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Discussion of 'Factors influencing undrained strength of fine-grained soils at high water contents' by H. B. Nagaraj, M.V. Sravan and B. S. Deepa

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Introduction

The undrained shear strength of soil is a key engineering parameter which is often linked to liquidity index (e.g. Vardanega and Haigh, 2014). This linkage depends critically on the undrained strength of soil at the liquid limit, a subject on which there has been some debate. The determination of the liquid limit is carried out using one of two general methods: 'Casagrande cup' or 'fall cone', depending on national standards.

The authors have presented an interesting paper that shows (amongst other things) how fall-cone undrained shear strength values at the fall-cone liquid limit ($c_{\mathrm{uFC,LL}}$) can vary with changes in the water content at the fall-cone liquid limit ($w_{\mathrm{L,FC}}$). Data of the undrained strength at liquid limit when this is determined by the fall cone are rare, although since this test is itself a measurement of soil strength, examination typically shows a much narrower range of values than for strengths measured at the Casagrande-cup liquid limit ($w_{\mathrm{L,cup}}$).

It is hence surprising that the work of Nagaraj et al. (2018) shows such clear trends of varying undrained strength at the fall-cone liquid limit with water content. The paper hence prompts a debate as to whether the undrained strength at the fall-cone liquid limit can be sensibly assumed as a fixed value. This discussion seeks (in the context of the published paper) to explore this question.

Which liquid limit?

O'Kelly et al. (2018) recently reviewed values of liquid limit measured with the Casagrande cup device (Casagrande, 1932) and those measured with the BSI fall cone (BSI 1990). Many studies have been published comparing the two methods (e.g., Spagnoli 2012, O'Kelly et al. 2018 and Vardanega et al. 2018).

Hansbo (1957) showed that the penetration of a cone free-falling into a plastic material was linked to its strength (c_{u}) by:

$$c_{\mathrm{u}} = K\frac{mg}{h^2} \tag{1}$$

where c_{u} = undrained shear strength; K = cone factor; m = fall cone mass; g = acceleration due to gravity and h = fall cone penetration depth.

If the liquid limit is linked to a specific value of fall cone penetration for a particular cone (as is done in many codes, including for the 30°–80g cone setup adopted in BSI 1990), it is thus implicit that this should be linked to a fixed value of undrained shear strength. This is typically assumed to be around 1.7 kPa, though the precise value depends on assumptions as to the value of the cone factor K and also the assigned cone penetration depth value for the fall-cone liquid limit condition.

Conversely, Haigh (2012) showed based on analysis of the dynamic slope failure involved in the Casagrande test that the cup liquid limit corresponds to a specific strength of approximately 1 $\mathrm{m^2/s^2}$, the precise value being dependent on the cup device's base hardness, a property that varies widely (Haigh 2016). From this comparison, it is clear that while one might expect the undrained strength at the fall-cone liquid limit to be approximately constant, the undrained strength at the cup liquid limit would be expected to decrease with increasing value of the liquid-limit water content.

Undrained strength at liquid limit

Wroth and Wood (1978) reviewed various sources and selected 1.7 kPa as an appropriate value for the undrained strength at the liquid limit, essentially based on the mid-point in the range of values given by Youssef et al. (1965). As was standard practice at the time, this dataset utilised the Casagrande cup method for liquid limit determination. Because the cone method for liquid limit determination was developed to give for some 'typical' soil the same liquid limit values as derived using the cup method, the strength at the cone liquid limit should also be approximately 1.7 kPa. Conversely, at the plastic limit the assumption of a fixed strength cannot be made as it captures the plastic to brittle transition of the soil (Haigh et al. 2013; O'Kelly, 2013).

Nagaraj et al (2012) reviewed a large number of research papers that gave data for strength at liquid limit, mostly involving liquid limits measured using the cup method, with associated undrained strengths measured using the vane shear test. These data gave measured undrained strengths in the range 0.5 – 12 kPa, although mostly in the range of 0.7 – 2.7 kPa. The less widely reported undrained strengths measured using the vane shear test at the cone liquid limit had a narrower range of 0.8 – 4.8 kPa, although mostly in the range of 1.7 – 2.8 kPa. While the data shows a variation in the undrained strength measured even at the fall-cone liquid limit, this could be explained based on differences in strain rate and deformation mode between the vane shear and fall-cone tests, which might lead to variability for different soils, (cf. Haigh and Vardanega, 2012).

In the paper under discussion, (Nagaraj et al. 2018), however, the liquid limit values are determined throughout using the standard BSI fall-cone method (a 30° 80g cone penetrating 20mm), with the undrained strength at the liquid limit being measured using a second fall cone set up (i.e., a 60° 60g cone). In this manner, the usual explanations for the variation in undrained strength at the liquid limit seem to have been eliminated. It is thus surprising that such a large variation in undrained strength at liquid limit (1.0 – 2.8 kPa) is seen, and particularly that this shows such strong trends with water content at liquid limit rather than just showing a random variation that might be attributed to experimental error. If equation (1) is valid, the penetration of a 60° 60g fall cone at the water content at which a 30° 80g fall cone penetrates 20 mm (fall cone liquid limit) should be constant. The values of the cone factor K can change the precise strength that is attributed to this penetration, but cannot give variation between soils if they are assumed to be constant for a given cone setup. The results of the Nagaraj et al (2018) paper, hence seem to suggest that equation (1) does not apply for the data-set presented. It may be that the high levels of sand in the studied soil mixtures affect the assumption of the validity of equation (1).

If equation (1) does not apply, the reported fall-cone undrained strength values at cone liquid limit derived by the authors in this paper cannot be valid, (as they rely on equation (1) in their calculation), but the paper nevertheless has revealed a worrying incompleteness in the understanding of the fall-cone test in the geotechnical literature.

Notation

The following notation is used in this discussion paper:

c_u = undrained shear strength;

$c_{uFC,LL}$ = undrained strength at fall-cone liquid limit;

g = acceleration due to gravity;

h = fall cone penetration depth;

K = cone factor;

m = fall cone mass;

$w_{L,cup}$ = liquid limit measured with the Casagrande cup device;

$w_{L,FC}$ = liquid limit measured with the fall-cone apparatus.

References

BSI (1990). BS 1377-2:1990: Methods of test for soils for civil engineering purposes - Part 2: classification tests. BSI: London, UK.

Casagrande, A. (1932). Research on the Atterberg limits of soils. *Public Roads*, **13(8):** 121–136.

Haigh, S. K. and Vardanega, P. J. 2012. Discussion of “Re-examination of Undrained Strength at Atterberg Limits Water Contents” by H. B. Nagaraj, A. Sridharan & H. M. Mallikurjuna. *Geotechnical and Geological Engineering*, 30 (6): 1389-1391. https://doi.org/10.1007/s10706-012-9543-0

Haigh, S. K. 2012. Mechanics of the Casagrande liquid limit test. *Canadian Geotechnical Journal*, 49 (9), 1015–1023. https://doi.org/10.1139/t2012-066 [Corrigenda, 49 (9): 1116, https://doi.org/10.1139/t2012-081 and 49 (11): 1326, https://doi.org/10.1139/cgj-2012-0380].

Haigh, S.K. 2016. Consistency of the Casagrande liquid limit test. *Geotechnical Testing Journal*, 39 (1): 13–19. https://doi.org/10.1520/GTJ20150093

Haigh, S. K., Vardanega, P. J. and Bolton, M. D. 2013. “The plastic limit of clays.” *Géotechnique*, 63 (6): 435-440. http://dx.doi.org/10.1680/geot.11.P.123

Hansbo, S. 1957. A new approach to the determination of the shear strength of clay by the fall cone test. *Swedish Geotechnical Institute Proceedings*, 14: 5-47.

Nagaraj. H. N., Sridharan, A. and Mallikarjuna, H. M. 2012. Re-examination of Undrained Strength at Atterberg Limits Water Contents. *Geotechnical and Geological Engineering*, 30 (4): 727-736. https://doi.org/10.1007/s10706-011-9489-7

Nagaraj, H. B., Sravan, M. V. and Deepa, B. S. 2018. Factors influencing undrained strength of fine-grained soils at high water contents. *Geomechanics and Geoengineering: An International Journal*, 13 (4): 276-287. https://doi.org/10.1080/17486025.2018.1445873

O'Kelly, B.C. 2013. Atterberg limits and remolded shear strength-water content relationships. *Geotechnical Testing Journal*, 36 (6): 939–947, https://doi.org/10.1520/GTJ20130012

O’Kelly, B. C., Vardanega, P. J. and Haigh, S. K. 2018. Use of fall cones to determine Atterberg limits: a review. *Géotechnique*, 68 (10): 843-856, https://doi.org/10.1680/jgeot.17.R.039 and Corrigendum, 935, https://doi.org/10.1680/jgeot.2018.68.10.935.

Spagnoli, G. 2012. Comparison between Casagrande and drop-cone methods to calculate liquid limit for pure clay. *Canadian Journal of Soil Science*, 92 (6): 859-864. https://doi.org/10.4141/cjss2012-011

Vardanega, P. J. and Haigh, S. K. 2014. The undrained strength-liquidity index relationship. *Canadian Geotechnical Journal*, 51 (9): 1073-1086. https://doi.org/10.1139/cgj-2013-0169

Vardanega, P. J., O’Kelly, B. C., Haigh, S. K. and Shimobe, S. 2018. Classifying and characterising fine-grained soils using fall cones. *ce/papers*, 2 (2-3): 821-826, https://doi.org/10.1002/cepa.772

Wroth C. P. and Wood D. M. 1978. The correlation of index properties with some basic engineering properties of soils. *Canadian Geotechnical Journal,* 15 (2):137–145. https://doi.org/10.1139/t78-014

Youssef, M. S., El Ramli, A. H. and El Demery, M. (1965). Relationships between shear strength, consolidation, liquid limit and plastic limit for remolded clays. In: *Proceedings of the Sixth International Conference on Soil Mechanics and Foundation Engineering*, University of Toronto Press, Canada, vol. 1, pp. 126–129.