Can Time-To-Contact Be Used To Model A Helicopter Autorotation?

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ABSTRACT

The emergency autorotation manoeuvre used to land a conventional helicopter is a difficult task to accomplish successfully. It is even more difficult to perform in degraded visual conditions. There is an identified need to develop a cueing system to assist pilots should they need to perform an autorotation. In order to develop such a cueing system, a guidance strategy is needed such that pilots are able to follow it. Nature-inspired time to contact (tau) theory provides a powerful way to model the guidance task undertaken by an observer. This paper analyses the longitudinal flight data in the tau-domain obtained from a series of simulated autorotation manoeuvres conducted using the HELIFLIGHT-R full-motion simulator at the University of Liverpool. The analysis identifies several tau-based guidance strategies that will be used for the development of an autorotation cueing system.

1. INTRODUCTION

Background

A helicopter autorotation manoeuvre is usually employed to perform a safe landing following some catastrophic event (average probability of failure per flight hour of 2×10^{-7}) [1], such as an engine failure, the loss of the tail rotor, or a transmission failure. Autorotation manoeuvres can be divided into several discrete phases; steady-state descent, flare, push over and landing [2]. From a piloting perspective, it is a complex flight manoeuvre, requiring several piloting tasks to be coordinated simultaneously to ensure a successful landing. Due to this complexity, a successful autorotation cannot always be guaranteed. Even well trained, professional pilots can still encounter difficulties when dealing with such a demanding manoeuvre. The situation is further complicated if the pilot has to perform the autorotation manoeuvre in degraded visual conditions. It is therefore considered highly desirable to develop an autorotation guidance aid to assist pilots during the autorotation manoeuvre.

The autorotation manoeuvre is a high-dimensional problem, involving various constrained states and coupled nonlinear dynamics [3]. In previous research efforts, the authors implemented an autorotation cueing system based on a real-time expert controller in the HELIFLIGHT-R full-motion simulator at the University of Liverpool [4, 5]. When the controllers were used to automate the autorotation manoeuvre, the controllers performed well in terms of delivering a safe landing that met some pre-defined desired/adequate landing criteria. The same control algorithms were used to provide cues to the pilot to perform the manoeuvre manually. The cueing system comprised a head-up cockpit display containing visual markers, which indicated desired and actual collective pitch and longitudinal cyclic positions throughout the entire manoeuvre; from engine failure to main gear touchdown. Although the guidance provided could be mastered with practice, in some informal testing by engineering pilots, it was found that the commanded desired collective and longitudinal cyclic inputs generated by the real-time expert controller were difficult to follow simultaneously and accurately. An alternative way to cue the pilots has therefore been investigated.

Nature-inspired Time-to-contact/Tau (τ) Theory provides a powerful explanation as to how guidance is achieved by observers in the natural world [6]. Ref [7] showed that τ-based guidance strategies were used by pilots in the flare manoeuvre for fixed-wing aircraft, for example.

To be able to follow any developed autorotation guidance laws, the pilot must be able to follow the demanded control inputs generated by them. To meet such a requirement, it is considered that the more 'naturally' the symbols move, the easier it will be for the pilot to follow them. If the dynamic phases of the autorotation manoeuvre can be described and modelled using Tau Theory, it could then be utilized to drive a set of suitable 'natural' pilot cueing algorithms. Tau theory is also relatively simple, mathematically speaking, and so should, therefore, be easily implementable in real-time [6].

Autorotation Manoeuvre

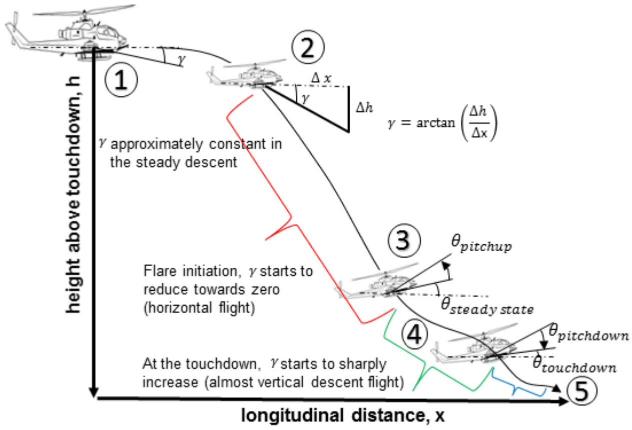

Figure 1. Straight-In Autorotation.

The Federal Aviation Administration (FAA) has issued autorotation manoeuvre guidance as summarized below [8, 9]. When an engine failure occurs, (position 1 in Figure 1), the pilot must firmly but promptly lower the collective pitch control to its fully down position to maintain main rotor speed. The pilot must also apply appropriate cyclic control

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inputs to achieve the manufacturer's recommended autorotation forward airspeed. Once a satisfactory steady-state descent condition has been achieved, position 2, the pilot must adjust the aircraft's attitude using cyclic control to maintain the manufacturer's recommended autorotation or best gliding speed. Rotor speed must be maintained by adjusting the collective pitch control. At approximately 200 to 150 feet above the landing surface, or at the altitude recommended by the manufacturer (position 3), the pilot should begin to reduce forward airspeed and decrease the rate of descent using aft cyclic by making the pitch-up motion. This is commonly known as the 'flare'. Care must be taken in the execution of the flare so that the cyclic control is not moved rearward so abruptly so as to cause the helicopter to climb, nor should it be moved so slowly as to not arrest the rate of descent, which may allow the helicopter to settle so rapidly that the tail rotor strikes the ground. When forward motion decreases to the desired ground speed, which is usually the slowest possible speed (position 4), the pilot should then move the cyclic control forward to reduce the pitch attitude of the helicopter during the landing. The altitude above the ground at this time should then be approximately 8 to 15 feet (depending on manufacturer's recommendations). The pilot should then allow the helicopter to descend vertically (position 5), increasing collective pitch, as necessary, to check the descent and cushion the landing.

Motion gaps during the autorotation manoeuvre

To perform a search for tau-guided motion during the autorotation manoeuvre, a number of questions first needed to be answered:

1. What so-called motion-gaps is the pilot controlling during the autorotation manoeuvre?
2. If there is scope for controlling more than one motion gap, what gap pairs would it be sensible to control simultaneously?
3. If the pilot is controlling multiple motion gap closures, is the pilot coupling the taus of those gaps?
4. If one spatial gap is being used, can this be modelled by coupling it with an intrinsic tau guide?

There are a number of potential longitudinal spatial gaps that can be closed during the autorotation manoeuvre. A sample of these are shown in Figure 1, namely, 1) the height (h) of the helicopter above the final landing spot; 2) the longitudinal distance (x) of the helicopter to go to the landing spot; 3) the change in helicopter pitch angle from θ_{flare} to $\theta_{touchdown}$ and 4) the flight path angle (γ) from start of the flare to touchdown as shown in Figure 1.

The paper is organised as follows. Section 2 provides an introduction to Tau theory. Section 3 outlines the flight simulation set-up and provides detailed tau analysis. Section 4 provides a guide on how the tau parameter can be used to develop autorotation cueing system. Finally, Section 5 ends the article with some concluding remarks.

2. PRELIMINARIES ON TAU THEORY AND THE TAU-GUIDE

Tau theory for motion control is, as the name suggests, based upon the fundamental optical invariant, time-to-contact (τ) and was originally conceived to be in the optical field. The proposition is founded on the principle that purposeful actions are accomplished by coupling the motion under the control of an observer with either externally or internally generated guidance sources: the so-called motion guides [10, 11]. For aircraft flight, in terms of visual guidance, it is posited that the pilot's overall strategy is to overlay or close the gap between the perceived optical flow field and the required flight trajectory [12]. The pilot then works directly with the available optical variables to achieve prospective control of the aircraft's future trajectory.

Time to contact, τ is defined as per Eq. (1):

$$\tau(t) = \frac{x(t)}{\dot{x}(t)} \qquad (1)$$

Here x is the motion gap to be closed, and $\dot{x}$ is the instantaneous gap closure rate. The term "motion gap" refers to a perceived difference between the observer's current and desired target states as shown in Figure 2.

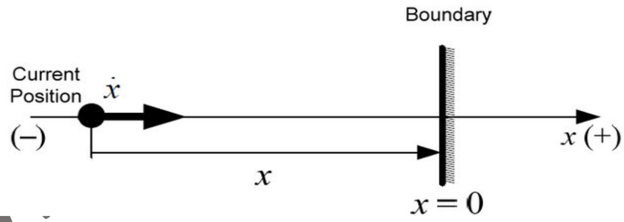

Figure 2. Kinematics of closing a perceived motion gap [12].

Previous research [7, 13] has shown that the rate of change of tau with time, $\dot{\tau}$, Eq. (2), is a useful variable to describe the motion of a vehicle.

$$\dot{\tau}_x = 1 - \frac{x\ddot{x}}{\dot{x}^2}; \quad \tau_{\dot{x}} = \frac{\ddot{x}}{\dot{x}} \qquad (2)$$

The hypothesis here is that the observer directly perceives the rate of change of τ the motion gap and uses this information as the basis for the necessary control inputs to achieve the desired motion. Maintaining $\dot{\tau}$ constant during a decelerating approach can be interpreted as keeping the τ's of the gap (τ_x) and the gap closure rate $(\tau_{\dot{x}})$ in a constant ratio [6]. Appendix A describes in more detail, the interpretation of the motion, based upon the corresponding $\dot{\tau}$ values.

Guidance of an observer's motion can also be achieved by using τ coupling: that is, keeping the tau of one optically available parameter in proportion with the tau of another variable. Tau coupling can take two forms: ***extrinsic*** (x and y are physically observable) or ***intrinsic*** (x is physically observable whereas g is posited to be generated by the observer's central nervous system). For extrinsic tau coupling, x and y are the externally perceived spatial variables (e.g., the height of the aircraft above ground and the distance to go to the desired touchdown point). The intrinsic tau guided motion occurs when movements are self-guided and there is no second extrinsic motion gap to couple onto, for example when playing the piano. In this case, there is a physical gap to close (between finger and piano keys), but the gap closure must be coupled to the rhythm of the tune being played, which is internally generated [10]. Under such circumstances, the motion gap is hypothesized to be coupled onto a so-called ***intrinsic motion guide***. The intrinsic τ guide is modelled using the relationship:

$$\tau_x = k\tau_g \tag{3}$$

The general intrinsic tau-guide (τ_G) model, for guiding the motion of an object that is approaching or receding from a destination and that starts at rest or starts with some initial velocity is given in Ref. [14] by:

$$\tau_G(t) = \frac{t(T+t)}{T+2t} \tag{4}$$

It has also been shown, in earlier work within Ref [10], [13], [15] that, for a ***constant acceleration guided*** (CAG) motion, the general intrinsic tau (τ_G) takes the form:

$$\tau_{g_{CAG}}(t) = \frac{k}{2}\left(t - \frac{T^2}{t}\right) \tag{5}$$

Here, t is the current time during the motion and T is the total duration of the motion ($0 < t \leq T$). Examples of motion that can be generated by varying the values of the coupling constant k in Eq. (5) are shown in Figure 3. For the detailed derivation of τ_g, see Lee and Padfield et al. [6, 11]. It can be observed that τ-coupled motion is only dependent upon the coupling parameter k and the total time of the manoeuvre, T. The dressing "$\hat{}$" indicates that the temporal variables are normalized by T, which is the duration of the manoeuvre, such that $-1 < \hat{t} \leq 0$.

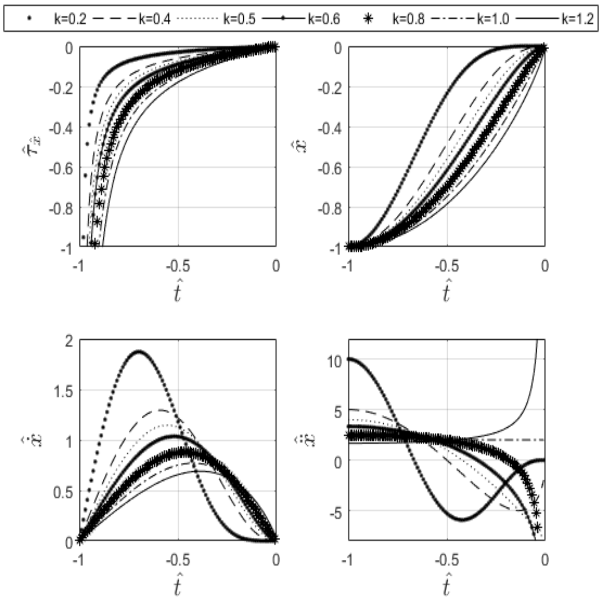

Figure 3. Motion τ, gap distance, closure rate and acceleration when following a constant acceleration guide such that $\tau_x = k\ \tau_{g_{CAG}}$.

The identified motion gaps are analysed in the tau-domain and compared with the shape of rate of change of tau ($\dot{\tau}$) and tau guide trajectories (τ_g). In order to get accurate k and T values, the Positive Wavelet Analysis (PWA) method of Ref [16] was used.

Using Positive Wavelet Analysis for Tau Coupling

The successful implementation of τ theory to flight control primarily relies on an accurate calculation or estimation of the value of the coupling parameter k and the manoeuvre time T. In early investigations, the parameter k was simply obtained through an optimization process, such as the classical linear least-square-error (LSE) method [2, 6]. This approach suffers from deficiencies. For instance, the LSE method has been found to be sensitive to the selected period T, over which the data are optimized. Its numerical stability is also vulnerable to the boundary conditions of the time period selected for the motion under investigation. In addition, when the original data are incomplete, or combined with oscillatory behaviour, the LSE method is not usually able to provide satisfactory results.

To address the above issues, PWA was used to estimate the coupling value k and the total manoeuvre time T For the data collected in this study. For more details on the use of PWA for tau coupling, the reader should consult Ref. [16]. Calculation of the Tau coupling parameters using PWA is performed using the following three steps:

1. Decompose the motion data into individual but possibly different guidance elements.
2. Perform a positive wavelet transformation on the motion gap.
3. Perform an approximate reconstruction of the motion gap.

3. Experimental Search for Tau-based Guidance Strategies for the autorotation Manoeuvre

In order to assess whether or not there is an identifiable autorotation strategy in the τ domain, a series of pilot-in-the-loop (PIL) autorotation manoeuvres were performed using the University of Liverpool's HELIFLIGHT-R full-motion simulation facility [17]. The FLIGHTLAB non-linear generic rotorcraft flight dynamics model (which is based upon the Black Hawk UH-60) was used in these simulated flight tests. During the simulation, the lateral states of the helicopter were frozen and the test pilot were asked to only focus on controlling the longitudinal states. The test pilot was an ex-Royal Navy rotary wing pilot, a graduate of the Empire Test Pilot School and a current Training Captain with a national flag-carrier airline. The pilot was asked to repeat the autorotation task 10 times. To try to ensure the repeatability of the experiment, the initial conditions were kept constant. Each test point was started from a trimmed straight-and-level flight condition at an altitude of 1000ft at 62 knots. Figure 4 shows the variation of the fundamental longitudinal states of the helicopter during each of the flight test points.

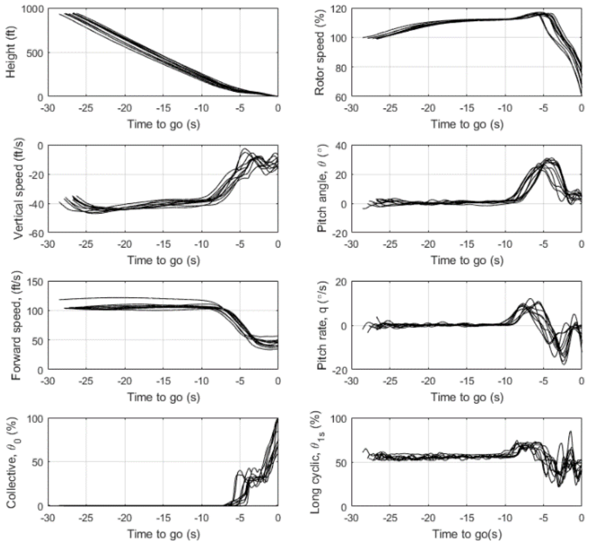

Figure 4. Longitudinal states during the autorotation manoeuvre using the FLIGHTLAB Generic Rotorcraft flight model.

The x-axes of the charts are labelled as "Time to go". This respects the convention in tau analyses whereby $t = 0$ indicates the time that the gap is closed (in this case, main gear touchdown). Therefore, 'negative' time indicates the remaining task time until touchdown.

Guidance Strategies in the Tau Domain for Height and Longitudinal Distance Gap Closure

A number of analyses were carried out to investigate how the tau of height and longitudinal distance changed over the duration of the autorotation manoeuvre. More specifically, the following simple hypothesis are tested:

$$\begin{aligned}\dot{\tau}_h &= c_h \\ \dot{\tau}_x &= c_x\end{aligned} \qquad (6)$$

Here, c_h and c_x are constants. See, Appendix A for the interpretation of their values. In order to understand whether or not the pilots used any tau-based strategies in the gap closure for height and longitudinal distance, the taus of the respective variables and their time derivatives are calculated as shown in Figure 5 and Figure 6.

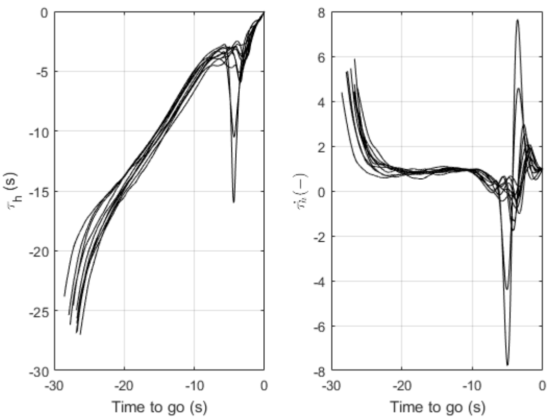

Figure 5. τ_h and $\dot{\tau}_h$ for the full manouevre.

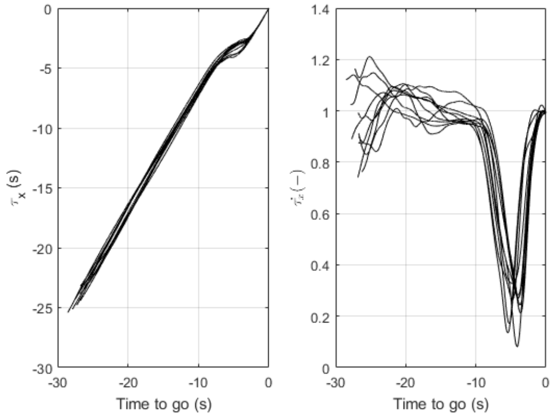

Figure 6. τ_x and $\dot{\tau}_x$ for the full manouevre.

It can be seen in Figure 5 and Figure 6 that, as soon the pilot establishes the steady-state descent condition (constant descent rate and forward velocity), the value of $\dot{\tau}_h$ and $\dot{\tau}_x$ approaches 1. This is consistent with a constant velocity motion. This happens because the pilot did not have any spatial gap to close, rather the pilot focused on maintaining constant decent rate and forward velocity.

The most dynamic part of the autorotation manoeuvre is the flare phase. It is because the helicopter is close to the ground and there are several spatial gaps to close quickly and simultaneously. Figure 7 shows the zoomed-in plots of the variation of $\dot{\tau}_h$ and $\dot{\tau}_x$ during this part of the manoeuvre.

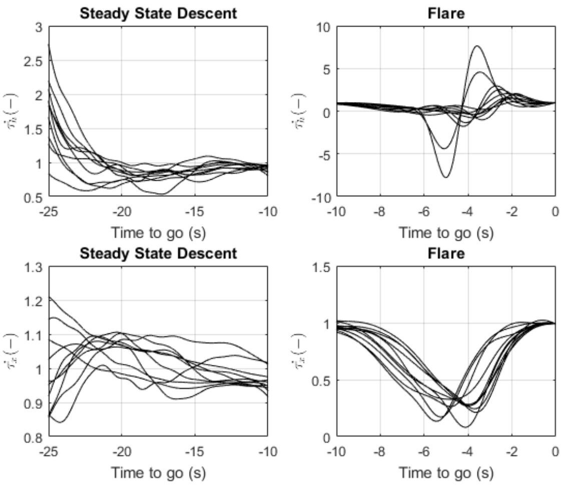

Figure 7. Zoomed in $\dot{\tau}_h$ and $\dot{\tau}_x$ during steady state and flare phase.

A constant $\dot{\tau}_h$ and $\dot{\tau}_h$ of approximately 1 is maintained before the flare is initiated. The $\dot{\tau}_x$ the profile is 'reasonably' consistent over the series of runs where $\dot{\tau}_x$ reduces from 1 to a value close to 0 with a constant rate of change between approximately 0.2 and 0.25/sec over a period of approximately 4-5 seconds. As the pilot levels the helicopter for landing, $\dot{\tau}_x$ returns to 0 over a period of approximately 3 seconds. $\dot{\tau}_x = 0$ constitutes an exponential decay of the trajectory [7]. During the flare rate of change of $\dot{\tau}_h$ is approximately 0.3/sec for the first 3 seconds after the flare is initiated. $\dot{\tau}_h$ then varies significantly from run to run. One possible reason for this is when the collective check input is applied. However, ideally $\dot{\tau}_h$ should be held constant at a low value below 0.5 in order to provide deceleration (see Appendix A for the interpretation of $\dot{\tau}$ values), until the helicopter begins to level off. Afterwards, $\dot{\tau_h}$ increases rapidly to 2 or more as when the longitudinal cyclic is pushed forward to level the aircraft, the descent rate increases. Finally $\dot{\tau_h}$ comes back to 1 by increasing collective to cushion the landing at approximately constant decent rate.

Guidance Strategies in the Tau Domain for Pitch Angle Gap Closure

During the steady-state descent, the pitch angle remains constant. Therefore, the pitch angle analysis in the tau domain focused on the flare phase of the manoeuvre. The tau of pitch angle (τ_θ) is calculated as:

$$\tau_\theta = \frac{\theta}{q} \qquad (7)$$

The tau of pitch angle (τ_θ) analysis is carried out in a piecewise manner, due to the zero crossing of the pitch rate (q) when the maximum pitch angle is achieved during any nose-up motion. τ_θ was analysed in two stages; first, the pitch-up gap closure and second, the pitch-down gap closure.

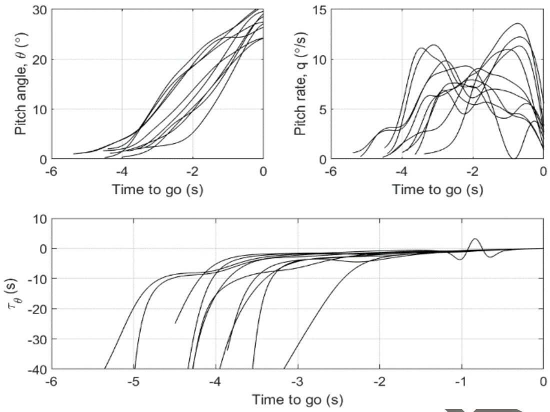

Figure 8. Tau of pitch angle during pitch-up motion.

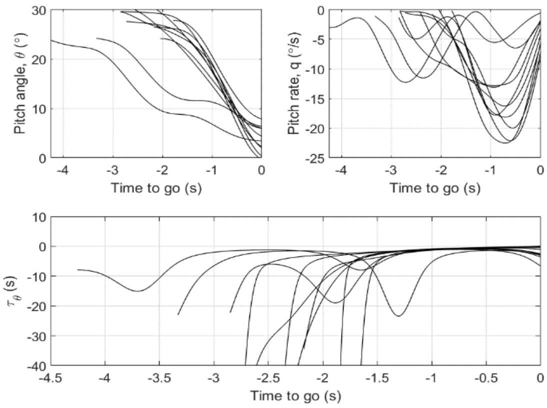

Figure 9. Tau of pitch angle during pitch down motion.

It can be seen from Figure 8 and Figure 9 that the form of the τ_θ motion closely resembles the constant acceleration guide (CAG) as presented in Figure 3. The oscillatory nature in the pitch rate during the flare phase is due to the pilot's attempt to stabilise the pitch angle. The oscillatory nature of τ_θ is due to the oscillatory pitch rate, q. Traditionally a classical least-square error (LSE) optimization algorithm would be used to calculate the coupling value k over a pre-chosen time period T. However this technique leads to inaccurate values of k and is highly dependant on the chosen time period T.

For a more accurate calculation of tau coupling terms (k and T) for pitch angle, PWA was used. As an example case, Figure 10 shows a result from a single run. Here the roughly chosen time period T_{chosen} for the pitch up motion is shown.

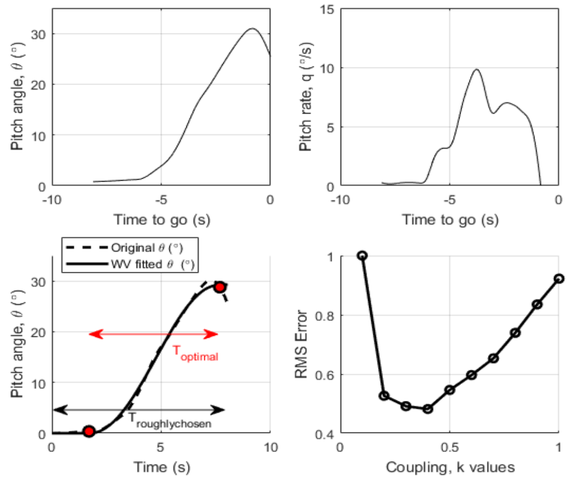

Figure 10. Positive Wavelet analysis of pitch angle gap closure.

It can be seen in the Figure 10 that the T_{chosen} is about 8 seconds. The PWA algorithm tries to fit the tau guide wavelet to the data using different k values. The algorithm outputs the optimal $k_{optimal}$ values and time period $T_{optimal}$ based on the lowest root-mean-square (RMS) value. For the example run case shown in Figure 10, the optimal value for the time period $T_{optimal}$ and coupling constant $k_{optimal}$ is 6.1 seconds and 0.4 . The coupling values for the $\tau_\theta - \tau_{CAG}$ for all the runs are given in Table 1 and Table 2.

Table 1. Coupling for a pitch up motion.

Run No.	Time period, $T_{optimal}$ (s)	Coup-ling, k	RMS Error	h at the start of pitch up (ft)	τ_h at the start of pitch up motion (s)
1	6.1	0.3	0.012	189	-4.48
2	6.1	0.4	0.013	221	-5.57
3	6.1	0.2	0.014	192	-4.92
4	4.1	0.9	0.010	174	-4.59
5	4.1	0.8	0.006	199	-4.80
6	4.1	0.7	0.003	192	-4.93
7	4.1	0.8	0.007	211	-5.41
8	5.1	0.3	0.009	196	-5.09
9	3.1	0.8	0.011	172	-4.56
10	4.1	0.7	0.010	207	-5.55

Table 2. Coupling for a pitch down motion.

Run No.	Time period, $T_{optimal}$ (s)	Coup ling, k	RMS Error	h , at the start of pitch down (ft)	τ_h at the start of pitch down motion (s)
1	2.1	0.7	0.005	33	-2.54
2	2.1	0.8	0.020	28	-1.44
3	2.1	0.9	0.026	41	-2.05
4	2.1	0.8	0.024	34	-1.89
5	1.1	0.7	0.019	17	-1.65
6	2.1	0.6	0.006	50	-2.18
7	2.1	0.4	0.026	49	-2.34
8	2.1	0.4	0.021	30	-2.38
9	1.1	0.6	0.030	22	-1.42
10	1.1	0.9	0.051	21	-1.23

From Table 1 it can be seen that pilot consistently starts the flare manoeuvre at approximately $\tau_h =$ -5 seconds. It is noticeable that when the time period is longer, the coupling value is smaller. This means that when the pilots have a longer time to complete the pitch-up manoeuvre, the peak deceleration occurs at the earlier portion of the manoeuvre. The complete interpretation of the significance of the coupling value is given in Appendix B.

In Table 2 it can be seen that pitch-down motion starts at approximately τ_h= –2.5 seconds (at an altitude of 30-40 feet) when the pilots try to achieve the appropriate pitch attitude for landing. The pitch-down motion in the flare takes place over a very short time (approximately from -5 seconds time to go to -1 second). Therefore the pilots spends a very short amount of manoeuvre time in this pitch-down motion phase, hence the coupling values are larger compared to the pitch-up motion. This means that the peak deceleration takes place towards the end of the pitch-down manoeuvre.

4. Use of Time-To-Contact Model To Develop Autorotation Cueing

One possible way to cue to pilot using the time-to-contact model for height and longitudinal gap closure is to select $\dot{\tau}_h$ and $\dot{\tau}_x$ profiles in such a way that the $\dot{\tau}_h$ and $\dot{\tau}_x$ ("ideal") profiles obtained from the PIL simulation matches as shown in Figure 11.

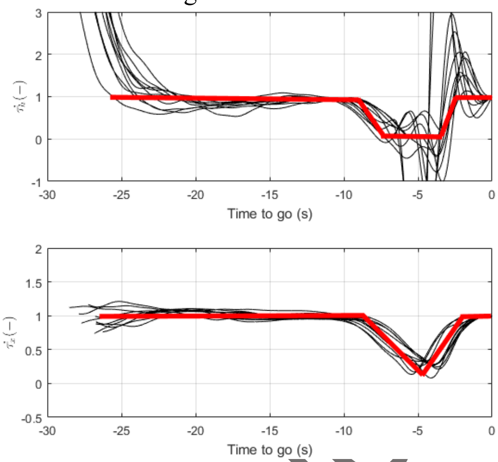

Figure 11. Ideal (in red) $\dot{\tau}_h$ and $\dot{\tau}_x$ strategies for autorotation.

Two control inputs, longitudinal cyclic and collective, are available for the pilot to control two $\dot{\tau}$-strategies. The proposed strategy can be mapped back to the control input strategy to determine how and when the controls inputs should be applied which can be used to cue the pilots as discussed in Ref [5]. The desired control input positions and actual control stick position can symbolically be presented to the pilots.

Pitch angle during the steady-state phase remains constant between ±5 degrees. Therefore, the desired constant pitch angle can be cued to the pilot in the steady-state decent phase. In the flare phase of the manoeuvre, the pilots can be cued with desired pitch angle ($\theta_{desired}$). $\theta_{desired}$ can be found from the τ_θ analysis. The τ_θ can be analysed as a pitch up and pitch down gap closure. The τ_θ closely resembles the constant acceleration intrinsic τ_{CAG} tau guide. Systematic analysis using the PWA can be done to calculate the tau coupling k and time period T. The desire pitch angle trajectory can be found using:

$$\hat{\theta}_{desired} = -\left(1 - \left(\frac{t}{T}\right)^2\right)^{1/k} \qquad (8)$$

Here $\hat{\theta}_{desired}$ is the normalized desired pitch angle. Figure 12 show an idealized pitch angle profile during the autorotation manoeuvre.

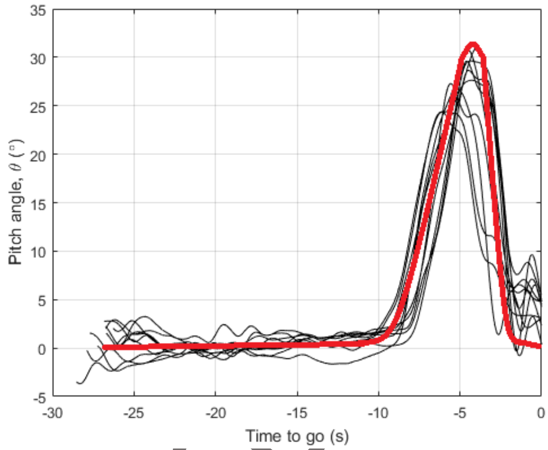

Figure 12. Ideal pitch angle trajectory (in red) derived from τ_θ analysis.

The $\theta_{desired}$ strategy can be mapped back to the control input strategy to determine how and when the control inputs should be applied which can be used to cue the pilots. Alternatively, pilots can be directly cued based on the desired and actual pitch angle position.

5. CONCLUSIONS

Tau analysis have been performed on the simulated flight-test data obtained during autorotation manoeuvre in order to find out that if tau can be used to develop pilot cueing systems. The analysis focused on the three longitudinal spatial variables, namely height, longitudinal distance and pitch angle. Tau theory requires that some form of spatial gap be closed. Hence, the reported tau analysis in this paper focused on the analysis of each phase of the flight (steady-state decent, flare and landing). To address the questions earlier, in an autorotation manoeuvre the pilot is controlling the height, longitudinal distance and pitch angle motion gaps. Further analysis is needed to identify if the pilot is coupling multiple motion gaps. Pitch angle gap closure can be modelled as intrinsic constant acceleration tau guide. It has been identified that there are two types of tau-based strategies that can be used for developing cueing systems:

1. Select $\dot{\tau}_h$ and $\dot{\tau}_x$ profiles in such a way that the "ideal" $\dot{\tau}_h$ and $\dot{\tau}_x$ profiles obtained from the PIL simulation matches.
2. Generate desired pitch angle $\theta_{desired}$ using the $\tau_\theta - \tau_{gCAG}$ coupling value k and manoeuvre time T obtained from the PWA method.

For the first type, the proposed strategy can be transformed into control input position, which can be employed within a cockpit display to drive display symbology to indicate desired collective pitch and longitudinal cyclic positions throughout the entire manoeuvre, from autorotation entry to touchdown.

For the second type, the $\theta_{desired}$ can be directly displayed to the pilot in the form a flight director. Alternatively, the difference between the actual pitch angle and $\theta_{desired}$ can be mapped to control input and the pilots can be cued with the control stick positions (similar to type 1).

For the development of the tau-based autorotation cueing system, the designer can choose any one of the strategies or a combination of these two strategies for each phase of the manoeuvre.

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APPENDIX

Appendix A

Table 3. Summary of $\dot{\tau}$ results.

Value of tau dot ($\dot{\tau}$)	Implied movement of the helicopter	Effect of keeping acceleration-deceleration constant	Effect of keeping tau dot constant
$\dot{\tau} > 1$	Accelerating	Collides ($\dot{\tau}$ decreases to 1)	Collides
$\dot{\tau} = 1$	Constant velocity	Collides ($\dot{\tau}$ constant)	Collides
$0.5 < \dot{\tau} < 1$	Decelerating	Collides ($\dot{\tau}$ increases to 1)	Controlled collision (braking increases)
$\dot{\tau} = 0.5$	Decelerating	Stop at	Stops at (braking constant)
$0 < \dot{\tau} < 0.5$	Decelerating	Stops short ($\dot{\tau}$ decreases)	Stops at (braking decreases)

Appendix B

Motions following a constant acceleration guide:

i. Motions begin with an abrupt acceleration, which then subsides, with maximum velocity occurring further into the manoeuvre as k is increased. When $k = 0 \cdot 4$, the maximum velocity occurs mid-way, in time, through the manoeuvre.

ii. When $k = 0 \cdot 5$, there is a finite deceleration at the end of the manoeuvre; when $k > 0 \cdot 5$, an infinite deceleration is required to close the gap, which, in practice, means a collision will occur.

iii. As the manoeuvre comes to a close $t \to 1$, $\widehat{\tau'} \to k$, asymptotic to the constant deceleration guided motion, noting that k (constant acceleration) is actually half of k (constant deceleration).

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