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To cite this version:
Ilias Petropoulos, Christelle Wervaecke, Didier Bailly, Thibaut Derweduwen. Numerical investigations of the exergy balance method for aerodynamic performance evaluation. AIAA AVIATION 2019, Jun 2019, DALLAS, United States. 10.2514/6.2019-2926. hal-02333523

HAL Id: hal-02333523
https://hal.archives-ouvertes.fr/hal-02333523
Submitted on 25 Oct 2019

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Numerical investigations of the exergy balance method for aerodynamic performance evaluation

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The exergy framework presents an interesting complement to traditional drag extraction methods for the performance evaluation of aerodynamic devices. On the one hand it remains valid for novel configurations for which no thrust/drag distinction is applicable (e.g. Boundary Layer Ingestion), while it can also account for the influence of thermal effects. At the same time, it can provide an additional understanding on the physical phenomena occurring within the flowfield and their interactions, via the destruction of exergy or its transfer from one form to another. The present work aims at providing a further insight into some numerical aspects of the exergy balance approach. A strategy towards the limitation of some of its sensitivities is also presented. This specifically addresses the quantification of spurious generation of anergy, which is directly associated to the spurious generation of entropy in numerical computations.

I. Nomenclature

\[\begin{align*}
p, T &= \text{static pressure and temperature} \\
\rho &= \text{density} \\
u, v, w &= \text{x-, y-, z-component of the velocity vector} \\
e &= \text{mass-specific internal energy} \\
h, s &= \text{mass-specific enthalpy and entropy} \\
\varepsilon &= \text{mass-specific flow exergy} \\
F_x &= \text{net streamwise force} \\
k_{\text{eff}} &= \text{effective thermal conductivity} \\
q_{\text{eff}} &= \text{effective heat flux by conduction on a heating surface} \\
n &= \text{normal vector} \\
\tau_{\text{eff}} &= \text{effective viscous stress tensor} \\
\Phi_{\text{eff}} &= \text{effective dissipation rate per unit volume} \\
(\bullet)_{\infty} &= \text{thermodynamic or kinematic quantity at the free-stream reference state} \\
S_o, S_w, S_p, S_b &= \text{integration surfaces} \\
V_o &= \text{integration volume} \quad (\partial V_o = S_o) \\
x_{\text{tp}} &= \text{downstream position of the Trefftz plane (limit of the control volume } S_o)\end{align*}\]

II. Introduction

Numerical methods have an ever-increasing role within the aerodynamic design process of modern aircraft. Due to the increasing complexity of these configurations, solver techniques have significantly evolved over the past decades and nowadays allow the accurate and efficient calculation of a very wide range of fluid flows. Within a research or industrial cycle however, the accurate and efficient evaluation of systems’ performance (efficiency, aerodynamic performance coefficients,...) is of an equally crucial importance.

This has lead to multiple families of numerical techniques being developed for the calculation of such performance measures. As regards aerodynamic drag, far-field methods in particular have been proven to be advantageous in terms of sensitivity to numerical error compared to a near-field evaluation. Moreover, such techniques permit the decomposition...
of drag into several components depending on the nature of drag sources [1, 2], which in turn can provide precious physical insight to aerodynamic designers.

Design restrictions themselves can arise from regulations, particularly for civil aircraft in terms of pollutant emissions or acoustic signature (e.g. Counter-Rotating Open Rotors). At present however, the design of civil aircraft is largely governed by aerodynamic efficiency. In the search of improved designs, several industrial trends have emerged. These range from being variations of existing concepts, such as ultra-high by-pass ratio (UHBR) dual-stream engines, or more unconventional concepts (Prandtl box-wing planes, distributed propulsion system, blended wing-body etc.).

Another promising concept is the so-called Boundary Layer Ingestion (BLI) [3, 4]. In traditional civil aircraft, engines are deliberately rather segregated from the fuselage to reduce as much as possible the impact of engine integration. BLI designs on the other hand include engines which are highly integrated in the fuselage. This aims at ingesting and directly re-energizing a part of the fuselage’s boundary layer, rather than using the engines to separately overpower the momentum defect in the wake. Due to this mechanism, BLI designs can in principle achieve a target net streamwise force with higher efficiency due to a reduced axial/transversal kinetic energy deposition to the free-stream flow.

BLI configurations however entail additional complexities due to the engine integration. The design of the engines themselves is particularly complex as, apart from aerodynamic performance, fan blades must be able to structurally withstand and perform under significant distortion. Furthermore, the aerodynamic performance evaluation of such aircraft is not straightforward because no distinction can be made between thrust and drag, thus making far-field drag extraction methods less interesting on such configurations.

The performance of such novel concepts can be evaluated using more "global" measures, such as done in the power balance method [5]. Another approach for the evaluation of such novel configurations has been developed in ONERA based on the notion of exergy [6–8]. As opposed to the separation of energy into mechanical and thermal (partially convertible to work), the notion of exergy is used to refer to the theoretical work potential of both mechanical and thermal form. This will be discussed in further detail in Section III. It should also be noted that although it may have been largely motivated by it, the exergy analysis is not restricted to the concept of BLI. Furthermore, it is related to the power balance in terms of mechanical energy, but differs in that it can also account for and quantify the influence of thermal effects. It can thus be particularly interesting for the evaluation of aero-thermo-propulsive configurations, such as heat exchanger devices integrated in propulsion systems.

Following the development of a prototype method, this approach has since been applied to a number of academic or industrial configurations [6–8, 10]. A novel implementation of the exergy balance method has been developed to achieve the efficiency required for industrial production and facilitate the development of new functionalities. The present paper itself aims at demonstrating the capabilities of the exergy analysis and at investigating its eventual sensitivities. More specifically, the main focus is put on the identification and quantification of numerical effects, which is essential for the method’s industrialization.

### III. Exergy analysis

The notion of exergy is used to refer to the work potential of a system with respect to its difference from the reference conditions of a heat reservoir. In other words, it represents a measure of the theoretical potential of work which can be extracted from the system by returning it to the reference conditions.

Exergy can naturally appear in many forms (magnetic, chemical, physical,...), similarly to energy. Without loss of generality, the focus is primarily put on physical and chemical exergy for aerodynamics applications. By neglecting gravitational potential exergy and also considering a perfect gas hypothesis (thus neglecting chemical exergy), the definition of exergy can be written:

\[
\varepsilon = h_i - h_{i,\infty} - T_{\infty} (s - s_{\infty})
\]

(1)

where \(s\) denotes entropy and \(h_i\) denotes specific total enthalpy (\(h_i = h + \frac{1}{2}V^2\)). For external aerodynamics applications, the reference conditions are usually taken as the atmosphere free-stream flow.

### A. Formulation of the exergy balance

The method aims at evaluating the time-averaged changes in exergy. It is thus oriented towards the post-processing of Reynolds-averaged Navier-Stokes (RANS) solutions. Based on the momentum relation, mass conservation and thermodynamic relations, a general exergy balance equation is obtained [6, 7]:

\[
\dot{\varepsilon}_{prop} + \dot{\varepsilon}_q = \dot{W} + \dot{\varepsilon}_m + \dot{\varepsilon}_{th} + \dot{\mathcal{A}}_\theta + \dot{\mathcal{A}}_{\varphi T} + \dot{\mathcal{A}}_\omega
\]

(2)
The balance of Eq. (2) accounts for aerodynamic, propulsion system, as well as thermal phenomena occurring within a control volume. It can thus be used for the performance evaluation of aero-thermo-propulsive systems through exergy (i.e. work potential) loss or transformation from one form to another.

The term $W_{\Gamma}$ is written:

$$W_{\Gamma} = V_{\infty} \int_{S_o} \left[ \rho u (V \cdot n) + (p - p_{\infty}) n_x \right] dS = -F_x V_{\infty}$$

where $\Gamma$ is the weight specific aircraft energy height, i.e. the sum of its potential and kinetic energy. This term is obtained from the momentum relation and is equal to the power (i.e. the rate of work) of the resultant net streamwise force, calculated in the far-field. As such, it should be underlined that the above expression provides no distinction between thrust and drag. More precisely, this term represents the potential utilized by the aircraft to maintain a steady path.

The two terms $\dot{E}_{prop} + \dot{E}_q$ on the left-hand side of Eq. (2) represent the inflow of exergy to the system through propulsive, non-adiabatic or permeable surfaces. $\dot{E}_{prop}$ represents the flow of exergy via the propulsion system:

$$\dot{E}_{prop} = -\int_{S_p} \rho \delta h (V \cdot n) \ dS + T_{\infty} \int_{S_p} \rho \delta s (V \cdot n) \ dS$$

where $\delta (\bullet) = (\bullet) - (\bullet)_{\infty}$ and $S_p$ is the surface enclosing the propulsion system. The term $\dot{E}_q$ represents the flow of exergy via thermal conduction:

$$\dot{E}_q = -\int_{S_b} (q_{\text{eff}} \cdot n) \ dS + \int_{S_b} \frac{T_{\infty}}{T_w} (q_{\text{eff}} \cdot n) \ dS$$

where $T_w$ denotes the local temperature on the body walls and $q_{\text{eff}}$ denotes the effective heat flux by conduction ($q_{\text{eff}} = -k_{\text{eff}} \nabla T$).

The right-hand side terms of Eq. (2) represent the decomposition of exergy into different forms within the control surface $S_o$. The term $\dot{E}_m$ represents the rate of mechanical exergy outflow:

$$\dot{E}_m = \int_{S_o} \frac{1}{2} \rho u^2 (V \cdot n) \ dS + \int_{S_o} \frac{1}{2} \rho (v^2 + w^2) (V \cdot n) \ dS + \int_{S_o} (p - p_{\infty}) [(V - V_{\infty}) \cdot n] \ dS$$

The two first integrals in Eq. (6) are respectively the streamwise and transversal kinetic exergy deposition rate. The first ($\dot{E}_u$) is related to jets or the wake, while the second ($\dot{E}_{vw}$) is related to transversal kinetic energy such as the lift-induced vortices associated with induced drag. The last term ($\dot{E}_p$) is a boundary pressure-work rate related to pressure and velocity differences on the surface $S_o$ with respect to the reference state.
The rate of thermal exergy outflow \( \dot{E}_{th} \) is given by:
\[
\dot{E}_{th} = \int_{S_o} \rho \delta e (\mathbf{V} \cdot \mathbf{n}) \, dS + \int_{S_o} \rho_{\infty} (\mathbf{V} \cdot \mathbf{n}) \, dS - T_{\infty} \int_{S_o} \rho \delta s (\mathbf{V} \cdot \mathbf{n}) \, dS
\]
and is decomposed in three parts respectively representing the rate of thermal energy outflow, the rate of isobaric surroundings work and the outflow rate of anergy (def. below).

The last three terms in Eq. (2) represent the generation of anergy within the control volume. This term is used to refer to the loss of work potential through irreversible processes. In the exergy framework, anergy mechanisms are often also referred to as mechanisms of loss or destruction of exergy.

The rate of anergy generation by viscous dissipation is given by:
\[
\dot{A}_\Phi = \int_{V_o} \frac{T_{\infty}}{T} \Phi_{\text{eff}} \, dV
\]
where \( \Phi_{\text{eff}} = (\bar{T}_{\text{eff}} \cdot \nabla) \cdot \mathbf{V} \) is the effective dissipation rate per unit volume. The viscous mechanisms contained in \( A_\Phi \) transform differences in kinetic energy to thermal energy, thus leading the system towards a mechanical equilibrium with homogeneous velocity/pressure field.

The term \( A_{VT} \) is the rate of anergy generation by thermal conduction:
\[
A_{VT} = \int_{V_o} \frac{T_{\infty}}{T^2} k_{\text{eff}} (\nabla T)^2 \, dV
\]
and represents the loss of work potential through thermal diffusion due to temperature differences. Finally, \( A_w \) is the rate of anergy generation by shockwaves:
\[
A_w = T_{\infty} \int_{\partial V_w} \rho \delta s (\mathbf{V} \cdot \mathbf{n}) \, dS
\]
where the normal \( \mathbf{n} \) is shown in Fig. 1. The anergy terms of Eqs. (8)-(10) are positive and can be collectively referred to as the total rate of anergy generation \( A_{tot} = A_\Phi + A_{VT} + A_w \).

**B. Physical Interpretation**

Eq. (2) represents the balance within the control volume between: the inflow of exergy into the system on the left-hand side and its eventual transformation into work, its deposition to the free-stream flow or the loss of work potential on the right-hand side. In a powered configuration, the inflow of exergy (left-hand side of Eq. (2)) is provided to produce a net streamwise thrust (i.e. \( F_x \)). This exergy is however always supplied in excess, as part of it is transferred to the free-stream flow in the form of mechanical (\( \dot{E}_m \)) or thermal (\( \dot{E}_{th} \)) exergy, whereas another part of this work potential is lost through irreversible processes (\( A_{VT}, A_\Phi, A_w \)). It should be noted that, even though the mechanical and thermal exergy outflow terms (i.e. \( \dot{E}_m, \dot{E}_{th} \)) do not represent exergy losses, they still represent work which is not given for the required streamwise force.

A good design would thus achieve a target streamwise force with a minimum kinetic/thermal exergy deposition to the free-stream and a minimum loss of work potential. Equivalently, this means that a good design requires a minimum inflow of exergy (\( \dot{E}_{prop}, \dot{E}_q \)) to achieve the target streamwise force.

Unlike traditional aerodynamic force extraction methods, the exergy decomposition is by definition dependent on the volume \( V_o \) (i.e. on \( \partial V_o = S_o \)). Contrary to aerodynamic forces, exergy changes from one form to another in the proximity of aerodynamic bodies and further downstream. This is because thermal mixing and viscous mechanisms lead to the gradual (physical and numerical) dissipation of exergy as the system tends towards equilibrium with the reference state. As shown in Fig. 1, the control volume is limited by a downstream position of the Trefftz plane. A variation of this downstream limit can give an insight into flow physics via the loss of exergy, or its transfer from one form to another downstream of the aerodynamic body. This mechanism is illustrated in Fig. 2 as a downstream decrease of mechanical/thermal exergy components and an increase of anergy terms.

For practical purposes, the terms of Eq. (2) are nondimensionalized by:
\[
(\bullet) = (\bullet) / \left( \frac{1}{2} \rho_{\infty} V_{\infty}^3 A_{\text{ref}} \right)
\]
where \( A_{\text{ref}} \) denotes a reference surface. The non-dimensional values of the exergy terms obtained via Eq. (11) are thus given in power counts (p.c.), in analogy to traditional drag counts.
C. Post-processing method

As already discussed in Section II, post-processing codes are required to keep up with the rapid evolutions of CFD solvers in terms of flexibility and computational efficiency in modern architectures. To attain this objective, the implementation of the exergy balance method presented in Section III.A is coupled with Cassiopée
tm, a partly open-source library of functions developed at ONERA for the pre- and post-processing of CFD solutions [11].

The exergy analysis is contained in a separate post-processing module called FFX (Far-Field Exergy), developed at ONERA. CFD solutions including computational mesh and connectivity information are based on the CGNS standard (CFD General Notation System) [12]. The interface is implemented in Python, offering significant flexibility (scripting, co-processing, coupling with other modules,...). Internal computations are handled in Python/C++, allowing the combination of flexibility and computational performance. The current version of the software allows the exergy post-processing of CFD solutions on structured or unstructured grids, with the flow solution being located either at cell centers or at vertices. It can also handle several treatments and boundary conditions relative to aeronautic applications such as Actuator Disk models, Fan/OGV modelization with Body-Force terms, heat exchanger boundary conditions etc. Some results of applications of FFX to industrial cases can be found in Refs. [9,10,13]. For the convergence sensitivity study presented in Section IV.A the computation of the exergy decomposition with FFX was performed by memory coupling with the solver and on-the-fly post-processing during the CFD computation.

The presented results of a far-field decomposition of the drag force were performed with the FFD (Far-Field Drag) software developed at ONERA. This is based on an implementation of the far-field drag decomposition method of ONERA [1,2] on a similar software architecture with FFX.

IV. Numerical applications

In the following, numerical investigations of the exergy balance approach are performed by application to a series of aerodynamics computations. These include sensitivity analyses (convergence of the computation, mesh refinement), as well as investigations aiming at further increasing the precision of the formulation (definition of integration volumes using physical criteria). The RANS solutions presented below were computed with the ONERA-Airbus-SAFRAN elsA CFD solver [14].

A. Sensitivity to convergence level and grid density

The present section presents a series of sensitivity analyses of the exergy balance method with respect to the precision of the RANS CFD solution. This is regarded from two separate aspects: the level of convergence and the grid density.
The first case is a NACA0012 airfoil in an inviscid transonic flow \((M_\infty = 0.8, \alpha = 0^\circ, p_\infty = 101325 \text{ Pa}, T_\infty = 300 \text{ K})\). Computations were performed with the Jameson spatial discretization scheme \((k_1 = 1/2, k_2 = 1/64)\). The case is solved on a series of high-quality O-type analytical Euler grids composed of \(128 \times 128, 256 \times 256\) and \(1024 \times 1024\) cells (denoted as \(n_i \times n_j\)), originating from the study presented in Ref. [15]. The far-field extent is at approximately 150 \(c\), where \(c\) is the airfoil chord (see Fig. 3). It is reminded that for solutions of the Euler equations, viscous and heat transfer terms \((\dot{E}_h, \dot{A}_{VT}, \dot{A}_d)\) are zero. In this case, the resultant net streamwise force acting on the airfoil is equal to drag since no thrust is provided on non-powered configurations.

Fig. 3 presents the evolution of the exergy terms with respect to the level of convergence of the residual of the mass-conservation equation for the inviscid NACA0012 case at \(M_\infty = 0.8\) on different grids. The downstream limit of the control surface \(S_\alpha\) was taken at \(x_\alpha = 2.0\). Errors due to numerical approximation, especially on coarse meshes, can lead to the exergy balance of Eq. (2) not being exactly verified on the discrete level. Since each term is calculated directly and independently from the others, the difference between \(W^T\) and the sum of the other terms can also be regarded as a measure of accuracy. Furthermore, the mass-conservation residual is a quite representative measure of convergence, as conservation in the flow field can be more important for terms calculated in the far-field than near-field ones. It is shown that \(W^T\) is more sensitive than the other terms at early stages of convergence. The convergence of all terms towards the values of the converged solution is almost linear with respect to the reduction of the mass-conservation residual on this case. The overall accuracy of the numerical method is however satisfactory, the value of \(W^T\) being within below 1% of error with respect to the reference values after a residual reduction of approximately five orders of magnitude on the two coarser grids and three orders of magnitude on the fine grid. As for mechanical exergy and wave anergy terms, a reduction by three to four orders of magnitude has
Table 1  Far-field drag and exergy decomposition for the inviscid NACA0012 case at $x_{tp} = 2.0$. Drag and exergy terms are presented in drag and power counts respectively ($\times 10^{-4}$).

<table>
<thead>
<tr>
<th>Grid</th>
<th>$n_i$</th>
<th>$C_{D,nf}$</th>
<th>$C_{D,ff}$</th>
<th>$C_{D,sp}$</th>
<th>$W\Gamma$</th>
<th>$\hat{E}_u$</th>
<th>$\hat{E}_{vv}$</th>
<th>$\hat{E}_p$</th>
<th>$\hat{E}_{th}$</th>
<th>$\tilde{A}_w$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\infty} = 0.4$</td>
<td>$n_i = 128$</td>
<td>5.93</td>
<td>0.00</td>
<td>5.93</td>
<td>-6.01</td>
<td>0.65</td>
<td>0.50</td>
<td>-1.24</td>
<td>0.10</td>
<td>0.00</td>
<td>6.00</td>
</tr>
<tr>
<td></td>
<td>$n_i = 256$</td>
<td>1.49</td>
<td>0.00</td>
<td>1.49</td>
<td>-1.50</td>
<td>0.51</td>
<td>-1.21</td>
<td>0.09</td>
<td>0.00</td>
<td>1.50</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$n_i = 1024$</td>
<td>0.09</td>
<td>0.00</td>
<td>0.09</td>
<td>-0.09</td>
<td>0.60</td>
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<td>-1.20</td>
<td>0.10</td>
<td>0.00</td>
<td>0.09</td>
</tr>
<tr>
<td>$M_{\infty} = 0.8$</td>
<td>$n_i = 128$</td>
<td>92.84</td>
<td>84.74</td>
<td>8.09</td>
<td>-92.96</td>
<td>5.30</td>
<td>1.29</td>
<td>-8.22</td>
<td>2.33</td>
<td>82.37</td>
<td>9.89</td>
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<tr>
<td></td>
<td>$n_i = 256$</td>
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<td>83.75</td>
<td>1.85</td>
<td>-85.61</td>
<td>5.07</td>
<td>1.31</td>
<td>-8.13</td>
<td>2.30</td>
<td>82.97</td>
<td>2.08</td>
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<td></td>
<td>$n_i = 1024$</td>
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<td>83.55</td>
<td>-0.02</td>
<td>-83.53</td>
<td>5.03</td>
<td>1.31</td>
<td>-8.12</td>
<td>2.30</td>
<td>83.83</td>
<td>-0.82</td>
</tr>
</tbody>
</table>

Table 2  Far-field drag and exergy decomposition for the viscous NACA0012 case at $x_{tp} = 2.0$. Drag and exergy terms are presented in drag and power counts respectively ($\times 10^{-4}$).

<table>
<thead>
<tr>
<th>Grid</th>
<th>$n_i$</th>
<th>$C_{D,nf}$</th>
<th>$C_{D,w}$</th>
<th>$C_{D,ff}$</th>
<th>$C_{D,sp}$</th>
<th>$W\Gamma$</th>
<th>$\hat{E}_m$</th>
<th>$\hat{E}_{th}$</th>
<th>$\tilde{A}_\phi$</th>
<th>$\tilde{A}_{VT}$</th>
<th>$\tilde{A}_w$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{\infty} = 0.3$</td>
<td>256</td>
<td>91.17</td>
<td>0.00</td>
<td>90.84</td>
<td>0.32</td>
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<td>4.67</td>
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<tr>
<td></td>
<td>512</td>
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<td>90.86</td>
<td>0.03</td>
<td>-90.88</td>
<td>4.25</td>
<td>0.17</td>
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<td>1.06</td>
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<td>0.81</td>
</tr>
<tr>
<td></td>
<td>1024</td>
<td>90.73</td>
<td>0.00</td>
<td>90.74</td>
<td>0.00</td>
<td>-90.72</td>
<td>4.11</td>
<td>0.17</td>
<td>85.02</td>
<td>1.07</td>
<td>0.00</td>
<td>0.36</td>
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<td>$M_{\infty} = 0.8$</td>
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<td>59.36</td>
<td>160.12</td>
<td>2.94</td>
<td>-162.70</td>
<td>3.52</td>
<td>3.26</td>
<td>79.93</td>
<td>9.30</td>
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<td></td>
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<td>157.92</td>
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<td>3.02</td>
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<td>157.32</td>
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<td>2.89</td>
<td>80.84</td>
<td>9.52</td>
<td>60.69</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Table 2 shows values of exergy balance terms on the converged computations. The rate of energy generation due to shockwaves is close to the value of the far-field wave drag component $C_{D,w}$. The values of these two terms can be...
Fig. 5  Convergence of the exergy balance with respect to the reduction of the residual of the mass-conservation equation for the viscous NACA0012 case at $M_\infty = 0.3$ (top) and at $M_\infty = 0.8$ (bottom) on different grids.

comparable under an appropriate nondimensionalization, even though the terms themselves have a different physical content. This however cannot be straightforwardly generalized to any regime, e.g. to high-Mach-number flows. As regards the convergence of drag, a far-field drag computation is again shown to be superior to a near-field evaluation. The accuracy of the exergy balance (i.e. the discrete exergy balance residual) converges with grid refinement. Moreover, the difference between the residual of the exergy balance compared to spurious drag is higher than for the inviscid case.

Fig. 6 presents the evolution of some terms of the exergy balance for variation of the downstream limit of the control volume. These variations show little sensitivity to mesh density on this case. As already discussed in Section III.B, such graphs allow to monitor the evolution of exergy terms downstream of the aerodynamic body. More specifically, they portray the reduction of kinetic/thermal exergy in the wake and the increase of rate of anergy generation due to irreversible processes (viscous dissipation, thermal conduction). More localized physical insight can be obtained by visualizations of the volumic field of $\mathcal{A}_R$, $\mathcal{A}_T$ (see Fig. 6). These can be used to identify flow regions where losses of exergy (i.e. work potential) are dominant, either due to numerical discretization error or due to physical phenomena. In the NACA0012 airfoil case, loss of exergy is concentrated in the shockwave, boundary layer and the airfoil wake. Spurious production of anergy is also observed near the airfoil leading edge (cf. discussion of Section IV.B).

As a following step, the wing-body configuration of the Common Research Model (CRM) is considered. This is in order to evaluate the extensibility of the observations on airfoil cases to a more complex and realistic three-dimensional aeronautic configuration. Exergy analyses have already been presented on this configuration during the development of the formulation presented in Section III.A [6,8]. The original version of the grids corresponds to the multi-block unified baseline grids of the 5th AIAA CFD Drag-Prediction Workshop [17]. The current computations were performed on a series of grids adapted to the experimentally measured wing twist at the design point [18]. The number of cells for the $L^2/L^3/L^4$ grid levels is $2,156,544 / 5,111,808 / 17,252,352$ respectively ($Y^+ = 1.33/1.00/0.67$). The conditions at the aerodynamic design point correspond to Mach number $M_\infty = 0.85$, $C_L = 0.5$ and $Re = 5 \times 10^6$. The aerodynamic force coefficients for the three grid levels are presented in Table 5.

8
Fig. 6  Left: Evolution of exergy outflow and anergy generation terms for variation of the downstream limit $x_{tp}$. Right: field of rate of anergy generation due to viscous dissipation and thermal conduction. NACA0012 solution on the 1024 × 256 grid at $M_{\infty} = 0.8$.

<table>
<thead>
<tr>
<th>Grid level</th>
<th>Near-Field Drag</th>
<th>Far-Field Drag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_L$</td>
<td>$C_M$</td>
</tr>
<tr>
<td>L2'</td>
<td>0.4997</td>
<td>-0.0868</td>
</tr>
<tr>
<td>L3'</td>
<td>0.4999</td>
<td>-0.0884</td>
</tr>
<tr>
<td>L4'</td>
<td>0.4995</td>
<td>-0.0896</td>
</tr>
</tbody>
</table>

Table 3  Far-field drag decomposition for the CRM configuration. Drag coefficients are presented in drag counts (×10$^{-4}$).

Fig. 7  Convergence of near-field drag coefficients and exergy balance with respect to the reduction of the residual of the mass-conservation equation for the CRM case on the L3' grid.

The convergence of near-field drag components (pressure and friction drag) and the exergy decomposition is shown in Fig. 7 for the L3' grid. The downstream limit of the control volume has been taken at $x_{tp} = 90$, downstream of the fuselage trailing edge. The value of $W^\Gamma$ is more sensitive at the early stages of convergence. The error with respect to the converged value of $W^\Gamma$ is less than 1% after a reduction of the mass-conservation equation residual by five orders of magnitude. On the other hand, a reduction by three orders of magnitude is sufficient to obtain the same accuracy for the near-field drag components and the other terms of the exergy decomposition. In terms of converged absolute values, $W^\Gamma$ is within 0.2 counts of the near-field drag coefficient. The convergence of the friction drag component is shown to be faster than that of pressure drag. Still, the difference between $W^\Gamma$ against the exergy deposition and anergy...
Table 4  Exergy decomposition on different grid levels for the CRM case at $x_{tp} = 90$.

<table>
<thead>
<tr>
<th>Grid level</th>
<th>$W \dot{T}$</th>
<th>$\dot{E}_u$</th>
<th>$\dot{E}_{vw}$</th>
<th>$\dot{E}_{th}$</th>
<th>$\dot{A}_w$</th>
<th>$\dot{A}_\Phi$</th>
<th>$\dot{A}_{\nabla T}$</th>
<th>$\dot{E}_{\text{tot}}$</th>
<th>$\dot{A}_{\text{tot}}$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2’</td>
<td>-257.97</td>
<td>6.40</td>
<td>83.43</td>
<td>-1.98</td>
<td>1.84</td>
<td>4.37</td>
<td>122.72</td>
<td>11.96</td>
<td>87.86</td>
<td>139.05</td>
</tr>
<tr>
<td>L3’</td>
<td>-256.18</td>
<td>6.25</td>
<td>84.82</td>
<td>-1.95</td>
<td>1.82</td>
<td>4.73</td>
<td>125.86</td>
<td>12.51</td>
<td>89.13</td>
<td>143.10</td>
</tr>
<tr>
<td>L4’</td>
<td>-254.63</td>
<td>6.42</td>
<td>86.34</td>
<td>-1.99</td>
<td>1.86</td>
<td>5.22</td>
<td>128.89</td>
<td>13.03</td>
<td>90.77</td>
<td>147.14</td>
</tr>
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</table>

Table 4 shows the exergy decomposition for the CRM configuration on the three grid levels. The residual in the exergy balance equation is important, but is reduced as the grid is refined. This is not associated to the accuracy of a particular term but a distributed contribution of $W \dot{T}$, the collective mechanical exergy deposition rate ($\dot{A}_{\text{tot}}$) and total anergy generation ($\dot{A}_{\text{tot}}$). For mechanical exergy, grid convergence is most likely tied to the reduction of numerical dissipation of the wing tip vortex on the finer grids (i.e. the term $\dot{E}_{vw}$). As regards the rate of anergy generation, grid convergence is most likely tied to the more accurate calculation of primitive variable gradients (viscous dissipation, thermal conduction, shockwave). On all three grid levels, the absolute value of $W \dot{T}$ is within 1 count from that of the near-field drag coefficient.

As in the airfoil case, exergy analysis can be used to identify flow regions where loss of exergy (i.e. loss of work potential) is dominant either due to viscous dissipation or thermal mixing (see Fig. 8). On the same figure, a downstream extension of the control volume can portray the reduction of mechanical exergy components and increase of anergy generation due to irreversible processes within the volume.

B. Quantification of spurious anergy generation

The study of Section IV.A has demonstrated that numerical errors (inadequate convergence, numerical discretization) can lead to the exergy balance not being verified on the discrete level. The error in the verification of the exergy balance can be partly attributed to non-physical entropy generation. The objective of the present study is the detection and quantification of this spurious entropy, with the aim of limiting the dependence of an accurate exergy analysis to the mesh quality and numerical discretization.
A possibility would be the use of integration volumes detected by physical criteria associated to specific flow phenomena, rather than integrating in an enlarged volume which encloses the aerodynamic body (see Fig. 1). This strategy is similar to the quantification of spurious drag for far-field drag extraction methods [2,16]. As such, integration volumes are based on the detection of three types of regions depending on the flow mechanisms contained therein: viscous, thermal and shockwaves. The term $\mathcal{A}_w$ is already integrated in such a localized surface $\partial\mathcal{V}_w$ (Eq. (10)).

In the present analysis, the rate of viscous anergy generation (term $\mathcal{A}_\Phi$) is integrated in a reduced volume $\mathcal{V}_\Phi$ rather than the complete volume $\mathcal{V}_o$ (see Eq. (8)). For flows involving thermal transfer, the definition of a thermal integration volume based on physical criteria has also been developed. The term $\mathcal{A}_T$ is then integrated in the volume $\mathcal{V}_T$, rather than the complete volume $\mathcal{V}_o$ (see Eq. (9)). Following the detection of these two volumes, the integration of $\mathcal{A}_\Phi$, $\mathcal{A}_T$ in $\mathcal{V}_o \setminus \mathcal{V}_\Phi$ and $\mathcal{V}_o \setminus \mathcal{V}_T$ respectively is used as a first measure of spurious anergy generation. Naturally, this strategy relies on an accurate definition of $\mathcal{V}_\Phi$ and $\mathcal{V}_T$.

The first case investigated is a NACA0012 airfoil in viscous flow. The reference conditions correspond to the viscous computations presented in Section IV.A. Additional computations for a non-adiabatic wall boundary condition are also investigated, corresponding to a wall temperature of $T_w = 350$ K and $T_w = 450$ K (see Fig. 9). It is reminded that the term $\dot{E}_q$ in the exergy balance is non-zero in these cases due to the non-adiabatic airfoil surface (see Eqs. (2), (5)).

Results from the integration in reduced volumes defined based on physical criteria are shown in Tables 5, 6 for the subsonic and transonic viscous NACA0012 case. These show that the viscous and thermal rate of anergy generation can

<table>
<thead>
<tr>
<th>Grid</th>
<th>Term ($\times 10^{-4}$)</th>
<th>Volume</th>
<th>Adiabatic wall $\mathcal{A}_\phi$</th>
<th>$x_p = 2.0$</th>
<th>$x_p = 15.0$</th>
<th>$T_w = 350$ K</th>
<th>$x_p = 2.0$</th>
<th>$x_p = 15.0$</th>
<th>$T_w = 450$ K</th>
<th>$x_p = 2.0$</th>
<th>$x_p = 15.0$</th>
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<tbody>
<tr>
<td>$ni = 256$</td>
<td>$\mathcal{A}_\phi$</td>
<td>$\mathcal{V}_w$</td>
<td>82.68</td>
<td>85.77</td>
<td>73.32</td>
<td>76.31</td>
<td>60.27</td>
<td>63.11</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{A}_T$</td>
<td>$\mathcal{V}_w$</td>
<td>82.65</td>
<td>85.74</td>
<td>73.29</td>
<td>76.28</td>
<td>60.24</td>
<td>63.08</td>
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<tr>
<td>$ni = 512$</td>
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<td>$\mathcal{V}_o$</td>
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<td>1.11</td>
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<td>68.08</td>
<td>358.42</td>
<td>376.27</td>
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<tr>
<td>$\mathcal{A}_T$</td>
<td>$\mathcal{V}_o$</td>
<td>1.01</td>
<td>1.11</td>
<td>65.04</td>
<td>68.08</td>
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<td></td>
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<tr>
<td>$ni = 1024$</td>
<td>$\mathcal{A}_\phi$</td>
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<td>84.59</td>
<td>87.74</td>
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<td>78.34</td>
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<tr>
<td>$\mathcal{A}_T$</td>
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<td>371.27</td>
<td>389.55</td>
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Table 5 Effect of reduction of integration volumes for the viscous and thermal rate of anergy generation based on physical criteria for the NACA0012 case at $M_{\infty} = 0.3$. Results from the integration in reduced volumes defined based on physical criteria are shown in Tables 5, 6 for the subsonic and transonic viscous NACA0012 case. These show that the viscous and thermal rate of anergy generation can...
be integrated within the “physical” integration volumes, as long as the latter are appropriately defined so as to contain the viscous and thermal phenomena respectively. The value differences are lower than 0.05 counts for both $\mathcal{A}_\Phi$ and $\mathcal{A}_T$. Views of the three-dimensional integration volumes for the CRM case are shown in Fig. 10. Table 6 shows the values of $\mathcal{A}_\Phi$ and $\mathcal{A}_T$ integrated on the reduced volumes on the CRM case. As for the NACA0012 computations, the reduction of integration volumes results in small differences compared to the integration in the complete control volume. This further validates the precision of the physical detection criteria, but does not suffice to compensate for the exergy balance residual identified in the analysis of Section IV.A.

At a following step, spurious entropy production is directly calculated from the flow solution via the entropy flux. The rate of spurious anergy generation is defined as:

$$\mathcal{A}_{sp} = T_\infty \int_{V_{sp}} \nabla \cdot (\rho \delta s \mathbf{V}) \, d\mathbf{V}$$  \hspace{1cm} (12)$$

The divergence of the entropy flux is shown in Fig. 11 for the NACA0012 airfoil in subsonic and transonic regime. Physical production of entropy is localized in boundary layers, the wake and shockwaves. In both regimes however, spurious entropy production is also observed close to the airfoil leading edge. This issue has been long identified by multiple authors and is also related to the production of spurious irreversible drag in this region [16]. On the transonic
Table 7  Effect of reduction of integration volumes for the viscous and thermal rate of anergy generation based on physical criteria for the CRM case.
both in subsonic and transonic regime. On RANS airfoil calculations, several observations can be made. First, on the subsonic case with adiabatic wall, the exergy balance residual is well-balanced by $A_{sp}$. The exergy balance residual itself is however augmented on cases with non-adiabatic walls. This can be related to the creation of stronger thermal gradients in the airfoil wake that are not accurately captured on coarser grids. As regards RANS transonic cases, $A_{sp}$ accounts for only a part of the residual with the difference being even more important on the CRM case.

V. Conclusions

Several numerical investigations have been performed regarding the sensitivity of exergy analysis to numerical errors, specifically to the level of convergence and the grid density. The term $W^f$, associated to the net streamwise force acting on the aircraft, has been found to be sensitive at earlier stages of convergence. No similar sensitivity has been identified for the rest of the terms of the exergy balance. Furthermore, it was shown that the exergy balance equation may not be satisfied at the discrete level due to numerical approximation errors inherent in numerical computations. The residual of this balance was found to reduce with refinement of the grid, as physical phenomena represented by the exergy formulation are more accurately captured. In an attempt to balance this residual, a strategy has been presented for the identification and quantification of spurious anergy generation (i.e. loss of work potential due to irreversible processes). This is based on the definition of integration volumes based on physical criteria. The resulting term has been found to at least partly compensate the residual in the discrete exergy balance equation. Nonetheless, it does not suffice to balance the exergy balance residual, suggesting that the latter can be the result of a more complex interaction of physical and numerical effects. Further refinements of this strategy thus appear interesting, since exergy analysis itself remains a powerful and promising tool for aerodynamic design and performance assessment.

Acknowledgments

The present research work was partially funded by the French Directorate General for Civil Aviation (DGAC) through the SUBLIME convention. The studies presented have used the elsA software, whose development is partially funded by its three co-owners: ONERA, Airbus, SAFRAN.

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