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ASSESSMENT OF CFD METHODS TAKING INTO ACCOUNT LAMINAR-TURBULENT TRANSITION FOR AEROELASTICITY OF LAMINAR WINGS

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Abstract: High fidelity aeroelastic simulations of laminar wings require an accurate prediction of the aerodynamic forces taking into account non-linear phenomena due to the laminar-turbulent transition. This paper proposes an evaluation of CFD RANS based methods associated with transition criteria or models for flight conditions of interest from an aeroelastic point of view (low and high incidences, transonic Mach numbers). This evaluation is carried out through comparisons with wind tunnel tests. The first step consists in assessing different models for steady transonic flow fields around a 2D laminar airfoil. The influence of crossflow transition on the aerodynamic response to a 3D wing oscillating in pitch is addressed in a second step for low speed flight conditions.

1 INTRODUCTION

Decreasing drag has been a motivation for aircraft manufacturers for a long time, and extended laminarity was identified very early as an efficient way. The fighter airplane P51 Mustang was indeed one of the first airplanes equipped with intentionally designed laminar wing in the early 40s. But the main difficulty lay in the capability to manufacture sufficiently smooth surface finishes. Today, it has been overcome and wing surfaces meeting laminarity requirements can be produced. Furthermore laminarity can be natural (Natural Laminar Flow or NLF) or helped by technologies aimed at flow control (Hybrid Laminar Flow Control) [1]. On the other hand the motivation of decreasing the environmental footprint adds nowadays to that of improving the aerodynamic performance. Laminar wings are then currently more and more investigated by both aircraft manufacturers and researchers as shown by the Hondajet airplane [2] and the recent flight tests performed by Airbus with the A340-300 Blade within the framework of the European project CLEANSKY2. But this kind of wings may present specific characteristics such as high aspect ratio, and low sweep angle, characteristics that are potentially favorable to high structural flexibility and thereby to aeroelastic instabilities. Moreover the aerodynamic behavior is also different from that of classic wings, with non-linearities occurring at low incidences and transonic Mach numbers. These non-linearities induced by the laminar to turbulent flows transition might have an influence on the aeroelastic stability of the airplane as presented by Tichy et-al [3] and shown experimentally by Hebler [4] and Poirel et-al [5]. Aeroelastic numerical simulations require then accurate predictions of aerodynamic forces for a wide range of flight conditions and taking into account the latter non-linear phenomena.

CFD is today currently used to perform accurate fluid-structure coupling simulations required for the prediction of the flight shape and thus the aerodynamic performance, of the load and gust responses [7]-[10], and of the aeroelastic stability of conventional airplanes [6] for a wide range of flight conditions. Garrigues presented the use of high fidelity numerical simulations for aeroelasticity in Dassault Aviation [11]. (U)RANS modelling seems to be the best compromise between the needed accuracy for taking into account laminar to turbulent flow transition and the computational time. Several ways of modelling the transition within a RANS simulation have been proposed. The intermittency variable is added to the conservative and turbulent variables and act as a weighting function of the turbulent quantities (turbulent viscosity or Reynolds stress tensor for examples). It can vary from 0 for laminar areas to 1 for turbulent areas. Criteria based on local or non local data, and models based on transport equations have been developed to predict natural transition according to its nature (Tollmien-Schlichting instabilities, cross flow instability, attachment line transition, bypass transition…). Such criteria or models yield most of time the values of the intermittency variable. This paper presents the assessment of such transition models implemented into the CFD code *elsA* (ONERA-Airbus-Safran property [12]) for 2 cases: subsonic and transonic steady flows around a laminar airfoil, and low speed unsteady flows past a 3D wing with a high sweep angle. For both cases, numerical results are compared with experimental data.

2 NUMERICAL MODELS

CFD-based methods using RANS models have proven to be suitable for aeroelastic high fidelity simulations of conventional transonic airplanes. But flows around "laminar wings" are laminar on a significant part of their surfaces inducing non-linear phenomena as shown by Tichy et-al [3]. There is therefore a need of specific numerical methods able to predict accurately lift, drag and moments taking into account non-linear phenomena due to the laminar to turbulent transition. Furthermore, aimed at aeroelastic simulations, such methods should also account for deformable meshes and time evolving conditions.

Several kinds of transition models associated with RANS and turbulence models are available today. The first developed were local criteria that determine if the flow at the investigated position is laminar or turbulent. Local means here that the knowledge of boundary layer quantities only at the investigated position is required. Such criteria provide most of time the value of a critical Reynolds number from empirical correlations between boundary layers quantities and the external turbulence level [13]-[15]. Moreover, non local transition criteria, i.e. taking into account the boundary layer history, have also been developed. The AHD (Arnal-Habiballah-Delcourt) criterion [17][18], based on systematic linear stability theory, is used to model transition induced by Tollmien-Schlichting instabilities. Separation induced transition is modelled by Gleyzes (GL) [19] and Roberts [20] criteria and crossflow transition is accounted for by C1 criterion [21]. The software *elsA* features strong transition prediction capability to predict transition for a wide range of natural transition mechanisms [16] by proposing several criteria mentioned above. But only the implementation of the AHD criterion associated with the Gleyzes, Roberts and C1 crossflow ones presented in [22] is selected in the present paper. This implementation, denoted "transition lines method" consists in assuming that streamlines at boundary layer edge follow the mesh lines. This implementation gave satisfactory results on aircraft configuration [23] and helicopter blade [24]. Another way consists in applying the intermittency directly to the production term of the turbulence model and in deriving this coefficient from a correlation-based algebraic function relying on local flow data, as proposed by Cakmakcioglu [25]. On the other hand, transition models based on transport equations have been developed with the aim of being compatible with general purpose CFD codes and usable in the case of complex 3D geometries. The most popular are the models proposed by Langtry and Menter, in which two additional transport equations for the intermittency and a transition onset criterion based on momentum-thickness Reynolds number are solved [26]. More recently, Fehrs proposed a one equation model to improve transition prediction for low external turbulence and high Reynolds Number flows [27]. One can mention a last kind of transition model called “parabola method” [28] which was recently implemented by means of transport equations [29]. But this method is still very CPU time consuming.

3 STEADY TRANSONIC FLOWS AROUND A LAMINAR AIRFOIL

Although aeroelastic simulations involve fluid-structure coupling, the first step in the assessment process consists in evaluating the abilities of the CFD methods to predict accurately the steady aerodynamic forces for a wide range of transonic flight conditions. Experiments were performed in the Onera’s S2MA wind tunnel (Modane-Avrieux center) with a laminar airfoil designed and manufactured by Dassault Aviation within the European funded project CLEANSKY-SFWA ITD (Smart Fixed-Wing Aircraft) in 2012 [30]. Pressure, drag and transition location measurements were carried out for several Reynolds and Mach numbers (from 0.3 to 0.8), and for incidences ranging from negative ones to stall. Those experiments aimed at investigating the influence of the laminar to turbulent transition due to Tollmien-Schlichting (TS) instabilities. This study is focused on the tests performed for a Reynolds number of $2.55\text{x}10^6$ and a Mach number of 0.66. This case is challenging since the flow is subsonic at low incidences but becomes transonic with increasing angle of attack. Furthermore the experiments exhibited for these flow conditions a specific non-linear behavior of lift and drag for lower incidences than stall, and a very large range of transition location especially on the upper surface. The lift polar curve exhibits indeed a kind of bucket similar to that of the CAST 10-2 airfoil for low transonic Mach numbers presented by Hebler [4].

Numerical simulations using different flow models were carried out in order to assess their capability to predict the non-linear behavior due to the laminar to turbulent transition. The first modeling implemented into the code VIS07 [31] [32] consists in solving the (un)steady aerodynamic equations according to a viscous-inviscid interaction strategy. The numerical method is based on a viscous-inviscid splitting and on the “Defect Formulation” developed by Le Balleur. The transition is taken into account naturally by the viscous solver based on boundary layers models. The code VIS07 builds its own viscous and inviscid meshes and has automatic mesh adaptation capabilities. The other flow models are based on RANS with turbulence models (Spalart-Allmaras or Menter’s k-ω SST) and transition models. They are implemented in the multi-block structured code *elsA* [12]. Transition was accounted for using either an association of the AHD, Gleyzes and Roberts criteria (association named “AHD” in the following), or the two transport equations Menter-Langtry $\gamma\text{-Re}_\theta$ model.

The mesh used with RANS models was extracted from the study achieved in [30] and meets all specifications required by the RANS, turbulence and transition models. It has just been slightly modified to make the mesh lines normal to wall in an area close to the airfoil. It is made of a structured C-block and contains about 120000 cells. The first cell layer is such that y^+ is less than 1, and the neighboring cell size ratio is less than 1.1 in the boundary layer (Figure 1).

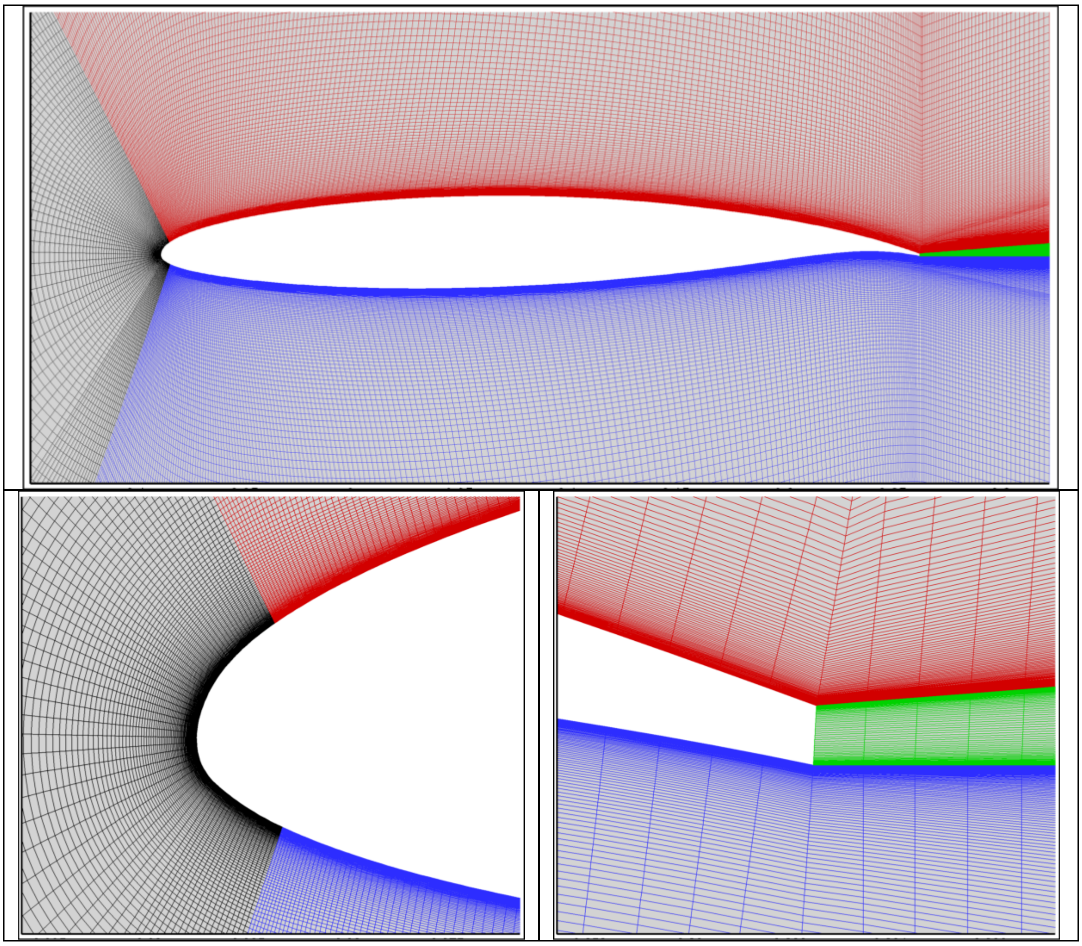

Figure 1: Mesh built around the laminar airfoil

Figure 2 shows the evolutions of the lift and drag coefficients with respect to the incidence, evolutions determined by the three sets of simulations (viscous-inviscid coupling, RANS with AHD and RANS with Menter-Langtry model) and by fully turbulent RANS simulations. Since the updating of the aerodynamic conditions (Mach and incidence) due to the walls of the wind tunnel section is not well known, and the numerical simulations were performed with infinite atmosphere conditions, corrections were applied to the experimental incidences to get the applied numerical ones, such that the lift coefficient at the null angle of attack from every numerical simulation is equal to the experimental one. No correction was brought to the Mach number. For such aerodynamic conditions, the experiments exhibit a linear evolution of the lift coefficient for angles of attack less than 2°. For higher incidences, the lift increases with the angle of attack but less than a linear model would predict (grey line in Figure 2). This non-linear behavior is well captured by the numerical simulations based on viscous-inviscid interactions (blue curve titled “VIS”) and on RANS with the AHD transition criteria (red curve titled “elsA-AHD”). The impact of the turbulence model (Spalart-Allmaras or Menter’s k-ω SST) associated with the AHD criterion or used for fully turbulent simulations is also small for low incidences and becomes significant only for incidences close to stall. Another point is that those simulations overestimate the slope of the linear part of the polar curves. The simulations performed considering the flows fully turbulent (green curve titled “elsA-fully turb”) provide a lift level in accordance with the experiments with a slightly lower estimation of the linear than that computed taking into account the transition, but as expected they do not capture the non-linear behavior due to the free transition. The transition model of

Menter-Langtry (orange curve titled "elsA-ML") underestimates also significantly this nonlinear phenomenon, and predicts a behavior close to that predicted by fully turbulent simulations.

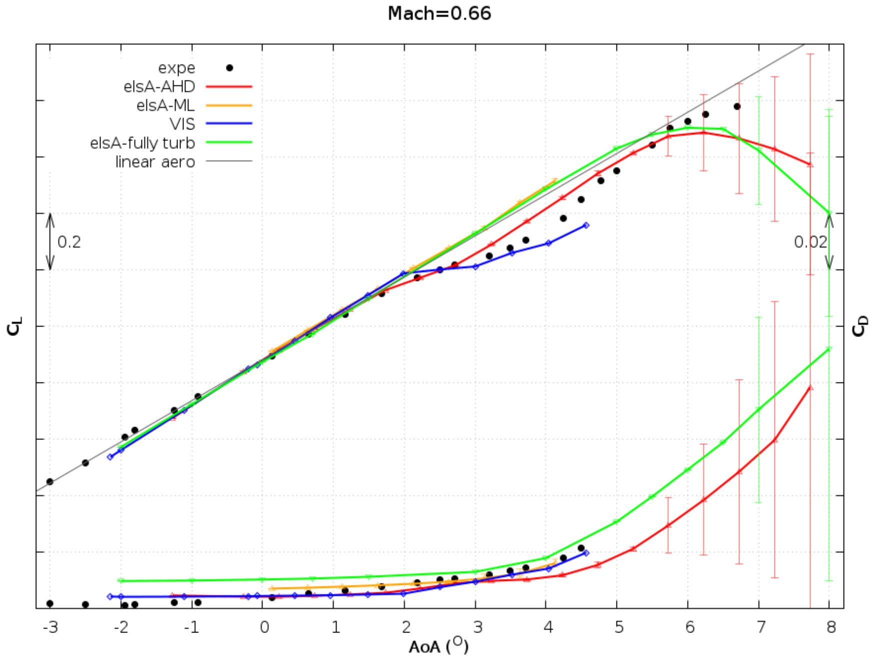

Figure 2: Lift-drag polar curves for the 2D laminar airfoil (the vertical bars represent the amplitude of the oscillations of the numerical solutions)

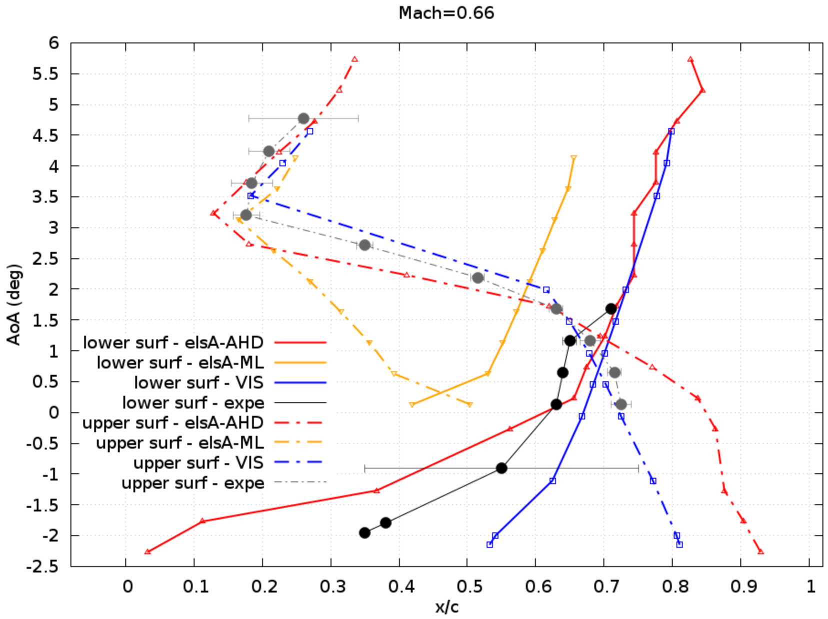

Figure 3: Evolution of the transition location against the angle of attack

Figure 3 shows the transition location on the upper and lower surfaces with respect to the angle of attack for each kind of numerical simulations. As for the lift anf drag coefficients, the RANS with the AHD criterion and viscous-inviscid interactions codes provided transition locations in rather good agreement with the experiments, whereas significant discrepancies can be noticed for the locations predicted using RANS with Menter-Langtry model (too upstream on both the upper and lower surfaces).

The nature of the flow around the airfoil is laminar in a large area for low angles of attack: up to 75% chord on the upper surface and 70% chord on the lower surface. But transition on upper surface moves rapidely with increasing angle towards the leading edge when a supersonic bulb appears for an incidence greater than 1.5deg, incidence at which the lift loss when compared to a linear aerodynamics model arises. A shock appears between 2.5deg and 3deg, incidences for which the transition moves rearward and is located at the shock (Figure 4 and Figure 5). This behavior resulting from simulations using elsA with the AHD criteria is similar to the experimental one, but for lower incidences than the experimental ones. The amplitude of the lift loss is also slightly underestimated.

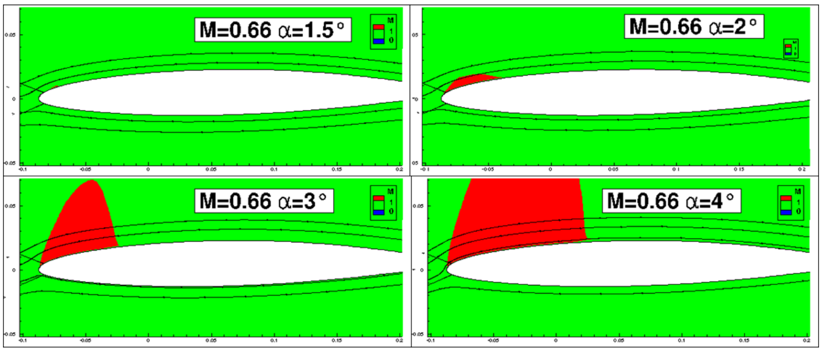

Figure 4: Mach contours for several incidences

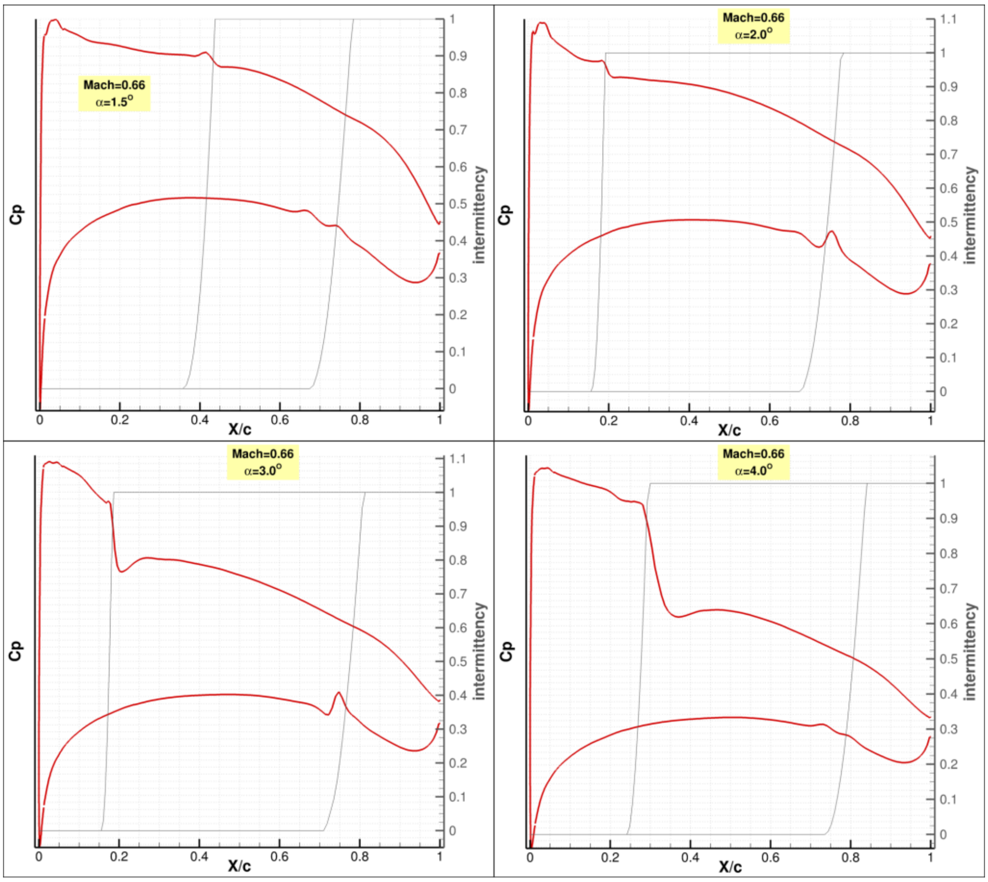

Figure 5: Pressure and intermittency coefficient distributions computed using *elsA* with the AHD transition criteria for 4 angles of attack

Experiments have exhibited for this laminar airfoil at Mach number equal to 0.66 a specific non-linear behavior of lift and drag due to laminar-turbulent transition and a large range of transition location especially on the upper surface. Such a behavior can be predicted by numerical simulations only if transition is taken into account. The Menter-Langtry γ-Re_θ transition model was designed and developed aimed at being used associated with the Menter k-ω SST turbulence model in CFD codes. It is therefore the most useful transition model for aeroelastic simulations but its implementation in *elsA* leads to lift prediction against AoA not in good agreement with experiments for the computed case. Results from the numerical simulations using the viscous-inviscid interactions formulation are the closest to the experiments and provide a good prediction of the non-linear behavior, but this numerical technology lacks robustness when strong shocks occur, and is not suited for aeroelastic simulations of 3D complex geometric cases. The CFD techniques based on RANS and transition criteria allows also capturing non linear phenomena due to the transition. The criteria are based on physics modeling and the combination of them (AHD, Gleyzes and Roberts) allow the detection of the transition resulting from several phenomena. But they depend on several model parameters making them difficult to tune to get a converged accurate solution. Furthermore their implementation, denoted "transition lines method" requires the user to specify the location of the stagnation point and the direction of the streamlines which are supposed to follow mesh lines. The latter requirements are not useful for an aeroelastic simulation during which these data can vary during the simulation.

To circumvent the drawback of the transition criteria due to their implementation according to the "transition lines method", recent developments focused on transport equations. The AHD, Gleyzes and crossflow-C1 criteria were implemented by means of four additional transport equations [33]. This transition model provides a lift evolution similar to that determined by the "transition lines method" implementation (Figure 6) with however some discrepancies (slightly higher lift) for angles of attack greater or equal to the incidence from which the non-linear phenomenon due to transition arises. This new implementation of the criteria (AHD, Gleyzes and Roberts) based on transport equations are then potentially the most interesting for aeroelastic simulations.

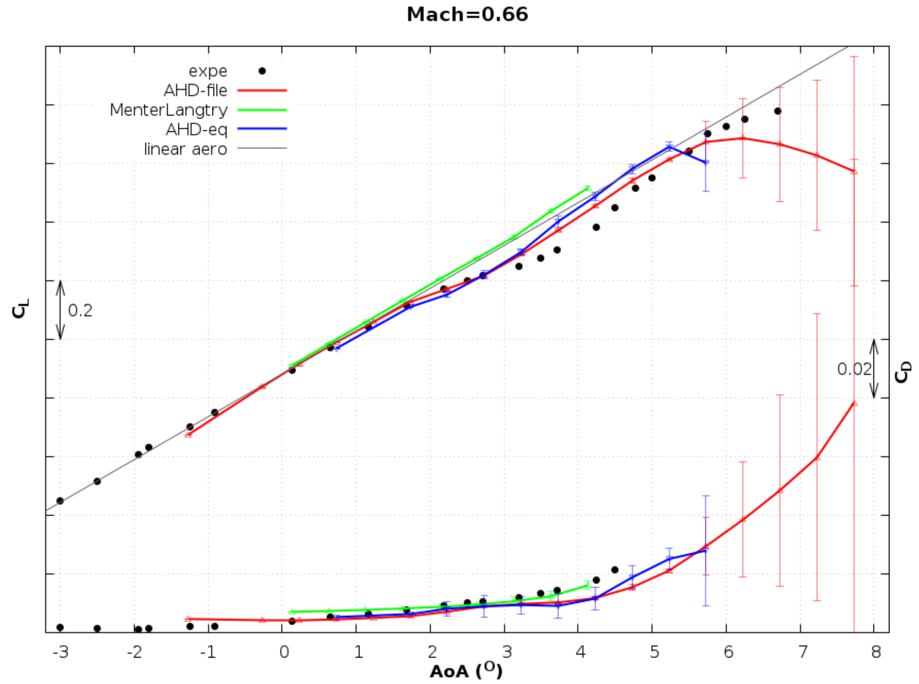

Figure 6: Lift-drag polar curve determined by RANS computations with three transition models: the AHD transition lines method implementation (red curve), the Menter-Langtry transport equations model (green curve) and the AHD criteria based on transport equations (blue curve).

4 UNSTEADY LOW SPEED FLOW AROUND A 3D SWEPT WING

Crossflow instability transition occurs at low frequencies which could be of the order of aeronautical structures natural frequencies. It could thus potentially interact with the aeroelastic behavior of a structure. Wind tunnel tests aimed at studying the influence of the crossflow transition on unsteady flow fields for a harmonic pitching motion of a swept wing were performed in the TRIN1 low-speed wind tunnel at Onera.[34] The experimental model is a straight wing whose section airfoil is the symmetric profile ONERA-D of chord 0.35m. In order to trigger laminar to turbulent crossflow transition, the wing was installed in the wind tunnel test section with a sweep angle of 60°. The pitching motion is achieved by a rotation of the wing around its mid span axis (Figure 7).

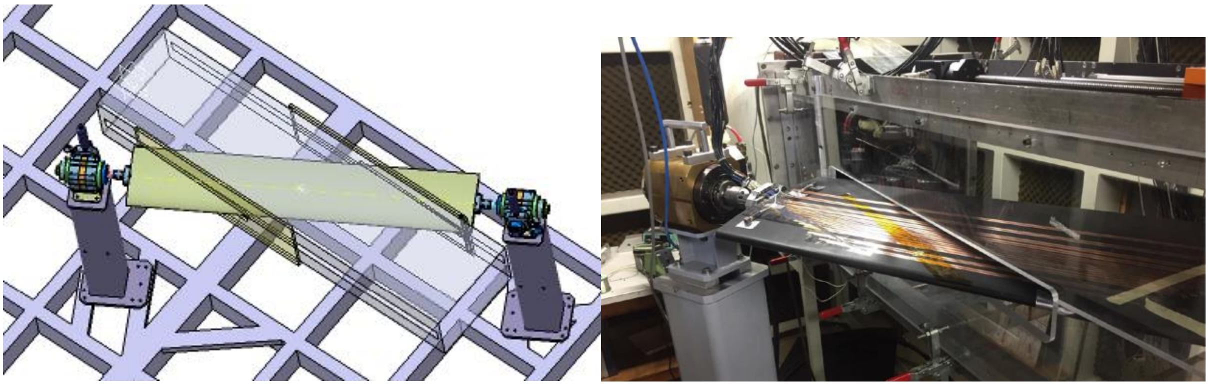

Figure 7: The experimental wing model in the wind tunnel test section

Numerical simulations were performed using RANS models with the “mesh lines method” implementation of the combination of AHD-Gleyzes-Roberts-C1 criteria for the aerodynamic conditions: inlet flow velocity equal to 70 m/s, and mean AoA equal to -8°.

Due to the sizes of the experimental model and of the test section, the influence of the top and bottom walls of the test section can not be neglected. A 3D structured mesh based on a O-topology wind-wise, and taking into account the test section walls was then built with about 6.6 million cells (520 cells on the airfoil and 240 in the normal direction), a first cell size of 5µm and at least 50 cells in the boundary layer (Figure 8).

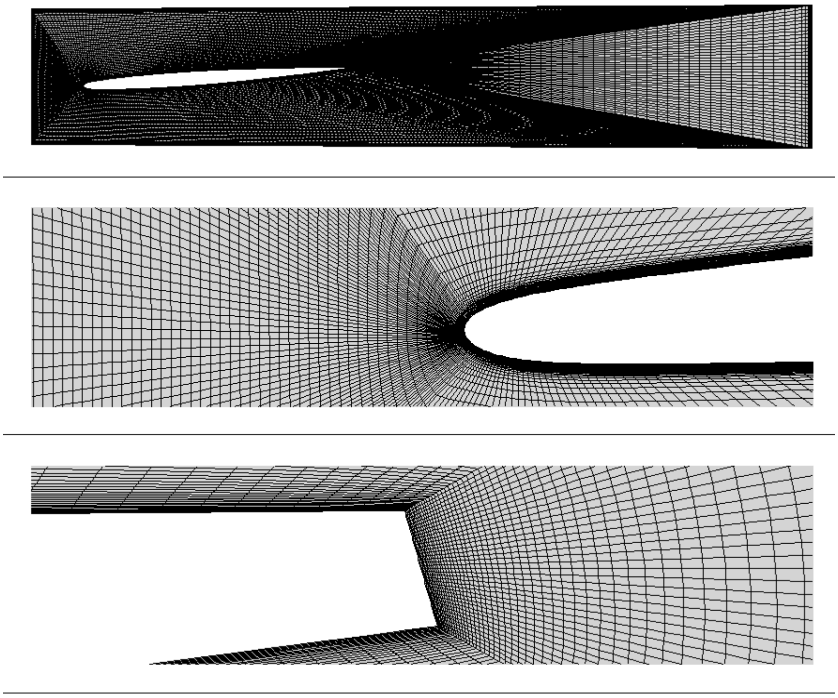

Figure 8: mesh of the ONERA-D wing (top: global view of a plane close to root, middle: zoom on the leading edge and bottom: zoom on the trailing edge)

First of all, it was checked by disabling or enabling the C1 criterion that for these aerodynamic conditions the laminar-turbulent transition on the upper surface is triggered by crossflow instabilities. As can be seen in Figure 9, the transition is predicted close to the trailing edge when the C1 criterion is not used and about mid chord when it is activated. The flow on the lower surface remains always turbulent.

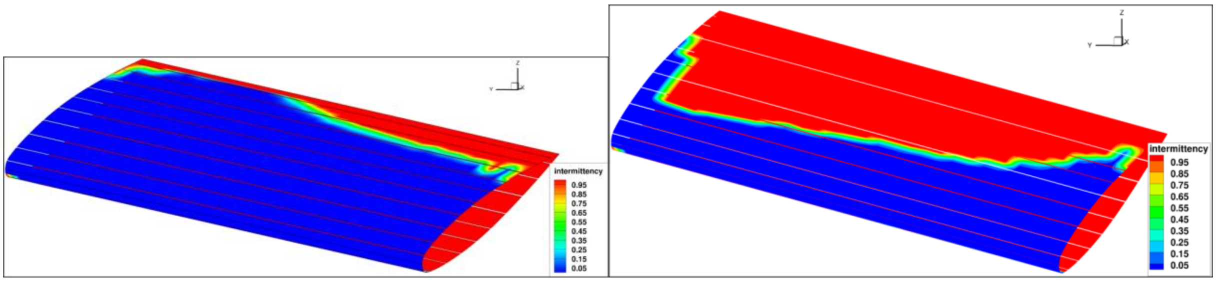

Figure 9: intermittency coefficient distributions – left: only TS criterion, right: TS and crossflow criteria

Inlet and outlet boundary conditions were tuned to obtain a good agreement between the steady experimental and numerical pressure distributions as shown in Figure 10 representing the pressure distribution on two wing sections. Computations were carried out using both the Spalart-Allmaras and Menter’s k-ω SST turbulence models. For these aerodynamic conditions, the turbulence model has a small impact on the pressure and intermittency distributions, but has a significant influence on the boundary layer quantities. It can also be noticed that a fully turbulent simulation yields equal steady lift and superposed steady pressure distributions.

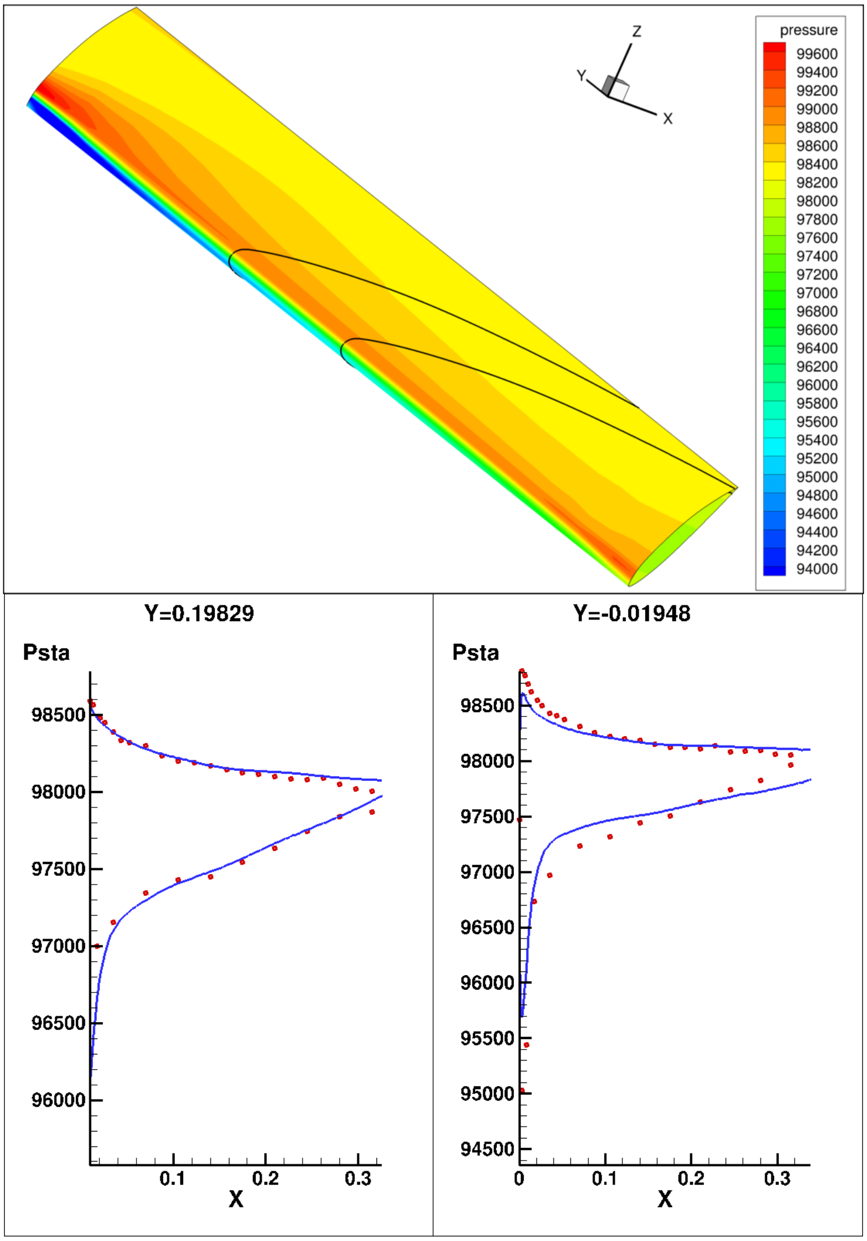

Figure 10: pressure distribution on the upper surface (top) and on the 2 sections (bottom) represented with blue solid lines. Experimental data are represented by red dots.

A dynamic analysis was performed for a forced pitching motion whose amplitude and frequency are equal respectively to 1° and 5 Hz. Unsteady experiments have shown that the mean location of the transition on the upper surface, measured using hot films is slightly upwind the steady location (Figure 11), denoting thus unsteady nonlinear phenomena acting on the transition position. Nevertheless the discrepancy is of the order of the tolerance of the sensors. More accurate measurements should confirm the occurrence of these phenomena.

Unsteady numerical simulations using the transition criteria exhibited also such a nonlinear phenomenon but of higher amplitude. Figure 12 represents the steady and unsteady (at several time snapshot) distributions of the intermittency coefficient. The steady value of this quantity is indeed not within the range of unsteady variations, and the experimental discrepancy between the mean and steady locations is less than 2.5% chord, whereas the numerical discrepancy is about 10%.

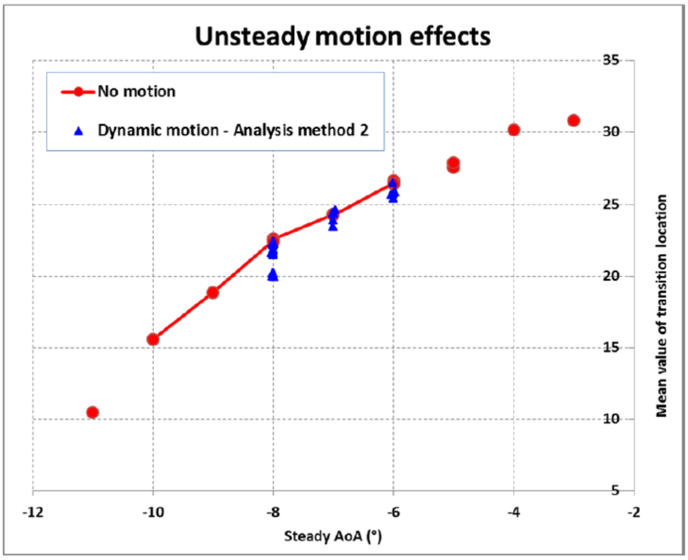

Figure 11: experimental mean and steady transition location

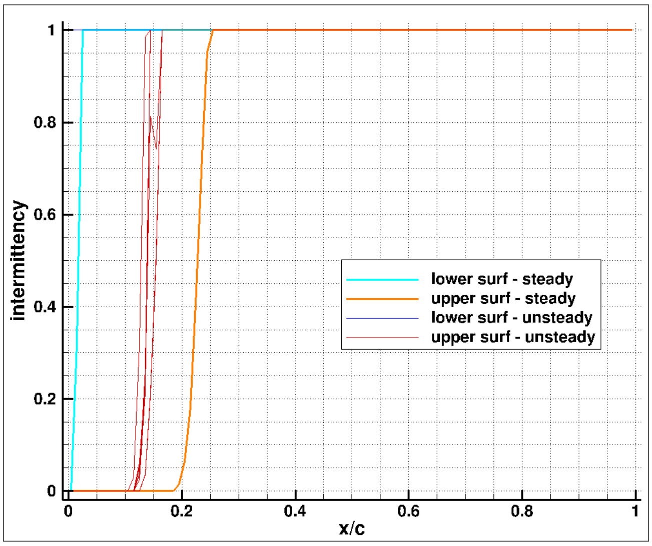

Figure 12: Numerical intermittency coefficient distributions on a section airfoil (steady and unsteady at several instants)

5 CONCLUSIONS

A lot of efforts are today focused on the evaluation of extended laminar areas of an aircraft with the objective of decreasing significantly drag and fuel consumption. But laminar parts of an aircraft, especially the wings, may induce some particularities such as high aspect ratios or low sweep angles, which may tend to more flexible structures and may thereby be favorable to aeroelastic instabilities. Furthermore experiments have shown that the aerodynamic behavior of such wings is different from that of classic wings with non-linearities due to laminar-turbulent transition occurring at low incidences and transonic Mach numbers. Numerical aeroelastic simulations require then high fidelity methods able to compute accurately aerodynamic forces taking into account transition. This paper proposes an evaluation of the CFD based methods associated with transition models or criteria for flight conditions of interest from an aeroelastic point of view (from low to high incidences and from low speed to transonic Mach numbers). This evaluation was carried out through comparisons with wind tunnel tests. The first step consisted in assessing different aerodynamic models for steady transonic flow fields around a 2D laminar airfoil. Simulations were performed using a viscous-inviscid coupling formulation, a RANS formulation associated with the Menter-Langtry γ-Re_θ transition model and the latter RANS formulation associated with transition criteria. Wind tunnel tests exhibited a non-linear lift evolution with increasing incidence due to the displacement of the transition toward the leading edge on the upper surface. This phenomenon was well captured by the simulations using the viscous-inviscid coupling and RANS with criteria formulations. Nevertheless the used viscous-inviscid coupling formulation was developed only for 2D flows and its extension to 3D would require unreasonable effort. The second formulation consists of an association of several criteria according to the nature of the instability triggering the transition. It leads to accurate prediction of transition, but its first implementation according the "transition lines method" is not suited to simulations with deforming meshes and time evolving conditions. Furthermore these transition criteria are very sensible to model or numerical parameters and lack robustness for aeroelastic simulations. But the second implementation based on transport equations make the criteria as well suited as the Menter-Langtry model to unsteady simulations with deformable meshes while being as accurate as their first implementation.

Unsteady aerodynamic flows were addressed in a second step. Low speed wind tunnel tests were carried out to investigate the crossflow transition effects on the unsteady aerodynamics of a pitching oscillating wing. They exhibited a non-linear behavior of very low amplitude of the transition location. Numerical simulations using a RANS formulation associated with the C1 criterion predicted a similar phenomenon whose amplitude were overestimated. They also showed that the transition for such low speed conditions has no influence on the pressure distributions.

These two comparisons between experiments and numerical simulations showed the capabilities of the numerical models to capture non-linear phenomena due to transition. As perspectives, the transition models evaluation has to be extended to forced motions but for transonic conditions. The effect of transition has also to be investigated in case of fluid-structure coupling with a highly flexible structure.

6 REFERENCES

[1] Risse K. and Stumpf E. (2014), "*Conceptual aircraft design with hybrid laminar flow control*", CEAS Aeronautical Journal Vol. 5 issue 3 pp 333-343

[2] Fujino M., Yoshizaki Y. and Kawamura Y. (2003), "*Natural-Laminar-Flow Airfoil Development for a Lightweight Business Jet*", Journal of Aircraft, vol. 40, No. 4, pp 609-615

[3] Tichy L., Mai H., Fehrs M, Nitzsche J. and Hebler A. (2017). "*Risk analysis for flutter of laminar wings*", IFASD Como Italy, June 2017

[4] Hebler A. (2017), "*Experimental assessment of the flutter stability of a laminar airfoil in transonic flow*", IFASD Como Italy, June 2017

[5] Poirel D. and Mendes F. (2014), "*Experimental Small-Amplitude Self-Sustained Pitch-Heave Oscillations at Transitional Reynolds Numbers*", AIAA Journal, Vol. 52 No. 8

[6] Girodroux Lavigne P., Dugeai A., Sens A.S., Krueger W.R., Dimitrov D., Nitzsche J., Ritter M. and Stickan B. (2013), "*Investigations of Transonic Aircraft in the Joint DLR/ONERA Project "Non-Linear Aeroelastic Simulation, NLAS II"* ", IFASD Bristol GB, June 24-27 2013.

[7] Liauzun C. (2010), “*Aeroelastic response to gust using CFD techniques*”, Proceedings of 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels and Minichannels, FEDSM2010-ICNMM2010, Montreal, Canada, August 2-4 2010.

[8] Heinrich R. and Reimer L. (2013), “*Comparison of different approaches for gust modelling in the CFD code TAU*”, IFASD Bristol GB, June 24-27 2013.

[9] Bekemeyer P., Thormann R. and Timme S. (2017), “*Rapid Gust Response Simulation of Large Civil Aircraft using Computational Fluid Dynamics*”, The Aeronautical Journal Volume 121, issue 1246, pp1795-1807, December 2017

[10] Huvelin F., Dequand S., Lepage A. and Liauzun C. (2018), « *On the validation and use of high fidelity numerical simulaitons for gust response analysis* », AerospaceLab Journal issue 14, http://www.aerospacelab-journal.org

[11] Garrigues E. (2018), “*A review of industrial aeroelasticity practices at Dassault Aviation for military aircraft and business jets*”, AerospaceLab Journal issue 14, http://www.aerospacelab-journal.org

[12] Cambier L., Heib S. and Plot S. (2013), “*The Onera CFD software: input from research and feedback from industry*”, Mechanics & Industry, 14, pp159-174

[13] Dunham J. (1972), “*Prediction of Boundary Layer Transition on Turbomachinery Blades*”, AGARD Meeting Boundary Layer in Turbomachines, Paris

[14] Abu-Ghannam B.J., Shaw R. (1980), “*Natural Transition of Boundary Layers. The effects of Turbulence, Pressure Gradient, and Flow History*”, J. Mech. Engr. Science, 22(5): 213-228

[15] Drela M., Giles M.B. (1987), “*Viscous-Inviscid Analysis of Transonic and Low Reynolds Number Airfoils*”, AIAA Journal, 25(10): 1347-1355

[16] J. Perraud, H. Deniau, and G. Casalis, ‘*Overview of transition prediction tools in the elsA software*’, in ECCOMAS 2014, 2014.

[17] Arnal, D., “*Transition prediction in transonic flow*,” Symposium Transsonicum III, Springer, 1989, pp. 253–262.

[18] Perraud, J., and Durant, A., “*Stability-Based Mach Zero to Four Longitudinal Transition Prediction Criterion*,” Journal of Spacecraft and Rockets, 2016, pp. 730–742.

[19] C. Gleyzes, J. Cousteix, and J. L. Bonnet, ‘*Theoretical and experimental study of low Reynolds number transitional separation bubbles’*, in Conference on Low Reynolds Number Airfoil Aerodynamics, Notre Dame, IN, 1985, pp. 137–152.

[20] W. B. Roberts, ‘*Calculation of Laminar Separation Bubbles and Their Effect on Airfoil Performance*’, AIAA Journal, vol. 18, no. 1, pp. 25–31, 1980.

[21] Arnal, D., Habiballah, M., and Coustols, E., “*Théorie de l’instabilité laminaire et critères de transition en écoulement bi et tridimensionnel*,” La Recherche Aérospatiale, Vol. 2, 1984, pp. 125–143.

[22] Cliquet, J., Houdeville, R., and Arnal, D., “*Application of Laminar-Turbulent Transition Criteria in Navier-Stokes Computations*,” AIAA Journal, Vol. 46, No. 5, 2008, pp. 1182–1190.

[23] Hue, D., Vermeersch, O., Duchemin, J., Colin, O., and Tran, D., “*Wind-Tunnel and CFD Investigations Focused on Transition and Performance Predictions of Laminar Wings,*” AIAA Journal, Vol. 56, No. 1, 2018, pp. 132–145.

[24] Richez, F., Nazarians, A., and Lienard, C., “*Assessment of laminar-turbulent transition modeling methods for the prediction of helicopter rotor performance*,” 43rd European Rotorcraft Forum, 2017.

[25] Cakmakcioglu S. C., Bas O. and Kaynak U. (2017), "*A correlation-based algebraic transition model*", Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science

[26] Langtry R. B., Menter F. R. (2009), "*Correlation-Based Transition Modeling for Unstructured Parallelized Computational Fluid Dynamics Codes*", AIAA Journal Vol. 47, No. 12

[27] Fehrs M. (2018), "*One-Equation Transition Model for Airfoil and Wing Aerodynamics*", New Results in Numerical and Experimental Fluid Mechanics XI, 136, pp199-208 Springer

[28] Perraud, J., Arnal, D., Casalis, G., Archambaud, J.-P., and Donelli, R., "*Automatic transition predictions using simplified methods*," AIAA journal, Vol. 47, No. 11, 2009, pp. 2676–2684.

[29] Bégou, G., Deniau, H., Vermeersch, O., and Casalis, G., "*Database Approach for Laminar-Turbulent Transition Prediction: Navier–Stokes Compatible Reformulation*," AIAA Journal, 2017, pp. 3648–3660.

[30] Hue D., Vermeersch O., Bailly D., Brunet V. and Forte M. (2015), "*Experimental and Numerical Methods for Transition and Drag Predictions of Laminar Airfoils*", AIAA Journal Vol. 53 No. 9, pp2694-2712

[31] Le Balleur J.C. (1981), "*Strong matching method for computing transonic viscous flows including wakes and separations. Lifting Airfoils.*", La Recherche Aérospatiale, 3,March, pp. 21–45.

[32] Le Balleur J.C. (1992), "*Viscous-inviscid calculation of high-lift separated compressible flows over airfoils and wings*", In Symposium AGARD/FDP High lift aerodynamics, Banff Canada, AGARD-CP-515, Paper 26.

[33] Pascal, L., Delattre, G., Deniau, H., J. Cliquet and G. Bégou, "*New implementation of stability-based transition models by means of transport equations*", AIAA Aviation 2019, to be published

[34] Lepage A., Amosse Y., Brazier J.P., Forte M., Vermeeersch O. and Liauzun C. (2019), « *Experimental investigation of the laminar-turbulent transition and crossflow instability of an oscillating airfoil in low speed flow* », IFASD Savannah USA

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