

The formation of drops by nozzles and the breakup of liquid jets

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Abstract

Shows by means of high-speed motion pictures that the breakup of a jet passes successively through three phases (breakup according to Rayleigh — breakup according to Weber-Haenlein — atomization), and that the transition from one phase to the next occurs at increasing higher Reynolds No's. as a function of a dimensionless term, containing viscosity, surface tension, density, and orifice diameter $Z = \eta/\sqrt{\sigma\rho d}$ where η is the absolute viscosity, σ is the surface tension, ρ is the density and d is the orifice diameter.¹

Translator's note: Translation from German of: W. von Ohnesorge. "Die Bildung von Tropfen an Düsen und die Auflösung flüssiger Strahlen [The formation of drops by nozzles and the breakup of liquid jets]". *Zeitschrift für Angewandte Mathematik und Mechanik* 16(6) (Jan. 1, 1936), pp. 355–358. ISSN: 1521-4001. DOI: 10.1002/zamm.19360160611.

The margin indicates the page of the original text. All underlining added by the translator. Underlining indicates phrases for which the exact translation is unclear.

Translated by Ben Trettel (<https://trettelresearch.com/contact.html>) via Google Translate and Reverso Context. Last updated on 2019-10-20. SVN revision #2368.

¹Translator's note: There is no abstract in the original. Abstract from K. J. DeJuhasz. *Spray Literature Abstracts*. In collab. with E. A. Scicchitano. Vol. 1. 4 vols. New York, NY: American Society of Mechanical Engineers, 1959. 384 pp. URL: <https://hdl.handle.net/2027/uc1.b3688902>. OCLC: 1704784, 7414497. LCCN: 59004809, pp. 253–254.

The various types of drop formation in the disintegration of liquid jets injected into the atmosphere, which emerge partly under gravity, partly under pressure, have been recorded by means of a high-frequency cinematographic apparatus. As the speed is increased, four cases are distinguished: p. 355

0) Slow dripping from the nozzle under gravity without jet formation,

I) Breakup of a cylindrical jet through by axisymmetric surface perturbations (according to Rayleigh),

II) Breakup by screw-symmetric perturbations of the jet (wavy breakup according to Weber and Haenlein),

III) Atomization of the jet.

The attached table provides information about the magnitude of the involved forces, in which the capillary forces are taken as one: p. 356

process	inertial forces	gravitational forces	capillary forces	direct effect of viscosity	indirect effect of viscosity
Case 0	$\rightarrow 0$	significant	1	$\rightarrow 0$	0
Case I	moderate	very low	1	very low	$\rightarrow 0$
Case II	increasing	~ 0	1	significant	moderate
Case III	very strong	$\rightarrow 0$	1	strong	significant

As the speed of drop formation increases, both inertial and viscous forces gain in influence as gravitational forces decrease. The indirect effect of viscosity means vortex shedding by internal viscosity.

The theoretical explanation of the low-speed drop formation and the jet breakup through the effect of axisymmetric oscillations goes back to Rayleigh, who bases his theory on the assumption of inviscid liquids and potential flow, thus taking gravity and capillarity into account. We owe a refinement of the theory by considering the efforts of C. Weber². The mathematical treatment of dynamic processes presents insurmountable difficulties. For the general equations of motion of Navier-Stokes for viscous fluids, there are no exact solutions today, as long as inertia and viscosity interact in approximately the same order of magnitude. In such cases, the laws of mechanical similarity provide reliable information, in particular for the evaluation and processing of the necessary tests.

²ZAMM 11 (1931), p. 136. Translator's note: English translation available as: C. Weber. *Breakup of a Liquid Jet*. Ed. by B. Trettel. Trans. by C. H. Hutchinson, W. J. Love, and The Technical Library Research Service. Oct. 20, 2019. DOI: 10.26153/tsw/3371.

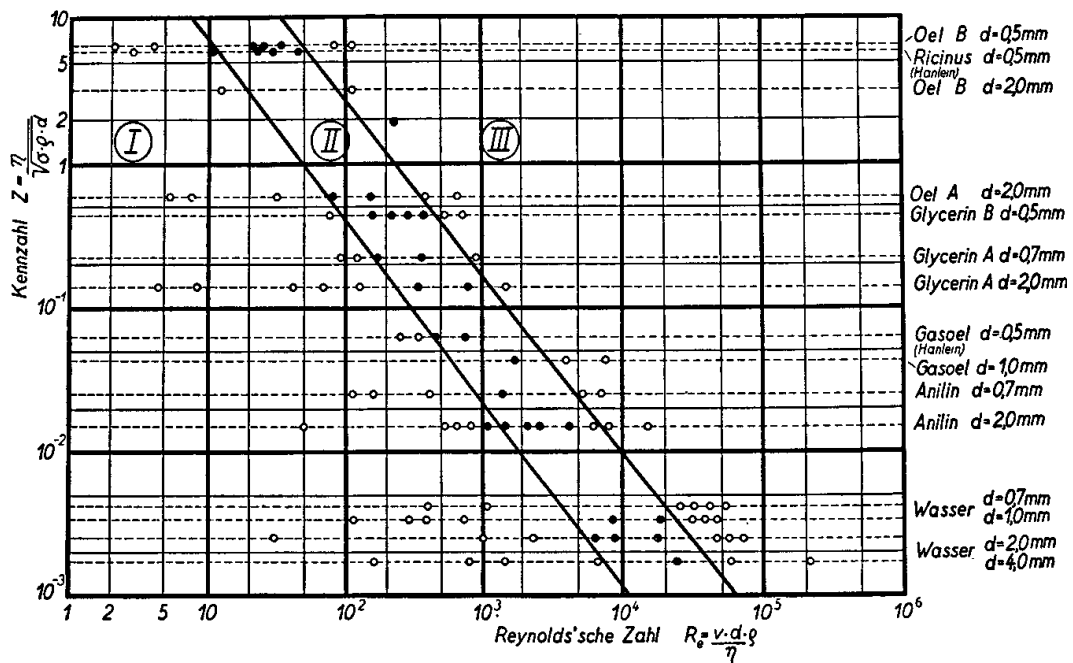


Figure 1: Presentation of the test results in the graph $Z = Z(Re)$. The breakup of the jet takes place in one of the three characteristic forms within the regions I, II or III.

For the static case 0, the dimensionless identifier r/a is significant, where r is the radius of the nozzle bore and a is the “Laplace constant” calculated from the material values σ and density ρ : $a = \sqrt{\frac{\sigma}{\rho g}}$. The static meridian curve of the axisymmetric droplet can be calculated by numerical integration and thus the weight of the falling droplet can be determined with a large approximation Check within the practically important range $0.01 < r/a < 1.0$.

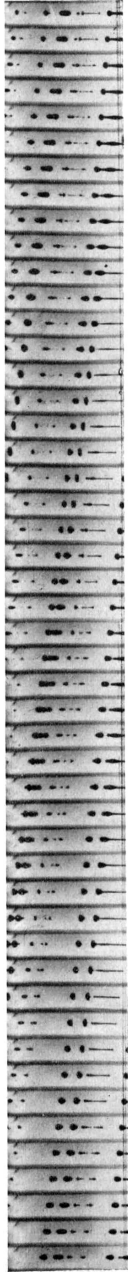
In the case of jet breakup and atomization, Weber’s similarity law of capillarity and Reynolds’ similarity law of viscosity are taken into account, while the Froudean law is meaningless. At different density of the surrounding medium, Reynolds’ law must be fulfilled for the external medium. The realization of Weber’s and Reynolds’ law both succeeds in our case, because on the one hand the two numbers have similar structure as power laws, on the other hand the capillarity of the liquids varies Check only within relatively narrow limits. For the interaction of both laws we form a new dimensionless index by eliminating the velocity p. 358

$$Z = \frac{\eta}{\sqrt{\sigma \rho d}},$$

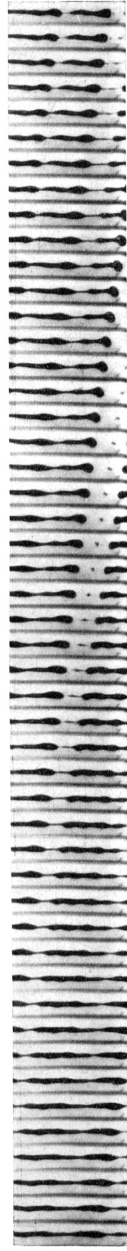
in which all the characteristic material coefficients are: surface tension σ , density ρ , absolute viscosity η , and the diameter d of the nozzle bore, which we use as a function of the Reynolds number Re using the test results (figure 1). In the graph $Z = Z(Re)$, each liquid, with the nozzle diameter held constant, corresponds to a straight line parallel to the abscissa. The type of jet breakup identified by the Reynolds number is then assigned a particular point on the line $Z = \text{constant}$. The experiments show that the regions of processes I, II, and II are separated by straight lines, which denote power laws. Region I is in the range of small Reynolds numbers, especially for liquids with high internal viscosity.

Process II takes place after a critical area has been exceeded for all the fluids examined within a relatively narrow range. The atomization process III starts suddenly. The jet shows a smooth surface in the immediate vicinity of the nozzle outlet with axisymmetric swellings, which degenerate into helical transverse displacements of increasing amplitude. In the bent out parts, intermediate droplets form, and finally the jet shatters completely. With only slightly increased Reynolds numbers, the atomizing cone jumps back to the nozzle outlet. Check

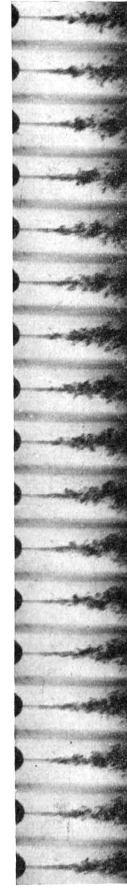
The cinematographic recordings with frequencies between 200 and 12,000 pictures per second have been made in the Institute for Technical Flow Research, headed by Prof. Föttinger, at the Technical University of Berlin, with the high-speed camera developed there. Check



(a) Breakup of a cylindrical jet through the effect of of axisymmetric vibrations (water). $Re = 1000$. Frame rate: 650 sec^{-1} .



(b) Jet breakup through the effect of screw-symmetric vibrations (water). $Re = 9800$. Frame rate: 1600 sec^{-1} .



(c) Start of jet atomization (aniline). $Re = 6200$. Frame rate: 4200 sec^{-1} .

Figure 2