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Maximizing Output Power in a Cantilevered Piezoelectric Vibration Energy Harvester by Electrode Design

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Abstract. A resonant vibration energy harvester typically comprises of a clamped anchor and a vibrating shuttle with a proof mass. Piezoelectric materials are embedded in locations of high strain in order to transduce mechanical deformation into electric charge. Conventional design for piezoelectric vibration energy harvesters (PVEH) usually utilizes piezoelectric material and metal electrode layers covering the entire surface area of the cantilever with no consideration provided to examining the trade-off involved with respect to maximizing output power. This paper reports on the theory and experimental verification underpinning optimization of the active electrode area of a cantilevered PVEH in order to maximize output power. The analytical formulation utilizes Euler-Bernoulli beam theory to model the mechanical response of the cantilever. The expression for output power is reduced to a fifth order polynomial expression as a function of the electrode area. The maximum output power corresponds to the case when 44% area of the cantilever is covered by electrode metal. Experimental results are also provided to verify the theory.

1. Introduction

For the purpose of vibration energy harvesting, cantilevers with a piezoelectric layer sandwiched between two electrode layers, and resting on on top of a substrate, are widely used due to its simplicity and moderately high power density [1, 2, 3], as shown in figure 1. Currently, top and bottom electrode layers usually cover the entire piezoelectric layer to enable transduction of mechanical response to electrical power [4]. However, due to the distribution of strain in the piezoelectric layer while vibrating, the volumetric strain is higher near the clamped end and very little near the free end of the cantilever [5]. Due to the non-uniformly distributed strain along axis x, there should be an optimal value for the area of electrode, or in other words the active piezoelectric area. In this paper, the optimal area of active piezoelectric layer for a maximum power output is calculated from the Euler-Bernoulli beam theory and the result is experimentally verified by a MEMS scale cantilevered PVEH.

2. Theoretical derivation

In this section, the optimal area of electrode layer is theoretically derived for a maximum power from a cantilevered energy harvester. Figure 2 shows the structure of a cantilever with some parameters for calculation. The length, width, thickness of the piezoelectric and substrate layers are L, H, W and h respectively. It is assumed that the width of the electrode layer is also W, but its length starts from the clamped end is a variable x, which is the value that we aim to find to maximize the power output.

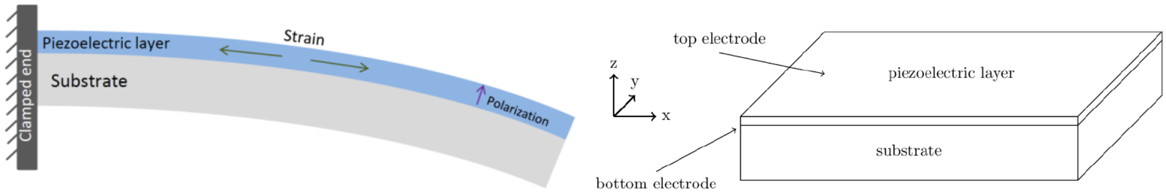

Figure 1. Cantilevered PVEH without proof mass

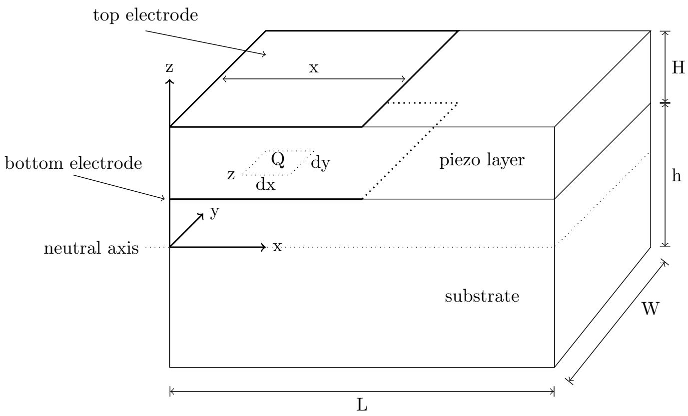

Figure 2. Schematic of a cantilevered PVEH for calculation

The calculation starts from the Euler-Bernoulli Beam Theory [6], which gives an approximate relation between displacement along z-axis for a specific point of beam at (x) and the applied external force, This equation is given by equation 1 [7]:

$$EI\frac{d^4\omega(x)}{dx^4} = q(x) \tag{1}$$

In the equation 1, the parameters E and I represent the Young's modulus and Second moment of area of the entire cantilever respectively; $\omega(x)$ is the displacement (m) of a point at x, and $q(x)$ is the external excitation force per unit length (N/m). Assuming that the excitation force is $F = F_0 \sin\omega_0 t$ and the force is uniformly distributed along x-axis, so we have:

$$q = \frac{F}{L} = \frac{F_0}{L}\sin\omega_0 t \tag{2}$$

By integrating the equation 1 and applying the Dirichlet Boundary Conditions (at the clamped end: $\omega' = 0$ and $\omega = 0$; at the free end: $\omega''' = 0$ and $\omega'' = 0$), we have:

$$\omega(x) = \frac{1}{24}Ax^4 - \frac{1}{6}ALx^3 + \frac{1}{4}AL^2x^2 \tag{3}$$

where $A = \frac{q}{EI}$. For a symmetrical bending, the tensile stress experienced by the beam can be expressed as $\sigma_{(x,y,z)} = \frac{Mz}{I}$, where M is the bending moment which is given by $M = -EI\frac{d^2\omega(x)}{dx^2}$, I is the second moment of area, so we have the stress given by:

$$\sigma_{(x,y,z)} = -zE\frac{d^2\omega(x)}{dx^2} = -z\frac{q}{I}(\frac{1}{2}x^2 - Lx + \frac{1}{2}L^2) \tag{4}$$

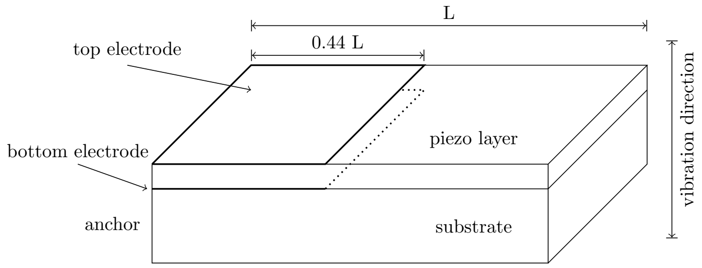

Figure 3. Schematic of a cantilevered PVEH with optimal electrode length

So the amount of charge generated by the strain is expressed as:

$$Q_{(x,y,z)} = d_{31}\sigma_{(x,y,z)} = -zd_{31}\frac{q}{I}(\frac{1}{2}x^2 - Lx + \frac{1}{2}L^2) \tag{5}$$

This is the charge generated per area $dxdy$ at location z, as shown in figure 2. The total surface charge can be calculated by integrating equation 5.

$$Q_{total} = -d_{31}\frac{F_0}{L}\frac{W(h+H)}{2I}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2 + \frac{1}{2}L^2x)\sin\omega_0 t \tag{6}$$

When the cantilever is vibrating at its natural frequency, the equivalent electrical circuit for the cantilever can be equivalent to a current source I_P connected with a capacitor C_P and a resistor R_P in parallel [8]. The generated power by the harvester is the power consumed by the internal impedance (capacitor and resistor in parallel) from the current source. The current source I_P can be calculated from the derivative of charge to time:

$$i_p = \frac{dQ_{total}}{dt} = i_0\cos\omega_0 t \ \left(with\ i_0 = -d_{31}\frac{F_0\omega_0}{L}\frac{W(h+H)}{2I}(\frac{1}{6}x^3 - \frac{1}{2}Lx^2 + \frac{1}{2}L^2x)\right) \tag{7}$$

The internal impedance can be deduced from the capacitance and resistor in parallel:

$$Z_p = C_p//R_p = \frac{\frac{R_p}{2\pi C_p}}{R_p + \frac{1}{j\omega_0 C_p}} = \frac{\rho}{1 + j\omega_0\varepsilon_r\varepsilon_0\rho}\frac{H}{xW} \tag{8}$$

By applying the second moment of area $I = \frac{W(h+H)^3}{12}$, It can be seen that the generated power by the harvester is:

$$\Rightarrow P_0 = B(\frac{1}{36}x^5 - \frac{1}{6}Lx^4 + \frac{5}{12}L^2x^3 - \frac{1}{2}L^3x^2 + \frac{1}{4}L^4x)\ \left(B = d_{31}^2F_0^2\omega_0^2\frac{18H}{WL^2(h+H)^4}\frac{\rho}{1 + j\omega_0\varepsilon_r\varepsilon_0\rho}\right) \tag{9}$$

Equation 9 gives the expression of the generated power by the harvester. From the plot of this expression shown in figure 5, the output power is at its maximum value when $x \approx 0.44L$. From the expression of stress along x-axis in equation 4, it can be found that the stress at $x = 0.44L$ is around 31% of the maximum stress value.

3. Experiment

In order to experimentally verify the theoretical calculation of the active area for optimal output power, a MEMS-scale cantilevered harvester without tip mass is fabricated. The size of the cantilever is 3.5 mm × 3.5 mm and the top electrode is split into 8 segments as shown in figure 4 (left). From region 1 to region 8, they sequentially occupy 20%, 10%, 10%, 10%, 10%,

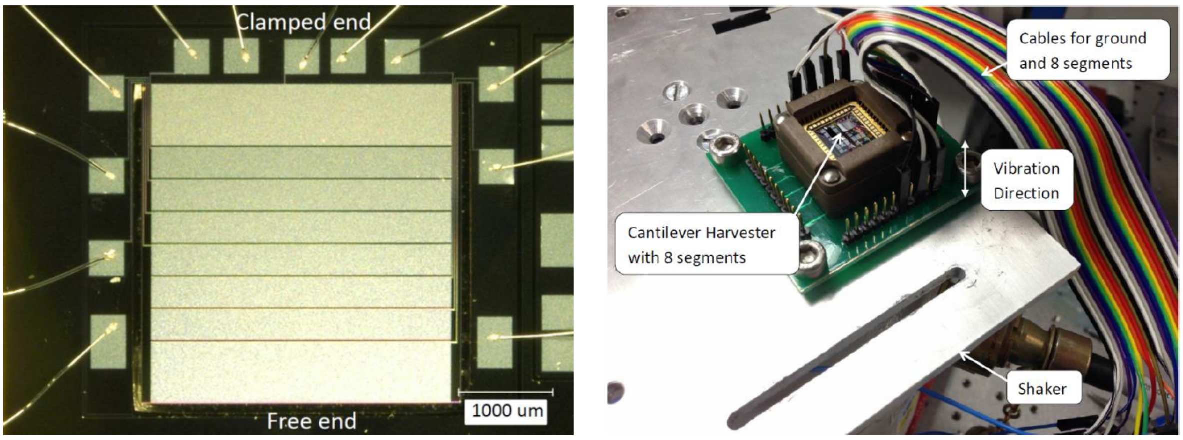

Figure 4. Optical micrograph of MEMS cantilever with 8 electrode regions (left) and the experimental setup (right)

Table 1. Experimental output power comparison of cantilever with 8 regions (frequency: 1208 Hz, acceleration: 0.5 g)

Electrode area	Measured capacitance (nF)	Matched load resistor (kΩ)	Measured Power (nW)
0%	0	-	0
20%	0.464	280	140.01
30%	0.858	160	180.63
40%	1.128	115	214.07
50%	1.401	95	222.01
60%	1.673	75	213.16
70%	1.945	65	199.51
80%	2.217	55	189.55
100%	2.689	50	153.6

10%, 10% and 20% respectively, of the total length of the cantilever. The device in the figure contains 12 electrode pads where there are 8 pads for 8 regions and 4 pads for ground. In order to minimize the effect of capacitance of ground pads, only one of them is used in experiments.

The MEMS device to be tested is clamped in a chip socket, which is fixed on a shaker, see figure 4 (right). The natural frequency of the cantilever is 1208 Hz and the peak input acceleration is around 0.5 g. Experiments are performed with gradually increased top electrode area by adding regions from region 1 to 8.

For each active electrode area, the load resistor is varied to determine the value that matches the internal impedance. Table 1 shows the measured results and the figure 5 illustrates how the output power varies with different active electrode area (comparison of theoretical results and experimental results). By fitting the 8 points with a polynomial trend line, the maximum value can be found at around 48%. The discrepancy with respect to the theoretical value (44%) (solid line shown in figure 5)can be attributed to the parasitic capacitance in the ground pad and also due to fabrication tolerances.

4. Conclusion

A theoretical calculation and experimental verification in this paper are performed to find an optimal active piezoelectric layer area for maximizing output power of a cantilevered PVEH. The results show that maximizing active area does not always increase output power; in the

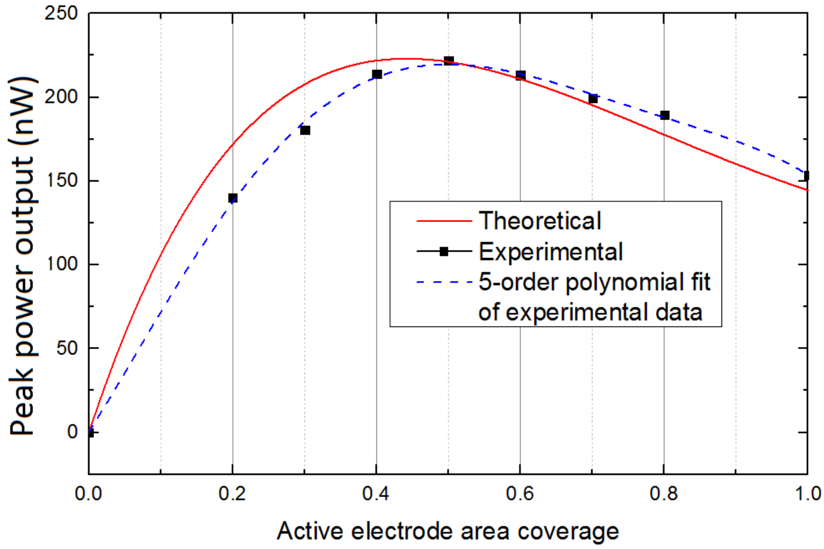

Figure 5. Measured peak output power of the cantilevered PVEH compared to model predictions

contrast, power can be reduced if the low-strain area is covered. For designing a piezoelectric vibration energy harvester at either macro-scale or MEMS-scale, the active layer does not necessarily need to cover the entire piezoelectric layer. This work shows that the piezoelectric area corresponding to less than 31% of the maximum bending stress should not be included in the active electrode region. This design approach can also be applied to other structural topologies and mode shapes for PVEHs.

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