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Computational Complexity and Tort Deterrence

Joshua C. Teitelbaum*

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Abstract

Standard economic models of tort deterrence assume that a tortfeasor's precaution set is convex—usually the non-negative real numbers, interpreted as the set of feasible levels of spending on safety. In reality, however, the precaution set is often discrete. A good example is the problem of complex product design (e.g., the Boeing 737 MAX airplane), where the problem is less about how much one spends on safety and more about which combination of safety measures one selects from a large but discrete set of alternatives. I show that in cases where the precaution set is discrete, the problem faced by a tortfeasor under strict liability and negligence is computationally intractable, frustrating their static deterrence effects. I then argue that negligence has a dynamic advantage over strict liability in that negligence can move a tortfeasor's behavior in the direction of socially optimal care over time more rapidly than strict liability.

JEL codes: C61, K13.

Keywords: computational complexity, \mathcal{NP} -hard, negligence, strict liability, supermodularity, tort law.

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1 Introduction

In many ways the economic approach to law embraces Oliver Wendell Holmes' "bad man" theory of the law (Holmes 1897). Holmes argued that if "you want to know the law and nothing else, you must look at it as a bad man," to whom the notion of legal duty means "a prophecy that if he does certain things he will be subjected to disagreeable consequences by way of imprisonment or compulsory payment of money" (Holmes 1897, pp. 459 & 461). In other words, if you want to understand the law, you must focus on the deterrence effects of legal rules.

Nowhere is the focus on the deterrence effects of legal rules more prominent than in the economic analysis of tort law. The economic model of tort deterrence posits that the tortfeasor, known as the injurer, chooses safety precautions, or care, when engaging in a risky activity to minimize his expected liability plus his cost of care (see, e.g., Shavell 1987, ch. 2). Under the economic model, a liability rule is optimal if the solution to the injurer's care-taking problem under that rule is socially optimal.

Standard formulations of the economic model of tort deterrence constitute the injurer as the unboundedly rational bad man. Unbounded rationality implies that the injurer can compute the solution to his care-taking problem. This in turn implies that an optimal liability rule can achieve socially optimal deterrence, for it can induce the injurer to take socially optimal care (see, e.g., Cooter and Ulen 2012, ch. 7).

Beginning with Simon (1955, 1957), economists have questioned the assumption of unbounded rationality and explored the implications of bounded rationality for standard economic analysis. Legal economists have followed suit (see, e.g., Jolls, Sunstein, and Thaler 1998; Korobkin and Ulen 2000). An important aspect of bounded rationality is limited computational capacity. This aspect of bounded rationality refers not only to humans' limited cognitive ability or skill with respect to computation, but also to the theoretical and practical limits of computability even when aided by machines (see, e.g., Simon 1976, 1990). While the former limit is the subject of behavioral economics and psychology, the latter limits are the subjects of computability theory and computational complexity theory, respectively.

In this paper, I examine the computational complexity of the injurer's care-taking problem under strict liability and negligence, the two basic liability rules of Anglo-American tort law, and explore the implications thereof for the deterrence effects of these rules. I start by defining several concepts from computational complexity theory,

including what it means for a problem to be computationally tractable. Informally, a problem is computationally tractable if it can be solved in a reasonable amount of time even as the scale of the problem becomes large.

In the heart of the paper, I analyze the static deterrence effects of strict liability and negligence using the unilateral care model with fixed activity level.¹ I probe the standard assumption that the injurer's precaution set is convex—usually the non-negative real numbers, interpreted as the set of feasible levels of spending on safety. In reality, the precaution set is often discrete. A good example is the problem of complex product design (e.g., the Boeing 737 MAX airplane), where the problem is less about how much one spends on safety and more about which combination of safety measures one selects from a large but discrete set of alternatives.² Nevertheless, the assumption of "convex care" seems innocuous enough. "All models are approximations," after all; their assumptions "are never exactly true" (Box, Luceño, and Paniagua-Quiñones 2009, p. 61). At the same time, however, because "all models are wrong the scientist must be alert to what is importantly wrong" (Box 1976, p. 792). I show that (i) the assumption of convex care is instrumental to ensuring that the injurer's care-taking problem under strict liability and negligence is computationally tractable, and that (ii) if the precaution set is discrete and large, the injurer's care-taking problem is computationally intractable. In other words, I show that the convex care assumption underwrites the unbounded rationality assumption, and that relaxing this assumption frustrates the static deterrence effects of strict liability and negligence.

The basic intuition behind these results is the following. Under the standard, convex care version of the model, the injurer's care-taking problem is a convex optimization problem—the minimization of a convex function over a convex set. By contrast, the discrete analog of the injurer's problem entails the minimization of a supermodular set function over a discrete set. There are known methods for solving convex optimization problems in a reasonable amount of time even when the feasible set is large. This is not the case for the problem of minimizing a supermodular set function over a discrete set. The only known method for solving this problem is the

¹The unilateral care model with fixed activity level is the foundational model upon which other economic models of tort deterrence are built. In cases of unilateral care, the injurer, but not the victim, can take care to reduce the expected loss to the victim from the injurer's activity. In cases of unilateral care with fixed activity level, the injurer can reduce the expected loss only by taking care and not by modulating his activity level.

²Other examples include many important medical care investments; see Arlen (2010, p. 993 n. 97).

brute-force method—i.e., evaluating the function at every subset of the feasible set—which takes an unreasonable amount of time when the feasible set is large because the number of subsets grows exponentially with the size of the feasible set.

Lastly, I consider the dynamic deterrence effects of strict liability and negligence in the case of discrete care. I argue that, notwithstanding their static ineffectiveness, the injurer's behavior under both rules moves in the direction of socially optimal care over time through a learning-by-experimentation process. I then argue that negligence, due to its information-generating property (see, e.g., Schäfer and Müller-Langer 2009, § 1.6),³ accelerates the injurer's learning process relative to strict liability, giving negligence a dynamic advantage over strict liability in the case of discrete care. I conclude that negligence is more robust than strict liability to computational complexity and the reality of limited computational capacity (bounded rationality).

This paper contributes to two strands of the law and economics literature. The first is the strand that focuses on the question of whether strict liability or negligence is superior in terms of tort deterrence. The pioneers of this strand include Calabresi (1961, 1970), Posner (1972a,b), Brown (1973), Diamond (1974a,b), Green (1976), Landes and Posner (1980, 1987) and Shavell (1980, 1987). Surveys of this strand are provided by Shavell (2007), Schäfer and Müller-Langer (2009), and Arlen (2017).

The second is the behavioral strand that explores the implications of bounded rationality for the standard economic analysis of tort deterrence. Jolls, Sunstein, and Thaler (1998) and Korobkin and Ulen (2000) were among the early calls for the modification of standard law and economics models to reflect bounded rationality. Zamir and Teichman (2018) provide a textbook treatment of the emergent field of behavioral law and economics, which includes a chapter on the behavioral analysis of tort law. Faure (2010), Halbersberg and Guttel (2014), and Luppi and Parisi (2018) provide surveys of behavioral models of tort law.

To my knowledge, this is the first paper in the law and economics literature to explore how computational complexity impacts tort deterrence. Legal economists have long recognized that "people's decision-making capabilities are relevant to the design of tort law" (Zamir and Teichman 2018, p. 330). In their early paper on strict liability, for instance, Guido Calabresi and Jon Hirschoff argue that the choice among tort liability rules should depend not on the theoretical ability of injurers and victims

³See also Ott and Schäfer (1997), Feess and Wohlschlegel (2006), and Chakravarty, Kelsey, and Teitelbaum (2019).

to optimize, but rather on their actual abilities taking into account the "psychological or other impediments" to optimizing (Calabresi and Hirschhoff 1972, p. 1059; see also Faure 2008).⁴ Subsequently, legal economists have studied the implications for tort law of various aspects of bounded rationality, including ambiguity (Teitelbaum 2007; Chakravarty and Kelsey 2017; Franzoni 2017) and unawareness (Chakravarty, Kelsey, and Teitelbaum 2019). As far as I am aware, however, no other paper in the literature has studied the implications of computational complexity for tort law.

This paper also contributes to the literature on legal complexity (see, e.g., Ehrlich and Posner 1974; Schuck 1992; Kaplow 1995; Ruhl and Katz 2015). Papers in this literature study various kinds of legal complexity, including the intricacy of legal rules and the legal system. To my knowledge, the only other paper in this literature that studies computational complexity is Kades (1997). In contrast to this paper, Kades does not focus on the tractability of compliance problems arising under tort law. Rather, he focuses on the tractability of adjudication problems in selected cases arising under bankruptcy law, commercial law, contract law, corporate law, criminal law, property law, and tax law. He also invokes computational complexity to explain judges' aversion to multiparty disputes and the existence of private property.⁵

The remainder of this paper is organized as follows. Section 2 is a primer on computational complexity theory. Sections 3 and 4 present the static and dynamic analysis of tort deterrence, respectively. Section 5 offers concluding remarks.

2 Computational Complexity

Computational complexity theory is a subfield of computer science that studies the tractability of computational problems, including decision problems (i.e., yes-no problems) and optimization problems. In this section, I semi-rigorously define several concepts from computational complexity theory that are relevant for the tort deterrence

⁴Legal economists have also long recognized that people's limited computational capacity is relevant to contracts. In his oft-cited paper on the transaction cost approach to the study of economic organization, for example, Oliver Williamson argues that "incomplete contracting is the best that can be achieved" because "organizational man," unlike "economic man," is "boundedly rational" and subject to "limits in formulating and solving complex problems" (Williamson 1981, pp. 553-554).

⁵There are a number of papers in the economics and computer science literatures that study the computational complexity of economic models, including, for instance, papers that study the tractability of computing Nash equilibria in games (Daskalakis 2009; Roughgarden 2010) and of the consumer's utility maximization problem (Gilboa, Postlewaite, and Schmeidler 2010, 2015; Echenique, Golovin, and Wierman 2011).

Table 1: Polynomial Time versus Exponential Time

Input size (n)	Running time	
	Polynomial-time algorithm (n^2 steps)	Exponential-time algorithm (2^n steps)
50	Less than a second	Less than a second
75	Less than a second	Less than a year
80	Less than a second	More than a decade
85	Less than a second	More than three centuries
90	Less than a second	More than 10,000 years
95	Less than a second	More than 350,000 years
100	Less than a second	More than 12 million years

Note: Assumes one calculation per step and 200,000 trillion calculations per second.

analysis in Sections 3 and 4. For a more rigorous introduction to these concepts, see, e.g., Garey and Johnson (1979), Schrijver (2003), or Kleinberg and Tardos (2006).

2.1 Algorithms and Efficiency

An *algorithm* is a step-by-step procedure for solving a computational problem. The *time complexity function* of an algorithm, denoted by $\tau(n)$, gives the maximum number of elementary steps that the algorithm requires to produce its output, expressed as a function of the size of its input, denoted by n . Algorithms are classified according to the rate at which $\tau(n)$ grows with n . Algorithms for which $\tau(n)$ grows with n at a polynomial rate (or slower) are said to run in *polynomial time*. Polynomial-time algorithms are considered to be *fast* or *efficient*. Algorithms for which $\tau(n)$ grows with n at a faster rate (e.g., exponential) are considered to be inefficient.

The efficiency of polynomial-time algorithms is manifested by a comparison with exponential-time algorithms. Table 1 displays the running times for selected input sizes $n \leq 100$ of a polynomial-time algorithm that requires n^2 steps and an exponential-time algorithm that requires 2^n steps, assuming one calculation per step and 200,000 trillion calculations per second (the peak speed of the world's fastest supercomputer). The polynomial-time algorithm runs in less than a second for all input sizes $n \leq 100$ (and, indeed, for all $n \leq 447, 213, 595$). By contrast, the running time of the exponential-time algorithm increases from less than a second for $n = 50$ to more than 12 million years for $n = 100$.

2.2 Problems and Tractability

In light of the efficiency of polynomial-time algorithms, problems that can be solved in polynomial time are considered to be *easy* or *computationally tractable*. In what follows, I describe the concept of \mathcal{NP} -hardness, which is the defining property of problems that are considered to be computationally intractable.

I begin with two classes of decision problems known as \mathcal{P} and \mathcal{NP} . \mathcal{P} is the class of decision problems that can be solved in polynomial time (i.e., *efficiently solved*). \mathcal{NP} is the class of decision problems for which it can be verified in polynomial time whether a proposed solution is correct (i.e., *efficiently verified*). Every problem in \mathcal{P} is necessarily also in \mathcal{NP} (i.e., $\mathcal{P} \subseteq \mathcal{NP}$), for if a problem can be efficiently solved that efficiently verifies whether a proposed solution correct. Whether $\mathcal{P} = \mathcal{NP}$ (i.e., whether every decision problem that can be efficiently verified can also be efficiently solved) is an open question—indeed, it is a Millennium Prize Problem (Jaffe 2000). It is conjectured and presumed that $\mathcal{P} \neq \mathcal{NP}$.

A decision problem is \mathcal{NP} -complete if it is in \mathcal{NP} and every other problem in \mathcal{NP} is reducible to it. One problem is *reducible* to a second problem if the existence of an efficient solution to the second problem would imply the existence of an efficient solution to the first problem. In this sense, an \mathcal{NP} -complete problem is at least as hard as every other problem in \mathcal{NP} . It follows that an \mathcal{NP} -complete problem can be efficiently solved if and only if $\mathcal{P} = \mathcal{NP}$. However, because an \mathcal{NP} -complete problem is in \mathcal{NP} , it can be efficiently verified whether a proposed solution to an \mathcal{NP} -complete problem is correct, whether or not $\mathcal{P} = \mathcal{NP}$.

I now come to the concept of \mathcal{NP} -hardness. A computational problem (not necessarily a decision problem) is \mathcal{NP} -hard if every problem in \mathcal{NP} is reducible to it. By definition, every \mathcal{NP} -complete problem is \mathcal{NP} -hard, but not every \mathcal{NP} -hard problem is \mathcal{NP} -complete.⁶ Moreover, because every problem in \mathcal{NP} is reducible to an \mathcal{NP} -hard problem, an \mathcal{NP} -hard problem is at least as hard as every problem in \mathcal{NP} , including all \mathcal{NP} -complete problems, which are the hardest problems in \mathcal{NP} . It follows that an \mathcal{NP} -hard problem can be efficiently solved only if $\mathcal{P} = \mathcal{NP}$, or, equivalently, that no \mathcal{NP} -hard problem can be efficiently solved unless $\mathcal{P} = \mathcal{NP}$.⁷ Because

⁶Indeed, another way to define an \mathcal{NP} -complete problem is an \mathcal{NP} -hard problem that is in \mathcal{NP} .

⁷Even if $\mathcal{P} = \mathcal{NP}$, this would imply only that some, but not necessarily all, \mathcal{NP} -hard problems can be efficiently solved.

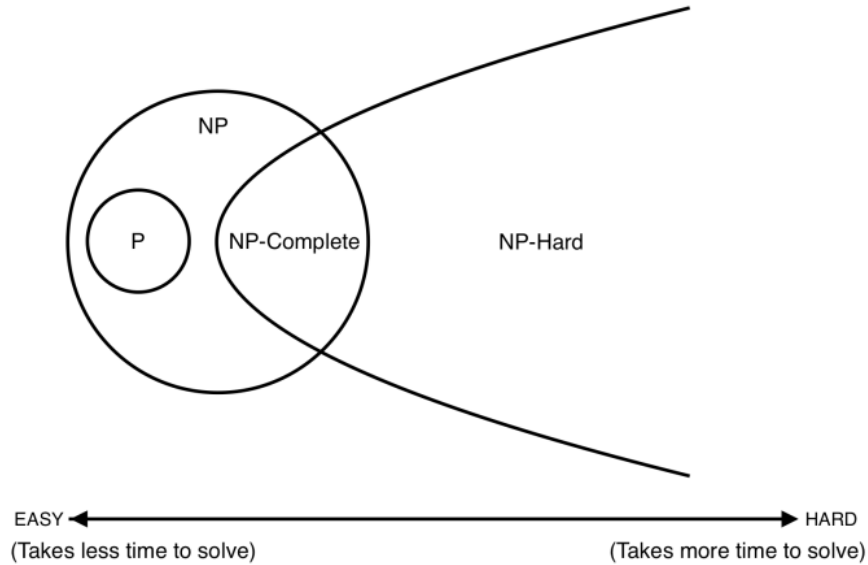


Figure 1: Relationship between \mathcal{P} , \mathcal{NP} , \mathcal{NP} -complete, and \mathcal{NP} -hard.

it is conjectured and presumed that $\mathcal{P} \neq \mathcal{NP}$, all \mathcal{NP} -hard problems, including all \mathcal{NP} -complete problems, are considered to be *computationally intractable*.

Figure 1 depicts the relationship between the classes \mathcal{P} , \mathcal{NP} , \mathcal{NP} -complete, and \mathcal{NP} -hard under the consensus presumption that $\mathcal{P} \neq \mathcal{NP}$.

2.3 Optimization Problems and Decision Problems

In Sections 3 and 4, I analyze the tractability of two types of computational problems: optimization problems and decision problems. An optimization problem has the following form: Given a set of feasible alternatives and an objective function defined on the feasible set, find a subset of the feasible set that minimizes (or maximizes) the value of the objective function. Every optimization problem has a corresponding decision problem of the following form: Given a set of feasible alternatives, an objective function defined on the feasible set, and a given value of the objective function, determine whether there is a subset of the feasible set such that the value of the objective function is not greater than (or not less than) the given value.

The following are two well-known examples of optimization problems.

Problem 1 (Traveling Salesman) Let $\mathcal{C} = \{1, \dots, n\}$ be a finite set of cities and let $d(i, j) \in \mathbb{Z}_+$ denote the distance between two cities $i, j \in \mathcal{C}$.⁸ In addition, let \mathcal{S}_n denote the set of all permutations of the elements of \mathcal{C} , representing the set of feasible routes that visit each city once. Find a route $\pi = (\pi(1), \dots, \pi(n)) \in \mathcal{S}_n$ that minimizes the total distance of the round trip defined by the route, given by $f(\pi) = \left[\sum_{i=1}^{n-1} d(\pi(i), \pi(i+1)) \right] + d(\pi(n), \pi(1))$.

Problem 2 (Maximum Directed Cut) Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph, where \mathcal{V} is the set of vertices and \mathcal{E} is the set of directed edges (i.e., ordered pairs of vertices), and let $c(e) \in \mathbb{Z}_+$ denote the capacity of an edge $e \in \mathcal{E}$.⁹ For every subset of vertices $A \subseteq \mathcal{V}$, let $\delta(A) = \{e = (u, v) \in \mathcal{E} \mid u \in A, v \in \mathcal{V} \setminus A\}$ denote the set of edges that cross the cut $\{A, \mathcal{V} \setminus A\}$. Find a subset $A \subseteq \mathcal{V}$ that maximizes the total capacity of the edges that cross the cut $\{A, \mathcal{V} \setminus A\}$, given by $f(A) = \sum_{e \in \delta(A)} c(e)$.

The decision problems that correspond to these optimization problems are:

1. Given $b \in \mathbb{Z}_{++}$, is there a route $\pi \in \mathcal{S}_n$ such that $f(\pi) \leq b$?
2. Given $b \in \mathbb{Z}_{++}$, is there a subset $A \subseteq \mathcal{V}$ such that $f(A) \geq b$?

These decision problems are both known to be \mathcal{NP} -complete (see, e.g., Garey and Johnson 1979).¹⁰ This implies that the corresponding optimization problems are \mathcal{NP} -hard. Indeed, any optimization problem is \mathcal{NP} -hard if the corresponding decision problem is \mathcal{NP} -complete. This follows from the fact that the corresponding decision problem is reducible to the optimization problem. More generally, two common ways of establishing that an optimization problem is \mathcal{NP} -hard are: (i) show that the corresponding decision problem is \mathcal{NP} -complete or (ii) show that the optimization problem generalizes a known \mathcal{NP} -hard optimization problem (which implies that the latter is reducible to the former).

2.4 Approximation Algorithms

If an optimization problem is \mathcal{NP} -hard, implying that it cannot be efficiently solved (assuming $\mathcal{P} \neq \mathcal{NP}$), there nevertheless may exist an efficient algorithm that approximates the problem within some constant factor. Such an algorithm is called an

⁸It is assumed that $d(i, j)$ can be evaluated in polynomial time for all $i, j \in \mathcal{C}$.

⁹It is assumed that $c(e)$ can be evaluated in polynomial time for all $e \in \mathcal{E}$.

¹⁰In fact, the Minimum Cut decision problem, the undirected version of Minimum Directed Cut decision problem, was one of the 21 problems that Karp (1972) famously proved are \mathcal{NP} -complete.

α -approximation algorithm, where α denotes the factor within which the algorithm is guaranteed to approximate the optimal value of the problem. For example, the Traveling Salesman optimization problem, in the instance where d satisfies the triangle inequality, is known to have a $\frac{3}{2}$ -approximation algorithm (Christofides 1976); that is, there exists a polynomial-time algorithm that outputs a route $\tilde{\pi}$ such that $f(\pi^*) \leq f(\tilde{\pi}) \leq \frac{3}{2}f(\pi^*)$, where π^* denotes an optimal route (i.e., a route that minimizes the total distance of the round trip).¹¹

More generally, if x^* is a solution to an optimization problem with objective function f , an α -approximation algorithm for the problem is a polynomial-time algorithm whose output \tilde{x} satisfies (i) $f(x^*) \leq f(\tilde{x}) \leq \alpha f(x^*)$ for some $\alpha \geq 1$, in the case of a minimization problem, or (ii) $\alpha f(x^*) \leq f(\tilde{x}) \leq f(x^*)$ for some $0 \leq \alpha \leq 1$, in the case of a maximization problem. If there exists an α -approximation algorithm for an optimization problem, we say that the problem can be *efficiently approximated* within a factor of α . If, in addition, the factor α is "reasonable" (a subjective judgment that depends on the specific instance of the problem), we say that the problem is *reasonably approximable*. If, however, an optimization problem cannot be efficiently approximated within a factor of α (assuming $\mathcal{P} \neq \mathcal{NP}$), we say that it is \mathcal{NP} -hard to approximate the problem within a factor of α .

3 Tort Deterrence

I analyze tort deterrence using the unilateral care model with fixed activity level. As noted above, the unilateral care model with fixed activity level is the foundational model upon which other economic models of tort deterrence are built. Although the analysis could be extended to richer models that contemplate, for example, bilateral care or variable activity level, it would not change the main takeaways of the analysis.

There are two agents: an injurer and a victim. Both are risk neutral expected utility maximizers. The agents are strangers and not in any contractual relationship. Transaction costs are sufficiently high to preclude Coasian bargaining.

The injurer engages in a risky activity. In the event of an accident, the victim incurs a loss. The injurer, but not the victim, can take precautions against an accident. The injurer's set of feasible precautions is the *precaution set*.

¹¹The distance d satisfies that triangle inequality if $d(i, k) \leq d(i, j) + d(j, k)$ for all $i, j, k \in \mathcal{C}$.

The governing liability rule determines whether the injurer is liable to the victim for her loss in the event of an accident. I consider the two basic liability rules of tort law: negligence and strict liability. Under negligence, the injurer is liable to the victim if the injurer failed to exercise due care (a legal standard set by the court) when engaging in his activity. Under strict liability, the injurer is liable to the victim whether or not the injurer exercised due care when engaging in his activity.

A liability rule is *optimal* if the solution to the injurer's problem under that rule is socially optimal. A liability rule is *effective* if the injurer's problem under that rule is computationally tractable or reasonably approximable. After all, if the injurer's problem cannot be efficiently solved or approximated within a reasonable factor, the rule cannot effectively regulate the injurer's behavior. Only a liability rule that is both optimal and effective can achieve socially optimal deterrence.

3.1 Convex Care

The standard "convex care" version of the model makes the following assumptions (cf. Shavell 1987).

(C1) The precaution set is convex—usually \mathbb{R}_+ , interpreted as the set of feasible levels of spending on safety.¹²

(C2) The injurer chooses a level of care $x \in \mathbb{R}_+$ having cost $c(x) \geq 0$, where the cost of care function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is monotone increasing and convex.

- Monotone increasing: $c(x) \leq c(y)$ for all $x \leq y$ in \mathbb{R}_+ .
- Convex: $c(x+z) - c(x) \leq c(y+z) - c(y)$ for all $x \leq y$ in \mathbb{R}_+ and $z \in \mathbb{R}_+$.¹³

In other words, the injurer's cost of care increases with additional care at an increasing rate, reflecting increasing marginal cost of care.

(C3) The victim's expected loss (i.e., the probability of an accident multiplied by the amount of the victim's loss in the event of an accident) is $\ell(x) \geq 0$, where the expected loss function $\ell : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is monotone decreasing and convex.

¹²A set $X \subseteq \mathbb{R}^n$ is convex if every convex combination of every pair of elements of X lies in X , i.e., if $\lambda x + (1 - \lambda)y \in X$ for all $x, y \in X$ and $\lambda \in [0, 1]$ (see, e.g., Boyd and Vandenberghe 2004).

¹³This definition, which characterizes convexity in terms of increasing differences, follows from the standard definition that a function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is convex if $c(\lambda x + (1 - \lambda)y) \leq \lambda c(x) + (1 - \lambda)c(y)$ for all $x, y \in \mathbb{R}_+$ and $\lambda \in [0, 1]$. See, e.g., Simchi-Levi, Bramel, and Chen (2005, prop. 2.2.6).

- Monotone decreasing: $\ell(x) \geq \ell(y)$ for all $x \leq y$ in \mathbb{R}_+ .
- Convex: $\ell(x+z) - \ell(x) \leq \ell(y+z) - \ell(y)$ for all $x \leq y$ in \mathbb{R}_+ and $z \in \mathbb{R}_+$.

In other words, the victim's expected loss decreases with additional care at a decreasing rate, reflecting decreasing marginal benefit of care (i.e., diminishing marginal returns to care).

(C4) Both c and ℓ can be evaluated in polynomial time at all $x \in \mathbb{R}_+$. This is an unstated assumption in prior expositions of the standard model.

In addition, the cost of care and expected loss functions are usually assumed to be twice differentiable. If c and ℓ are twice differentiable, then assumption (C2) implies that $c'(x) \geq 0$ and $c''(x) \leq 0$ at all $x \in \mathbb{R}_+$, and assumption (C3) implies that $\ell'(x) \leq 0$ and $\ell''(x) \geq 0$ at all $x \in \mathbb{R}_+$.

3.1.1 The Social Problem

The social problem is to find a level of care $x \in \mathbb{R}_+$ that minimizes the total cost of the injurer's activity (the injurer's cost of care plus the victim's expected loss):

$$\underset{x \in \mathbb{R}_+}{\text{minimize}} \quad c(x) + \ell(x).$$

I assume that the social problem has a unique interior solution. By definition, the level of care $x^* \in \mathbb{R}_+$ that solves the social problem is *socially optimal*.

Given assumptions (C1)–(C4), the social problem is a standard convex optimization problem—it entails the minimization of a convex function over a convex set.¹⁴ It follows that the social problem is computationally tractable, for it can be efficiently solved using known polynomial-time algorithms for convex optimization problems. For instance, it can be efficiently solved using subgradient methods if the total cost function, $s(x) \equiv c(x) + \ell(x)$, is non-differentiable (see, e.g., Bertsekas 2016) or interior-point methods if the total cost function is differentiable (see, e.g., Boyd and Vandenberghe 2004). In the non-differentiable case, x^* satisfies the condition $0 \in \partial s(x^*)$, where $\partial s(x)$ is the subdifferential of s at x .¹⁵ In the differentiable case, x^* satisfies

¹⁴Observe that $c(x) + \ell(x)$ is convex because the sum of two convex functions is convex.

¹⁵Intuitively, the subdifferential of s at x is the set of "derivatives" of s at x (i.e., the set of slopes of the lines that intersect s only at x). If s is differentiable at x then $\partial s(x) = s'(x)$. Otherwise $\partial s(x)$ is an interval. For example, suppose $s(x) = |x|$. Note that s is differentiable at all $x \in \mathbb{R}$ except $x = 0$. Then $\partial s(x) = -1$ for all $x < 0$, $\partial s(x) = 1$ for all $x > 0$, and $\partial s(x) = [-1, 1]$ at $x = 0$.

the condition $0 = s'(x^*)$. In both cases, x^* is the level of care at which the marginal cost of care equals the marginal benefit of care (i.e., the marginal reduction in expected loss). This can be readily seen in the differentiable case, where the condition $0 = s'(x^*)$ is equivalent to the condition $c'(x^*) = -\ell'(x^*)$.

3.1.2 The Injurer's Problem

Under strict liability, the injurer's problem is

$$\underset{x \in \mathbb{R}_+}{\text{minimize}} \quad c(x) + \ell(x).$$

This problem is identical to the social problem, because strict liability forces the injurer to internalize the total cost of his activity. Under strict liability, therefore, the solution to injurer's problem is socially optimal and the injurer's problem is computationally tractable. Hence, strict liability is both optimal and effective.

Under negligence, the injurer effectively faces strict liability if he chooses a level of care $x < \bar{x}$, where $\bar{x} \in \mathbb{R}_+$ is the due care standard set by the court, and he faces no liability if he chooses $x \geq \bar{x}$. Accordingly, the injurer's problem under negligence has two stages. The first is

$$1. \underset{x \in [0, \bar{x}]}{\text{minimize}} \quad c(x) + \ell(x).$$

Let $\hat{x} \in \mathbb{R}_+$ denote the solution, which I assume is unique. The second stage is

$$2. \text{ choose } \begin{cases} x = \hat{x} & \text{if } c(\hat{x}) + \ell(\hat{x}) < c(\bar{x}) \\ x = \bar{x} & \text{if } c(\hat{x}) + \ell(\hat{x}) \geq c(\bar{x}) \end{cases}.$$

That is, the injurer chooses $x = \hat{x}$ if it is better (i.e., cheaper in expectation) to take \hat{x} and face potential liability for an accident than to take due care and avoid potential liability; otherwise he chooses to take due care, $x = \bar{x}$.

If the court sets the due care standard equal to the socially optimal level of care, $\bar{x} = x^*$, then negligence is also optimal and effective. Negligence with $\bar{x} = x^*$ is optimal because x^* is the solution to the injurer's problem under this rule. To see this, observe that if $\bar{x} = x^*$ then $\hat{x} = x^*$ and thus $c(\hat{x}) + \ell(\hat{x}) = c(x^*) + \ell(x^*) < c(x^*) = c(\bar{x})$,

which implies that the injurer chooses $x = x^*$.¹⁶ Negligence with $\bar{x} = x^*$ (or, for that matter, any $\bar{x} \in \mathbb{R}_+$) is effective because both stages of the injurer's problem are computationally tractable: the first stage is a standard convex optimization problem, which can be efficiently solved, and the second stage amounts to verifying whether \hat{x} is a solution to the decision problem, "Given $c(\bar{x}) \in \mathbb{R}_+$, is there a level of care $x \subseteq \mathbb{R}_+$ such that $c(x) + \ell(x) < c(\bar{x})$?", which can be efficiently verified per assumption (C4).

Because strict liability and negligence with $\bar{x} = x^*$ are both optimal and effective, both liability rules can achieve socially optimal deterrence. The following theorem recaps the preceding results.

Theorem 1 *In the case of convex care, strict liability and negligence with $\bar{x} = x^*$ are both optimal and effective. Hence, both strict liability and negligence can achieve socially optimal deterrence.*

3.2 Discrete Care

In the standard model, the assumption that the precaution set is convex is instrumental to ensuring that the injurer's problem is computationally tractable. In reality, however, the precaution set is often discrete. In what follows, I re-examine tort deterrence using a "discrete care" version of the unilateral accident model with fixed activity level.

The discrete care version of the model makes the following assumptions.

- (D1) The precaution set is discrete—denoted \mathcal{N} , interpreted as the set of feasible safety measures. I assume that \mathcal{N} is finite but "large," to rule out trivial cases and to make the discrete case as comparable as possible to the convex case (where the precaution set is uncountably infinite).
- (D2) The injurer chooses a subset of care $X \subseteq \mathcal{N}$ having cost $c(X) \geq 0$, where the cost of care function $c : 2^{\mathcal{N}} \rightarrow \mathbb{R}_+$ is monotone increasing and supermodular.¹⁷

- Monotone increasing: $c(X) \leq c(Y)$ for all $X \subseteq Y \subseteq \mathcal{N}$.

¹⁶Negligence with $\bar{x} < x^*$ is never optimal (because the solution to the injurer's problem under this rule is $x = \bar{x} < x^*$). Negligence with $\bar{x} > x^*$ may be optimal for some c and ℓ , but only negligence with $\bar{x} = x^*$ is optimal for all c and ℓ .

¹⁷Note that $2^{\mathcal{N}}$ denotes the set of all subsets of \mathcal{N} .

- Supermodular: $c(X \cup \{i\}) - c(X) \leq c(Y \cup \{i\}) - c(Y)$ for all $X \subseteq Y \subseteq \mathcal{N}$ and $i \in \mathcal{N} \setminus Y$.¹⁸

In other words, the injurer’s cost of care increases with additional care at an increasing rate, reflecting increasing marginal cost of care. In this sense, a monotone increasing supermodular set function is the discrete analog of a monotone increasing convex function.

(D3) The victim’s expected loss is $\ell(X) \geq 0$, where the expected loss function $\ell : 2^{\mathcal{N}} \rightarrow \mathbb{R}_+$ is monotone decreasing and supermodular.

- Monotone decreasing: $\ell(X) \geq \ell(Y)$ for all $X \subseteq Y \subseteq \mathcal{N}$.
- Supermodular: $\ell(X \cup \{i\}) - \ell(X) \leq \ell(Y \cup \{i\}) - \ell(Y)$ for all $X \subseteq Y \subseteq \mathcal{N}$ and $i \in \mathcal{N} \setminus Y$.

In other words, the victim’s expected loss decreases with additional care at a decreasing rate, reflecting decreasing marginal benefit of care (i.e., diminishing marginal returns to care). In this sense, a monotone decreasing supermodular set function is the discrete analog of a monotone decreasing convex function.

(D4) Both c and ℓ can be evaluated in polynomial time at all $X \subseteq \mathcal{N}$.

3.2.1 The Injurer’s Problem

As before, the injurer’s problem under strict liability coincides with the social problem—minimize the total cost of the injurer’s activity:

$$\underset{X \subseteq \mathcal{N}}{\text{minimize}} \quad c(X) + \ell(X).$$

Like before, I assume this problem has a unique interior solution. Because the subset of care $X^* \subseteq \mathcal{N}$ that solves this problem is socially optimal, strict liability is optimal.

The injurer’s problem under negligence is

$$\begin{aligned} & 1. \underset{X \subseteq \bar{X}}{\text{minimize}} \quad c(X) + \ell(X); \\ & 2. \text{ choose } \begin{cases} X = \hat{X} & \text{if } c(\hat{X}) + \ell(\hat{X}) < c(\bar{X}) \\ X = \bar{X} & \text{if } c(\hat{X}) + \ell(\hat{X}) \geq c(\bar{X}) \end{cases}, \end{aligned}$$

¹⁸See, e.g., Bach (2013).

where $\overline{X} \subseteq \mathcal{N}$ is the due care standard set by the court and $\widehat{X} \subseteq \mathcal{N}$ is the unique solution to the first-stage minimization problem. If the court sets $\overline{X} = X^*$, then X^* is the solution to the injurer’s problem under negligence. The logic here is analogous to the logic in the case of convex care. Hence, negligence with $\overline{X} = X^*$ is also optimal.¹⁹

In general, however, neither strict liability nor negligence is effective in the case of discrete care. I reach this conclusion on the basis of the following two propositions.

Proposition 1 *In the case of discrete care, it is \mathcal{NP} -hard to approximate the injurer’s problem, whether under strict liability or negligence, within any constant factor.*

Proof. The proof is by reduction from Maximum Directed Cut. Let $f : 2^{\mathcal{V}} \rightarrow \mathbb{R}_+$ denote the cut capacity function in the Maximum Directed Cut optimization problem defined in Section 2. It is known that f is submodular, non-negative, and not necessarily monotone or symmetric (see, e.g., Feige, Mirronki, and Vondrák 2011). In addition, let $\overline{C} = \sum_{e \in \mathcal{E}} c(e)$ denote the total capacity of all edges in the graph.

For every subset of vertices $A \subseteq \mathcal{V}$, let $g(A) = \overline{C} - f(A) + \eta$, where $\eta > 0$. Observe that $g : 2^{\mathcal{V}} \rightarrow \mathbb{R}_+$ is a supermodular set function.²⁰ Note further that g is non-negative and not necessarily monotone or symmetric.

It is known that it is \mathcal{NP} -hard to approximate the Maximum Directed Cut optimization problem within a factor greater than $\frac{12}{13}$ (Håstad 2001, thm. 8.3). This implies that it is \mathcal{NP} -hard to distinguish between the following two mutually exclusive instances of the Maximum Directed Cut decision problem:

1. There exists a subset $A \subseteq \mathcal{V}$ such that $f(A) = \overline{C}$.
2. There does not exist a subset $A \subseteq \mathcal{V}$ such that $f(A) > \frac{12}{13}\overline{C}$.

Note that in the first instance the minimum value of g is η , while in the second instance the minimum value of g exceeds $\frac{1}{13}\overline{C}$.

Suppose there exists an α -approximation algorithm for the injurer’s problem—the minimization of a real-valued supermodular set function over a discrete set where the objective function is non-negative and not necessarily monotone or symmetric. Then we could apply this algorithm to the problem $\max_{A \subseteq \mathcal{V}} g(A)$. In the first instance, the

¹⁹As before, negligence with $\overline{X} \subset X^*$ is never optimal, negligence with $\overline{X} \supset X^*$ may be optimal for some c and ℓ , but only negligence with $\overline{X} = X^*$ is optimal for all c and ℓ .

²⁰If f is a submodular set function then $-f$ is supermodular. Moreover, the sum of a supermodular set function and a constant is supermodular. Note that $\overline{C} + \eta$ is a constant.

algorithm would return a subset \hat{A} such that $g(\hat{A}) \leq \alpha\eta$. In the second instance, it would return a subset \hat{A} such that $g(\hat{A}) > \frac{1}{13}\overline{C}$. Because η is arbitrary, it can be chosen so that $\alpha\eta < \frac{1}{13}\overline{C}$. This would make it possible to distinguish between the two instances of Maximum Directed Cut, because in the first instance the algorithm would yield $g(\hat{A}) \leq \alpha\eta < \frac{1}{13}\overline{C}$, while in the second instance it would yield $\alpha\eta < \frac{1}{13}\overline{C} < g(\hat{A})$. This, however, contradicts the fact that it is \mathcal{NP} -hard to distinguish between the two instances. It follows, therefore, that there does not exist an α -approximation algorithm for the injurer's problem, which is equivalent to the statement that it is \mathcal{NP} -hard to approximate the injurer's problem within any constant factor. ■

Remark. Mittal and Schulz (2013, thm. 8) prove a similar result for the minimization of an integer-valued supermodular set function. Their proof is by reduction from the E4-Set Splitting optimization problem.²¹ The proof of Proposition 1, which is by reduction from the Maximum Directed Cut optimization problem, generalizes their result to a real-valued supermodular set function.

Proposition 2 *In the case of discrete care, the injurer's problem, whether under strict liability or negligence, is \mathcal{NP} -hard.*

Proof. The result follows immediately from Proposition 1. If it is \mathcal{NP} -hard to approximate the injurer's problem within any constant factor, then it is \mathcal{NP} -hard to approximate the injurer's problem within a factor of 1, which is equivalent to the statement that the injurer's problem is \mathcal{NP} -hard. ■

Remark. The foregoing results on the hardness of supermodular minimization stand in contrast to the fact that submodular minimization is easy (Grötschel, Lovász, and Schrijver 1981). The basic reason is that the convex closure of a submodular set function has a closed form that is easy to compute, while this generally is not the case for supermodular set functions (see, e.g., Bach 2013).²² Consequently, one can easily solve a submodular minimization problem by leveraging the fact that the minimum of a set function is equivalent to the minimum of its convex closure, while this generally is not the case for a supermodular minimization problem.

²¹For a statement of the E4-Set Splitting optimization problem, see, e.g., Håstad (2001, def. 2.9).

²²The convex closure of a real-valued set function f on \mathcal{N} is the greatest real-valued convex function on $\mathbb{R}^{|\mathcal{N}|}$ that everywhere lowerbounds f . The convex closure of a real-valued submodular set function is known as the Lovasz extension (Lovász 1983).

Propositions 1 and 2 establish that the injurer’s problem, whether under strict liability or negligence, is neither computationally tractable nor reasonably approximable. Hence, neither strict liability nor negligence is effective in the case of discrete care. The following theorem recaps the preceding results.

Theorem 2 *In the case of discrete care, although both strict liability and negligence with $\bar{X} = X^*$ are optimal, neither liability rule is effective. Hence, neither strict liability nor negligence can achieve socially optimal deterrence.*

4 Dynamics of Tort Deterrence

The main takeaway of the previous section—that neither strict liability nor negligence can achieve socially optimal deterrence in the case of discrete care—is based on a static analysis. In this section, I consider the dynamics of tort deterrence in the case of discrete care. I argue that (i) the injurer’s behavior, under either liability rule, moves in the direction of socially optimal care over time through a learning-by-experimentation process, but that (ii) negligence has a dynamic advantage over strict liability in that negligence accelerates the injurer’s learning process.

The starting point is the observation that the decision problems corresponding to the injurer’s problem under strict liability and negligence are in \mathcal{NP} .

Proposition 3 *In the case of discrete care, the decision problem that corresponds to the injurer’s problem, whether under strict liability or negligence, is in \mathcal{NP} .*

Proof. The decision problem that corresponds to the injurer’s problem under strict liability is: Given $b \in \mathbb{R}_+$, is there a subset of care $X \subseteq \mathcal{N}$ such that $c(X) + \ell(X) \leq b$? The decision problem that corresponds to the injurer’s problem under negligence is: Given $c(\bar{X}) \in \mathbb{R}_+$, is there a subset of care $X \subseteq \bar{X}$ such that $c(X) + \ell(X) \leq c(\bar{X})$? Take either decision problem and suppose we are given a proposed solution $Y \subseteq \mathcal{N}$. Per assumption (D4), it can be efficiently verified whether $c(Y) + \ell(Y) \leq b$ or $c(Y) + \ell(Y) \leq c(\bar{X})$, as the case may be. Hence, each decision problem is in \mathcal{NP} . ■

Now, consider how the injurer’s behavior evolves over time as he repeatedly engages in his risky activity, periodically causing an accident. Let $t = 0$ denote the time before the injurer first engages in his activity, let $t = 1$ denote the time after the first

accident but before the injurer next engages in his activity, let $t = 2$ denote the time after the second accident but before the injurer next engages in his activity, and so forth. In addition, assume that: (i) each time there is an accident, the circumstances of the accident suggest a subset of care $Y_t \subseteq \mathcal{N}$ that would have prevented the accident; (ii) each time there is an accident, the victim brings suit against the injurer before the court; and (iii) the injurer, the victim, and the court all have access to the same computational methods.

Suppose first that the governing liability rule is strict liability. At $t = 0$, the injurer chooses a subset of care $X_0 \subseteq \mathcal{N}$. However, because the injurer's problem is neither computationally tractable nor reasonably approximable, it is likely that $X_0 \neq X^*$. At $t = 1$, the injurer can efficiently verify whether $c(Y_1) + \ell(Y_1) \leq c(X_0) + \ell(X_0)$, i.e., whether Y_1 is superior to X_0 . This follows from Proposition 3. If Y_1 is superior to X_0 , the injurer adopts $X_1 = Y_1$; otherwise he stands pat at $X_1 = X_0$. At $t = 2$, the injurer can efficiently verify whether Y_2 is superior to X_1 . If it is, the injurer adopts $X_2 = Y_2$; otherwise he stands pat at $X_2 = X_1$. And so forth. In this way, as $t \rightarrow \infty$, the injurer's behavior moves in the direction of socially optimal care X^* .

Suppose next that the governing liability rule is negligence, with due care standard \bar{X}_0 at $t = 0$. Note that because the social problem is neither computationally tractable nor reasonably approximable, it is likely that $\bar{X}_0 \neq X^*$. Note further that because the injurer and the court have access to the same computational methods, the injurer always chooses to take due care: $X_t = \bar{X}_t$ at all t . Thus, we need only consider the evolution of the due care standard. At $t = 1$, the court can efficiently verify whether Y_1 is superior to \bar{X}_0 . This follows from Proposition 3 and the fact that the social problem is identical to the injurer's problem under strict liability. If Y_1 is superior to \bar{X}_0 , the court adopts $\bar{X}_1 = Y_1$; otherwise it stands pat at $\bar{X}_1 = \bar{X}_0$. At $t = 2$, the court can efficiently verify whether Y_2 is superior to \bar{X}_1 . If it is, the court adopts $\bar{X}_2 = Y_2$; otherwise it stands pat at $\bar{X}_2 = \bar{X}_1$. And so forth. In this way, as $t \rightarrow \infty$, the injurer's behavior moves in the direction of socially optimal care X^* .

So far the analysis suggests that the evolution of the injurer's behavior over time is the same under strict liability and negligence. Each time there is an accident, nature proposes a solution Y_t and the injurer can efficiently verify whether Y_t is superior to the status quo and adapt his behavior accordingly. It is a learning-by-experimentation process, akin to the process of learning the probability distribution of an unfair coin through repeated flips.

The dynamic advantage of negligence emerges when we add other injurers to the story who engage in the same risky activity but whose accidents are unobserved by our injurer. We can illustrate the point by adding just one other such injurer. Suppose that the other injurer's accidents occur at periods $t = \frac{1}{2}, t = \frac{3}{2}$, and so forth, but that the two injurers are otherwise identical (same precaution set, same cost of care function, etc.). Return first to the case where strict liability is the governing liability rule. Because our injurer does not observe the other injurer's accidents, he does not observe the sequence $Y_{\frac{1}{2}}, Y_{\frac{3}{2}}, \dots$. He therefore cannot learn from the other injurer's accidents, implying that his learning process is the same as before, with adaptations in his behavior possible only at periods $t = 1, 2, \dots$.

Now return to the case where negligence is the governing liability rule. Although our injurer does not observe the sequence $Y_{\frac{1}{2}}, Y_{\frac{3}{2}}, \dots$, the court does. Consequently, the court's learning process is faster than before, with adaptations in the due care standard possible at periods $t = \frac{1}{2}, 1, \frac{3}{2}, 2, \dots$. This accelerates our injurer's learning process, for he observes the sequence $\bar{X}_{\frac{1}{2}}, \bar{X}_1, \bar{X}_{\frac{3}{2}}, \bar{X}_2, \dots$, making adaptations in his behavior more frequent.²³ In this way, negligence can move the injurer's behavior in the direction of socially optimal care more rapidly than strict liability.

5 Conclusion

Standard economic models of tort deterrence assume that an injurer's precaution set is convex. In reality, however, the precaution set is often discrete. I show that in cases with discrete care, the injurer's care-taking problem under strict liability and negligence is computationally intractable, frustrating their static deterrence effects. I then argue that while both liability rules can be effective in terms of dynamic deterrence, negligence has a dynamic advantage over strict liability in that negligence, due to its information-generating property, can move the injurer's behavior in the direction of socially optimal care over time more rapidly than strict liability.

My analysis leads me to two main conclusions. The first is that the standard assumption of convex care, though seemingly innocuous, is pivotal to constituting the injurer as Holmes' unboundedly rational "bad man"—*homo law-and-economicus*.²⁴ Insofar as discrete care is more realistic, the injurer may be more like H.L.A. Hart's

²³Indeed, the more injurers we add to the story, the greater is the rate of this acceleration.

²⁴The term *homo law-and-economicus* was coined by Gordon (1997, p. 1014).

"puzzled man . . . who is willing to do what is required, if only he can be told what it is" (Hart 1961, p. 40). The second conclusion is that negligence is more robust than strict liability to the reality of bounded rationality and limited computational capacity. While this does not imply that negligence is superior to strict liability in all circumstances, it may help to explain why negligence is the general basis for accident liability under modern Anglo-American tort law.

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