Sensitivity analysis of optimal routes, departure times and speeds for fuel-efficient truck journeys

David Watling
Institute for Transport Studies
University of Leeds
Leeds, U.K.
d.p.watling@its.leeds.ac.uk

Richard Connors
Institute for Transport Studies
University of Leeds
Leeds, U.K.
r.d.connors@its.leeds.ac.uk

Haibo Chen
Institute for Transport Studies
University of Leeds
Leeds, U.K.
h.chen@its.leeds.ac.uk

Abstract—Embedded within the vehicle "routing" problem of determining the order in which customers are served, is the route choice problem of which sequence of roads to use between a pair of pick-up/drop-off locations, and this latter is the focus of the paper. When the objective is something other than travel time, such as fuel consumption, an additional control dimension is that of speed, and in a time-varying context the question of optimal speed determination is no longer a local one, due to potential downstream interactions. This also brings in the possibility to adjust departure times. Recently this problem, of joint route, departure time and speed determination for fuel minimization in a time-varying network, was shown to be efficiently solvable using a Space-Time Extended Network (STEN). In the present paper, we explore the sensitivity of the optimal solutions produced to: i) the fidelity of the within-day traffic information; ii) the currency of between-day traffic information in comparison with historical mean conditions; iii) the availability of historical information on variability for risk-averse routing; and iv) competition from other equally-optimal or near equally-optimal solutions. We set out the methods by which each of these tests may be achieved by adaptation of the underlying STEN, taking care to ensure a consistent reference basis, and describe the potential real-life relevance of each test. The results of illustrative numerical experiments are reported from interfacing the methods with real-time data accessed through the Google Maps API.

Keywords— network, route choice, dynamics, variability, fuel

I. INTRODUCTION

Heavy Goods Vehicles continue to be a major contributor to both CO2 emissions and fuel consumption, in spite of major improvements in vehicle technology [1]. Freight transport by all modes accounts for around a third of greenhouse gas emissions, with HGVs estimated to be responsible for up to a half of these [2]. In the present paper we consider ways in which the behaviour of trucks—in terms of the sequence of roads they follow, the time-of-day they travel, and the speeds they aim to attain—may be modified using real-time predictive information in order to reduce such impacts, with a particular emphasis on fuel consumption.

Our focus differs from many previous studies of fuel optimization for trucks, which have focused either on instantaneous optimization—namely real-time control of a vehicle's powertrain system to reduce instantaneous emissions [3] [4]—or (at the other extreme) planning the sequence and/or timing of pickups/deliveries on a tour, as a variant of classical vehicle routing problems [5] [6].

The context we consider is an intermediate one between these two scales of analysis whereby, in the light of real-time information, (re-)planning is made of a single leg of the vehicle's tour (a leg being between a single pair of delivery/pickup locations). Importantly, given the time-varying nature of traffic congestion, this analysis considers any downstream impacts on that leg, with the result being a choice of route for the current leg, the departure time for the leg, and the 'target speed' on the links of the route.

As an example of the downstream impacts we may try to avoid, when considering fuel consumption for the whole leg, it may in some cases be better to go faster than the instantaneously fuel-optimal speed on one link, in order that the truck passes a downstream link before the onset of a recurrently-congested period, in which (if the truck did not avoid this period) stop-and-start traffic would burn more fuel than is lost on the upstream link. As an alternative example, when notification of an incident ahead occurs, it may be preferable from a fuel perspective to reduce the current speed (so that the incident impacts might pass), or to switch to an alternative route, or to delay the start-time of the leg if it has not yet commenced. We refer to this class of optimization problems as the 'tactical' level context, since it sits between the problems of instantaneous optimization and determining the optimal sequence/scheduling of pickups/drop-offs.

In our own recent research [8], we considered this tactical level context in a deterministic setting, assuming perfect predictive real-time information to be available. It was shown how the problem of jointly optimizing route, departure time, speeds and stops could be reformulated as a shortest path problem over a Space-Time Extended Network (STEN). In computational experiments it was shown how such a problem could be solved in a fraction of second, even at a high temporal resolution when the STEN is of size tens of millions of links.

In the present paper, we develop this method and formulation further, in order to allow a panel of systematic tests to be performed to understand the sensitivity of the optimal solutions generated to various aspects of the problem and data specification. In order to do so, we pay special attention to the way in which any solutions may be mapped, post-optimization, to a common reference-specification in order to obviate ambiguity in between-specification comparisons. At the same time, we discuss the real-life relevance of the comparative tests, and conclude by reporting some illustrative numerical experiments. The structure of the paper is as follows. We begin, in section II, by summarising the key elements of our previously-defined optimization formulation, which we aim to extend and develop in the present paper. In section III, we set out the panel of tests we wish to perform, describe how they may be implemented and discuss their relevance to the real-world. In section IV, the results of applying the method to a case study are presented,

This research was supported by funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 713788 [7].

before discussing conclusions and future research directions in section V.

II. Formulation of Optimization Problem

In the present section we summarize the main pertinent elements of the formulation presented in [8], which we shall in later sections extend/develop. The decision problem considered is that of a truck travelling on a particular leg between a given pair of pickup/delivery locations. Given that the truck type, load, height and width are known for this leg, it is possible to define a permissible network for this leg, respecting any physical restrictions, and this is defined as a graph G={V,E}, where V is the set of vertices/nodes and E is the set of directed links joining the nodes. It is supposed that time, be it clock time or travel time, is measured in some given discrete units contained in the set T.

Although the formulation can apply equally to any point during the trip leg, in the present paper we shall restrict attention to the case where real-time decisions are made at the start of the leg. This is interesting as it allows an extra dimension of departure time choice. A second restriction we shall make, relative to the general case, is that we shall not consider here the possibility of breaks/stops in the journey. Thus, the decision dimensions are route, departure time, and link speeds (travel times), the latter subject to the constraint that clearly no link's selected travel time can be lower than the minimal travel time permitted either by truck speed limits or (time-dependent) congestion.

Suppose, then, at some given discrete clock-time $k \in T$, based on real-time predictive information and given any relevant speed limits for the truck we are routing, $\tau_{ijt,\min}$ denotes the minimum time to traverse the link from node $i \in V$ to node $j \in V$ for a truck of this type entering the link at time $t \in T$. Note that importantly, these predicted minimum travel times are measured in the same discrete units as clock-time. In addition, given that we shall consider possibilities in which it may be optimal to go slower than the minimal travel time suggests, we specify also $\tau_{ijt,\max}$, again in the common discrete time units and denoting the maximum travel time from i to j when leaving i at time t. These maximum values are user-specified (as opposed to the minimum values, which are informed by data); while specifying larger values allows a wider range of possibilities to be explored, there is a computational trade-off in that this will expand the size of the STEN created.

For the case in which we are routing from node $i^* \in V$ at a fixed departure time $d \in T$, to arrive ultimately at node $j^* \in V$, over a physical network G={V,E}, where the predicted minimum and maximum travel times at any clock time are contained in the vectors $\boldsymbol{\tau}_{\min}$ and $\boldsymbol{\tau}_{\max}$, then a STEN $(\tilde{V}, \tilde{E})$ is created by the mapping:

$$(\tilde{V}, \tilde{E}) = f(V, E, i^*, j^*, d, \boldsymbol{\tau}_{\min}, \boldsymbol{\tau}_{\max})$$

which is defined by the following procedure:

Step 1: Create a new vertex set $\tilde{V}$ by:

i) creating a space-time vertex (i, t) for each non-source node $i \in V \setminus \{i^*\}$ and each clock-time $t \in T$;

ii) creating a single space-time vertex (i^*, d) for the given source and departure time;

iii) creating a single, common, dummy ultimate sink vertex $j_{\text{SINK}} \notin V$ with a free (undefined) time, i.e. a space-time vertex (j_{SINK}, free).

Step 2: Create a new link set $\tilde{E}$ by:

i) creating a link for each non-source node $i \in V \setminus \{i^*\}$ and each link $(i, j) \in E$, by adding a space-time travelling link from each space-time vertex $(i, t) \in (V \setminus \{i^*\}, T)$ to $(j, t + \theta)$, for all integers θ such that

$$\tau_{ijt,\min} \le \theta \le \tau_{ijt,\max}$$

ii) creating a link from the space-time source node (i^*, d) and each link $(i^*, j) \in E$, by adding a space-time travelling link from (i^*, d) to $(j, d + \theta)$, for all integers θ such that

$$\tau_{i^*jd,\min} \le \theta \le \tau_{i^*jd,\max} \ ;$$

iii) creating a link from each space-time sink node (j^*, t) ($t \in T$) to the dummy ultimate sink node (j_{SINK}, free).

The STEN is used as the way of imposing the space-time constraints on what is physically possible; by definition, any route through the STEN is feasible in space and time. We may then use this as the network definition in a shortest path algorithm, and can choose any criterion to 'weight' the space-time links. For example, if we choose travel time as the weight, then a shortest path through the STEN is a minimum time path. In fact the procedure we outline is over-complex in such a case, since under travel time minimisation we know that only the space-time links from (i, t) to $(j, t + \tau_{ijt,\min})$ will ever be potentially used. The value of the procedure outlined is that it maintains space-time feasibility with criteria other than travel time minimisation; the particular one we focus on in the present paper is that in which the weights represent the amount of fuel consumed, which is related to the travel time implied by each space-time link. A shortest path in the STEN is then one that minimises fuel with respect to both route and speeds.

While the procedure above is for a fixed departure time, it may be extended to additionally embed choice of departure time by:

- a time-expansion of the source-node, which means we simply remove the special treatment of source nodes in steps 1(i)/(ii) and 2(i)/(ii) above;
- creating a single super-origin with links connected to the time-expanded source nodes.

III. Sensitivity Tests: Methods and Interpretation

In the present section we set out the issues to be explored trough sensitivity testing, the systematic methods used for each, and the real-life interpretations/applications.

A. Benefit of high resolution within-day data

The first issue considered is the influence of the temporal resolution at which the data on within-day variation in travel times is available. An important point to note first is that this is a different issue from the question of the temporal discretisation chosen for the STEN described in Section II, which is instead more of an 'algorithmic' question: the finer the detail in the STEN, the better the base data is reflected, but the larger the STEN becomes, which has a computational cost. Therefore, as the question of the temporal resolution of the input data is a different one, it is important to run any comparative experiments on the same, finest grain STEN (as we do later, in Section IV).

The experiments to be run require both an averaging step (to translate from a finer to a coarser scale time resolution for the data) and an interpolation step (to map to a common time resolution for the STEN). However, in order to obtain a consistent basis for comparison, the optimization results are all evaluated on a common time-resolution. This last step takes a little care to implement, since in general it will not be possible to exactly follow the optimal speeds (travel times) as determined at the coarse scale (when the coarse-level suggests a minimum travel time less than the minimum defined at the finer scale), even though clearly we can follow the same route and departure time. Thus the process for running the model on a coarser time-resolution than the base data resolution runs like this:

1. Choose a time-resolution for the STEN, finer than the base data resolution, and fix this across all tests (in the later experiments we use a one minute resolution).
2. Choose within-day data time resolution, and map the (finer) base data resolution to it by averaging.
3. Project the within-day data time-resolution in step 2 to the common STEN time-resolution through linear interpolation.
4. Find optimal departure time, route and speeds according to STEN formulation from step 3, and based on some specified optimization objective.
5. Evaluate the optimal solution created in step 4 by projecting it to a "closest" feasible solution at the finer grain time-resolution, through a sequential process. Traversing the links of the route in sequential order, then firstly calculate the entry time to that link, given the travel times chosen for upstream links. Then for the link under consideration, choose the travel time (speed) as defined by the optimal solution from step 4, unless this is less than the minimum time allowed for this entry time at the finer level, in which case set the travel time to the minimum travel time at the finer level.

The real-life relevance of such tests is that data may not always be available at a fine resolution, and therefore it is interesting to explore the impact of this resolution, and whether the impacts may differ for different routing objectives. Conversely, these tests may be described as revealing the importance of finely defined time-varying data, as a means of understanding "the value of information" in a within-day sense.

B. Impact of using between-day averaged data

The second issue explored is the influence/importance of between-day variation in the time-dependent travel time profiles. In particular, the question we explore is: what is the difference between running the optimization on day-averaged travel time profiles versus the profiles for a particular day? A similar consideration arises as to that which was described in section III.A, namely that in order to ensure a common basis for comparison, some care is needed in evaluating the optimal solutions with respect to a common basis. In particular, we shall explore what happens when the optimal departure time, route and speeds from the day-averaged data is applied to the day-specific cases. The process for evaluating the day-averaged strategy is:

1. Create day-averaged travel time profiles from the underlying day-specific data.
2. Find optimal departure time, route and speeds according to STEN formulation with the average profiles from step 1, and based on some specified optimization objective.
3. For each day in turn, project the solution from step 2 to a "closest" feasible solution at the day-specific level, through a sequential process. Traversing the links of the route in sequential order, then firstly calculate the entry time to that link, given the travel times chosen for upstream links. Then for the link under consideration, choose the travel time (speed) as defined by the optimal solution from step 2, unless this is less than the minimum time allowed for this entry time from the day-specific data, in which case set the travel time to the minimum travel time from the day-specific data.

The real-life relevance of this work is to understand what happens when day-specific data are not available, and we must then route based on historical conditions. Alternatively an "uninformed" operator routing without real-time information might be said to be acting somewhat like the day-averaged case, at least for choice of physical route. These experiments might be described as aiming to reveal "the value of information" in a between-day sense.

C. Impact of risk-averse planning

This set of experiments is run in the same way as for the case described in section III.B, except that here some upper percentile value (across days) is used for the travel time at each link entry-time. However, contrary to the case considered in section III.B, understanding historical percentiles is not the behaviour of an uninformed operator. Indeed, any operator must have some initial route for the day ahead, and this case might therefore be like a risk-averse informed operator might behave. Thus, these tests may be said to be indicative of "the value of information" in a distributional between-day sense.

D. Competition from (near) equally optimal solutions

While running a shortest path algorithm on a STEN provides a particular solution, it is quite possible that other feasible solutions exists which are equally optimal, or near equally optimal. This issue is explored by creating a series of STENs, each constrained to a small part of the departure time interval, and calculating optimal route and speed profiles for each constrained departure interval. The value of the objective function (and other metrics) at these constrained optimal solutions may then be compared across the range of departure times.

The real-life relevance of this work is related to the fact that generating a set of good/near-optimal potential solutions (with respect to measures such as fuel and travel time) is particularly beneficial given that there will be other important trade-offs to be made, for example related to the impact of delayed arrival times on customer-satisfaction, perishable goods, or just-in-time delivery processes, or to the influence of driver work shift durations on the overall transportation costs.

IV. Sensitivity Tests: Results

We consider a 40T truck travelling from a depot in Northern England to the Eurotunnel. The network, comprising motorway-standard links, is illustrated in **Figure 1** with origin [yellow] and destination [green] marked. There are 51 nodes and 66 directional links. The origin and destination are fixed for all tests.

Travel times for the network links were obtained from the Google Maps live traffic API which was polled every 15 minutes (for each link) for one week, to get real-time predicted travel times on each link. While this does not provide actual travel time data, it is a useful proxy.

Since the shortest link free flow time is approximately 2 minutes, we discretise on this basis (for the purposes of creating the STEN); the 15 minute link travel time data are resampled (with linear interpolation) at 1 minute intervals.

In all of the experiments reported below, one of two routes turns out to be optimal, and so we identify and number these routes here for ease of later reference.

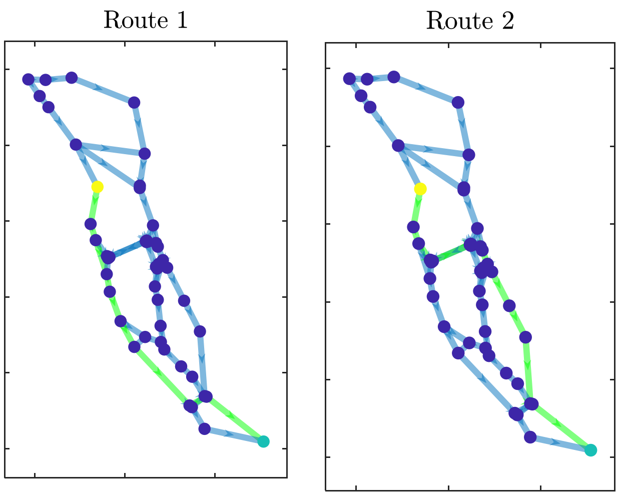

Figure 1: Network and Optimal Routes

The CMEM fuel rate model presented in Franceschetti et al (2013) has optimal speed 55.19km/h. We adopt the functional form of this model, but alter its parameters to increase the optimal (minimum fuel use) speed to 77.79 km/h. This results in the fuel minimization objective function more often facing constraints due to congestion. Our data shows 28% of recorded link speeds are below 77.79km/h (whereas only 6% are below 55.19km/h) so when minimizing fuel the optimal speed is compromised on many/most routes. The resulting fuel rate model is shown in **Figure 2**.

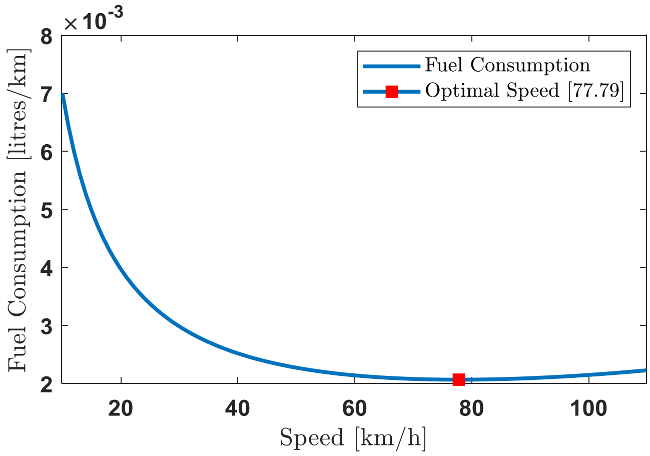

Figure 2. Assumed relationship between speed and fuel consumption

A. Benefit of high resolution within-day data

To investigate the impact of using lower resolution link travel time data, we consider route optimization using the departure time window of 05:00 – 07:00 on Monday 5th Nov. Using full resolution (15min) data we compute the routes minimizing travel time and minimizing fuel use. We generate lower resolution data by splitting the day into W hour width time-bins, using the mean link travel time within that time-bin. In all cases the link travel time data are interpolated to give 1 minute discretization as the basis for the STEN. The link travel time data for link 1 is shown in **Figure 3**.

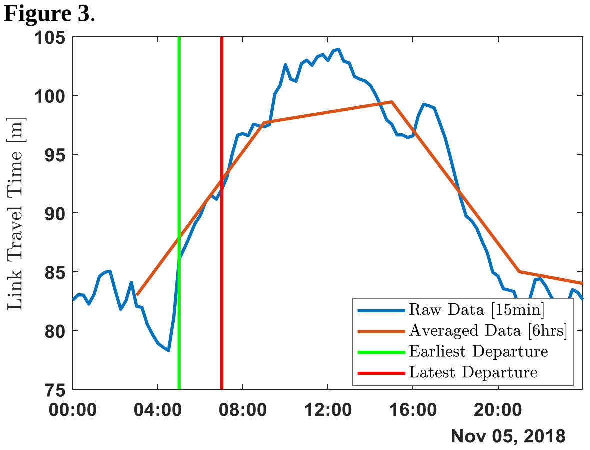

Figure 3. Example of link travel time data interpolated from 15min and 6hr resolutions, with departure time window marked. Note 6hr data suggests infeasibly low travel time when departing at 12:00.

Optimization using lower resolution data gives an optimal route i.e. a departure time, a sequence of links to travel and the desired speed on each link. This results in a predicted travel time and predicted fuel use (italics in **Table 1** and **Table 2**). However, analysing this route using the highest resolution data may reveal that desired travel times are not always feasible. This gives an "actual" travel time and fuel consumption. The discrepancies, particularly in travel time, can be significant.

Table 1. Minimize Travel Time with Less Refined Data

Data time-bin width	Solution/Metrics			
	Departure Time	Travel time	Fuel consumption	Route
15min	06:59	450.00	77.21	1
2hr actual	06:59	458.98	77.23	1
2hr Predicted	06:59	457.00	77.24	1
4hr actual	06:48	459.70	77.21	1
4hr predicted	06:48	454.00	77.21	1
6hr actual	05:01	478.03	77.14	1
6hr predicted	05:01	463.00	77.06	1
8hr actual	05:54	482.65	76.38	2
8hr predicted	05:54	449.00	75.54	2

Table 2. Minimize Fuel Consumption with Less Refined Data

Data time-bin width	Solution/Metrics			
	Departure Time	Travel time	Fuel consumption	Route
15min	06:59	489.00	75.60	2
2hr actual	06:51	488.91	75.74	2
2hr Predicted	06:51	482.00	75.54	2
4hr actual	06:27	495.66	75.97	2
4hr predicted	06:27	481.00	75.38	2
6hr actual	05:02	513.06	76.10	2
6hr predicted	05:02	488.00	75.45	2
8hr actual	05:26	507.88	76.26	2
8hr predicted	05:26	478.00	75.27	2

B. Impact of using between-day averaged data

We average the link travel time data over 5 days (Mon – Fri), for each link, at each 15 minute time point. Using these "mean day" link travel times we compute the optimal routes minimizing travel time and then minimizing fuel. We then consider these routes executed each day in turn, taking the departure time from the origin and attempting to follow the link travel times/speeds when possible along the prescribed mean-day optimal routes. We compare these results with the routes optimized on each day. The results are shown in **Table 3** and **Table 4**, where in all cases departure time is constrained to be 05:00 – 07:00.

Table 3. Minimum Travel Time on each day vs Mean Day

Day	Minimise Travel Time Optimise on Each Day				Minimise Travel Time Mean-Day Solution			
	Dep. time	Travel time	Fuel	Route	Dep. time	Travel time	Fuel	Route
μ					06:55	454	77.13	1
5	06:59	450	77.21	1	06:55	456.62	77.12	1
6	06:57	452	77.19	1	06:55	456.29	77.13	1
7	06:59	451	75.82	2	06:55	464.53	77.29	1
8	06:57	446	77.21	1	06:55	455.74	77.12	1
9	05:01	440	75.45	2	06:55	473.13	77.40	1

Table 4. Minimum Fuel Route on each day vs mean Day

Day	Minimise Fuel Optimise on Each Day				Minimise Fuel Mean-Day Solution			
	Dep. Time	Travel time	Fuel	Route	Dep. time	Travel time	Fuel	Route
μ					06:59	483	75.43	2
5	06:59	489	75.60	2	06:59	487.84	75.62	2
6	06:59	490	75.61	2	06:59	488.16	75.64	2
7	05:01	474	75.38	2	06:59	485.44	75.52	2
8	06:59	484	75.35	2	06:59	484.00	75.43	2
9	05:01	474	75.16	2	06:59	503.08	75.91	2

C. Impact of risk-averse planning

For each link, at each 15minute time point, we construct a link travel time timetable using the 80th percentile of link travel time data over 5 days (Mon – Fri). Using these link travel times we compute optimal routes minimising travel time and minimising fuel. As above, we follow these routes on each day in turn, using the departure time and attempting to follow the link travel times/speeds when possible along the prescribed route. If the desired speed is not feasible, due to congestion on that day, the closest feasible speed is used. We compare these results with the routes optimised on each day. The results are shown in **Table 5** and **Table 6**, again with in all cases departure time constrained to be 05:00 – 07:00.

Table 5. Risk Averse Minimum Travel Time

Day	Minimise Travel Time Optimise on Each Day				Minimise Travel Time 80th Percentile Solution			
	Dep. Time	Travel time	Fuel	Route	Dep. Time	Travel time	Fuel	Route
μ					05:01	463.00	77.34	1
5	06:59	450	77.21	1	05:01	464.95	77.33	1
6	06:57	452	77.19	1	05:01	464.95	77.33	1
7	06:59	451	75.82	2	05:01	465.04	77.32	1
8	06:57	446	77.21	1	05:01	464.72	77.33	1
9	05:01	440	75.45	2	05:01	464.46	77.33	1

Table 6. Risk Averse Minimum Fuel Consumption

Day	Minimise Fuel Optimise on Each Day				Minimise Fuel 80th Percentile Solution			
	Dep. Time	Travel time	Fuel	Route	Dep. Time	Travel time	Fuel	Route
μ					06:59	492	75.67	2
5	06:59	489	75.60	2	06:59	493.53	75.69	2
6	06:59	490	75.61	2	06:59	493.36	75.69	2
7	05:01	474	75.38	2	06:59	493.00	75.67	2
8	06:59	484	75.35	2	06:59	493.00	75.67	2
9	05:01	474	75.16	2	06:59	506.53	76.04	2

In **Table 6**, minimising fuel, the results for Friday 9th are notable. The risk averse solution gives consistent performance across days 5th – 8th , but is highly suboptimal on 9th when both travel time and fuel use increase markedly. The cause of this can be determined by comparing the link travel times (hence speeds) from the solution optimised for day 9, with that attained by the 80th percentile solution. Link 13 highlights the problem (see **Figure 4**). Severe congestion on Friday 9th is avoided by the day-optimised solution (departure time, shown by black dot, moves earlier to 05:01) whereas the 80th percentile solution departing at 06:59 enters link 13 (red dot) and the slow traffic dictates highly suboptimal link speed, increasing fuel use and travel time on this day.

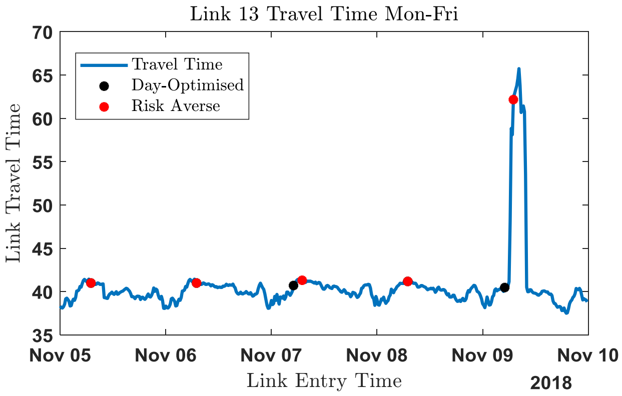

Figure 4. Link 13 travel time. Link entry times shown by black dots (day optimized) and red dots (80th percentile).

D. Competition from (near) equally optimal solutions

Only two routes appear as optimal solutions (for either objective) and many of the optimal results shown above are similar. This prompts examination of whether or not route optimization is worthwhile on these data, of if many (near) equally optimal options exist. In each 10min departure time window for the period 04:00 – 16:00, we compute the optimal travel time and optimal fuel consumption, first when constrained to route 1, and then constrained to route 2. Note that even with fixed departure time, the choice of objective (minimizing travel time vs fuel consumption) changes the optimal speed profile, which is then optimised considering the entire STEN for this route.

The plots in **Figure 5** and **Figure 6** are for Monday 5th. For comparison the plots in **Figure 7** and **Figure 8** are for Friday 9th. Travel times on route 1 & 2 are shown in blue, fuel consumption in orange. When optimizing fuel use, figures 6 & 8 show that route 2 is always optimal, significantly outperforming route 1. The travel time penalty incurred by optimizing for fuel use is seen comparing the blue lines across **Figure 5** and **Figure 6**, and across **Figure 7** and **Figure 8**. When seeking to optimise travel time, both routes 1 and 2 need to be considered; the blue curves cross each other several times, particularly in **Figure 5**. It is also notable that at some departure times there are significant differences in the route travel times.

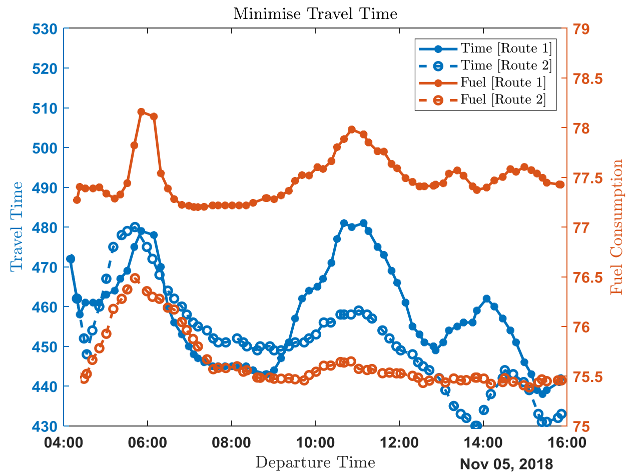

Figure 5. Variation in route performance, minimizing travel time with fixed departure times: Mon 5th Nov.

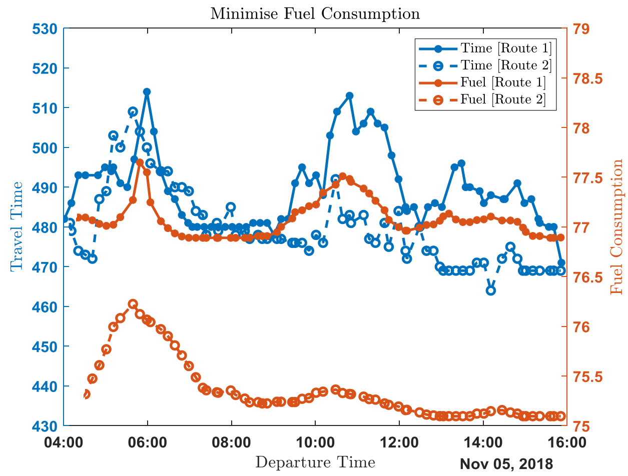

Figure 6. Variation in route performance, minimizing fuel with fixed departure times: Mon 5th Nov.

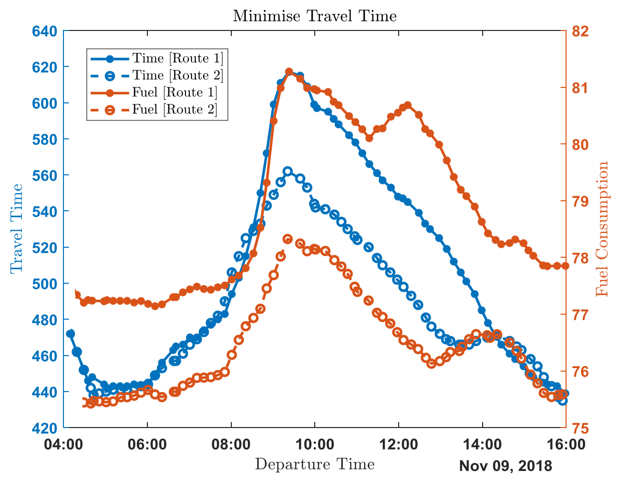

Figure 7. Variation in route performance, minimizing travel time with fixed departure times: Fri 9th Nov.

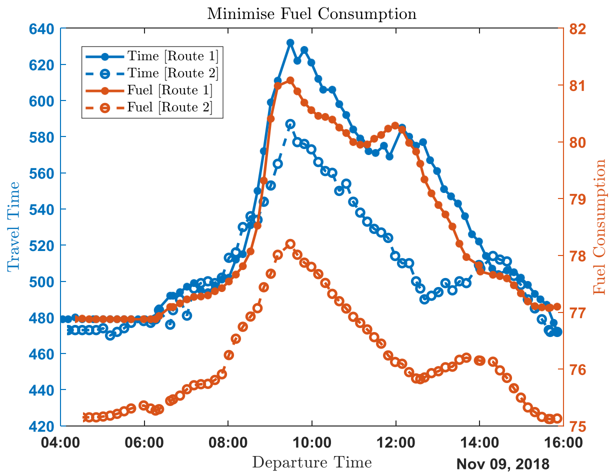

Figure 8. Variation in route performance, minimizing fuel with fixed departure times: Fri 9th Nov

V. Conclusions

Complementary to previous works on green vehicle routing around a sequence of customers, the present paper has considered the influence (on fuel consumption and travel time) of the particular series of roads traversed between a pair of pick-up/drop-off locations, along with departure time and speeds. While an efficient STEN formulation for this problem has been previously proposed, the sensitivity of the optimal solutions so produced to various aspects of the problem definition has not been previously explored. In the paper, we have explained by careful construction of tests, in some cases requiring optimal solutions to be mapped to a common reference-frame through a second STEN formulation, it is possible to obtain meaningful numerical comparisons that avoid the 'noise' of potentially confounding factors. The numerical tests we have reported are intended to be illustrative of the methods, rather than providing definitive results. Nevertheless, they have generated several suggestive hypotheses, which might be further explored in more comprehensive studies in the future.

In terms of further research directions, there are several aspects of the analysis that merit further investigation. The tests reported have used a quite widely-used fuel consumption model, which in its base form depends on both speed and acceleration. Our work has used only the speed element, yet it would be possible to implement the same methods by assuming standard within-link driving profiles that use the link travel time as a boundary condition, which would better reflect realistic fuel consumption influences. A restriction of the work reported here is that we have neglected the influence of breaks in the journey, either mandatory ones required due to driver working legislation, or discretionary ones that may be good to take from a fuel-consumption perspective when (for example) waiting for incident effects to clear. Finally, the routing problem we consider is only one component of an overall routing system, where we have simply assumed predictive journey information to be available. It would be highly instructive to combine the optimization methods described with real-time journey prediction methods, in order to obtain a "whole-system" evaluation of such fuel-reducing strategies.

Acknowledgment

We acknowledge the use of data from the Google Maps API. This research was funded by the EU Horizon 2020 optiTruck project https://optitruck.eu/

References

[1] BEIS, "2015 UK Greenhouse Gas Emissions, Final Figures. Statistical Release: National Statistics.," Department for Business, Energy and Industrial Strategy., U.K., 2017.

[2] FTA, "Logistics Carbon Reduction Scheme, First Annual Report: Recording, reporting and reducing CO2 Emissions from the logistics sector.," Freight Transport Association, U.K., 2010.

[3] L. Damiani, M. Repetto and A. Pini Prato, "Improvement of powertrain efficiency through energy breakdown analysis," Applied Energy, pp. 252-263, 2014.

[4] A. Schepmann and A. Vahidi, "Ultracapacitor power assist with preview-based energy management for reducing fuel consumption of heavy vehicles," International Journal of Powertrains, vol. 5, no. 4, pp. 375-394, 2016.

[5] A. Franceschetti, D. Honhon, T. Van Woensel, T. Bektaş and G. Laporte, "The time-dependent pollution-routing problem," Transportation Research Part B: Methodological, vol. 56, pp. 265-293, 2013.

[6] S. Dabia, E. Demir and T. Van Woensel, "An exact approach for a variant of the pollution-routing problem," Transportation Science, vol. 51, no. 2, pp. 607-628, 2016.

[7] European Commission, "optiTruck: Optimal fuel consumption with Predictive Power Train Control and calibration for intelligent trucks," H2020 Programme, Project Reference 713788, Brussels, 2016.

[8] D. P. Watling, R. D. Connors, D. S. Milne and H. Chen, "Optimization of route choice, speeds and stops in time-varying networks," Institute for Transport Studies, University of Leeds, 2019.