Kinematic Analysis and Optimization of Bicycle Suspension

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Abstract *Bicycle suspension is increasingly used to improve off-road performance and to facilitate the use of smaller wheels for folding bicycles. Despite the advantages of suspension, unwanted activation due to pedalling and braking forces can result in energy losses and reduced control. This paper presents a kinematic analysis of the effects of pedalling forces on bicycle suspension. This analysis results in a Suspension Activation Ratio (SAR) which is the ratio of the suspension activation force to the pedalling force, it is dependent on the gear selected so a given bicycle will have a different SAR for each gear. The analysis has been experimentally verified. The SAR may therefore be used as a performance metric to compare suspension designs and an objective function for suspension design optimization where the SAR is minimized for all possible gear ratios. Suspension geometry thus optimized shows agreement with optimal pivot positions indicated by previous studies. Previous work has involved dynamic simulation and experimentation to estimate these energy losses; however it is difficult to apply this analysis to rapidly evaluate different suspension designs for performance evaluation or design optimization. The kinematic design approach presented here provides the first step in suspension design which should precede dynamic design to optimize spring and damping rates.*

1. Introduction

Bicycle pedalling forces may cause activation of the rear bicycle suspension resulting in energy loss. Trials of riders completing cross country race courses confirm that the presence of rear suspension results in energy loss by showing increased oxygen uptake [1] and increased lap times [2] when compared to a bike without rear suspension.

Force applied at the pedals results in moments acting about the suspension pivot due to an increased vertical ground reaction force and a horizontal propulsion force, in addition to the chain tension force. Therefore, even if the bottom bracket (pedal/crank axle) is part of the rear suspended frame section, suspension activation may still occur. It is however possible for these forces to be balanced and therefore not activate the suspension regardless of pedalling frequency or suspension spring and damping rates. This paper considers the use of a kinematic equation for the Suspension Activation Ratio (SAR) which is the ratio of the suspension activation force to the pedalling force. It is dependent on the geometry of the bicycle frame, suspension and transmission. A positive SAR will result in compression of the suspension under pedalling forces and a negative SAR will result in extension or lock-out of the suspension. The objective is therefore to achieve a SAR of zero.

Previous work has taken a dynamic approach to minimize energy loss. Wang & Hull developed a dynamic model of the human body seated on a bicycle using vibratory tests to determine stiffness and damping values for joints and limbs [3] and use this to determine that power dissipation due to suspension activation was 1.3% of the total input energy [4]. The model was also used to optimize suspension pivot position, with the height but not the horizontal position of the pivot effecting efficiency. The optimal position depends on gear selection and was found to be 11cm above the bottom bracket for a 32-tooth front chain ring (gear) [5]. Needle & Hull compared this model with experimental data using a bicycle with an adjustable pivot point and concluded that the optimal pivot position was 8.4cm above the bottom bracket [6]. Good & McPhee developed a dynamic model with the rider and bicycle as a single rigid body [7] and used this to find an optimal pivot position of 11.6cm above and 2.7cm rearwards of the bottom bracket [8].

Both Wang & Hull and Good & McPhee found that the front suspension was not significantly activated by seated pedalling on level ground.

Karchin & Hull have argued that it is when pedalling in a standing position on steep inclines that suspension activation is most noticeable and investigated the difference in the optimal pivot position for both a seated or standing riding position. They found that there was a significant difference between the optimal pivot points of the two riding positions, with a standing position resulting in a higher amount of energy dissipated [9]. This paper considers the seated riding positon which is easier to model with fewer assumptions and which accounts for the majority of the time pedalling.

Nomenclature

The relevant parts of a bicycle referred to in this paper are labelled in Figure 1.

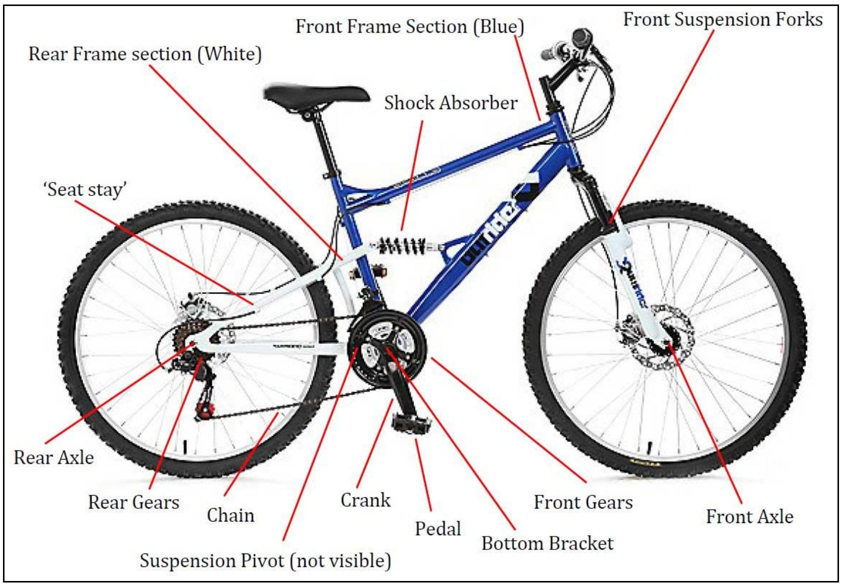

Figure 1: Parts of a Full Suspension Mountain Bicycle

General Terms:

CoG = Centre of gravity

SAR = Suspension Activation Ratio

Equation Variables:
F = input or output force (N)
l = length, or pitch (of chain) (m)
N = number of teeth on gear
r = radius (m)
R = reaction force (N)
T = torque (Nm)
x = x-axis dimension (m)
z = z-axis dimension(m)

Subscripts:
BB = bottom bracket
CH = chain
COG = centre of gravity
CR = crank
FA = front axle
FG = front gear
FW = front wheel
P = pedal
PR = propulsion
RA = rear axle
RG = rear gear
RW = rear wheel
SP = suspension pivot
SU = suspension

Kinematic Model

The kinematic model assumes that all components are rigid bodies and that suspension displacement will result in negligible changes in geometry. There are 4 forces acting on the suspension; the suspension activation force (F_{SU}); the propulsion force (F_{PR}); the reaction force at the rear wheel due to pedalling (R_{RW}); and the chain tension force (F_{CH}). The free body diagram of the rear suspension and transmission can be seen in Figure 2.

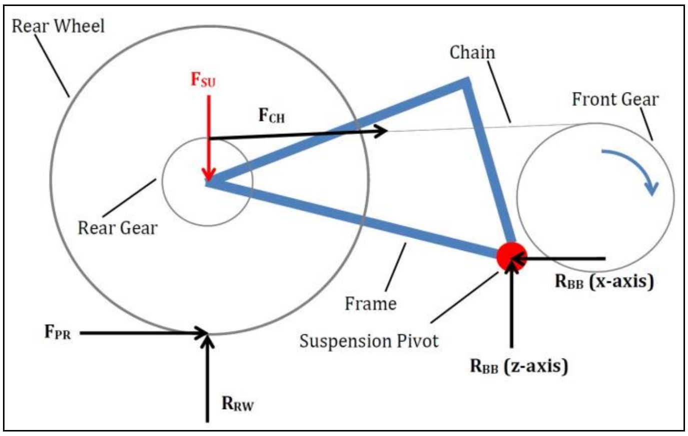

Figure 2: Free Body Diagram of Rear Suspension and Transmission

The geometry of the bicycle is defined as x and z-axis coordinates with the x-axis extending forwards from the rear axle and the z-axis extending upwards from the ground. The SAR is found by first considering the balance of moments about the suspension pivot to show that

$$F_{SU} \cdot x_{SP} = F_{CH} l_{CH-SP} + R_{RW} x_{SP} - F_{PR} z_{SP} \qquad (1)$$

where x_{SP} and z_{SP} are the x and z coordinates of the suspension pivot, and $l_{CH\text{-}SP}$ is the perpendicular distance between the chain line and the suspension pivot.

Dividing both sides of this equation by x_{SP} and F_P gives the Suspension Activation Ratio (SAR)

$$SAR = \frac{F_{SU}}{F_P} = \frac{F_{CH}}{F_P}\frac{l_{CH-SP}}{x_{SP}} + \frac{R_{RW}}{F_P} - \frac{F_{PR}}{F_P}\frac{z_{SP}}{x_{SP}} \qquad (2)$$

Each of the force ratio terms can be expressed in terms of the bicycle geometry. The chain force ratio (F_{CH}/F_P) is given by

$$\frac{F_{CH}}{F_P} = \frac{l_{CR} 2\pi}{l_{CH} N_{FG}} \qquad (3)$$

where l_{CR} is the crank length, l_{CH} is the chain pitch and N_{FG} is the number of teeth on the chain wheel (front gear).

The propulsion force ratio (F_{PR}/F_P), derived from the chain force, the size of the rear gear and the radius of the rear wheel, it is given by

$$\frac{F_{PR}}{F_P} = \frac{l_{CR} N_{RG}}{N_{FG} \cdot z_{RA}} \tag{4}$$

where N_{RG} is the number of teeth on the rear gear sprocket and z_{RA} is the height of the rear axle from the ground.

Considering the bicycle and rider as a rigid body and resolving forces the rear wheel reaction force ratio (R_{RW}/F_P) is given by

$$\frac{R_{RW}}{F_P} = \frac{l_{CR} \cdot N_{RG} \cdot z_{CoG}}{N_{FG} \cdot z_{RA} \cdot x_{FA}} \tag{5}$$

where z_{CoG} is the height of the center of gravity above the ground and x_{FA} is the x-axis position of the front axle.

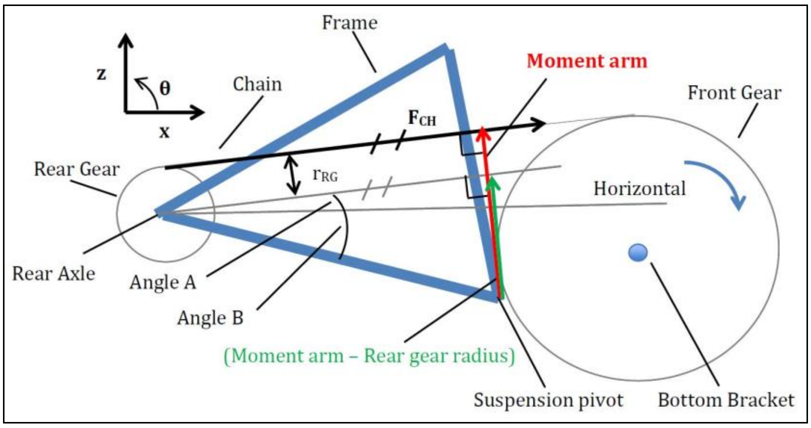

Figure 3: Chain Moment Arm

The chain force (F_{CH}) acts on the suspension through a moment arm ($l_{CH\text{-}SP}$) which is the perpendicular distance between the chain line and the suspension pivot. The chain line is the tangent between the front and rear sprockets. This is calculated by first finding the angles A and B as shown in Figure 3. *A* is the angle between the chain line and the horizontal (x-axis), it is given by

$$Angle\ A = \tan^{-1}\left(\frac{z_{BB} - z_{RA}}{x_{BB}}\right) + \tan^{-1}\left(\frac{r_{FG} - r_{RG}}{\sqrt{x_{BB}^{\ 2} + \left(z_{BB} - z_{RA}\right)^2}}\right) \tag{6}$$

where x_{BB} and z_{BB} are the *x* and *z* coordinates of the bottom bracket, z_{RA} is the height of the rear axle, and r_{FG} and r_{RG} are the front and rear gear radii respectively.

The front and rear gear radii are given by

$$r_{FG} = \frac{l_{CH} N_{FG}}{2\pi} \quad (7) \qquad r_{RG} = \frac{l_{CH} N_{RG}}{2\pi} \quad (8)$$

B is the angle between the horizontal (x-axis) and the line between the rear axle and suspension pivot, it is given by

$$Angle\ B = \tan^{-1}\left(\frac{z_{RA} - z_{SP}}{x_{SP}}\right) \quad (9)$$

where x_{SP} and z_{SP} are the *x* and *z* coordinates of the suspension pivots

The chain force moment arm ($l_{CH\text{-}SP}$) is then given by

$$l_{CH-SP} = r_{RG} + \sqrt{x_{SP}^2 + \left(z_{SP} - z_{RA}\right)^2}\,\sin\left(Angle\ A + Angle\ B\right) \quad (10)$$

Due to the complexity of the geometry involved in the chain force moment arm calculation the equation was verified by comparing the calculated value for various geometries with values calculated by a constraint based sketching program within a parametric solid modelling type of computer aided design software. This showed that there was a close agreement although the length of the moment are differed by up to 0.3 mm.

Substituting into equation (2) it is now possible to calculate the Suspension Activation Ratio (SAR). The full substitution is not shown as the resulting equation is very long and no significant simplification is possible. It is therefore more practical to calculate the individual terms and then substitute the resulting values into the full equation. This approach is also more suitable for calculation by spreadsheet as used in the experimental verification and design optimization given below.

Experimental Verification

A bicycle was equipped with instruments to measure the pedal force and suspension displacement. The suspension was calibrated so that the suspension activation force could be found from the suspension displacement. It was assumed that the suspension velocity was sufficiently low that damper forces were minimal. The front fork was locked with a clamp so that no movement of the front suspension took place.

The pedal force was measured using a strain gauge on the crank arm which was calibrated by applying known loads to the pedal. The signal from the strain gauge was transmitted through a cable via a slip ring. The strain gauge assembly comprised of a 4-arm active bridge, with 2 strain gauges on the top surface of the crank, and 2 on the bottom surface. Whilst 2 strain gauges would be in tension, the opposing 2 would be in compression during pedaling. The signals of the absolute outputs of the 4 gauges can be summed. The advantage of such an assembly is a stronger signal (and therefore lower signal to noise ratio) and more repeatable measurements under varying temperature.

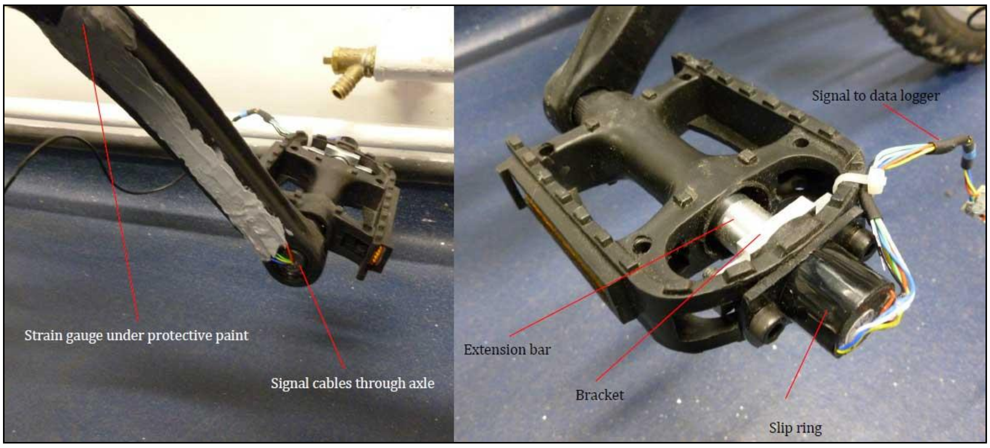

Figure 4: Strain Gauge on Crank used to Measure Pedalling Force

The suspension activation force was estimated by measuring the displacement of the suspension with a linear displacement potentiometer and comparing this with displacements recorded for known static loads.

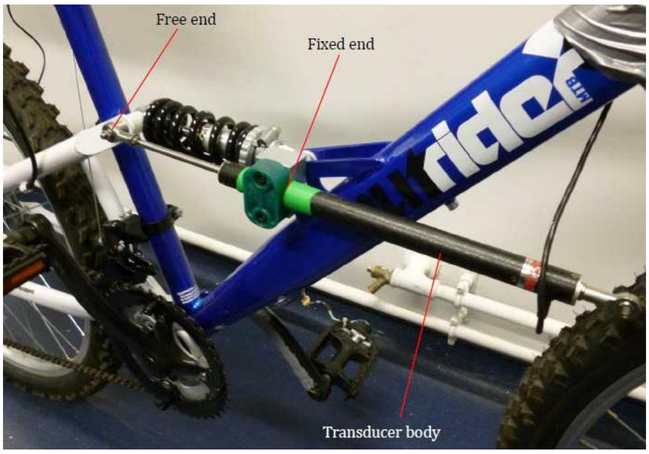

Figure 5: Linear Transducer used to Measure Suspension Displacement

The data logger used was the NI 6009UBS data logger (15). LabView software was used to record data and a Laptop was carried in a rucksack whilst performing tests. The data capture sample rate was set to 150Hz.

The input parameters for the kinematic model were measured on the bicycle and are shown in Table 1. The CoG for the complete system (rider, bicycle and data logging equipment) was found by measuring the ground reaction force at the front wheel with the bicycle at two known inclination angles. By taking moments about the rear wheel for both angles it was then possible to calculate the CoG position.

Table 1: Parameters Measured for Test Bicycle

Parameter		**Value (m)**
BB Height	z_{BB}	0.314
BB Position	x_{BB}	0.445
Rear Axle Height	z_{RA}	0.327
Suspension Pivot Height	z_{SP}	0.307
Suspension Pivot Position	x_{SP}	0.405
Front wheel position (wheelbase)	x_{FA}	1.1
Crank Length	l_{CR}	0.17
Chain Pitch	l_{CH}	0.0127
Height of CofG	z_{COG}	1.0662

Testing was carried out on level ground with the rider in a seated position. The peak pedal force and suspension activation force were used to calculate the SAR. For each gear ratio the SAR was recorded 6 times and the average and standard deviation were calculated. These values were then compared with the theoretical SAR values calculated using the parameters in Table 1. Figure 6 shows that there was a reasonable agreement between the theoretically calculated and experimentally measured SAR values. The error bars for the experimental results represent two standard deviations for the repeatability of the test results; they do not include uncertainties in the systematic errors of the test equipment or any reproducibility data. There was also no attempt to quantify uncertainties in the values used for the theoretical calculation. In particular the CoG is variable. It therefore seems reasonable to conclude that the kinematic equation appears to be correct and where the calculated values fall slightly outside of the experimental variation this may be due to the suspension velocity dependent damping forces or other unquantified uncertainties in the measurements used.

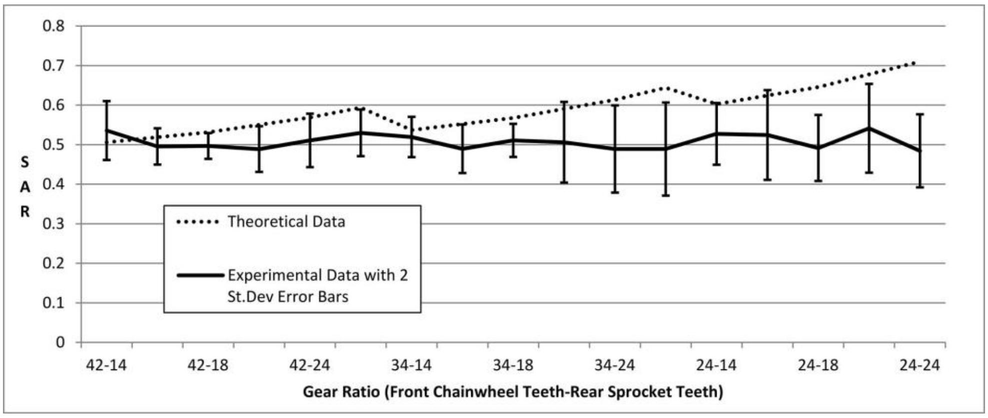

Figure 6: Comparison between Theoretical and Experimental SAR Values

Analysis

The kinematic equation for the SAR has been experimentally verified and shows that for this bicycle suspension arrangement there is no significant difference between the SAR in different gears. Further analysis of the three moments acting on the suspension to generate the suspension activation force, Figure 7, shows that the moments due to the rear wheel ground reaction and propulsion forces vary strongly depending on the gear selected but effectively cancel each other out. Perhaps surprisingly the chain moment is not significantly affected by the gear selected. Further analysis of the data shows that although the chain tension is strongly affected by the gear selected, the change in the moment arm cancels out this effect.

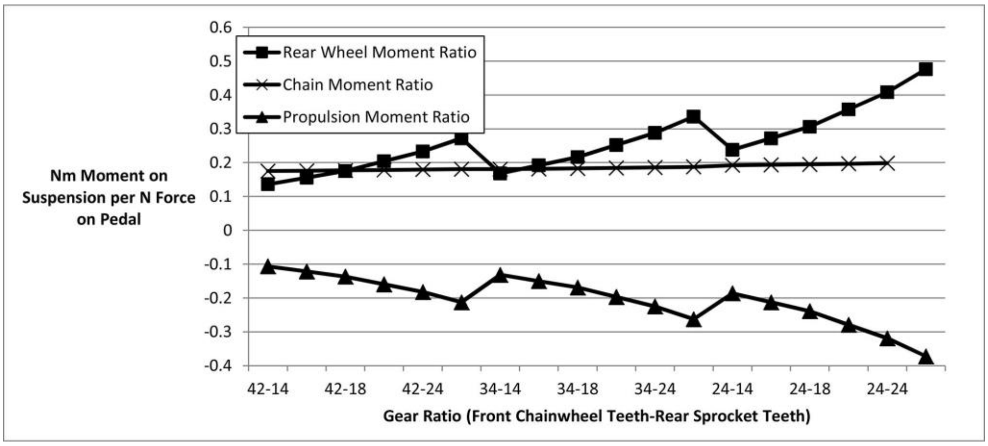

Figure 7: Analysis of Moments acting on Suspension

Kinematic analysis shows that the SAR is valid for riding on both flat and inclined surfaces but verification has only been carried out on the flat.

Design Optimization

The SAR is a useful metric to evaluate the performance of existing suspension designs. Its full value lies however in its use as an objective function to optimize suspension pivot location. The objective function to minimize is the sum of the SAR's for each gear ratio which can be summarized as

$$\min \sum_{i=1}^{n} \left(SAR_{\left(\frac{N_{FG}}{N_{RG}}\right)_i} \right)^2 \quad (11)$$

With the current suspension pivot position (0.405, 0.307) this results in a sum of squares value of 6.54. After optimizing the pivot position the sum of squares was reduced to 0.14 and the SAR remains within 0.11 of zero. The optimized suspension pivot position (0.328, 0.362).

This position is approximately 35mm above and 117 mm rearward of the bottom bracket. Both Wang & Hull and Good and McPhee's models have found the optimal pivot position to be above and rearward of the bottom bracket.

5. Conclusions

A general kinematic equation has been derived and experimentally verified for the SAR which is relevant to a seated riding position. Optimized pivot positions, obtained using the SAR as an objective function, agree with dynamic models and testing for cross country mountain bikes. This further confirms the validity of this approach. Small differences between the experimental and theoretical results were not fully explained by experimental repeatability; however other sources of uncertainty in both experimental and theoretical results were not quantified so this is not a great concern. Further work should involve a full uncertainty analysis with particular consideration given to the variable nature of the CoG used in the kinematic equation. Extension of the method to include consideration of riding in a standing position could follow from this and full verification over all conditions including standing while riding up an incline could then be carried out.

This work only considers pedaling induced forces on the rear suspension, the approach could be extended to also consider braking forces. It would then be possible to carry out a combined optimization to minimize the effect of both pedaling and braking on the suspension. Another area in which the analysis could be extended would be the inclusion of internal gears within the rear wheel hub or bottom bracket.

Previous approaches have used complex and specialized models for mountain bikes. The generic equation for the SAR can be used as a metric to quickly assess the suspension performance of different mountain bike designs and to optimize designs for less conventional designs, for example small wheeled folding bikes. A spreadsheet containing the calculations required to calculate the SAR is available online [10].

6. Acknowledgements

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7. References

1. Nielens, H. and T.M. Lejeune, *Energy Cost of Riding Bicycles with Shock Absorption Systems on a Flat Surface.* International Journal of Sports Medicine, 2001. **22**: p. 400-404
2. Herrick, J.E., J.A. Flohr, D.L. Wenos, and M.J. Saunders, *Comparison of Physiological Responses and Performance Between Mountain Bicycles With Differing Suspension Systems.* International Journal of Sports Physiology and Performance, 2011. **6**(4): p. 546-548
3. Wang, E.L. and M.L. Hull, *A Dynamic System Model of an Off-Road Cyclist.* Journal of Biomedical Engineering, 1997. **119**(3): p. 248-253
4. Wang, E.L. and M.L. Hull, *A Model for Determining Rider Induced Energy Losses in Bicycle Suspension Systems.* Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 1996. **25**(3): p. 223 - 246
5. Wang, E.L. and M.L. Hull, *Minimization of Pedaling Induced Energy Losses in Off-road Bicycle Rear Suspension Systems.* Vehical System Dynamics, 1997. **28**: p. 291-306
6. Needle, S.A. and M.L. Hull, *Off-road bicycle with adjustable suspension kinematics.* Journal of Mechanical Design, Transactions of the ASME, 1997. **119**(3): p. 370-375
7. Good, C. and J. McPhee, *Dynamics of mountain bicycles with rear suspensions: modelling and simulation.* Sports Engineering, 1999. **2**(3): p. 129–143
8. Good, C. and J. McPhee, *Dynamics of mountain bicycles with rear suspensions: design optimization.* Sports Engineering, 2000. **3**: p. 49-55
9. Karchin, A. and M.L. Hull, *Experimental optimization of pivot point height for swing-arm type rear suspensions in off-road bicycles.* Journal of Biomechanical Engineering, 2002. **124**(1): p. 101-106
10. Muelaner, J.E. *Spreadsheet for Calculation of the Suspension Activation Ratio (SAR)*. 2014 [cited; Available from: http://betterbicycles.org/design-parameters/suspension/.