# Supplementary information for Fast Objective Coupled Planar Illumination Microscopy 

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## Supplementary Note 1: Aberrations for non-focal imaging

We analyze the effect on image quality when imaging outside of the native focal plane of an imaging system - referred to here as non-focal imaging. We begin by considering a conventional microscope imaging system configured to acquire an image of the native focal plane of the objective (diagrammed below, top). Modern microscopes are usually implemented with two lenses: an objective lens that faces the sample, and a tube lens that focuses light collected by the objective onto a camera sensor (or alternatively an eyepiece). The objective lens is designed to achieve diffraction-limited imaging of a single (native) object plane. Likewise a conventional tube lens forms a well-resolved image at a single image plane. Images of non-native object planes are also formed at non-native image planes (diagrammed in magenta) but in a conventional microscope these planes are not imaged because the camera sensor is coincident with the native plane.

Conventional Imaging


Non-focal Imaging


Some recently-developed microscopes bring these non-native planes into focus at the camera sensor by using a tube lens with a tuneable focal length (diagrammed below, bottom). Modern tuneable lenses can be adjusted very quickly, opening up the possibility of achieving high-speed 3D imaging simply
by scanning through a range of focal lengths. The drawback of this approach is that images of non-native planes suffer degraded image quality due to aberrations. Tuning the focal length of the tube lens corrects for the lowest order (defocus) aberration, but cannot correct for higher order aberrations (spherical, astigmatism, coma, etc). This is not due to imperfections in the tube lens: if the tube lens in isolation was fully aberration-corrected at each focal length in its range then the performance of the system would still be limited by the objective lens because it is not tuneable. Thus regardless of which tube lens is used, an objective lens that is not tuneable can only form an aberration-corrected image of a single object plane, as established by the Abbe sine condition.

Before reviewing the sine condition and exploring its implications for a nonfocal imaging system we note that if the tube lens could be tuned to correct not just defocus but all the aberrations of the combined tube lens/objective system, then one could indeed achieve diffraction-limited volume imaging. However such a system presents an extreme engineering challenge, especially considering the NA values and fields of view for modern microscope objectives. Current tuneable lenses are far from meeting this requirement. Thus in the analysis below we quantify the severity of spherical aberration when imaging in nonnative planes with a static objective lens and a tube lens that can be tuned to correct for system defocus. Since light is focused at infinity in the region between the two lenses, the distance between the lenses does not affect our analysis, and the system behaves as a single-lens system subject to the sine condition, diagrammed in the following page.


Consider a point source along the optic axis separated by $\Delta z$ from the native object focal plane. Rays are emitted from this object over a range of angles and can be traced to the native image focal plane.

The sine condition states that in an aberration-corrected system each ray passing through the native object and image planes satisfies

$$
\begin{equation*}
n \sin \theta=M n^{\prime} \sin \theta^{\prime} \tag{1}
\end{equation*}
$$

where $n$ is the refractive index of the object immersion medium, $n^{\prime}$ that of the imaging immersion medium (here taken to be 1 for air), $M$ the local linear magnification near the axis, and $\theta, \theta^{\prime}$ are the ray angles in the object and image space, respectively. For a ray of angle $\theta$, let $h$ denote the height of the strike position of each ray in the object focal plane, where $h=\Delta z \tan \theta$. We can calculate the position $\Delta z_{\theta}^{\prime}$ where this intersects the optic axis:

$$
\begin{align*}
\Delta z_{\theta}^{\prime} & =\frac{h^{\prime}}{\tan \theta^{\prime}}  \tag{2}\\
& =\frac{M h}{\frac{\sin \theta^{\prime}}{\sqrt{1-\sin ^{2} \theta^{\prime}}}}  \tag{3}\\
& =\frac{M \Delta z \tan \theta}{\frac{\frac{1}{M} n \sin \theta}{\sqrt{1-\frac{1}{M^{2}} n^{2} \sin ^{2} \theta}}}  \tag{4}\\
& =\frac{M^{2}}{n} \Delta z \frac{\sqrt{1-\frac{1}{M^{2}} n^{2} \sin ^{2} \theta}}{\sqrt{1-\sin ^{2} \theta}} \tag{5}
\end{align*}
$$

Note that the strike position is a function of $\theta$, and therefore in general we do not have perfect focus. The only time the angle-dependence disappears (and thus aberrations are avoided) is when the ratio of square-roots cancels, which requires $|M|=n$. This is the condition derived more generally by the Maxwell perfect-imaging theorem. (We can also see that the longitudinal magnification is $\frac{M^{2}}{n}$.)
$\stackrel{n}{W}$ When this condition does not hold, the fractional range of strike positions
(up to the maximum angle consistent with the numerical aperture NA) is

$$
\begin{equation*}
\frac{\sqrt{1-\frac{\mathrm{NA}^{2}}{M^{2}}}}{\sqrt{1-\frac{\mathrm{NA}^{2}}{n^{2}}}}-1 \tag{6}
\end{equation*}
$$

For $M=20$ and water immersion, at NA 0.5 we get approximately $8 \%$, and at NA 1 we get a $50 \%$ spread.

The axial RMS spot radius offers a more informative summary of the imaging performance. This quantity is the square root of the mean squared error (MSE) in strike position for every ray within the collection cone of the objective lens. For a ray with angle $\theta$ emitted from a source at $\Delta z$ the squared error is

$$
\begin{equation*}
\left(\Delta z_{\theta}^{\prime}-z_{0}^{\prime}\right)^{2}=\left(\Delta z_{\theta}^{\prime}-\frac{M^{2}}{n} \Delta z\right)^{2} \tag{7}
\end{equation*}
$$

For each angle $\theta$ an ideal point source at $\Delta z$ emits a circle of rays. It can easily be shown with trigonometry that the total light collected at angle $\theta$ is proportional to $\tan (\theta)$. Therefore we scale the error at each angle $\theta$ accordingly to get the sum squared error and then divide to calculate MSE:

$$
\begin{align*}
\mathrm{MSE} & =\frac{\int_{0}^{\theta^{\prime}} \tan (\theta)\left(\Delta z_{\theta}^{\prime}-\frac{M^{2}}{n} \Delta z\right)^{2} \mathrm{~d} \theta}{\int_{0}^{\theta^{\prime}} \tan (\theta) \mathrm{d} \theta}  \tag{8}\\
& =\frac{\int_{0}^{\theta^{\prime}} \tan (\theta)\left(\Delta z_{\theta}^{\prime}-\frac{M^{2}}{n} \Delta z\right)^{2} \mathrm{~d} \theta}{-\ln \left(\cos \left(\theta^{\prime}\right)\right)} \tag{9}
\end{align*}
$$

The upper limit, $\theta^{\prime}$ is the maximum angle that the objective can collect as specified by the NA. Note that the above calculates the RMS spot radius in image space; in order to convert this to object space units divide by the axial magnification.

Supplementary Note 2: Microscope parts
(ordered approximately from excitation to collection of emission)

| Number in diagram | Part description | Vendor | Custom? | Part number | Price (USD) | Count | *discontinued, estimated price |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| not shown | Laser system w/5 wavelengths, AOTF | Spectral (now Andor) | no | Laser Merge Module (LMM) | 90000 | 1 * |  |
| 1 | Pigtailed fiber collimator | OZ Optics | no | LPC-01-405/640-3/125-P-1-6AC-40-3S-3-2 | 146.5 | 1 |  |
| 2 | Pigtailed fiber collimator holder | (machine shop) | yes |  | NA | 1 |  |
| 3 | Lightsheet lens module ("cigarette") | (machine shop) | yes |  | NA |  |  |
| not shown | Lightsheet achromat lens | Edmund Optics | no | 45-262 | 87.5 | 1 |  |
| not shown | Lightsheet cylinder lens | Tower Optical | yes* |  | 99 | 1 |  |
| not shown | Lightsheet achromat lens (thin sheet) | Edmund Optics | no | 45-207 | 59 | 1 |  |
| not shown | Lightsheet cylinder lens (thin sheet) | Edmund Optics | yes** | 48-373 | 375 | 1 |  |
| 4 | Rotation collar | (machine shop) |  |  | NA | 1 |  |
| 5 | Clamp for lightsheet lens holder | (machine shop) |  |  | NA | 1 |  |
| not shown | $10 x, 0.3$ N.A. objective (UMPLFLN10X/W) | Olympus | no | 1-U2M583 | 774.25 | 1 |  |
| 6 | 20x, 0.5 N.A. objective (UMPLFLN20X/W) | Olympus | no | 1-U2M585 | 1464.64 | 1 |  |
| not shown | 40x, 0.8 N.A. objective (LUMPLFLN 40X/W) | Olympus | no |  | 2500 | 1 |  |
| 7 | Objective holder (RMS) | (machine shop) | yes |  | NA |  |  |
| 8 | Mini-dovetail stage (2-axis) | Lightspeed Technologies Inc. | no | MDE266 | 304 | 1 |  |
| 9 | Mini-dovetail stage (3-axis) | Lightspeed Technologies Inc. | no | MDE269 | 573 | 1 |  |
| 10 | dovetail stabilizer | (machine shop) | yes |  | NA | 1 |  |
| 11 | Front piezo plate | (machine shop) | yes |  | NA | 1 |  |
| 12 | Piezo positioner, 800um range | Piezosystem Jena | no | NanoSX800 | 11205 | 1 |  |
| not shown | Piezo amplifier (digital control) | Piezosystem Jena | no | 30DV300 | 5616 | 1 |  |
| 13 | Rear piezo plate | (machine shop) | yes |  | NA |  |  |
| not shown | Precision broadband mirror | Edmund Optics | no | 48-017 | 395 | 1 |  |
| not shown | 200 mm tube lens | Thorlabs | no | ITL200 | 450 | 1 |  |
| not shown | Knife edged mirror | Thorlabs | no | MRAK25-G01 | 125.46 | 1 |  |
| not shown | 50/50 beamsplitter ( $25 \times 36 \mathrm{~mm}$ ) | Thorlabs | no | BSW10R | 110 | 1 |  |
| not shown | Filter cube w/insert | Thorlabs | no | DFM | 304 | 1 |  |
| not shown | Extra filter cube insert | Thorlabs | no | DFMT1 | 201 | 1 |  |
| not shown | 0.9 x telecentric relay lens | Edmund Optics | no | 62-902 | 2265 | 2 |  |
| not shown | CMOS camera | PCO | no | Edge 4.2 | 16400 | 2 |  |
| not shown | Stages for aligning cameras | Thorlabs | no | DTS25 | 179.5 | 4 |  |
| not shown | DAQ board (PCI-6259) | National instruments | no | 779072-01 | 1592.1 | 1 |  |
| not shown | Physiology stage surface | Thorlabs | no | PHYS24BB | 2500 | 1 |  |
| not shown | Lab jack | Newport | no | 281 | 999.94 | 1 |  |
| not shown | Breadboard for connecting lab jack | Thorlabs | no | MB1224 | 259 | 1 |  |
| not shown | XY microscope translation stage | Scientifica | no | N/A (Quote ref: QLS-30894) | 4042.5 | 1 |  |
| not shown | RAID hard drives | Seagate | no | 4221403 | 78.19 | 20 |  |
| not shown | Breadboard for vertical mounting of system | Thorlabs | no | MB1824 | 400 | 1 |  |
|  |  |  |  | Price total: | 164194.69 |  |  |

## Cylindrical lens specifications

Center Thickness $\quad 1 \mathrm{~mm} \pm 0.1 \mathrm{~mm}$
Edge Thickness $\quad 1.37 \mathrm{~mm}$
Effective Focal Lengtr -6.25 mm
Back Focal Length -6.91 mm
Focal Length Tol $\pm 3 \%$
Radius $\quad-3.24 \mathrm{~mm}$
Surface quality $\quad 60 / 40$ both sides
Coating MgF2
**ylindrical lens specifications (thin sheet)
Same as Edmund Optics 48-372, only diameter was reduced to 5 mm

view from left

view from rear left








| 1 | $3.0 \times 0.5$ |
| :--- | :--- |
| 2 | counterbore, 3 mm depth |
| 3 | alignment pin hole, 6 mm depth |
| 4 | 4.1 mm dia through |
| 5 | M32 threaded hole |









## Supplementary Figure 1





 of width $100 \mu$ s to reduce sampling noise. (d) Similar to (c), but the median difference between corresponding samples in each cycle is shown.

## Supplementary Figure 2









 reverse stacks of a bidirectional recording. Note that optimal timings cannot be predicted by the depth in the sample, as would be expected from a simple error in sensor gain.

## Supplementary Figure 3






 is predictive of axial displacement of the focal plane.

## Supplementary Figure 4






 system. Thus the effective size of the field of view is reduced by a small amount when compared with a single-camera system.

## Supplementary Figure 5

a



b


Supplementary Figure 5 Heartbeat-mediated correlations in the $\Delta \mathrm{F}$ signal (a) Comparison of time-domain correlations in the isolated heartbeat frequency bands (left) and another nearby set of frequency bands. The former correlation matrix contains a much higher proportion of strong positive and negative correlations, as would be expected with a broadly distributed heartbeat-induced motion signal. (b) The sizes of spectral deviations at the frequencies of the putative heartbeat signal are highly correlated across neurons, suggesting that the two frequencies are components of a single signal. For each of the 629 segmented neurons we measured the size of the peak in the PSD at each frequency. Peak amplitiude was measured by dividing the PSD amplitude at each frequency by the mean amplitude of the two surrounding frequency bins (thus peak size is not defined for the maximum and minimum frequency bins). Plotted is the correlation between peak sizes across frequencies. Correlations near the diagonal are expected to be strong because they are between similar frequencies. However the strong correlation between the 7.5 Hz and 2.5 Hz peak size stands out. Correlation is also visible at 5.0 Hz (another harmonic of 2.5 Hz )
a

b

c






 artifact results from sub-pixel motion.

