

A thesis submitted for the degree of Master of Engineering  
in the Faculty of Engineering at the University of Liverpool.

STRAIN DISTRIBUTION IN TENSION MEMBERS

November 1963

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**TABLES AND DIAGRAMS.**

212. \* GAIN-CONTROL

213. \* DEFLECTION

214. \* REPOSITION-GAUGE

215. \* CALIBRATION

216. \* DIAL-GAUGE

217. \* RANGE-GAUGE

218. \* DIMENSIONAL INSTRUMENTS

219. \* TRIP-ROCK

THE INSTRUMENTS SHOWN IN VOLUME 2 OF THE SERIES

IN IMAGES

(a) POSITION OF THE IMAGE SLITS

To the right ———— \* \*

Upwards ———— \* \*

(b) POSITION OF IMAGE SLITS

Clockwise ———— \* \*

ABBREVIATIONS USED IN VOLUME 1 AND 2

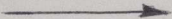
OF THE THESIS


GAL.	=	GALVANOMETER
DEF.	=	DEFLECTION
R.G.	=	RESISTANCE-GAUGE
CALIB.	=	CALIBRATION
D.G.	=	DIAL-GAUGE
M.H.	=	MAIHAK-GAUGE
H.E.	=	HUGGENBERGER EXTENSOMETER
K.E.	=	KNIFE-EDGE

SIGN CONVENTION USED IN VOLUME 2 OF THE THESIS


TO INDICATE

(a) DIRECTION OF THE STRAIN SLOPE

To the right  = +

Upwards  = +

(b) DIRECTION OF TWIST APPLIED

Clockwise  = +

### ACKNOWLEDGEMENTS

I would like to express my sincere thanks to Professor J.B.B.Owen, D.Sc., F.R.Ae.S., A.M.I.C.E., of the Department of Civil Engineering, University of Liverpool, for his personal interest, kindness, advice and guidance in the work presented here. I am also indebted to other members of the department for their help and encouragement. In particular I would like to thank Mr. J. Pace, incharge of electronic-laboratory and Mr. G. Goodall, workshop supervisor, for their help and personal interest in my work.

## SUMMARY

The long term objective of the present investigation was the study of the load distribution in a tension joint with a view to reducing stress concentrations. For control purposes, the strain distribution in a rectangular mild steel plate, loaded as axially as possible in a special machine, was first of all investigated experimentally using resistance strain gauges. It was found that the strain readings obtained on a plain rectangular plate did not vary linearly across transverse cross-sections. A non-linear strain variation of a similar nature was briefly noted by Batho and Samawai (ref. 1), but not investigated. In a specimen with a circular hole, the writer found that the strain concentrations observed were abnormally high in the region of the hole and were also asymmetric. The main work of the thesis was therefore directed to find the possible causes for the observed variations which if ignored might give misleading results in more elaborate joint tests.

The tests described indicate that the non-linear strain variation was not due to initial strains in the plate, Luder bands, the way the plate was loaded, the plate shape and size, variation in the strain sensitivity of resistance gauges, gauges being defective or the thickness of durifix used to stick the gauges. A series of tensile tests on araldite specimens with or without circular holes, gave symmetrical strain readings across cross-section. These results suggest that the non-linear strain variation across cross-sections of the mild steel

specimens used, was due to the material not being homogeneous enough over the effective length of the resistance strain gauges used (about  $1/4''$  to  $1/2''$ ) for St. Venant's theory to be strictly applicable. These variations may be significant causes of scatter in fatigue and impact test results.

It was decided to first investigate the behavior of rectangular mild steel plates, with or without holes, when subjected to tensile loads. These rectangular plates were loaded in the elastic-range and beyond into the plastic-range, to study how plasticity altered the strain distribution. The plates were also subjected to cycles of loading and unloading. These tests were conducted in a special tensile testing machine, which consisted of a test specimen through crossed knife-edges that giving more control than is obtainable in standard testing machines. A detailed description of this machine is given in Chapter 3. Specimens tested in this machine should be subjected to tension, at least at a small but fixed stress. Strain values at a given cross-section of the specimen might then be expected to be constant or vary linearly. But the strain values registered by resistance gauges on the same cross-section differed from one another and the variation was of the order of 10% out of linear. To continue with joint tests until the cause of this has been studied and understood and in order to shed further light on the reason for this discrepancy, rectangular mild steel plates of

CHAPTER 11.1. INTRODUCTION AND MAIN RESULTS OBTAINED

The objective of this investigation was the reduction of stress-concentrations in joints, subjected to tensile loads. Fatigue tests results on such joints have often given considerable scatter and in order to provide a reliable datum from which studies could be extended, it was decided to first investigate the behaviour of rectangular mild steel plates, with or without holes, when subjected to tensile loads. These rectangular plates were loaded in the elastic-range and beyond into the plastic-range, to study how plasticity altered the strain distribution. The plates were also subjected to cycles of loading and unloading.

Most tests were conducted in a special tensile testing machine, which transmits loads to a test specimen through crossed knife-edges thus giving more control than is obtainable in standard testing machines. A detailed description of this machine is given in Chapter 3. Specimens tested in this machine should be subjected to tension, at worst at a small but fixed offset. Strain values at a given cross-section of the specimen might then be expected to be constant or vary linearly. But the strain values registered by resistance gauges on the same cross-section differed from one another and the variation was of the order of 1% out of linear. To continue with joint tests until the cause of this has been studied was undesirable and in order to shed further light on the reason for this discrepancy, rectangular mild steel plates of



different shapes and sizes were tested, using different types of resistance-gauges and loading machines. But the gauges indicated a non-linear strain pattern in every case, irrespective of the shape and size of the test specimen and the loading machine. Similar results were obtained with Huggenberger extensometers .

A steel plate with a circular hole, symmetrically placed at the centre, also gave asymmetrical strain variation over cross-sections and this was quite marked at the hole, when it was tested in axial tension, using the special tensile testing machine. These strain readings at the hole were further checked by Huggenberger extensometers which also gave similar results. Resistance-gauges, which gave larger strain values when the specimen was loaded within the elastic-range, also showed larger strain values when loaded beyond the elastic-range into the plastic range .

Rectangular mild steel plate with two circular holes did not give results which were asymmetrical to such a large degree as the specimen with one hole. This plate was also subjected to loads within and beyond the elastic limit. In this case, the strain behaviour was as expected.

Attempts were made as detailed in Chapters 4, 5 and 6, to investigate possible sources of the non-linear strain-pattern observed. A suitably chosen  $\pm 2\%$  correction of observations often improved linearity over cross-sections, but this did not give axially consistent resultant loads and offsets of load.

In a further endeavour to find the cause of non-linear results, rectangular araldite plates with or without circular holes were tested in tension using the photoelastic technique to indicate the strain pattern. These tests gave a linear strain behaviour which agreed with theory. The araldite specimen with a circular hole, symmetrically placed at the centre, also gave the expected strain pattern when it was tested in tension, in the special tensile testing machine, and strains were measured with resistance strain gauges. As a further check the tests were repeated on the rectangular mild steel plate A (Chapter 8) but these still again gave non-linear results.

The final conclusion derived from these experiments and calculations is that the non-linear strain variation in case of mild steel specimens is due to the material not being homogenous enough for the St. Venant's theory to be strictly applicable.

Timoshenko (ref. 16, p.125) by using the elementary theory of bending of curved bars, derives the values given in Table 2.1 for the stresses at the edge of a hole. These stress values depend on the ratio  $R/r$ , the ratio between the diameter  $R$  of a circle, the stresses at the circumference of which are not materially affected by the presence of a hole, to the diameter  $r$  of the hole, see Fig. 1.1.

The increase in the stress-concentration which occurs in the table beyond the value obtained when  $R/r = 6$  is physically unacceptable and is due to errors resulting from the use of the elementary theory of curved bars.

## CHAPTER 2.

### STRESS-CONCENTRATIONS DUE TO ROUND HOLES

The stress-concentration due to holes in plates has a bearing on the stress distribution in rivetted joints. The relevant available literature on stress-concentrations was therefore studied.

#### 2.1. STRESS-CONCENTRATION WITHIN ELASTIC-RANGE

Kirsch (ref.37, p.80) gives an analytical solution for the stress distribution around a circular hole in an infinite plate, subjected to a tensile field and loaded within the elastic range. His solution gives the stress-concentration at the edge of a circular hole, as three times the field stress. If the width of a plate is not less than four times the diameter of the hole in it, Timoshenko (ref. 37, p.81) shows that Kirsch's result is less than 6% in error.

Timoshenko (ref. 38, p.385) by using the elementary theory of bending of curved bars, derives the values given in Table 2.1 for the stresses at the edge of a hole. These stress values depend on the ratio  $D/d$ , the ratio between the diameter  $D$  of a circle, the stresses at the circumference of which are not materially affected by the presence of a hole, to the diameter  $d$  of the hole, see Fig. 2.1.

The increase in the stress-concentration which occurs in the table beyond the value obtained when  $D/d = 6$  is physically unacceptable and is due to errors resulting from the use of the elementary theory of curved bars.

Stresses around a circular hole near a straight boundary of a semi-infinite plate (Fig.2.2), under tension parallel to this boundary were analysed by G.B. Jeffery (ref. 23), using bi-polar co-ordinates. A corrected result and a comparison with photoelastic tests were given later by R. D. Mindlin (ref. 28).

Howland (ref.18) investigated the elastic stresses in the neighbourhood of a circular hole on the centre line of a strip subjected to axial tension. He found that the stress-concentration at the hole was 4.32 times the mean stress, (i.e. 1.44 times Kirsch's value of 3), when the diameter of the hole was a half the width of the specimen. Howland's solution is rapidly convergent with holes of up to this size, and he states that Coker, Chalko and Satake's results (ref. 4) agree satisfactorily with his analysis. The holes used by the writer in his experiments were 0.44 times the width of the plates. For such holes, Howland's analysis gives a stress-concentration of 4.

The case of a plate of finite dimensions subjected to axial tension and containing two circular holes, lying on the centre-line of the plate has not yet been analysed, but Howland (ref. 20) extended his analysis to study the stresses in a plate of infinite dimensions, containing two circular holes, lying in the direction of the stress field. He found that the values of the stress-concentration factors at the edges of the holes were considerably reduced. The value of stress-concentration factor was reduced by about 50% at points A and B, Fig. 2.1, at the edges

of  $7/8$  inch diameter holes, in a plate of infinite dimensions. (The value of stress-concentration factor of 4 in case of a single hole, the diameter of which was 0.44 times the width of the plate, reduced to 2 in case of two holes).

Recently, Ling (ref. 26) has also analysed the problem of strain-distribution in a plate of infinite dimensions subjected to a tensile field, and containing two circular holes. He found the value of stress-concentration factor was 2.6 at points, such as A and B, Fig. 2.1, with  $7/8$  inch diameter holes. This result is 30% higher than that obtained from Howland's analysis.

## 2.2. STRESS-CONCENTRATION BEYOND THE ELASTIC INTO THE PLASTIC-RANGE

Very little theoretical work has yet been done, which deals with the problem of strain-distribution in a plate of finite dimensions, containing a circular hole, symmetrically placed at the centre and subjected to loads beyond the elastic into the plastic range. In 1945 Faerberg (ref. 13) gave an approximate solution of this problem on the basis of the theory of elasto-plastic bending of a curved beam and he found that the plastic zone which starts to form at the points A and B, Fig.2.3., at a tensile stress of one third the yield stress, extends rapidly around the hole contour, as the value of the mean tensile stress increases from a third to 0.65 times the yield stress. When the mean tensile stress is greater than 0.65 times the yield stress, the plastic zones grow in a

direction perpendicular to the tensile stresses and simultaneously become smaller at the contour of the hole itself. He also found that with increasing values of the tensile stress, the deformations near the hole increased considerably more rapidly than at the outside edges of the plate.

At loads beyond the elastic limit, the strain-distribution in a plate of finite dimensions, containing two circular holes, has not yet been analysed.

## EXPERIMENTAL INVESTIGATIONS

### 2.3 (a) ELASTIC RANGE

Coker, Chakko and Satake (ref.4) investigated experimentally the stress distribution in tension members, due to presence of a hole and their results as already stated are in agreement with those obtained from Howland's analysis. Frocht (ref. 14, p.234) also studied this problem using the photoelastic-technique and found that his results were also in agreement with those of Howland (i.e. he obtained a stress-concentration factor of about 3.3 at the edge of a 0.515 inch diameter hole, symmetrically placed at the centre of a 1.041 inch wide plate; the Howland's analysis would indicate a value of 3.36).

### 2.4 (b) PLASTIC-RANGE

Very little experimental work has been done, for the case of plates,

containing circular holes and subjected to loads beyond the elastic into the plastic-range. Faerberg (ref. 13) studied the strain-distribution in a rectangular duralium plate, 800x180x5 m.m., containing a circular hole 15 m.m., in radius, using a 2 m.m. Huggenberger-tensometer, when subjected to loads beyond the elastic into the plastic-range. The experimental values were found almost exactly the same as his theoretical results up to mean tensile stress of about one half the yield stress. But beyond this value the experimental results were not in agreement with theory, as it is evident from Fig. 104 (ref. 36, page 235).

#### 2.5. INVESTIGATION OF THIS THESIS

In the present thesis, the problem of strain-distribution in plain rectangular mild steel plates and also those containing one or two circular holes, has been studied in the elastic and plastic-range, using the resistance gauges at the holes and also at cross-sections away from the holes and applying loads via crossed knife-edges, using a special tensile testing machine (see Chapter 3), in an endeavour to apply controllable loads and avoid bending.

These tests gave anomalous results and so for comparison further tests were made on araldite specimens of dimensions similar to those of steel specimens, using the photoelastic technique. The results obtained were found in agreement with that of available theory.

A rectangular araldite specimen, containing a circular hole,

symmetrically placed at the centre, was tested in tension, in the special tensile testing machine, using resistance strain gauges at the hole, to measure strain. The results obtained from this test were symmetrical but did not agree with that of available theory. These results were about 40% lower than that obtained from Howland's analysis.

and that the applied load. The line of action of the load applied by them is often uncontrolled and uncontrollable. Even in the case of tensile testing machine, shown in Fig. 3-1, although the load is applied through the knife-edge marked X, the possible line of action of the load at the lower end is indeterminate. Even at the upper end the resultant load can fall anywhere along the 20" length of the knife-edge.

The special tensile testing machine described in paragraphs 3.1 to 3.3 had been designed and constructed, before the writer arrived, with the objective of loading a specimen in axial tension in a controlled manner. In this end, load was applied through vertical knife-edges, marked X in Fig. 3.1(a), at the top and the bottom of the machine. The plates supporting the knife-edges are welded to R.S.J's and across each other at right angles in places displayed vertically about 2" apart.

With no specimen in place, the bottom portion A of the machine rests on channel section B, while the top portion C of the machine hangs freely from the top channel D.



CHAPTER 3SPECIAL TENSILE TESTING MACHINE3.1 INTRODUCTION

Conventional types of machines used for tensile tests bend, shear and twist the specimen tested. The line of action of the load applied by them is often unknown and uncontrollable. Even in the case of Buckton testing machine, shown in Fig. 3.1, although the load is applied through the knife-edge marked K3, the precise line of action of the load at the lower end is indeterminate. Even at the upper end the resultant load can fall anywhere along the 20" length of the knife-edge.

The special tensile testing machine described in paragraphs 3.2 to 3.8, had been designed and constructed, before the writer arrived, with the objective of loading a specimen in axial tension in a controlled manner. To this end, load was applied through crossed knife-edges, marked K in Fig. 3.2(a), at the top and the bottom of the machine. The plates supporting the knife-edges are welded to R.S.J's and cross each other at right angles in planes displayed vertically about 2" apart.

With no specimen in place, the bottom portion A of the machine rests on channel section S, while the top portion B of the machine hangs freely from the top channel C2.

The specimen is inserted in Blocks B1 and B2, which have V-taper type jaws and located in position by pins, which pass through the jaws and blocks. These pins are used for initial centering of the specimen, as explained later, and if the procedures, detailed in paragraph No. 3.4., are adopted in experimental work with this machine, a state of pure axial tension should be achievable. The machine is designed for a maximum load of 10 ton, and can be used for testing specimens up to a maximum length of 20", width of  $1\frac{3}{4}$ " and thickness of  $\frac{1}{4}$ ".

Steel wires which run diagonally, shown in Fig. 3.2(a) and (b), serve to brace the machine externally.

A screw jack J, which can either be operated by hand or by a 3 H.P. motor is bolted to the base B1, Fig. 3.2(c). The top of the screw of the jack is placed in the channel-section 3. Two 1 inch diameter rods (drilled throughout their length, so that they can be adjusted to any required height), pass vertically through the channel section 3 and subsequently into R.S.J.'s. One of the plates on which the half-edges bear is welded to the bottom of this R.S.J. The other R.S.J., which is of the same dimension as 3, is placed at right angles and is between the channel section 3 and R.S.J. This R.S.J. is bolted, using 1 inch diameter rods to bottom block, B1, which has V-taper type of jaws. The other plate on which the half-edges bear is welded at the top of this

DETAILS OF THE SPECIAL MACHINE

3.2. BOTTOM-PORITION A

The base of the machine consists of two channel sections, placed back to back but about 1 inch apart, as shown in Fig. 3.2(a). Two such sections C1 are placed at right angles to each other. At the ends of these sections, as shown in Fig. 3.2(a) and (b), rest four cylindrical columns C, which subsequently support the channel sections C2 at the top and which are of the same dimensions as those at the bottom. Through these columns run bars screwed at their ends and used to preload the columns and deal with recoil forces. Steel wires which run diagonally, shown in Fig.3.2(a) and (b), serve to brace the machine externally.

A screw jack J, which can either be operated by hand or by a 2 H.P. motor is bolted to the base C1, Fig. 3.2(a). The top of the screw of the jack is pinned to the channel-section S. Two 1 inch diameter rods (threaded throughout their length, so that they can be adjusted to any required height), pass vertically through the channel section S and subsequently hold R.S.J.E. One of the plates on which the knife-edges bear is welded to the bottom of this R.S.J. The other R.S.J.F, which is of the same dimensions as E, is placed at right angles and in between the channel section S and R.S.J.E. This R.S.J.F. is bolted, using 1 inch diameter rods to bottom block, B1, which has V-taper type of jaws. The other plate on which the knife-edge bear is welded at the top of this

R.S.J.F. In this way, knife-edges are made to cross each other at right angles. They rest in V-grooves in plates, which are welded to a circular block, shown in Fig. 3.3, one at the top and other at the bottom, such that they lie at right angles to each other. With no specimen in the blocks, R.S.J. F rests on channel section S along with the block B1, as it is evident from Fig. 3.2(a).

### 3.3. TOP-PORITION B

It consists of pairs of channel sections C2, placed at right angles to one another, as shown in Fig. 3.2(a). They rest on columns C, which in turn rest on base C1.

The position of the bolts at top channel section C2, Fig. 3.2(a), is determined such that the straight lines joining the points P and R, Q and S respectively, pass along the centre line of the bolts at top channel C<sub>2</sub> and channel Section S. This also fixes the position of block B2, such that it is in alignment with block B1. The proper alignment of the top and bottom blocks B1 and B2 is very essential for applying loads in pure axial tension.

These bolts hanging from channel section C2, Fig. 3.2(a), hold R.S.J. G, to the top of which is welded one of the plates on which the knife-edges bear. The other R.S.J.H, which lies at right angles and in between the top channel section C2 and R.S.J. G, holds the load indicating bar and block B2. The other plate on which the knife-edge

bear is welded to the bottom of this R.S.J. These knife-edges rest in V-grooves in plates which are welded to a circular block, shown in Fig. 3.3.

#### 3.4. WORKING PRINCIPLE OF THE MACHINE

The specimen is inserted in blocks B1 and B2 Fig. 3.2(a). These have V-taper type of jaws. The specimen is first held in position by pins, which pass through the jaws and the blocks. The pins passing through accurately centred holes in the jaws and specimen are used for initial centering of the specimen, and the jaws should only grip the specimen, after a sufficient load required for aligning the specimen, has been applied through the pins. It was, however, found by the writer that the jaws gripped the specimen, before it was properly aligned. Special care was therefore taken to keep the jaws apart initially for this "lining up" to occur. This, the writer found, could be accomplished by inserting rubber pieces in between the jaws, as shown in Fig.3.4.

When the specimen is loaded in tension, the jack screw goes down. This in turn pulls the R.S.J.E downwards. Load is then transferred through the knife-edges to R.S.J.F., which pulls block B1. Thus, the load is applied to the specimen in tension, as the top portion remains hanging freely from the top channel C2.

Theoretically, when the points of intersections of the crossed

knife-edges and the centre of the specimen are in a straight line, as shown in Fig. 3.2(a), the load should be transferred axially to the specimen. The alignment can be checked, by putting a straight-edge across the faces of the blocks. If the blocks are not in alignment, this can be achieved by moving the crossed knife-edges. The knife-edges relative to the specimen can be moved  $\frac{1}{8}$ " across the thickness of the specimen but no movement across the specimen width is possible.

### 3.5. SETTLING UP OF THE SPECIAL MACHINE

This machine had not been used before the writer started his work. It was found in initial testing that the top and bottom blocks of the machine were not in alignment, after the machine had been initially set up. They could not be aligned, even by moving the crossed knife-edges. The following procedure was therefore adopted for centering the top and bottom blocks :

A flexible piece of wire, strong enough to hold the weight of the bottom portion of the machine, was held by the jaws, and the jack was un-screwed from the base of the machine. The load was, then applied, which lifted the jack upwards. At this stage, the bottom portion of the machine was hanging freely by this flexible wire. Thick steel plates were now tightened along the faces of the top and bottom blocks, by the help of C-clamps, as shown in Fig. 3.5, which aligned these blocks. The idea of using a thin flexible wire was to allow the blocks

to align themselves under the pressure of the steel plates, without hinderance.

The machine was then unloaded and the bottom portion was allowed to come down to the base under its own weight. The jack was then bolted to the base at this position. After this adjustment had been made, the top and bottom blocks were found in alignment, when checked by putting the straight-edge along the faces.

It is important to continually check the freedom of movement between the various parts of the machine. This is achieved by keeping at least  $\frac{1}{8}$  inch gap at load, between channel section S and R.S.J.F and R.S.J.E and block B1. Similarly in the top-portion of the machine there should be a gap between R.S.J. G and top of load indicating bar and R.S.J.H and the top-channel C<sub>2</sub>. These gaps can be arranged by adjusting the threaded rods to the required lengths.

### 3.6. LOAD MEASURING DEVICE

A mild steel Proving Ring, the dimensions of which are given in Fig. 3.6, had already been designed to be used as a load indicator. The dial-gauge on the ring used was capable of measuring 1/10,000 of an inch deflection. The ring was also fitted with resistance-gauges.

The material of the proving-ring showed signs of yielding, when the specimen under test was subjected to a load of about 7 ton. Thus, it was of no use as a load indicator up to the 10 ton range of the machine, unless it could be strain-hardened. The proving-ring was

then loaded to gradually increasing tensile loads, using a 100 ton Buckton universal testing machine and a special rig, Fig. 3.6. First yielding occurred at a load of about 7 ton and the proving-ring was then loaded and reloaded several times within this range, until it regained elasticity. It was, then subjected to higher and higher loads, and loaded and unloaded at each increment of load, until elasticity was regained. In this way, the proving-ring was loaded up to 15 tons, and appeared elastic. But it gave unsatisfactory results as a load indicator, because the loading and subsequent unloading curves were loops, as shown in Fig. 3.7 (a). The dial-gauge readings also gave the same kind of loop, as is evident from Fig. 3.7(b).

This proving-ring was therefore replaced by a 2" x 2" tensile bar which was fitted with resistance strain-gauges.

The details of the various parts of this load indicating bar, which was designed for a load of 15 tons, is given in Fig. 3.8. It consists of a mild steel rectangular block A ( $3'' \times 2'' \times 6\frac{1}{2}''$ ), which is fitted in top block B<sub>2</sub> and is kept in position by a pin. The other square block-B ( $1\frac{1}{2}'' \times 1\frac{1}{2}'' \times 16\frac{1}{2}''$ ) is screwed at one end, inside the block-A, and is bolted to the 4" x 4" x 15" plate C, at its other end.

Two resistance gauges are attached to each face of bar B, one acts as an active and the other as 'dummy'. In calibrating the bar, each pair of active and compensating gauges were connected in separate circuits. This was loaded to gradually increasing tensile loads, using



a 100 ton Buckton universal testing machine and the special rig, shown in Fig. 3.6. This rig was the same, as that used in case of the proving-ring, except that of a 4" x 4" x 15" bar, which has now replaced the 4" x 1½" x 15" top bar of the proving-ring. The readings obtained from the various resistance-gauges are recorded in table 3.1 and plotted in Fig. 3.9(a), (b), (c) and (d). All four sets of gauges gave different readings, when subjected to the same load. The mean of gauge pairs varied by about 2.6%. This variation, which is of the same nature as that obtained from tests on rectangular mild steel specimens, reported in Chapters 4, 5 and 6. This load indicating bar was 1½" thick as compared with the 1/8" and 5/16" thickness of the test specimens referred to and the results suggest that variations in the thickness of the glue layer attaching the gauges was not the cause of the non-linear observations obtained - variations in the distance of the gauge from the neutral axis have a very smaller effect with the 1½" load indicating bar than with the test specimens.

All eight gauges were then connected in one circuit as shown in Fig. 3.10, in an endeavour to eliminate bending strains. The readings obtained are recorded in table 3.2 and plotted in Fig. 3.11 (a) and (b). Calibration was done using two different voltage supply, i.e. 4 volt and 2 volt, to the bridge circuit as given in table 3.2. The results obtained during loadings and subsequent unloadings coincided. The combined reading obtained at 2 volt supply to the bridge circuit which

was also used in separate circuits, was about 4% lower than the mean of individual slope. This variation confirms the non-linear character of cross-sectional readings. The resistance gauges were protected from moisture by enclosing in a sealed envelope with silica gel crystals.

### 5.7. BEHAVIOUR AND USE OF THE SPECIAL MACHINE

This machine was found to be sensibly accurate for loading the specimens in axial tension, if the top and bottom blocks B1 and B2 were in alignment and the knife-edges were properly placed in the grooves. But it has some limitations to its use, such as, that the maximum length of a test specimen is 20", width  $1\frac{3}{4}$ " and thickness  $\frac{1}{4}$ ". It is not easy to often take the specimen out of the machine, if some alterations are to be made.

The following points should be taken into consideration, while doing the experiments :

- (1) Rubber pieces should be placed in between the jaws to keep them apart at initial loads.
- (2) The machine should not be unloaded too much as it pushes the top knife-edges out of their seats.
- (3) It should be checked before starting the experiment that the knife-edges are properly placed in their grooves. Otherwise there is a possibility of an offset of loading being applied to the specimen.

- (4) The top and bottom blocks should be checked for alignment before starting the experiment.
- (5) It was sometimes found difficult to get the pins out of the specimen, if they got sheared, even by the technique of screwing them out, against a steel plate, shown in Fig. 3.12. It is desirable to avoid shearing the pins.

### 3.8. SUGGESTION FOR IMPROVEMENTS IN SPECIAL MACHINE

Although not incorporated the following improvements might well be made to this special machine :

- (1) The seats for the knife-edges should be made deep in order to prevent the knife-edges slipping out of their seats.
- (2) An easier method of moving the knife-edges in their grooves to achieve axiality of loading should be devised.

CHAPTER 4TESTS ON PLAIN RECTANGULAR MILD STEEL PLATES4.1. INTRODUCTION

In order to check the strain distribution produced in nominally tensile tests, a series of axial tension tests were conducted on plain rectangular mild steel plates. The strain readings obtained on the first mild steel specimen A, were not as expected. This specimen was tested in the special tensile testing machine, and British Thermostat gauges were first attached and then Tinsley gauges to it. The observed strains were non-linear across cross-sections of the specimen. The specimen was made from ordinary 'commercial' mild steel and was annealed, before testing, as explained in paragraph 4.3 to avoid possible effects of initial stresses on strain observations. The non-linear strain variation across cross-sections cannot then be associated with the presence of initial stresses in the specimen.

In order to investigate possible causes of this non-linearity, steel to EN 3A was used instead of 'commercial' mild steel, and plates B, C, and D of different shapes and sizes were tested, using as well as the specially designed machine, a 100 ton Buckton Universal testing machine and a 6.5 ton Denison machine. Also a different make of resistance-gauges were used viz Armstrong Whitworth Aircraft gauges, in order to investigate whether the non-linearities in strain readings were due to the gauges used. The results obtained from resistance-gauges

were also checked by Huggenberger extensometer readings.

Specimen B, which was of rectangular shape and about the same size as specimen A was annealed before testing, to avoid possible effect of initial stresses on the strain observations. Avro resistance gauges were attached to it. The strain readings obtained again gave a non-linear strain variation across cross-sections, as in case of specimen A. The results were checked by Huggenberger extensometers placed between the resistance-gauges and these also gave results of the order expected from resistance-gauges. The change of the material and the resistance-gauges had thus no effect on the non-linear strain patterns, which were of similar nature to those obtained on specimen A.

To check whether the shape of the specimens were responsible for the non-linear strain variation, specimen C had dimensions based on those specified in B.S. 15 for standard tensile test specimens. Avro resistance-gauges were attached to it. It was tested in the as 'received' condition and not annealed after cutting. It was made from the same batch of mild steel that was used in specimen B and therefore was expected to be free from initial stresses, except those produced by cutting and machining. It was first tested in a 100 ton Buckton Universal testing machine and then in the special tensile testing machine, in order to see whether the special machine used in the tests was the cause of the non-linear strain variation or not. But the results obtained from tests in both the machines, were still non-linear.

The resistance-gauge readings were also checked by Huggenberger extensometers which gave results of the order expected from resistance-gauges. The non-linear strain variation was therefore not due to either a characteristic peculiar to the specially designed machine or the shape or the process of annealing the specimens.

Finally, a small rectangular mild steel plate, made from the same batch of steel as that used in specimens B and C, but about half the length of specimens A or B, was tested in a 6.5 ton Denison machine to check the effect of specimen length. It was not annealed and Avro resistance-gauges were attached to it. The results obtained from tests were non-linear across a gauged cross-section, as in case of specimens A, B and C.

In the tests the plates were first loaded within and then beyond into the plastic-range. They were loaded and unloaded several times in order to see whether this affected strain readings but non-linearity across cross-sections persisted.

#### 4.2. TESTS ON SPECIMEN A

Specimen A was a plain  $1\frac{3}{4}$ " x 20" x  $\frac{5}{16}$ " mild steel plate. Near the ends of the specimen  $\frac{1}{4}$ " diameter holes were drilled as shown in Fig. 4.1. Pins through these holes and corresponding holes in the special tensile testing machine jaw-grips, applied the initial loads which were intended to centre the specimen. Fig. 4.1. gives as well as the dimensions of the specimen, the positions of the resistance-gauges which were attached to it. British thermostat resistance-gauges were first attached to the specimen, as shown in Fig. 4.1 (a). They had an average resistance of 126.5 ohms and manufacturers stated their gauge-sensitivity factor to be 2.04. These gauges eventually broke due to a sudden jerk, probably produced by slip under load, of the specimen in the jaws (see paragraph 4.5). After this occurred, these resistance-gauges were replaced by Tinsley gauges. These had an average gauge resistance of 98.4 ohm and the firm stated their gauge-sensitivity to be 2.17. In both cases the set of gauges nearest to the ends of the specimen were about twice the width of the specimen from the end of the gripping jaws. The central gauges were 4.57 times the width of the specimen from the end of the jaws.

Timoshenko and Goodier (ref. 3.7., p.52) show that in such a specimen at twice the width of specimen from a concentrated load, the

temperature which takes about three days. This process should have left the specimen free from initial stress, and was used for all specimens.

stress distribution is almost uniform. St. Venant's principle might then be expected to apply to strain-observations at the gauge positions of Fig. 4.1 (a) and (b), so that the variation of strain over any one cross-section would be linear.

The specimen was tested in axial tension, using the special tensile testing machine. The resistance-gauges were connected to a 24-channel strain-gauge set. The bridges were initially each balanced and changes of galvanometer readings were taken as measures of strain. Resistance calibration was achieved by placing a known resistance across the active-gauges.

#### TEST PROCEDURE

##### 4.3. ANNEALING

The specimen, before the gauges were attached, was annealed in order to get rid of initial stresses, which might have affected strain observations.

The specimen was inserted at room temperature in an Afco. electric furnace and the temperature of the furnace was then raised to 600C, which it usually reaches in 6 to 7 hours. The specimen was kept at this temperature for about 6 to 8 hours. Then the furnace was switched off, but the specimen left inside until the furnace had cooled to room temperature which takes about three days. This process should have left the specimen free from initial stress, and was used for all specimens.



#### 4.4. STICKING AND PROTECTING THE RESISTANCE-GAUGES FROM MOISTURE

The specimen was first cleaned using emery paper, then degreased with carbon-tetra chloride. Pencil lines were then drawn on the specimen to locate where the gauges were to be struck. The resistance gauges which had been previously coated with durifix which had been allowed to dry, were again coated with durifix and pressed by thumb on to the specimen which had also been treated with durifix in the same way as the gauges. By pressing from the centre to both sides of the gauges, an endeavour was made to squeeze out any air trapped between the gauges and the specimen. The gauges were held pressed on to the specimen for about 24 hours by means of wooden blocks, with rubber pieces on top of the gauges as shown in Fig. 4.2. These wooden pieces were held together by light loads applied from screwed rods. Twenty four hours after sticking, the clamps were removed and the wires soldered to the gauges.

The specimen was then gently warmed in a stream of hot air to get rid of moisture, the gauges wrapped with cotton pads, and the entire gauged length of the specimen wrapped in P.V.C. The connecting wires came out of the wrapping through a B.I.C. plastic band as shown in Fig. 4.3.

The gauges were then checked for resistance and resistance to earth, i.e. to the specimen. When incorporated into a wheatstone bridge

network, the reading under no load remained constant, i.e. there was no zero "drift".

The above mentioned procedures for fixing and protecting the resistance-gauges were adopted in case of active and also the temperature compensating gauges. The temperature compensating gauges, each approximately of the same resistance value as the active-gauges, were attached to a separate mild steel plate.

#### 4.5. EXPERIMENTAL RESULTS

Fig. 4.4. shows the results for British Thermostat gauge No. 1 of Fig. 4.1(a). These results are sensibly linear during loading and subsequent unloading. Similar results were obtained with all the other resistance-gauges and from the straight lines obtained, the values shown in table 4.1, columns 2(b) and 5(b) were obtained.

The stress values given in columns 3 and 6 are derived from the resistance calibration of each gauge. A resistance of 250K ohm across a gauge (resistance 126.5 ohm) gave a galvanometer deflection of 4.7 cm. There was a 2 volts battery supply to the bridge.

The firm's sensitivity factor for these gauges was 2.04. It follows that the strain equivalent of a resistance of 250K ohm across 126.5 ohm gauge is

$$\begin{aligned} &= 126.5/250,000 \times 2.04 \\ &= 2.48 \times 10^{-4} \end{aligned}$$

and that this is equivalent to a galvanometer deflection of 4.7 cm.

1 cm. deflection on the galvanometer is therefore equivalent to :

$$= 2.48 \times 10^{-4} / 4.7$$

$$= 0.528 \times 10^{-4}$$

$$= 0.685 \text{ t.s.i.}, \text{ when } E = 13000 \text{ t.s.i.}$$

Theoretically, one should expect the strain-values of table 4.1 to be sensibly equal at all cross-sections, and any variation across a cross-section to be linear. But the results in columns 3 and 6 are different and further (as shown in column 8 of the table, in which, because of the equal spacing of the gauges, all values at anyone cross-section should be equal), indicate a non-linear strain variation across each cross-section of the specimen.

Fig. 4.5 shows pictorially the observations at cross-sections No. (i), (ii) and (iii) and indicates possible correction which would have to be made to obtain sensibly linear strain variation over the cross-section. The corrections in the readings from gauges 3 and 14, 7 and 18, 11 and 22 are very large. These results indicate the presence of an offset of the load, along the plane X-X and Y-Y, as shown in Fig. 4.5, in which arrows indicate the direction of the strain-slope. The values of the offsets of the load along the plane X-X and Y-Y at cross-sections No. (i), (ii) and (iii), corresponding to mean bending strain at the respective cross-sections, are calculated as follows :

For cross-section No. (i), the offset of the load along the plane X-X

$$\begin{aligned}
 &= \frac{0.85/2}{7.6} \times \frac{I}{YA} \\
 &= \frac{0.85/2}{7.6} \times \frac{1.75}{12} \times \frac{5}{16} \times \frac{5}{16} \times \frac{5}{16} \times \frac{32}{5} \times \frac{16}{1.75 \times 5} \\
 &= \underline{0.0029"}
 \end{aligned}$$

and along the plane Y-Y

$$\begin{aligned}
 &= \frac{0.2/2}{7.6} \times \frac{I}{YA} \\
 &= \frac{2}{76} \times \frac{5}{16} \times \frac{1.75^3}{12} \times \frac{16}{1.75 \times 5} \times \frac{8}{5} \\
 &= \underline{0.0051"}
 \end{aligned}$$

The offset of the load along the plane Y-Y, at cross-section No. (i) is not in agreement with what might be expected from the results, at cross-sections No. (ii) and (iii). This cannot be explained by the presence of an offset of the load. Although the offset of the load along the plane X-X, is in the same direction at each cross-section, but the results are not consistent. The values of the offsets in inches at cross-sections No. (i), (ii) and (iii) along the plane X-X and Y-Y are given in Fig. 4.5, which cannot be explained because the results are not consistent at the cross-sections. It is thus difficult to explain even the corrected linearised strain variation as due to the presence of an offset of the load likely to be produced by the machine.

While the experiment was in progress and the specimen was under load, a clicking sound was heard. On investigation, the gauges at cross-sections (ii) and (iii) were found to be open circuited. One wonders

whether these gauge failures were due to strain waves of excessive combined amplitude at the gauges.

The gauges were now replaced by gauges manufactured by Tinsley. Eighteen resistance-gauges were used as shown in Fig. 4.1(b).

#### 4.6. PURE BENDING TEST WITH TINSLEY GAUGES

In view of the anomalous results obtained with British Thermostat gauges and in order to calibrate the new gauges, specimen-A was tested in pure bending as shown in Fig. 4.6, using roller supports. Two sets of rollers were used at both ends to provide freedom of movement in case one set of rollers got jammed, during the experiment. The loads were applied using a four point loading system, which subjected the specimen to a constant bending moment over its central section. A dial-gauge capable of measuring deflection to the nearest  $1/10000$  of an inch was used to measure the deflection of the beam.

In an endeavour to check the strains indicated by the resistance-gauges, Mairhak vibrating-wire gauges were clamped at the positions shown in Fig. 4.1(b). The Mairhak-gauges were held on the specimen, as indicated in Fig. 4 of Appendix A.

It was found, however, that these 20 m.m. gauge length Mairhak strain-gauges and the associated frequency measuring apparatus were too insensitive to act as a check on the resistance-gauges.

The resistance-gauges were connected to a 24-channel strain-gauge set, as in the previous case. The bridges were each initially balanced

The readings of columns 3 and 3, of Table 4.4 (a) and (b) vary 31.

and changes of galvanometer readings were taken as measures of the strain. Resistant calibration was achieved by placing a known resistance across the active-gauges. Temperature compensation was achieved by using for each active gauge, a similar gauge attached to a separate mild-steel plate. These "dummy" gauges replaced the British Thermostat gauges, which were previously attached to this plate.

The results obtained are given in table 4.2. Fig. 4.7. shows the results for gauges No. 5 and 14, which were on opposite sides of the specimen and are typical. The strain-gauge sensitivity factors, given in table 4.2, were calculated by the usual procedure detailed in paragraph of Appendix C. The variance of these results is 2.75% i.e., 1 in 3 strain gauges can be expected to have a sensitivity factor differing by more than 2.75% from the mean of 1.98 (The firm quoted 2.04).

The strain-gauge sensitivity factors calculated can be altered considerably by choosing different straight lines to represent observations. A typical example is given in Fig. 4.8, where two possible straight lines, such as No. (i) and (ii) can be drawn and which gives gauge-factor values of 1.9 and 2.03, respectively. It is thus possible by appropriate adjustment of the slopes of lines, to make all the strain-gauge sensitivity factors equal.

The Maihak-gauge readings obtained are given in Table 4.3(a) and (b). The comparative values of strains obtained from resistance-gauges, Maihak gauges and dial-gauges are given in Table 4.4(a) and (b).

The readings of columns 3 and 5, of Table 4.4 (a) and (b) vary sensibly linearly with the load, as shown in Fig. 4.9 (a) and (b). The three points in parenthesis in column 5, Table 4.4 (a), are not in the straight line for gauge no. 1889. This might possibly be due to the effect of backlash in the gauge at low loads, or gauge no. 18889 not operating in the test giving column 5.

The Mairhak-gauges were clamped first near resistance-gauge No. 2 and 11 (Fig. 4.4) on opposite faces of the specimen. For a bending moment change of 100 lb. inch, the strain values obtained by calculation with  $E = 13000$  (wt.s.i.), from the dial-gauge readings, from the resistance-gauges, using the manufacturer's sensitivity factor and Mairhak-gauges are shown diagrammatically at the base of table 4.4 (a). These should all be the same but differ considerably.

The Mairhak-gauges were next clamped near the resistance gauge No. 5 and 14 and were subjected to the same amount of bending as in the previous case. The comparative results are shown at the base of table 4.4 (b).

Decreasing the assumed value of  $E$  to  $1.5 \times 13000 / 1.585 = 12800$  ton per square inch, which is possible, brings into agreement the strain indicated by the dial-gauge measurement of curvature and that obtained from bending theory.

Apart from the Mairhak-gauge readings of column 5 of table 4.4(a), the strains indicated by these gauges differ by only 4%. This is within the order of accuracy of the measurement of the frequency change.

The resistance gauges, if the firm's gauge factor is corrected by 15%, the mean difference of the strain values they indicate and that obtained from the dial-gauge observations in table 4.4(a) and (b), still differ by 2% to 6%. These corrected values are well within observational accuracy. These results indicate that Maihak-gauges cannot be used as a check to the resistance-gauges and it is therefore, difficult to say that the variation in the resistance-gauge readings was due to the gauges being defective.

The evidence from these bending calibration tests is not precise enough to come to definite conclusions regarding the magnitude of gauge factors. That 1 in 3 may differ from 1.98 by more than 2.75% is perhaps the best result available.

#### TENSION TEST ON SPECIMEN A

##### 4.7. WITHIN ELASTIC-RANGE

After initial calibration of the gauges, the regauged specimen-A, was again subjected to axial tension, using the special tensile testing machine. The results obtained from the resistance-gauges are given in table 4.5. Calculations of strain values have been made first with the gauge factor value given by manufacturer's, columns 3(b) and 6(b) and then with the actual values obtained for each gauge from the pure bending test, column 3(a) and 6(a). But the results in columns 3 and 6 are different and indicate as shown in column 8 of table 4.5, in which all values at any one cross-section should be equal, a non-



linear strain variation across each cross-section of the specimen. But this variation in strain readings can be made linear, as shown pictorially in Fig. 4.10, if corrections of the order of  $\frac{1}{2}$  2% are made in the observed values of strain. Strain readings obtained from resistance-gauges No. 5 and 14, 8 and 17, cross-section No. (ii) and (iii), respectively, however need a far larger correction to obtain linearity. These results indicate the presence of an offset of the load along the plane X-X and Y-Y, as shown in Fig. 4.10, in which the arrows indicate the direction of the strain slope. The offset of the load along the plane X-X, at cross-section No. (i), is not in agreement with what might be expected from the results, at cross-sections No. (ii) and (iii). This again cannot be explained by the presence of an offset of the load. Although the offset of the load along the plane Y-Y, is in the same direction at each cross-section, but the result at cross-section No. (i) is not in agreement with what might be expected from those at cross-section No. (ii) and (iii). The values of the offsets in inches along the plane X-X and Y-Y are given in Fig. 4.10.

In an endeavour to investigate further the observed non-linearities in strain observations, strain values were calculated by taking suitable slopes on the graph between the loads applied to the specimen and the corresponding strains, as shown in Fig. 4.11. These strain values are given in table 4.6. But in this case also, the results in columns 3

and 6 are different and indicate, as shown in column 8, in which all values at any one cross-section should be equal, a non-linear strain variation across each cross-section of the specimen. These observations can be made linear, as shown in Fig. 4.12, if approximately  $\pm 2\%$  corrections are made as previously. The offsets of the load obtained, given in Fig. 4.12, are not consistent at cross-sections No. (i), (ii) and (iii). The non-linear strain variation across cross-sections, therefore, does not seem to be due to the offsets of the load likely to be produced by the special machine. In order to investigate the cause of the non-linearity, specimen B made from a special steel, was therefore tested, reported in paragraphs 4.8 to 4.11.

#### TEST ON SPECIMEN B

##### 4.8. DESCRIPTION OF THE SPECIMEN

Specimen B was a rectangular mild steel plate 2" x 20" x  $\frac{1}{8}$ ". The steel from which it was made, unlike specimen A which was ordinary 'Commercial' mild steel, was deliberately chosen to conform to EN3A of B.S. 970 (1955), p.41, bright finished and normalised. The chemical composition is :

C = .16 ----- .15/.25

Si = Trace --- .05/.35

Mn = .47 ----- .40/.90

S = .035 ---- less than .06

P = .025 ---- " " .06

and the strength properties

Y.P. = 16.01 ton

U.T.S. = 25.94 ton

and Elongation = 35% on 2"

Details of the specimen with position of the eighteen Avro 99.3 ohm resistance gauges, with a gauge-sensitivity factor of 2.25 given by manufacturers, attached to it, are shown in Fig. 4.13. The ends of the specimen had to be reduced to  $1\frac{1}{2}$ " wide, as the jaws of the special-tensile-testing machine were not wide enough to take a 2" wide specimen. The set of gauges nearest to the ends of the specimen were about two times the width of specimen from the end of the jaws.

The specimen was annealed after making it of the required shape. The procedure adopted for annealing the specimen and fixing and protecting the resistance-gauges from moisture, was that detailed in paragraphs 4.3. and 4.4. respectively.

#### 4.9. GAUGE-FACTOR CHECK

Gauge factor values given by the firm were checked by attaching two of the gauges from the batch, to a beam which was loaded by means of four point loading system, as shown in Fig. 4.14. The specimen was loaded and unloaded twice, until loading and subsequent unloading curves coincided. The results are shown in table 4.7, and plotted in Fig. 4.15, for gauge No. 1. A dial-gauge capable of measuring  $1/10000$  of an inch

was used to measure the deflection at the centre of the beam.

The gauge factor calculated by the usual procedure detailed in paragraph 28, appendix C was 2.4 as compared 2.25 quoted by the suppliers (Avro-Whitworth), i.e. 7% different. The value of Young's Modulus, calculated from these results is  $31 \times 10^{+6}$  p.s.i.

#### TENSION TEST ON SPECIMEN B

##### 4.10 WITHIN ELASTIC RANGE

The specimen was tested in axial tension, using the special tensile testing machine. The gauges were connected to a 24 channel strain-gauge set, as previously. The bridges were initially each balanced and changes of galvanometer readings were taken as measures of the strains. Resistance calibration was achieved by placing a known resistance across the active-gauges.

The resistance gauges were checked for "drift" at zero load, before starting tests. The maximum drift noted in  $5\frac{1}{2}$  hours, was not more than 1 mm. which corresponds to a stress of about 280 p.s.i.

Rubber pieces, which helped in centering the specimen as explained in Chapter 3, paragraph 3.4 were placed in between the jaws to keep them apart at initial loads.

The strain readings obtained from the resistance-gauges when loading was within the elastic-range are given in columns 2(b) and 5(b) of table 4.8. Gages 8, 9 and 18 although electrically continuous and

apparently working, were insensitive to strain. The results they gave, will be ignored.

All these gauge readings when plotted against the applied load, were found to give straight lines, except those of gauge No. 2. This gave a small loop during loading and subsequent unloading. In calculating the strains from gauges which gave small loops, a line as shown in Fig. 4.16 (a), from zero to the maximum load, has been taken to indicate the strain. The results in columns 3 and 6, Table 4.8. are different and indicate, as shown in column 8 of table 4.8, in which all the values at each cross-section should be equal, a non-linear strain variation, across any one cross-section of the specimen. But, if arbitrarily  $\pm 2\%$  corrections are made in the observed values of strains, the results can be made linear, with the exception of those obtained from gauges No. 2 and 11, section (i), shown pictorially in Fig. 4.16(b). The readings from gauges No. 2 and 11 are very high and a correction of the order of about  $11\%$  has to be made to bring them into line with other readings. These results indicate the presence of an offset of the load along the plane X-X and Y-Y, as shown in Fig. 4.16 (b), which is possible. But, because the results at X-sections No.(iii) are not known, it is therefore, rather difficult to arrive at definite conclusions, regarding the possibilities of the presence of such an offset.

#### 4.11. TESTS INTO THE PLASTIC-RANGE

The specimen was then loaded beyond the elastic-range into the plastic-range and local yielding started at a load of about 5 ton, i.e. a mean stress of 12 t.s.i. The strain observations were now checked by Huggenberger-Extensometers, located as shown in Fig. 4.13. The specimen was loaded and unloaded several times within the above range of stress, i.e., 12 t.s.i., until loading and subsequent unloading curves coincided. All the gauge readings when plotted against the applied load were then found to give straight lines, except those of gauge No. 2. This gave a small loop during loading and subsequent unloading, as in the elastic-range. In calculating the strain from this gauge, a line as shown in Fig. 4.17 (a), from zero to the maximum load has been taken to indicate the strain. The results given in columns 3 and 6 Table 4.9, are different and indicate as shown in column 8 of table 4.9, in which all the values at each cross-section should be equal, a non-linear strain-variation across each cross-section of the specimen. But, as shown pictorially in Fig. 4.17(b), if arbitrarily  $\pm 2\%$  corrections are made in the observed values of strain, the results can be made linear, with the exception of those on gauges No. 2 and 11, section (i).

The Huggenberger Extensometer readings, given in table 4.10, are of the order expected from the Resistance-gauges, shown pictorially in Fig. 4.18(a). The corrected resistance-gauge results indicate the

presence of a small offset of the load along the plan X-X and Y-Y, as shown in Fig. 4.18(b), for cross-section No. (1).

At this stage in the testing, one wonders, whether the non-linear strain observations across the cross-sections of the specimen was due to the presence of local Luder's bands. To shed further light on this, the specimen was subjected to large values of strains, until it stretched 6% in its length. (It had to be reduced in length at the ends to make it 19", because the maximum length of a tensile specimen, that can be tested in the special tensile testing machine is only 20"). The resistance-gauges were now all found to be open circuited and so strain changes were measured using the Huggenberger extensometer. The strain readings obtained from them were now found to be making small loops, in each case, during loading and subsequent unloading. In calculating the strain rate, a line, as shown in Fig. 4.19(a), from zero to the maximum load, has been taken to indicate the strain. The results obtained from these gauges are recorded in table 4.11, and shown pictorially in Fig. 4.19(b), when the specimen was subjected to a mean stress of 17.4 t.s.i. The results in columns 3 and 6 are again different and indicate as shown in column 8 of table 4.11, in which all the values at any one cross-section should be equal, a non-linear strain variation across each cross-section of the specimen. This rules out the possibility of the non-linear strain behaviour due to the presence of Luder's bands.

The effect of moving the knife edges loads relative to the specimen was also roughly studied. The results obtained are given in table 4.12. The changes produced, were in the sense expected but even large movements did not produce changes of the order of anomalies. The comparative picture of strain readings obtained by moving the crossed knife-edges to various positions is shown in Fig. 4.20. This result suggests that the calculated offset of load was not actually present.

#### TEST ON SPECIMEN C

##### 4.12 DESCRIPTION OF THE SPECIMEN

Specimen C was a rectangular mild steel plate, the dimensions of which were based on B.S.15 (1948) specifications for tensile testing of metals. This shape was chosen so as to ascertain whether the presence of taper usual in specimens used in material tests would appreciably alter the strain patterns obtained. It was of the same batch of mild steel which was used in specimen B. Details of the specimen with position of eighteen Avro 99.5 ohm resistance gauges with a gauge sensitivity factor of 2.25, as indicated by manufacturers, attached to it, are shown in Fig. 4.21, (a) and (b). The ends of the specimen had to be reduced to  $1\frac{1}{2}$ " wide, as the jaws of the special tensile testing machine were not wide enough to take a 2" wide specimen. The set of the gauges nearest to the ends were about 2.5. times the width of the specimen from the end of the jaws.



The specimen was not annealed after making it of the required shape in order to discover whether the process of annealing the specimens was the cause of the non-linear strain variation. It was tested in the as 'received' condition, because the steel used was expected to be reasonably free from initial stresses.

The strain-readings obtained from resistance-gauges were checked by Huggenberger extensometers readings, which were clamped at positions shown in Fig. 4.21(a).

### TENSION TEST ON SPECIMEN C

#### 4.13 WITHIN ELASTIC-RANGE

The specimen was first tested in a 100 ton Buckton universal testing machine and then in the special tensile testing machine. The gauges were connected to a 24 channel strain-gauge set, as in previous cases. The bridges were initially each balanced and changes of galvanometer readings were taken as measure of strain. Resistance calibration was achieved by placing a known resistance across the active-gauges.

The resistance-gauges were checked for "drift" at zero load, before starting the experiment. The maximum drift noted in 4 hours time was not more than 0.05 mm., which corresponds to a stress of less than 160 p.s.i.

Rubber pieces, which helped in centering the specimen, as in the previous cases, were placed in between the jaws of the special tensile

testing machine, to keep them apart at initial loads.

All the gauge readings when plotted against the applied load were found to give straight lines as shown in Fig. 4.22(a), when tested in Buckton, but some of them, i.e. gauge No. 6 and 15, gave small loops, when tested in the special tensile testing machine, during loading and subsequent unloading. In calculating strains from gauges which gave small loops, a line, as shown in Fig. 4.22 (b) from zero to the maximum load has been taken to indicate the strain. The strain readings obtained from the resistance-gauges when the specimen was tested in Buckton and the special tensile testing machine, are given in tables 4.13 and 4.14, respectively. The results in columns 3 and 6 are different and indicate, as shown in column 8 of tables 4.13(a) and 4.14(a), in which all the values <sup>at</sup> each cross-section should be equal, a non-linear strain variation across each cross-section of the specimen. But if arbitrarily 12% corrections are made in the observed values, the results can be made linear, as shown pictorially in Figs. 4.23(a) and 4.24(a) with the exception of those on gauges No. 7 and 18 in Fig. 4.23(a) and 8 in Fig. 4.24(a), section (iii). These readings can be adjusted to bring them into line with other results, if corrections of the order of about 8% are made in their values. The results shown in Fig. 4.123(a), when the specimen was tested in Buckton, indicate an offset of the load along the plane X-X and Y-Y. The offset of the load, indicated by the results at X-section No. (iii), both along the plane X-X and Y-Y

strain values obtained from resistance-gauges are given in tables 4.13(a) and 4.14(a).

The results in columns 3 and 6 are different and indicate, as

are not in agreement with those, at X-sections No. (i) and (ii). The observations even when corrected can thus not be explained by the presence of an offset of the load.

The strain readings obtained from resistance-gauges were checked by Huggenberger extensometer readings, given in tables 4.13(b) and 4.14(b), which also gave the same non-linear strain behaviour across the cross-sections of the specimen. The comparative pictures of strain readings obtained from resistance-gauges and Huggenberger extensometers, when the specimen was tested in Buckton and the special tensile testing machine, are shown in Figs 4.23(b) and 4.24(b), respectively.

#### 4.14. TESTS INTO THE PLASTIC-RANGE

The specimen was now subjected to loads beyond the elastic into the plastic-range, using the special tensile testing machine and yielding started at a load of about 3 ton i.e., a mean stress of 16 t.s.i. The strain observations were also checked by Huggenberger extensometers readings, located at the positions shown in Fig.4.2(a). The resistance-gauges at cross-section No. (iii), showed more extensive yielding than that at other sections. This was probably due to local yielding. All the gauge readings when plotted against the load now gave small loops during loading and subsequent unloading. In calculating the strain rate in these cases, a line as shown in Fig. 4.25, from zero to the maximum load has been taken to indicate the strain. The strain values obtained from resistance-gauges are given in table 4.15(a). The results in columns 3 and 6 are again different and indicate, as

shown in column 8 of table 4.15(a), in which all values at each cross-section should be equal, a non-linear strain variation across the cross-sections of the specimen. But they can be made linear, if corrections of about 2% are made in the observed values of strain, as shown in Fig. 4.26. These results indicate an offset of the load along the plane X-X and Y-Y, as shown in Fig. 4.26, in which the arrows indicate the direction of the strain slope and which is possible. But, because the results at X-section No. (iii) are not known, it is therefore difficult to arrive at definite conclusions, regarding the presence of such an offset.

The values of residual strains obtained, after the specimen had been subjected to cycles of loading and unloading, are given in table 4.15(b). These values also indicate the presence of banding across the cross-sections of the specimen.

## CONCLUSIONS

It is quite evident from the results obtained that the shape of the specimen and the type of the machines, used for tests, had little effect on the observations of the strain distributions. These were non-linear across cross-sections in each case.

## TEST ON SPECIMEN D

### 4.15. DESCRIPTION OF THE SPECIMEN

Specimen D was a rectangular mild steel plate 9" x 2" x  $\frac{1}{8}$ ". It was deliberately made about half the length of Specimens A, B or C and was tested in a 6.5 ton Denson, in order to discover whether the length

of the specimens or the type of machines used in the previous tests, were the causes of the non-linear strain variation. It was also, of the same mild steel which was used in specimens B and C. Ten Avro, 97.5  $\Omega$  resistance-gauges with a gauge-sensitivity factor of 2.25 as indicated by manufacturers, were attached to it. Details of the specimen with position of resistance-gauges is shown in Fig.4.27(a). The set of gauges nearest to the ends were about 1.5 times the width of the specimen from the ends of the jaws.

The specimen was not annealed and was tested in the as 'received condition'.

#### 4.16 TENSION TEST ON SPECIMEN D

The specimen was tested in a 6.5 ton Denison, shown in Fig.4.27(b). The resistance-gauges in this case, were also connected to the 24-channel strain-gauge set, as in previous cases. The bridges were initially each balanced and changes of galvanometer readings were taken as measure of strain. Resistance calibration was achieved by placing a known resistance across the active gauges.

The resistance gauges were checked for "drift" at zero load, before starting the tests and they gave no drift. The strain readings obtained from the resistance-gauges are given in table 4.16. All these gauge readings when plotted against the applied load, gave straight lines. The results in columns 3 and 6 are different and indicate, as shown in column 8 of the above table, in which all the values should be equal, a non-linear strain variation across the

cross-section . But, if corrections of the order of  $\pm 2\%$  are made in the observed values of strains, the results can be made linear, as shown pictorially in Fig. 4.28. These results indicate the presence of an offset of the load along the plane X-X and Y-Y, shown in Fig. 4.28, in which the arrows indicate the direction of the strain slope. The value of the offset of the load has been calculated, considering the mean bending strains at the cross-section . But if separate values of the bending strains are considered which varies from 0.05 cm to 0.4 cm. along the plane X-X, the presence of the offset of the load cannot be justified. Similar is the case with the values of bending strains along the plane Y-Y, which are also not consistent across the cross-section.

#### 4.17 TEST INTO THE PLASTIC-RANGE

The specimen was now loaded beyond the elastic-range into the plastic-range and yielding started at a load of about 4 ton, i.e. a mean stress of 16 t.s.i. All the gauge readings when plotted against the applied load now gave small loops. The strain readings recorded in table 4.17, have been calculated by taking a line from zero to the maximum load, as shown in Fig. 4.29. The results of columns 3 and 6 are different and indicate, as shown in column 8, in which all the values should be equal, a non-linear strain variation across the cross-section of the specimen. But, if arbitrarily  $\pm 2\%$  corrections are made in the observed values, the results can be made to vary linearly, as shown in Fig. 4.30. The values of the offset of the load obtained along the

plane X-X, considering the average value of the bending strain can be justified. But that obtained along the plane Y-Y, considering the average value of the bending strain, cannot be justified, because the values of the bending strains are not consistent across the cross-section.

In an endeavour to investigate the non-linear strain variation, the specimen was subjected to cycle of loadings and unloadings. The results now obtained for a load of 3.5 ton, i.e. a mean stress of 14 ton per sq. inch, are given in table 4.18. These results were calculated from the straight line portion of the loading curve, shown in Fig. 4.31(a), for gauge No. 2 and are still showing the same non-linear strain variation across the cross-section. This cannot be due to the presence of Luder's bands, because the specimen has already been subjected to cycle of loadings and unloadings and these must have covered the entire length of the specimen.

The residual strains built up during loadings and subsequent unloadings, have been calculated from the results, such as that shown in Fig. 4.31(b) and are given in table 4.19. These values again confirm the pattern of strain readings obtained from the previous tests.

which should have been equal, as well as the large inhomogeneous and inconsistent effect of loading in the specimens, which they indicate was connected to vary linearly over cross-sections, could be consistent with "local" lack of homogeneity in the mild steel specimens, being the cause of the non-linear strain readings.

#### 4.18. CONCLUSIONS

The experimental results obtained on plain rectangular mild steel plates, tested in sensibly axial tension, indicate a non-linear strain variation across the cross-sections of the specimen, irrespective of the shape and size of the specimen and the type of machine used for tests. These non-linear strain variations can be made linear, if appropriate corrections of the order of  $\pm 2\%$  are made in the observed values of strains. By moving the crossed knife-edges of the special tensile testing machine, in an endeavour to produce offset loading, the strain readings were affected in the sense expected but the changes were small compared with the offset initially present as indicated by strain gauge readings. The results obtained, thus indicate either (i) that the strain readings obtained do not give an accurate measure of the actual strains, but checks with different gauges seem to indicate that this was unlikely, because they also gave non-linear strain readings, or (ii) that over a strain gauge length of about  $\frac{1}{4}$ " to  $\frac{1}{2}$ " the annealed mild steel used is not the homogeneous material it is usually considered to be. The random scatter of strain observations, which should have been equal, as well as the large improbable and inconsistent offset of loading in the specimen, which they indicate when corrected to vary linearly over cross-sections, would be consistent with "local" lack of homogeneity in the mild steel specimens, being the cause of the anomalous strain readings.



## CHAPTER 5.

### TESTS ON RECTANGULAR MILD STEEL PLATE WITH A CIRCULAR HOLE, SYMMETRICALLY PLACED AT THE CENTRE

#### 5.1. INTRODUCTION

The strain readings obtained from tests on plain rectangular mild steel plates, reported in Chapter 4, were non-linear across cross-sections, irrespective of the shape and size of the specimens and the type of machines used. The non-linearity was also not due to the resistance-gauges used being defective, because the results obtained from Huggenberger extensometers placed between the resistance-gauges, were of the order expected from resistance-gauges. It was therefore, decided to continue tension tests on a rectangular mild steel plate E with a circular hole, symmetrically placed at the centre, in order to see how the non-linear strain variation obtained in case of plain rectangular mild steel specimens, effects the stress-concentration at the hole. This information could be of great help in the study of load distribution in joints. The specimen was tested in the special tensile testing machine, described in Chapter 3. It was annealed before testing, as explained in paragraph 4.3, to avoid possible effects of initial stresses on the strain observations.

The test results obtained from resistance-gauges indicate a symmetrical strain variation across cross-sections of the specimen, which was quite marked at the hole. The strain readings obtained

from resistance-gauges at the hole were checked by Huggenberger extensometers, placed between the resistance-gauges. They also gave asymmetrical strain readings, which were of the order expected from resistance-gauges.

In order to find out the cause of asymmetrical strain variation, various tests, reported in paragraphs 5.4 to 5.16 were made, but results obtained indicate that probably the material used was not homogeneous enough for the St. Venant's theory to be strictly applicable.

In the tests, the plate was first loaded within the elastic and then beyond into the plastic-range. It was loaded and unloaded several times in order to see whether this affected strain readings, but asymmetrical strain variation across cross-sections persisted.

The resistance-gauges from which the data obtained in paragraphs 4.3 and 4.4 were taken were also taken at positions 'a' near the hole as shown in Fig. 5.1 (a). These gauges were used to check the strain-pattern obtained from resistance-gauges. The gap between the resistance-gauges which had to be left to attach the detachable Huggenberger extensometer gauges, were sealed with 2.1.6 plastic in order to protect the resistance-gauges from moisture.

The first set of resistance-gauges around the ends of the specimen were about 1.5 times the width of the specimen from the end of

## 5.2. DESCRIPTION OF SPECIMEN E

Specimen E was a rectangular mild steel plate 2" x 19" x  $\frac{1}{8}$ ", containing a  $\frac{7}{8}$ " diameter hole, symmetrically placed at the centre. It was made from the same batch of mild steel as that used in specimens B and C (Chapter 4). Near the ends of the specimen  $\frac{1}{4}$ " diameter holes were drilled as shown in Fig. 5.1(a). Pins through these holes and corresponding holes in the special tensile testing machine jaw grips, applied the initial loads which were intended to centre the specimen, as in previous tests. The ends of the specimen had to be reduced to a width of  $1\frac{1}{2}$ " because the jaws of the special tensile testing machine were not wide enough to take a 2" wide specimen.

The specimen was annealed after cutting and drilling. The procedure adopted for annealing the specimen and fixing and protecting the resistance-gauges from moisture, was that detailed in paragraphs 4.3 and 4.4.

Huggenberger extensometers readings were also taken at positions 'a' near the hole as shown in Fig. 5.1 (a). These gauges were used to check the strain-pattern obtained from resistance-gauges. The gap in between the resistance-gauges which had to be left to attach the demountable Huggenberger extensometer gauges, were sealed with B.I.C. plastic in order to protect the resistance-gauges from moisture.

The first set of resistance-gauges nearest the ends of the specimen were about 1.5 times the width of the specimen from the end of

the jaws, as shown in Fig. 5.1 (a). The next set of gauges were about 2.5 times and gauges at the hole were about 3.5 times the width of the specimen, from the end of the jaws.

Timoshenko and Goodier (ref 37, page 52) show that at twice the width of the specimen from a concentrated load, the stress-distribution is uniform. St. Venants principle might then, be expected to be almost applicable to the first set of gauges, which are twice the width of the specimen from the hole and about one and a half times the width from the end of the jaws.

The beam was loaded to 100 lbs. by equal increments of 10 lbs. and subsequently unloaded to 0. The readings of resistance-gauges and dial-gauges are given in table 5.1. The null point method of balancing the bridge was used. Fig. 5.2, for gauge No. 1, is typical of the results obtained.

The slope of the graph gives the gauge-sensitivity factors given at the bottom of table 5.1. The mean value of 2.34 was used for the calculation of stresses in the tests reported later here. It will be noted that figure quoted by the manufacturer's was 2.15, a large difference (some 15%), as compared with variances here of about 1% in the six tested.

### 5.3. GAUGE FACTOR CHECK

Gauge sensitivity factor values given by the firm were checked. Two resistance-gauges were taken from each of the three packets, containing 100 resistance-gauges. One of the resistance gauges from each pair, was attached to the tension side and other compression, of the mild steel beam, which was loaded by means of a four point loading system, as shown in Fig. 4.14. A dial-gauge capable of measuring deflections to nearest 1/10000 of an inch was used to measure the beam deflection.

The beam was loaded to 100 lb., by equal increments of 10 lb. and subsequently unloaded to 0. The readings of resistance-gauges and dial-gauges are given in table 5.1. The null point method of balancing the bridge was used. Fig. 5.2, for gauge No. 4, is typical of the results obtained.

The slope of the graph gives the gauge-sensitivity factors given at the bottom of table 5.1. The mean value of 2.54 was used for the calculation of stresses in the tests reported later here. It will be noted that figure quoted by the manufacturer's was 2.25, a large difference (some 4%), as compared with variance here of about 1% in the six tested.

TENSION TEST ON SPECIMEN E5.4. WITHIN ELASTIC-RANGE

The specimen was tested in axial tension using the special tensile testing machine. The gauges were connected to a 24 channel box, 6 channel box and 2 separate boxes. These boxes were connected as shown in Fig. 1 of appendix A, so that only one galvanometer was used for measuring the bridge out of balance, which was the measure of strain taken (the null point method of measuring the values of strain was not used).

The resistance-gauges were checked for "drift" at zero load before starting the tests. The maximum drift noted in 3 hours was not more than 0.05 cm. which corresponds to a stress of about 150 p.s.i. Over an interval of one hour which is larger than the usual period of one test, the drift was negligible.

Rubber pieces were placed in between the jaws to keep them apart at initial loads, which helped in centering the specimen, as explained in Chapter 3, paragraph 3.4.

Most of the gauge-readings when plotted against the applied load were found to give straight lines, as shown in Fig. 5.3. for gauges No. 7, 9, 10, 23, 24, 25 and 26, but for gauge No. 8, a mean line was drawn. From these graphs, the change of galvanometer readings for a load change from 0.154 to 0.616 ton (viz 0.462 ton) are, as shown in column 2(b) and 6(b) of table 5.2(a), in which horizontal lines

separate different cross-sections.

Resistance-gauges marked with asterisk in table 5.2(a) gave higher calibration values than the other gauges, for the same value of resistance, placed across each in turn. This was, because, these gauges were connected to the 6 channel box which had fixed arms of a different value. The results are shown diagrammatically in Fig. 5.4(a) and the gauge readings marked with an asterisk have been multiplied by 2.85/3.75.

#### 5.5. RESULTS AT THE CROSS-SECTIONS AWAY FROM THE HOLE

The results obtained at cross-sections No (i), (ii), (iv) and (v) of Fig. 5.1, indicate a non-linear strain variation over each cross-section, as shown in Fig. 5.4(a). Arbitrarily correcting the observed values by appropriate amount of up to some  $\pm 2\%$  to the "corrected values" shown in these diagrams, makes the strain distribution over these cross-sections vary linearly, but observations from gauges No. 28 and 31 were very much too high to fall into this linear pattern. Even with these corrections, the mean of the strains observed at each cross-section varied, as shown in Fig. 5.4 (a), from 1.325 to 1.2 (i.e. 10%) and more than might be expected. An arbitrary value of 1.15 cm. has been assigned to gauge No. 22, which was not working. This value was arrived at, to fit in linearly with other values at the cross-section.

Resistance-gauges No. 1, 2 and 3 at cross-sections No. (i), gave

higher strain values than their respective partners No. 19, 18 and 17, on the other face of the specimen shown in Fig. 5.4(a). Also, resistance-gauge No. 1 gave higher strain reading than that given by gauge No. 2 and which in turn gave higher reading than gauge No. 3. Similarly gauge No. 19 gave higher strain reading than that of gauge No. 18 and 17.

These results suggest the presence of an offset of the load along the plane X-X and Y-Y.

Similar results were obtained at cross-sections No. (ii), (iv) and (v), which also indicate the presence of the offset of the load in the above mentioned directions. The values of the offset of the load obtained at these cross-sections are not consistent, which cannot be explained.

For bending about Y-Y, the values of the bending strains, at cross-sections No. (i), (ii), (iv) and (v) are 0.5, 0.4, 0.45 and 0.6 cm. If St. Venants bending theory holds at these cross-sections, then the variation in the bending strain along the length of the specimen must be linear, because the specimen is loaded only at its ends. The observed variation is however not linear and so either St. Venants theory is not applicable or the bending strain observations are inaccurate and they indicate only an offset of load giving a mean bending strain equivalent to 0.5 cm., i.e. a load offset equivalent to 0.1".

For bending about X-X, the bending-strains at cross-sections No. (i), (ii), (iv) and (v) are 0.55, 0.7, 0.1 and 0.35 cm., respectively.



They also do not vary linearly along the length of the specimen, but an average value of 0.4 cm., corresponds now to a very small offset of the load equivalent to 0.0031".

If arbitrarily  $\pm 2\%$  corrections are made to the observed readings, the results can be made to vary linearly. The high readings of gauges 28 and 30 may be indicative of gauge sensitivity varying more than  $\pm 2\%$ , but there is little other evidence of unsatisfactory gauges. The gauges have no "drift" at zero load and gave a straight line variation during loading and subsequent unloading.

#### 5.6. RESULTS AT THE HOLE

The results at cross-section No. (iii) containing the circular hole, are not in agreement with what might be expected from the results obtained at cross-sections No. (i), (ii), (iv) and (v). Thus resistance-gauges No. 7, 8, 9 and 10, would be expected to give higher strain-readings than their respective partners No. 26, 25, 24, and 23, but gave lower readings. The gradient from gauge 10 to gauge 7 is however in the direction expected from other sections. The results obtained from the diagonally opposite resistance-gauge No. 8 and 24, 9 and 25 respectively, added together in order to effect some compensation for the effect of bending, are in ratio 1.09 : 1. Nevertheless, these results at the hole, are checked by Huggenberger extensometers readings, shown in Fig. 5.4(b). The strain readings at the hole thus do not seem to follow a pattern which fits in well with other observations away from the hole.

The specimen was now subjected to a load of 0.77 ton, i.e. a mean stress of 3.08 t.s.i.. Resistance gauges at the edge of the hole, i.e. gauge No. 8, 9, 24 and 25 gave loops during loading and subsequent unloading, while the readings obtained from the rest of the gauges, were still linear, when plotted against the applied load. These loops do not indicate any sign of yielding, as it is evident from Fig. 5.5(a) and (b). The change of galvanometer reading for a load change from 0.154 to 0.77 ton (vs 0.616 ton) at various cross-sections of the specimen, are given in column 2(b) and 6(b) of table 5.3(a). In calculating strain from gauges, which gave loop during loading and subsequent unloading, strain values, shown in Fig. 5.5.(a) and (b), are taken from straight line portion of the curves. Resistance-gauge No. 22 which was not working, has been assigned an arbitrary value of 3.6 cm., shown in Fig. 5.6 (a).

#### 5.7. RESULTS AT CROSS-SECTIONS AWAY FROM THE HOLE

The results at the cross-sections away from the hole, indicate a non-linear strain-variation, as in the previous test, but can be explained by St. Venant bending, shown pictorially in Fig. 5.6(a), if arbitrarily 12% corrections are made in the observed values. The strain readings indicate an offset of the load, along the plane Y-Y and X-X, equivalent to an average value of 0.125" and 0.0042" respectively. The mean observed strains, shown in Fig. 5.6(a), at cross-

sections No. (i), (ii), (iv) and (v) are about in the same ratio, as that in the previous test. This suggests that the strain readings are consistent.

### 5.8. RESULTS AT THE HOLE

The results obtained at cross-section No. (iii), containing the circular hole, are similar to those in the previous test. These results are checked by Huggenberger extensometers readings, shown in Fig. 5.6(b). Huggenberger extensometers readings, gave loops as shown in Fig. 5.7, when plotted against the applied load, during loading and subsequent unloading, but straight line portion of the curve has been taken to indicate the strain.

The specimen was now subjected to a load of 1.08 ton, i.e. a mean stress of 4.3 t.s.i. Resistance-gauges at the hole, i.e. gauges No. 7, 8, 9, 10 and 23, 24, 25, and 26 gave loops during loading and subsequent unloading. This time, the loops were larger than in the previous case, as shown in Fig. 5.8(a) and (b). Resistance-gauge No. 6 and 31, gave absurd results, shown in Fig. 5.9 and which suggest that probably the gauges were not working and the strain-readings obtained from them would be ignored in future. Strain readings obtained from the resistance-gauges at the cross-sections of the specimen, for a load change from 0.158 ton to 1.08 ton (viz 0.922 ton) are given in table 5.4(a). In calculating strain from gauges which gave loops, strain values shown in Fig. 5.8 (a) and (b), are

taken from straight line portion of the curves. Resistance-gauge No. 22, which was not working, has been assigned an arbitrary value of 4.35 cm., shown in Fig. 5.10 (a).

#### 5.9. RESULTS AT THE CROSS-SECTIONS AWAY FROM THE HOLE

The results at the cross-sections away from the hole are indicating a non-linear strain-variation, as in the previous tests, but can be made linear, if arbitrarily  $\pm 2\%$  corrections are made in the observed values. The offset of the load along the plane Y-Y and X-X, is equivalent to an average value of 0.104" and 0.004" respectively. The mean observed strains, shown in Fig. 5.10(a), at cross-sections No. (i), (ii), (iv) and (v), are in the same ratio as that in the previous tests.

#### 5.10. RESULTS AT THE HOLE

The results at the cross-section No. (iii), containing the circular hole, are similar to those obtained in the previous tests. These readings are checked by Huggenberger extensometers readings, shown in Fig. 5.10(b). Huggenberger extensometers readings gave loops but strain values, are taken from the straight line portion of the curves.

In the above tests, the ratio of the mean strain at the cross-sections away from the hole, i.e. No. (i), (ii), (iv) and (v) are

fairly consistent. The results at cross-sections No. (iii), containing the circular hole, also have about the same ratio at each test. This indicates that the strain-gauges are behaving consistently.

In order to shed further light on the observed strain variation across the cross-sections of the specimen, strain readings were obtained at the hole, by moving the crossed knife-edges of the special tensile testing machine, in an endeavour to produce an offset loading. The strain readings at the hole, were altered in the sense expected, each time, the knife-edges were moved to a new position, but the changes produced were not large, as shown in table 5.5. Strain readings were also obtained, bringing the top end of the specimen to the bottom jaws and vice versa, and also turning the specimen through  $180^{\circ}$ . But the resistance-gauges, which gave higher strain readings, were still showing higher values, irrespective of the position of the knife-edges. These results were also checked by Huggenberger extensometers readings, which were of the order expected from the resistance gauges results, as shown pictorially in Fig. 5.11.

The non-linear variation, therefore, seems unlikely due to any offset of the load, likely to be produced by the loading machine. If the Huggenberger extensometers readings had not agreed with values obtained by resistance-gauges, these would have been more suspect and lack of homogeneity in the steel specimen remains as the most likely cause of the non-linear and other anomalous results obtained.

### 5.11. CALCULATION OF STRESS-CONCENTRATION FACTORS AT THE HOLE

The stress-concentration value at the hole can be calculated as follows :-

The value of the mean-stress applied to the specimen under investigation, was taken as the mean-value of stress at cross-sections No. (i), (ii), (iv) and (v) as shown in table 5.2(a) which is :-

$$= \frac{1}{4} (1.87 + 1.83 + 1.75 + 1.8) = 1.81$$

and the mean-value of stress at the hole at points K1' and K2', (shown in Fig. 5.1 (a)) as

$$= \frac{1}{4} (3.57 + 2.25 + 5.55 + 4.9) = 4.07$$

and

$$= \frac{1}{4} (1.335 + 0.63 + 3.94 + 3.02) = 2.231,$$

respectively.

The mean stress-concentration values at points K1' and K2' is then

$$4.07/1.81 = 2.25 \text{ and } 2.231/1.81 = 1.23,$$

respectively.

### 5.12 COMPARISON WITH THEORETICAL RESULTS

From Howland's theory (ref 18), the stress-concentration factor at the edge of the hole, which has a diameter of 0.44 times the width

\* in what follows the word "stress" is used in abbreviation for "Exstrain" unless otherwise stated.

of the plate, is equal to 4, as seen from Fig. 5.12. The stress-concentration values at points K1' and K2' of 2.25 and 1.23, obtained from the average values from resistance-gauges, are about the same as the theoretical results shown in Fig. 5.13. But the stress-concentration values obtained from the results of the individual gauges at points K1' and K2', shown in Fig. 5.14, indicate a large variation from that of the theoretical results.

As shown in Fig. 5.15, the values of the stress-concentrations around the hole vary from a negative value of -1.5 at K3 to a positive value of 4 at K1. The resistance-gauge at the edge of the hole, subtends an angle of about  $11.5^\circ$  to the centre of the hole, as shown in Fig. 5.15. The strain-values recorded by the resistance-gauges at the edge of the hole, would therefore be the mean of the strain between points K1' and K4', which lie at the centre of the resistance-gauge and subtend an angle of  $0^\circ$  and  $11.5^\circ$  respectively, to the centre of the hole (Fig. 5.15). Therefore, theoretically, one would expect the strain-values at points K1', which are at a distance of about  $3/32$ " from the edge of the hole, i.e. the centre of resistance-gauge, to be 2.4, instead of 2.5 as obtained from Howland's Analysis (ref. 18). But the strain values obtained experimentally, in the case under review, at points K1' is 2.25 instead of 2.4, i.e. about 6% lower than the theoretical result.

actual local values depart considerably from the mean values just as the strain measurements around a supposedly uniformly stressed plain specimen were observed.

The values of "cross-stresses"  $xx/T$  and  $yy/T$ , where  $T$  is the applied tensile stress, on a longitudinal sections through the hole, for the case of a plate, containing a circular hole, the diameter of which is 0.5 times the width of the specimen, were calculated by R.C.J. Howland and are given in table XV, reference 18. The writer calculated the values for a plate, containing a circular hole, the diameter of which is 0.4 times the width of the plate and these are given in table 5 of Appendix C. The values of the constants used in the equations for calculating the cross-stresses, i.e.  $xx/T$  and  $yy/T$  were those, given by R.C.J. Howland, table-X, reference 18.

The graphs so obtained for plates with hole diameter/width ratios,  $\lambda$ , of 0.4 and 0.5, are shown in Fig. 5.16. A curve sketched between them, gave the value of cross-stresses for the case of a plate containing a circular hole, the diameter of which is 0.44 times the width of the plate.

The values obtained from Huggenberger extensometers at point K was 0.344. This point is about 0.85" from the point K3 and the theoretical value is about 0.38.

### 5.13 CONCLUSIONS

Mean values of stress-concentration factors thus agree reasonably well with those which can be obtained from Howland's theory, but the actual local values depart considerably from the mean values just as the strain measurements across a supposedly uniformly stressed plain specimens were non-uniform.



### 5.14. TEST INTO THE PLASTIC-RANGE

The specimen was now loaded beyond the elastic-range. Resistance-gauges at the hole, showed yielding at a load of about 1.54 ton, i.e. a mean stress of 6.16 t.s.i., in the specimen while the results obtained from the resistance-gauges at other sections, were still linear, when plotted against the applied load. The load was gradually increased at each cycle of loading. The strain readings obtained, from the resistance-gauges at the hole showed yielding, each time the specimen was subjected to a higher load than that of the previous cycle. But, each time on unloading, an almost linear stress/strain line, as shown in Figs. 5.17 and 5.18, for gauges No. 8 and 25, was obtained.

The strain-readings obtained from resistance-gauges at various cross-sections of the specimen, for a load change, from 0.16 to 1.70 ton (viz. 1.54 ton), are given in table 5.6(a). The results obtained at the hole gave loops during loading and subsequent unloading. In calculating strains for these cases, straight line portion of the curves, as shown in Fig. 5.18, has been taken to indicate the strain.

### 5.15 RESULTS AT THE CROSS-SECTIONS AWAY FROM THE HOLE

The results obtained from the resistance-gauges at cross-sections No. (i), (ii), (iv) and (v), indicate a non-linear strain variation across the cross-sections. These results, if corrections of the order of 2% are made in the observed values of strains, as shown pictorially in Fig. 5.19(a), vary linearly across each cross-section. The mean observed strains at these cross-sections are in the ratio of 1:0.988:1:0.935, which are very nearly the same as previously obtained, in the

elastic-range. With the exception of section (ii), the calculated offset of load along the Y-Y direction is sensibly constant (0.06") and that along X-X small (2 to 4 thousands of an inch).

#### 5.16 RESULTS AT THE HOLE

The pattern of strain readings obtained at the hole, is the same as that in the elastic-range. Corresponding strain readings obtained on each face are different, thus those from resistance-gauges No. 8 and 25, 9 and 24 are in the ratio of 1.6 and 1.46 respectively. The results at the hole were checked by Huggenberger extensometers readings, shown in Fig. 5.19(b), which are of the order expected from resistance-gauges results.

The plastic residual strains built up during loadings and subsequent unloadings are given in table 5.7(a), and were obtained from the results, such as that shown in Figs. 5.17 and 5.18 and Figs. 5.20 and 5.21(a) and (b), for gauge nos. 7 and 26, 8 and 25 respectively. The values of the residual strains at the hole were also checked by Huggenberger extensometers readings, shown in Fig. 5.21(a) and (b), which are in agreement with those of resistance-gauge results, as shown in Fig. 5.21(c). These results indicate, far greater residual strains at the stations 23, 24, 25 and 26 than at the corresponding stations on the other face. They tend to confirm that the anomalous strain behaviour recorded in the elastic-range was actually present since, where the strains were high considerable yielding has occurred and

left large residual strains on unloading.

The specimen was now subjected to a load of 2.18 ton, i.e. a mean stress of 8.72 t.s.i. The results obtained for a load change from 0.16 to 2.18 ton (viz 2.02 ton) are given in table 5.8. The resistance-gauges at the hole were all found open-circuited at this load, while the results obtained from the resistance-gauges at cross-sections No. (i), (ii), (iv) and (v), were linear when plotted against the applied load. These results indicate a non-linear strain-variation across the cross-sections away from the hole, but as before, excepting the results for gauges No. 28 and 31, they can be adjusted to be linear, if corrections of the order of 12% are made in the observed values of strains, as shown pictorially in Fig. 5.22. Gauge No. 28 and 31 gave loops during loading and subsequent unloading and large corrections are necessary in their readings to bring them in line with other results.

In a further endeavour to find the cause of this non-linear strain variation, specimen was stretched about 5% of its length (the extension was measured using a dividers). The specimen was afterwards subjected to a load of 2.66 ton, i.e. a mean stress of 10.4 t.s.i. and the results obtained for a load change from 0.16 to 2.66 ton (viz 2.5 ton) are given in table 5.9(a). The results at the hole were obtained, using the Huggenberger-extensometers and are given in table 5.9(b). The results at cross-sections No. (i), (ii), (iv) and (v) are non-

linear, but can be made linear, as shown in Fig. 5.23, if corrections of about  $\pm 2\%$  are made in the observed values. The values of the offset of the load obtained given in Fig 5.23, are not consistent at cross-sections No. (i), (ii), (iv) and (v), which cannot be explained by an offset of the load likely to be produced by the special machine.

The possibility of the non-linear strain variation over the cross-sections being due to Luder's bands can now be ruled out because these must have covered the entire length of the specimen, when it was stretched 5%.

#### 5.17 COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS - PLASTIC RANGE

In an axially loaded tensile specimen with a circular hole, ref. 13, the plastic zone will begin to form at the points A and B, Fig. 2.3. It will be of the shape, shown hatched in the figure and will not necessarily completely surround the hole. An approximate solution for the strain distribution has been worked out by Faerberg (ref. 13) on the basis of the theory of elasto-plastic bending of a curved beam. He found for an idealised material representative of duralium plate, with a linear elastic followed by a linear plastic stress/strain curve, that the plastic zone which starts to form at points A and B at a mean tensile stress of one third the yield stress, extends rapidly around the hole contour as the value of the tensile stress increases from a third to 0.65 times the yield stress. When

the tensile stress if greater than 0.65 times the yield stress, plastic zones grow in a direction perpendicular to the tensile stresses and simultaneously become smaller at the contour of the hole itself. He also found that with increasing values of the tensile stress, the deformations near the hole increased considerably more rapidly than at the outside edges of the plate. His experimental values for the point A are almost the same as his theoretical values, for mean tensile stresses up to 0.5 times the yield stress.

In the tests made by the writer, the strains were not symmetrical across "cross-sections" so that the growth of the plastic zones described by Faerberg could not be verified. The deformations near the hole increased considerably more rapidly than they did on the outside edges of the plate, which can be seen from Fig. 5.18 and 5.20(a). The permanent set remaining after the specimen has been subjected to a tensile stress of 6.8 t.s.i. are given below, for cross-section No. (iii).

Column No.	1	2	3	4	5	6	7
Line No.	R.G. No	Permanent Residual Strain in cm.	R.G. No.	Permanent Residual Strain in cm.	Sum of Column 2 & 3	line $\frac{(n+1)}{2}$ Line n	Average value of Column 6
1	7	8.5	26	19.8	28.3	1.86	
2	8	21.3	25	31.35	52.7		3.4
3	9	8.65	24	20	28.65		
4	10	0.2	23	5.6	5.8	4.95	

The average value of the permanent deformation near the edge of the hole is about 3.4 times that on the outside edge of the plate. This agrees with the Faerberg's results, shown in Figs 102 and 103 (ref. 15).

The values of the strains, which were used to vary linearly, at arbitrarily 2 of cross-sections are not in the observed values of strains. The values of the effect of the load required to produce these linear strain variations are actually not consistent with a line of loading passing through the hole.

The results at cross-sections (ii) and (iii), containing the circular hole were also obtained and are not in agreement with what might be expected from the results of cross-sections (i), (iv), (v) and (vi). This cannot be explained by the presence of an effect of the load. Further, the results at (ii) and (iii) were obtained by moving the crossed half-weights of the optical strain measuring system, in an endeavor to produce an effect of the load. The strain readings were affected in the same direction, but though produced, were not of the order of magnitude. The cross-sections, which gave higher strain readings, were still showing strain variation, at the two positions of the crossed half-weights. The maximum strain variation across the cross-sections of the specimen, therefore, were not in any effect of the load, which is a result of the loading system.

The maximum strain variation, which was widely seen in the variation in the permanent strain readings of the cross-sections

## 5.18 CONCLUSIONS

The results obtained at cross-sections No. (i), (ii), (iv) and (v) away from the hole, indicate a non-linear strain-variation across the cross-sections of the specimen. These results can be made to vary linearly, if arbitrarily  $\pm 2\%$  corrections are made in the observed values of strains. The values of the offset of the load required to produce these linear strain variations are axially not consistent with a line of loading passing through the knife-edges.

The results at cross-section No. (iii), containing the circular hole were asymmetrical and also not in agreement with, what might be expected from the results at cross-sections No. (i), (ii), (iv) and (v). This cannot be explained by the presence of an offset of the load. Further, the results at the hole were checked by moving the crossed knife-edges of the special tensile testing machine, in an endeavour to produce an offset of the load. The strain readings were affected in the sense expected, but changes produced, were not of the order of anomalies. The resistance-gauges, which gave higher strain readings, were still showing similar results, at the new positions of the crossed knife-edges. The non-linear strain variation across the cross-sections of the specimen, therefore, seems unlikely due to any offset of the load, likely to be produced by the loading machine.

The non-linear strain variation, also seems unlikely due to the variation in the gauge-sensitivity factors of the resistance-gauges used between the specimen and the plate. Thus, the non-linear strain variation cannot be due to the lack of straightness of the specimen.

used, because the results at the hole were checked by Huggenberger extensometers readings, which confirmed the resistance-gauge results.

The resistance-gauges which gave higher strain readings in the elastic range, also gave higher values of plastic residual-strains, after the specimen had been subjected to loads which took it into the plastic-range. The results obtained from resistance-gauges were also consistent in each test and no appreciable "drift" was recorded at zero load. The non-linear strain variation is not likely to be due to defective resistance-gauges.

The specimen was annealed, as explained in paragraph 4.3 before starting the tests. The presence of large initial strains in the material is thus an unlikely cause of the non-linear strain variation. The specimen was also stretched about 5% of its length, but the results were still non-linear over the cross-sections of the specimen. The non-linear variation due to Luder's band, can now be ruled out because these must have covered the entire length of the specimen, when it was stretched 5%.

The strain-pattern obtained at cross-section No. (iii) which is not in agreement with, what might be expected from the results at cross-sections No. (i), (ii), (iv) and (v), indicates the presence of local bending at this cross-section. But the specimen was tested on a flat plate, before the resistance-gauges were attached to it and to the naked eye it appeared to have contact and day light was not seen between the specimen and the plate. Thus, the non-linear strain variation cannot be due to the lack of straightness of the specimen.



The values of the strain per ton obtained, from the resistance-gauges, at the cross-sections of the specimen in tests I and II, in the elastic-range and that, after the specimen had been subjected to loads which took into the plastic-range are shown in columns 2 and 4, table 5.10. These results indicate a change in their values from test I to test II, in an asymmetric manner, i.e. rise in some case and vice-versa. This again indicates, that probably the strain was not linearly distributed across cross-sections of the specimen, in different tests.

In the light of above arguments, it seems that probably the steel specimen was not homogenous enough for St. Venant's theory to be strictly applicable and that the strains over  $1/4$ " gauge lengths will be sensibly different from those calculated on the assumption that the material is homogeneous although mean values are found to agree sensibly with such theory.

Two resistance-gauges were attached to the specimen and the results obtained from them were further checked by Roggenberger extensometers.

The results obtained from resistance-gauges indicate asymmetrical strain variation across cross-sections of the specimen, which was not quite as marked at the hole as that to be seen of specimen B. The results obtained from resistance-gauges at the hole were also checked by Roggenberger extensometers, placed between the resistance-gauges.

## CHAPTER 6

### TESTS ON RECTANGULAR MILD STEEL PLATE WITH TWO EQUAL CIRCULAR HOLES

#### 6.1. INTRODUCTION

In the study of joints, it is very important to see, how the stress-concentration is affected by the presence of more than one hole, which are near enough to influence each other. In order to study this effect and also that of asymmetrical strain variation obtained in case of specimen E, containing a circular hole, a rectangular mild steel plate F, containing two equal circular holes, was therefore, tested in axial tension in the special tensile testing machine. The specimen was made from the same batch of mild steel, that was used in specimen E. It was annealed before testing, as explained in paragraph 4.3, to avoid possible effects of initial stresses on the strain observations. Avro resistance-gauges were attached to the specimen and the results obtained from them were further checked by Huggenberger extensometers.

The results obtained from resistance-gauges indicate asymmetrical strain variation across cross-sections of the specimen, which was not quite as marked at the hole as that in case of specimen E. The results obtained from resistance-gauges at the hole were also checked by Huggenberger extensometers, placed between the resistance-gauges.

They gave results which were of the order expected from resistance-gauges.

In the tests, the plate was first loaded within the elastic and then beyond into the plastic-range. It was loaded and unloaded several times in order to see whether this affected strain readings, but the asymmetrical strain variation across cross-sections persisted.

## 6.2. DESCRIPTION OF SPECIMEN F

Specimen F was a rectangular mild steel plate 2" x 18" x  $\frac{1}{8}$ ", containing two circular holes, each  $\frac{7}{8}$ " in diameter, as shown in Fig. 6.1(a) and (b). It was made from the same batch of mild steel, as that used in specimens B C Chapter 4.

Near the ends of the specimen,  $\frac{1}{4}$ " diameter holes were drilled, as shown in Fig 6.1(a) to align the specimen in the special testing machine and the ends of the specimen had to be reduced to a width of  $1\frac{1}{2}$ ", because the jaws of the special tensile testing machine were not wide enough to take a 2" wide specimen.

The specimen was annealed after cutting and drilling. The procedure adopted for annealing the specimen and the fixing and protecting the resistance-gauges from moisture, was that detailed in paragraphs 4.3 and 4.4.

Details of the specimen with the position of the thirty Avro resistance gauges, attached to it, are shown in Fig. 6.1 (a). The

manufacturers quoted a sensitivity factor of 2.25 and an average value of resistance of 98.9 ohm, for these gauges.

Huggenberger extensometers readings were also taken at positions 'a' near the holes, as shown in Fig. 6.1(a). These gauges were used to check the strain pattern obtained from the resistance-gauges. The gap in between the resistance-gauges at the holes, which had to be left to attach the demountable Huggenberger extensometers, were sealed with B.I.C. plastic, in order to protect the resistance-gauges from moisture.

The set of resistance-gauges at the cross-section No. (i), were about twice the width of the specimen and those at the cross-section No. (ii), were 3.5 times the width of the specimen, from the end of the top jaws. The resistance-gauges at the cross-section No. (v), were 1.25 times the width of the specimen from the end of the bottom jaws, as shown in Fig.6.1.(a).

The strain readings obtained from the resistance-gauges at the cross-sections of the specimen are given in column 2(b) and 6(b) of table 6.1(a) for a load change, from 0.25 ton to 1.5 ton (viz 1.25 ton), i.e. a mean stress of 5 tons.

The strain values obtained from the resistance-gauges at the holes and also from resistance-gauges No. 4, 10, 21 and 25 gave 1-upt during loading and subsequent unloading. In these cases, strain

TENSION TEST ON SPECIMEN F6.3 WITHIN ELASTIC-RANGE

This specimen was tested in axial tension, using the special tensile testing machine. The resistance-gauges were connected to a 24 channel box and 6 channel box, which were connected to each other in one circuit, as explained in Appendix C so that only one galvanometer was used to measure the bridge out of balance, which measured the strain (the null point method of measuring the values of strain was not used).

The resistance-gauges were checked for "drift" at zero load, before starting the tests, but no drift was recorded, even after 3 hours.

Rubber pieces were placed in between the jaws to keep them apart at initial loads, which helped in centering the specimen, as explained in Chapter 3, paragraph 3.4.

The strain readings obtained from the resistance-gauges at the cross-sections of the specimen are given in columns 2(b) and 6(b) of table 6.1(a) for a load change, from 0.25 ton to 1.5 ton (viz 1.25 ton), i.e. a mean stress of 5 t.s.i.

The strain values obtained from the resistance-gauges at the holes and also from resistance-gauges No. 6, 10, 21 and 25 gave loops during loading and subsequent unloading. In these cases, strain

values tabulated, are taken from straight line portion of the curves shown in Fig. 6.2. The strain values obtained from the other resistance-gauges Nos. 7, 8, 9 and 22, 23, and 24 gave straight lines, when plotted against the applied stress and loading and subsequent unloading curves coincided as shown in Fig. 6.3. Similar results were obtained from resistance-gauge Nos. 1, 15, 16 and 30.

#### 6.4. RESULTS AT CROSS-SECTIONS NO. (ii) AND (iv), CONTAINING THE CIRCULAR HOLES

The strain-values obtained from the resistance-gauges at corresponding points in these cross-sections are sensibly the same. The results obtained from resistance-gauges No. 3 and 4, 19 and 18, at cross-section No. (ii), indicate the strain slope of 0.10 cm., in the direction indicated by the arrows, shown in Fig 6.4, while the results obtained from resistance-gauges No. 2 and 5, 20 and 17, indicate strain-slope in the reverse direction from that at the edge of the hole. These opposed strain-slopes cannot be due to an offset of the applied load. The results at cross-section No. (iv), are of the same nature.

The results obtained from the resistance-gauges Nos. 3, 4, 18, 19 and 12, 13, 27, 28, at the edges of the holes, differ by less than  $\pm 4\%$ . The results at the hole, were checked by Huggenberger extensometers, which gave as indicated in Fig. 6.5 results of the order expected from resistance-gauges.

### 6.5. RESULTS AT CROSS-SECTIONS NO. (i), (iii) and (v)

The results at cross-sections No. (i) and (v), indicate the presence of bending about the X-X axis. The bending strains at cross-section No. (i) and (v) are the equivalent of  $\pm 0.17$  cm. and  $\pm 0.12$  cm., respectively. But these results are the differences of quantities of about twenty five times their value, so that it seems allowable to take their average value of  $\pm 0.15$  cm., equivalent to a strain of  $0.17 \times 10^{-4}$ , to represent the bending strain at these cross-sections. The mean value of strain at both the cross-sections viz 3.48 cm. and 3.52 cm, is about the same and a bending strain of  $\pm 0.15$  cm. corresponds to an offset of  $0.00089''$ . The calculated mean strain from the average value of  $\frac{3.52+3.48}{2} = 3.5$  cm., with  $E = 13000$  t.s.i. is  $3.9 \times 10^{-4}$ .

The strain values obtained across cross-sections No. (iii) of the specimen, i.e. in between the holes, can be made more symmetrical across the cross-section, if appropriate corrections of the order of  $\pm 2\%$  are made in the observed values of strains, as shown in Fig. 6.4. These results indicate a small compression at the point  $K_1$ , shown in Fig. 6.6(a). The strain values obtained at the points  $K_2$  and  $K_4$  are both about equal, i.e. about 4.25 cm., while that obtained at  $K_3$  is 6.1 cm., i.e. 45% higher than that at the points  $K_2$  or  $K_4$ , as shown in Fig. 6.6 (b).

The values of the stress-concentration factors obtained at points K1, K2 and K3 are 0.042, 1.2 and 1.74, respectively.

The stress-concentration value at the holes has been calculated as follows :-

(i) The value of mean-stress applied, is taken as the mean-value of stress at cross-sections No. (i) and (v), which is about 3.5 cm.

(ii) The mean values of stresses at the holes at points K5 and K4 are taken as the average of the values at corresponding points, 3, 4, 18, 19, 12, 13, 27, 28 and 2, 5, 17, 20, 11, 14, 26 and 29 viz 7.23 cm. and 4.33 cm., respectively.

(iii) The stress-concentration at points K5 and K4 is  $7.23/3.5 = 2.06$  and  $4.33/3.5 = 1.23$  times the mean stress, respectively.

#### 6.6. COMPARISON WITH THEORETICAL RESULTS

No theoretical solution is available for a plate of finite width subjected to tension and containing two circular holes, but the stress-concentration in an infinite plate has been calculated by CHIH-BI LING (ref.26). For the hole pitch of 1.5" used in the present work, the stress-concentration factor at the edge of the holes, given by Chih-Bi Ling's theory is 2.7, as indicated in Fig.6.7 and by Howland's solution 2, seen in Fig. 6.8, for plates of infinite dimensions and containing circular holes. An outline of the solutions of Ling and Howland are given in Savin's book (ref.36



pages 134 to 139, and 145 - 149 respectively).

Neither writer mentions the presence of small compression in between the two holes, found here experimentally. The stress concentration at a point  $1/8"$  from the edge of the hole, which is approximately the centre of the resistance-gauges Nos. 3, 4, 12, 13, 19, 27, and 28, is found to be 1.25 from Howland's results, as shown in Fig. 6.8, while the result obtained from the present experiments is 2.14, i.e. about 71% higher. As shown in Fig. 6.8, the values of stress-concentration factors across cross-section No. (iii) have been calculated by interpolation from Howlands results and the value of stress-concentration factors, so obtained are 0.047 and 1.0 at points K1 and K2 respectively. The mean experimental values, obtained here are 0.042, and 1.2 respectively.

## 6.7 LOADED BEYOND THE ELASTIC INTO THE PLASTIC-RANGE

The specimen was now subjected to larger loads, which produced plastic-strains. Material at the edges of the holes, started yielding at a mean stress of 6.85 t.s.i.. The yielding was largest at points K5, i.e. at the edges of the holes. Although in the elastic-range, the strain at points K3 was higher than that at points K4, plastic strain became greater in this test at K4 than at K3, as it is evident from Fig. 6.9 and 6.2 for gauges Nos 2 and 6 respectively. This was, because the plastic-zones, which first form at the edges of the holes, i.e. at the points K5 have a considerable affect on the strains at the points K4. The plastic-zones at the edges of the holes, grow in the direction perpendicular to that of the tensile forces and simultaneously become narrower, i.e. does not extend so far along the length of the specimen, as Faerberg found (ref 13), for the case of a plate, containing a circular hole and subjected to loads which produced appreciable plastic-strains.

The specimen was now unloaded and on reloading again to the same mean stress of 6.85 t.s.i., the specimen was found to have regained elasticity. The loading and subsequent unloading curves, within this range, followed the unloading curve of the previous case, as shown in Fig. 6.2.

The strain-readings obtained from the resistance-gauges at the

various cross-sections of the specimen, for a load change, from 0.25 ton to 1.715 ton (viz 1.465 ton), are given in columns 2(b) and 6(b) of table 6.2(a).

6.8. RESULTS AT CROSS-SECTIONS NO. (ii) AND (iv), CONTAINING THE CIRCULAR HOLES.

The ratio of the mean strains, at 3, 4, 18 19 and 12, 13, 27, 28 at the edges of the holes is 1.01, as compared with 1.02 in the previous tests. This suggests that the strain-readings are consistent, but in this test also, the results obtained, for example from gauge-4 is greater than that at gauge-3, while that at gauge-2 is greater than that at gauge-5, i.e. indicate an inconsistent change of strain over the cross-section, a change which cannot be explained by an offset applied load. Similar results were obtained at cross-section No.(iv). The strain differences at corresponding points are not more than 10% of the observed values. A variation of strain-gauge sensitivity between gauges of this order would thus be necessary to explain the discrepancy in the results. The results at cross-section No. (ii) were also checked by Huggenberger extensometers readings, shown in Fig. 6.10(a). The readings were of the order expected from resistance-gauges results.

6.9. RESULTS AT CROSS-SECTIONS NO. (i), (iii) AND (v).

The results at cross-sections No. (i) and (v), indicate the presence of bending and the value of the bending strain at each cross-section. No. (i) and (v) is equivalent to  $\pm 0.075$  cm. The mean value of the strain at these cross-sections is 3.87 cm. and 4.08 cm., respectively (i.e. differing by about 5%).

The results at cross-section No. (iii) can be made to vary linearly over the cross-section, if corrections of the order of  $\pm 2\%$  are made in the observed values of strains, as shown in Fig. 6.10(b). The strain values at K2 and K4 are nearly equal, as in the previous case.

The average stress-concentration factors at the points K2, K3, K4 and K5 are 1.2, 1.74, 1.27, 2.07 respectively.

The plastic-residual strains obtained after each unloading, are given in table 6.3(a) and were calculated from the results, as those of Fig. 6.11(a) and (b), for gauges Nos. 4 and 18. The values at one of the holes was also obtained by Huggenberger-Extensometers. The results obtained from them, are shown in Fig. 6.12(a) and (b), but are not reliable because the pointer of the gauges had to be reset, when appreciable yielding occurred.

The plastic residual strains obtained from resistance-gauges Nos. 8 and 23 were compressive and also those obtained from resistance-gauge No. 7 and 9, 21 and 25, as shown in Fig. 6.13 after the specimen has been subjected only up to a mean stress of 6.85 t.s.i.. But, when it was subjected to a slightly higher mean stress of about 7.1 t.s.i., resistance-gauge Nos. 7 and 9, 21 and 25, now gave residual tensile strains,

but resistance-gauge No. 8 and 23 were still showing compressive residual strain.

The loading and subsequent unloading graphs in the plastic range, as shown in Fig. 6.11(a) and Fig. 6.2, indicate that the material was yielding, even at a mean stress of 1 t.s.i.. This was because enough time was not given, between loading and the previous unloading, and the material could not regain elasticity. These results are in agreement with those obtained by S. Timoshenko (ref. 37).

### 6.10 CONCLUSIONS

The strain measurements obtained at the edges of the holes, vary by about  $\pm 10\%$  from one another, in the elastic and also in the plastic range. The results also indicate a reduction of about 4% in the value of stress concentration at the edges of the holes, compared to that obtained in case of one hole.

The results at cross-section No. (iii), which is in between the holes, indicate the presence of compression at point K1, shown in Fig. 6.6(a). The strain readings, at the points K3 are higher than that at the points K4, by about 45%. The stress-concentration factors at points K4 and K5 at the holes, viz 2, 5, 17, 20 and 11, 14, 26, 29 and 3, 4, 18, 19 and 12, 13, 27, 28, obtained from the experimental results are 36.5% and 71% in excess of those obtained from the theoretical analysis of R.C.J. Howland (ref. 20) for plates of infinite dimensions and containing circular holes.

The plastic residual-strains built up, during loading in the plastic range and subsequent unloading rose to a larger value (viz  $36.8 \times 10^{-4}$ ) at the points K4 (i.e. at gauges No. 2, 5, 17, 20, 11, 14, 26 and 29).

than that obtained at the points K3 (viz  $11.2 \times 10^{-4}$ ) from gauges No. 6, 10, 21 and 25. The plastic-zones, which first develop at the points K5, i.e. at the edges of the holes, markedly affect the results at the points K4, while the results at the points K3 are not so affected and thus indicate lower residual plastic strains remaining at K3. These results agree qualitatively with the results of Faeborg (ref. 13) which he obtained for the case of a finite plate, containing a circular hole and subjected to loads into the plastic-range.

The strain-slope per ton, given in columns 2(a) and 6(a) of table 6.1(a), in the elastic-range, is nearly the same, as that obtained after the specimen has been subjected to loads, which took it into the plastic-range, given in columns 2(a) and 6(a) of table 6.2(a). This suggests that the behaviour of the material, in the elastic-range and also, after it has been subjected to loads beyond the elastic into the plastic-range remained nearly the same.

of the results obtained with the steel specimens tested, it was therefore, decided to check the stress-concentrations around a hole in an available specimen 2, of the same dimensions as that of the steel specimen 1. Tests were also made on other available specimens.

The photoelastic fringe pattern would correspond to the stress strain through the thickness. The resistance-gauges would give the surface values only which might or might not correlate with the mean. Any difference in the test results obtained with resistance-gauges on the steel and available specimens could be expected to be due to the change in

CHAPTER 7TESTS ON RECTANGULAR ARALDITE PLATES WITH AND WITHOUT CIRCULAR HOLES7.1 INTRODUCTION

The experimental results, obtained on mild steel specimen E, containing a circular hole and reported in Chapter 5, gave concentrations of stress which were different from one another and from the theoretical values. Also the strains as measured by the resistance-gauges were not linear at sections where they might have been expected to be so. The distribution over cross-sections was non-linear and not obviously related to the concentrations which might have been expected to occur at holes.

Observations around holes, in photoelastic specimens, reported by Frocht (ref. 14, p. 231) gave results in agreement with theory. In view of the results obtained with the steel specimens tested, it was therefore, decided to check the stress-concentration around a hole in an araldite specimen H, of the same dimensions as that of the steel specimen E. Tests were also made on other araldite specimens.

The photoelastic fringe pattern would correspond to the mean strain through the thickness. The resistance-gauges would give the surface values only which might or might not correlate with the mean. Any difference in the test results obtained with resistance-gauges on the steel and araldite specimens could be expected to be due to the change in

the material, one being more uniform than the other or the strain-gauges giving more consistent measurements of strains when attached to the one rather than the other.

## 7.2. TECHNIQUE OF CASTING THE ARALDITE SPECIMENS

The araldite used for making the specimens tested, was cast by the writer. The details of the casting technique are given in paragraphs 6 and 7 of Appendix B. The ingredients of araldite were mixed by a mixer, the speed of which could be so adjusted to avoid the formation of air voids in the mixture. The usual procedure of mixing the ingredients by hand with a stick was found to be most unsatisfactory, because it produced many air voids.

The moulds used for casting the specimens, were designed so that little machining and drilling had to be done to the cast specimens. The araldite mixture, the ingredients of which had already been thoroughly mixed, was introduced into the mould from the bottom, shown in Fig. 6, of Appendix B, in an endeavour to avoid air voids getting into the final cast. If air was trapped in the mixture, the air bubbles could go up with the mixture and out of the casting. This would not have been possible, had the mixture been poured from the top. The inside of the mould was lubricated by Silicone Releasal in order to avoid the mould sticking to the cast specimens. Araldite specimens with one or two holes were casted in this way without drilling and with no air voids.

patterns obtained at various loads. They were obtained in by hand.  
Fig. 15 shows a photograph of the fringe pattern obtained at a load of



### 7.3. TESTS ON ARALDITE SPECIMENS

Specimen G, the dimensions of which were based on B.S. 15 specifications for tensile tests and a specimen based on the dimensions recommended by Frocht (ref. 14, p. 160), for calculating the model and material fringe values, were cast and tested in axial tension, using the loading frame, shown in Fig. 8 of Appendix B. The load was applied to the specimen via pins, which passed through the specimens and the loading frame. The specimens were subjected to gradually increasing tensile loads. The load intervals between successive extinctions was constant and gave the principal stress difference between fringe orders. These were 277 p.s.i. for the Frocht specimen and 288 p.s.i. for the B.S. 15 specimen. The complete extinction across specimen G and Frocht control specimen indicate that the strain was uniform across the cross-sections, away from the ends of these specimens.

Specimen H, which had a circular hole, symmetrically placed at the centre, was tested in axial tension, using the loading frame, detailed in paragraph 9 of Appendix B. The formation of fringes was studied for equal increments of load on the specimen. The fringe-order values were calculated by counting the number of fringes passing through a particular point during the application of the load (The position of an isotropic point was not known). Fig. 15 of Appendix B shows the fringe patterns obtained at various loads. These were sketched in by hand. Fig. 16 shows a photograph of the fringe pattern obtained at a load of

260 lbs. on the specimen. Because it was not possible to get a clear picture of isoclinic lines from the araldite specimen,  $0^\circ$  isoclinic lines which are essential for evaluating the vertical and horizontal stresses  $p$  and  $q$  respectively, were obtained from tests on a perspex specimen having the dimensions of the araldite specimen H.

Specimens I and J, which had two circular holes (the diameters of which were 0.44 and 0.5 times the width of the specimens), were tested in a similar manner. The fringe patterns obtained at various loads are shown in Fig. 25 and 34 of Appendix B. Photographs of the fringe patterns obtained at a load of 260 lbs. on the specimens are shown in Fig. 26 and 35 of Appendix B. The  $0^\circ$  isoclinic lines were again obtained from tests on perspex specimens of the same dimensions.

#### 7.4. INTERPRETATION OF PHOTOELASTIC RESULTS

The procedure followed for calculating the values of the vertical and horizontal stresses  $p$  and  $q$  respectively, for specimens H, I and J, from the fringe patterns of Figs 16, 26 and 35 (Appendix B), is given in Appendix B. The stress distribution

- (a) across cross-sections (i) perpendicular and (ii) parallel to the applied tensile stress and passing through the centre of the holes,
- (b) around the edges of the circular holes,
- (c) along the edge of the specimens parallel to the applied tensile stress,

was calculated using the methods indicated by Frecht (Ref. 14, p.231).

The writer extended this procedure to find out the values of stresses  $p$  and  $q$ , in specimens I and J, across cross-section perpendicular to the applied tensile stress and passing between the holes.

Rectangular araldite specimen H, containing a circular hole and tested in axial tension, gave symmetrical strain variation across cross-sections. The stress-concentration value obtained at points K1 i.e. at the edges of hole, from the fringe-pattern is about 12.5% higher than that obtained from the theoretical analysis of Howland (ref. 18).

Rectangular araldite specimens I and J, containing two circular holes, also gave symmetrical strain pattern. The stress-concentration values, obtained from the fringe-patterns at the edges of holes, points K5, Figs. 24 and 35 of Appendix B, are not in agreement with those obtained from Howland's analysis for plates of infinite dimensions and containing circular holes (ref. 20). The results indicate a reduction of about 23% to 22% in the maximum stress-concentration, compared with that obtained from tests on specimen H, which contained a circular hole.

#### 7.5 STRAIN-GAUGE TESTS ON SPECIMEN H

Specimen H was tested in axial tension, in the special tensile testing machine and stress-concentrations at the hole were measured with resistance-gauges. The results obtained from tests, at points K1', i.e. the centre of the resistance-gauges from the edge of the hole, indicate a reduction of about 40% in the values of maximum stress-concentration, compared with that obtained from Howland's analysis. The

reason for this is not known.

## 7.6 CONCLUSIONS

The results obtained on rectangular araldite specimens indicate symmetrical strain pattern across cross-sections. This suggests that the araldite cast by the writer is more a homogeneous than the steel used by him. The non-linear strain variation across cross-sections, obtained from tests on mild steel specimens, seems more likely to be due to the non-homogeneity of the mild steel.

A circular hole, symmetrically placed in rectangular plates, which it was thought, was axially loaded through crossed-hair-edges, were also asymmetric and differed from the average by up to 15%.

## 8.2 CAUSE OF DIFFERENCES

That these differences were due to the presence of residual stresses in the bars can be discounted because the bars were annealed at 500°C for 6 to 8 hours and allowed to cool in the furnace for 2 to 3 days. The differences were thus unlikely to be due to initial strains in the test specimens. The discrepancies were also not due to the resistance gauges "drifting" i.e. changing their zero values which were fairly steady. The resistances of the gauges were also consistent and of the value expected. Their resistance to stretch was high. Strain measurements on loading in the elastic-range and subsequent unloading coincided.

## CHAPTER 8

### DISCUSSION OF THE EXPERIMENTAL RESULTS

#### 8.1. ANOMALOUS RESULTS

The results obtained from tension and bending tests on rectangular mild steel plates, using wire-resistance gauges, gave unexpected variations in the measured strains. Differences of some 10% to 15% were found between gauge readings which theoretically might have been expected to be equal. Measurements of stress-concentrations around a circular hole, symmetrically placed in a rectangular plate, which it was thought, was axially loaded through crossed-knife-edges, were also asymmetric and differed from the average by up to 45%.

#### 8.2. CAUSES OF ANOMALIES

That these differences were due to the presence of residual stresses in the bars can be discounted because the bars were annealed at 6000 for 6 to 8 hours and allowed to cool in the furnace for 2 to 3 days. The differences were thus unlikely to be due to initial strains in the test specimens. The discrepancies were also not due to the resistance gauges "drifting" i.e. changing their zero values which were fairly steady. The resistances of the gauges were also consistent and of the value expected. Their resistance to earth was high. Strain measurements on loading in the elastic-range and subsequent unloading coincided.

The tension testing machine used was of special design, in that loads were applied via crossed knife-edges. The objective of this design was to have control of the line of action of the load applied to the specimen and so avoid bending. The position of the knife-edges relative to the specimen could be moved so as to load the test specimen off centre. When this was done, a slight change in strain reading was observed in the sense expected, but the pattern of strain observations, remained very nearly the same (see Figs. 4.20 and 5.11, Chapter 4 & 5).

The position of the specimen with a circular hole was changed in the machine, the end which was in the top jaws being placed in the bottom jaws of the machine. This did not produce any appreciable change in the observed strain-pattern. This suggests that local effects from the machine are not responsible for the anomalous results. In this test demountable Huggenberger lever type extensometers were placed between the stick on resistance-gauges. These tended to confirm the general nature of the strain-pattern obtained from the resistance-gauges (see Fig 5.11).

As a further check, the effect of twisting the specimen grip relative to each other was investigated and is described in paragraph 8.3. Other gauged specimens were also tested in a 100 ton Buckton universal testing machine and 6.5 ton Denison machine. These strain gauge readings also showed unexpected variations, which were also confirmed by Huggenberger extensometer readings. All these machines have in common V-taper type of jaws and the effect of the jaw grip in the special machine, referred to is described in paragraph 8.4.

A uniform bending test on one gauged specimen indicated that the strain-sensitivity factor of the gauges on this particular specimen varied some 10% to 15%. But the range of observed values were so small, that possible errors can bring down the percentage variation to a negligible amount. Also the galvanometer which was used to measure the bridge out of balance was found to be some 4% to 6% non-linear over part of its range, as seen from table 8.7. Between  $\pm 6$  cms, it was however sensibly linear ( a null point method of measuring out of balance was not used in the tests, as galvanometer of the type used were known to be sensibly linear). These two latter effects may contribute to the anomalous readings of strain obtained.

### SPECIAL TESTS.

In order to shed some light on the causes of the anomalous behaviour noted and to check results obtained, tests already made on plate A were repeated and further special tests made.

#### 8.1. EFFECT OF TWIST ON SPECIMEN IN SPECIAL MACHINE.

In the testing machine which loads the specimen through the knife-edges, the possibility exists of a small amount of twist occurring of one end of the specimen relative to the other.

Twisting the end grips at no load and also under load, relative to each other indicated that a large twist was necessary to produce variations of the order of the anomalies. The anomalies are then unlikely to be due to twist in this machine. Nevertheless, the measure strain accompanying the twist were also asymmetrical across a transverse

sections and exhibited anomalies, as shown in table 8.1(a) and (b).

#### 8.1. EFFECT OF POSSIBLE BENDING OF SPECIMEN IN SPECIAL MACHINE

The presence of bending might have been the cause of the variations in the strain-gauge readings. The effect bending was checked by moving the bottom portion of the plate, block and jaw laterally. This was easy to do in this machine at zero load and produced considerable changes in the strain gauge readings, as shown in table 8.1(a). These readings were obtained by applying a known lateral displacement. The changes in the readings of resistance-gauges equi-distant from the neutral axis were not always of equal amounts, as might be expected from St. Venant's bending. A possibility of the departure from St. Venant's bending might be in the nature of the grip provided by the V-taper type of jaws used. It might have been possible that they grip the specimen unevenly as illustrated in Fig. 8.1. To accentuate such an effect rubber pieces were placed between the bottom jaws and the specimen, as shown in Fig. 8.2. These rubber pieces were eventually crushed, but no large differences were found on the strain pattern observed as seen from table 8.2(a) and (b). This could not be interpreted as corresponding solely with St. Venant's bending and torsion.

Next copper strips were placed on one side only of the bottom jaws, as shown in Fig. 8.3. These copper pieces did change the readings, in the sense expected but again the strain observations given in table 8.3, could not be explained by St. Venant bending and torsion only. In a



further test the jaws at the top and bottom of the specimen were held off the specimen by four pieces of mild steel which were thicker than the specimen. The entire load was now transferred through pins to the specimen, as shown in Fig. 8.4. In this test strain-gauge readings given in table 8.4 were not equal, even in the central portion of the specimen and their variation cannot be explained by St. Venant bending and torsions

These experiments indicate that either (i) Strain patterns which do not conform to St. Venant's theory are continually present, (ii) strain-gauge sensitivities and galvanometer non-linearity are giving misleading results, or (iii) the material used was not homogeneous enough for the St. Venant's theory to be strictly applicable.

#### 8.5. EFFECT OF GAUGE-FACTOR VARIATIONS AND NON-LINEARITY OF GALVANOMETER

Using roller supports, plate A was also tested in pure bending, as shown in Fig. 4.6, in order to avoid the axial tension. There was variations in the gauge readings, as seen from table 8.5, which could be interpreted as gauge-sensitivity-factor variations for these resistance-gauges of some 10% to 15%. Closer inspection of the results showed that possible corrections could reduce these variations to a negligible amount. The readings obtained were also found to be different from what they were when this specimen was first calibrated. The variations between these two set of readings, given in table 8.6, columns 1 and 2, was some 8% to 10%. Diverse and perhaps varying gauge

sensitivity to strain to which the gauges were subjected, may then be a cause of the deviations from St. Venant bending which were observed.

Galvanometer corrections, with appropriate strain-gauge sensitivity factors did in a number of cases produce corrected results which obeyed St. Venant's theory, but the procedure was not very satisfactory and suggests that it is not easy to be sure that cumulative errors due to galvanometer reading, strain-gauge sensitivity and galvanometer non-linearity have not been incurred.

If St. Venant's theory holds in the tests which have been made and the anomalous results have been obtained because gauge factors vary from gauge to gauge, then calculations can be made as to what the relative gauge factors were. First in the initial calibration test, when specimen A was bent by a four point loading system and secondly when this test was repeated some months later, the results obtained are as shown in columns 2 and 3 of table 8.8. In this table gauge No. 1 has arbitrarily been assigned a factor of 1. In the case of gauge 13 two identical tests between which the apparatus was not even fully unloaded, gave relative gauge-factors of 1.075 and 0.95. All other gauges gave sensibly consistent results in these tests. It can therefore be concluded that gauge No. 13 is not reliable. There is also little correlation between the relative gauge-factors obtained in both tests. Load was actually applied to the specimen by clamping plates on each side at the ends. Such loading might approximate to that given by V-grips and lack of correlation in the gauge factors in the two tests might well

be due to St. Venant's theory not applying.

Tests made with the load applied

- (i) through pins centrally located near the ends and
- (ii) through copper strips which were offset and so produced considerable bending,

give a possible means of calculating the relative gauge sensitivities, if it is assumed that the strain-distribution across the cross-sections is linear. Then for example, the mean of strain measurements taken at positions (1) and (10) Fig. 8.5, should be the same at equal loads in both test conditions (i) and (ii). If a and b are the gauge factors for these gauges, the results of table 8.3. and 8.4. Column 3, give

$$3.7a = + 3.4b = 4.9a + 2.8b$$

which gives,

$$a/b = 4.9-3.7/3.4-2.8 = 1.2/0.6 = 2/1$$

a ratio well outside the range of values previously obtained. Using this procedure, in some cases, even negative values of relative gauge factors were obtained. For example, considering the central section of the specimen, if c and d are the relative gauge factors of gauge 4 and 13, then from the results of table 8.3 and 8.4, Column 3,

$$3.85c + 3.4d = 4.35c + 3.55d$$

$$c/d = 3.55-3.4/ 3.85-4.35 = -0.15/0.5 = -3/10$$

These answers indicate that the approach is giving erroneous results. Possible reasons for this are

- (a) that the loads assumed the same in tests (i) and

at which the strain readings are the nominal values shown in Fig.

8.10 (ii) to be equal are not really so,

(b) That these are reading errors,

(c) that the strain distribution is not linear.

If the load applied in these tests was not of the same magnitude and the mean of the readings on each cross-section is taken as an indication of the applied load, then three different values of the relative applied loads are obtained viz 1.06 (for gauges 1 to 3, 10 to 12) 1.09 (for gauges 4 to 6, 13 to 15) and 1.13 (for gauges 7 to 9, 16 to 18). Arbitrarily making these respective corrections and following the procedure already obtained, the relative gauge-factors now obtained are as indicated in column 4, table 8.8. These values are more sensible than the figures of 2 and  $\frac{3}{10}$  previously obtained but there is little justification for making a load correction in this arbitrarily way. If a mean correction of 1.09 is made, i.e. if the load thought to be equal in both tests were 9% different, relative gauge-factors can be re-calculated. Some of the differences arising in the calculations are small and if observational corrections of the magnitude shown in column 5, table 8.8, are made in the necessary direction, all the relative gauge-factors now be made unity. Five of the eighteen corrections are larger than likely observational errors. A further attempt was now made to calculate the true value of the relative applied loads in the tests referred to. This was done by assuming that the loads applied were in the ratio of K:1. Then if a, c, d and f are the gauge factors of the resistance-gauges No. 1, 3 10 and 12 respectively,

at which the strain readings are the numerical values shown in Fig.

8.10(a).

Observed Strain value	3.7a		3.45c		4.9a		3.3c	
gauge No.	1	2	3		1	2	3	
gauge No.	12	11	10		12	11	10	
Observed value of strain	3.5d		3.45f		4.1d		2.8f	

(Fig. 8.10(a))

Then, if strains vary linearly, the mean values must be equal,

$$1. \text{ i.e. } K ( 3.7a + 3.45f ) = ( 4.9a + 2.8f ) \quad \dots \dots \dots (1)$$

$$= K ( 3.7a + 3.45f ) = K ( 3.5d + 3.45c ) \quad \dots \dots \dots (2)$$

$$= 4.9a + 2.8f \quad = 4.1d + 3.3c \quad \dots \dots \dots (3)$$

These constitute three distinct equations and contain five unknown quantities.

If  $a = c = d = f = 1$ , we have as mean strains, the average of

$$( 3.7 + 3.45 + 3.45 + 3.5 ) = 14.1$$

$$\text{and } ( 4.9 + 3.3 + 2.8 + 4.1 ) = 15.1$$

These give a mean value of  $K = 15.1/14.1 = 1.07$ . If we assume

that  $K = 1.07$ , then from (1),

$$1.07 ( 3.7a + 3.45f ) = 4.9a + 2.8f$$

$$\text{whence } a/f = 0.9/0.95$$

The possible error in observations is of the order of  $\pm 0.05$  so that this result can be modified to make  $a = f$ . Also from (2) and (3),

$$1.07(3.5d + 3.45c) = 4.1d + 3.3c$$

whence  $c/d = 0.35/0.4$  and this can again be modified to make

$c = d$ . We have thus demonstrated that  $a = f$  and  $c = d$  to the order of

the accuracy of experimental observations. These results give

$$\text{from (1) } K = 7.7/7.15 = 1.075$$

$$\text{from (2) } d = 7.15a/6.95 = 1.03a$$

$$\text{and from (3) } d = 7.7/7.4 Xa = 1.04a$$

We have thus arrived at a possible solution of (1), (2) and (3) with  $K = 1.07$ ,  $a = f$ ,  $c=d=1.035a$ .

If we had made lesser corrections to our estimated ratios of  $a/f$  and  $c/d$  we might have had

$$f = 1.02a \text{ and } d = 1.02c$$

Now (1), (2) and (3) give

$$K = 7.76/7.22 = 1.073$$

$$d = 7.22/7.02 = 1.03a$$

$$\text{and } d = 7.76/7.47 = 1.04a$$

i.e.  $K$  and  $d/a$  have not altered appreciably and are thus not very sensitive to the corrections made in arriving at the ratios of  $a/f$  and  $c/d$ .

If we take the values,  $K=1.07$ ,  $a=f=1$ ,  $c=d=1.035a$ , the observed relative strain values correct to the values shown in Fig. 8.10(b) which vary linearly, are indicated by the numbers in the diagram.

Observed strain (relative)	3.7a ← 0.13 → 3.57a	4.9a ← 0.48 → 3.42a
gauge No.	1 ← 2 <sup>7.15</sup> → 3	1 ← 2 <sup>7.7</sup> → 3
gauge No.	12 ← 11 <sup>7.15</sup> → 10	12 ← 11 <sup>7.7</sup> → 10
Relative-strain	3.62a	3.45a
Corrected Strain	<u>3.58</u> ← 0.13 → <u>3.45a</u>	<u>4.28</u> ← 0.48 → <u>2.8a</u>
Diff. in Strain reading (top to bottom)	0.12	0.12
		0.62
		0.62

(Fig. 8.10(b) )

In a like manner the observed strains can be corrected to give a linear variation over other cross-sections. The relative gauge-factors now obtained are those shown in column (6) of table 8.8 and the observational corrections which have to be made to obtain these results are those shown in column (7). The load ratio values are those in column (8) and these cannot vary. They give a mean  $K = 1.1$  and with this value and the corrections shown in column (10), the relative gauge-factors now obtained are those shown in column (9).

It is difficult to see why observational errors should be conveniently positive or negative or larger than  $\pm 0.05$  cm., or indeed that the load was 10% different from the value it was intended to apply. If these corrections are not acceptable, then departures from a linear strain distribution must have been present. If the corrections are acceptable, they imply that the mean sensitivity factor for these gauges was  $0.99 \times 1$  and the standard deviation 0.0336. The variance of these results is 3.4%, i.e. 1 in 3 strain gauges can be expected to have a sensitivity-factor differing by 3.4% from the mean of 0.99.

The non-linear strain readings obtained from the tests, across cross-sections given in tables 8.2(a) and (b), 8.3 and 8.4 can be made to vary linearly, if arbitrarily  $\pm 2\%$  corrections are made in the observed values of strains. The values of the offset of load obtained, given in Figs. 8.6, 8.7, 8.8 and 8.9 are not consistent at cross-sections. This suggests that errors of  $\pm 2\%$  are not the cause of non-linear observations.

In a further attempt to discover the cause of the non-linear strain variation across cross-sections, specimen A was again tested in the special tensile testing machine, and the strain readings were recorded using a galvanometer which was four times as sensitive as the one used in previous tests. But the strain readings obtained, given in table 8.9 again gave a non-linear strain variation across cross-sections. The results can be made to vary linearly, as shown in Fig. 8.10(e), if arbitrarily  $\pm 2\%$  corrections are made in the observed values of strains. But, the values of the offset of the load obtained, given in Fig. 8.10(c) are not consistent at cross-sections. This suggests that errors of  $\pm 2\%$  are not the cause of non-linear observations.

In order to re-check, whether the sticking of the Tinsley resistance-gauges, was the cause of non-linear strain variation, resistance-gauges at the central section of the specimen, i.e. cross-section No. (11), which gave a large degree of non-linearity, were now replaced by British Thermostat gauges. The specimen was kept in the machine, while the resistance-gauges were replaced, and this time the resistance-gauges were not attached to the specimen by the writer. The position of Tinsley gauges where British Thermostat gauges had to be attached, were marked on the specimen and an attempt was made to attach the new gauges, exactly at the same positions, so that the results can be compared.

The strain readings obtained from British Thermostat gauges, were of the same nature as that obtained previously from tests with Tinsley gauges.



Also the results now obtained from the other Tinsley gauges, given in table 8.10, are the same as those obtained previously. The individual results obtained from the British Thermostat gauges are not exactly the same as those obtained from the Tinsley gauges, as seen from table 8.11, which were previously attached in these positions. This might have been due to effective elements of the British Thermostat gauges not being attached exactly in the same positions as the Tinsley gauges were previously attached. This again confirms the fact that mild steel used, was not homogeneous across cross-sections, as it is normally assumed to be, because a little change in the position of the gauges seems to affect the readings.

Finally, all eighteen active resistance-gauges on the specimen were connected in series and the voltage supply to the circuit was slowly raised, in order to see which of any gauges, burnt out before the other and so indicate a difference in the adhesion to the specimen. The voltage supply to the circuit was slowly raised to 260 volts, the current through the gauges is then about 125 mA, but no gauge burnt out. All remained electrically continuous. This high current carried suggests that the gauges were properly stuck to the specimen and the non-linearity was not due to improper sticking.

The non-linear strain variations across transverse cross-section, of a similar nature were found by Batho and Samawai (Ref.1) in their joint test, but not investigated. They found that the strain was a maximum at the

centre and a minimum near the edges of each side, the difference between the maximum and the minimum being 12%. The ratio of the strains on the two faces of the plate was 1.22 (i.e. about 22% variation).

The experimental results obtained on plain rectangular mild steel specimens, gave non-linear strain variations across the cross-sections. This was not associated with the presence of loadings, because considerable movement of the crossed knife-edges, through which specimens were loaded in the special tensile testing machine, produced changes which were not of the order of the stresses, although the stresses were in the sense expected. Also the specimens were possibly straight.

The gauges gave consistent readings during loading and subsequent unloading and Suggsberger extensometers, which were used to check strain readings, gave results of the order expected from resistance-gauges. The non-linear strain variations were then probably not due to defective resistance-gauges being used in the tests, or due to variation in the strain sensitivity factors of the gauges.

The non-linear strain variation was also not due to the shape and size of the specimens tested or the type of machine used to load the specimens. All tests showed this feature.

Most of the specimens tested were annealed before tests were made on them, and therefore, initial local stresses in the material were

CHAPTER 9CONCLUSIONS9.1 NON-LINEARITY OF RESULTS

The experimental results obtained on plain rectangular mild steel specimens, gave non-linear strain variations across transverse cross-sections. This was not associated with the presence of bending, because considerable movement of the crossed knife-edges, through which specimens were loaded in the special tensile testing machine, produced changes which were not of the order of the anomalies, although the changes were in the sense expected. Also the specimens were sensibly straight.

The gauges gave consistent readings during loading and subsequent unloading and Huggenberger extensometers, which were used to check strain readings, gave results of the order expected from resistance-gauges. The non-linear strain variations were then probably not due to defective resistance-gauges being used in the tests, or due to variation in the strain sensitivity factors of the gauges.

The non-linear strain variation was also not due to the shape and size of the specimens tested or the type of machine used to load the specimens. All tests showed this feature.

Most of the specimens tested were annealed before tests were made on them, and therefore, initial local strains in the material seem

unlikely to be the cause of the non-linear strain variation. Some of the specimens were stretched by about 5% of their length, in an attempt to get Luder bands spread over the whole length of the specimens, but still, the strain readings obtained, were non-linear across the cross-sections.

Rectangular mild steel specimen E, with a circular hole, symmetrically placed at the centre, also gave asymmetrical strain variation across cross-sections. These were quite marked at the hole.

Rectangular mild steel specimen F, containing two circular holes, also gave asymmetrical strain variations, but asymmetry was not as marked as in specimen E.

Photoelastic, resistance-gauge and Huggenberger extensometer observations on rectangular araldite specimens with one and two circular holes, symmetrically placed at the centre, tested in tension, were sensibly symmetrical across transverse cross-section.

The linear variation of strain over the transverse sections of the araldite specimens and the non-linear strain variation for the mild steel specimens, seems from the preceding observations most likely to be due to even normalised mild steel being a more locally heterogeneous material than it is usually considered to be. This should be borne in mind in any detail investigation of strains on mild steel specimens.

## 9.2. OTHER RESULTS OF TESTS

The average stress concentration obtained with mild steel specimen E, which had a circular hole, symmetrically placed at the centre, agrees

approximately with the theoretical analysis of Howland. But the individual results differed from the average by up to about 45% and indicates that the non-linearity of about 15% observed in the tests on plain specimens is here accentuated some three or four times. The stress-concentration values obtained by photoelastic technique, on an araldite specimen with a circular hole, on the other hand agreed with Howland's theory.

The stress-concentration obtained in mild steel specimen F, with two circular holes, did not agree with the corresponding theoretical analysis of Howland for a plate of infinite dimensions, and may have a bearing on the scatter often found in fatigue and impact tests in which failures start from local areas of relatively high concentration of strain. But the results obtained indicate a reduction in the maximum stress-concentration of about 6%, compared with that obtained on mild steel specimen with a circular hole.

The maximum stress-concentration values, obtained near the edges of the holes, in araldite specimens with two circular holes, were some 20% less than those obtained with araldite specimen having only one hole. The results suggests that in the design of tension joints it may be possible to so space the holes as to reduce stress-concentrations.

The results obtained on mild steel specimen F with two circular holes, tested in axial tension, indicate that the strain readings in the elastic-range were higher at points near the edge of the specimen, at cross-sections perpendicular to the applied tensile stress and passing

between the holes, than those at points near the edge of the specimen at cross-section perpendicular to the applied tensile stress and passing through the centre of the holes. But the plastic residual strain, built up during repeated loading into the plastic-range on the contrary rose to larger values at points near the edge of the specimen, at cross-sections passing through the centre of the holes, than that at points near the edge of the specimen at the cross-section passing between the holes. This was because the plastic zones, which first develop at points near the edge of the holes, markedly affect the results at points at the edge of the specimen at cross-sections passing through the centre of the hole, while the results at points near the edge of the specimen, at the cross-section between the holes are not so affected and thus result in lower residual plastic strains remaining at these points.

(b) The contact resistance was nearly constant and had an average value of one milliohm.

(c) The maximum thermal e.m.f. generated when the switch has been operated several times is less than 1  $\mu$  v.

(d) The installation resistance between any two adjacent studs and between any stud and the collection ring is not less than 5,000 megohms.

The fixed arm resistances were supplied by electrical resistances strain-gauges which are attached to metallic pieces. The selector switch and fixed arm resistances were put in a box in order to reduce the effect

APPENDICES(A) INSTRUMENTATION1. 24 CHANNEL STRAIN-GAUGE SET

A schematic circuit diagram of the wheatstone bridge when used with a galvanometer and a multiway apex unit is shown in Fig. 1(a). The apex unit available had 24 channels which can be brought into service separately by using a 24 point selector switch. Helical potentiometers having 10 turns and 2 ohms resistance were used in this unit. With these potentiometers it was possible to balance the bridge very accurately and without difficulty.

The selector switch used had the following characteristics:-

- (a) Snap action for changing from one position to the other.
- (b) The contact resistance was nearly constant and had an average value of one milli ohm.
- (c) The maximum thermal e.m.f. generated when the switch has been operated several times is less than  $1 \mu v$ .
- (d) The installation resistance between any two adjacent studs and between any studs and the collection ring is not less than 5,000 meg. ohms.

The fixed arms resistances were supplied by electrical resistance strain-gauges which are attached to metallic pieces. The selector switch and fixed arm resistances were put in a box, in order to reduce the effect

of any temperature changes in the surroundings.

In the case when 32 active resistance-gauges were used, another box having 6 channels and 2 separate boxes, each having one channel, were connected to 24-channel box, by using a switch in the circuit, so that only one galvanometer was necessary. The circuit details are given in Fig. 1(a) and the apparatus is shown in Fig 1(b).

The sensitivity of each gauge depends on the magnification of the length of lever. The gauges used had a gauge length of 2" and magnification of the order of 1000. The scale could be read to about tenth of a scale division. This corresponds to a strain of  $10^{-4}$  on a strain of about 100 micro-strain.

Only a small strain is necessary to stretch the gauge, so that the method of attachment made only sufficient pressure to hold it in position. A number of attempts to methods to mount the gauge are generally a simple straight arrangement, as shown in Fig. 2(a) and (b) that is needed.

The main advantage of this design was that it is light and can easily be attached to a test stand. In using the tests, it was found that there was a little "backlash" during loading and subsequent unloading which can lead to quite misleading results. This occurred because unloading the "backlash" is negligible, and pressure and supports that are due to the differences loading with the gauges.



2. HUGGENBERGER EXTENSOMETER

A gauge, employing lever magnification, has been developed by Huggenberger. The principle of the operation is shown diagrammatically in Fig. 2.

The sensitivity of each gauge depends on the magnification due to the length of levers. The gauges used had a gauge length of  $\frac{1}{2}$ " and a magnification of the order of 1900. The scale could be read to about a tenth of a scale division, which corresponds to a strain of  $10 \times 10^{-6}$  or a stress of about 300 p.s.i. in steel.

Only a small force is required to actuate the gauge, so that the method of attachment needs only sufficient pressure to hold it in position. A variety of equipment is available to mount the gauge, but generally a simple clamping arrangement, as shown in Fig. 3(a) and (b) is all that is needed.

The main advantages of this gauge are that it is light and can easily be attached to a test piece. But during the tests, it was found that there was a little 'backlash' during loading and subsequent unloading, which can lead to quite misleading results. But repeated loading reduced the 'backlash' to negligible proportions and suggests that it is due to the knife-edges bedding into the specimen.

### 3. BRIEF DESCRIPTION OF THE MAIHAK-GAUGES AND ITS PRINCIPLE OF WORKING

This is in principle a vibrating wire strain-gauge and is useful for long term tests on structures, because of its inherent stability. The gauge consists of a stretched steel wire, held at its two ends to the member under test and made to vibrate at its natural frequency. If the member suffers a change in strain, the wire undergoes a change in its tension and consequently its frequency changes.

The law connecting frequency and strain is of the form

$$n^2 = \text{constant} \times e/L^3$$

where

$n$  = frequency

$e$  = strain

and  $L$  = the free length of the wire.

The 'Maihak' gauge is available with gauge lengths varying from 20 mm. to 100 mm. and those used in the tests mentioned in Chapter 4, were of type MDS 15, having a gauge length of 20 mm.

With the measuring equipment available, these gauges are theoretically capable of detecting a strain of  $1 \times 10^{-6}$  (30 p.s.i. in steel). But it was found rather difficult to record such a small amount of strain, because even the temperature variation and the error that can easily be made in recording the frequency change, can give readings which correspond to a strain of about  $5 \times 10^{-6}$ .

The frequency change that can be measured with these gauges is about

900 cycles/sec (i.e. from  $f = 900$  to 1800 cycles/sec) which corresponds to a strain of about  $6000 \times 10^{-6}$ . The range of strain that can be measured varies from about  $100 \times 10^{-6}$  to  $6000 \times 10^{-6}$ , with an accuracy of  $\pm 5\%$ .

In order to eliminate errors resulting from an operator synchronising the frequencies inaccurately by audible means, an electrical means of comparison has been used similar to that developed by the staff of the Building Research Station (ref. 43). The vibrations of the gauge and a signal from a built-in oscillator of variable frequency are transmitted electrically to a cathode tube, each vibration producing a linear trace. The individual traces are perpendicular to each other, and consequently when the two vibrations have the same frequency, the figures produced by combining the signals is a circle, or an ellipse if the two amplitudes are different. Initially the two vibrations are synchronised, and when strain is applied, a calibrated dial is turned until the vibrations are again synchronised. The dial reading is converted into a change of frequency, and the value of strain obtained. This system suffers from the disadvantage that only static strains can be measured.

The advantages of this gauge are that it is robust and highly sensitive, with a useful working range both in tension and compression. It is stable and can be used for long term measurements.

The disadvantage of this gauge is that auxiliary apparatus of special design is required.

The calibration of the 'Maihak' gauges and the oscillator set

used in the tests, was already available in the department.

Wahlberg-gauges were attached to the specimen under tests as indicated in Fig. 4. Two rectangular mild steel plates A were connected to each other by means of  $1/4"$  diameter threaded bars B. The Wahlberg gauges were tightened against the specimen by adjusting one of the screw rods C. The other was just kept against the other Wahlberg gauge.

In this way, it is possible to apply a suitable pressure to both Wahlberg-gauges, until satisfactory response was achieved as oscillator set and frequency at constant strain was steady. This arrangement was found to be easy to use in pure bending tests, but it can also be used in tension and compression tests.

## (a) PHOTOELASTIC INVESTIGATION

### 1- PHOTOELASTIC MATERIAL

A proper choice of the material, to be used in the photoelastic investigations, is very essential, because the results depend on the suitability of the material. Some of the essential points to be borne in mind while selecting the material, are as follows:-

- (a) PHOTOELASTICITY - The material should be such that it can be machined by means of ordinary workshop tools, without producing a lot of machining stress.

#### 4. DESIGN USED FOR ATTACHING THE MAIHAK-GAUGES TO THE SPECIMEN

Maihak-gauges were attached to the specimen under test, as indicated in Fig. 4. Two rectangular mild steel plates A were connected to each other by means of  $1/4$ " diameter threaded bars B. The Maihak gauges were tightened against the specimen by adjusting one of the screwed rods C. The other was just kept against the other Maihak gauge.

In this way, it is possible to apply a suitable pressure to both Maihak-gauges, until satisfactory response was achieved in oscillator set and frequency at constant strain was steady. This arrangement was found to be easy to use in pure bending test, but it can also be used in tension and compression tests.

#### (B) PHOTOELASTIC INVESTIGATIONS

##### 5. PHOTOELASTIC MATERIAL

A proper choice of the material, to be used in the photoelastic investigations, is very essential, because the results depend on the workability of the material. Some of the essential points to be borne in mind while selecting the material, are as follows :-

- (a) MACHINABILITY - The material should be such that it can be machined by means of ordinary workshop tools, without producing a lot of machining stress.

- (b) HIGH OPTICAL SENSITIVITY - The materials should have high fringe orders for stresses well within the elastic limit and in fairly thin models, so that  $(p-q)$  can be determined at each point with satisfactory accuracy by the mere counting process without resort to special instruments such as the Babinet-Soleil Compensator.
- (c) ABSENCE OF UNDUE OPTICAL OR MECHANICAL CREEP - Fringe-order should not depend on small change in time, but on the load and the model. Under constant loads there exists an optical creep which manifests itself in small, continuous increases in the fringe order as time goes on even though the loads are constant.
- (d) FREEDOM FROM INITIAL STRESSES
- (e) ISOTROPY
- (f) LINEAR STRESS STRAIN AND LINEAR STRESS FRINGE RELATIONS
- (g) PROPER HARDNESS

Some of the materials used for the photoelastic investigation by various investigators are (i) Celluloid (ii) Bakelite (iii) Marblette (iv) Xyloite (v) Trolon (vi) Araldite, as well as glass and gelatin.

Araldite D, is very suitable for photoelastic investigations, as it provides, very nearly, all the requisites of a good photoelastic material, and it was therefore, selected for the present tests. The

material used for casting the specimens was "Araldite MY753 and hardener HY951 (manufacturer's references Nos). Araldite MY753 is in liquid form of medium viscosity.

In the case under review, paragon was selected for making the mould, because it is easy to put to a required shape. While making the mould, the main consideration was to arrive at a design, so that the making of the cast specimens could be avoided.

The main rectangular plates  $4" \times 24" \times 1/8"$ , the rectangular strips  $1" \times 24" \times 1/8"$ , and a small rectangular piece  $1" \times 2" \times 1/8"$  were made of paragon. The circular discs,  $1/8"$  in diameter and  $1/4"$  thick were also made of paragon to be used for making specimens with circular holes.

The two main rectangular plates were put on top of each other and in between them, were placed the two rectangular strips, small rectangular piece and the circular discs (one or two depending on the number of holes to be provided in the cast specimens), as shown in Fig. 4, leaving a space of  $2"$  in between the rectangular strips. They all were fastened together by means of clamps, as shown in Fig. 4, which were attached to the stand. The mould was held in position, such that the end with rectangular piece  $1" \times 2" \times 1/8"$  was at the bottom and that, the open end was at the top as shown in Fig. 4.

A hole just under  $1/4"$  in diameter was drilled in the rectangular piece  $1" \times 2" \times 1/8"$  and through which passed a metal tube. The rubber tube which was connected to the funnel, was in turn connected to the metal

PROCEDURE FOR CASTING THE SPECIMENS6 DESCRIPTION OF THE MOULD

Moulds, for casting the araldite specimens, can be made, either of metal or perspex. In the case under review, perspex was selected for making the moulds, because it is easy to cut to a required shape. While making the moulds, the main consideration was to arrive at a design, so that the machining of the cast specimens could be avoided.

Two main rectangular plates 4" x 24" x 3/8", two rectangular strips 1" x 24" x 1/4", and a small rectangular piece 1" x 2" x 1/4" were made of perspex. Two circular discs, 7/8" in diameter and 1/4" thick were also made of perspex to be used for casting specimens with circular holes.

The two main rectangular plates were put on top of each other and in between them, were placed the two rectangular strips, small rectangular piece and the circular discs (one or two depending on the number of holes to be provided in the cast specimens), as shown in Fig 6, leaving a space of 2" in between the rectangular strips. They all were fastened together by means of clamps, as shown in Fig. 6, which were attached to the stands. The mould was held in position, such that the end with rectangular pieces 1" x 2" x 1/4" was at the bottom and that, the open end was at the top, as shown in Fig. 6.

A hole just under 1/4" in diameter was drilled in the rectangular piece 1" x 2" x 1/4" and through which passed a metal tube. The rubber tube which was connected to the funnel, was in turn connected to the metal



tube, as shown in Fig. 6.

Plasticine was used on the outer surface of the mould, as shown in Fig. 6 to stop the leakage of the araldite mixture, through the spertures in between the main plates and the strips. It was found to be most suitable, because Bostik sealing Strip dissolved in the araldite mixture and turned it to white colour, which was then of no use as a photoelastic material.

The inside of the mould was lubricated with Silicone Releasel, in order to avoid the mould sticking to the cast specimens.

#### 7 MIXING AND POURING OF ARALDITE

Araldite MY 753 and hardner HY 951, were mixed together at room temperature, in the ratio of 100 parts by weight of araldite and 8 to 10 parts by weight of hardner. The ingredients were mixed together, by means of a variable speed mixer, the speed of which can be so adjusted, as to avoid the formation of the air voids in the mixture. It usually takes 30 to 45 minutes for proper mixing of the two ingredients. If they are thoroughly mixed, the final mixture would be of shining colour. It is very essential that the two ingredients should be properly mixed, otherwise the final cast will not set properly.

The final mixture was poured in the funnel, which was connected to a rubber tube, as shown in Fig. 6 and which in turn was connected to a metal tube, at the bottom of the mould. The araldite mixture was poured into the mould from the bottom, in an endeavour to avoid the air-voids, getting into the final cast. If air was trapped in the mixture, the air

bubbles could go up with the mixture, and out of the casting which would not have been possible, had the mixture been poured from the top.

When the araldite mixture reached the required height in the mould, rubber tube was cut at the bottom and tightened by means of a small clamp, which was left in that position, until the final cast was properly set. This usually takes about 24 hours.

Araldite specimens with one or two holes were casted in this way without drilling them and with no air voids present. The specimen with one hole, had to be made of the shape, shown in Fig. 14, at the ends, so that it could be tested in the special tensile testing machine. Also specimens, such as that based on B.S. 15 specifications and one used for calculating the material and model fringe values, were machined to make them of the required shape. But, special care was taken to avoid machining stresses being developed in the specimens. The specimens were machined slowly and whenever they became too warm, machining was stopped until the specimen again became cold. In this way the development of machining stresses in the specimens could be avoided. Cutting tools should be correctly ground and feather edges removed, otherwise the workpiece will tend to chatter or chip. The cutting speed was maintained medium to high and the feed was low.

## DESCRIPTION OF THE APPARATUS

### (8) PHOTOELASTIC BENCH

The set up of the bench used in the present photoelastic investigation, was that shown in Fig. 7. This was available. It was made of light angle sections. At the centre of the frame, run two rods, as shown in Fig. 7, to which the various optical components are attached and can slide on the rods. The bench is made of two parts and in between them is the loading frame.

### (9) LOADING-FRAME :-

The frame used for loading the specimens is that shown in Fig. 8. The load was applied via pins which passed through the specimen and the loading-frame, as shown in Fig. 8. In this way, it was possible to load the specimen, sensibly in axial tension. The position of the specimen can be adjusted by means of handles A and B, so that it can be brought on the axis of the light and a clear picture of the fringes can be obtained.

The specimens were placed at the positions, shown in Fig. 8 in the tests reported here later, and were attached at points Q and R by means of pins, which were 13" from the fixed end P. The bar carrying the load pan P, was fixed at point P, which was 45" from the centre line of load pan P.

A load of 1-lb in the pan would then produce a load of  $45/13 = 3.46$ -lb. on the specimen.

## 10. CALCULATION OF THE MODEL AND MATERIAL FRINGE VALUES

The dimensions of the specimen, used for calculating the model and material fringe values of the araldite used in the present photoelastic investigations, are those recommended by Frocht (reference 14, page 160), and shown in Fig. 9(c). The specimen was  $1/4$ " thick.

The specimen was subjected to gradually increasing loads in tension using the loading frame, which was placed in the polariscope at the positions, shown in Fig. 7. The formation of the fringes were studied for each increment of one pound load in the load pan P, (3.46-lbs on the specimen). The fringe-patterns obtained for each increment of 3.46-lb. load on the specimen, were drawn by hand. Those for fringe orders up to three are shown in Fig. 9.

At a load of 17.5-lb on the specimen, the entire length and width of the specimen was covered with a black fringe and that was the 1st order fringe (reference 14, page 137). Then, at a load of about 24.2-lbs, the entire length and width of the specimen, on the contrary, became white. This was 1.5 order fringe. On further subjecting the specimen to a load of 34.6-lbs., the entire length and width of the specimen was again covered with a black-fringe and that was a 2nd. order fringe. In this way, the fringe order values were obtained.

A change of fringe order of unity was thus seen to be caused by a load of 17.3-lb on the specimen i.e. a principal stress difference of  $17.3 \times 16 = 277$  p.s.i.

### 11. DETERMINATION OF THE VALUES OF THE MODULUS OF ELASTICITY

The model and material fringe values  $F$  and  $f$ , are therefore equivalent to  $1/2 \times 277 = 138.5$  p.s.i. and  $1/4 (138.5) = 34.6$  p.s.i. respectively.

The specimen, shown in Fig. 10 of the dimensions specified in 2.5.13 for tensile tests was tested in tension, to calculate the values of modulus of elasticity and Poisson's ratio. It was noted from the same test of availability, that was used in the photoelastic investigations reported here later.

Two resisto-strain-gauges, one at right angles to the other, as shown in Fig. 10 were attached on both sides of the specimen, such that resisto-strain-gauge No. 1 and 3; 2 and 4 were directly on opposite sides of each other. Resisto-strain-gauge No. 1 and 3 measured the longitudinal strain and those No. 2 and 4, the lateral strain.

The procedure adopted for fixing and protecting the resisto-strain-gauges from moisture, was that outlined in paragraph 4.4, except that, care was taken not to keep the specimen in the oven of hot air for a long time.

The resisto-strain-gauges were checked for drift, i.e., changing zero position at no load, but no appreciable drift was recorded even after 3 hours.

The results obtained from the resisto-strain-gauges, when the specimen was tested in a 100 ton Buxton universal testing machine (used in its 50 ton range) and a 6.5 ton Instron, are given in Table 7, columns 2 and 3, respectively. The results obtained from gauges in both machines, gave loops during loading and subsequent unloading, but the straight line portion

11. CALCULATION OF THE VALUE OF THE MODULUS OF ELASTICITY  
AND POISSON'S RATIO

An araldite specimen, shown in Fig. 10 of the dimensions specified in B.S.15 for tensile tests was tested in tension, to calculate the values of modulus of elasticity and Poisson's ratio. It was made from the same batch of araldite, that was used in the photoelastic investigations, reported here later.

Two resistance-gauges, one at right angles to the other, as shown in Fig. 10 were attached on both sides of the specimen, such that resistance-gauge No. 1 and 3, 2 and 4 were directly on opposite sides of each other. Resistance-gauge No. 1 and 3 measured the longitudinal strain and those No. 2 and 4, the lateral strain.

The procedure adopted for fixing and protecting the resistance-gauges from moisture, was that detailed in paragraphs 4.4, except that, care was taken not to keep the specimen in the stream of hot air for a long time.

The resistance-gauges were checked for drift, i.e. changing zero position at no load, but no appreciable drift was recorded even after 3 hours.

The results obtained from the resistance-gauges, when the specimen was tested in a 100 ton Buckton universal testing machine (used in its 20 ton range) and a 6.5 ton Denison, are given in Table 1, columns 2 and 3, respectively. The results obtained from tests in both machines, gave loops during loading and subsequent unloading, but the straight line portion

of the curves, as shown in Fig. 11 (a) and (b) respectively for gauges No. 1 and 2, were taken to give the strain values tabulated.

### CALCULATIONS

#### TEST IN 100 TON BUCKTON UNIVERSAL TESTING MACHINE

#### AND A 6.5 TON DENLSON MACHINE

Load applied to the specimen = 0.07 ton

Area of the specimen =  $1/2 \times 1/4$  square inch

Stress is therefore, equivalent to  $0.07 \times 2240 \times 1/8$

= 1255 p.s.i.

A resistance of 135.5K ohm across a gauge (resistance 100 ohm) gave a galvanometer deflection of 1.4 cm. There was a 2 volt battery supply to the bridge. The firm's sensitivity factor for these gauges attached to B.S. 15 specimen, was 2.25. It follows that the strain equivalent of a resistance of 135.5K ohm across 100 ohm gave a galvanometer deflection of 1.4 cm.

1 cm., deflection on the galvanometer is therefore, equivalent to

$$100/135500 \times 2.25 \times 1.4$$

$$= 2.35 \times 10^{-4}$$

The values of the longitudinal and lateral strains obtained from the average values of resistance-gauges Nos 2 and 3, 2 and 4 i.e. 15 cm. and 5.68 cm., respectively, from tests in a 100 ton Buckton Universal testing machine, are equivalent to  $35.2 \times 10^{-4}$  and  $13.4 \times 10^{-4}$ , respectively. The values of the modulus of elasticity and Poisson's ratio are therefore, equivalent to  $1255/35.2 \times 10^{-4} = 0.354 \times 10^6$  and

$$\frac{13.4 \times 10^{-4}}{35.2 \times 10^{-4}} = 0.38, \text{ respectively.}$$

Similarly, the values of the modulus of elasticity and Poisson's ratio, obtained from tests in a 6.5 ton Denison machine are equivalent to  $0.35 \times 10^6$  and 0.40 respectively.

The average values of the modulus of elasticity and Poisson's ratio (from tests in Buckton and Denison Machines), are therefore equivalent to  $0.35 \times 10^6$  and 0.39 respectively.

## 12. TEST ON SPECIMEN G

The dimensions of the specimen G, given in Fig. 12(c), were based on B.S. 15 specifications. It was made from the same batch of araldite, that was used in the previous tests, for calculating the model and material fringe values and modulus of elasticity.

The specimen was subjected to gradually increasing loads in tension using the loading frame, placed in the polariscope, shown in Fig. 7. The formation of the fringes were studied for each increment of one pound of load in the load pan P, shown in Fig. 8, i.e. a load of 3.46 lb on the specimen. The specimen was subjected to a maximum load of  $31 \times 3.46$  lb., and the fringe-patterns obtained at various loads were drawn by hand. These are given in Fig. 12(a), (b), (c), (d) and (e) for 0, 38-lb., 55.5-lb., 72.5-lb., and 107-lb. load on the specimen. At a load of 38-lb. on the specimen, the entire length and width of the specimen was covered with a black fringe, as shown in Fig. 12(b) and that was the 1st order fringe. At a load of 55.5-lb. the entire length and width of the specimen, on the contrary became white and that was 1.5 order fringe. On further loading



to 72.5 lb., when the load became sufficient enough to produce retardation of two wave lengths, again black fringe covered the entire length and width of the specimen and that was 2nd. order fringe. Fig. 13 shows the photographs of the fringe pattern, obtained at zero and 125-lb. load on the specimen which corresponds to a fringe order value of 0 and 3.5, respectively.

The load required on the specimen to produce one fringe order change is about 36-lb. equivalent to a principle stress difference of 288 p.s.i.

The fringe patterns obtained indicate symmetrical strain variation across cross-section and were linear with the load.

### 13. TEST ON SPECIMEN H

Specimen H was a rectangular araldite plate 2" x 18" x 1/4" containing a circular hole 7/8" diameter hole, symmetrically placed at the centre. It was made from the same batch of araldite that was used in the previous tests. It was tested in tension, first by photoelastic technique and then in the special tensile testing machine, using resistance-gauges, which were attached at positions, shown in Fig. 14.

### 14. PHOTOELASTIC-TECHNIQUE

The specimen was subjected to gradually increasing loads in tension using the loading-frame; detailed in paragraph 9. The formation of the fringes were studied for each increment of 3.46 lb. load on the specimen. The fringe order values were calculated by counting the number of fringes, passing through a particular point during the application of the load. (The position of an isotropic point was not

known). The fringe patterns obtained at various loads were drawn by hand. These are given in Fig. 15 (a), (b), (c), (d) and (e) for zero, 17.3lb., 34.6lb., 52lb., and 260-lb. load on the specimen, respectively. A photograph of the fringe-pattern obtained at 260-lb. load is shown in Fig. 16.

It was not possible to get a clear picture of isoclinic lines from the araldite specimen, that obtained is shown in Fig. 17 for  $0^\circ$  isoclinics, a perspex specimen of the dimensions similar to those of the araldite specimen, was made and tested in tension. The perspex specimen gave a clear picture of isoclinic lines, as shown in Fig. 18(a) and (b) for  $0^\circ$  and  $85^\circ$  isoclinics, respectively, at 260-lb. load on the specimen. The positions of the cupic points M, can easily be determined from  $0^\circ$  isoclinic lines obtained from the perspex specimen. The separate values of the vertical and horizontal stresses p and q, are determined with the help of the cupic points M, as explained in paragraphs 15 to 18.

15. SEPARATE STRESSES P & Q, ALONG THE AXIS OF SYMMETRY OA PERPENDICULAR TO THE APPLIED TENSILE STRESS.

First, plot the (p-q) curve, shown in Fig. 19 which is obtained from the photoelastic stress pattern, shown in Fig. 16. The position of cupic points M is taken from the zero degree isoclinic lines, shown in Figure 18(a).

Next, calculate and lay off the average stress  $\delta a$  in the fringes,

on a line perpendicular to the line OA, which in the case under consideration is represented by segment AA' or OO' and equals to 3.34 fringes.

This stress  $\delta a$  is computed from the equation

$\delta a = P/2AF$  where P, A and F denote the applied load, the net area of the section of symmetry, and the shear fringe value of the model, respectively.

Now draw tangents to (p-q) curve at points B and D which are designated by  $(Tpq)_b$  and  $(Tpq)_d$ , respectively. Point D is directly over the cusp-point M, Fig 19. Then calculate the slopes of the q curve at O and is given by

$$\tan (\beta q)_o = - (p-q) / r_o = n_o / r_o$$

We note that  $\tan (\beta q)_o$  is in fringes per arbitrary unit length.

Thus, if OA is taken as one unit then

$$\begin{aligned} r_o = r/d1 &= 0.437/0.563 \\ &= 0.775 \text{ unit of OA.} \end{aligned}$$

Noting that the fringe order  $n_o$  is 8.5, we obtain for

$$\tan (\beta q)_o = 8.5/0.775 = 11 \text{ fringes per unit length of OA}$$

At point A, the radius of curvature is infinite, and therefore

$$\tan (\beta q)_o = 0$$

Further since the boundary at O is convex, it follows from article 7.3

(reference 14. p.28) that  $dq$  at O has the same sign as p and that it

remains of the same sign throughout the section OA, i.e. the tension. The

q curve, Fig. 19, therefore lies above X-axis and p curve, that is the curve of the normal stress, is everywhere above (p-q) curve except at points B and C where the two curves meet. For obviously,  $p = (p-q) + q$  and if q is positive, p is greater than (p-q).

We next determine the direction of the tangent to the p curve at B i.e.  $(Tp)_B$ . In this, we follow the procedure reported by Frocht (ref.14, p.223). Through point B, then draw a line  $(Tp)_B$  parallel to  $(Tpq)_D$  which denotes the tangent drawn to the (p-q) curve at D, Fig. 19.

The tangent  $(Tpq)_D$  is parallel to  $(Tp)_C$ . The angle between  $(Tp)_B$  and  $(Tp)_C$  drawn from B is denoted by  $\beta$ . The bisector of this angle locates point E and the segment ED is the first approximation to the maximum value of q.

Since point A is also a cupic point, the p and (p-q) curves have a common tangent at C. Now draw a smooth curve BE<sup>n</sup>C as shown in Fig. 19., which is the first approximation to the required p curve. Measure the area under this curve, using a planimeter and compare it with that under AA'OO' curve, i.e. the average stress curve. The two areas show a considerable difference, the approximate p curve is adjusted until equilibrium is satisfied. In this case three attempts had to be made to arrive at a result which is within 5% (i.e. area under p curve is only 5% higher than that under the average stress curve AA'OO'). After determining p curve, q curve can now be determined, because the values of q stresses at points C, M and A are known, being zero, maximum (i.e. equivalent to ED) and zero, respectively. The p and q curves thus obtained are shown in Fig. 19. The method of drawing p curve is only approximate,

but the results obtained are sensibly accurate.

16. SEPARATE STRESSES P AND Q ALONG THE AXIS OF SYMMETRY  
OA PARALLEL TO THE APPLIED TENSILE STRESS

In order to determine the principal stresses  $p$  and  $q$ , along the axis of symmetry OA parallel to the applied tensile stress, shown in Fig. 20, first draw  $(p-q)$  curve obtained from the fringe-pattern, shown in Fig. 16.

The boundary stresses at O and A and the slopes of the stress curves at these points are known. Thus at O, the vertical stress  $p$  vanishes and the horizontal stress  $q$  is compressive. The magnitude of stress  $q$  at O is given by  $OB'$  of the  $(p-q)$  curve, since here the value of  $p$  is zero.

Similarly at point A, where pure tension begins, horizontal stress  $q$  vanishes and vertical stress  $p$  is given by

$$p = F/2AF = 260/2 \times 2 \times 1/4 \times 138.5 \\ = 1.88 \text{ fringes.}$$

The slope of  $p$  curve at O is calculated by (ref. 14, p. 59)

$$\partial p / \partial s = - (p-q) / l_2 = 2.8/r$$

The slopes of the  $p$  curve and  $q$  curve at A, are evidently both zero, since the stress trajectory through A are straight lines. The slope of  $q$  curve at B', B Fig. 20 is found from the relation

$$q = p - (p-q) \\ \partial q / \partial s_1 = \partial p / \partial s_1 - \partial (p-q) / \partial s_1 \text{ or } \tan (\beta_q)_{b'} = \tan (\beta_p)_o - \tan (\beta_{pq})_b$$

The angles  $(\beta_q)_{b'}$ ,  $(\beta_p)_o$  and  $(\beta_{pq})_b$  are measured from the line OA.

The graphical construction used for determining the tangent to  $q$  curve at B' is as follows :-

First, the position of point C, which is at a distance  $r$ , the radius of the circle from point O is marked on the line OA. Then draw rectangles  $OBC_1C$  and  $OBC_2C$ . Join the lines  $OC_1$  and  $BC$ , which give the tangent to  $p$  curve at points O and B denoted by  $(T_p)_O$  and  $(T_p)_B$ , respectively. Next, draw the tangent to the  $(p-q)$  curve at B denoted by  $(T_{pq})_B$ . The distance  $KL=RS$ , shown in Fig. 20, between the tangents  $(T_{pq})_B$  and  $(T_p)_B$ , determine the tangent to  $q$  curve at point  $B'$ . The curve of  $q$  stresses, which starts as a compression at point  $B'$ , Fig. 20 change into a tension at some point between  $B'$  and A. The final curve is determined by adjusting the tensile area resting on FA, to equal to the compressive area  $OB'F$ . After two or three trials, the two areas can be made equal and the final curve thus determined represent  $q$  curve. From the values of  $q$  stresses thus determined the  $p$  stresses can easily be calculated. The final curves of  $p$  and  $q$  stresses are shown in Fig. 20.

## 17 STRESSES AROUND THE EDGE OF THE HOLE

The vertical and horizontal stresses  $p$  and  $q$ , respectively, around the edge of the hole, are determined as follows :-

First draw the  $(p-q)$  curve, as shown in Fig. 21 obtained from the fringe-pattern of Fig. 16. It is evident from Fig. 21, that the value of  $(p-q)$  curve is 8.5, zero and 2.8 fringe orders at points A, B and C respectively.

The hole is free from the initial stresses and therefore only one of the principal stresses, either  $p$  or  $q$  exist at a time, around the edge of the hole. We know that the value of  $q$  stress is zero at point A and therefore  $(p-q)$  curve gives the value of  $p$  stress at this point.

The p curve follows the (p-q) curve from point A to point B, where its value is zero. From point B to point C there are only q stresses, as shown in Fig 21, while p stress is zero. There are four points, such as point B around the edge of the hole where the values of both, the p and q stresses are zero.

### 18. STRESSES ALONG THE EDGE OF THE SPECIMEN, PARALLEL TO THE APPLIED STRESS

The stresses along the edge of the specimen, parallel to the applied tensile stress, are determined as follows :-

We know that the values of q stresses at the edge of the specimen are zero and therefore (p-q) curve gives the value of the vertical stresses p. The curve, shown in Fig. 22., is the curve of p stresses along the edge of the specimen from the centre of the hole to point A where the pure tension begins. It is obtained by plotting (p-q) curve in terms of fringe-order values, those touching the edge of the specimen, as we move from the centre of the hole to point A.

### 19. CALCULATIONS

#### Stress concentration-factors

#### at points K1 and K2, Fig.14

Fringe-order value at the edge of the hole, i.e. at points K1	=	8.5
Fringe-order value at points K2	=	1.5
Average stress value in terms of fringe-order	=	$\frac{P}{2AF} = \frac{260}{2 \times 2 \times 1/4 \times 138.5} = 1.88$

Stress-concentration factors at points K1 and K2 are therefore,

equivalent to  $8.5/1.88 = 4.5$  and  $1.5/1.88 = 0.8$ , respectively.

## 20. COMPARISON WITH THEORETICAL RESULTS

The theoretical values of stress-concentration, at points K1 and K2, obtained, as shown in Fig. 5.12, from Howland's analysis, for the case under review are 4 and 0.85 times the average stress, respectively (i.e. the theoretical value at points K1 is about 12% lower than that obtained from photoelastic results, while that at points K2 is about equal).

The theoretical values obtained from Howland's analysis, along the axis of symmetry OA, parallel to the applied tensile stress, shown in Figure 5.16., are in agreement with those of photoelastic results, shown in Fig. 20. For example at point K3, Fig. 20, which is at a distance of  $2r$  from point O, the theoretical value of  $p$  stress is 0.45 times the average stress, while that obtained from experimental results is 0.48 times the average stress (i.e. 6% higher than the theoretical value).

The theoretical values of stress-concentration around the edge of the hole, shown in Fig. 5.15, also agree with those of the experimental values. For example at point C, the theoretical value of stress-concentration is 1.5 times the average stress, while that obtained from experimental results, shown in Fig. 21., is 1.49 the average stress. Point B, where both the vertical and horizontal stresses  $p$  and  $q$  are zero, lie between  $50^\circ$  and  $60^\circ$ , Fig. 21, both in case of theoretical and experimental curves.



21. COMPARISON WITH RESULTS OBTAINED ON MILD STEEL PLATE E

The average value of stress concentration factors obtained, in case of mild steel plate E, tested in the special tensile testing machine, at points K1' and K2', shown in Fig. 5.1(a), i.e. the centre of the resistance-gauges, are 2.25 and 1.23. The photoelastic results at these points, obtained from Fig. 19, are  $5.25/1.88 = 2.75$  and  $2.25/1.88 = 1.2$  (i.e. the photoelastic result at points K1', is about 22% higher than that of steel specimen).

The above results indicate that the photoelastic results are different from those obtained on mild steel specimen E, but agree fairly well with the theoretical results, obtained from Howland's analysis (ref. 18).

22. TEST IN SPECIAL TENSILE TESTING MACHINE WITH RESISTANCE GAUGES ATTACHED AT THE HOLE

Ten Avro 100  $\Omega$  resistance-gauges with a gauge sensitivity factor of 2.25, given by manufacturers, were now attached to the araldite specimen, which had already been tested by photoelastic-technique, at positions shown in Fig. 14. The resistance-gauges, which were attached at the hole, were cut from the sides, in order to provide space between the resistance-gauges, to be used by Huggenberger extensometers. This operation does not affect the workability of the resistance-gauges, as was proved by an experiment, done by the writer and described in appendix C. The Huggenberger extensometers were used to act as a check to the resistance-gauges readings.

The procedure adopted for cutting the resistance gauges, was as

follows :-

The resistance-gauge was placed on a perspex plate with a light source at the bottom, as shown in Fig. 23(a), so that the wires of the resistance gauge could be seen. The resistance-gauge was then slightly pressed against the perspex plate with an iron role and cut from the sides, using a sharp knife. Care was taken that the wires of the resistance-gauge were not affected.

The procedure adopted for fixing and protecting the resistance-gauges from moisture was that detailed in paragraphs 4.3 and 4.4. The resistance-gauges were checked for drift before starting the tests, but no drift was recorded even after 2 hours.

Resistance-gauges No. 5 and 10 at cross-section No. (i) were at a distance of about two times the width of the specimen and those at cross-section No. (ii) were about 3.5 times the width of the specimen from the end of the jaws of the special tensile testing machine. The St. Venant's theory might then be expected to apply at these cross-sections.

The results obtained from the resistance-gauges, when the specimen was tested in axial tension in the special tensile testing machine are given in table 2. The strain-readings obtained, gave loops during loading and subsequent unloading, as shown in Fig. 23(b), for gauge No. 5 and 4, but straight line portion of the curve has been taken to give the strains tabulated. These results give the figures shown in column 9, lines 2 to 5 of table 2 and indicate symmetrical strain behaviour of the specimen at the sections through the hole. This symmetry was not obtained with any of the steel specimens and suggests that the araldite was behaving as a more homogeneous material .

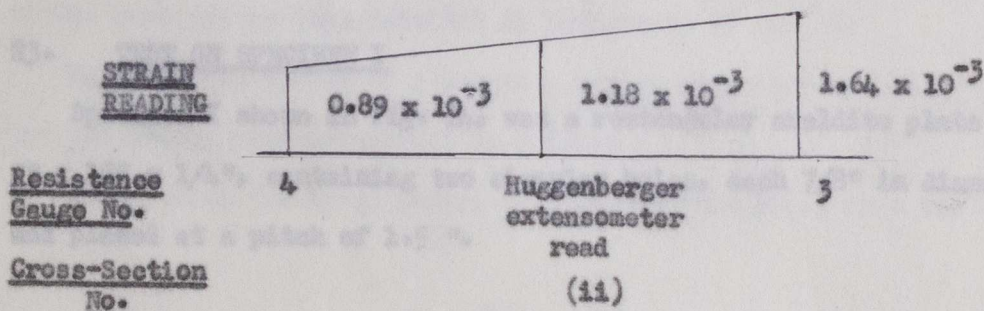
The resistance-gauges results were checked by Huggenberger extensometers readings, which were attached at positions shown in Fig. 14. The strain-readings obtained from these gauges, not only confirm the resistance-gauge results, but also indicate a symmetrical strain pattern. Huggenberger extensometers gave equal readings, on both faces of the specimen, given in table 4.

TABLE 4

Cross-Section No.	Position of Huggenberger extensometer	Huggenberger extensometer reading	Strain	Position of Huggenberger extensometer	H.E. Reading	Strain
(11)	Between resistance gauge No. 3 & 4.	1.12	$1.18 \times 10^{-3}$	Between resistance gauge No. 8 and 9	1.12	$1.18 \times 10^{-3}$

The strain-readings obtained from the average reading of resistance-gauges No. 4 and 9, 3 and 8 are  $0.89 \times 10^{-3}$  and  $1.64 \times 10^{-3}$ , respectively.

The comparative picture of the strain readings, obtained from resistance-gauges and Huggenberger extensometers, is shown below :-



The results obtained from tests on araldite specimen with resistance gauges at the hole and tested in the special tensile testing machine are not in agreement with those obtained from fringe-pattern and mild steel specimen E. The comparative results obtained from tests on mild steel specimen E and araldite specimen H, are shown in table 5.

TABLE 5.

RESULTS OBTAINED FROM	<u>STRESS-CONCENTRATION FACTORS</u>	
	AT POINTS K1'	AT POINTS K2'
1. The theoretical analysis of Howland.	2.5	1.2
2. The Fringe-pattern	2.75	1.2
3. The araldite specimen tested in special tensile testing machine with resistance gauges attached at the hole.	1.5	0.82
4. Mild steel specimen E	2.25	1.23

The results obtained from tests by photoelastic technique and that in the special tensile testing machine, with resistance-gauges at the hole, indicate a symmetrical strain pattern across cross-section perpendicular to the applied tensile stress and passing through the centre of the hole.

23. TEST ON SPECIMEN I

Specimen I shown in Fig. 24, was a rectangular araldite plate 2" x 18" x 1/4", containing two circular holes, each 7/8" in diameter and placed at a pitch of 1.5".

The specimen was subjected to gradually increasing loads in tension, using the loading frame shown in Fig. 8. The formation of the fringe were studied, for each increment of one pound load in load pan P, i.e. a load of 3.46 lb. on the specimen. The specimen was subjected to a maximum load of 260 lb. The fringe-patterns obtained at various loads were drawn by hand. These are given in Fig. 25, for 0, 34.6-lb., 121-lb., and 260-lb. load on the specimen. Fig. 26 shows the photograph of the fringe-pattern, obtained at 260-lbs. load on the specimen.

The isoclinic lines were obtained from tests on a perspex specimen, which was of dimensions, similar to that of araldite specimen, because a clear picture of isoclinic lines could not be obtained from the araldite specimen. Fig. 27 shows  $O^{\circ}$  isoclinic-lines at 260 -lb., load on the perspex specimen.

The position of cupic-point M, was determined from  $O^{\circ}$ -isoclinic lines.

The values of vertical and horizontal stresses p and q on the axis of symmetry OA, parallel and perpendicular to the applied tensile stress, were obtained, as shown in Figs. 28 and 29. in the same way as that in case of specimen H. Stresses p and q around the edge of the hole, and along the edge of the specimen, shown in Fig. 30 and 31, were also determined in the same way as that detailed in paragraphs 17 and 18.

The separate values of stresses p and q, at cross-section O-O, between the two holes, were determined as follows :-

First, draw the (p-q) curve, as shown in Fig. 32(a) from the

fringe-pattern of Fig. 26. The position of the cupic point  $M$  is determined from the  $0^\circ$  isoclinic-line, which subsequently determines the position of maximum  $q$  stress. The magnitude of  $p$  and  $q$  stresses at points  $O$  are known, i.e. the values of  $q$  stress is zero at points  $O$  and therefore the  $(p-q)$  curve gives the value of  $p$  stress at these points. The slope of the tangents to the  $p$  and  $q$  curves, at points  $O$  are also known, being horizontal, because of the radius of curvature being infinity at these points. Further, the values of  $(p-q)$  curve is zero, as shown in Fig. 32(a), from point  $A$  to  $A$  and therefore the tangents to  $p$  and  $q$  curves are again horizontal at these points.

The graphical construction, used for determining the  $p$  curve is the same, as that used for determining the stresses along the axis of symmetry  $OA$ , perpendicular to the applied tensile stress and passing through the centre of holes. The  $p$  curve is determined by adjusting the area under  $p$  curve, until it is equivalent to the average stress-curve. After two or three trials, the two areas were within 4% (i.e. the area under  $p$ -curve was only 4% higher than that under the average-stress curve).

The value of  $q$  max is determined from  $p$  curve, shown in Fig. 32(a) which is equivalent to  $ED$ . The magnitude and slope of the  $q$  curve at points  $O$  and  $A$  are known and also the position of cupic point  $M$ , where the value of  $q$  stress is maximum, being equivalent to  $ED$ . A smooth curve, as shown in Fig. 32(a) was then drawn to give  $q$  curve.

This is an approximate method of drawing p and q curves, but the results obtained are sensibly accurate. Fig. 32(b) gives the values of p and q stresses, between the holes along the cross-section parallel to the applied tensile stress and passing through the centre of the holes.

#### 24. CALCULATIONS

##### Stress concentration factors at points K5 and K4, Fig. 24.

Fringe order value at the edge of the hole, i.e. at points K5 = 6.5

Fringe-order value at point K4 = 1.5

Average stress value in terms of fringe order =  $P/2AF = 260/2 \times 2 \times 1/4 \times 138.5 = 1.88$

Stress-concentration factors at points K5 and K4 are therefore, equivalent to  $6.5/1.88 = 3.45$  and  $1.5/1.88 = 0.8$ , respectively.

#### 25. COMPARISON WITH THEORETICAL RESULTS

The theoretical values of stress-concentrations, obtained at points K1, K2, K4 and K5, from Howland's analysis for plates of infinite dimensions and containing circular holes (ref. 20) are 0.04, 1.0, 2, 0.9 times the average stress, respectively.

The comparative results obtained from theoretical analysis, of Howland, Araldite specimen I and those obtained on mild steel specimen F, are given in Table 6.

TABLE 6

RESULTS OBTAINED FROM-	STRESS-CONCENTRATION FACTORS			
	AT POINTS K1	K2	K4	K5
1. The theoretical analysis of Howland	0.4	1.0	0.9	2.0
2. The Fringe pattern	0.0	1.6	0.8	3.45
3. Mild steel specimen F	K1	K2	K4'	K5'
	0.042	1.2	1.23	2.14
4. Fringe-pattern	0.0	1.6	1.23	2.29
5. Theoretical Analysis of Howland	0.04	1.0		1.25

The above results indicate that the theoretical results are not in agreement with those obtained from the fringe pattern. The results obtained on mild steel specimen F are about the same as those obtained from fringe-pattern.

The tests results obtained, indicate a symmetrical strain pattern across cross-sections perpendicular to the applied tensile stress and passing through the centre of the holes and between the holes.

#### 26. TEST ON SPECIMEN J

Specimen J, shown in Fig. 33, was a rectangular araldite plate 1.75" x 18" x 1/4", containing two circular holes, (the diameters of which were 0.5 times the width of the plate). The holes were at a pitch of 1.5".



The specimen was tested in tension, as in the previous tests on specimen I. The fringe-pattern obtained at various loads were drawn by hand. These are shown in Fig. 34 for 0, 34.6-lb, 121-lb., and 260-lb. load on the specimen. Fig. 35 shows the photograph of the fringe-pattern obtained at a load of 260-lb. on the specimen.

The isoclinic lines, shown in Fig. 36 were obtained from tests on a perspex specimen of dimensions similar to those of araldite specimen.

The graphical constructions, shown in Figs. 37, 38, 39, 40, 41, and 42, were made in the same way as that in case of specimen I. Fig. 41 indicates a small amount of compression at point K1.

## 27. CALCULATIONS

### Stress-concentration factors

#### at points K5 and K4, Fig. 33

Fringe order value at the edge of the hole, i.e. at points K5 = 7.5

Fringe order value at point K4 = 1.5

Average stress value in terms of fringe-order =  $P/2AF = 260/2 \times 1.75 \times 1/4 \times 138.5 = 2.15$

Stress-concentration factors, at points K5 and K4, are therefore, equivalent to  $7.5/2.15 = 3.5$  and  $1.5/2.15 = 0.7$ , respectively.

The tests results obtained, indicate a symmetrical strain pattern across cross-sections perpendicular to the applied tensile stress and passing through the centre of the holes and between the holes.

(c) GENERAL28. CALCULATION OF THE GAUGE-SENSITIVITY FACTOR

The gauge-sensitivity factor values of the resistance gauges can be calculated as follows :-

The gauge-sensitivity factor is the ratio between the change of gauge resistance to the strain causing the change, i.e.

$$F = dR/R \times 1/\text{strain}$$

- (1) The strain values for various loads applied to the specimen in bending can be calculated from dial-gauge readings.

The portion of the beam between the dial-gauge supports bends into an arc of a circle when it is subjected to a four point loading system. From Fig. 43, in which  $R$  is the radius of the curvature of the beam to the neutral axis and  $\delta$  the measured deflection.

$$\begin{aligned} (R - d/2)^2 &= (R - d/2 - \delta)^2 + (L/2)^2 \\ &= (R - d/2)^2 + \delta^2 - 2(R - d/2)\delta + L^2/4 \\ L^2/4 &= 2R\delta - d\delta - \delta^2 \text{ where } \delta \text{ is small and } d \text{ is} \\ &\text{small compared with } R. \end{aligned}$$

This gives

$$2R = L^2/4\delta \dots\dots\dots(1)$$

The strain is then  $d/2 \times 1/R = 4d\delta/L^2$  on substitution from equation (1).

- (ii) The value of change of gauge resistance per ohm, i.e.  $dR/R$  can be calculated, considering the wheatstone bridge circuit and knowing the galvanometer resistance and its deflection per micro amp.

Suppose that initially the apex points of the bridge are at zero potential, and the battery terminals at a potential  $E/2$  and  $-E/2$ , Fig. 44.

Now suppose the active gauge is strained and that its resistance changes a small amount  $dR$ . The battery voltage remains constant, but the potential of the splices changes small amounts to values say  $\delta$  &  $\theta$ , where both  $\delta$  &  $\theta$  are small. The current flowing in the various arms of the bridge are now as shown in Fig. 44 and by ohm's law ;

$$(E/2 - \delta) = i_1 (R + dR) \dots\dots\dots (2)$$

$$(\delta + E/2) = (i_1 - i_g) R \dots\dots\dots (3)$$

$$(E/2 - \theta) = i_3 R_f \dots\dots\dots (4)$$

$$(\theta + E/2) = (i_g + i_3) R_f \dots\dots\dots (5)$$

$$(\delta - \theta) = i_g \cdot R_g \dots\dots\dots (6)$$

Subtracting (2) from (3) gives

$$2\delta = -i_g \cdot R - i_1 \cdot dR \dots\dots\dots (7)$$

Also from (2)

$$i_1 = (E/2 - \delta)/R(1 + dR/R) = E/2R + \text{small terms} \dots\dots (8)$$

Substitution in (7) gives

$$2\delta = -i_g \cdot R - (E/2) \cdot (dR/R) \dots\dots\dots (9)$$

Similarly subtraction of (4) from (5) gives

$$2\theta = i_g \cdot R_f \dots\dots\dots (10)$$

and subtraction of (10) from (9) gives

$$2(\delta - \theta) = -i_g (R + R_f) - (E/2) (dR/R) \dots\dots (11)$$

$$= 2 i_g \cdot R_g \text{ from (6)}$$

whence

$$i_g = -E/2 \cdot 1/R + R_f + 2R_g \times dR/R$$

the galvanometer current, thus measures the value of  $dR/R$ . The galvanometer used in the tests reported in chapters 4, 5, 6, 7 and 8 had the following specifications :-

Resistance = 9.4 ohms

Periodic time = 2 seconds

Deflection per  
micro-amp. = 45 mm.

Considering gauge No. 1 (chapter 4, Specimen A)

$$d/2 = 5/32''$$

$$L = 12''$$

$$\text{and } \delta = 136.5/10^4 \text{ inch, for a load of 40-lb.}$$

$$\text{the strain is then :- } = 4d\delta/L^2$$

$$= 4 \times 5 \times 136.5 / 16 \times 144 \times 10^4$$

$$= 1.185 \times 10^{-4}$$

The value of change of gauge-resistance per ohm is then

$$dR/R = 2 i_g/E (R + R_f + 2R_g)$$

$$= 2 \times 1/10^6 \times 4.5 \times 2 (98.4 + 110 + 19.8)$$

$$\text{where } i_g = 1/10^6 \times 4.5 \text{ per cm.}$$

$$= 228.2/10^6 \times 4.5 \text{ per cm. deflection of galvanometer.}$$

$$dR/R \text{ (for load of 40-lb)} = 4.6 \times 228.2/10^6 \times 4.5$$

It is however, better to apply a known resistance across the gauge and record the galvanometer deflection. Hence calculate equivalent resistance change of strain.

The gauge sensitivity factor value is therefore, equivalent to

$$\begin{aligned}
 & dR/R \times 1/\text{strain} \\
 = & 4.6 \times 228.2 \times 10^4 / 10^6 \times 4.5 \times 1.185 \\
 = & 1.975
 \end{aligned}$$

The gauge-factor values of the resistance gauges are given in table 4.2, chapter 4.

## 29. CALCULATION OF STRAIN VALUES FROM MAIHAK-GAUGE READINGS

The strain-values from Maihak-gauge readings can be calculated as follows :-

We know that

$$\text{Strain} = K \times (\text{change in } f^2/1000)$$

where K = Gauge factor value of Maihak-gauge (which had already been calculated in the Department and are  $3 \times 10^{-6}$  for M.G. No. 18889 and  $2.5 \times 10^{-6}$  for M.G.No. 18892)

The values of  $f^2/1000$  were obtained from the calibration chart which was available.

Then, for example, for gauge No. 18892, we have the following readings on frequency meter,

at zero load 895.2 and at 40 lb. 899.8, with a possible error of  $\pm 0.5$ .

The values of  $f^2/1000$  for the above readings from the calibration chart are 1521 and 1471. The change in  $f^2/1000$  for a load of 40lb. is then,

$$= (1521 - 1471)$$

$$= 50$$

This gives the strain  $= K \times (\text{change in } f^2/1000)$   
 $= 2.5 \times 10^{-6} \times 50$   
 $= 125 \times 10^{-6}$  on using the calibration  
 constant obtained in the  
 department.

The stress change  $= 125 \times 10^{-6} \times 30 \times 10^6$   
 $= 3750 \text{ p.s.i.}$ , where  $E$  is taken as  $30 \times 10^6 \text{ p.s.i.}$

The stress given by the dial-gauge, corresponding to this  
 $= 3550 \text{ p.s.i.}$

The stress calculated from the applied bending moment.  
 $= 3500 \text{ p.s.i.}$

The other values given in table 4.5 (a) and (b) of chapter 4 were  
 calculated in this way.

### 30. PURE BENDING TEST

Four resistance strain-gauges, two as supplied by the manufacturers  
 and the other two, which had been cut from the sides, were attached to a  
 mild steel beam, which was loaded by means of a four point loading system,  
 as shown in Fig. 4.14. Two of the resistance-gauges, one as supplied by  
 the manufacturers and the other, which had been cut from the sides, were  
 attached to the tension side and the other two to the compression side.

The procedure adopted for cutting the resistance-gauges from the sides,  
 was that explained in paragraph 22 of Appendix B.

The resistance-gauges were checked for drift at zero load before  
 starting the tests, but no drift was recorded even after one and half  
 hour.

The beam was loaded to 16 lb. by an equal increment of 2-lb, and subsequently unloaded to 0. The strain-readings obtained from resistance-gauges No. 1 and 3, which were attached to tension and compression sides, respectively are plotted in Fig. 45. The results obtained from these gauges are coinciding during loading and subsequent unloading, as shown in Fig. 45 and suggest that the workability of the resistance gauges are not affected, if they are cut from the sides, as long as the effective portion of a gauge i.e. the gauge-wires, is not affected from such an operation.

31. STRESSES IN A STRIP OF FINITE WIDTH, WHICH IS WEAKENED BY A CIRCULAR HOLE

The solution of this problem which is due to R.C. J. Howland (ref.18) will now be outlined.

We consider a strip of isotropic elastic material, bounded in the X, Y-plane by the lines  $X=tb$ , and of uniform thickness perpendicular to this plane.

The hole will be supposed to have its centre at the origin, and to be of radius  $a$  ( $<b$ ). Polar co-ordinates  $(r,\theta)$  will also be used, and it will be convenient to take the initial line along the Y-axis and positive direction of  $\theta$  clockwise, Fig. 46.

Then the relation between the two systems of co-ordinates is

$$\begin{aligned} X &= r \sin\theta \\ Y &= r \cos\theta \quad \dots\dots\dots(1) \end{aligned}$$

We shall also write

$$\lambda = a/b \quad \xi = x/b \quad \eta = y/b \quad \rho = r/b \quad \dots\dots (2)$$

so that  $\xi$ ,  $\eta$  and  $\rho$  are co-ordinates measured in a unit equal to half the width of the strip, and  $\lambda$  is the radius of the hole measured in the same unit. The value of  $\lambda$  greater than 0.5, will not be considered.

The material will be supposed to be in a state of generalised plane stress, specified by a stress-function  $X$ . This function must satisfy the following conditions :-

(a) At points within the material,

$$\nabla^4 X = 0, \quad [1. \text{e.} \quad \frac{\partial^4 X}{\partial U^4} + \frac{2\partial^2 X}{\partial U \partial V} + \frac{\partial^4 X}{\partial V^4} = 0] \quad \dots (3)$$

(b) the stresses

$$XX = \frac{1}{b^2} \frac{\partial^2 X}{\partial \xi^2}, \quad YY = \frac{1}{b^2} \frac{\partial^2 X}{\partial \eta^2}, \quad XY = -\frac{1}{b^2} \frac{\partial^2 X}{\partial \xi \partial \eta}$$

tend to definite values at infinity; in particular, in the tension problem,  $XX \rightarrow T$  (Const.),  $YY \rightarrow 0$ ,  $XY \rightarrow 0$  when  $X \rightarrow \pm \infty$ .

(c) On the straight edges,  $\eta = \pm 1$ .

$$\frac{\partial^2 X}{\partial \xi^2} = \frac{\partial^2 X}{\partial \xi \partial \eta} = 0$$

(d) At the rim of the hole,  $\rho = \lambda$

$$\begin{aligned} r\theta &= \frac{1}{b^2} \left[ \frac{1}{\rho} \frac{\partial^2 X}{\partial \theta^2} + \frac{1}{\rho} \frac{\partial X}{\partial \rho} \right] = f(\theta) \quad \dots (4) \\ r\theta &= -\frac{1}{b^2} \frac{\partial}{\partial \rho} \left( \frac{1}{\rho} \frac{\partial X}{\partial \theta} \right) = \phi(\theta) \end{aligned}$$

Where  $f(\theta)$  and  $\phi(\theta)$  are given functions, both even and both symmetrical about  $\theta = \frac{1}{2}\pi$ ; in particular, in the tension problem, the rim of the hole is free from stress and  $f(\theta)$  and  $\phi(\theta)$  both vanish.

To satisfy these conditions, we write

$$X = X_0^1 + X_0 + X_1 + X_2 + \dots (5)$$

where the terms of the series are each, separately, solutions of the bi-harmonic equation (3) and have, in addition, the following properties :-



$X'_0$  gives the stresses at infinity and none on the edges  $\eta = \pm 1$ .

$X'_0 + X$  satisfies the conditions at the rim of the hole and at infinity, but not on the edges i.e. it is the solution for an infinite plate. This will be supposed known,  $X_0$  being expressible in the form

$$X_0 = -D_0(0) \log \rho + \sum_{n=1}^{\infty} \left( \frac{D_{2n}(0)}{\rho^{2n}} + \frac{E_{2n}(0)}{\rho^{2n-2}} \right) \cos 2n\theta \quad \dots\dots (6)$$

$X_1$  cancels the stresses due to  $X_0$  on the edges  $\eta = \pm 1$ , but introduces stresses on the rim of the hole;  $X_2$  cancels these, but again produces stresses on the edges.

More generally  $X_{2r} + X_{2r+1}$  gives zero stresses on  $\eta = \pm 1$ , while  $X_{2r-1} + X_{2r}$  gives zero stresses over  $\rho = \lambda$ . For definiteness we add the conditions that the stresses due to  $X_{2r}$  shall tend to 0, when  $\rho \rightarrow \infty$ , while those due to  $X_{2r+1}$  shall be finite at the origin and throughout the finite part of the strip.

All the stress functions are then fully determinate and  $X$  will satisfy the required conditions, provided that the series in (5) and its derivatives up to the fourth are uniformly convergent.

If the series is truncated after  $X_{2r+1}$  it will give a value of  $X$  satisfying all the conditions except those on the edges  $\eta = \pm 1$ . If the residual tractions due to  $X_{2r}$  are small enough, this value of  $X$  is, for practical purposes, the value required. Similarly, if the series is truncated after  $X_{2r+1}$ , the resulting value of  $X$  satisfies all the conditions except those at the rim of the hole  $\rho = \lambda$ . If the additional tractions due to  $X_{2r+1}$  are small enough, the solution is again sufficient in practice.

In neither case is there any problem of convergence; but it is clear that, if the method is to be practicable, it is necessary that there should be terms fairly early in the series which correspond to tractions of negligible amount. It is found that if  $\lambda$  does not exceed 0.5, it is not necessary to proceed beyond  $X_3$ , while if  $\lambda$  is less than 0.25, it is possible to stop at  $X_2$ . Values of  $\lambda$  much larger than 0.5 lead to very labourious computations.

### 32. THE TENSION PROBLEM

To obtain the solution for a tension  $T$  applied to the strip, the rim of the hole as well as the edges of the strip being free from stress, we start with

$$X'_0 = \frac{1}{4} b^2 T \rho^{-2} (1 + \cos 2\theta)$$

$$X_0 = \frac{1}{4} b^2 T \left[ -2\lambda^2 \log \rho - \left( 2\lambda^2 - \frac{\lambda^4}{\rho^2} \right) \cos 2\theta \right]$$

the corresponding stresses being

$$r r_0 = \frac{1}{2} T \left[ \left( 1 - \frac{\lambda^2}{\rho^2} \right) - \left( 1 - 4 \frac{\lambda^2}{\rho^2} + 3 \frac{\lambda^4}{\rho^4} \right) \times \cos 2\theta \right]$$

$$\theta \theta_0 = \frac{1}{2} T \left[ \left( 1 + \frac{\lambda^2}{\rho^2} \right) + \left( 1 + 3 \frac{\lambda^4}{\rho^4} \right) \cos 2\theta \right]$$

$$r \theta_0 = \frac{1}{2} T \left( 1 + 2 \frac{\lambda^2}{\rho^2} - 3 \frac{\lambda^4}{\rho^4} \right) \sin 2\theta$$

Now we have

$$D_0(0) = \frac{1}{2} b^2 T \lambda^2, \quad D_2(0) = \frac{1}{4} b^2 T \lambda^2$$

$$E_2(0) = -\frac{1}{2} b^2 T \lambda^2$$

and the other coefficients being zero, we obtain from equations (32)

and (34), (ref. 18, page 59)

$$L_{2n}(0) = b^2 T \lambda^2 \left[ \frac{1}{2} (\eta_{\alpha_0} - \eta_{\beta_1}) + \frac{1}{4} \eta_{\alpha_1} \lambda^2 \right]$$

$$M_{2n}(0) = b^2 T \lambda^2 \left[ \frac{1}{2} (\eta_{\gamma_0} - \eta_{\delta_1}) + \frac{1}{4} \eta_{\gamma_1} \lambda^2 \right]$$

From these it is easy to calculate the values of the coefficients of  $X_1$  and to proceed to those of  $X_2$ , retaining  $\lambda$  as a parameter. The calculation has been carried out for  $\lambda = 0.1, 0.2, 0.3, 0.4, 0.5$ , and the coefficients are recorded in tables VI to IX (ref 18. p.70) where the following abbreviations have been used.

$$\begin{aligned} d_0(r) &= D_0(r)/b^2 T & d_{2n}(r) &= D_{2n}(r)/b^2 T \\ e_{2n}(r) &= E_{2n}(r)/b^2 T & l_{2n}(r) &= L_{2n}(r)/b^2 T \\ m_0(r) &= M_0(r)/b^2 T & m_{2n}(r) &= M_{2n}(r)/b^2 T \end{aligned}$$

The approximation is in each case carried out until the greatest residual stress on the edge of the strip is 1% of T or less. Since the distribution of stress near the hole is the matter of greatest interest, the approximation is always ended with a stress function of even order so that the hole is free from residual tractions. The final value of X (summing the various stress functions) may be written

$$X = \frac{1}{4} b^2 T \rho^2 (1 + \cos 2\theta) + b^2 T [-d_0 \log \rho + m_0 \rho^2 + \sum_{n=1}^{\infty} \left\{ \frac{d_{2n}}{\rho^{2n}} + \frac{e_{2n}}{\rho^{2n-2}} + (l_{2n} + m_{2n} \rho^2) \rho^{2n} \right\} \cos 2n\theta].$$

$$\text{where } d_0 = \sum_r (d_0)^r, \quad d_{2n} = \sum_r d_{2n}(r)$$

$$e_{2n} = \sum_r e_{2n}(r), \quad l_{2n} = \sum_r l_{2n}(r)$$

$$m_0 = \sum_r m_0(r), \quad m_{2n} = \sum_r m_{2n}(r)$$

The stresses are

$$\begin{aligned} r r &= T \left[ \frac{1}{2} (1 - \cos 2\theta) + 2m_0 - \frac{d_0}{\rho^2} - 2 \sum_{n=1}^{\infty} \left\{ \frac{n(2n+1)d_{2n}}{\rho^{2n+2}} + \frac{(n+1)(2n-1)e_{2n}}{\rho^{2n}} + \right. \right. \\ &\quad \left. \left. n(2n-1)l_{2n}\rho^{2n-2} + (n-1)(2n+1)m_{2n}\rho^{2n} \right\} \cos 2n\theta \right] \\ \theta \theta &= T \left[ \frac{1}{2} (1 + \cos 2\theta) + 2m_0 + \frac{d_0}{\rho^2} + 2 \sum_{n=1}^{\infty} \left\{ \frac{n(2n+1)d_{2n}}{\rho^{2n+2}} + \frac{(n-1)(2n+1)e_{2n}}{\rho^{2n}} + \right. \right. \end{aligned}$$

$$n(2n-1) \left[ \frac{1}{2n} \rho^{2n-2} + (n+1)(2n+1) \frac{a_{2n} \rho^{2n}}{\rho} \right] \cos 2n\theta ]$$

$$r\theta = T \left[ \frac{1}{2} \sin 2\theta + 2 \sum_{n=1}^{\infty} \left\{ n(2n-1) \left( \frac{1}{2n} \rho^{2n-2} - \frac{a_{2n}}{\rho} \right) + n(2n+1) \left( \frac{a_{2n} \rho^{2n}}{\rho} - \frac{d_{2n}}{\rho^{2n+2}} \right) \right\} \sin 2n\theta \right]$$

These equations, together with table X (ref. 18, page 73) which gives the coefficients for various values of  $\lambda$ , give the complete solution for the stresses in the neighbourhood of the hole.

The values of stresses at the circumference of the hole and at cross-section A-A, Fig. 46, perpendicular to the applied tensile stress, were calculated by R.C.J. Howland for values of  $\lambda = 0.1, 0.2, 0.3, 0.4,$  and  $0.5$  and are given in table XII and XIV. Tables XV and XVI, gives the values of stresses for  $\lambda = 0.5$  at cross-section OA, Fig. 46, at parallel to the applied tensile stress and at the edge of the strip, respectively.

The values of stresses along the axis OA, parallel to the applied tensile stress, were also calculated by the writer for  $\lambda = 0.4$ , given in table 3. The values of the coefficients used in the above calculations, were those given in table X, (Ref. 18).

The value of stress obtained from Howland's analysis, at the edge of a circular hole, at  $\theta = 0$ , Fig. 46, the diameter of which is 0.5 times the width of the plate, is 4.32 times the applied tensile stress, i.e. 1.44 times the Kirsch's results

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