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Abstract

In the context of wage discrimination the effects of a changing population composition on wages have been analysed only when labour supplies are fixed. This note introduces variable supplies, with labour supply varying at the extensive margin. Contrary to the case of fixed labour supplies, we find that wages of the discriminated and the non-discriminated group can change in the same direction. The effects on the wage ratio however, are in the same direction as in the case of fixed supplies. The reason that wage levels can change in the same direction is that in addition to the relative labour supply effect, an aggregate labour supply effect can be of the opposite sign and dominate the relative labour supply effect.

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JEL codes: J21, J71

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I. Introduction

If different population groups have different productive characteristics, changes in population composition shift aggregate labour supply and wages change accordingly. However, changes in population composition can affect wages even when population groups have identical productive characteristics, when labour market discrimination is present. Becker (1971a, pp. 51-52) was the first to show how wages depend on population composition in the presence of employer discrimination. In Becker's analysis, as the population proportion of the discriminated group increases, its wage falls and the wage of the non-discriminated group increases, resulting in a larger wage differential between the two groups. Becker's analysis relied on perfectly inelastic labour supplies and was therefore more relevant in the short-run. As it becomes increasingly recognised that Becker's models are also relevant in the long run, we need to have an understanding of the effects of population composition with variable labour supply. Does the elasticity of labour supply matter, or can Becker's results be extended to the long run? These questions bear directly on any analysis of the effects of the changing racial and ethnic population composition in the US in the past 30 years which saw the proportion of non-Hispanic white population falling from 80 percent in 1980 to 64 percent in 2010 (US Census Bureau, 1995, 2011). This note argues that a variable labour supply substantially alters the analysis of the effects of a changing population composition in the presence of labour discrimination. With variable labour supply, the effects of a changing population composition on *relative* wages have the same direction as in the case of fixed labour supply, but the effects on wage *levels* are different.

An early criticism of Becker's taste-based discrimination models was that they were only plausible in the short run because in the long run market forces were

expected to drive discriminators out of business (Arrow 1972). Subsequent empirical and theoretical work however has shown that these models may be applicable over longer periods of time. Goldberg (1982) showed that with employer discrimination modelled as additional utility derived from hiring a specific type of worker (referred to as nepotism), discriminating employers would not be driven out of business because the asking price for their capital would be too high for those with no discriminatory preferences. Charles and Guryan (2007) argue that prejudiced employers can stay in business in the long run when prejudice is modelled as portable across economic roles. The empirical analysis of Charles and Guryan (2008), covering the period 1972-2004, confirmed the Becker model prediction that wage differentials would vary with the preferences of the marginal employer and showed that a large part of state level black-white wage differentials is explained by differences in prejudices. Flabbi (2010) found that discriminatory preferences toward women in the US are more resilient than previously thought.

In this note we closely follow Goldberg's (1982) articulation of Becker's (1971a) employer discrimination model and allow labour supply to vary at the extensive margin, i.e. participation into employment. The focus on the extensive margin is empirically relevant and theoretically convenient. Unlike its effect on hours, the effect of wages on participation can be confidently assumed monotonic. Moreover it is widely accepted that the labour supply responsiveness at the extensive margin dominates that of the intensive margin (Heckman 1993, p. 117; Cahuc and Zylberberg 2004, p. 38).

II. The model

Consider two types of workers, *M* and *F*, with identical productive capacity. Employers dislike employing workers of type *F*, with this distaste expressed as follows. When the market wage for the *F* workers is w_F, employers value it as $(1+d_F)w_F$ with $d_F \geq 0$. The term d_F is Becker's discrimination coefficient (Becker, 1971a, p.14). Following Becker (1971b, p. 71, n. 4), employers' preferences are expressed as:

$$U = \Pi - d_F w_F L_F \qquad (1)$$

Where Π denotes profits and L_F the level of employment of *F* workers. The employers' problem is to maximise utility subject to:

$$Q = f(L) = f(L_M + L_F) \qquad (2)$$

$$\Pi = Q - (w_F L_F + w_M L_M) \qquad (3)$$

Where the price of output is taken as the numeraire and $f' > 0$, $f'' < 0$. From the first order conditions we have:

$$f' = w_M \quad \text{if } L_M > 0, \text{ and}$$

$$f' < w_M \quad \text{if } L_M = 0 \qquad (4)$$

$$f' = w_F(1+d_F) \quad \text{if } L_F > 0, \text{ and}$$

$$f' < w_F(1+d_F) \qquad \text{if } L_F = 0 \tag{5}$$

Conditions (4) and (5) imply that in a competitive labour market with a continuous distribution of d_F across employers and for a given set of wages for *M* and *F*, a firm will hire either only *M* or only *F* workers. If an employer's d_F is such that $w_M > w_F(1+d_F)$, the relative market wage differential between *M* and *F* is higher than this employer's discrimination coefficient, which implies that the *F* workers are relatively cheaper and therefore the firm workforce will be all *F*. For such a firm the marginal cost of hiring *F* workers is always below the marginal cost of hiring *M* workers so only *F* workers are hired. Similarly, if an employer's d_F is such that $w_M < w_F(1+d_F)$, then only *M* workers are hired.

If d_F has a density $h(d_F)$, then $X = \frac{1}{d_F + 1}$ has a density $g(x)$ which, in principle, can be derived from $h(d_F)$ (see Goldberg (1982), p. 310). Suppose individuals either work or not, and if they do, they work a fixed number of hours. If the cumulative distribution function of reservation wages of group *k* is given by $S_k(w_k)$, *k* = *F*, *M*, then $S_k(w_k)$ is the employment rate of group *k* at wage w_k. The equilibrium wages of groups *F*, *M* are determined by:

$$p_M S_M(w_M) = \int_0^{w_F/w_M} R(w_M)g(x)dx \tag{6}$$

$$p_F S_F(w_F) = \int_{w_F/w_M}^{1} R(\frac{w_F}{x})g(x)dx \tag{7}$$

Where p_k is the population proportion of group k, $k = F, M$, $R(\cdot) = [f']^{-1}(\cdot)$ is a firm's labour demand, and the mass of workers and firms are both equal to one.

Equations (6) and (7) indicate that the F/M wage ratio regulates the clearing of the markets for each group. In equilibrium the aggregate supply of M workers equals the sum of the demands of those firms with $x < \frac{w_F}{w_M}$. The aggregate supply of the F workers equals the sum of the demands of those firms with $x > \frac{w_F}{w_M}$. The general economic problem is the simultaneous clearing of the markets for two inputs which are imperfect substitutes, with the degree of substitutability variable at the firm level.

Goldberg (1982) discusses how firm size varies with discrimination preferences, and Becker (1971) analyses how equilibrium wages change as the population proportions of the two groups change when labour supplies are perfectly inelastic.

III Analysis

To analyse how wages change as population proportions change keeping total population constant, we totally differentiate (6) and (7) with respect to p_M and solve for $\frac{dw_M}{dp_M}$ and $\frac{dw_F}{dp_M}$ (see Appendix for details). We obtain:

$$\frac{dw_F}{dp_M} = \frac{A_1A_6 + A_3A_4}{A_5A_3 + A_2A_6} \qquad (8.1)$$

$$\frac{dw_M}{dp_M} = \frac{A_1A_5 - A_2A_4}{A_5A_3 + A_2A_6} \qquad (8.2)$$

Where:

$$A_1 = S_M(w_M)$$

$$A_2 = \frac{R(w_M)}{w_M} \cdot g(\frac{w_F}{w_M})$$

$$A_3 = R'(w_M) \cdot \int_0^{w_F/w_M} g(x)dx - p_M S'_M(w_M) - R(w_M) g(\frac{w_F}{w_M}) \frac{w_F}{w_M^2}$$

$$A_4 = S_F(w_F)$$

$$A_5 = (1-p_M) S'_F(w_F) - \int_{w_F/w_M}^{1} R'(\frac{w_F}{x}) \frac{1}{x} g(x)dx + R(w_M) g(\frac{w_F}{w_M}) \frac{1}{w_M}$$

$$A_6 = R(w_M) g(\frac{w_F}{w_M}) \frac{w_F}{w_M^2}$$

The denominator in (8.1) and (8.2) can be re-arranged as:

$$[(1-p_M) S'_F(w_F) - \int_{w_F/w_M}^{1} R'(\frac{w_F}{x}) \frac{1}{x} g(x)dx + R(w_M) g(\frac{w_F}{w_M}) \frac{1}{w_M}] \cdot$$
$$\cdot [R'(w_M) \cdot \int_0^{w_F/w_M} g(x)dx - p_M S'_M(w_M)] -$$
$$-[(1-p_M) S'_F(w_F) - \int_{w_F/w_M}^{1} R'(\frac{w_F}{x}) \frac{1}{x} g(x)dx] \cdot R(w_M) g(\frac{w_F}{w_M}) \frac{w_F}{w_M^2}$$

Both terms of this expression are negative as long as $R'(\cdot) < 0$ and $S'(\cdot) \geq 0$. $R'(\cdot) < 0$ follows from $R(\cdot) = [f']^{-1}(\cdot)$ and $f'' < 0$. $S'(\cdot) \geq 0$ is true as long as $S(\cdot)$ is a differentiable cdf.

The signs of these derivatives therefore depend on their numerators. The numerator of (8.1) can be re-arranged as:

$$R'(w_M)\cdot\int_0^{w_F/w_M} g(x)dx - p_M S'_M(w_M) - R(w_M)g(\frac{w_F}{w_M})\frac{w_F}{w_M^2}\cdot\frac{S_F(w_F)-S_M(w_M)}{S_F(w_F)}$$

The sign of this expression is ambiguous. The first two terms are negative but the sign of the last term depends on the relative employment rates of the two groups. If $S_F(w_F) \geq S_M(w_M)$, then $\frac{dw_F}{dp_M} > 0$. But this does not have to be the case. This will certainly *not* be the case if *M* and *F* have the same cdf for their reservation wage because the *M* will have a higher wage than the *F*. But if the two groups have different cdfs for their reservation wages, this condition is possible. In particular, it is likely that the group that suffers discrimination has lower assets, which could imply that at any given wage the *F* employment rate is higher than the *M* employment rate. Then, if the effect of lower assets is greater than the effect of lower wages, we will have $S_F(w_F) > S_M(w_M)$.

Relative employment rates also matter for the effect of a changing p_M on w_M. The numerator of (8.2) can be re-arranged as:

$$(1-p_M)S'_F(w_F) - \int_{w_F/w_M}^{1} R'(\frac{w_F}{x})\frac{1}{x}g(x)dx + R(w_M)g(\frac{w_F}{w_M})\frac{1}{w_M}[\frac{S_M(w_M)-S_F(w_F)}{S_M(w_M)}]$$

This expression is positive if $S_M(w_M) \geq S_F(w_F)$, i.e. the opposite of what is required for $\frac{dw_F}{dp_M} > 0$. If $S_M(w_M) \geq S_F(w_F)$ then $\frac{dw_M}{dp_M} < 0$.

The effects of a changing p_M on wages are summarised in Table 1. Since $p_F = 1 - p_M$, the effects of a changing p_F have the opposite sign. These effects are

summarised in the last two columns of Table 1. The effects of a changing population composition on wage levels that Becker (1971a) derived with perfectly inelastic labour supplies are a special case of the analysis above, and can be derived setting $S'_M(\cdot) = S'_F(\cdot) = 0$. Note also that the results of Becker's analysis correspond to the case where $S_M(w_M) = S_F(w_F)$.

Table 1

Effects of a changing population proportion on wage levels

	Effects of dp_M		Effects of dp_F	
	$\frac{dw_F}{dp_M}$	$\frac{dw_M}{dp_M}$	$\frac{dw_F}{dp_F}$	$\frac{dw_M}{dp_F}$
$S_M(w_M) < S_F(w_F)$	+	+ or –	–	+ or –
$S_M(w_M) > S_F(w_F)$	+ or –	–	+ or –	+
$S_M(w_M) = S_F(w_F)$	+	–	–	+

Turning to the effects of changing population composition on the wage *ratio*, substituting $\frac{dw_F}{dp_M}$ and $\frac{dw_M}{dp_M}$ from (8.1) and (8.2) we have:

$$\frac{d}{dp_M}(\frac{w_F}{w_M}) = \frac{1}{w_M}\cdot\frac{dw_F}{dp_M} - \frac{w_F}{w_M^2}\cdot\frac{dw_M}{dp_M} = \frac{1}{w_M}\cdot[\frac{dw_F}{dp_M} - \frac{w_F}{w_M}\cdot\frac{dw_M}{dp_M}] =$$

$$= \frac{1}{w_M(A_5A_3 + A_2A_6)}\cdot[A_1A_6 + A_4A_3 - \frac{w_F}{w_M}\cdot(A_1A_5 - A_2A_4)]$$

We have already shown above that $A_5A_3 + A_2A_6 < 0$. After re-arranging, the term in the brackets becomes:

$$S_F(w_F)R'(w_M)\cdot\int_0^{w_F/w_M} g(x)dx - p_M S_F(w_F)S'_M(w_M) -$$

$$-(1-p_M)\cdot S_M(w_M)S'_F(w_F)\frac{w_F}{w_M} + \frac{w_F}{w_M}\cdot S_M(w_M)\int_{w_F/w_M}^{1} R'(\frac{w_F}{x})\frac{1}{x}g(x)dx$$

Given our assumptions this expression is negative, so $\frac{d}{dp_M}(\frac{w_F}{w_M}) > 0$ and

$\frac{d}{dp_F}(\frac{w_F}{w_M}) < 0$.

Compared to fixed labour supplies then, variable labour supplies do not preserve the direction of the effects of population composition changes on wage *levels* but preserve the direction of the effects on wage *ratios*. To understand why it is possible (if not likely) for a given change in population composition to change wages of the two groups in the same direction, it is important to note how aggregate labour supply *across both groups* changes. When the employment rates of the two groups differ, aggregate labour supply increases as the population proportion of the group with the higher employment rate increases. As aggregate labour supply increases, it depresses the wages of *both* groups. For example, if $S_M(w_M) > S_F(w_F)$ and p_M

increases, the *M* workers become too expensive for the marginal employers. This puts downward pressure on the *M* wage and upward pressure on the *F* wage. At the same time however, aggregate labour supply across both groups increases, depressing the wages of both groups. In the case of the M workers both effects are in the same direction. In the case of F workers they are not. If for the *F* workers the aggregate labour supply dominates the relative labour supply effect, then their wage will fall, just as the *M* wage falls.

IV Conclusion

In his analysis of the effects of a changing population composition in the context of employer discrimination, Becker (1971a) showed that with fixed labour supplies, if the population proportion of the discriminated group increased, their wage would decrease, the wage of the non-discriminated group would increase, and therefore the relative wage gap would increase. We have shown that with the variable labour supplies this analysis does not hold. Changes in population composition may change the wages of the two groups in the same direction, though this is not necessary. The reason is that with variable labour supplies, aggregate labour supply across both groups changes. The direction of this change depends on which group's population proportion increases and which group has the higher employment rate. If the population proportion of the group with the higher employment rate increases, then aggregate labour supply increases, putting downward pressure on the wages of both groups. The effects of changes in population composition on relative wages however, are the same regardless labour supplies being variable or not. The *relative* wage of the group whose population proportion increases, will always fall.

This analysis sheds new light on the theoretical significance of empirical analyses of the labour market effects of population composition. The Charles and Guryan (2008) finding that higher population proportions of blacks increase the black-white wage gap is consistent with long run (as well as short run) labour supplies. The Abowd and Killingsworth (1984) finding that higher population proportions of blacks have no effect on white employment rates is not consistent with fixed but could be consistent with variable labour supplies. With fixed labour supplies higher population proportions of blacks would lower black wages and increase white wages. The lower black wages would lower black employment rates and the higher white wages would raise white employment rates. A higher population proportion of blacks would therefore be expected to have a positive effect on the employment rates of whites. But with variable labour supplies and conditional on the employment rates, the aggregate labour supply effect could cancel out the relative labour supply effect and result in no overall effect on white employment rates.

We should also note that we assumed that discriminatory preferences were constant as population composition changed. Although there is some evidence that discriminatory preferences change very slowly overtime (Flabbi, 2010), it would be useful to investigate whether the predictions of our analysis can be extended to a context where discriminatory preferences change in response to changes in population composition.

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Appendix

Totally differentiating (6) and solving for $\frac{dw_M}{dp_M}$ we have:

$$\frac{d}{dp_M}[p_M S_M(w_M)] = \frac{d}{dp_M}[\int_0^{w_F/w_M} R(w_M)g(x)dx] \Rightarrow$$

$$\Rightarrow S_M(w_M) + p_M S'_M(w_M)\frac{dw_M}{dp_M} =$$

$$= R'(w_M)\frac{dw_M}{dp_M}\cdot\int_0^{w_F/w_M} g(x)dx + R(w_M)g(\frac{w_F}{w_M})[\frac{1}{w_M}\cdot\frac{dw_F}{dp_M} - \frac{w_F}{w_M^2}\cdot\frac{dw_M}{dp_M}] \Rightarrow$$

$$\Rightarrow \frac{dw_M}{dp_M} = \frac{S_M(w_M) - R(w_M)g(\frac{w_F}{w_M})\cdot\frac{1}{w_M}\cdot\frac{dw_F}{dp_M}}{R'(w_M)\cdot\int_0^{w_F/w_M} g(x)dx - p_M S'_M(w_M) - R(w_M)g(\frac{w_F}{w_M})\frac{w_F}{w_M^2}} \qquad \text{(A.1)}$$

Totally differentiating (7) and solving for $\frac{dw_M}{dp_M}$ we have:

$$\frac{d}{dp_M}[(1-p_M)S_F(w_F)] = \frac{d}{dp_M}[\int_{w_F/w_M}^1 R(\frac{w_F}{x})g(x)dx] \Rightarrow$$

$$\Rightarrow -S_F(w_F) + (1-p_M)S'_F(w_F)\frac{dw_F}{dp_M} =$$

$$= [\int_{w_F/w_M}^1 R'(\frac{w_F}{x})\frac{1}{x}g(x)dx]\frac{dw_F}{dp_M} - R(w_M)g(\frac{w_F}{w_M})[\frac{1}{w_M}\cdot\frac{dw_F}{dp_M} - \frac{w_F}{w_M^2}\cdot\frac{dw_M}{dp_M}] \Rightarrow$$

$$\Rightarrow \frac{dw_M}{dp_M} =$$

$$= \frac{-S_F(w_F) + \frac{dw_F}{dp_M}[(1-p_M)S'_F(w_F) - \int_{w_F/w_M}^1 R'(\frac{w_F}{x})\frac{1}{x}g(x)dx + R(w_M)g(\frac{w_F}{w_M})\frac{1}{w_M}]}{R(w_M)g(\frac{w_F}{w_M})\frac{w_F}{w_M^2}}$$

(A.2)

Equating (A.1) and (A.2) and solving for $\frac{dw_F}{dp_M}$ gives (8.1).