Information Leakage in a Music Score

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When I was invited to contribute to a Festschrift in honor of Catuscia Palamidessi, I immediately thought I should not miss the opportunity to express my friendly feelings and admiration for the research career of a good friend of 30 years.

Initially, we had some common research axes on Logic Programming and Concurrent Constraint Programming. The GULP conference on these topics in Italy was a high place where Catuscia had no equal to defend, between Pisa and Florence, what should be the true Italian pronunciation, but with too subtle arguments for a foreigner. In 1990, the ICLP conference in Jerusalem and Eilat was a memorable mark of Catuscia's passion for Concurent Logic Programming. In 2000, our small CSE Penn State-Inria workshop on "Concurrency and Logic" was premonitory of a new life for Catuscia at Inria. In the last decade however, our research interests evolved in different directions, respectively on quantitative information flows and quantitative biology.

When reading the recent papers of Catuscia on information leakage in information flows, in the quest of finding something to write related to those topics, a music unexpectedly came to my mind, with two concurrent voices playing obsessively on some jazzy rhythm, responding each other, and producing some bits of information about a secret message subject to interpretation, as always in music.

It is my pleasure to dedicate that little piece of music, called *"Concur in C"*, to Catuscia for her 60th birthday. The music score is given below[1].

But could we try to apply Catuscia's theory on information leakage in information flows to the notes of each voice, in the hope of revealing something more on the meaning of this mysterious musical message remained enigmatic to the composer?

In information flow theory, a *channel*, or a *voice* in our musical interpretation, is a triple (X, Y, C), where X and Y are finite sets (of secret input values and produced output music notes) and the channel matrix C is an $|X| \times |Y|$ matrix giving the probability of getting output y when the input is x. Given a prior distribution π on X, the joint distribution $p(x, y) = \pi[x]C[x, y]$ on random variables $X \times Y$ is the (unique) joint distribution that gives marginal probabilities $p(x) = \sum_y p(x, y) = \pi[x]$, conditional probabilities $p(y|x) = p(x, y)/p(x) = C[x, y]$ (if $p(x)$ is nonzero), and similarly $p(y)$ and $p(x|y)$.

The marginal distribution of the music notes in a voice are given by the frequencies of the notes in that voice. In *Concur in C*, the upper and lower voices happen to play the same number of notes, 186, with slightly different

[1] *"Concur in C"* can be heard with several orchestrations (piano, flute-saxophone, flute-organ-bass, big band) on `http://lifeware.inria.fr/~fages/music/`

distributions and slightly higher Shannon's entropy ($-\sum_{i=1}^{186} p_i(y).\log_2(p_i(y))$), i.e. more information, in the upper voice:

number of occurrences	C	C# Db	D	D# Eb	E	F	F# Gb	G	G# Ab	A	A# Bb	B	total
upper voice	32	0	0	54	2	14	20	40	0	0	22	2	186
$p^u(y)$	0.17	0	0	0.29	0.01	0.07	0.11	0.22	0	0	0.12	0.01	entropy 2.56
lower voice	41	0	0	36	0	22	27	39	0	0	21	0	186
$p^l(y)$	0.22	0	0	0.19	0	0.12	0.15	0.21	0	0	0.11	0	entropy 2.53

In music, it is expected that the upper voice contains more information than the lower voice usually dedicated to accompaniment. But here, the two voices play similar melodies in counterpoint which results in similar entropy values for both voices. It would be interesting to test this approach on more elaborate scores.

Thank you Catuscia for having once again inspired me, in an unexpected fashion, this self-surprising gift with curious questions about the quantification of information in music.

There has always been something magic in you.

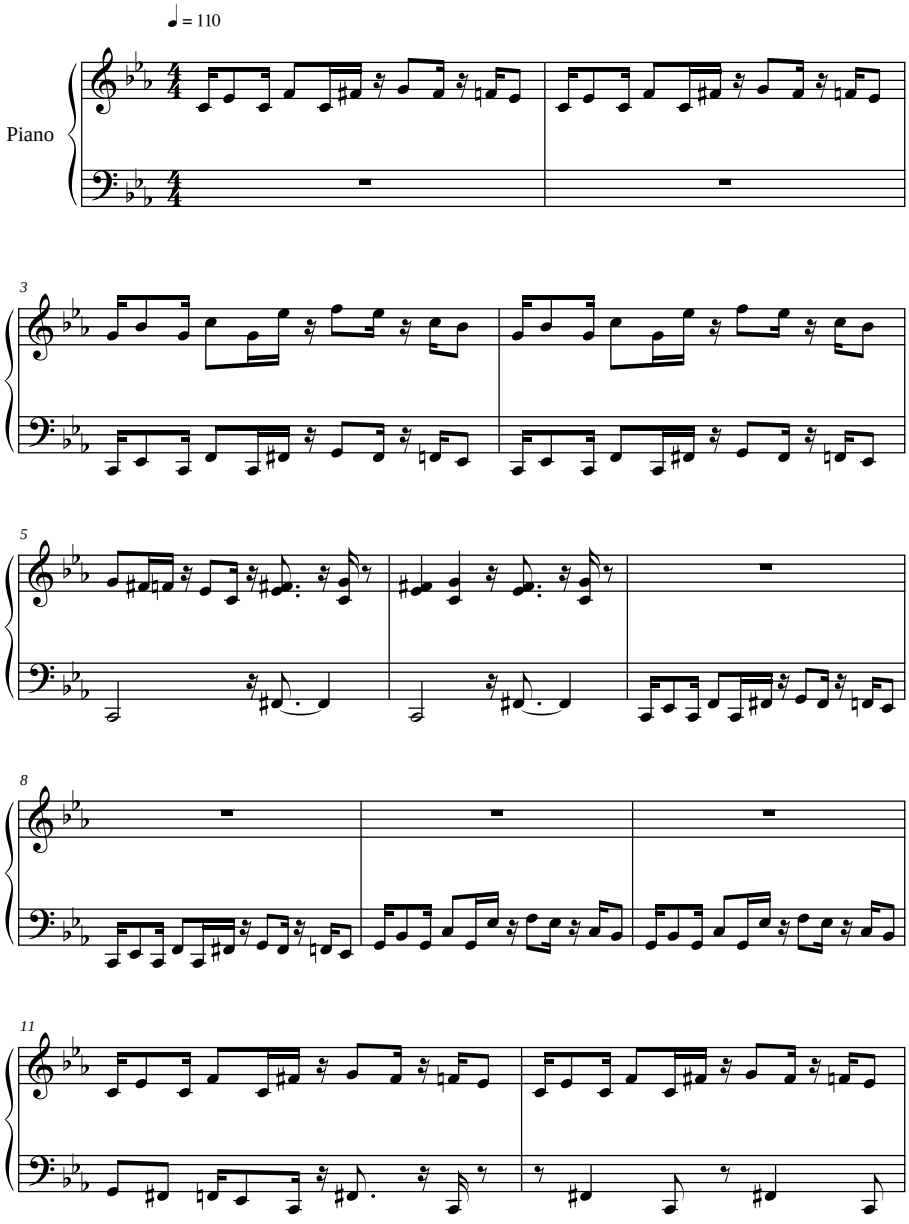
Happy 60th Birthday Catuscia!

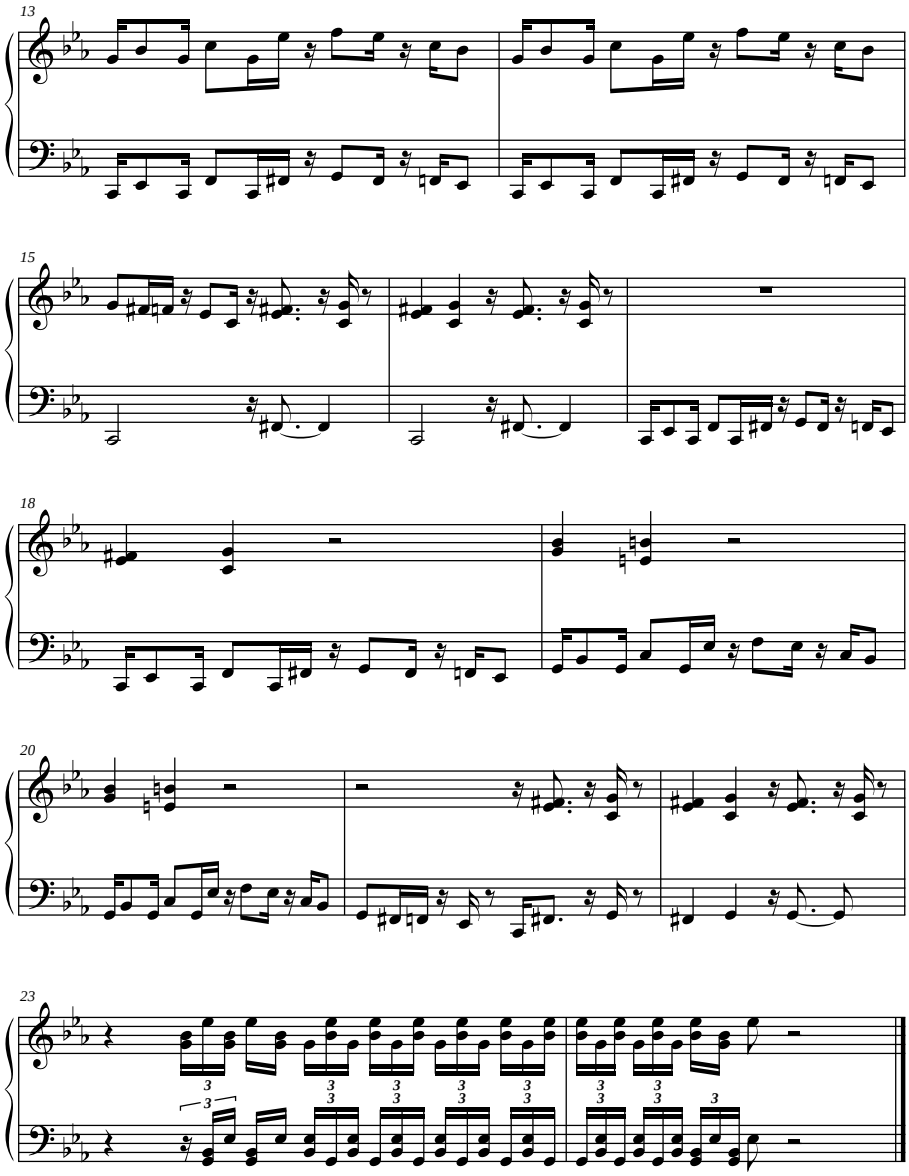
Acknowledgments: special thanks go to Hubert Garavel for having improved the layout of my music score, and for his tolerance regarding my parallel fifths.

Concur in C

To Catuscia Palamidessi for her 60th birthday

F. Fages

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