EXPERIMENTAL BIFURCATION ANALYSIS OF A WING PROFILE

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Abstract: The prediction of flutter instabilities is very critical in aeroelstic wing design, as it limits the aircraft operational envelope. Aeroelastic structures that have nonlinear characteristics, as in highly flexible wings, can exhibit limit cycle oscillations in the vicinity of the flutter boundary. However, comprehensive characterization of these nonlinear oscillations can be challenging without a well established nonlinear mathematical or numerical model. In the present paper, control-based continuation (CBC) technique is used to characterize the nonlinear oscillatory dynamics of a physical aeroelastic system undergoing pre and post flutter oscillations, without the use of a mathematical model. The aeroelastic system was represented by a two-dimensional wing with pitch and heave degrees of freedom, tested in the low turbulence wind tunnel of the University of Bristol. The aim of this research is to demonstrate the capability of the CBC technique to trace unstable periodic behavior through stabilizing unstable limit cycle oscillations. The results allowed to produce a full bifurcation diagram for a fluttering wing profile, despite the noisy turbulent flow environment of the wind tunnel.

1 INTRODUCTION

It is well known that a wing subjected to an air flow will start to oscillate at some critical velocity, due to the interaction of structural, inertial and aerodynamic forces acting on it. This dynamic aeroelastic phenomenon is an aeroelastic instability know as 'flutter', which can potentially lead to catastrophic situations, as for the NASA Helios prototype [9]. As a result, the design of aircraft wings as well as other aeroelastic systems need to carefully consider flutter cases, as the speed at which this instability occurs limit the operational and design envelopes. Different Methods have been developed in the past to predict the critical flutter velocities based on linear assumptions or pre-flutter dynamic behavior from flight or wind tunnel tests. However, these methods were often constrained to only estimating the the critical flight speed after which flutter can occur, rather than assessing the global dynamic behavior in the vicinity of the instability region.

In the presence of nonlinearity, the dynamics of aeroelstic systems in the region of flutter instabilities can lead to a richer behavior, which may contain stable as well as unstable persistent limit cycle oscillations (LCO's) [8]. These conditions can be manifested earlier than the classical flutter speed limits, if the system is subjected to relatively large-enough perturbations. These conditions, are known as subcritical flutter conditions and can lead to sudden and significant variations in the behavior of the physical system, leading to hysteresis like phenomena for example. The understanding of the complex nonlinear dynamics in such cases can be facilitated by the use certain nonlinear analysis tools such as numerical continuation and bifurcation methods [2, 8]. These techniques allow to produce bifurcation diagrams, which can portray a more complete picture of the rich nonlinear dynamics of the system. Conventionally, bifurcation diagrams are constructed by tracing the steady state solutions of the nonlinear system, as one of the system's parameters is varied. This is usually done more efficiently using continuation methods [7].

Currently, numerical continuation is a common technique used to perform the bifurcation analysis with numerical models [7]. Within aerospace engineering, examples of bifurcations are self-excited oscillations of an aeroelastic structure include wing flutter [8], landing gear systems [14], rotorcraft blades [2], etc. However, this studies relied on numerical or mathematical nonlinear dynamical model representations to describe the governing equations of the system. On the other hand, in the absence of an adequate numerical model, conducting physical experimental studies become a necessity, which justifies the importance of developing accurate methods for conducting the bifurcation analysis experimentally. Previously, numerical [1, 8] and experimental [10, 11] investigations were performed to characterize the aeroelastic behavior of wing profiles and related LCOs occurrences, but without robustly investigating the occurrence of unstable LCOs.

In theory, unstable LCOs could be traceed experimentally, if an appropriate experimental continuation algorithm is developed. However, The main challenges of applying numerical continuation methods directly to interrogate physical experiments and so to produce full bifurcation diagrams are the controllability and observability of the states of interest as well as the ability of locally stabilizing unstable dynamics. The latter can be dangerous in physical experiments, contrary to numerical simulations.

Control-based continuation (CBC) techniques are one solution that allows numerical continuation to be applied in an experimental scenarios [5, 15, 16]. It uses a non-invasive feedback control and can produce bifurcation diagrams for physical systems without any kind of numerical model. So far applications have mainly considered single-degree-of-freedom systems in presence of low levels of noise and characterized by dynamics with a known frequency of oscillation. Examples of these applications are the parametrically excited pendulum [13], a nonlinear energy harvester [5], a bilinear spring [6], and a nonlinear tuned-mass-damper [3].

This paper aims to extend the current CBC technique to investigate the aeroelstic wing dynamics exhibiting nonlinear stable and unstable LCOs. In this case, a rigid wing with pitch and heave degrees of freedom, subjected to airflow is used to operate in subcritical flutter condition, i.e. existence of stable and unstable LCOs. The study also aims to test the potential of CBC in determining the dynamics of system with self-excited behavior, which presents oscillations with a frequency not known a priori. This paper will first describe the methodology adopted and the experimental set up. Then, the experimental results obtained will be presented and discussed.

2 METHODOLOGY

In order to conduct this study, a flutter rig, with pitch and heave degrees of freedom (DoF), was designed to exhibit limit cycle oscillations at relatively low flow speeds (between 10 and 30m/s ideally). This condition was obtained by ensuring the wing center of gravity is slightly shifted from its shear axis to provide a level of coupling between the degrees of freedom. A spring with hardening effect was used in the pitch DoF to bound the LCOs to a relatively low acceptable amplitude. This analysis will first briefly discuss the subcritical flutter condition of the rig when operated, indicating the existence of unstable limit cycle solutions. The the control-based continuation is then introduced to trace both stable and unstable solutions as the wind speed is varied.

2.1 Description of the experimental setup

To observe flutter an experimental rig was built at the University of Bristol. The rig consists of four main sub-assemblies; the wing section, the support frame structure, the sensors, the springs and impact absorbers. The schematic representation of the rig is presented in Figure1. The 600 mm long test wing was placed in the middle of the 700 mm x 800 mm working section of the wind tunnel. The wing is suspended by a main shaft, which goes through its quarter-chord position. The wing has a NACA0015 aerofoil section with a chord length of 30.5 mm. On each side of the wing, elliptic-shaped acrylic side-plates are attached to ensure the two-dimensional aerodynamic flow behavior. The wing shaft is mounted onto two aluminum plates through a set of ball bearing block. These bearings allow the wing and the shaft to rotate in the pitch DoF. The two aluminum plates on the other hand, can slide vertically trough four linear bearings located at the corners, guided by vertical steal rods. This structure allows for the heave DoF. The heave rods are tightly clamped onto the frame, which is in-turn clamped and aligned with the wind tunnel metal structure.

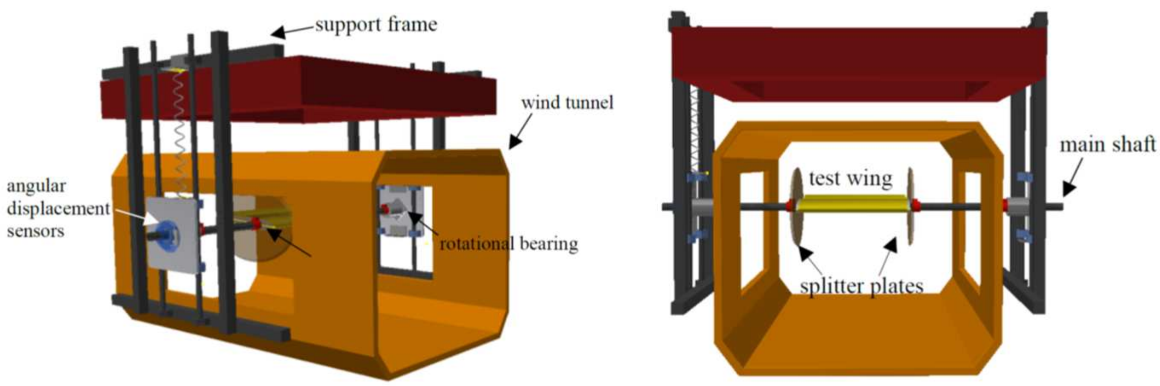
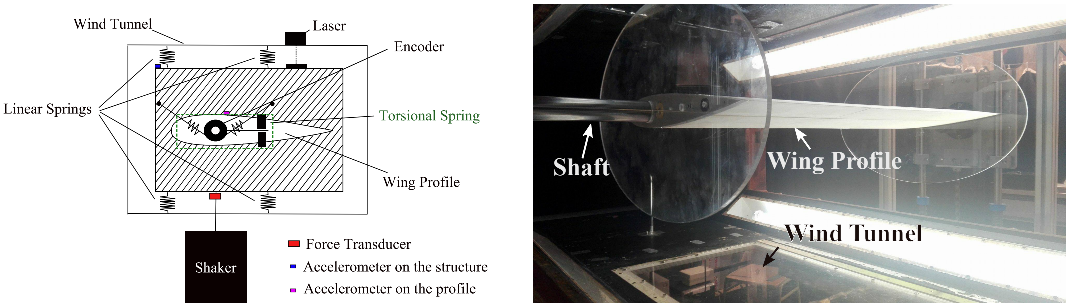


Figure 1: A schematic view of the flutter rig mounted in the Low Turbulent wind tunnel of the University of Bristol.

The sensors were required to measure two variables: pitch angle and plunge displacement. For the pitch angle, a magnatic rotary absolute encoder was used, whereas for the measurement of vertical displacement a laser position sensor was attached. The measurements data was acquired using a Beaglebone Black with a data acquisition cape [4] at a sample rate of 5 kHz. Stiffness in the heave degree of freedom was achieved through linear springs attached between the aluminum plates and steel brackets bolted onto the frame. Impact absorbers were designed



(a) (b)
Figure 2: The experimental apparatus. (a) the spring arrangement, (b) wing profile.

to prevent any major damage to the rig in case of excessive plunge motion. Furthermore, the pitch stiffness was obtained trough a torsional spring arrangement achieved with a combination of linear springs and specially designed spring plate, which has hardening characteristics. Figure 2(a) shows the linear springs acting in the heave and pitch motion as well as a nonlinear (hardening) torsional spring, which prevented excessive and unbounded oscillations.

For the control-based continuation, a control force was applied directly in the heave motion and indirectly in the pitch sense, since the DoFs are coupled. The APS Electro-Seis Shaker (a voice-coil-type actuator) [17] was used to impose the required control force. Real-time control was performed using the same Beaglebone Black with a data acquisition cape [4] at a sample rate of 5 kHz.

2.2 Implementation of the control-based continuation CBC

The CBC method is implemented in Matlab and communicates with the real-time controller asynchronously. The control is performed using the Beaglebone Black board [4], and Figure 3 shows the flow of information on a schematic level. Using Matlab, selected control gain values and target amplitude are passed to the board where the control signal is generated given the error in terms of the actual and target states. The Beaglebone board is adopted also for the acquisition and signal conditioning can be applied.

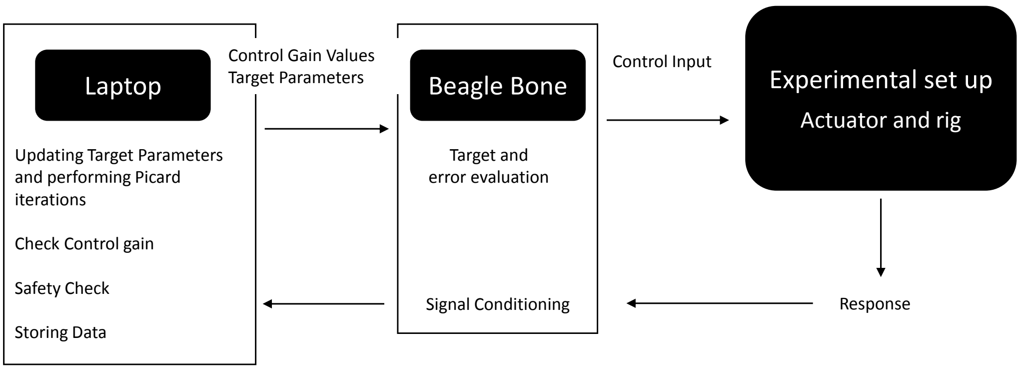


Figure 3: Flow chart illustrating the information transfer in the experiment subsystems.

The key point of the CBC strategy is the adoption of a non-invasive control that can stabilize the solution. When a steady state solution is found and the control is non-invasive, then the determined solution is the one characterizing the system in open loop subjected to external forces present. Here, the control action is a proportional and derivative control and the non-invasiveness is obtained by the selection of a suitable control target (see Figure 4. Labeling with e and $\dot{e}$ the error and its derivative, the control law is $u(t) = K_p e(t) + K_d \dot{e}(t)$ and it is not invasive if $u(t) = 0$, experimentally this needs to be relaxed a bit because of the presence of noise.

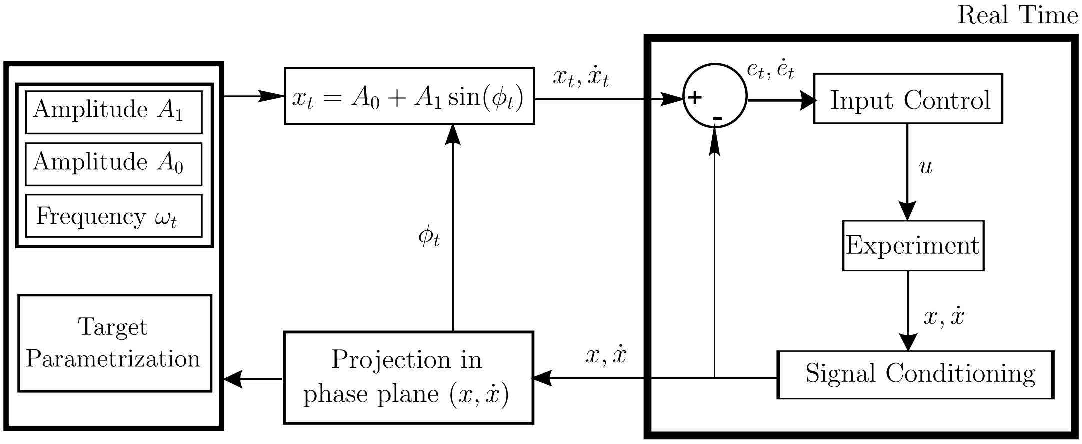


Figure 4: Control scheme for the frequency locked CBC.

In the present application, flutter oscillations are self-excited and the typical way of performing CBC (where the period is known) is not directly applicable since fluctuations in the period of oscillation caused by disturbances in the airflow can cause a loss of synchrony between the control target and the actual state of the system. To overcome this problem, an alternative parametrization of the control target to avoid explicit time dependence was implemented

In order to check when the control is non-invasive, i.e. the obtained time response of the system is similar to the open loop response, the variation of the normalized error as the target amplitude varies is considered. Figure 5 shows an example of the stated variation and the related point on the bifurcation diagram. When a minimum occurs, the selected target amplitude is said to be the one for which the dynamic of the system is similar to the open loop case, i.e. LCOs of the open loop system are found. In the present application, the target amplitude was achieved in terms of the heave DoF.

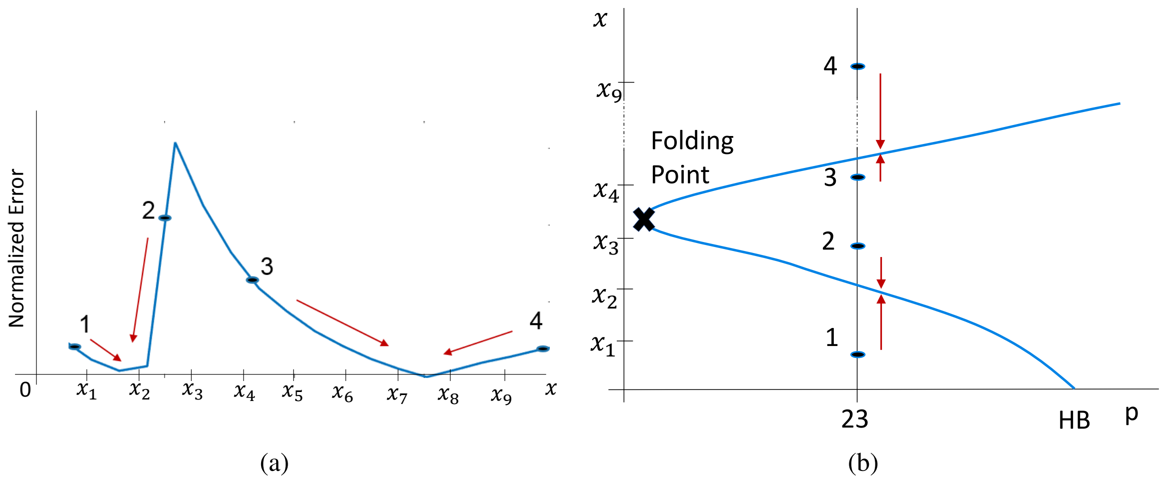


Figure 5: CBC concept of finding solution: (a) normalized error trend, (b) bifurcation diagram.

3 RESULTS AND DISCUSSION

Without the use of any control techniques, varying the velocity of the flow in the wind tunnel and letting the system be perturbed by the flow itself, it was possible to identify the stable branches (equilibria and LCOs) of the bifurcation diagram of the system of interest. This is shown in Figure 6, where the steady state solutions of the considered flutter rig at different velocities are shown. By increasing the wind tunnel speed from about 14m/s in an intermittent and steady manner, the wing loses stability just after 24 m/s (black dot points). This point identifies the classical flutter speed of the rig, which from a bifurcation point of view is known as a Hopf point. After this point the wing starts undergoing stable self-oscillation (LCOs). If the flow velocity is then decreased, starting from the stable oscillating conditions, a further jump occurred at a velocity just above 16 m/s and a stable equilibrium condition is reached (red circle). The presence of jump (Figure 6) suggests the occurrence of an other bifurcation point and the existence of an unstable LCO branch, similar to what is known from literature, for example [1, 8]. The CBC technique is therefore, intended allow a comprehensive characterization of such unstable LCO branch.

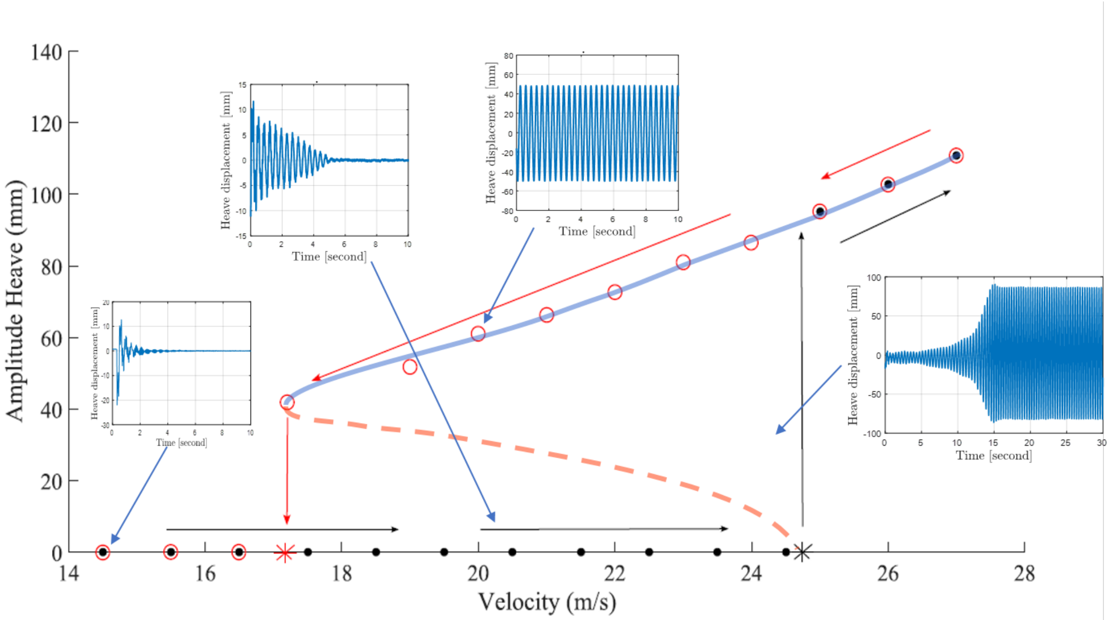
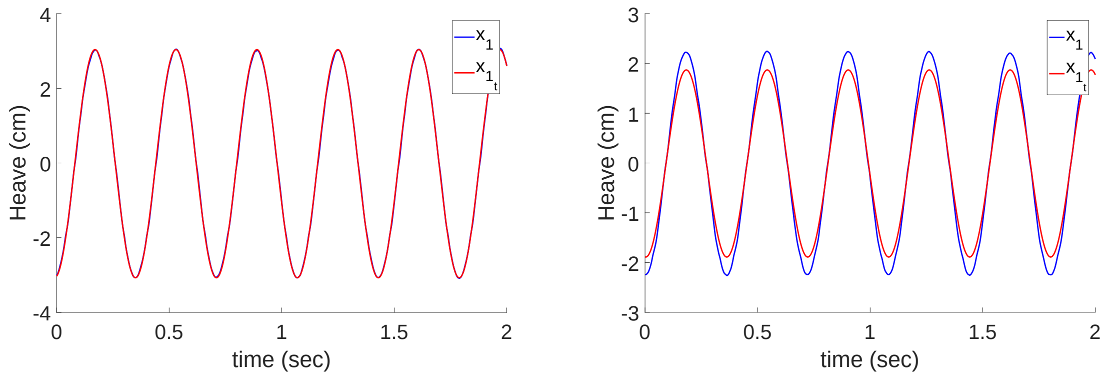


Figure 6: Steady state solutions in open loop of the considered flutter rig. Black dots denotes stable solutions (equilibria and LCOs) found by increasing the flow speed, while hollow red circles denotes stable solutions (equilibria and LCOs) found by decreasing the flow speed. The blue line indicates the amplitude of the stable LCO branch. The red dashed line indicated the possible unstable LCO branch. The black star denote the Hopf bifurcation point (the flutter speed point). Example time responses are shown at selected flow speeds.

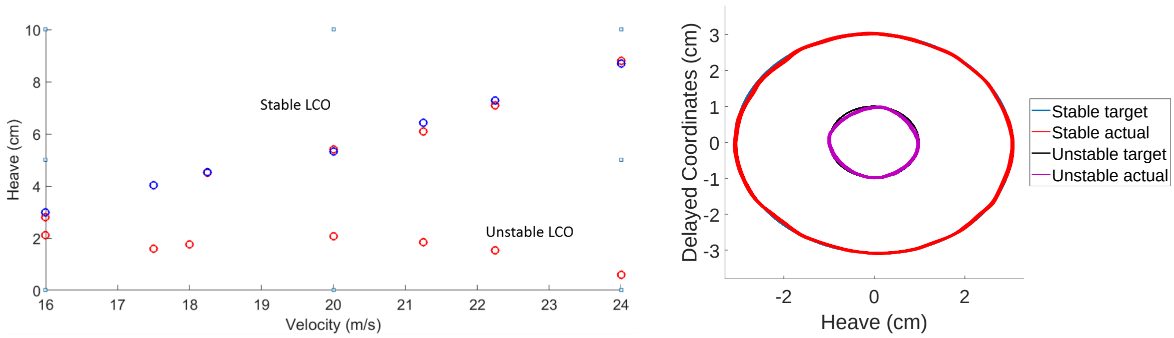
The application of the CBC to the flutter rig provided very promising results. It was possible to identify amplitude targets for which the control is not invasive. Figure 7 shows controlled dynamics for the heave motion obtained having selected two different amplitude targets for the heave motion. The control is non-invasive, i.e. $u = 0$, for one of the selected target (Figure 4(a)).

Varying the velocity and the target amplitude, initial bifurcation diagrams were produced. With this respect, Figure 5(a) shows the experimental results where the CBC method was able to stabilize the unstable limit cycle oscillations that emerge from a subcritical Hopf bifurcation at a critical flow speed of above 24m/s. The blue points were obtained with small perturbations



(a) (b)

Figure 7: Time histories obtained at a velocity equal to 23 m/s keeping active the control. (a) control is not invasive, (b) control is invasive. x_1 amd x_{1_t} are the actual heave motion and the target.



(a) (b)

Figure 8: Experimental results using open loop test and CBC. (a) Bifurcation diagram, open loop LCOs are shown in blue circles while LCOs obtained by CBC are denoted by red circles, (b) Phase plot at approx. V=17.5m/s.

using a light hammer impact applied in heave direction. The empty points were obtained using the CBC techniques in two different runs.

Figure 5(b) shows an example phase plots in terms of heave and heave delayed coordinate. The results are obtained at a velocity equal to 17.5 m/s. The control was acting in a non-invasive way, merely of stabilizing the unstable LCO characterizing the system in open loop. When the control is switched off the dynamic is attracted to the stable LCO for roughly 70% of the times to the stable equilibrium solution 30% of the times. If the same experiment is performed at lower or higher velocity than 17.5 m/s the time response trajctory will be attracted to either the stable equilibrium or stable LCO oscillation depending on the strength of their basin of attraction.

4 CONCLUSIONS

This paper presented a study into experimental nonlinear flutter charcterization in a wind tunnel setting, without the use of numerical model. The flutter rig was described with the experimental setup. Furthermore, example results of the rig nonlinear open loop dynamics were presented and discussed. Control-based continuation (CBC) was used to produce the full bifurcation diagram, continuing stable and unstable equilibrium solutions as well as stable and unstable limit cycle branches. The tests demonstrated that CBC can be applied in challenging environments where there are high levels of noise, in both measurement and flow environment. The improved CBC has given promising results considering multiple degrees-of-freedom systems subjected to fluctuations in the period of oscillation and lack of synchrony between the control target and the actual state of the system.

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5 REFERENCES

[1] A. Abdelkefi, R. Vasconcellos, A. H. Nayfeh and M. R. Hajj (2013) An analytical and experimental investigation into limit-cycle oscillations of an aeroelastic system, *Nonlin. Dyn.* **71**: 159173.

[2] D. Rezgui and M. H. Lowenberg. October 23, 2018. On the nonlinear dynamics of a rotor in autorotation: a combined experimental and numerical approach. *Phil. Trans. R. Soc. A* **373**: 20140411. http://dx.doi.org/10.1098/rsta.2014.0411

[3] D. A. W. Barton (2017) Control-based continuation: Bifurcation and stability analysis for physical experiments, *Mech. Sys. Sig. Proc.***84 (B)**: 54-64.

[4] D. A. W. Barton, A real-time controller (RTC) based on the Beagle Bone Black, https://github.com/dawbarton/rtc/, accessed in September 2018.

[5] D. A. W. Barton and S. G. Burrow (2010) Numerical Continuation in a Physical Experiment: Investigation of a Nonlinear Energy Harvester, *J. Comp. Nonlin. Dyn.* **6 (1)**, Oct 04, 2010.

[6] E. Bureau, F. Schilder, I. F. Santos, J. J. Thomsen and J. Starke (2014) Experimental bifurcation analysis of an impact oscillatordetermining stability, *J. Sound Vib.* **333 (21)**: 5464-5474.

[7] H. Dankowicz and F. Schilder (2013) Recipes for continuation, SIAM.

[8] G. Dimitriadis (2017) Introduction to Nonlinear Aeroelasticity, John Wiley & Sons Ltd.

[9] T. E. Noll, J. M. Brown, M. E. Perez-Davis, S. D. Ishmael, G. C. Tiffany and M. Gaier (2004) Investigation of the Helios Prototype Aircraft Mishap. https://www.nasa.gov/pdf/64317main_helios.pdf, downloaded in September 2018.

[10] E. Papatheou, X. Wei, S. Jiffri, M. Prandina, M. Ghandchi Tehrani, S. Bode, K. Vikram Singh, J. E. Mottershead and J. E. Cooper (2012) Flutter control using vibration test data: theory, rig design and preliminary results. Int. Seminar on Modal Analysis (ISMA2012), Leuven, BE, 17 - 19 Sep, 2012.

[11] J. Pattinson, M. H. Lowenberg and M. G. Goman (2013) A Multi-Degree-of-Freedom Rig for the Wind Tunnel Determination of Dynamic Data. *J. Aircraft* **50 (2)** : 551-566.

[12] J. Sieber and B. Krauskopf. (2008) Control based bifurcation analysis for experiments, *Nonlin. Dyn.* **51**: 365377.

[13] J. Sieber, A. Gonzalez-Buelga, S. A. Neild, D. J. Wagg and B. Krauskopf (2008) Experimental Continuation of Periodic Orbits through a Fold, *Phys. Rev. Lett.* **100, 244101**.

[14] I. Tartaruga, J. E. Cooper, M. H. Lowenberg, P. Sartor and Y. Lemmens (2018) Uncertainty and Sensitivity Analysis of Bifurcation Loci Characterizing Nonlinear Landing-Gear Dynamics *J. Aircraft* **55 (1)**: 162-172.

[15] L. Renson, D. A. W. Barton and S. A. Neild (2017) Experimental Tracking of Limit-Point Bifurcations and Backbone Curves Using Control-Based Continuation, *International Journal of Bifurcation and Chaos* **27 (1)**, 2017.

[16] J. Sieber, B. Krauskopf, D. Wagg, A. Gonzalez-Buelga, S. Neild (2010) Control-based continuation of unstable periodic orbits, *J. Comput. Nonlin. Dyn.* **6**.

[17] http://apsdynamics.com/en/products/vibration-exciter/ball-bearing-exciter.html?file=files%2Fspektra%2Fimages%2Fcontent%2Fschwingungs-und-stosserreger%2FDatenblaetter_EN%2FAPS_113_HF_Data_Sheet_en.pdf, downloaded in September 2018