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Robust sampled-data implementation of PID controller

Anton Selivanov and Emilia Fridman

***Abstract*— We study a sampled-data implementation of the PID controller. Since the derivative is hard to measure directly, it is approximated using a finite difference giving rise to a delayed sampled-data controller. We suggest a novel method for the analysis of the resulting closed-loop system that allows to use only the last two measurements, while the existing results used a history of measurements. This method also leads to essentially larger sampling period. We show that, if the sampling period is small enough, then the performance of the closed-loop system under the sampled-data PID controller is preserved close to the one under the continuous-time PID controller. The maximum sampling period is obtained from LMIs derived using an appropriate Lyapunov-Krasovskii functional. These LMIs allow to consider systems with uncertain parameters. Finally, we develop an event-triggering mechanism that allows to reduce the amount of sampled control signals used for stabilization.**

I. INTRODUCTION

Proportional integral derivative (PID) controllers are extremely popular in the control engineering practice. These controllers depend on the output derivative that can hardly be measured in practice. Instead, the derivative can be approximated using the finite difference $\dot{y} \approx (y(t)-y(t-\tau))/\tau$. This gives rise to a time-delayed controller, which was studied, e.g., in [1], [2] using the frequency domain approach.

In this paper, we study the sampled-data implementation of PID. This problem has been recently considered in [3]. One of the ideas (originated in [4], [5]) was to use the Taylor's expansion for the delayed term with the remainder in the integral form. The remainder is then compensated by an appropriate Lyapunov-Krasovskii term. The data sampling was studied using the time-delay approach [6].

Here, we significantly improve the results of [3]. The key novelty is that the sampling error is calculated for the derivative approximation, while in [3] it was calculated for the delayed measurement. This idea allows to write the stability conditions in terms of the controller gains used in the original continuous PID, while [3] used the sampling-dependent gains obtained after substituting the approximation into the continuous PID (see Remark 3). One of the consequences is that the sampled-data implementation of PID requires to use only the last two measurements, while [3] used a history of measurement whose length was increasing for a decreasing sampling period. Moreover, we obtain significantly larger sampling periods compared to [3] (see Example 1). The stability conditions are formulated in terms of LMIs that are affine with respect to the system parameters. This allows to use them to study systems with uncertain parameters.

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Finally, we develop an event-triggering mechanism that allows to reduce the amount of sampled control signals used for stabilization [7], [8]. Some of the ideas presented here have been generalized in [9] to study time-delayed and sampled-data implementation of control depending on high-order output derivatives.

The analysis will employ the following inequalities.

Lemma 1 (Exponential Wirtinger's inequality [10]): Let $f\colon [a,b]\to\mathbb{R}^n$ be an absolutely continuous function with a square integrable first order derivative such that $f(a)=0$ or $f(b)=0$. Then, for any $\alpha\in\mathbb{R}$ and $0\le W\in\mathbb{R}^{n\times n}$,

$$\int_a^b e^{2\alpha t} f^T(t)Wf(t)\,dt \le e^{2|\alpha|(b-a)}\frac{4(b-a)^2}{\pi^2}\int_a^b e^{2\alpha t}\dot{f}^T(t)W\dot{f}(t)\,dt$$

Lemma 2 (Jensen's inequality [11]): If $f\colon [a,b]\to\mathbb{R}$ and $\rho\colon [a,b]\to[0,\infty)$ are such that the integration concerned is well-defined, then

$$\left[\int_a^b \rho(s)f(s)\,ds\right]^2 \le \int_a^b \rho(s)\,ds\int_a^b \rho(s)f^2(s)\,ds.$$

II. SAMPLED-DATA PID CONTROL

Consider the scalar system

$$\ddot{y}(t)+a_1\dot{y}(t)+a_2y(t)=bu(t) \tag{1}$$

and the PID controller

$$u(t)=\bar{k}_py(t)+\bar{k}_i\int_0^t y(s)\,ds+\bar{k}_d\dot{y}(t). \tag{2}$$

The controller (2) depends on the output derivative, which is hard to measure directly. Instead, the derivative can be approximated by the finite-difference

$$\dot{y}(t)\approx y_1(t)=\frac{y(t)-y(t-h)}{h},\quad h>0. \tag{3}$$

This approximation leads to the delay-dependent control

$$\begin{aligned} u(t)&=\bar{k}_py(t)+\bar{k}_i\int_0^t y(s)\,ds+\bar{k}_dy_1(t)\\ &=k_py(t)+k_i\int_0^t y(s)\,ds+k_dy(t-h), \end{aligned} \tag{4}$$

where[1] $y(t)=y(0)$ for $t<0$ and

$$k_p=\bar{k}_p+\frac{\bar{k}_d}{h},\quad k_i=\bar{k}_i,\quad k_d=-\frac{\bar{k}_d}{h}. \tag{5}$$

[1]Then $\dot{y}(t)$ with $t\in[0,h)$ is approximated by 0

We study the sampled-data implementation of the controller (4) that is obtained using the approximations

$$\begin{aligned}&\int_0^t y(s)\,ds \approx \int_0^{t_k} y(s)\,ds \approx h\sum_{j=0}^{k-1} y(t_j), \qquad t\in[t_k,t_{k+1}),\\ &\dot y(t)\approx \dot y(t_k)\approx y_1(t_k)=\frac{y(t_k)-y(t_{k-1})}{h},\end{aligned}$$

where $h>0$ is the sampling period, $t_k=kh$, $k\in\mathbb{N}_0$, are the sampling instants, and $y(t_{-1})=y(t_0)$. Substituting these approximations into (2), we obtain the sampled-data controller

$$\begin{aligned}u(t)&=\bar k_p y(t_k)+\bar k_i h\sum_{j=0}^{k-1} y(t_j)+\bar k_d y_1(t_k)\\ &=k_p y(t_k)+k_i h\sum_{j=0}^{k-1} y(t_j)+k_d y(t_{k-1}),\\ &\qquad\qquad t\in[t_k,t_{k+1}),\ k\in\mathbb{N}_0\end{aligned}\tag{6}$$

with k_p, k_i, and k_d defined in (5).

We will show that the sampled-data controller (6) stabilizes the system (1) if (2) stabilizes (1) and the sampling period $h>0$ is small enough. Moreover, we will derive LMIs that allow to find appropriate h.

First, we present the estimation error $\dot y(t)-y_1(t)$ in a convenient integral form.

Lemma 3: If $y\in C^1$ and $\dot y$ is absolutely continuous, then y_1 defined in (3) satisfies

$$y_1(t)=\dot y(t)+\kappa(t),\quad \kappa=\int_{t-h}^t \frac{t-h-s}{h}\ddot y(s)\,ds. \tag{7}$$

Proof: Taylor's expansion with the remainder in the integral form gives

$$y(t-h)=y(t)-\dot y(t)h-\int_{t-h}^t (t-h-s)\ddot y(s)\,ds.$$

Reorganizing the terms, we obtain

$$y_1(t)=\frac{y(t)-y(t-h)}{h}=\dot y(t)+\int_{t-h}^t\frac{t-h-s}{h}\ddot y(s)ds.$$

■

To study the stability of (1) under the sampled-data PID control (6), we rewrite the closed-loop system in the state space. Let

$$x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\\x_3(t)\end{bmatrix}=\begin{bmatrix}y(t)\\ \dot y(t)\\ (t-t_k)y(t_k)+h\sum_{j=0}^{k-1}y(t_j)\end{bmatrix}$$

for $t\in[t_k,t_{k+1})$. Introduce the errors due to sampling

$$\begin{aligned}v(t)&=x(t_k)-x(t),\\ \delta(t)&=y_1(t_k)-y_1(t),\end{aligned}\qquad t\in[t_k,t_{k+1}),\ k\in\mathbb{N}_0.$$

Using these representations and (7) in (6), we obtain

$$\begin{aligned}u(t)&=\bar k_p x_1(t_k)+\bar k_i x_3(t_k)+\bar k_d y_1(t_k)\\ &=[\bar k_p,\bar k_d,\bar k_i]x+[\bar k_p,0,\bar k_i]v+\bar k_d(\kappa+\delta).\end{aligned}\tag{8}$$

Then the system (1) under the sampled-data PID control (8) can be presented as

$$\begin{aligned}\dot x&=Ax+A_v v+B(\kappa+\delta),\\ y&=Cx,\end{aligned}\tag{9}$$

where

$$\begin{aligned}A&=\begin{bmatrix}0&1&0\\-a_2+b\bar k_p&-a_1+b\bar k_d&b\bar k_i\\1&0&0\end{bmatrix},\\ A_v&=\begin{bmatrix}0&0&0\\b\bar k_p&0&b\bar k_i\\1&0&0\end{bmatrix},\quad B=\begin{bmatrix}0\\b\bar k_d\\0\end{bmatrix},\quad C=\begin{bmatrix}1&0&0\end{bmatrix}.\end{aligned}\tag{10}$$

Theorem 1: Consider the system (1).

(i) For given sampling period $h>0$, controller gains $\bar k_p$, $\bar k_i$, $\bar k_d$, and decay rate $\alpha>0$, let there exist positive-definite matrices $P,S\in\mathbb{R}^{3\times3}$ and nonnegative scalars W, R such that[2] $\Psi\le 0$, where $\Psi=\{\Psi_{ij}\}$ is the symmetric matrix composed from

$$\begin{aligned}&\Psi_{11}=PA+A^TP+2\alpha P,\quad \Psi_{12}=PA_v,\\ &\Psi_{13}=\Psi_{14}=PB,\quad \Psi_{15}=A^TG,\quad \Psi_{22}=-\tfrac{\pi^2}{4}S,\\ &\Psi_{25}=A_v^TG,\quad \Psi_{35}=\Psi_{45}=B^TG,\\ &\Psi_{33}=-W\tfrac{\pi^2}{4}e^{-2\alpha h},\ \Psi_{44}=-Re^{-2\alpha h},\ \Psi_{55}=-G,\\ &G=h^2e^{2\alpha h}S+h^2\left[\begin{smallmatrix}0&0&0\\0&1&0\\0&0&0\end{smallmatrix}\right](\tfrac14 R+e^{2\alpha h}W)\end{aligned}$$

with A, A_v, B, and C given in (10). Then, the sampled-data PID controller (6) exponentially stabilizes the system (1) with the decay rate α.

(ii) Let there exist $\bar k_p$, $\bar k_i$, $\bar k_d$ such that the PID controller (2) exponentially stabilizes the system (1) with a decay rate α'. Then, the sampled-data PID controller (6) with k_p, k_i, k_d given by (5) exponentially stabilizes the system (1) with any given decay rate $\alpha<\alpha'$ if the sampling period $h>0$ is small enough.

Proof: (i) Consider the functional

$$V=V_0+V_v+V_\delta+V_y+V_\kappa \tag{11}$$

with

$$\begin{aligned}V_0&=x^TPx,\\ V_v&=h^2e^{2\alpha h}\int_{t_k}^t e^{-2\alpha(t-s)}\dot x^T(s)S\dot x(s)\,ds\\ &\quad-\frac{\pi^2}{4}\int_{t_k}^t e^{-2\alpha(t-s)}v^T(s)Sv(s)\,ds,\\ V_\delta&=Wh^2\int_{t_k}^t e^{-2\alpha(t-s)}\dot y_1^2(s)\,ds\\ &\quad-W\frac{\pi^2}{4}e^{-2\alpha h}\int_{t_k}^t e^{-2\alpha(t-s)}\delta^2(s)ds,\\ V_y&=Wh^2e^{2\alpha h}\int_{t-h}^t e^{-2\alpha(t-s)}\frac{s-t+h}{h}\ddot y^2(s)\,ds,\\ V_\kappa&=R\int_{t-h}^t e^{-2\alpha(t-s)}\frac{(s-t+h)^2}{4}\ddot y^2(s)\,ds.\end{aligned}$$

[2]MATLAB codes for solving the LMIs are available at https://github.com/AntonSelivanov/CDC18a

Wirtinger's inequality (Lemma 1) implies $V_v \geq 0$ and $V_\delta \geq 0$. Using the representation (9), we obtain

$$\dot{V}_0 + 2\alpha V_0 = 2x^T P[Ax + A_v v + B(\kappa+\delta)] + 2\alpha x^T Px,$$
$$\dot{V}_v + 2\alpha V_v = h^2 e^{2\alpha h}\dot{x}^T S\dot{x} - \frac{\pi^2}{4}v^T Sv,$$
$$\dot{V}_\delta + 2\alpha V_\delta = Wh^2\dot{y}_1^2 - W\frac{\pi^2}{4}e^{-2\alpha h}\delta^2.$$

Using Jensen's inequality (Lemma 2) with $\rho \equiv 1$, we get

$$\dot{V}_y + 2\alpha V_y = Wh^2 e^{2\alpha h}\ddot{y}^2 - Whe^{2\alpha h}\int_{t-h}^{t} e^{-2\alpha(t-s)}\ddot{y}^2(s)\,ds \leq Wh^2 e^{2\alpha h}\ddot{y}^2 - W\left[\int_{t-h}^{t}\ddot{y}(s)\,ds\right]^2.$$

Differentiating (7), we obtain

$$\dot{y}_1 = \int_{t-h}^{t}\frac{\ddot{y}(s)}{h}\,ds.$$

Therefore,

$$\dot{V}_y + 2\alpha V_y \leq Wh^2 e^{2\alpha h}\ddot{y}^2 - Wh^2\dot{y}_1^2.$$

Using Jensen's inequality (Lemma 2) with $\rho(s) = s - t + h$, we obtain

$$\dot{V}_\kappa + 2\alpha V_\kappa = R\frac{h^2}{4}\ddot{y}^2 - R\int_{t-h}^{t} e^{-2\alpha(t-s)}\frac{s-t+h}{2}\ddot{y}^2(s)\,ds \leq R\frac{h^2}{4}\ddot{y}^2 - Re^{-2\alpha h}\kappa^2.$$

Summing up, we have

$$\dot{V} + 2\alpha V \leq \psi^T\bar{\Psi}\psi + \dot{x}^T G\dot{x},$$

where $\psi = \mathrm{col}\{x, v, \delta, \kappa\}$ and $\bar{\Psi}$ is obtained from Ψ by removing the last two block-columns and block-rows. Substituting (9) for $\dot{x}$ and applying the Schur complement, we find that $\Psi \leq 0$ guarantees $\dot{V} \leq -2\alpha V$. Since $V(t_k) \leq V(t_k^-)$, the latter implies exponential stability of the system (9) and, therefore, of (1), (6).

(ii) The closed-loop system (1), (2) is equivalent to $\dot{x} = Ax$. Since (1), (2) is exponentially stable with the decay rate α', there exists $P > 0$ such that $PA + A^T P + 2\alpha P < 0$ for any $\alpha < \alpha'$. Choose $S = \frac{1}{h}I_2$, $R = \frac{1}{h}$, and $W = \frac{1}{h}$. Applying the Schur complement to $\Psi \leq 0$, we obtain

$$PA + A^T P + 2\alpha P + hF(e^{-2\alpha h}) < 0.$$

The latter holds for small $h > 0$. Thus, (i) guarantees (ii). ∎

Remark 1: Since the LMIs of Theorem 1 are affine in a_1, a_2, and b, they can be used to study uncertain systems of the form (1) with $a_1 \in [\underline{a}_1, \bar{a}_1]$, $a_2 \in [\underline{a}_2, \bar{a}_2]$, and $b \in [\underline{b}, \bar{b}]$. In this case, one needs to solve the LMIs of Theorem 1 for each combination of (a_1, a_2, b) with $a_1 \in \{\underline{a}_1, \bar{a}_1\}$, $a_2 \in \{\underline{a}_2, \bar{a}_2\}$, $b \in \{\underline{b}, \bar{b}\}$ applying the same decision variables. Using the descriptor approach [6], the LMIs of Theorem 1 can be modified to cope with system uncertainties better. In Example 2 considered below this leads to insignificant improvements ($h = 0.023$ using Theorem 1 as it is and $h = 0.024$ using Theorem 1 with a descriptor).

Remark 2: Using the ideas of [3], the results of this paper can be easily extended to the vector systems

$$\ddot{y}(t) + A_1\dot{y}(t) + A_2 y(t) = Bu(t)$$

under the sampled-data PID control

$$u(t) = \bar{K}_p y(t_k) + \bar{K}_i h\sum_{j=0}^{k-1} y(t_j) + \bar{K}_d y_1(t_k), t \in [t_k, t_{k+1}),$$

where $y \in \mathbb{R}^l$, $u \in \mathbb{R}^m$ and A_1, A_2, B, $\bar{K}_p$, $\bar{K}_i$, $\bar{K}_d$ are matrices of appropriate dimensions.

Remark 3: In [3], the system (1) was studied under the sampled-data feedback (cf. (6))

$$u(t) = k_p y(t_k) + k_i h\sum_{j=0}^{k-1} y(t_j) + k_d y(t_{k-q}), \quad t \in [t_k, t_{k+1}),\ k \in \mathbb{N}_0, \quad (12)$$

where q is an integer delay. In the analysis, the errors due to sampling $y(t_k) - y(t)$ and $y(t_{k-q}) - y(t - qh)$ were multiplied by $k_p = \bar{k}_p + \bar{k}_d/(qh)$ and $k_d = -\bar{k}_d/(qh)$ that grow when $qh \to 0$. Consequently, one had to increase the discrete delay q while reducing the sampling period h to maintain k_p and k_d bounded. Here, the errors due to sampling are multiplied by $\bar{k}_p$ and $\bar{k}_d$ that do not depend on h (see v and δ in (8)). This allows to use $q = 1$ and, therefore, smaller memory is required to implement (6) (see Example 1).

III. Event-triggered PID control

We introduce the event-triggering mechanism to reduce the amount of transmitted control signals [7], [8]. Namely, we consider the system

$$\ddot{y}(t) + a_1\dot{y}(t) + a_2 y(t) = b\hat{u}_k, \quad t \in [t_k, t_{k+1}),\ k \in \mathbb{N}_0, \quad (13)$$

where $\hat{u}_k$ is the event-triggered control: $\hat{u}_0 = u(t_0)$,

$$\hat{u}_k = \begin{cases} u(t_k), \text{ if (15) is true,} \\ \hat{u}_{k-1}, \text{ if (15) is false} \end{cases} \quad (14)$$

with $u(t)$ from (6) and the event-triggering condition

$$(u(t_k) - \hat{u}_{k-1})^2 > \sigma u^2(t_k). \quad (15)$$

Here, $\sigma \in [0, 1)$ is the event-triggering threshold.

Remark 4: We consider the event-triggering mechanism with respect to the control signal, since the event-triggering with respect to the measurements $\hat{y}_k = y(t_k) + e_k$ leads to an accumulating error in the integral term:

$$\int_0^{t_k} y(s)\,ds \approx h\sum_{j=0}^{k-1}\hat{y}_j = h\sum_{j=0}^{k-1} y(t_j) + h\sum_{j=0}^{k-1} e_j.$$

Theorem 2: Consider the system (13).

(i) For given sampling period $h > 0$, controller gains $\bar{k}_p$, $\bar{k}_i$, $\bar{k}_d$, event-triggering threshold $\sigma \in [0, 1)$, and decay rate $\alpha > 0$, let there exist positive-definite matrices

$P, S \in \mathbb{R}^{3\times 3}$ and nonnegative scalars W, R, ω such that[3] $\Phi \le 0$, where

$$\Phi = \left[\begin{array}{c:cc} \Psi & \begin{matrix} PB\bar{k}_d^{-1} \\ 0 \\ 0 \\ 0 \\ GB\bar{k}_d^{-1} \end{matrix} & \begin{matrix} \Phi_{17} \\ \Phi_{27} \\ \sigma\omega\bar{k}_d \\ \sigma\omega\bar{k}_d \\ 0 \end{matrix} \\ \hdashline * * * * * & -\omega & 0 \\ * * * * * & 0 & -\sigma\omega \end{array}\right],$$

$$\Phi_{17} = \sigma\omega \begin{bmatrix} \bar{k}_p \\ \bar{k}_d \\ \bar{k}_i \end{bmatrix}, \quad \Phi_{27} = \sigma\omega \begin{bmatrix} \bar{k}_p \\ 0 \\ \bar{k}_i \end{bmatrix}$$

with A, A_v, B, and C given in (10). Then, the event-triggered PID controller (6), (14), (15) exponentially stabilizes the system (13) with the decay rate α.

(ii) Let there exist $\bar{k}_p$, $\bar{k}_i$, $\bar{k}_d$ such that the PID controller (2) exponentially stabilizes the system (1) with a decay rate α'. Then, the event-triggered PID controller (6), (14), (15) exponentially stabilizes the system (13) with any given decay rate $\alpha < \alpha'$ if the sampling period $h > 0$ and the event-triggering threshold σ are small enough.

Proof: Introduce the event-triggering error $e = \hat{u}_k - u(t_k)$ for $t \in [t_k, t_{k+1})$. Then (13) under the event-triggered PID control (6), (14), (15) can be presented as

$$\begin{aligned} \dot{x} &= Ax + A_v v + B(\kappa + \delta + \bar{k}_d^{-1} e), \\ y &= Cx, \end{aligned} \tag{16}$$

for $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}_0$, with A, A_v, B, and C defined in (10). Consider the functional (11). For $\omega \ge 0$, the event-triggering rule (14), (15) guarantees

$$0 \le \omega\sigma u^2(t_k) - \omega e^2.$$

Thus, we have

$$\begin{aligned} \dot{V} + 2\alpha V &\le \dot{V} + 2\alpha V + [\omega\sigma u^2(t_k) - \omega e^2] \\ &\le \varphi^T \bar{\Phi} \varphi + \dot{x}^T G \dot{x} + \omega\sigma u^2(t_k), \end{aligned}$$

where $\varphi = \mathrm{col}\{x, v, \delta, \kappa, e\}$ and $\bar{\Phi}$ is obtained from Φ by removing the blocks Φ_{ij} with $i \in \{5, 7\}$ or $j \in \{5, 7\}$. Substituting (16) for $\dot{x}$ and (8) for $u(t_k)$ and applying the Schur complement lemma, we find that $\Phi \le 0$ guarantees $\dot{V} \le -2\alpha V$. The remainer of the proof is similar to that of Theorem 1. ∎

Remark 5: The results of Theorem 2 can be applied to uncertain systems in a manner similar to Remark 1.

IV. Numerical examples

Example 1: Following [2], [3], we consider (1) with $a_1 = 8.4$, $a_2 = 0$, $b = 35.71$. The system is not asymptotically stable if $u = 0$. The PID controller (2) with $\bar{k}_p = -10$, $\bar{k}_i = -40$, $\bar{k}_d = -0.65$ exponentially stabilizes (1). Let $\alpha = 5$ be the desired decay rate. The LMIs of Theorem 1 are feasible for $h = 0.019$, which is larger than $h = 4.7 \times 10^{-3}$ obtained in [3]. This leads to the controller gains $k_p \approx -44.21$, $k_i = -40$, and $k_d \approx 34.21$ calculated using (5). Moreover, [3] considered (12) with $q = 7$. In our case, $q = 1$, which leads to a smaller memory used in the implementation.

[3]MATLAB codes for solving the LMIs are available at https://github.com/AntonSelivanov/CDC18a

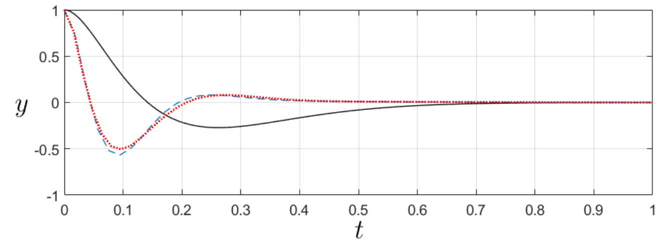

Fig. 1. Example 1: system (1) under continuous-time control (2) (black solid line), sampled-data control (6) (blue dashed line), event-triggered control (14) (red dotted line).

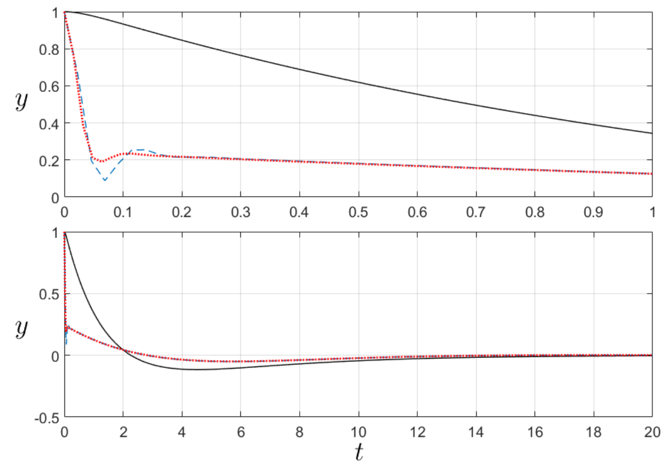

Fig. 2. Example 2: system (1) under continuous-time control (2) (black solid line), sampled-data control (6) (blue dashed line), event-triggered control (14) (red dotted line).

Consider now the system (13) with the same parameters. The LMIs of Theorem 2 are feasible for $h = 0.016$, $\sigma = 0.02$, implying that the event-triggered PID controller (6), (14), (15) stabilizes the system (13). Sampled-data control (6) requires to transmit $\lfloor 10/h \rfloor + 1 = 527$ control signals during 10 seconds of simulations. The event-triggered control requires to transmit on average 325.9 control signals. This value was found performing numerical simulations for 10 randomly chosen initial conditions satisfying $\|x(0)\|_\infty \le 1$. Thus, the even-triggering mechanism reduces the amount of transmitted control signals by almost 40%. However, due to a smaller sampling period, it requires to transmit more measurements. Nevertheless, the total amount of transmitted signals is reduced by almost 10%. Fig. 1 shows $y(t)$ for various types of control.

Example 2: Following [12], we consider (1) with

$$a_1 \in [0.01248, 9.251], \quad a_2 \in [5.862, 22.19], \\ b \in [0.03707, 0.04612].$$

The PID controller (2) with

$$\bar{k}_p = -516.6, \quad \bar{k}_i = -143.8, \quad \bar{k}_d = -765.5$$

stabilizes the system (1). The LMIs of Theorem 1 are feasible for $\alpha = 0.1$, $h = 0.023$ implying that the sampled-data controller (6) with $k_p \approx -3.38 \times 10^4$, $k_i = -143.8$, $k_d \approx 3.33 \times 10^4$ exponentially stabilizes the system (1) with the decay rate $\alpha = 0.1$. The LMIs of Theorem 2 are feasible for $\alpha = 0.1$, $h = 0.016$, $\sigma = 0.1$. Thus, the system (13) is exponentially stable under the event-triggered control (6), (14), (15). Sampled-data control (6) requires to transmit $\lfloor 20/h \rfloor + 1 = 870$ control signals during 20 seconds of simulations. The event-triggered control requires to transmit on average 103.1 control signals. This value was found performing numerical simulations for $a_1 = 2.674$, $a_2 = 10.97$, $b = 0.04107$ and 10 randomly chosen initial conditions satisfying $\|x(0)\|_\infty \leq 1$. Thus, the event-triggering mechanism reduces the amount of transmitted control signals by more than 88%. The total amount of transmitted signals is reduced by more than 20%. Fig. 2 shows $y(t)$ for various types of control.

V. CONCLUSION

PID controllers are widely used in the industry. For practical application, their sampled-data implementation is important. This paper provides an efficient method for such implementation. The results are formulated in terms of simple LMIs and are applicable to uncertain systems.

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