



Novel approach for adaptive coefficient tuning for the simulation of evaporating high-speed sprays injected into a high temperature and pressure environment

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	cylinder conditions. An additional discussion is performed for the validity range of existent models given that in the recent years there has been a shift towards more extreme thermodynamic conditions in the injection stage (reaching the limits of trans critical flows). Although in this work the framework was implemented in the RANS context because this is the tool of preference of digital engineering currently by automotive industries, the approach can be easily extended in LES.



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Novel approach for adaptive coefficient tuning for the simulation of evaporating high-speed sprays injected into a high temperature and pressure environment

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Abstract

Producing reliable in-cylinder simulations for quick turnaround engine development for industrial purposes is a challenging task for modern CFD, mostly because of the tuning effort required for the sub models used in the various frameworks (RANS and LES). Tuning is required because of the need of modern engines to operate under a wider range of conditions and fuels. In this paper

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4 capable of delivering a priori a coefficient matrix for each operating condition. This approach
5 produces excellent results for multiple comparison metrics like liquid and vapor penetration
6 lengths, radial and axial mass fraction and temperature distributions. In this paper we also show
7 for the first time that input model coefficients can potentially be linked to ambient boundary
8 conditions in a physically consistent manner. Changes in injection pressure, charge pressure and
9 charge density are considered. This paves the way for the tabulation of the constants in order to
10 eliminate lengthy tuning iterations between operating conditions and move towards adaptive
11 simulations as the piston moves changing the in-cylinder conditions. An additional discussion is
12 performed for the validity range of existent models given that in the recent years there has been a
13 shift towards more extreme thermodynamic conditions in the injection stage (reaching the limits
14 of trans critical flows). Although in this work the framework was implemented in the RANS
15 context because this is the tool of preference of digital engineering currently by automotive
16 industries, the approach can be easily extended in LES.

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37 Keywords: Design-of-Experiments, ECN Spray A, RANS, Diesel spray, Injection simulations
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1 **1 Introduction**

2 Digital product development, based on advanced numerical modelling, is progressively becoming an
3 integral part of the design of modern energy systems. Testing future combustion systems in a virtual
4 environment is a more time and cost-effective way of design optimisation in comparison to conventional
5 hardware-based methods. However, the success of the optimisation depends on the reliability of the
6 virtual tools which require rigorous validation to a wide range of operating conditions. For diesel
7 injection and combustion in Internal Combustion Engines (ICEs), this can be a challenging task due to
8 the large range of scales and phases involved in fuel injection dynamics. Simulating the full spray
9 injection, mixing, evaporation and combustion at thermodynamically extreme conditions (injection
10 pressures beyond 250MPa) that modern systems operate in, is both a scientifically and computationally
11 challenging task.

12 Detailed physical calculations (such as Direct Numerical Simulation (DNS) and to some extent Large
13 Eddy Simulations (LES) for turbulent motion and detailed chemical kinetic models for combustion defy
14 the purpose of using Computational Fluid Dynamics (CFD) as a time efficient virtual design tool for
15 commercial purposes. For engine developers, quick-turnaround simulations of global combustion
16 parameters such as heat release (and the associated pressure rise), spray penetration and lift-off length
17 (to guide bowl design to avoid wall impingement) and emission production at a range of operating
18 conditions are of paramount interest. Simulations which can capture characteristic trends against
19 operating conditions would allow faster engine mapping. Using methods that “ignore” some of the
20 scales of the problems under investigation (such as Reynolds-Averaged Navier-Stokes (RANS) and to
21 some extent LES for turbulent motion and tabulated or reduced chemical kinetic models for
22 combustion) can reduce the run time but introduce new uncertainties. These numerical models use a
23 range of parameters to encapsulate real physics or bridge “unknown” or “unresolved” processes.

24 Many experiments have been conducted in constant volume chambers. These have mostly been led by
25 researchers of the Sandia National Laboratories and contributors of the Engine Combustion Network
26 (ECN). The focus to date has been on turbulent spray flames under diesel-like combustion condition.

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3 27 The setup allows a high degree of optical access for advanced experimental diagnostics and well-
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5 28 characterised initial and boundary conditions for simulations, including detailed fuel injector
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7 29 characterisation (1-3). Their experiments showed the effects of operating conditions like charge
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9 30 pressures, densities, temperatures and injection pressures or fuel injection equipment specifications like
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11 31 nozzle orifice diameter on spray (4-9), jet (10-14), combustion (13-21) and emission (22-27)
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13 32 characteristics. In (4), Siebers outlines the effect of boundary conditions on liquid penetration. Liquid
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15 33 length is linearly dependent on orifice diameter and fuel temperature, independent of injection pressure,
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17 34 highly sensitive to gas densities & temperatures. These results are confirmed by other researchers like
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19 35 (6, 7, 28). A study by Pickett *et al* in (18) links the dependency of Ignition Delay (ID) to ambient gas
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21 36 densities, temperatures, oxygen concentrations and the fuel cetane number. Another study by Siebers
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23 37 *et al* in (16) and more recently by Benajes *et al* in (13) characterizes the response of flame lift-off to the
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25 38 above-mentioned operating conditions and fuel injection specifications.

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29 39 Corresponding computational simulations were carried out by various groups using RANS approaches
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31 40 (29), coupled with conditional moment closure (CMC) (30, 31), transported probability density function
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33 41 (TPDF) (32-35) or flamelet-type models (36-38), or LES approaches (39-49). Among others, work by
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35 42 Bolla *et al* (30) and Pei *et al* (32, 34) shows that while acceptable trends can be achieved for the bulk
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37 43 of the available experimental data without changing model input parameters, matching the results
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39 44 quantitatively is more difficult.

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43 45 Expecting that a low-fidelity simulation setup can be tuned to one key point and then without adjustment
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45 46 predict another, conversely asserts that a range of changing thermodynamic conditions can be captured
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47 47 by simplified sub-models. This however is not always the case and our studies have shown that by
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49 48 accepting limitations to these simplified sub-models and adjusting tuning constants to the new
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51 49 environment, these inaccuracies may be addressed. The determination of which coefficients have a
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53 50 significant impact on certain performance measures of interest is difficult. Common single parameter
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55 51 swings are often incorrect and misleading due to the multivariable interaction on various responses.
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57 52 Literature showing the effect of isolated parameter swings can be found in (50-52). The problem with
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59 53 the “single parameter swing approach” is that it is not necessarily useful for future predictive engine

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3 54 simulations. If after such parameter swing the simulated data matches pre-existent experimental data, it
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5 55 can be unclear whether the accuracy of the presented results is an indicator of good model performance
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7 56 in terms of physical representation, or the result of coefficient tuning and/or code numerics. In fact,
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9 57 when the simulated operating condition is altered while leaving the setup unchanged, a deterioration of
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11 58 the quality of simulation is often noticed. With this in mind, **three** important questions arise. We will
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13 59 address some of them in this work while others have already been investigated by the authors in previous
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16 60 work **or will be addressed in future publications:**

- 19 61 1. Are the most commonly used **sub models**, which were derived for classical droplet evaporation
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21 62 and breakup processes, valid for simulating the conditions related to real diesel injection
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23 63 (sometimes approaching trans-critical conditions)? In this work one scenario that we will
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25 64 investigate is whether the continuous injection of cool spray leads to a local cooling of the gas
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27 65 phase around the liquid droplets and subsequent reduction of both local charge temperature and
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29 66 pressure and whether this cooling effect is potent enough that only the initial droplets would
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31 67 fall into the super/trans critical regime while the following droplets would be trans/sub critical.
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33 68 2. Is there a single coefficient matrix for the various sub-models used in spray injection (namely
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35 69 turbulence, atomisation, evaporation, mixing) that can provide good match with experimental
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37 70 data at different operating conditions? This question highlights whether the mathematical
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39 71 fundamentals of the **sub models are** sophisticated enough to account for physical changes in the
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41 72 injection process.
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43 73 3. If such a coefficient matrix does not exist, are there any trends in the change of the coefficient
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45 74 values linked to physical processes and boundary conditions that can guide the a priori selection
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47 75 of the coefficient values? This question relates to the concept of intelligent tuning strategies of
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49 76 physically reasonable parameters to trigger similar trends between simulations and experiments
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51 77 if the sub models cannot adequately replicate the real process. Should such a tuning approach
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53 78 be necessary, we seek to identify pre-defined values of key tuning parameters depending on the
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55 79 boundary conditions and some “key benchmark points”. This opens the possibility of intelligent
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3 80 or even automated tuning. If such a pre-definition of tuning parameters is derivable, then
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5 81 lengthy tuning iterations will no longer be necessary.
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8 82 Initial work towards answering the second question has been performed by the authors in (53) and (54).
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10 83 The conclusions of Nsikane *et al* in (53) suggested that a single setup to match a range of operating
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12 84 conditions could not be found. While this does not prove that such a setup does not exist, the employed
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14 85 so called “Design of Experiment (DoE)” approach to statistically analyse hundreds of simulations
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16 86 certainly increased the confidence that targeted tuning will always be necessary. Subsequent work by
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18 87 Nsikane *et al* in (54) suggested that under some narrow circumstances and only if macroscopic spray
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20 88 characteristics (such as liquid and vapour penetration) are of interest, the simulation setup may remain
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22 89 unchanged. A closer inspection at microscopic spray characteristics (such as droplet statistics) would
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24 90 highlight that if the setup is kept unchanged, even for a narrow range, some physical changes could not
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26 91 be captured. This was shown to have little to no impact on the overall spray behaviour but still shows
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28 92 the importance of accurate tuning. Thus, with question 2) being answered with a confident “no”, further
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30 93 research by Nsikane *et al* in (55) used DoE response models on reactive cases to identify the key
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32 94 simulations constants which were sensitive and/or robust to certain changes in boundary conditions.
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34 95 This led to the development of an input parameter tabulation approach. A more detailed analysis of the
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36 96 results of the tabulated simulations will be discussed in future work. Questions 1 and 3 remain
37
38 97 unanswered and are the main subject of the current paper.
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43 98 The objective of this paper is twofold. First, the question of validity of the **sub models** is answered by
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45 99 investigating whether the simulated conditions commonly used for model tuning are indeed classical
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47 100 evaporation cases rather than supercritical sprays (Question 1). Secondly, it elaborates the use of DoE
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49 101 to understand the behaviour of model constants at changing boundary conditions (Question 3). A novel
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51 102 methodology is **suggested where automated parameter optimisation and subsequent setup refinement**
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53 103 deliver a setup for each condition which produces good results for multiple comparison metrics like
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55 104 liquid and vapour penetration lengths, radial and axial mass fraction and temperature distributions. The
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57 105 subsequent analysis then investigates the used simulation constants and analysed in which way they had
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106 to change to match the experiment and, most importantly, whether their change accurately reflect the
 107 changes in the underlying physical processes triggered by the change of operating conditions.

108 2 Experimental Data used for Comparison

109 2.1 Selection of Experimental Data

110 For this work, a set of experiments commonly known as the ECN Spray A has been selected (see Table
 111 1). For ECN “Spray A”, a diesel surrogate, n-dodecane, is injected vertically through a single-hole
 112 injector into a quiescent combustion chamber. Much effort has been put in to characterise the
 113 specifications of the injector by the authors in (1-3) and has been summarized in Table 2.

114 Table 1: Selection of ECN Spray A parametric variations (56)

Key point	Charge Temp (K)	Charge Density (kg/m ³)	Charge Pressure (MPa)	Inj. Pressure (MPa)	Injector (#)	Mixing regime classification $T_r\sqrt{P_r}$
1	900	22.8	6.05	150	210677 (Sandia NL)	2.49
2			6.07	100		
3			50			
4	1100	15.2	4.96	150		2.76
5	1400	7.6	3.19			2.82

116 Table 2: ECN Spray A injector specifications (57)

Fuel injection equipment	
Common rail fuel injector	Bosch solenoid-activated, generation 2.2
Nominal nozzle outlet diameter	90 μm
Nozzle K factor	1.5
Nozzle shaping	Hydro-eroded
Mini-sac volume	0.2 mm ³
Discharge coefficient	$C_d = 0.86$, using 10 MPa pressure drop

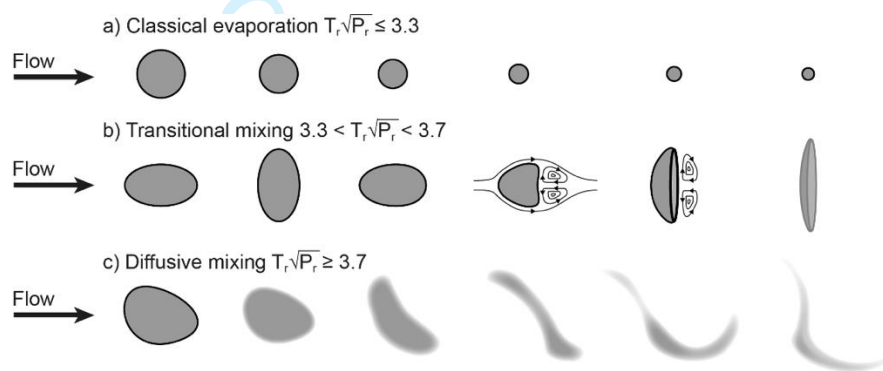
Spray full included angle	0° (single axial hole)
Common rail volume/length	22 cm ³ /23 cm *Use GM rail model 97303659
Distance from injector inlet to common rail	24 cm
Fuel pressure measurement	7 cm from injector inlet / 24 cm from nozzle
Approx. injector driver current	18 A for 0.45 ms ramp, 12 A for steady state
Fuel specifications	
Fuel	n-dodecane
Fuel temperature at nozzle	363 K (90°C)

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118 2.2 Droplet mixing regime of ECN Spray A

119 A shift of a liquid droplet from classical atomisation and vaporisation (subcritical) into the supercritical
 120 regime is characterised by the diminishing of droplet surface tension. This phenomenon occurs at a
 121 combination of high temperature and high pressures above the critical point of the fluid. In common
 122 diesel injection conditions, the liquid fuel all the way through the injector is subcritical. However, the
 123 ambient gas the liquid is injected into is typically in the supercritical regime of the fuel. As the fuel
 124 exits the nozzle, heat transfer processes will elevate the temperature of the fuel while simultaneously
 125 reducing the local temperature and pressure of the surrounding gas. There is still ambiguity among the
 126 scientific community as to whether a cool spray injected into a supercritical environment ultimately
 127 represents a supercritical spray or not. It is generally very difficult to observe the highly dynamic
 128 diffusive mixing (supercritical evaporation) process experimentally primarily due to technical
 129 limitation of the equipment (58-60). The absence of conclusive results has led to researchers try to
 130 identify supercritical characteristics of flows based on secondary evidence like macroscopic changes in
 131 the physical appearance of the plume (61-63). Crua *et al* in (64) eventually succeeded in capturing the
 132 droplet breakup and evaporation process and developed a conceptual model of the droplet mixing
 133 regimes shown in Figure 1. This work was based on a range of single component fuels injected through
 134 a single-hole injector into a quiescent vessel at various operating conditions. The authors show clear

135 evidence that despite the ambient conditions being supercritical, surface tension and primary
 136 atomization for n-dodecane can be observed, hence categorising the spray well within the ‘classical
 137 evaporation’ regime. They further show that even in cases where ambient conditions are so extreme
 138 that the fuel ultimately undergoes diffusive mixing, there is still a finite transition time which depends
 139 on local gas temperatures and pressures as well as on the fuel’s physical properties. This finding is of
 140 fundamental importance for this work because it justifies the use of the classical **sub models** which
 141 usually account for surface tension effects and would potentially not be valid for flows entering the
 142 “transitional mixing” regime or beyond. **Recent attempts of simulations of supercritical flows can be**
 143 **found for example by Chung *et al* in (65).**



144
 145 **Figure 1: Conceptual model of droplet mixing regimes (64)**

146 Despite the results from Crua *et al* in (64), some technical limitations to their approach are important to
 147 highlight. A visualisation of the individual droplets was only possible at the end of injection for
 148 relatively slow droplets and in an optically thin region of the spray. Physical processes within the core
 149 or the optically dense region of the spray remained unresolved. This is where CFD has the potential to
 150 offer some insight by assessing the temperature and pressure conditions in the optically dense region,
 151 so that a categorisation of the droplet mixing regimes can be attempted.

152 The ECN Spray A variations investigated here were also central conditions in the work by Crua *et al* in
 153 (64) where they developed a droplet mixing regime classification system (see Figure 2). The left image
 154 shows the nominal chamber pressure over charge temperature of a range of operating conditions.
 155 Following this, the axes were normalized by dividing the far-field values (T_g , P_g) by the fuel specific

critical temperature and pressure values T_c and P_c . A regression model with the best fit at $T_r\sqrt{P_r}$ allowed the automated classification seen on the right image. We have superimposed the key points selected for this work (see Table 1) onto Figure 2, which clearly shows that they would all clearly fall into the “classical evaporation” regime despite the ambient conditions being supercritical.

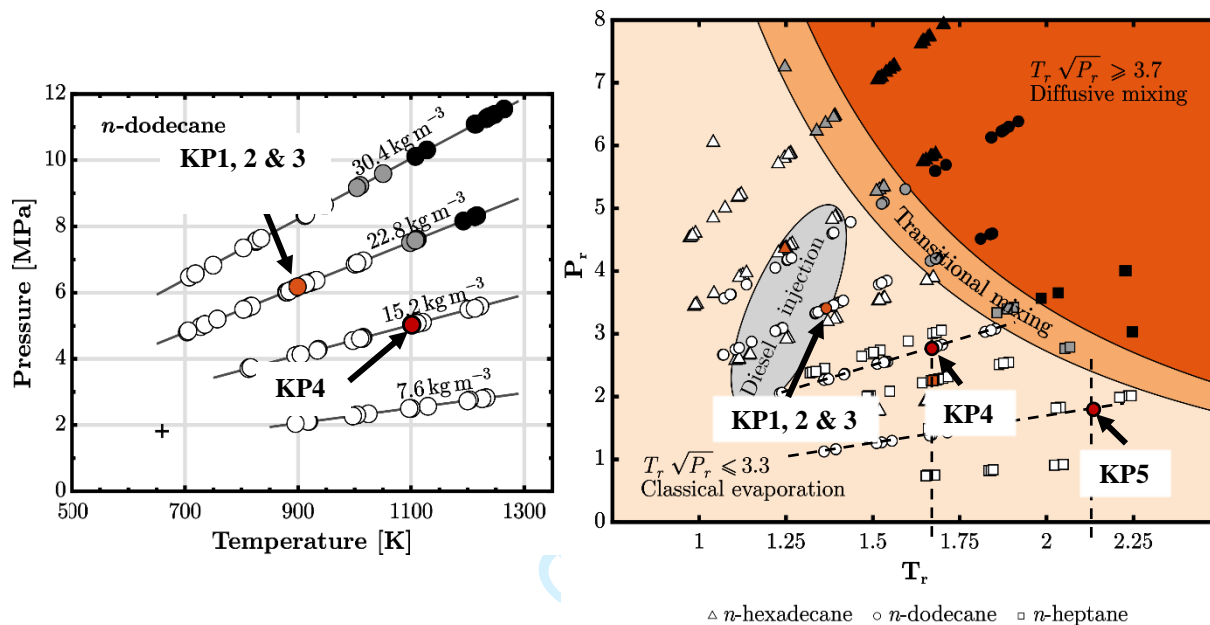


Figure 2: Gas pressure-temperature diagrams for n-dodecane (left). The left diagram is then projected onto a classification of mixing regime diagram on the right. Both P_r and T_r are calculated by dividing the imposed far-field (P_g, T_g) values by the critical values of the fuel (For n-dodecane $P_c = 18.2\text{bar}$, $T_c = 658\text{K}$). (Reproduced from (64), red symbols indicate the operating conditions examined in our work.)

3 Numerical Setup

The study is conducted with Ricardo Software’s commercially available CFD package VECTIS. It is a RANS based code with a long history of extensive industrial use for ICE’s and is well validated (66). The used sub-models are largely industry standard and can be also found in most other commercially available CFD solvers. This allows for the approach presented here to be applied on other CFD packages. Further, the same DoE optimization approach can also be taken in an LES framework.

The usual mesh and time-step independence studies were conducted to ensure model convergence. Independence is found for a mesh size of $0.45 \times 0.45\text{mm}$ ($\sim 1\,335\,096$ cells) and a time step of $5e-7\text{s}$ (53).

173 These settings can be carried over throughout the investigation. The selected **sub models** are listed in
 174 Table 3.

175 **Table 3: List of selected sub models**

Selected sub models	
Turbulence Model	Standard k- ϵ (67)
Spray Injection Method	Blob (Single size)
Droplet Tracking method	Eularian-Lagrangian
Droplet Breakup Model	KH-RT with Levich switching criterion (68, 69)
Droplet Drag Model	Putnam (70)
Droplet evaporation	Spalding correlation (71, 72)
Phase interaction	Droplet-droplet & droplet-turbulence (two-way coupling)

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177 **4 Design of Experiment**

178 Both in research and in industrial applications, experiments play a key role in

- 179 • Identifying the influence of input parameters on output parameters within a system
- 180 • Highlighting the sensitivity of the system towards changing conditions
- 181 • Finding a combination of input parameters which produce a desired output

182 When a system has too many influential and intertwined parameters to be unpicked in discrete
 183 investigations, a statistical approach to analyse the data can significantly reduce the burden. DoE is such
 184 an approach. In engine R&D, DoE is a common tool to visualise complex interactions and sensitivities
 185 in the system. It is important to note that DoE only highlights connections between independent (input)
 186 and dependent (response) variables but cannot give explanations to the fundamental processes. For a
 187 reliable connection between cause and effect to be made, a statistical relevant number of experiments
 188 must be conducted. The mathematical fundamentals of the simulation's sub models (see Appendix)
 189 show that the response of the CFD simulation relies on several user defined input variables. The DoE
 190 software used for this study is a Ricardo in-house tool called η Cal. In the following sections we will

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3 191 briefly describe the main steps followed in this work to build our DoE. The mathematical background
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5 192 of the η Cal tool is given in (73) and has been extended by Ricardo to deal with noisy data for engine
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7 193 test data. It should be underlined that part of the novelty of this work is that our effort is not only limited
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9 194 to identify links between independent (input) and dependent (response) variables as traditional DoE's
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11 195 do but also to unveil a physical explanation of these trends.

15 196 4.1 Simulation Design Matrix

17 197 *Input parameters*

21 198 A screening and selection process of the available sub models and their user definable parameters
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23 199 yielded a selection of 10 influential parameters. For the DoE approach to be considered statistically
24
25 200 relevant, 10 simulations per input parameter are required, hence 100 simulations for every DoE key
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27 201 point. Each of these runs has an input parameter combination which is defined by the software to ensure
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29 202 they are evenly distributed across the design space (stochastic process). The ranges in which the
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31 203 parameters vary were defined following the recommendations by the original authors and the VECTIS
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33 204 documentation. The DoE parameters, their range and phenomenological significance is collected in
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35 205 Table 4. Previous work (see (53)) showed how in these cases, an adjustment of C_2 was selected to
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37 206 achieve good results. In our more recent studies, both C_1 and C_2 are considered as DoE variables. The
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39 207 initial droplet diameter and the half cone angle are not part of any model, are however typically
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41 208 unknown and treated as simulation parameters. Although the coefficients are grouped, it is important to
42
43 209 realise that they are intertwined i.e. an initial condition like the droplet size will influence the mixing
44
45 210 and combustion.

49 211 **Table 4: Selected simulation constants and their physical implication**

Parameter	Range	Phenomenon	Group
Coefficient of Dissipation C_2 (-)	1.65 – 1.9	Destruction of Turbulence	Turbulence Coefficients
Drag scaling factor A_{drag} (-)	0.2 – 1.5	Liquid/Gas Momentum Transfer	
KH B_1 – Constant (-)	1 – 40	Primary Atomization	

KH B_0 – Constant (-)	0.3 – 0.8	Primary Atomization	Droplet Breakup Coefficients
RT C_{RT} – Constant (-)	0.3 – 2	Secondary Atomization	
RT - C_3 – Constant (-)	0.3 – 5.3	Secondary Atomization	
Levich A_{bu} – Constant (-)	5 – 12	Primary/Secondary Atomization	
Initial droplet diameter D_0 (μm)	60 – 90	Droplet Introduction	Initial conditions
Initial Half Cone Angle α_{cone} (deg)	2.5 – 7.5	Initial Dispersion	

212

213 *Response parameters*

214 To assess the quality of a simulation, the root-mean-square-error (RMSE) between the response
 215 parameter (like vapour penetration etc.) and the experimental data from the ECN is calculated. This
 216 allows a quantification of the similarity between the experimental and calculated curve progressions
 217 and provides the input for the stochastic process model (SPM). To avoid skewed results, extreme
 218 transients in the curves are avoided (i.e. ROI ramp up/down). The approach is described in Table 5. **The**
 219 **response parameters in this work are the liquid spray and vapor jet penetrations. The mass fraction**
 220 **distributions were initially also considered as target metrics, but due to the optimizer only allowing a**
 221 **limited number of target parameters, it was decided to use these data sets as secondary validation**
 222 **metrics. Once reliable local droplet size statistics of the dense spray become available, they could be**
 223 **added as target metric to calibrate the response to microscopic spray characteristics.**

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Table 5: Mathematical background for RMSE approach

Case	Time step	Metric Value	No. of time steps
Experiment	t	$x_{1,t}$	n_t
Simulation		$x_{2,t}$	

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n_t} (x_{1,t} - x_{2,t})^2}{n_t}} \quad \text{Eq 1}$$

225

4.2 DoE Optimisation

There are too many parameter combinations which could lead to a matching solution to be analysed manually. To narrow down viable solutions, a built-in optimizer is equipped with user defined target conditions is used. The optimizer creates a pareto diagram and compiles a list of combinations of input parameters which fulfil the target condition. Since there is not a best solution for these criteria, the optimizer will provide multiple solutions (~15 options). To further narrow down the number of solutions non-physical combinations are excluded. The non-physical combinations are considered to be those where multiple values of the DoE constants are gathered at the periphery of their range. A matrix with a single value at the extreme end of a range is still considered. The remaining few solutions (~5 options) are scrutinized by investigating the mass fraction and gas temperature distributions, the microscopic characteristics like droplet sizes, their transient regions of injection ramp up & down and how they behave to changing boundary conditions. A quality criterion would for example be that at discrete injection pressure increase (with unchanged chamber conditions) a simulation setup at each condition can be found that together exhibit a sweep in values which are related to an injection pressure swing.

A final refinement using the Stochastic Process Model (SPM) can then guide some minor single parameter adjustments to arrive at the best setup. Due to the uncertainty of the SPM it is possible that the optimiser offers a theoretically optimal solution (small RMSE's) that when simulated show shortcomings. Owing to the steep gradients of the RMSE sensitivity of some key simulation constants, the settings may need some adjustment slightly to produce good results. Since both results for both setups will be presented later, they will be referred to as "DoE Setup" and "Refined Setup".

This single parameter adjustment using the graphical representation of the SPM differs from the commonly used single parameter swing method criticised in the introduction in a way that will be described in detail in section 5.2. In short, using the graphical SPM interface, one can change the value of any simulation constant and observe the impact it would have on the liquid or vapor RMSE. It also shows the how sensitivities of all other constants change and whether any additional adjustments would become necessary to reduce the RMSE further.

4.3 Computational effort

The average duration per simulation over a 4ms injection duration on 20 cores Intel(R) Xeon(R) CPU E5-2650 v3 CPUs with 2.30GHz in this work is 2hrs. Other high quality simulations with which include detailed or reduced chemistry solvers, LES approaches for turbulence modelling and finer grids have shown to have runtimes that are higher by one, or in some cases two, orders of magnitudes (36, 74). This brief comparison highlights the potential time benefit of using simplified models over detailed solvers, though intelligent tuning becomes of utmost importance.

5 Results and Discussion

5.1 Microscopic Analysis of Baseline Spray A

Before we demonstrate the accuracy of the CFD calculations on macroscopic metrics, we must get a better insight into the droplet behaviour. It should be pointed out that in some cases a physically “wrong” model with extreme coefficient tuning might be able to reproduce some of the experimental results. In order to establish that this is not the case here we include the following results. These results are complimentary to the observations of section 2.2 and are used to show why the simulated conditions are indeed subcritical.

The left side of Figure 3 shows contour plots of charge temperature of the baseline (KP1) from the start of injection up to the steady state phase (liquid penetration stabilising) in 0.1ms increments. It also shows a qualitative representation of the droplet mean diameter and a quantitative contour plot of the droplet temperatures. Just after the start of injection (first row, 0.05 ASOI), the liquid jet of KP 1 starts reducing the charge temperature around 5mm downstream the nozzle. The droplets in this area begin to rapidly heat up by absorbing thermal energy from the surrounding air leading to their evaporation. Within this low temperature zone, some following droplets begin to coalesce instead of evaporating. These large droplets then penetrate through the surrounding air while evaporating downstream relatively slowly. Despite these cases being under evaporating conditions, some parallels can be drawn to the process described by Magnotti *et al* in (75) where under non-evaporating conditions shortly after

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3 277 injection some degree of droplet coalescence was observed. We assume that once the cooled initial
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5 278 stagnant air is removed by liquid jet induced turbulence, the low temperature zone stabilizes at higher
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7 279 temperatures preventing any further coalescing. As of 0.3ms ASOI, the steady state is established, and
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9 280 the droplets complete their breakup and evaporation process around 10mm downstream the nozzle.
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11 281 These images indicate that due to high droplet velocities, the droplets only begin to show considerable
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13 282 heating around 4mm downstream presumably due to thermal inertia. This is the first important point
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15 283 supporting the conclusions by Crua *et al* in (64) that essentially only droplets further downstream might
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17 284 reach to critical temperatures. A further investigation into the development of the temperature
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19 285 conditions of the gas phase around the spray injection region is shown on the right side of Figure 3. The
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21 286 radial temperature distributions of incremental slices at given time steps are plotted as a wireframe. The
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23 287 temperature is normalised with n-dodecane's critical temperature to match the characterization
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25 288 previously shown in Figure 2. The significance of this graph is that we can observe a local cooling along
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27 289 the centre axis leading to a drop of T_r . The consequence is effectively a shift to the left of the location
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29 290 of the key point in Figure 2 for the transient phase of the injection process. These findings also apply
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31 291 for all other invested key points.
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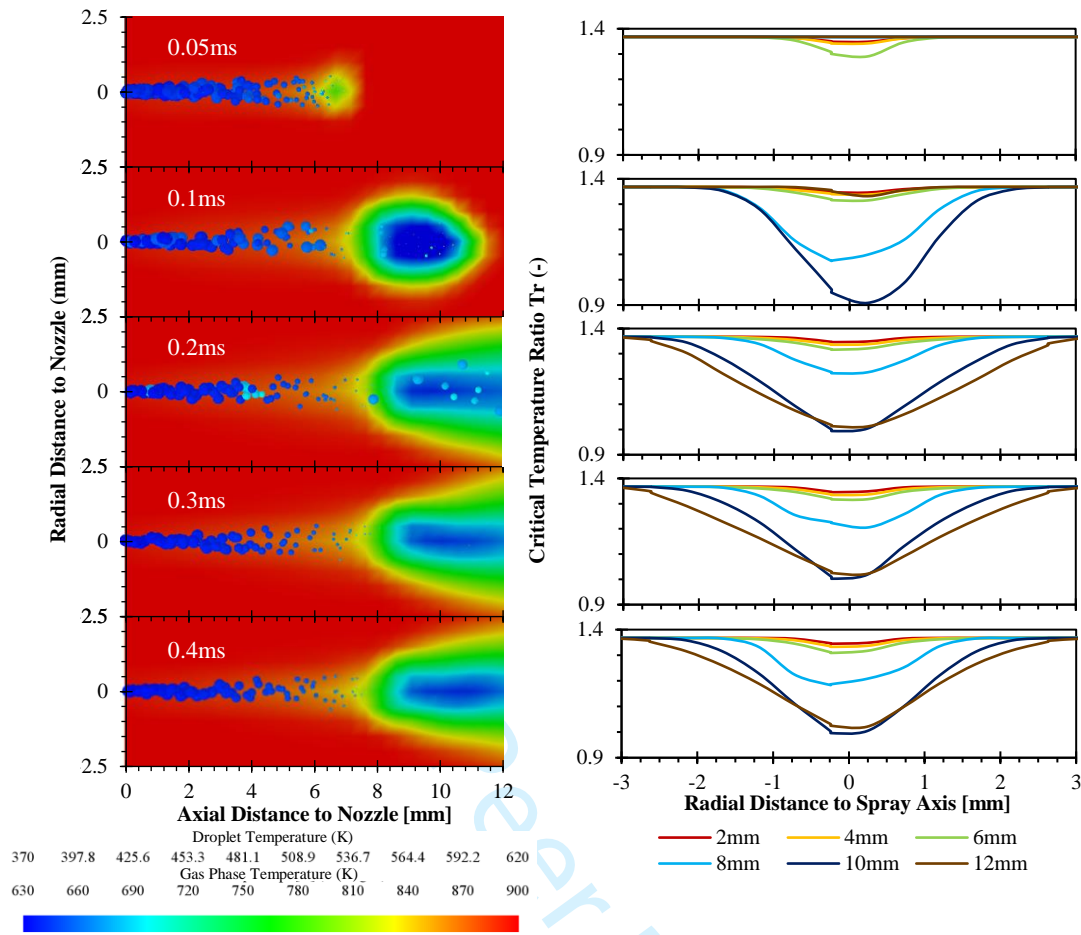


Figure 3: Charge temperature, droplet sizes (scaled qualitatively) and droplet temperatures (colour scale) at various time steps (left) and the corresponding radial critical temperature ratio distributions at six axial locations for the baseline ECN Spray A (KP1) (right)

5.2 DoE & Stochastic process model results

With the spray being placed well within the conventional evaporation regime it is justified to continue our approach with conventional spray models. The process outlined in section 4 is followed for all spray cases (KPs 1-5). At each key point, 100 simulations are run. Following the optimisation and manual refinement, a setup for each condition was found to match the experimental liquid and vapour penetration. The reasons for the refinement of the optimised solution are due to the uncertainty in the SPM and has been described in section 4.2.

Figure 4 shows the SPM's of all five key points at their refined setup. The significance of the constants on the x-axis are explained in Table 4. Each field in the rows represent the RMSE between the experiments and simulations (see Eq 1) as a function of the parameter in the column. The x-axes with

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3 306 the constant value in the column have been removed for confidentiality reason, which however does
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5 307 not obstruct the qualitative nature of the graph. The gradient represents the sensitivity of the RMSE
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7 308 towards a change of that parameter. The dotted lines represent 2σ confidence of the prediction.
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10 309 In Figure 4, the coefficient of dissipation C_2 shows to have a significant impact on the vapour
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12 310 penetration and only little on the liquid penetration. This makes C_2 the single most influential parameter
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14 311 to adjust the vapour penetration. This constant also shows a clear minimum which indicates that there
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16 312 is only a small range in which it may vary. The axial location of this minimum, which represents the
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18 313 constant's value, does not vary with operating condition.
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21
22 314 The drag scaling coefficient A_{drag} which influences the liquid/gas momentum transfer (Eq 4) is crucial
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24 315 for liquid and to some extent vapor penetration. For key points 1 – 4, we assume there is a minimum
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26 316 beyond the investigated range, leaving merit to extend the ranges in future work. However, not
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28 317 including the minima's does not hinder the results because the trade-off required to keep vapor RMSE
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30 318 low requires selecting a value within the range. The steep inclinations of the RMSE curves and different
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32 319 value for each condition highlight the condition sensitivity of this parameter. Generally, a reduction of
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34 320 RMSE sensitivity with decreasing density can be observed (compare absolute maximum liquid RMSE
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36 321 between KP 1, 4 & 5). Additionally, key point 5 (1400K, 7.6kg/m³, 150MPa) shows a clear increase
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38 322 for required absolute parameter value.
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42 323 The $KH - B_1$ constant, which influences the primary breakup time scale (Eq 7, appendix) is also
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44 324 paramount for liquid length calculations. However, unlike the characteristics of the drag scaling
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46 325 coefficient, $KH - B_1$ shows its minimum at approximately the same axial location and similar absolute
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48 326 RMSE sensitive at all key points. This means that although $KH - B_1$ is an influential parameter, the
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50 327 variations of the absolute value between conditions are small and therefore justify the parameter to be
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52 328 held constant across key points. In most cases (except KP 2), the minimum found for liquid RMSE
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54 329 appears to be an acceptable value for the vapor RMSE. The reasons for KP 2's deviation from this
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56 330 pattern are unclear at this point.
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3 331 The impact of the remaining coefficients remains small relative to the above. Changes of their value
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5 332 have little influence on the absolute RMSE of liquid and vapor penetration. This is not to say that they
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7 333 are not important as they do affect the microscopic characteristic of the spray plume. For example, work
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9 334 done in context of initial trialling of the tabulation on reactive cases has shown initial droplet sizes and
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11 335 the RT C_3 – constant to be influential for secondary droplet sizes which influence combustion
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13 336 characteristics for combusting cases. Given some quantitative droplet size measurements at any location
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15 337 of the spray plume, an additional target metric for the DoE could increase accuracy of the response of
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17 338 the simulation constants that are more influential for microscopic spray characteristics. While not
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19 339 influential here, the turbulent Schmidt number has shown to become more influential under realistic
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21 340 engine conditions with swirl motion and fuel injection through a multi hole injector.
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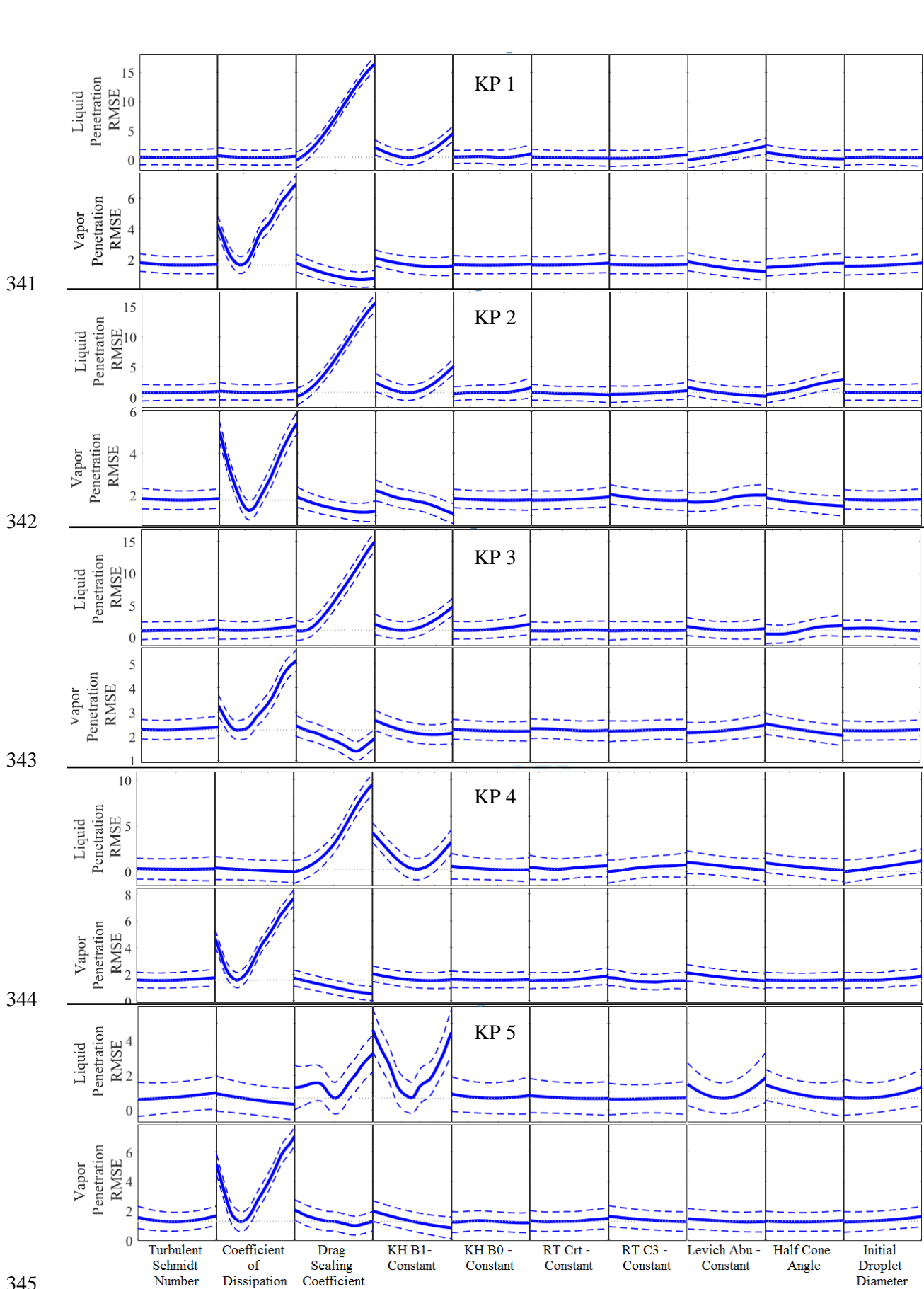


Figure 4: Graphical representation of the stochastic process model (SPM) that highlights sensitivities of the result on a change of parameter value for all key points (from KP 1 at the top to KP 5 at the bottom)

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5.3 Spray evaporation and mixture formation under inert conditions

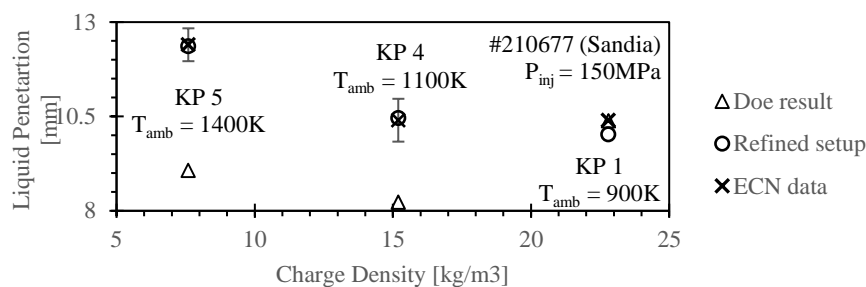
5.3.1 *Results of Design-of-Experiment approach on mixture formation under constant injection pressure and changing chamber conditions*

In this section we turn our attention to the key points (KP 4 & 5) which differ both in charge density and temperature from the baseline (KP 1). The experimental data shown in Figures 5 and 6 (hollow symbols) are not directly comparable as both density and temperature change simultaneously. Nevertheless, some interesting observations can be made from the progression of liquid and vapour penetration. In the case of an isolated charge temperature increase, it can be assumed the liquid penetration would decrease with increasing charge temperature due to increased droplet evaporation. Vice-versa, an isolated decrease of chamber density would increase liquid penetration (76). When these two effects happen simultaneously, the effects partially counteract each other until one of the effects becomes dominant.

The authors see strong indication of this phenomena occurring in the cases shown in Figure 5. As the charge temperature increases and charge density inversely decreases from KP 1 to 4, the liquid penetration slightly decreases. This decrease is potentially due to the change in the evaporation rate. When this progression continues to KP 5, the liquid penetration increases significantly (see Figure 6). The authors suggest that while between KP1 and 4 the temperature influence is slightly stronger, density effects become dominant between KP 4 and 5. The vapour penetration shows a clear sensitivity to reducing chamber density, presumably due to reduced aerodynamic resistance and subsequent dissipation. An isolated temperature increase under constant density is not thought to have much effect on the vapour motion. For KP 1, the ECN baseline at 900K, 22,8kg/m³ and 150MPa, no experimental error in the liquid penetration is stated, however is not expected to be significantly different to other four cases.

Figure 5 shows a comparison between the DoE optimised setup, the manually refined setup and the ECN test data of liquid penetration length over charge density. Figure 5 shows that the settings from

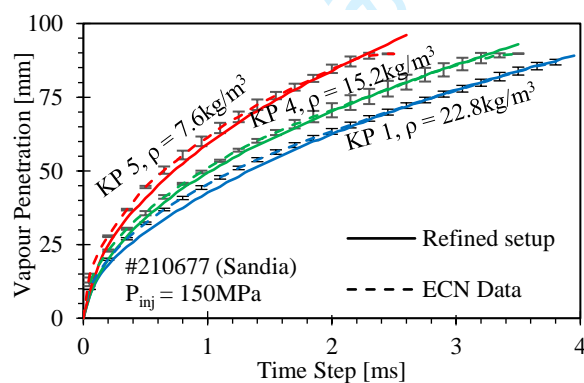
374 the “Refined setup” produce average liquid lengths within the supplied error bands of the experimental
 375 data.



376

377 **Figure 5: Simulation vs experimental data of liquid penetration of three different charge temperature and density**
 378 **conditions at constant injection pressure**

379 The overall smaller RMSE range (see Figure 4) for vapor penetration means the setup is more robust to
 380 changes of even the most sensitive constant. The effect on vapor penetration is not pronounced therefore
 381 Figure 6 only shows the results for the “Refined setup”. The temporal spray tip evolution is well
 382 captured at all three conditions, however show some merit to improvement in the time between 0.5 and
 383 1.5ms.

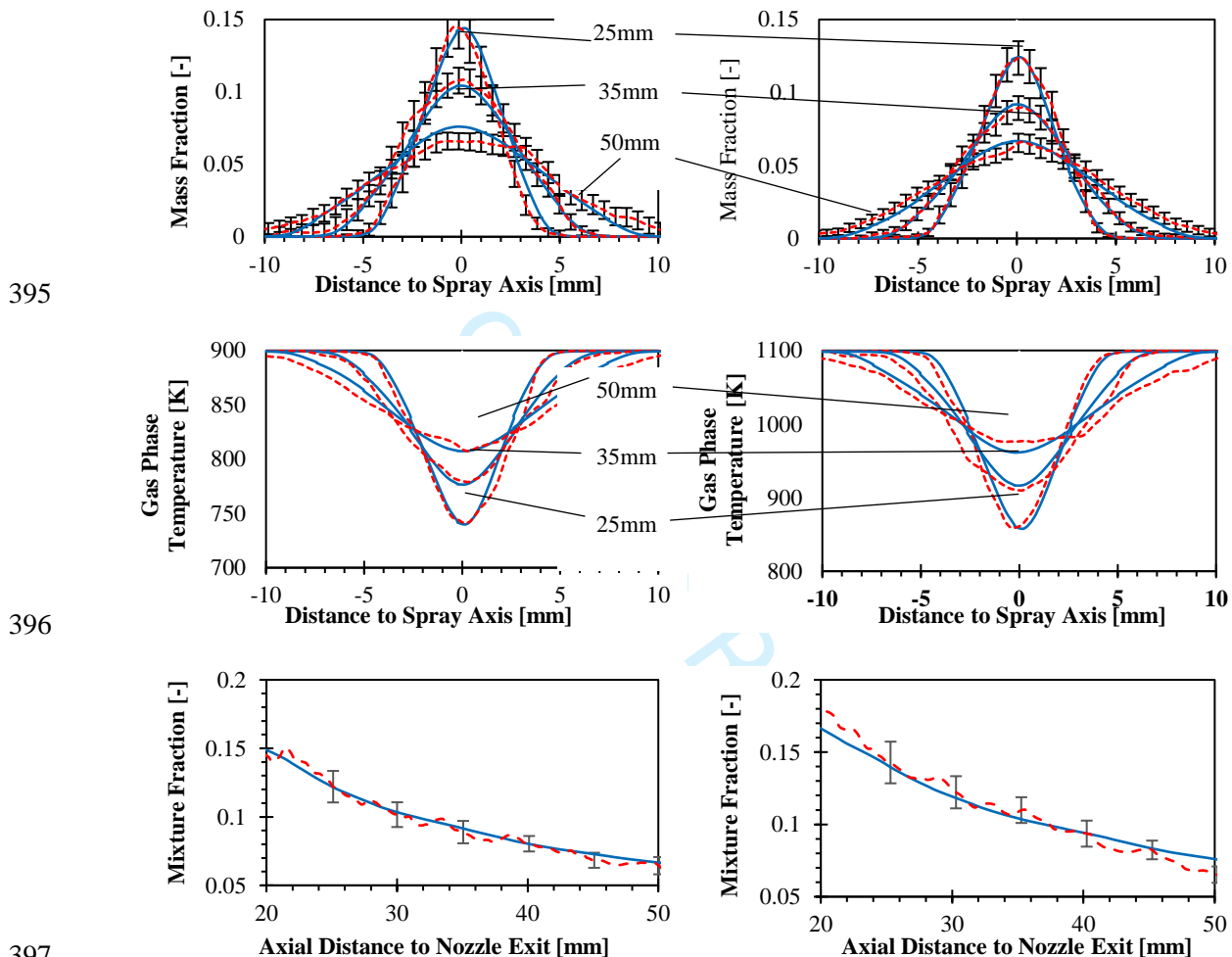


384

385 **Figure 6: Simulation vs experimental data of vapour penetration of three different charge temperature and density**
 386 **conditions at constant injection pressure**

387 To increase confidence in the simulated mixture preparation, the radial mass fraction and gas phase
 388 temperature distributions at three plume cross sections and axial mass fraction along the centreline of
 389 the plume are compared to available experimental data captured at a steady state time interval. The
 390 results are shown in Figure 7. In all metrics, the simulations perform well and, where available, lie

391 within the stated experimental error. No mass fraction and temperature measurements were taken for
 392 the 1400K key point (KP 5) because of increasing measurement uncertainties due to the experimentally
 393 challenging in-cylinder conditions. Where data is available, all metrics of the simulated data lie within
 394 or very close to the experimental error.



398 **Figure 7: Comparison of radial mass fraction and gas phase temperature distribution at 25, 35 and 50mm at 4ms (1st**
 399 **and 2nd row) and centre axis mass fraction distribution at 3.2ms (3rd row) between ECN test data (77) (red, dotted)**
 400 **and simulation (blue, solid) (KP 1, left column and KP4, right column)**

401 The analysis of the simulation setups required to match test data for these three conditions is
 402 summarised qualitatively in Figure 8. Upward or downward facing arrows signify a drop or an increase
 403 of the parameter value in comparison to the selected reference condition (KP 1). The symbol shown in
 404 bold means that the change is significant. A point indicates that the parameters remain unchanged. In
 405 Figure 4 the turbulence coefficient C_2 and primary breakup time-scale B_1 showed that they were

generally highly sensitive, but once the correct value is found, they may remain constant for all cases. The drag scaling factor A_{drag} was found to increase with decreasing density, resulting in less of the droplet momentum being passed onto the vapour phase. This increase is physically justifiable with a decrease in aerodynamic effects that occur at lower gas densities. Given that A_{drag} indicates the deviation of the droplets from sphericity, we can conclude that the need to increase A_{drag} with decreasing density would indicate that generated droplets are more spherically shaped rather than ellipsoid. In fact, for KP 5, where density is lowest, A_{drag} even approached unity while being far smaller for the other two conditions.

KP	A_{drag}	B_1	C_2	Droplet Diameter
1	Reference key point			
4	↑	•	•	↓
5	↑	•	•	↓

↑ value increase
 ↓ value decrease
 • near stagnant
Bold significant change

Figure 8: Identified relative adjustments for the main tuning coefficients between the key points 1, 4 and 5 (increasing charge temperatures/decreasing charge densities)

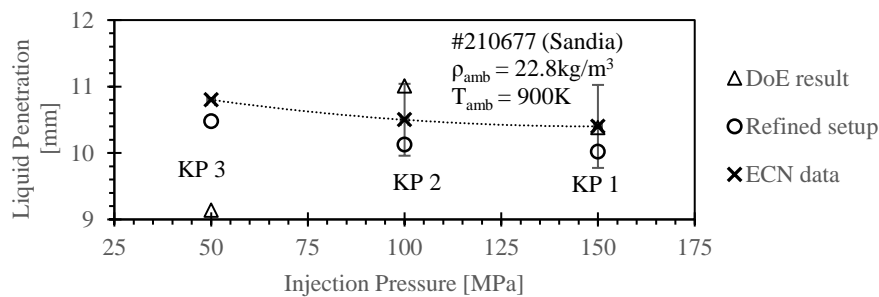
5.3.2 Results of Design-of-Experiment approach under changing injection pressure and constant chamber conditions

The experimental data shows that while the liquid penetration (see Figure 9) only slightly decreases with increasing injection pressure, the vapour penetration (see Figure 10) rises significantly, deeming the liquid/gas phase momentum transfer an influential process. The macroscopic liquid length is thought to be predominantly affected by air entrainment (or turbulent mixing)(4, 76). As the injection rate increases, so does the turbulent mixing, which continues to deliver energy that can break up the droplets around the same axial location.

The microscopic processes of the break up however do change with increasing injection pressures. Crua *et al* in (78) shows that the initial stages of injection are different between injection pressures. It is shown that at low injection pressures, surface tension is strong which allows slow but large droplet ligaments to be introduced. At higher injection pressures, break up forces exceed surface tension and

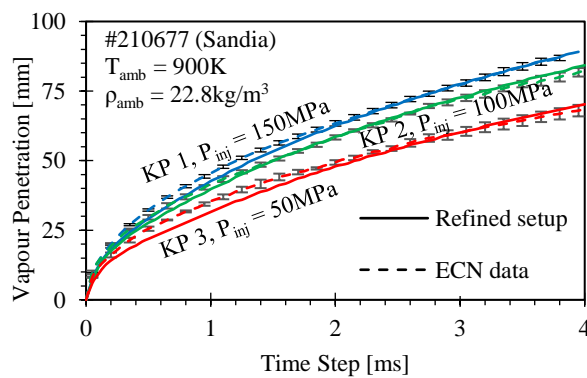
429 fast and small droplet-like ligaments exit the nozzle. Since the injection rate and velocities are reduced
 430 at lower pressures, less inertia is passed onto the vapor phase resulting in their slower progression.

431 Like in section 5.3.1, Figure 9 shows the comparison between the liquid penetrations of the raw DoE
 432 optimised setup, the refined setup and the ECN test data. The reasons for the difference between the
 433 DoE optimised setup and the refined setup have been described in the previous sections and apply here
 434 as well. The liquid penetration of the refined setup shows good average liquid penetrations for all three
 435 conditions and a comparable response of the absolute value to increasing injection pressures.



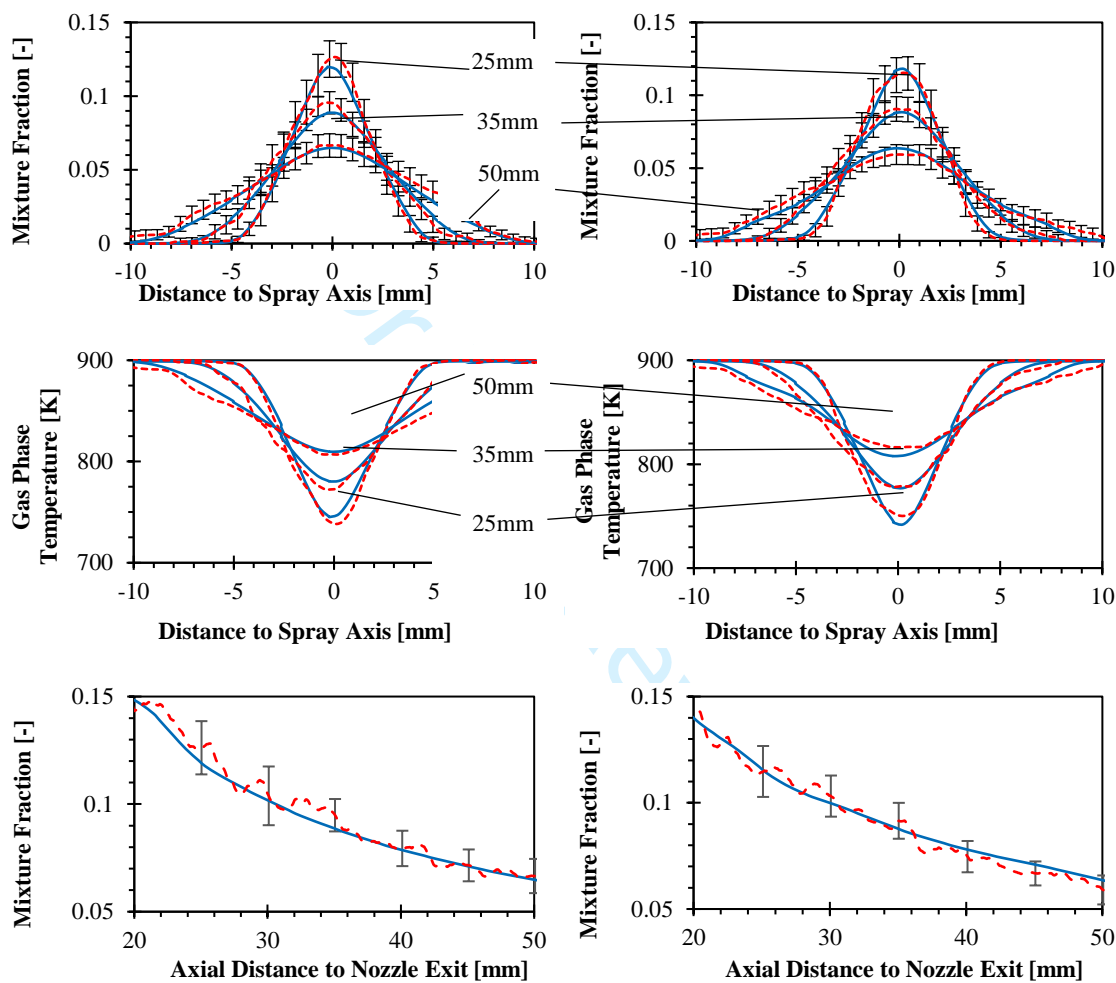
436
 437 **Figure 9: Simulated and experimental liquid penetration over injection pressure of KPs 1 – 3 under constant charge**
 438 **density and temperature conditions**

439 The refined setups vapor penetration (see Figure 10) of key points 1 and 2 are well captured throughout.
 440 It proved to be difficult to appropriately adjust the vapor penetration of KP 5. This indicates that there
 441 is merit for some further investigation into the gas/liquid momentum transfer at lower injection
 442 pressures in future work.



443
 444 **Figure 10: Simulated vs experimental vapour penetration of three injection pressure cases (KP 1 – 3) over the**
 445 **duration of 4ms under constant charge density and temperature conditions**

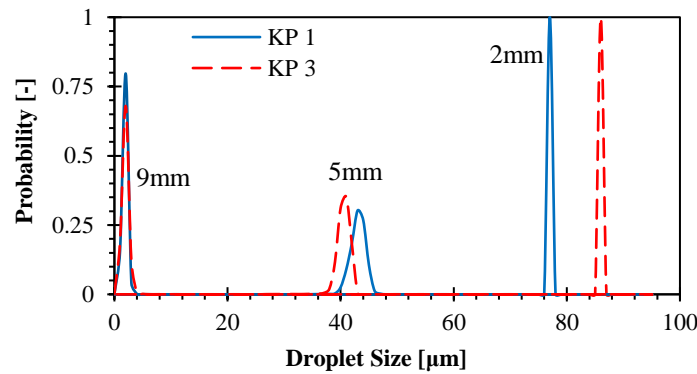
446 To increase confidence in the simulated mixture preparation, the radial mass fraction and gas phase
 447 temperature distributions at three plume cross sections and axial mass fraction along the centreline of
 448 the plume are compared to available experimental data captured at a steady state time interval. The
 449 results are shown in Figure 11. In all metrics, the simulations perform well and, where available, lie
 450 within the stated experimental error.



454 **Figure 11: Comparison of radial mass fraction and gas phase temperature distribution at 25, 35 and 50mm at 4ms**
 455 **(1st and 2nd row) and centre axis mass fraction distribution at 3.2ms (3rd row) between ECN test data (77) (red,**
 456 **dotted) and simulation (blue, solid) (KP2, left column and KP3, right column)**

457 Figure 12 shows the accumulated droplet size probability distribution function (PDF) at three locations
 458 in the spray core across the simulated injection duration. It shows that the droplet shrinking of KP1 is
 459 initially significantly slower than for KP3 but is completed at approximately the same liquid length.
 460 This counterintuitive behaviour can be explained by the finite time-scales. The droplets from KP1 have

461 a larger velocity and reach the 5mm monitoring slice before the ambient conditions could have a
 462 significant effect on them. Between the 5 and 9mm slice, these droplets are broken up and evaporated
 463 rapidly. This stands in contrast to the droplet progression of KP 3, where the slow but large blobs are
 464 continuously shedding mass predominantly through evaporation. The shrinking process continues at a
 465 similar pace all the way through to the final droplet breakup.



466
 467 **Figure 12: Droplet size probability distribution of KP1 (150MPa) vs KP3 (50MPa) at 2, 5 and 9mm slices**

468 The dominant simulation constants are the turbulence coefficient C_2 , the droplet deformation in form
 469 of the drag scaling factor A_{drag} and primary break-up time-scale B_1 . Like the key points discussed in
 470 section 5.3.1, the turbulence coefficient C_2 and primary break-up time-scale B_1 require no adjustment
 471 between conditions. The A_{drag} coefficient tends to decrease with increasing injection pressures. This
 472 combined with increased droplet sizes indicate that at lower injection pressures larger and more
 473 spherical droplets are injected while the opposite applies at higher injection pressures. This is in line
 474 with the physical processes described in this section.

KP	A_{drag}	B_1	C_2	Droplet Diameter	
1	Reference key point				
2	↑	•	•	↑	↑ value increase ↓ value decrease • near stagnant Bold significant change
3	↑	•	•	↑	

475
 476 **Figure 13: Identified relative adjustments for the main tuning coefficients between the key points 1, 2 and 3**
 477 **(decreasing injection pressure)**

6 Conclusions

Reliable simulations for in-cylinder processes are important to support adoption of increasingly digitalised development processes in the automotive industry. Various virtual engineering tools exist to reduce the dependency on expensive prototyping and testbed iterations, however still struggle either with physical accuracy (RANS) or with high computational effort (LES & DNS). With increasing computational power, approaches like DNS and LES have become more affordable, but only show their superiority over RANS in microscopic and simplified research-oriented environments or development of more radical research concepts.

This work is dedicated to addressing RANS' main weakness, its heavy tuning dependency, while preserving its main strength, the relative computational simplicity. It is shown that with appropriate "clever" tuning, RANS can deliver excellent results for industrially relevant metrics like vapour and liquid penetration as well as species mass fraction and temperatures in a time efficient manner which is of paramount importance when the design of real devices is under consideration. A novel methodology is presented where a DoE approach and subsequent automated optimisation have led to find simulation setups, which can match multiple metrics of interest at five varying ECN Spray A operating conditions in ~2hrs per simulation. To the best of our knowledge, this is the first time that this reverse engineering approach was conducted by running 100 simulations at related key points, defining the RMSE of each of the simulation towards experimental comparison data and then using an automated optimiser to minimise the error and show which combination of constants would produce a matching simulation. This approach can only overcome the status of curve fitting by investigating the used simulation constants along with the respective physical conditions as a coherent picture. Like this, the following findings were deduced:

1. The constant of dissipation C_2 in the standard $k-\epsilon$ model, the drag scaling coefficient A_{drag} , the primary breakup time scale B_1 and the initial droplet sizes are the key tuning parameters. It was required to tune these to match vapour and liquid penetration as well as radial and axial distributions of mass fraction and gas temperature at all conditions.

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3 504 2. The stochastic process models of all conditions confirm the known sensitivity of liquid and
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5 505 vapor penetration error towards a change of C_2 and B_1 . The novel finding here however is that
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7 506 the distinct minima's, which represent lower RMSE's, settle for the similar value regardless of
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9 507 boundary condition. This suggests that although the simulations are highly sensitive to these
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11 508 two parameters, once their optimal value is found, they may remain unchanged for further
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13 509 operating conditions.
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16 510 3. The drag scaling factor A_{drag} , which is crucial for liquid/gas phase momentum transfer, is highly
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18 511 sensitive towards changing chamber operating conditions, especially charge density and
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20 512 injection pressures.
- 21
22 513 4. The initial droplet sizes are a key simulation parameter and are highly sensitive towards
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24 514 injection pressures and charge densities.

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27 515 The findings in the points above allow for the questions defined in the introduction to be addressed.

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30 516 1. A study of recent literature suggests that liquid fuel may still be treated with classical
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32 517 evaporation equations even if the ambient conditions fall under the supercritical regime. The
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34 518 reason for this is the thermal inertia and rapid disintegration of droplets which breakup and
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36 519 evaporate the droplets before they reach supercritical conditions. This would solidify the claim
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38 520 that the conditions in this work are indeed subcritical and therefore permit the used traditional
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40 521 sub models.
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43 522 2. Previous work and our approach here covered a large range of combinations of simulation
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45 523 constants in a selected set of sub models and we found no indication that the various conditions
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47 524 could have a potential simulation setup in common. Highlighting that the ECN Spray A is a
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49 525 simplified spray injection case, we assume it would only make it even more unlikely to find a
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51 526 single setup to match various real diesel injection conditions.
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54 527 3. However, by comparing the DoE-derived simulation setups, we did identify some simulation
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56 528 constants which were robust to changing boundary conditions and others that had to be altered
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58 529 to match the condition. Most importantly, the sensitivity or robustness of the value of a constant
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60 530 could be traced back to its original physical expression.

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3 531 4. Point 3 indicates that there is then the potential to pre-define the simulation constants based on
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5 532 the prevailing boundary conditions.
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8 533 The study shown here forms the basis to a subsequent investigation using these settings to develop the
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10 534 tabulation for ~30 reacting Spray A variations that will be presented in a future publication. Our most
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12 535 recent efforts on an optical single cylinder and a standard production engine (both small-bore direct
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14 536 injected light duty Diesel engines with swirl and multi hole injectors) using the tabulation only required
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16 537 small changes in the turbulence coefficients to show good agreement with a range of experimental data
17
18 538 (79). A long-term objective of the project is the adoption of the tabulation into an algorithm that can
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20 539 auto-tune simulations based on input boundary conditions.
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22

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25
26
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34
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36
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743 **Appendix**744 **Mathematical Background of Sub Models**

745 The following is an overview of the governing equations for an individual droplet moving in a carrier
746 fluid. The two-phase flow is considered a dispersed liquid phase in the continuous gas phase based on
747 the Lagrangian approach

748 ***Turbulence Model***

749 The turbulent motion in this work is modelled using the Standard k-ε turbulence model. A full
750 description of all terms would be lengthy but can be found in Jones *et al* in (67). Here we focus on the
751 two terms which had a major impact on the quality of the simulation. The terms C₁ and C₂ are responsible
752 for scaling the dissipation rate of the turbulent kinetic energy. A higher C₂ increases dissipating effects
753 and consequently increases the diffusion of the gas phase. The transport equation for the turbulent
754 kinetic energy (Eq 2) and its dissipation rate (Eq 3) in the standard k-ε model are:

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho U_i k)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\mu' \frac{\partial k}{\partial x_i} \right] + G - \rho \epsilon \quad \text{Eq 2}$$

$$\frac{\partial(\rho\epsilon)}{\partial t} + \frac{\partial(\rho U_i \epsilon)}{\partial x_i} = \frac{\partial}{\partial x_i} \left[\mu' \frac{\partial \epsilon}{\partial x_i} \right] + \frac{\epsilon}{k} \left(C_1 G - C_2 \rho \epsilon + C_3 \rho k \frac{\partial U_i}{\partial x_i} \right) \quad \text{Eq 3}$$

755 Initial trials varying the turbulence dissipation coefficients showed a strong sensitivity of vapour
756 penetration. For this reason, they have been selected for closer investigation.

757 *Droplet introduction*

758 The droplets are introduced as a chain of spherical blobs that are grouped in parcels of droplets with
759 similar attributes and as such treated with the underlying equations. The diameters of these initial
760 droplets can either be defined by the user or left to be calculated by various initial droplet size
761 correlations. In this work, the authors have selected a user defined droplet introduction typically known
762 as “Table introduction”. This list of droplet sizes vs probability is flexible and introduces no new
763 variables. The disadvantage is that it does not consider any nozzle flow characteristics or charge
764 conditions. To further simplify the droplet introduction, only a single droplet size is introduced.
765 Multiple impact studies conducted throughout the study showed that there was no apparent benefit of
766 applying more complex droplet introduction methods and distributions.

767 *Momentum conservation*

768 The momentum equation for a droplet of mass m_d is described by Newton’s Second Law (Eq 4) in
769 which C_d is the aerodynamic drag coefficient, A_f is the projected area of the droplet in moving direction,
770 A_{drag} is a user defined tuning coefficient, ρ_g is the density of the surrounding gas and the relative
771 velocities between the droplets and the gas \vec{U} . This momentum contribution is then added into the energy
772 and momentum conservation equations as a source term. The initial screening of simulations constants
773 highlighted A_{drag} as highly influential, so it has been added to the list of coefficients to be investigated
774 with more detail.

$$m_d \frac{d\vec{V}}{dt} = \frac{1}{2} C_d A_f A_{drag} \rho_g |\vec{U}| \vec{U} \quad \text{Eq 4}$$

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3 775 The drag coefficient C_d is calculated by the Putnam model, which is expressed as shown in Eq 5. The
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5 776 model defines the C_d to be that of a sphere for the case the droplet Reynolds number are >1000 . Based
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7 777 on this, it has been hypothesized that a value of $0 < A_{drag} \leq 1$ in Eq 5 is physically reasonable as it
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9 778 accounts for the droplet drag coefficient for deformed droplets. Although values above 1 are
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11 779 theoretically possible, they would not be physically justifiable.

$$C_d = \begin{cases} \frac{24}{Re_d} \left(1 + \frac{1}{6} Re_d^{2/3}\right) & \text{for } Re_d \leq 1000 \\ 0.424 & \text{for } Re_d > 1000 \end{cases} \quad \text{Eq 5}$$

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23 781 ***Mass and energy conservation***

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26 782 During the evaporation of the droplet in a spray, it experiences simultaneous heat and mass transfer
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28 783 processes. By means of convection and conduction, the heat from the surrounding gas is transported
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30 784 into the droplet surface. The fuel vapour is returned to the gas stream via convection and diffusion. A
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32 785 detailed recollection of the underlying equations would be lengthy but can be found in the original
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34 786 papers (71, 80, 81).

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38 787 ***Spray break-up***

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41 788 ***Primary breakup modelling***

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44 789 In this work, an industry standard hybrid break-up model named KH-RT model (69) is used. It is based
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46 790 on the Kelvin-Helmholtz (KH) and Rayleigh-Taylor (RT) instability theory. A parent droplet with the
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48 791 radius r breaks up into new child droplets with the radius r_c following Eq 6. The tuning constant B_0 is a
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50 792 multiplier to linearly alter the size of the child droplet. The characteristic breakup time τ_{KH} is calculated
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52 793 as shown in Eq 7, with Ω_{KH} and Λ_{KH} being the maximum wave growth rate and its corresponding wave
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54 794 length. For the sake of brevity, the latter two parameters are not further elaborated. These equations
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56 795 show that B_0 and B_1 are the tuning factors responsible for the rate at which the parent droplet shrinks
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58 796 and defines the size of the child droplet as shown in Eq 8. These two coefficients are classic user
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797 definable input parameters that have shown to impact the simulated results and have therefore been
 798 added to the list of investigated constants.

$$r_c = B_0 \Lambda_{KH}$$

Eq 6

$$\tau_{KH} = \frac{3.788 B_1 r}{\Omega_{KH} \Lambda_{KH}}$$

Eq 7

$$\frac{dr}{dt} = \frac{r - r_c}{\tau_{KH}}$$

Eq 8

799 The switch between when primary and secondary breakup equations is defined by the breakup length
 800 L_b in the Levich model. It is calculated as shown in Eq 9. A_{bu} and B_{bu} are user tuning constants. The
 801 original authors recommended a value of 5.5 and 0 respectively. While A_{bu} scales the break up length
 802 based on the nozzle size and therefore appears to scale to some real boundary condition, B_{bu} being a
 803 simple addition is arguably arbitrary. Therefore, A_{bu} but not B_{bu} have been added to the list of
 804 coefficients to be investigated.

$$L_b = A_{bu} D_n \sqrt{\frac{\rho_l}{\rho_g}} + B_{bu}$$

Eq 9

805 *Secondary breakup modelling*

806 The RT model is then used in conjunction with KH to predict the secondary breakup of the droplets.
 807 The RT model predicts instabilities on the surface of the droplets that grow until a certain characteristic
 808 breakup time when the drop finally breaks up. Once waves begin to grow on the surface of the droplet,
 809 the wave growth time Ω_{RT} is tracked. This time is then compared to the breakup time. Usually, C_{RT} is a
 810 tuning factor and that kept at unity.

$$\tau_{RT} = \frac{C_{RT}}{\Omega_{RT}} \quad \text{Eq 10}$$

811 If the RT waves have been growing for a time greater than the breakup time, the drop is assumed to
812 break up. The approximated diameter D_d of this stable droplet size is influenced by C_3 . The correlation
813 is shown in Eq 11. Both C_{RT} and C_3 are influential parameters and will be investigated in more detail.

$$\frac{dD_d}{dt} = -\frac{D_d - 2\pi C_3}{\tau_{RT} K_{RT}} \quad \text{Eq 11}$$

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