported to CAD system, the realistic geometric solid model is shown. There is an example of the model reconstruction based on SAT file (Figure 10).

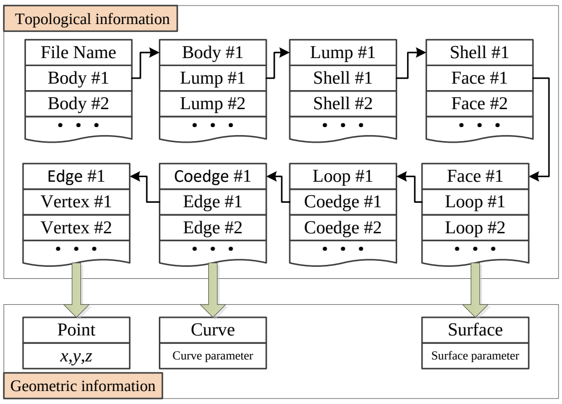


Figure 9. Data structure of SAT file.

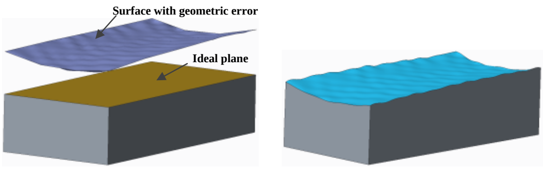


Figure 10. Model reconstruction based on SAT file

Figure 11 shows examples of the common realistic geometric solid model. In order to reflect the form errors more clearly, the heights of form errors are magnified by 100 times.

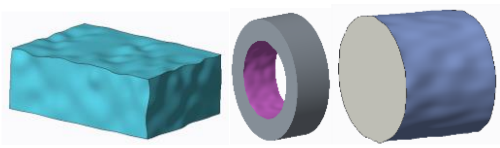


Figure 11. Common realistic geometric solid model

VI. Conclusion

The realistic geometric solid model contains the dimension, position deviation and the irregular form errors, which reflects the geometrical characteristics of the parts. Based on geometric errors and CAD model integration for precision assembly, the realistic geometric solid model of the parts can be obtained, which not only can describe the external shape, but also reflect the irregular geometric error. It provides the model basis for accurate simulation.

Acknowledgment

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